

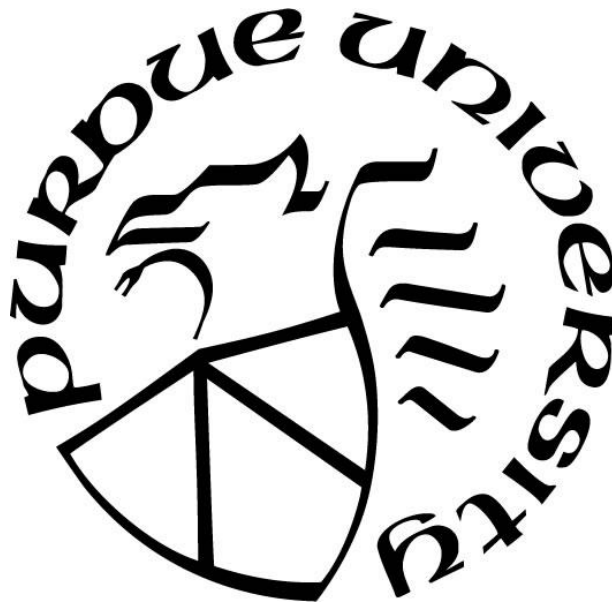
**LINEARIZING E-CLASS POWER AMPLIFIER BY USING
MEMORYLESS PRE-DISTORTION**

by
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A Thesis

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In Partial Fulfillment of the Requirements for the degree of*

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LIST OF ABBREVIATIONS

DPD	Digital Pre-distortion
AM/PM	Amplitude Modulation/ Phase Modulation
GaAs	Gallium Arsenide
GaN	Gallium Nitride
OFDM	Orthogonal Frequency Division Multiplexing
W-CDMA	Wideband Code Division Multiple Access
PAPR	Peak to Average Power Ratio
CEPT	European Conference of Postal and Telecommunications Administrations
FET	Field-Effect Transistor
IGBT	Insulated-Gate Bipolar Transistor
MOSFET	Metal-Oxide-Semiconductor Field-Effect Transistor
DSP	Digital Signal Processing
PWM	Pulse Width Modulation
PA	Power Amplifier
RMS	Root Mean Square
ACPR	Adjacent Channel Power Ratio
IF	Intermediate Frequency
EVM	Error Vector Magnitude
DUT	Device Under Test
ILA	Indirect Learning Architecture
DLA	Direct Learning Architecture
LMS	Least Mean-Square
RLS	Recursive Least-Squares
AWGN	Additive White Gaussian Noise
RPEM	Recursive Predictor Error Method
ILC	Iterative Learning Control

ABSTRACT

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Title: Linearizing E- Class Power Amplifier by Using Memoryless Pre-Distortion

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Radio Frequency Power Amplifiers (PA) are essential components of wireless systems and nonlinear in a permanent way. So, high efficiency and linearity at a time are imperative for power amplifiers. However, it is hard to obtain because high efficiency Power Amplifiers are nonlinear and linear Power Amplifiers have poor efficiency. To meet both linearity and efficiency, the linearization techniques such as Digital Predistortion (DPD) has arrested the most attention in industrial and academic sectors due to provide a compromising data between efficiency and linearity. This thesis proposed on digital predistortion techniques to control nonlinear distortion in radio frequency transmitters.

By using predistortion technique, both linearity and efficiency can obtain. In this thesis a new generic Saleh model for use in memoryless nonlinear power amplifier (PA) behavioral modelling is used. The results are obtained by simulations through MATLAB and experiments. We explore the baseband 13.56 MHz Power Amplifier input and output relationships and reveal that they apparent differently when the Power Amplifier shows long-term, short-term or memory less effects. We derive a SIMULINK based static DPD design depend on a memory polynomial. A polynomial improves both the non-linearity and memory effects in the Power Amplifier. As PA characteristics differs from time to time and operating conditions, we developed a model to calculate the effectiveness of DPD. We extended our static DPD design model into an adaptive DPD test bench using Indirect Learning Architecture (ILA) to implement adaptive DPD which composed of DPD subsystem and DPD coefficient calculation. By this technique, the output of PA achieves linear, amplitude and phase distortions are eliminated, and spectral regrowth is prevented.

The advanced linearity performance executed through the strategies and methods evolved on this thesis can allow a higher usage of the capability overall performance of existing and emerging exceptionally performance PAs, and therefore an anticipated to have an effect in future wireless communication systems.

Keywords

PA Efficiency, Behavioral model, Digital Pre-Distortion, Power Amplifier, Nonlinear, Volterra series, AM/ AM, AM/ PM, Static DPD, Adaptive DPD.

CHAPTER 1. INTRODUCTION

1.1 Wireless Link & Power Amplifiers

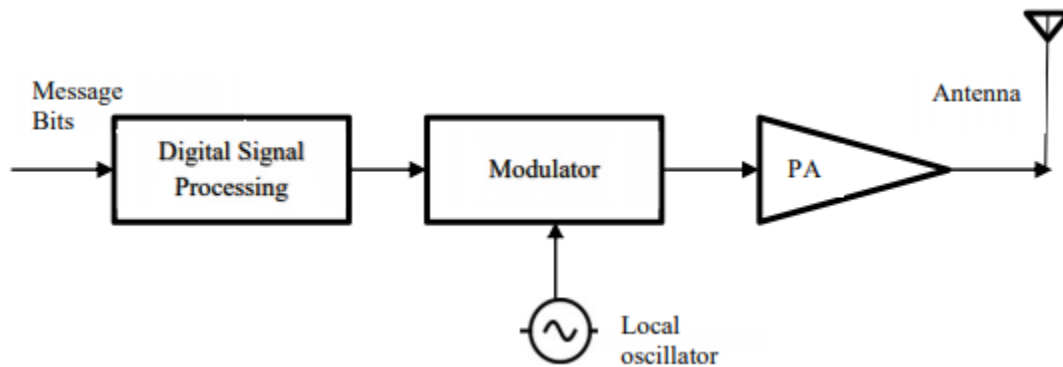


Figure 1.1 Block Diagram of Modern Transmitter [8]

Power amplifiers (PA) are one of the essential block in wireless systems [8], which plays a major role to overcome the loss between the transmitter and receiver. Figure 1.1 shows a block diagram of modern transmitter [8]. To achieve the adequate transmission power, Power Amplifiers are essentials in wireless or wireline systems. To get high efficient level we can increase the input power of power amplifier. However, PAs are non-linear when the input power is high, which builds a trade-off between efficiency and linearity. The last output in wireless system includes power amplifier to send the signal to the antenna. Depending on the frequency band, the power output and the desired efficiency, designers can select PA's fabricated with a wide range of technologies like: Gallium Arsenide (GaAs), Gallium Phosphide, Silicon Germanium, CMOS or Gallium nitride (GaN). Moreover, high speed of communication in wireless industry has controlled by 3GPP like Wideband Code Division Multiple Access (W-CDMA) which is using unstable envelope techniques. These techniques have high peak to average power ratio (PAPR) when increase the frequency spectrum efficiency. To overcome this non-linearity - digital predistortion (DPD) is one of the widely accepted techniques. In this paper we have examined and implemented different DPD techniques.

1.2 Importance of Linearity & Non-Constant Envelope

Now-a-days in wireless industry communication techniques used in 3G (W-CDMA) and 4G (OFDM) both uses unstable envelope techniques [8]. In figure 1.2 a 256 QAM OFDM signal sample has shown. As we said these techniques also has high peak to average power ratio which makes a conflict between power efficiency and linearity. In this paper we discussed about the conflict and the PA efficiency. And, importance of the linearity will be discussing in this section. When a Power Amplifier is behaving non-linearly, it starts to create harmonic distortion in the output which comes with a frequency spectral growth. However, due to strict frequency allocations this needs to be avoided.

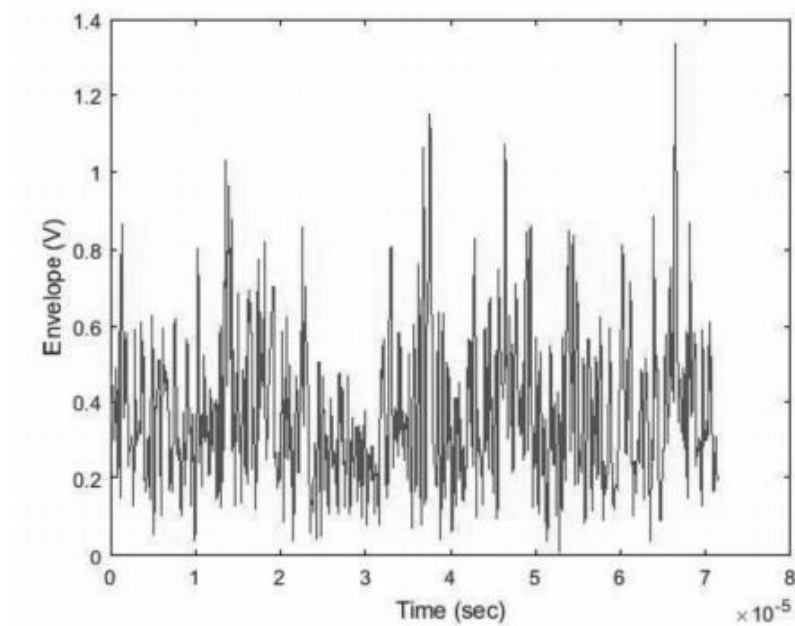


Figure 1.2 Sample 256 QAM, OFDM signal [8]

1.3 Non- Linearity Effect of Power Amplifiers

So far, we talked about the significance of linearity, but PA non- linearity has not been discussed yet. In this section the reasons of PA's non-linearity will be discussed. Due to high frequency PA in communication systems, nonlinear behavior happens which degrades signal. This is unlikely to the signals that pass through it and appears Harmonic Distortion, Gain compression, Inter-modulation distortion etc. A Power Amplifier should be driven in high power mode to obtain a high-power efficiency but after a definite amount of input power level each Power Amplifier tends

to hit the compression region. A power amplifier has promising gain while drive into low power level but for high power levels PA goes in to saturation and the gain does not meet as expected like low power level which means gain decreased, the point where the gain decreased 1dB [8]. From its constant value is called 1dB compression point.

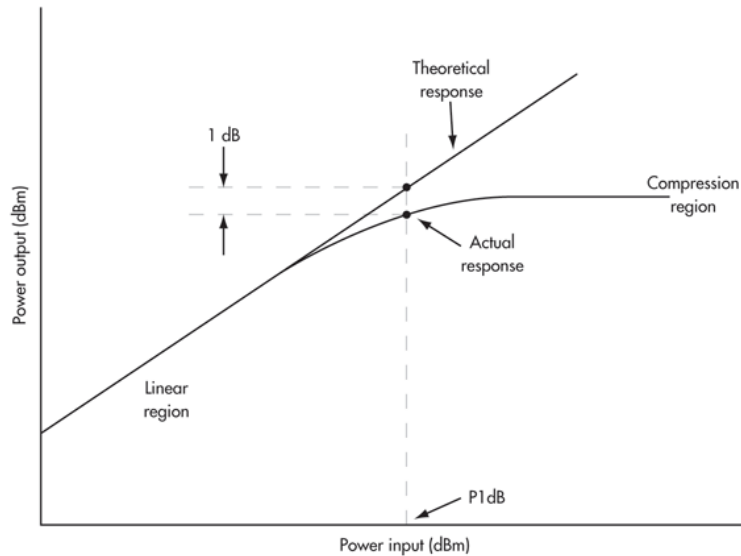
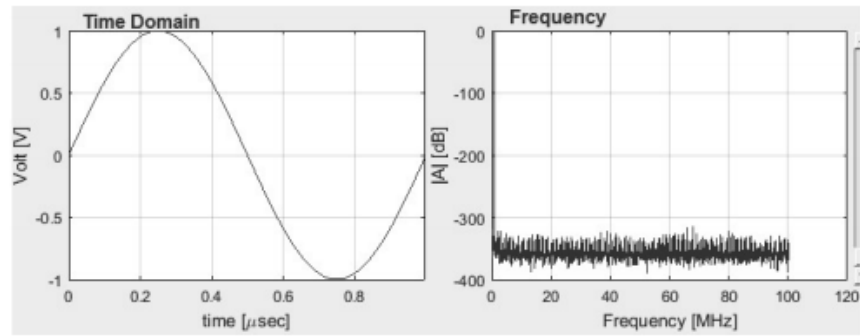


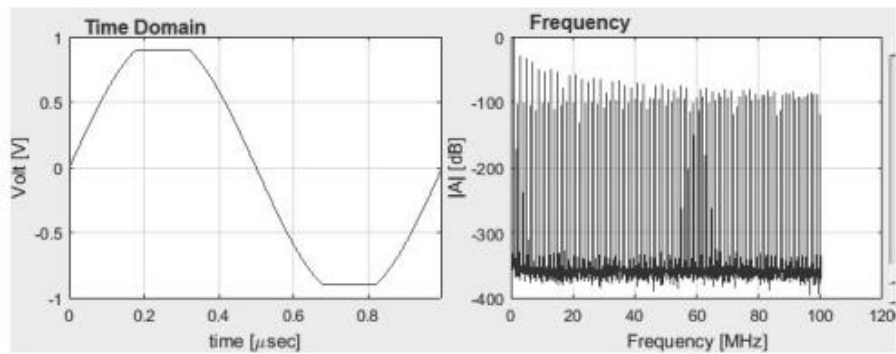
Figure 1.3 Input Power vs Output Power For 1 dB Compression Point [8]

This cutting in time domain has a spectral growth effect in frequency domain. Figure 1.4 (a) and figure 1.4 (b) are showing the effect of cutting, in figure 1.4(a) is shown a sin wave of 1MHz frequency and its frequency domain response. In figure 1.4(b) same wave but with five percent cutting and its frequency response has shown as well. When there is cutting, the signal grows in frequency domain. Additionally, in figure 1.4(c) [8] two different input signals comparison has shown. Here, Intermodulation products are more essentials and by using filters rest of the harmonics can be restrain.

(a)



(b)



(c)

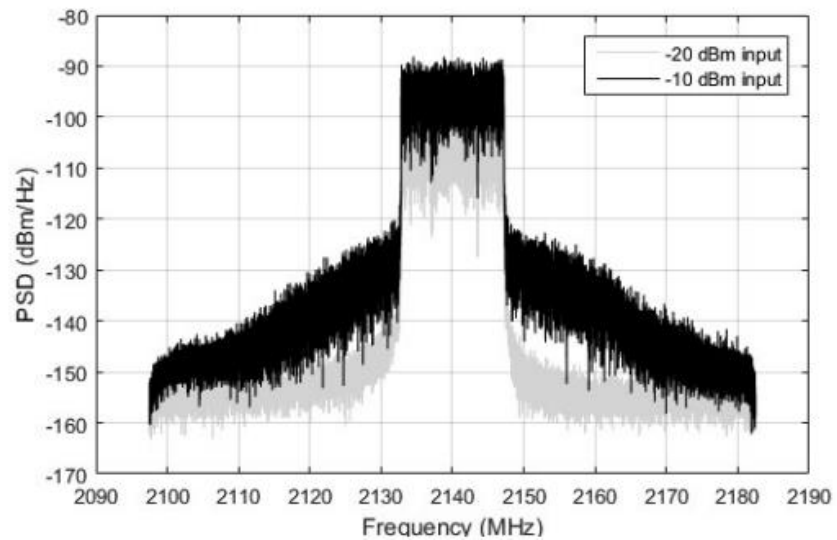


Figure 1.4 Comparison of Two Different Input Signals With Different Power Levels [8]

1.4 AM/AM & AM/PM Distortions

PA non-linearity can be obtained by amplitude to amplitude (AM/AM) conversion and amplitude to phase (AM/PM) conversion. AM/AM represents a digression from a constant gain when input power is increased to hit the compression region and AM/PM represents as the change of the carrier signal phase by an amplifier when input power is increased to hit the compression region [8]. AM/AM distortion happens while the amplifier set foot in the compression zone. In figure 1.3 the 1 dB compression point is shown. The high peaks of the input signal will drive the Power Amplifier to hit the compression region and the gain of PA degrades from its constant value gain. A DPD technique is a solution for this distorted amplitude and phase modulation problem.

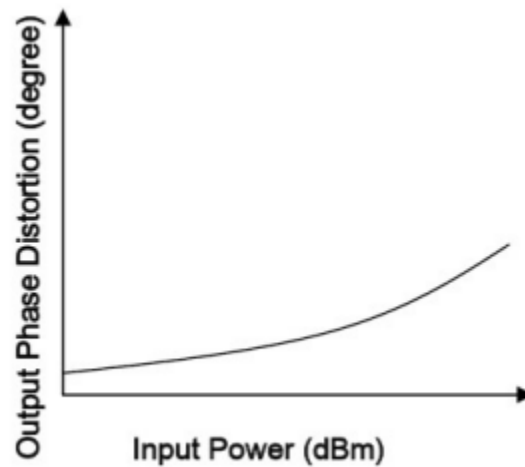


Figure 1.5 AM/PM Conversion

The static and adaptive effects of the Digital Predistortion to amplitude and phase modulation of the non-linear Power Amplifier will be discussed in the results section. In figure 1.6 AM/AM and AM/PM curves show effect of the compression region when PA enters there.

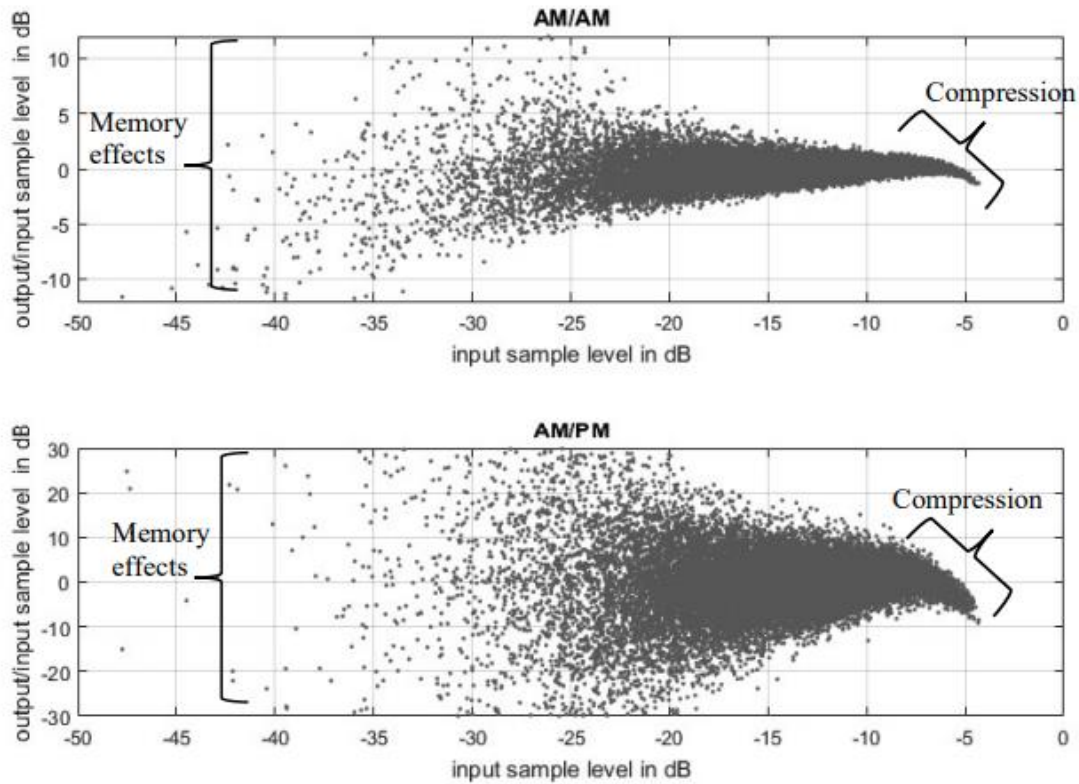


Figure 1.6 AM/AM and AM/PM Curves [8]

1.5 Thesis Outline

The thesis is organized as follows. In chapter 2, discussed about power amplifier classes and efficiency, chapter 3 includes 13.56MHz power amplifier design and 5 effects of Power Amplifier nonlinearity. Chapter 4 have discussed about three different Linearization techniques.

In chapter 5, we discussed about the behavioral model of a Power Amplifier by using five different methods, simulation methods and the method of DPD coefficient obtain.

In chapter 6 the digital predistortion modelling and equations are explained and discussed about the inverse learning structure.

In chapter 7 we discussed about the model and the simulation results including static and adaptive DPD. After this chapter, conclusions and future work for hardware implementation and wireless communication systems are discussed.

CHAPTER 2. INTRODUCTION TO POWER AMPLIFIER AND ITS EFFICIENCY

2.1 Power Amplifier Classes and Efficiency

The most common and known as best class of power amplifier due to low distortion level is called Class A amplifiers. Class A amplifier has highest linearity using single transistor process technology like BJT, FET or IGBT etc. These transistors connect with a common emitter for the both halves of the waveform, although it does not pass any base signal which represent that the output is never driven into saturation region. However, base biasing Q point operate in the middle of the line which makes the transistor never turns OFF.

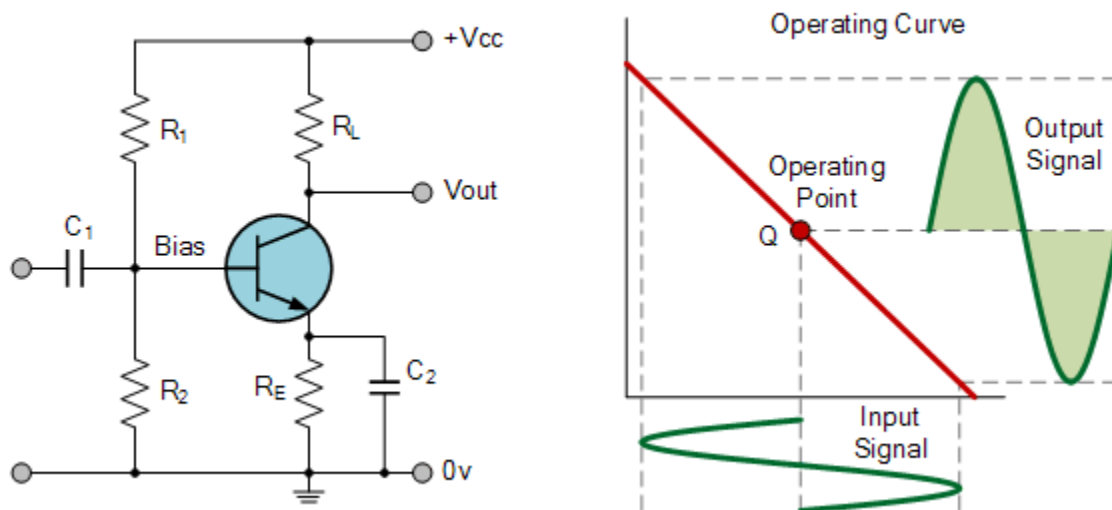


Figure 2.1 Class A Amplifier Design and Output

Class B amplifier came up with a solution of efficiency and heating problem which creates in Class A amplifier. However, Class B has higher efficiency, as one-half cycle is used as input, it creates a higher distortion than Class A amplifier. So, the input power is not utilizing fully. To overcome this problem, class B amplifier has been introduced.

The DC power is small here in class B as there is no DC biasing current which basically makes the efficiency higher for Class B.

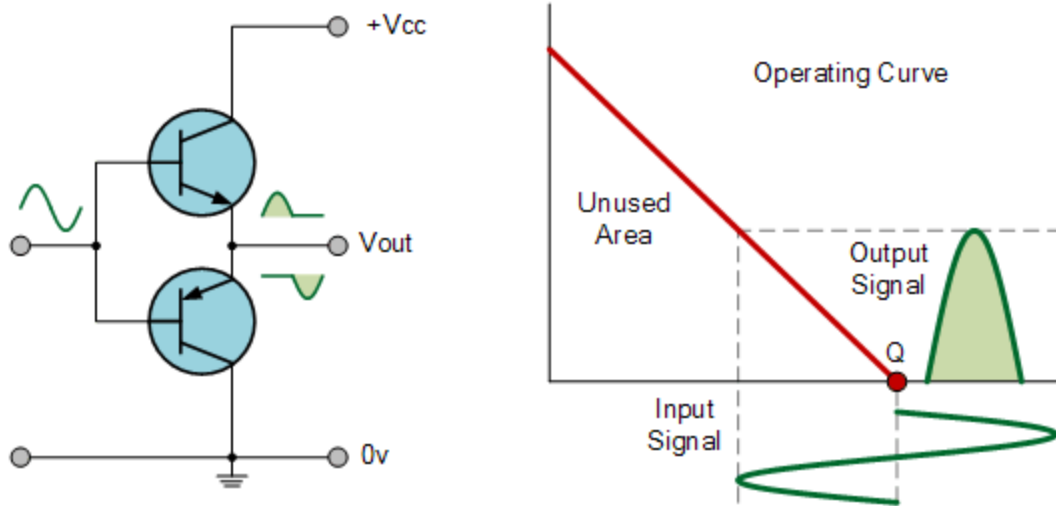


Figure 2.2 Class B Amplifier Design and Output

Class AB is one of the popular and common type of PA use in audio industry. Class AB is a summation of Class A and Class B amplifier. A current mirror can be implemented by using a pair of small signal diodes. Current mirror is a device whatever happens in one section of the transistor it will reflect on the other section of the diode. Current mirror helps to eliminate the cross over distortion because the bias voltage the diodes require it will go through those two transistors to turn those ON. And it will be ON for more than a half cycle and creates the conduction angle 180° and 360° based on the biasing point.

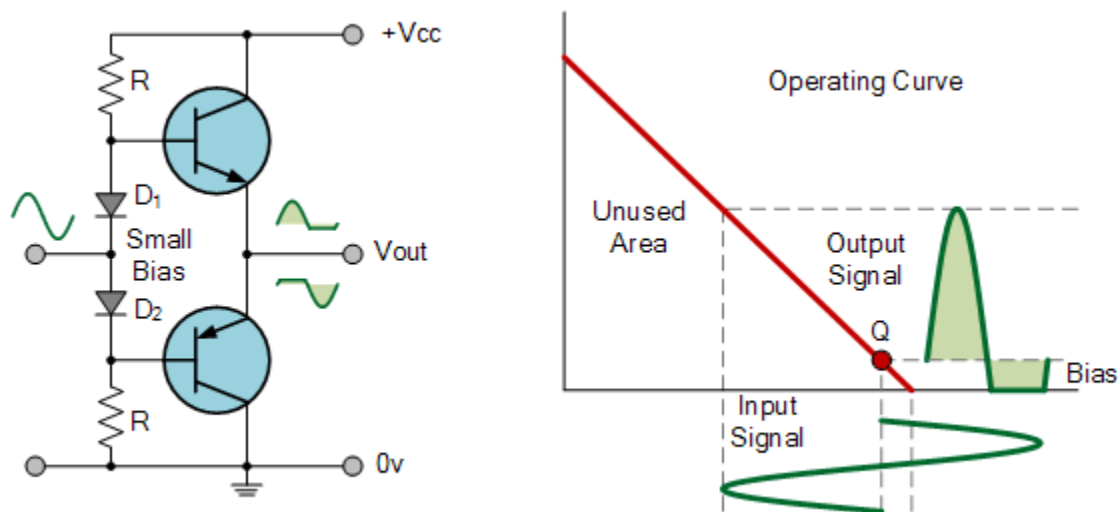


Figure 2.3 Class AB Amplifier Design and Output

The Class C Amplifier represents the greatest efficiency but has the lowest operating cycle and linearity. Although, due to heavy biasing of class C amplifier, it stays on for less than half of an input cycle and thus a conducting angle somewhere around vicinity of 90 degrees. Therefore, it creates high efficiency but on the other hand it causes high distortion in the output signal. So, it does not meet the criteria to use in audio industry.

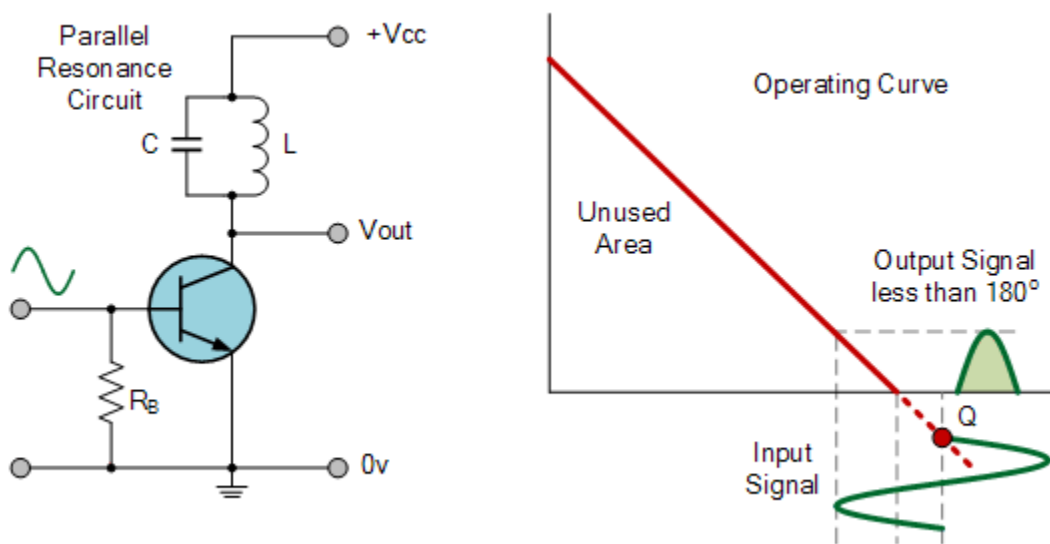


Figure 2.4 Class C Amplifier Design and Output

They are used in certain RF applications where efficiency is a key thing. They work in two operating modes, one is tuned and another one is un-tuned, and they also have low power dissipation. The output converts to a sinusoidal wave by using an LC resonance tank and connects with the collector of a transistor.

Power Amplifier designs in different high frequency relate with different switching techniques to achieve the efficiency with less distortion. Some amplifiers consist of an RLC circuit with a combination of Pulse Width Modulation (PWM) as an input and Digital signal processing (DSP) technique to reduce the power loss. There are also switch mode power amplifiers which promise a great efficiency with low distortion. Figure 2.5 and Table 1.1 show the conduction angle, efficiency and distortion level to provide a brief idea about all classes of PA's:

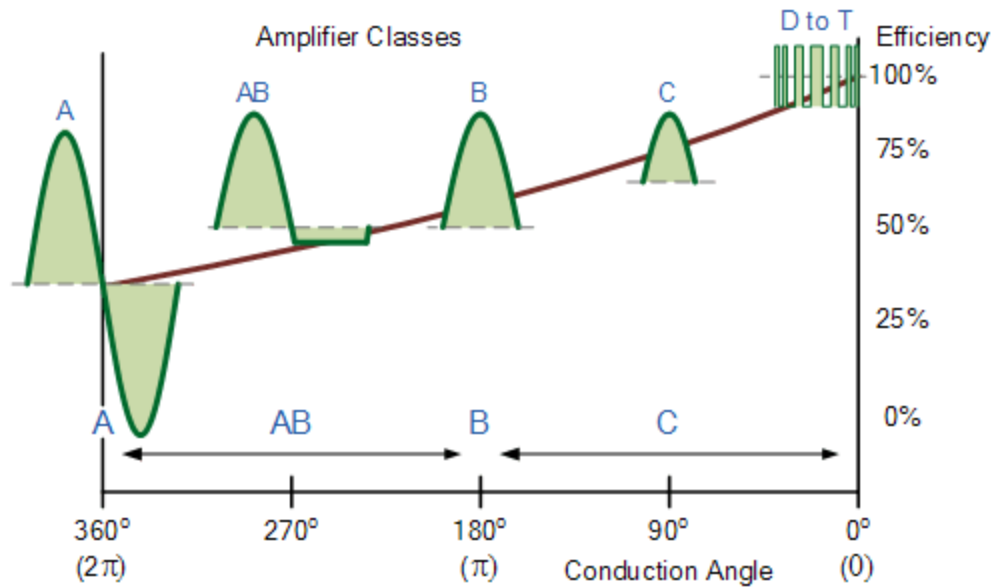


Figure 2.5 Amplifier's Efficiency and Conduction Angle

Table 1.1 Amplifier's Efficiency, Layout Area and Distortion

Amplifier	Maximum Efficiency	Layout Area	Distortion
Class A	50%	Small	Low
Class B	78.50%	Small	High
Class AB	<78.50%	Small	Moderate
Class C	Approaching 100%	Moderate	Moderate
Class D	Not Suitable for Narrowband RF		
Class E	Approaching 100%	Moderate	Low

CHAPTER 3. POWER AMPLIFIER DESIGN AND IT'S NON-LINEARITY

3.1 13.56MHz E- Class Power Amplifier Design and Simulation

In this section we have discussed about the analytical analysis of a Class-E PA, showing the relation between the circuit component and the input parameters. The solution represents many equations in analytical analysis section for the Class-E PA shown in Figure 3.1 due to value the duty cycle switching and the RF configuration. We have used MATLAB Simulink to design this Power Amplifier and later predistorted by using Simulink. We discussed about the capacitance current, nonhomogeneous other n^{th} number of order differential equations.

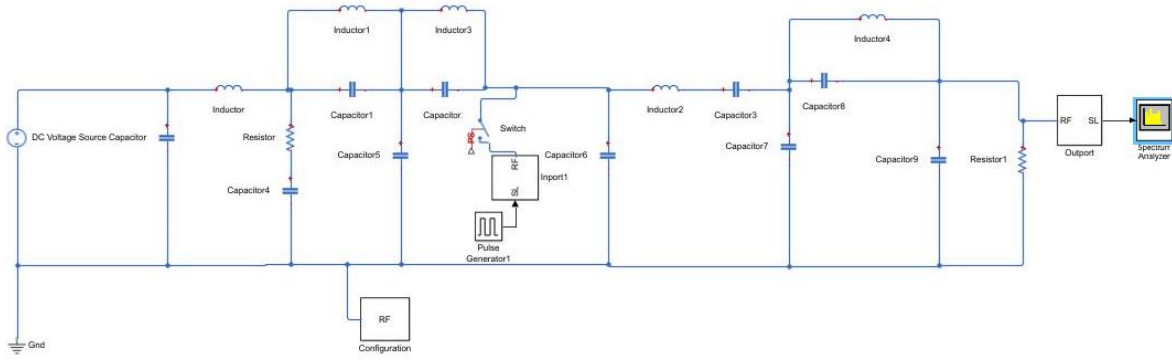


Figure 3.1 13.56 MHz E- Class Power Amplifier Schematic Design

3.1.1 Analytical Analysis

Two fundamental boundaries have used together to solve and relate the relation between the parameters and element of the circuit with the E class Power Amplifier conditions in ((3.1) and (3.2)) equations:

$$V_c \left(\frac{2\pi}{\omega} \right) = 0 \quad (3.1)$$

$$\frac{dV_c(t)}{dt} \Big|_{t=2\pi/\omega} = 0 \quad (3.2)$$

The load current is $I_R(t)$ which should be sinusoidal wave. In theory, it will work if Q consist with LC tank circuit in the schematic and this sine wave assumption can simplify the analysis part

$$I_R(t) = I_R \sin(\omega t + \varphi) \quad (3.3)$$

If the switch is closed, the time interval $0 < t < \pi/\omega$, and the capacitance voltage $V_c(t) = 0$ and the current through the capacitance (C) is, $I_c(t) = C \frac{dV_c(t)}{dt} = 0$ [1]. In this time interval, the switch current $I_S(t)$ is

$$\begin{aligned} I_S(t) &= I_L(t) + I_R(t) \\ &= \frac{V_{DD}}{L} t + I_L(0) + I_R \sin(\omega t + \varphi) \end{aligned} \quad (3.4)$$

$$\text{Where } I_L(0) = -I_R \sin(\varphi)$$

When the switch is opened, the time interval $d \cdot \pi/\omega < t < 2\pi/\omega$. Then, in the PA the current through capacitance C is [1]

$$I_c(t) = \frac{1}{L} \int_{d\pi/\omega}^t (V_{DD} - V_c(t)) dt + I_L \left(\frac{d\pi}{\omega} + I_R(t) \right) \quad (3.5)$$

Relation (3.5) can be re-arranged in the form of a linear, nonhomogeneous which has as solution

$$LC \frac{d^2 V_c(t)}{dt^2} + V_c(t) - V_{DD} - \omega L I_R \cos(\omega t + \varphi) = 0 \quad (3.6)$$

$$V_c(t) = C \cos(q\omega t) + V_{DD} - \frac{q^2}{1 - q^2} V_{DD} \cos(\omega t + \varphi) \quad (3.7)$$

$$\text{Where, } q = \frac{1}{\omega \sqrt{LC}} \quad (2.8)$$

$$p = \frac{\omega L I_R}{V_{DD}} \quad (9)$$

$$C = \left\{ \frac{q^2 \cos(2q\pi) \cos(\varphi)}{1 - q^2} p + \frac{\sin(2q\pi) q \sin(\varphi)}{1 - q^2} p - \cos(2q\pi) \right\} V_{DD} \quad (3.8)$$

The coefficients C came from the Class-E equations (3.1) and (3.2). It follows from (3.4) and (3.7) that $V_c(t)$ and $I_S(t)$ can be represent as V_{DD} and ω only if d, q, p and ϕ are known.

3.1.2 Achievable Waveforms

This 13.56 MHz Power Amplifier waveforms shows different peak voltage and peak current values though they have their own pros and cons values. Therefore, of Class-E waveforms provides a significance from an application point. Designing class E 13.56MHz using a single transistor where that transistor works as a switch, as in Figure 3.2, the following properties of Class-E waveform plays significance roles:

- The peak value of switching voltage supposed to be too low but it should not cross the breakdown voltage limit of the transistor.
- When the switch is turning OFF, on that moment transistor current should be low specifically while transistor drives by a sine wave signal.
- To minimize the power losses, the rms value of the transistor current, the inductor (L) and the capacitor (C) current, and the load current supposed to be low.

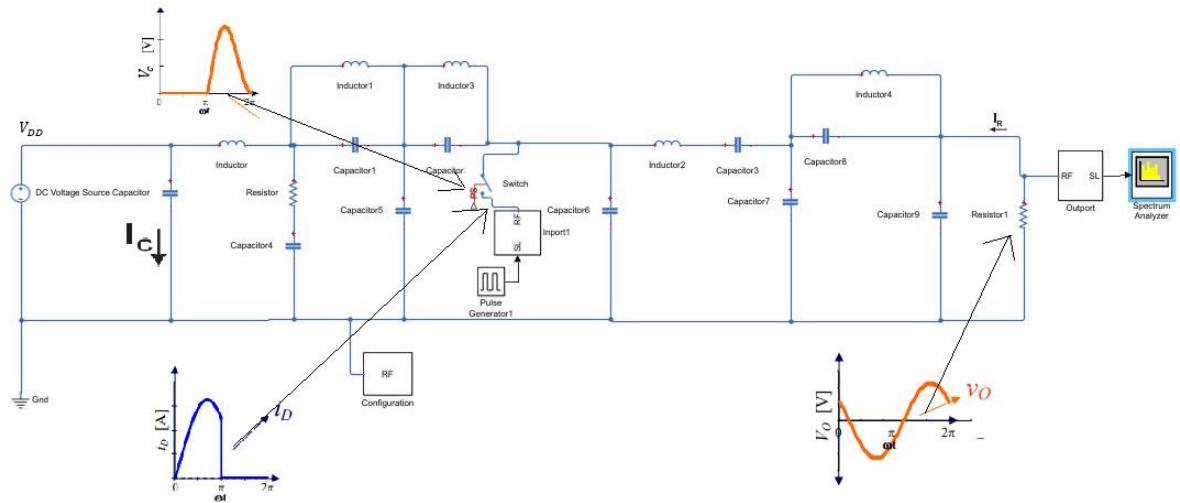


Figure 3.2 Class-E Waveforms With Ideal Components And Under Optimal Switching Conditions

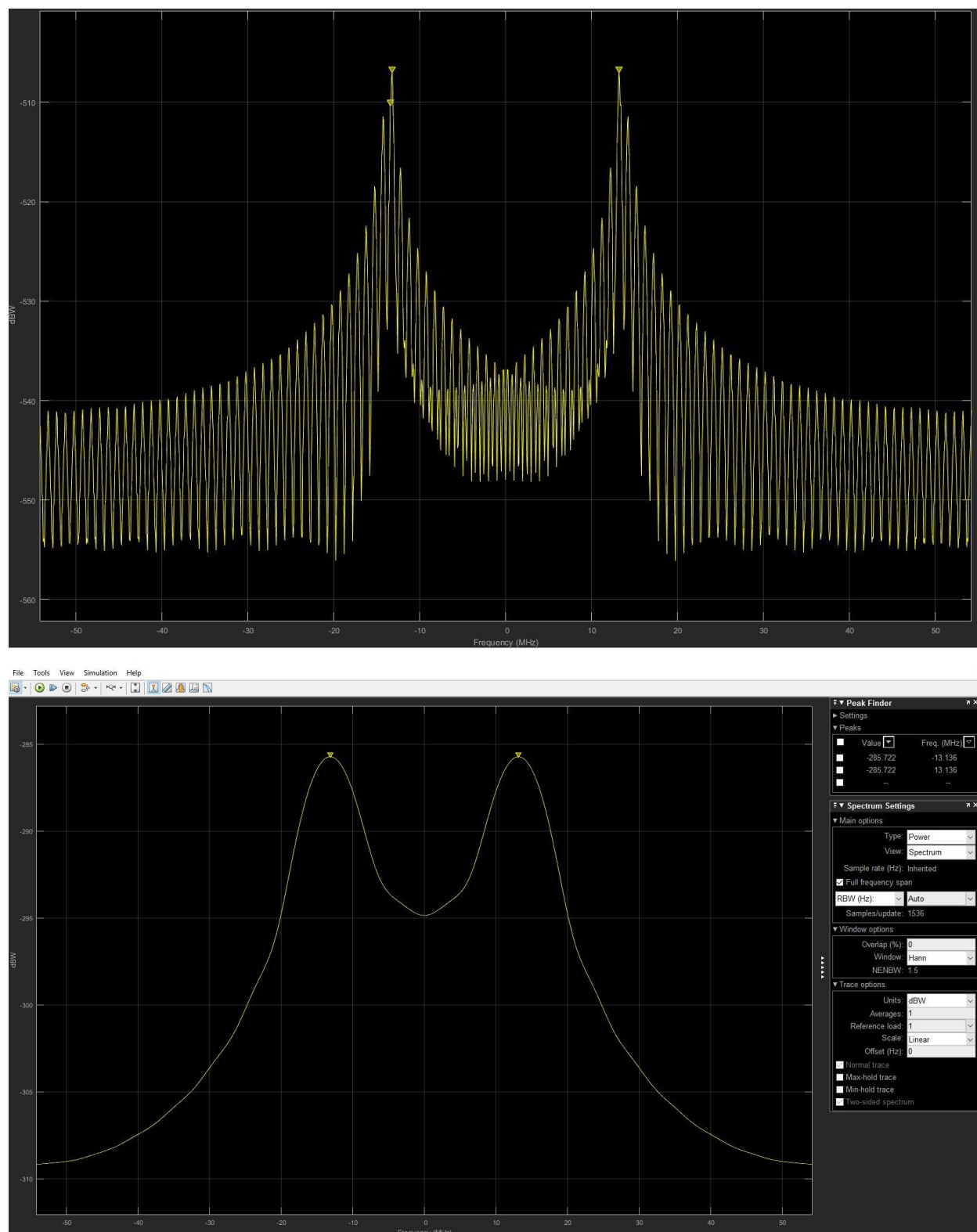


Figure 3.3 13.56MHz PA Output

3.2 Effects of Power Amplifier Nonlinearity

3.2.1 Harmonic Generation

Equation (3.1) is used to characterize the power amplifier behavior. Suppose we have a single tone input signal which is representing by V_i . So, $V_i(t) = \cos(2\pi ft)$. Based on Equation (3.1), applying $V_i(t)$ as input, the output $V(t)$ has frequency components (f) along with nf where n represents the integer [4]. This n integer number creates the n^{th} harmonic in the power amplifier. These harmonics occurs harmonic distortion, but they are not suppressed so it is not so complex technique to filter those from the tone of the spectrum [2].

3.2.2 Intermodulation Distortion

Equation (3.1), also shows the input and output nonlinear relation. If there is a two-tone signal input, $V_i(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$, going towards the amplifier, the output of the PA consists some frequency as $\pm m f_1 \pm n f_2$. These are called intermodulation products. Given $k = |m| + |n|$, $\pm m f_1 \pm n f_2$ products are called the k^{th} intermodulation product of the power amplifier. If these components are not suppressed, it causes distortion [2]. These components cannot be filtered out due to the intermodulation frequency components are not too far away from the fundamental components.

3.2.3 Spectral Regrowth

Spectral growth works like intermodulation distortion. Let's assume we are going to pass a sine wave signal through an amplifier and as an output low amplitude are unchanged and high amplitude gets distorted. So, if we look at the frequency domain we might have some pulse shape as an input but after passes through the nonlinear amplifier there are also some extra frequency component and that is the spectral growth because of the nonlinear operation of the amplifier. It is easier to find out if a single tone signal works as an input. But if any broadband signal passes through the nonlinear channel it causes adjacent channel interference and the shoulder of the spectrum will grow. Here creates an Adjacent Power Ratio (ACPR) which generally keeps low [2]. In today's wireless communication systems this power ratio is vastly using to figure out the linearity.

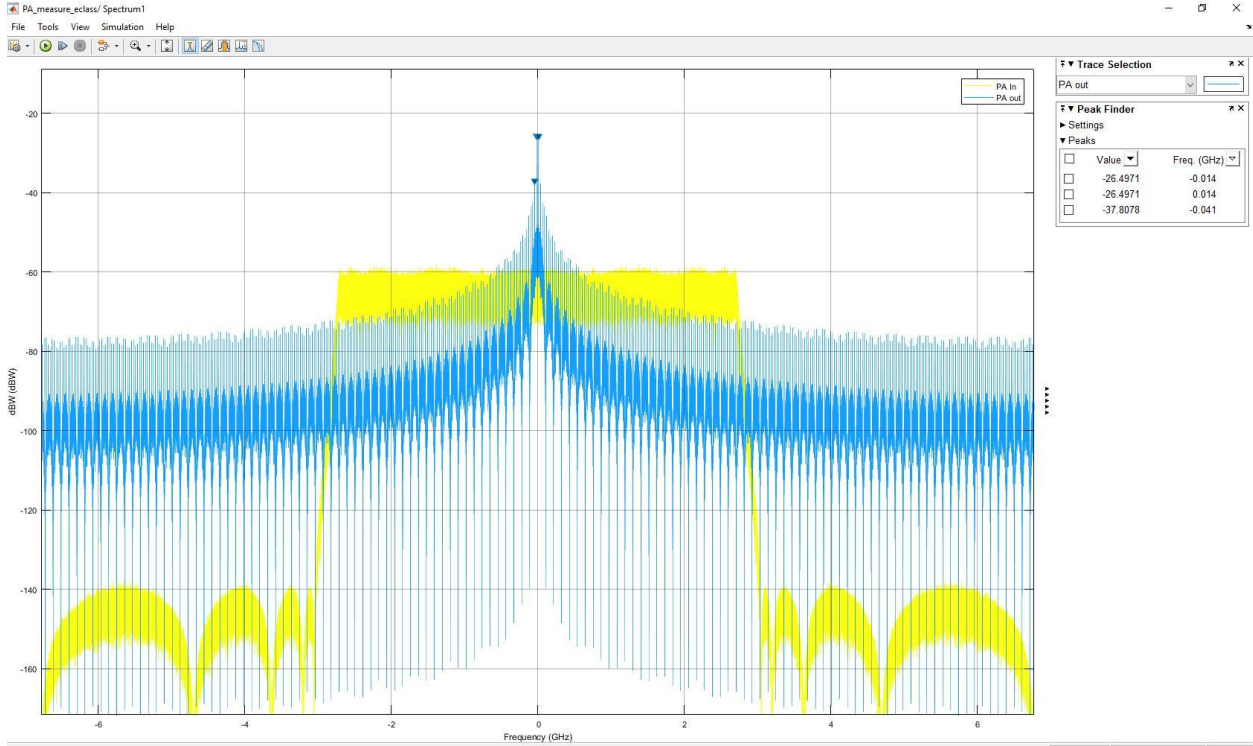


Figure 3.4 Spectral Regrowth

3.2.4 Cross Modulation

The nonlinear power amplifier channel works to transfer the signal of one carrier of frequency ω_1 , to another carrier of frequency ω_2 [4]. Assume that the input given in Equation (3.9) with carrier of frequency ω_1 which is modulated and carrier of frequency ω_2 which is unmodulated is passing through the PA

$$V_{in}(t) = V_1 \cos(\omega_1 t) + (1 + m(t)) \cos(\omega_2 t) \quad (3.9)$$

Here, $m(t)$ represents modulating waveform.

$$V_{out}(t) = \alpha_1 V_1 in(t) + \alpha_2 V_2 in(t) + \alpha_3 V_3 in(t) \quad (3.10)$$

This is the PA model up to 3rd order output where the output equation of the power amplifier consists component of ω_2 as $\alpha_1 V_1 + \alpha_3 V_3 + \alpha_3 V_1(1 + m(t))2 \cos(\omega_1 t)$ which represents frequency ω_2 as unmodulated signal of the input is transferring ω_1 which is modulated within the same bandwidth after passing through the nonlinear RF system. This phenomenon is called cross modulation.

3.2.5 Desensitization

Desensitization is an EMI form where receiver is not able to receive some weak signal and blocking by some unwanted strong signal which works in different frequency, however it seems to be able to receive while in the non-linear system there is no interface [4].

So, the input would be;

$$V_i(t) = V_1 \cos(\omega_1 t) + V_2 \cos(\omega_2 t) \quad (3.11)$$

Here the desired signal is ω_1 , the unwanted signal is ω_2 and $V_2 \gg V_1$. If we use equation (3.10), to obtain output, then the output signal has the wanted signal of frequency ω_1 as; $\alpha_1 V_1 + \alpha_3 V_3 + \alpha_3 V_1 V_2 \cos(\omega_1 t)$. Here, α_3 gives the PA saturation as it drives in negative. So that means ω_2 is dominating the wanted signal ω_1 . This phenomenon is known as desensitization.

CHAPTER 4. LINEARIZATION TECHNIQUES

4.1 Linearization Techniques

Now after looking at the effect of non-linearity, in this section we discussed about the removal techniques of power amplifier's nonlinearity. Those transmitter linearization techniques are:

- Feedback
- Feedforward
- Predistortion

In this section we will discuss about the predistortion techniques in detail.

4.1.1 Feedback

Feedback is a linearization technique can be applied either RF or Baseband signal. The block diagram has shown in Figure 4.1. Here, $V_i(t)$ is the input signal, $V_o(t)$ is the output signal. The output signal is feedback with a β gain and $V_r(t)$ signal is obtained. The difference of $V_i(t)$ and $V_r(t)$ gives the error signal $V_e(t)$. The error signal is given to the amplifier and linear output is obtained.

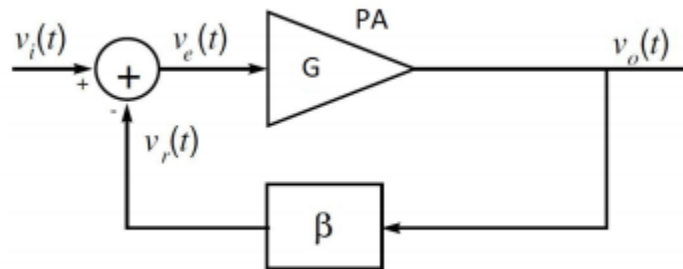


Figure 4.1 Feedback Linearization [4]

The main advantage of feedback linearization technique is that the closed loop gain gets less sensitive to the gain variation of the power amplifier. Furthermore, it is easy to add the necessary circuitry to the power amplifier for feedback [4]. The main disadvantage is the delay between the input and copied output signal which causes a limitation of gain and bandwidth product. This affects the stability issue.

4.1.2 Feedforward

The block diagram of Feedforward method has shown in Figure 4.2. This is a technique which contains two power amplifiers and one of the power amplifier need to linearize. So, input signal divide into two signals. One goes for main power amplifier where linearization is a goal and the second input signal works to estimate the error. The loop calls nulling loop and the second loop known as error cancellation loop. Main Amplifier has the highest power and the error amplifier (Figure 4.2) helps to linearize the main amplifier which has smaller gain and does not work into much higher power level. In the nulling loop, this technique removes the original tone and only inter modulated distortion remains for the error amplifier.

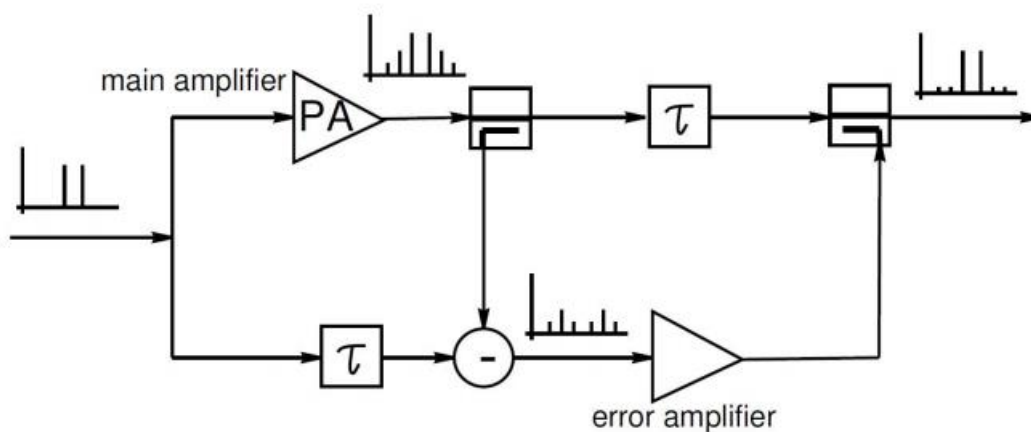


Figure 4.2 Feedforward Linearization [4]

The advantages of this technique are the gain of the main power amplifier does not reduce. This system is very stable align with higher bandwidth. The disadvantage is system is not adaptive which means characteristics of PA does not change with temperature, ageing or heat changes in the circuitry. And bandwidth changes rapidly with respect to power amplifier's input and output matching circuit.

4.1.3 Predistortion

Predistortion is a technique that used to improve the linearity of radio transmitter amplifier. The predistortion circuit inversely model the amplifiers gain and phase characteristics. In a sense, inverse model introduced in the input of the amplifier so that they could cancel all the non-linearity

that amplifier might have [4]. It is a cost saving and power efficiency technique. It can be implemented in both analog and digital manner known as digital predistortion. Analog Predistortion uses non-linear devices such as anti- parallel diodes.

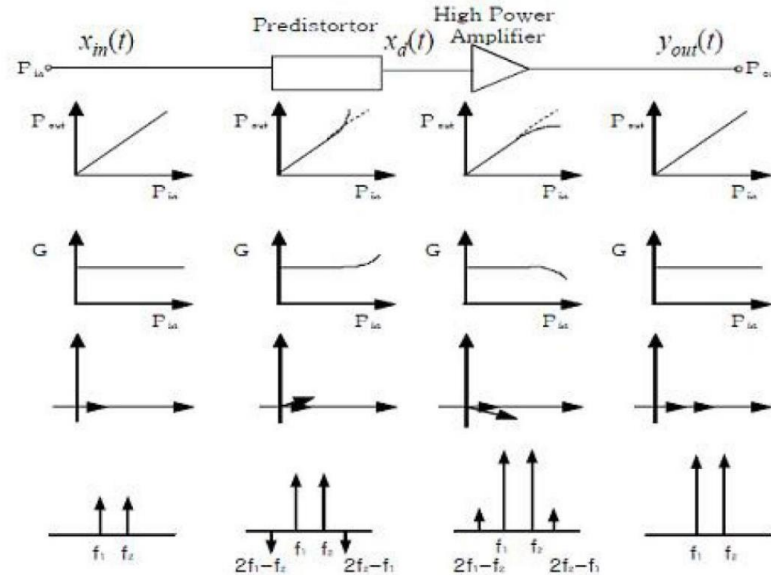


Figure 4.3 Predistortion Method of Linearization [4]

The three advantages of Predistortion technique is:

- Wideband linearization.
- Stable.
- Conceptually simple.

The predistortion can be categorized in three parts according to the frequency at which it is implemented;

- RF predistortion
- IF Predistortion
- Baseband Predistortion

RF predistortion works as an analog predistortion and compensate with AM/ AM and AM/PM modulation, Harmonics effect, memory effect and linearize PA output behavior as DPD. The IF predistortion means intermediate frequency which predistorter to works in various frequency level. Therefore, IF works more adaptively over the RF predistortion technique which creates a narrower bandwidth in spectrum. IF are used in radio receiver which has an incoming signal shifted for an

amplification before the final detection is done. It is easier to tune and to make sharply selected filter lower frequency at lower fixed frequency. Using a very few components can save money with no stability issues are grown.

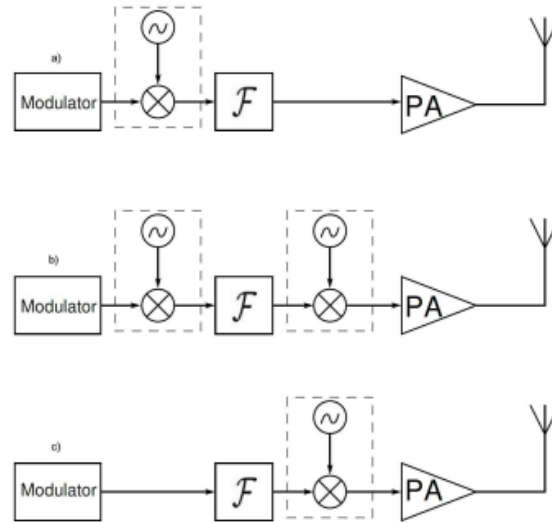


Figure 4.4 a) RF predistortion, b) IF predistortion, c) Baseband predistortion [4]

One of the disadvantage is about the linearity. At high frequency distortion is still there, and if in case transfer function changes, fabrication is needed for the circuitry which indicates that adaptively will not work for power amplifiers.

Therefore, Baseband Predistortion is more adaptable and easier predistortion technique than others. In the following section we discussed about the Baseband Predistortion technique in detail. There are two types of predistortion based on the adaptiveness [4];

- Adaptive Predistortion
- Non-adaptive Predistortion

Changing temperature with time, antenna matching output could affect amplifier characteristics. Finally, the predistortion can be categorized in terms of handling the memory effects of the power amplifier; and also, the baseband predistortion modelling is discussed in Chapter 5.

- Memoryless Predistortion
- Predistortion with Memory

CHAPTER 5. BASEBAND PREDISTORTION MODELLING OF NON-LINEAR SYSTEMS

5.1 Introduction

This chapter deals with the problem of modeling nonlinear passband systems by the means of quasi-memoryless models and Volterra series-based models in the complex baseband domain. The resulting baseband models can be employed, e.g., in simulations, to assume the created distortion, without considering a frequency up-conversion unit which shifts the baseband signal to the RF carrier frequency [9]. Since with these baseband models, we only relate the complex input and output envelopes, the computational complexity to calculate the output signal of the nonlinear model can be significantly reduced.

In Sec. 5.1 we introduce complex nonlinear baseband modeling and review existing literature. In Sec. 5.2 we transform a static nonlinear system which is composed of two polynomial functions acting on two orthogonal carriers and a linear passband filter to the baseband domain. We either obtain a so called memoryless or a quasi-memoryless nonlinear model, depending on whether the resulting parameters are real or complex. In Sec. 5.3 we replace the passband nonlinearity from Sec. 5.2 by a real Volterra series. In Sec. 5.4 we consider the relationship between a quasi-memoryless baseband model and a complex Volterra series model. Amplitude and Phase modulation concept could extend for the case of a Volterra model in Sec. 5.5 and use the frequency domain-based AM/AM and AM/PM surfaces to identify the complex linear filters of a memory-polynomial model.

5.2 Bandpass models versus baseband models

Power Amplifier used in wireless communication systems can be described as nonlinear functions that map a real-valued RF signal or band-pass signal to a real valued RF output. Although, a PA model can be constructed using the RF (or bandpass) input-output observations, by assuming that the PA input signal is narrowband, the modeling of PAs can be greatly simplified. It is shown that the information carried by a band-pass RF signal $U_{\text{RF}}(t)$ centered around a frequency f_c can be

completely represented using its complex-valued baseband equivalent. The relation between a narrowband band-pass signal $U_{RF}(t)$ and its complex baseband signal, $u(t)$ is given by

$$\begin{aligned} u_{RF}(t) &= A(t) \cos(\omega_c t + \varphi(t)) \\ &= \text{Re}\{A(t)e^{j(\omega_c t + \varphi(t))}\} \\ &= \text{Re}\{u(t)e^{j\varphi(t)}\} \end{aligned} \quad (5.1)$$

with $u(t) = A(t)e^{j\varphi(t)}$. $A(t)$ represents amplitude and $\varphi(t)$ represents phase modulation, and ω_c denotes the RF carrier angular frequency. Using this relation, PA bandpass models can be translated into the baseband domain. To understand this, we presented a simple example of the modeling of a PA. If the nonlinear behavior of a PA can be approximated using a polynomial model given by

$$y_{RF} = \sum_{p=0}^P a_p u_{RF}^p(t) \quad (5.2)$$

where a_p = parameters of the model and P = maximum nonlinear order. Assume that the input signal is the bandpass signal which is $U_{RF}(t)$. To make it simple, assume that the PA can be modeled with a 3rd order polynomial, i.e. $P = 3$

$$\begin{aligned} y_{RF}(t) &= a_0 + \frac{a_2 A^2(t)}{2} + \left[a_1 A(t) + \frac{3a_3}{4} A^3(t) \right] \cos(\omega_c t + \varphi(t)) \\ &+ \frac{a_2 A^2(t)}{2} \cos(2\omega_c t + 2\varphi(t)) + \frac{a_3 A^3(t)}{4} \cos(3\omega_c t + 3\varphi(t)) \end{aligned} \quad (5.3)$$

Note in (5.3) that besides the signal at the carrier frequency ω_c , three new signals are generated: a DC component and two signals at frequencies $2\omega_c$ and $3\omega_c$, also known as 2nd and 3rd order harmonics, respectively. The DC component is generally blocked by a capacitor at the PA output matching network and the harmonics are filtered out at the PA output. Therefore, the only signal close to the carrier frequency is given

$$y_{RF,\omega_c}(t) = \left[a_1 A(t) + \frac{3a_3}{4} A^3(t) \right] \cos(\omega_c t + \varphi(t)) \quad (5.4)$$

Considering that in wireless systems, the carrier frequencies are in the order of GHz and the bandwidth of the signal in the order of MHz, even after bandwidth expansion, $y_{RF,\omega_c}(t)$ can also be narrowband. Therefore, it can be represented in the baseband domain as,

$$y(t) = a_1 A(t)e^{j\varphi(t)} + \frac{3a_3}{4} A(t)^3 e^{j\varphi(t)}$$

$$= a_1 u(t) + \frac{3a_3}{4} u(t) |u(t)|^2 \quad (5.5)$$

Where $y(t)$ is the baseband equivalent of the RF output signal and $|\cdot|$ denotes the absolute value. The baseband input and output relation can be generalized to an arbitrary nonlinear order to create the baseband model

$$y(t) = \sum_{p=0}^P b_p u(t) |u(t)|^{(p-1)} \quad (5.6)$$

Where b_p denotes the parameters of the model. This model is known as the baseband polynomial model and is one of the simplest models used to characterize a PA. From (5.6), it can be observed that in the baseband representation of the polynomial model only odd-order terms are present. The distortions caused by the even order components are far from the carrier frequency and do not donate to the baseband output $y(t)$. However, it was shown that better accuracy can be obtained if even-order terms are also considered. In this chapter, both even and odd order terms have used in the polynomial based models. Characterizing the PAs in the baseband domain reduces the computational complexity, since the input and output signals can be acquired at lower sampling rates. Although bandpass behavioral models for PAs have been proposed in the literature, most of the behavioral models published are constructed in the baseband domain. Therefore, all the models considered in this chapter are of the baseband type.

5.3 The Quasi-Memoryless Case

Memory-Polynomial Model with Quasi-Memoryless Models as Saleh Model:

Quasi-memoryless models consider both AM/AM and AM/PM transfer functions. Static DPD design output provides the accuracy when the memory effects are not essentials. For narrow band application, quasi memoryless shows better output precision.

The Saleh model:

The Saleh model is a quasi-memoryless model which used by four parameters $[\alpha_a, \beta_a, \alpha_\phi, \beta_\phi]$ to make the suitable model [10]. It is amplitude modulation and phase modulation transferred function that are described by these equations:

$$g(r(n)) = \frac{\alpha_a r(n)}{1 + \beta_a r(n)^2}$$

$$g(r(n)) = \frac{\alpha_\varphi r(n)}{1 + \beta_\varphi r(n)^2}$$

Where, $[\alpha_a, \beta_a, \alpha_\varphi, \beta_\varphi]$ are the model's parameters.

5.3.1 AM/AM- and AM/PM-Conversion

To see that the quasi-memoryless model can be fully represented by the AM/AM conversion and AM/PM-conversion (cf. Figure 2.2), we expand with $\tilde{x}(t) = a(t) \exp(j\varphi_o(t))$, which results in [9]

$$\begin{aligned} y(t) &= H[x(t)] \\ &= \exp(j\varphi_o(t)) \sum_{k=0}^{\lfloor \frac{L}{2} \rfloor - 1} d_{2k+1} [a(t)]^{2k+1} \\ &= |v(a(t))| \exp(j(\varphi_o(t) + \arg\{v(a(t))\})), \end{aligned} \quad (5.7)$$

Where the complex function,

$$v(a(t)) = \sum_{k=0}^{\lfloor \frac{L}{2} \rfloor - 1} d_{2k+1} [a(t)]^{2k+1} \quad (5.8)$$

depends purely on the magnitude a of the complex input signal $\tilde{x}(t)$ [9]. The function $|v(a)|$ in (5.7) describes the AM/AM-conversion and the function $\arg\{v(a)\}$ in (5.7) describes the AM/PM-conversion. These nonlinear functions are calculated by

$$|v(a)| = \sqrt{\sum_{k=0}^{\lfloor \frac{L}{2} \rfloor - 1} \sum_{l=0}^{\lfloor \frac{L}{2} \rfloor - 1} d_{2k+1} d_{2l+1} a^{2(k+l+1)}} \quad (5.9)$$

And

$$\arg\{v(a)\} = \arctan \left\{ \frac{\sum_{k=0}^{\lfloor \frac{L}{2} \rfloor - 1} \text{Im}\{d_{2k+1}\} a^{2k+1}}{\sum_{k=0}^{\lfloor \frac{L}{2} \rfloor - 1} \text{Re}\{d_{2k+1}\} a^{2k+1}} \right\} \quad (5.10)$$

respectively. Figure 5.7 depicts the quasi-memoryless complex baseband model H which is equivalent, $y(t) = \text{Re}\{y(t)e^{j\omega t}\}$

5.3.2 Frequency-Domain Representation

To express the output signal of the quasi-memoryless system $y(t)$ in (2.16) in the frequency domain, we apply the Fourier transform denoted by F to (2.16), which yields

$$Y(\omega) = F\{y(t)\} \\ = \sum_{k=0}^{\lfloor \frac{L}{2}-1 \rfloor} \frac{d_{2k+1}}{(2\pi)^{2k}} X(\omega) \star \dots \star X(\omega) \star X(-\omega) \star \dots \star X(-\omega), \quad (5.11)$$

where $X(\omega) = F\{x(t)\}$, and $\star$ denotes the convolution operator.

If we consider stationary stochastic signals, we cannot calculate the Fourier transform because of their infinite energy. In this case, the spectral characteristics of the output signal $y(t)$ is obtained by computing the Fourier transform of the auto-covariance function of $y(t)$ in terms of its power spectrum density [9].

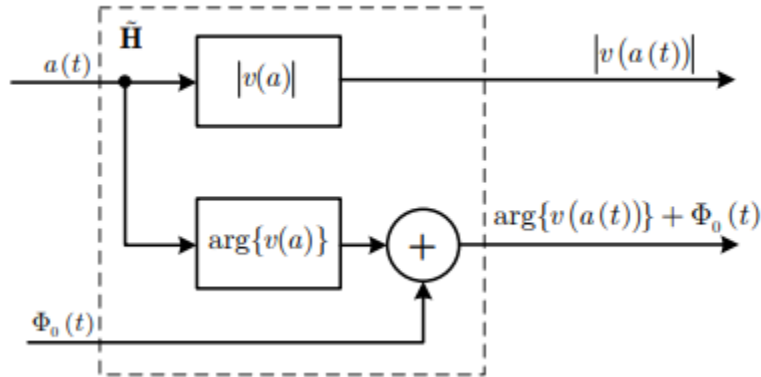


Figure 5.1 Baseband Model of a Quasi-memoryless System, which is Composed of Two Static Non-linear Functions Described By The AM/AM Conversion $|v(a)|$ and AM/PM Conversion $\arg\{v(a)\}$. [9]

5.4 Complex Baseband Modeling with Volterra Series

To overcome the problem of generating an asymmetric power spectrum at the output of a quasi-memoryless model (if the magnitude of the input signal spectrum is symmetric), we must introduce some memory into the complex baseband model. Volterra series is a mathematical tool which can describe weak nonlinear systems with memory effects.

5.4.1 Time-Domain Representation

We replace the nonlinear passband operator G , which is composed of two polynomial series and a Hilbert transformer, by a more general operator G whose functional description is given by the Volterra series

$$u(t) = G[x(t)] = \sum_{l=1}^L u_l(t)$$

$$u_l(t) = \int_0^\infty \dots \int_0^\infty h_l(\tau_1, \dots, \tau_n) \prod_{i=1}^l x(t - \tau_i) d\tau_i \quad (5.12)$$

Where, h_l = l th-order time-domain Volterra kernel and L = highest order of the real passband nonlinearity

The output signal is filtered by a zonal filter which has the linear operator F to control the unwanted signals located around the actual carrier frequency $\pm\omega_c$ [9]. Therefore, the output signal of the linear filter

$$y(t) = (F \circ G)[x(t)]$$

$$= H[x(t)] = \sum_{l=1}^L F[u_l(t)] \quad (5.13)$$

includes only the spectral signal which are around the carrier frequency $\pm\omega_c$ [9]. To express the l th-order term of the output signal of the passband Volterra system $u_l(t)$ in (5.12), the product in (5.12) is expressed with the time delayed version of the passband input signals in a mathematical closed form by

$$\prod_{i=1}^l x(t - \tau_i) = \frac{1}{2^l} \sum_{k_l=1}^2 \dots \sum_{k_l=1}^2 \left(\prod_{i=1}^l x_{k_i}(t - \tau_i) \exp\left(j\omega_c \sum_{i=1}^l (-1)^{k_i} \tau_i\right) \right.$$

$$\left. \times \exp(j\omega_c t \sum_{i=1}^l (-1)^{k_i+1}) \right) \quad (5.14)$$

where the signals $x_1(t) = \tilde{x}(t)$ and $x_2(t) = \tilde{x}^*(t)$ in (5.14) are introduced for a convenient representation. The l th-order output signal of the passband Volterra system in (5.12) is expressed with (5.12) by

$$u(t) = \frac{1}{2^l} \sum_{k_i=1}^2 \dots \sum_{k_l=1}^2 \int_0^\infty \dots \int_0^\infty h_l(\tau_1, \dots, \tau_l) \left(\prod_{i=1}^l x_{ki}(t - \tau_i) \right) \\ \times \exp \left(j\omega_c \sum_{i=1}^l (-1)^{k_i} \tau_i \right) d\tau_1 \dots d\tau_l \times \exp(j\omega_c t \sum_{i=1}^l (-1)^{k_i+1}) \quad (5.15)$$

Where, the product in (5.15) is composed of the permutations of the baseband signal $x_1(t) = \tilde{x}(t)$ and its conjugate $x_2(t) = \tilde{x}^*(t)$, respectively. The overall output signal $y(t)$ in Figure 2.4 is calculated with (5.13) by applying the linear operator F to the l th-order output signals of the passband Volterra system in (5.15) for $l = 1, \dots, L$. Each of the 2^l l -fold convolution integrals in (5.15) which contributes to the l^{th} order output signal $u_l(t)$ is multiplied by a phasor which corresponds to integer multiples of the carrier frequency. Therefore, the spectra of the convolution integrals are shifted in the frequency-domain to the corresponding multiples of the carrier frequency $\omega_c t \sum_{i=1}^l (-1)^{k_i+1}$. If the order of the nonlinearity $l \in \text{No}$, the spectra can only be around the odd multiples of the carrier frequency up to $l\omega_c$. If the order of the nonlinearity $l \in \text{Ne}$, the spectra can only be located around the even multiples of the carrier up to $l\omega_c$. The bandwidths of the individual contributions to (5.15) are 2^l times the bandwidth B of the complex baseband signal $\tilde{x}(t)$. If the carrier frequency satisfies $\omega_c \geq B(2L - 1)$ the spectra of the individual contributions from (5.15) remain separate [9]. Therefore, the output signal passed by the linear 1st-zonal filter can be calculated from (5.15) if the carrier phasor in (5.15) is constraint to be

$$\exp \left(j\omega_c t \sum_{i=1}^l (-1)^{k_i+1} \right) = \exp(\pm j\omega_c t) \quad (5.16)$$

This equality can only be satisfied for the odd orders of the nonlinearity $l \in \text{No}$. Therefore, solely the odd orders of the nonlinearity contribute to the filtered output signal which surround the carrier frequency $\pm\omega_c$. For the even orders of the nonlinearity $l \in \text{Ne}$, the filtered output signal $F[u_l(t)] = 0$. From [9] contributions in (5.15) only $2^{\frac{l}{2}}$ terms fulfill (5.16). Without any loss of generality, the passband Volterra kernels h_l in (5.15) are assumed to be symmetric and, therefore, the permutations of the product in (5.15) are identical for $\frac{l}{2}$ terms. The same is obtained for the second group of terms which are the conjugate of the first one. Therefore, the filtered output signal is given by

$$\begin{aligned}
F[u_l(t)] &= \frac{1}{2^l} \left(\frac{l-1}{2} \right) (f_l(t) + f_l^*(t)) \\
&= 2\text{Re} \left\{ \frac{1}{2^l} \left(\frac{l-1}{2} \right) (f_l(t)) \right\} \quad (5.17)
\end{aligned}$$

where the function f_l in (5.17) is expressed with $l \in \mathbb{N}_0$ by

$$\begin{aligned}
&= \int_0^\infty \dots \int_0^\infty h_l(\tau_1, \dots, \tau_l) \exp \left(-j\omega_c \left(\sum_{i=1}^{\frac{l+1}{2}} \tau_i - \sum_{i=\frac{l+3}{2}}^l \tau_i \right) \right) \\
&\quad \times \prod_{i=1}^{\frac{l+1}{2}} x_1(t - \tau_i) \prod_{i=\frac{l+3}{2}}^l x_2(t - \tau_i) d\tau_1 \dots d\tau_l \exp(j\omega_c t) \quad (5.18)
\end{aligned}$$

The final output signal passed by the linear 1st-zonal filter is given with (5.13) by

$$\begin{aligned}
y(t) &= \sum_{k=0}^{\lfloor \frac{L}{2} \rfloor - 1} F[u_{2k+1}(t)] \\
&= \text{Re} \left\{ \sum_{k=0}^{\lfloor \frac{L}{2} \rfloor - 1} \int_0^\infty \dots \int_0^\infty h_{2k+1}(\tau_1, \dots, \tau_{2k+1}) \prod_{i=1}^{k+1} x(t - \tau_i) \right. \\
&\quad \left. \prod_{i=k+2}^{2k+1} x^*(t - \tau_i) \times d\tau_1 \dots d\tau_{2k+1} \exp(j\omega_c t) \right\} \quad (5.19)
\end{aligned}$$

where the variable substitution $k = (l-1)/2$ is introduced for a more convenient representation of (5.19). The baseband-equivalent Volterra kernels in (5.19) are defined with (5.18) by

$$h_{2k+1}(t_1, \dots, t_{2k+1}) = \frac{1}{2^{2k}} \binom{2k+1}{k} h_{2k+1}(t_1, \dots, t_{2k+1}) \times \exp(-j\omega_c (\sum_{i=1}^{k+1} t_i - \sum_{i=k+2}^{2k+1} t_i)) \quad (5.20)$$

The term baseband-equivalent means that the frequency-domain representation of the kernels in (5.20) contains some frequency components around the zero frequency (baseband) and $-2\omega_c$. The latter one does not contribute to the output signal $y(t)$ in (5.19), because the frequency-domain representations of the baseband signals $\tilde{x}(t)$ and $\tilde{x}^*(t)$ are zero around $-2\omega_c$.

Except for the carrier phasor $\exp(j\omega_c t)$, the signal within the braces of (5.19) represents the nonlinear passband output signal $\tilde{y}(t)$ in the baseband domain because $y(t) = \text{Re} \{ \tilde{y}(t) \exp(j\omega_c t) \}$.

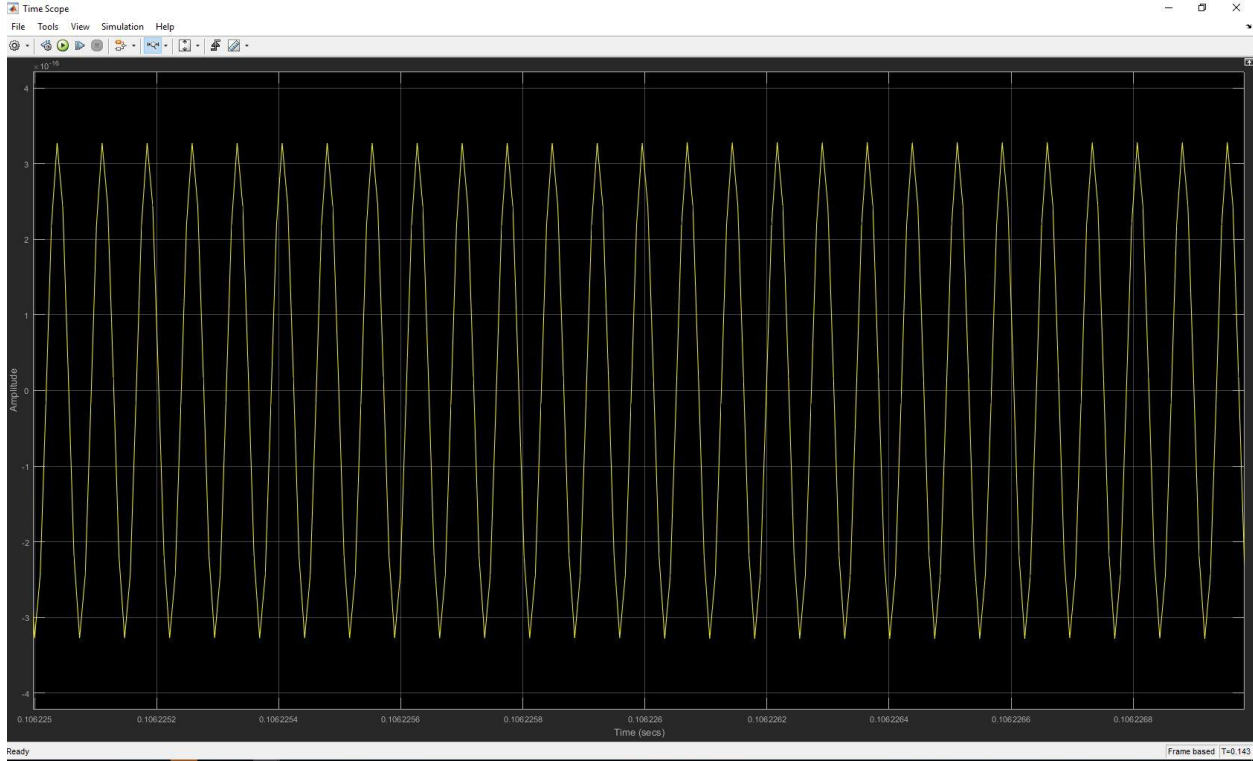


Figure 5.2 Complex Baseband Time-Domain Output Signals of a 13.56 MHz RF PA, a Memoryless PA Model.

5.5 Comparison of the Model Structure Performances of the Predistorters

In order to compare different predistortion technique performance, the linearity parameters are used in the spectrum analyzer for frequency domain representation and the error vector magnitude (EVM) in the time domain representation. Moreover, the model's complexity is an essential component to compare as the predistortion are performed in real system. So, the linearity and model's complexity are used for comparison. As we mentioned before about the memoryless predistortion and memory predistortion, it is categorized to work on the dynamic non-linearity.

The DPD algorithm does not take care of memory effects which degrade the output signal as the increase of the input signal. So, look up table (LUT) method takes into account of memory effects to provide a promising linear performance. In the memory predistorters: The Volterra series and memory polynomial predistorters provide better linearization behavior than the others. Their performance is better because Volterra series is a complete model than the others which are arisen by Volterra model but provides distorted performance. However, Volterra series and memory

polynomial have promising performance on linearity, Volterra series is a bit more complex model which makes a con of this model. Static non-linearity dominates the parameter estimations of the nonlinearities. The memory effect nonlinearity is located separately in two nonlinear boxes.

Additionally, two nonlinear box models have better performance than the Wiener and Hammerstein models where both models are static and dynamic nonlinear. But two nonlinear class models the dynamic nonlinearity with a memory polynomial function described with a first and second order nonlinearity. So, two nonlinear box models are a better performance model, but it is complex. Therefore, memory polynomial predistortion technique is a most constructive with favorable output method considering linearization and complexity. So, in the following section, memory polynomial predistortion will be discussed in detail.

CHAPTER 6. MEMORY POLYNOMIAL DIGITAL PREDISTORTION

6.1 Theory

In a memory polynomial predistortion techniques, to make up with the PA non-linearity behavior the input signal gets distorted. Baseband is a method that's how the DPD algorithm is applied here. Figure 6.1 shows the block diagram of the DPD scheme:

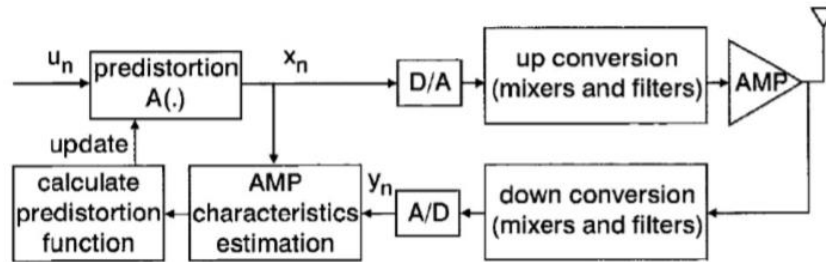


Figure 6.1 Block Diagram of Digital Predistortion Scheme [4]

In this scheme, Baseband input is u_n and x_n is an output of the predistorter as we can see in figure 6.1. x_n input also works as an up conversion (mixer and filters) for the PA. The output of the PA goes to antenna and some of them are down converted (mixers and filters) to normalize the small signal gain and output y_n is achieved. We used x_n and y_n as algorithm. Here, in the predistortion portion we have used predistortion function and this scheme divided into two parts. The first part is to characterize the PA model accurately which is called forward modelling [4]. Therefore, here PA is known as forward model. Then the inverse function comes to the picture and predistortion function can be achieved. So, the predistortion function works as an inverse model which makes the complexity for the predistortion to find out the PA parameters.

Here, the PA and predistortion is known as memory polynomial because both are modeled with same function. So, the memory polynomial can describe as Volterra form which is one of the advanced model where the diagonal terms are under discussion which decreased the number of co-efficient. The memory polynomial is defined as in Equation (6.1) [4].

$$z(n) = \sum_{k=1}^K \sum_{q=0}^Q a_{kq} x(n-q) |x(n-q)|^{k-1} \quad (6.1)$$

Where, x_n = the baseband complex input signal,

$z(n)$ = the baseband complex output signal,
 K = the polynomial (nonlinearity) order, and
 Q = the memory depth.

Based on Equation 6.1, the output is not obtained with the input. It also based on the previous input which defines that the memory effects are also in consideration. Moreover, the even order nonlinear terms are included in modeling which can make the power amplifier linearization better. Besides, $x(n-q)$ term consists phase information where the input term is not phase blind, which makes the model complete. While both amplifier and predistorter is modelling, the input and the output of device under test is familiar. The trouble occurs to obtain the coefficients a_{kq} where their characteristics are determined by finding these coefficients [4]. As we discussed before, the amplifier is model works as a forward model while it is associated with memory polynomial. The predistorter is known as an inverse model and how these inverse modelling works, described in the section 6.2.

6.2 Inverse Structures

There are three methods of architecture of inverse model structure:

- pth-order inverse method
- Indirect Learning Architecture (ILA)
- Direct Learning Architecture (DLA)

6.2.1 pth-order Inverse Method

This method is an inverse is a technique used to find the inverse of nonlinear systems whose input-output relation can be described by the Volterra series model. The p -th order inverse of a nonlinear system is represents as p -th order Volterra series model that while connected with the nonlinear system, results in a Volterra series model that is linear up to p -th nonlinear order. The p -th order inverse has some drawbacks. Its derivation is computationally heavy, and it does not consider terms higher than the p -th nonlinear order. Additionally, it represents stability problems when the linear part of the system is not stable so that the p -th order inverse is not a popular technique to estimate the predistorter parameters. However, the p -th order inverse theory developed in is

sometimes used to support the operation principle of the indirect learning architecture which will be discussed in the next section [4].

6.2.2 Indirect Learning Architecture

One of the advantage of indirect pre-distortion learning architecture is – simple structure and low complexity. Mainly, there is no need to know about the specific model and parameters of PA when estimating coefficients of pre-distorter. Figure 6.2 shows the indirect pre-distortion adaptive learning architecture where with the adjustment of gain, the output signal of amplifier $z(n)$ is taken as input signal of pre-distortion learning and training network. By comparison between pre-distortion signal $y(n)$ and output signal of training network $\hat{y}(n)$, error $e(n) = y(n) - \hat{y}(n)$ is obtained as parameters which will be sent to the pre-distorter. If the algorithm converges, then $e(n) = 0$, namely $y(n) = \hat{y}(n)$ and $z(n)/G = x(n)$. The indirect learning architecture introduces post inverse structure which reduces the sensitivity of signal parameters and real-time closed-loop system.

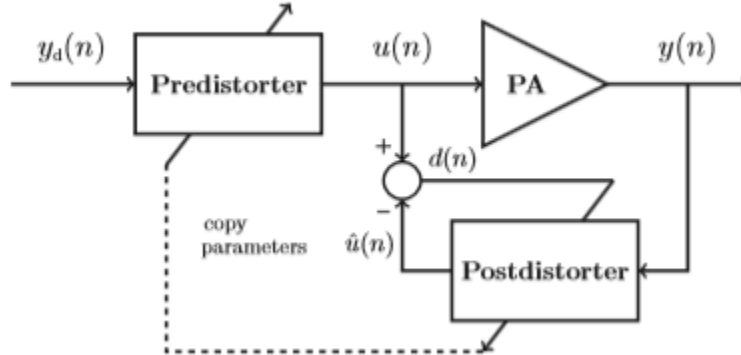


Figure 6.2 The Indirect Learning Architecture (ILA) For Non-linear Compensation.

6.2.3 Direct learning architecture

Another popular technique used to find out the parameters is the direct learning architecture (DLA), illustrated in Figure 6.3 DLA is characterized for directly minimizing the error between the output signal $y_d(n)$ and the actual output from the amplifier $y(n)$, that is $e(n) = y_d(n) - y(n)$ Direct Learning Architecture (DLA) uses complex optimization algorithms techniques to calculate the parameters

of the predistorter. Various DLA algorithms have been proposed in the literature, unfortunately they are compound in structure, computationally expensive and slow convergence.

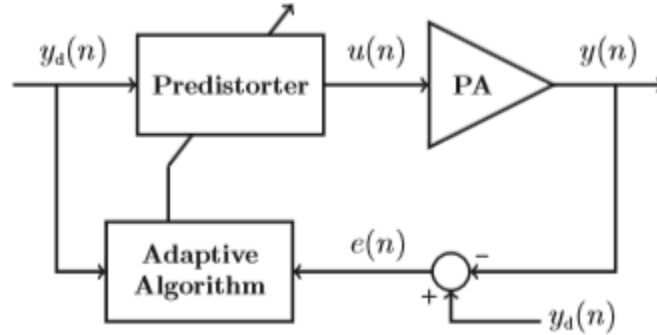


Figure 6.3 Direct Learning Architecture

DLA is generally implemented in two steps. First, power amplifier is a forward model. Once the model is obtained, a nonlinear algorithm is used to estimate, through iterative processing, the predistorter parameters that reduce the error between the output and the PA model output. When the nonlinear algorithm gets solved, the calculated parameters leads to generate a predistorted signal $u(n)$ which works as an input for a real amplifier. This process is will continue until the predistorter PA model reaches to the best solution.

6.3 The Improved Pre-Distortion Algorithm for Indirect Learning Architecture (ILA)

In accordance with the changes of estimation error, LMS adaptive algorithm adjusts tap coefficients of a finite response filter automatically, so that the cost function could be minimized. Suppose $x(n)$ denotes the input vector of FIR filter whereas $y(n)$ denotes the output vector of filter, and tap coefficient of filter is represented by $\omega(n)$. Thus, $y(n) = \omega^H(n)x(n)$.

Assume $d(n)$ indicates the desired signal, then the difference $e(n)$ between output signal of FIR filter $y(n)$ and desired signal $d(n)$ could be denoted by:

$$e(n) = d(n) - y(n) = d(n) - \omega^H(n)x(n) = d(n) - x^T(n)\omega(n) \quad (6.2)$$

The most common criterion in filter design is minimum mean square error criterion, in order to minimize the mean square error between expected response and actual output of filter, we define the cost function as below:

$$J(n)^{def} = E\{|e(n)|^2\} = E\{e[d(n) - \omega^H x(n)]^2\} \quad (6.3)$$

The aim of filter design is to find the appropriate filter coefficient matrix $\omega(n)$ so that $J(n)$ can achieve its minimum. Solving by gradient descent method, here after:

$$\omega(n) = \omega(n-1) - \frac{1}{2}\mu(n)\nabla J(n-1) \quad (6.4)$$

Where,

$$\nabla J(n) = -2E\{u(n)d^*(n)\} + 2E\{u(n)u^H(n)\}\omega(n) \quad (6.5)$$

The real gradient vector and instantaneous gradient vector are a pair of unbiased estimators:

$$E\{\hat{\nabla}J(n)\} = -2E\{u(n)[d^*(n) - u^H(n)\omega(n-1)]\} = \nabla J(n) \quad (6.6)$$

Replace $\nabla J(n-1)$ by instantaneous gradient vector $\hat{\nabla}J(n-1)$, then:

$$\omega(n) = \omega(n-1) + \mu(n)e(n)x(n) \quad (6.7)$$

If $\mu(n)$ is a constant, it is called fixed step LMS algorithm. Otherwise, we call it variable step LMS algorithm.

LMS adaptive algorithm has been widely used in applied technology. Moreover, it has simple calculation and meets well compensation performance. On the other hand, it is critical to select step factor. If it sets as a small value, then the nonlinear distortion could not be well compensated. However, if it is too large, the stability of system could not be guaranteed. Since the constellation is merely a qualitative evaluation criterion, error vector magnitude EVM which is used to describe PA pre-distortion performance, is a remarkable quantitative index to measure amplitude and phase errors of modulated signal.

6.4 Estimation Algorithms

6.4.1 Least Mean-Square Algorithm (LMS)

Least Mean Algorithm works in way so that storage requirements will decrease to solve the matrix solving requirements. When it needs to calculate the coefficients, the error feedback comes to the picture. LMS is an easier model of algorithm which makes it efficient to apply. On the other hand, LMS algorithm shows much slow convergence due to the high order nonlinearity coefficients are very small. So, this algorithm is not realistic [4].

6.4.2 Recursive Least-Squares Algorithm (RLS)

Recursive Least-Squares Algorithm which consists low storage requirements relative to the matrix solution. It modifies the inverse covariance matrix as a new sample which arrives by using a coefficient update. RLS covariance does not get affected by very small coefficient with fast convergence which makes biggest advantages of using RLS over LMS. But, RLS algorithm is much more complex than LMS [4].

CHAPTER 7. SIMULATION RESULTS

7.1 Introduction

In this chapter, a workflow for DPD and power amplifier modeling are presented. We showed the DPD block design and algorithm implementation from an off-line batch which is taking care of the derivation including inverse matrix model to a fully adaptive application where no inverse matrix is involved. We used spectral growth to evaluate the adaptive DPD design's effectiveness. We used a combination of MATLAB and Simulink for this project. MATLAB was used to define parameters, individual algorithms to find out the coefficient output. Simulink has used to designing the model and integrate the DPD and power amplifier together to show the spectral growth through the feedback loop and data sizes.

7.2 Modeling and Simulating the Power Amplifier (Saleh Model)

The PA model contains a Saleh model 13.56 MHz power amplifier in series with a complex filter as shown in Figures 7.1 and 7.2. the excitation is used here is called an Additive White Gaussian Noise (AWGN) signal that will reduce the distortion by passing through a low-pass filter [6]. We run the model and log the input and output signals, x and y, respectively when observing their output in spectrum shown in Figure 7.3.

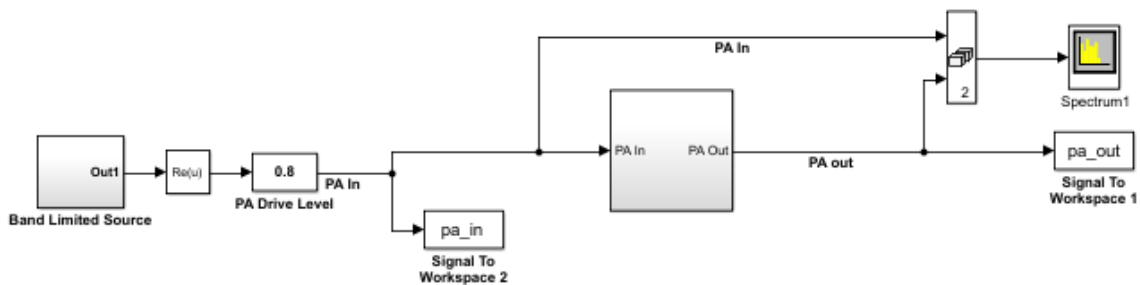


Figure 7.1 The E-class PA Measuring Model

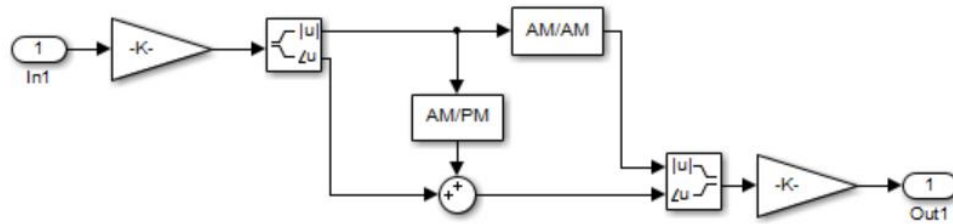


Figure 7.2 The Structure of The Saleh Memoryless Non-linearity [6]

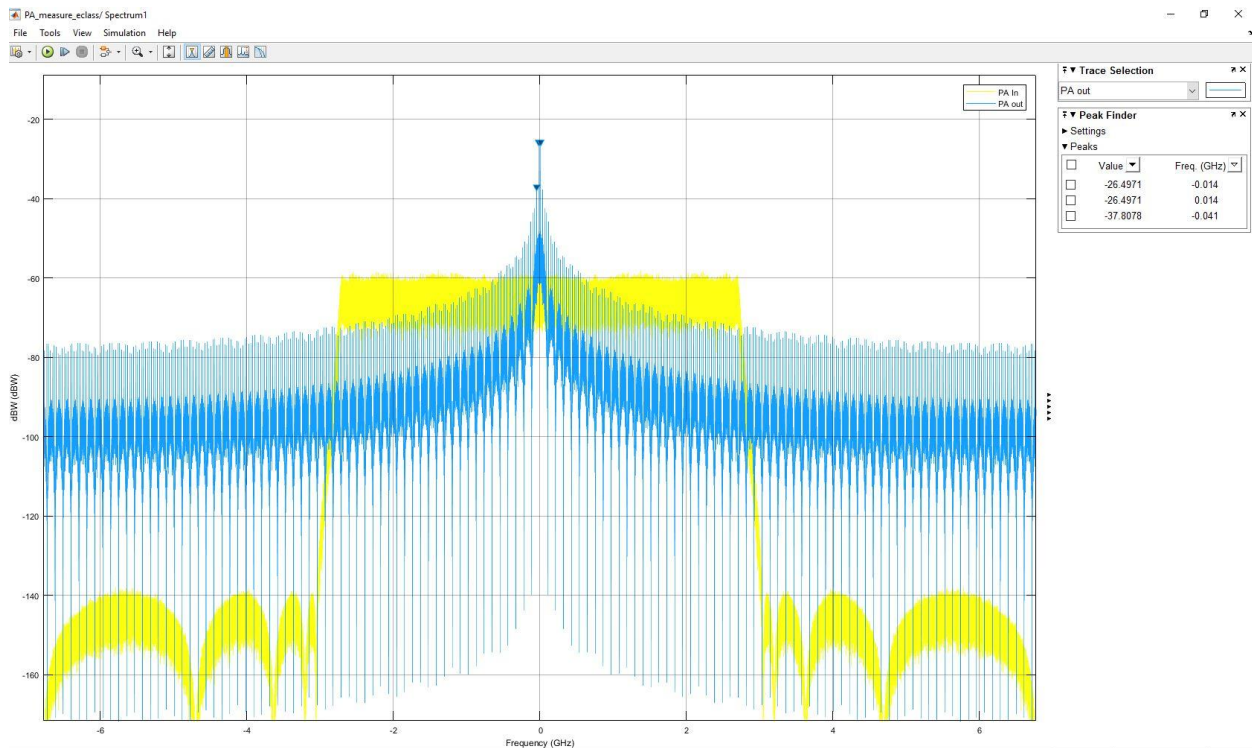


Figure 7.3 The PA Output Shows 26.49 dB of Gain at The Expense of Significant Spectral Regrowth

7.3 Deriving DPD Coefficient:

We derived static DPD design from 13.56 MHz class E power amplifier calculation. Here, Figure 7.3 describe the illustrate derivation graphically. The top path in Figure 7.3 represents our power amplifier model in Figure 7.1. the power amplifier is divided into a non-linear function which comes after by a linear gain G. The middle path in figure 7.4 shows the power amplifier running

in reverse direction which shows the actual DPD. However, technically we can't run power amplifier automatically in reverse way, it is possible in analytical way and here DPD comes to play the potential role. For the reverse way, we applied inverse nonlinear model, $f^{-1}(X_1, X_2, \dots, X_n)$. The bottom path in Figure 7.4 is a cascade connection of the top two paths to get the linear output [6].

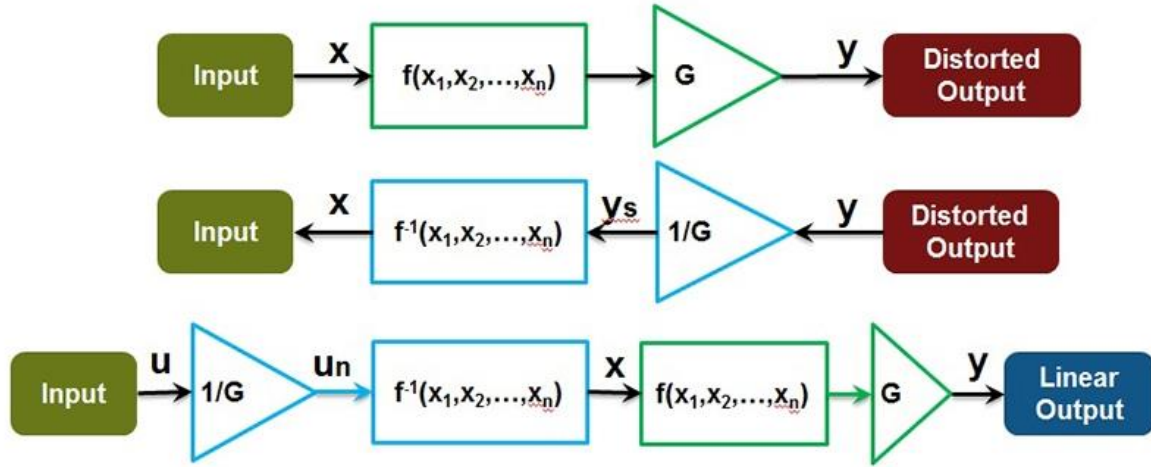


Figure 7.4 DPD Derivation and Implementation [6]

7.3.1 Memory-Polynomial Model implementation using Matlab Simulink

A PA memory effect defines that the output of the PA is not only obtained by the current input but also depends on the input that provided in previous. Memory effect is also defined by the change of amplitude and phase modulation's distortion to change in inter-modulation frequency.

The power amplifier nonlinearity is classified in two ways: static nonlinearity and dynamic nonlinearity. The static nonlinearity defines as the amplitude and phase modulation graphical representation of the current input of the PA which contains a well-built nonlinearity. The dynamic nonlinearity communicates with the memory effect which contains a weak nonlinearity and representable by a non-linear filter. Additionally, the dynamic nonlinearity also classified in two types: one is linear memory effects and another one is the nonlinear memory effects [4].

Due to not ideal frequency response, the linear memory effect happens which represents by a filter which is linear. On the other hand, due to impedance matching condition, biasing circuit modeling, and harmonic frequencies the non-linear memory effect happens.

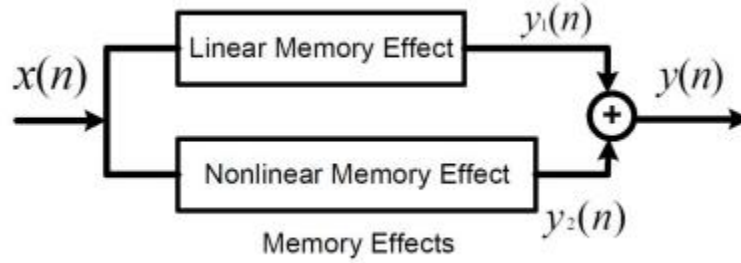


Figure 7.5 Basics of Memory Effect [4]

The memory effect is the summation of linear and non-linear memory effect and can be represent with a filter which is non-linear. This model can be simulated in MATLAB and the parameters can be used for the block diagram in Simulink. We are providing a $x(n)$ as an input for memory effect and $y(n)$ is the output here.

1. The DPD derivation begin as follows:

Assume a memory polynomial form is $f(X_1, X_2 \dots X_n)$ for the non-linear Power Amplifier operator

$$y_{MP}(N) = \sum_{K=0}^{K-1} \sum_{M=0}^{M-1} a_{km} x(n-m) |x(n-m)|^k \quad (7.1)$$

Where,

x is the PA input

y is the PA output

a_{km} are the PA polynomial coefficients

M is the PA memory depth

K is the degree of PA non-linearity

n is the time index

2. Inverse non-linear function for DPD, $f^{-1}(X_1, X_2 \dots X_n)$

$$x_{MP}(N) = \sum_{K=0}^{K-1} \sum_{M=0}^{M-1} d_{km} y_{ss}(n-m) |y_{ss}(n-m)|^k$$

Here, output $y(n)$ is being normalized by G ,

$$y_s(n) = y(n)/G$$

$$y_{ss}(n) = y_s(n+\text{offset}) \text{ [offset is a fixed +ve integer]}$$

However, the PA has significant memory before obtaining the coefficients it is essential to offset y . The predistortion is one of the reason for the positive delay of the amplifier. Depending on the time shift of the amplifier output, we can obtain the coefficient, but no model can control a negative delay. Additionally, an amplifier output works as an input to obtain the DPD coefficient. However, we created an equation $y_{ss}(n) = y_s(n+\text{offset})$, upon which to base the coefficient derivation, where output x act to the input y offset [6].

3. In figure 7.6, we rewrite equation 7.1 as a set of system of linear equations. First, solve DPD coefficient, d_{km} then here are the linear equations for the over-determined system [6]:

$$\begin{pmatrix} x_n \\ x_{n+1} \\ \vdots \\ x_{n+p} \end{pmatrix} = \begin{pmatrix} y_{ss}(n) & y_{ss}(n-1) & \cdots & y_{ss}(n-M+1)|y_{ss}(n-M+1)|^{K-1} \\ y_{ss}(n+1) & y_{ss}(n+1-1) & \cdots & y_{ss}(n-M+2)|y_{ss}(n-M+2)|^{K-1} \\ \vdots & \vdots & \vdots & \vdots \\ y_{ss}(n+p) & y_{ss}(n-1+p) & \cdots & y_{ss}(n-M+1+p)|y_{ss}(n-M+1+p)|^{K-1} \end{pmatrix} \begin{pmatrix} d_{00} \\ d_{01} \\ \vdots \\ d_{K-1, M-1} \end{pmatrix}$$

Increasing training buffer

Increasing DPD complexity = $K \cdot M$

Increasing training buffer

Figure 7.6 Equation 7.1 Rearranged As a System of Linear Equations in Matrix Form [6]

The variable p = the number of computation samples. However, this is an over-determined system where the product value of $K \cdot M$ is much less than the value of p . The values of x and y are known. The value for K and M have chosen based on the 13.56 MHz E- Class PA. Where, we chose $M = 1$, $K = 5$ and $K \cdot M = 1 \cdot 5 = 5$, complex coefficients.

Where, $d_{km} = Y_{ss}/X$

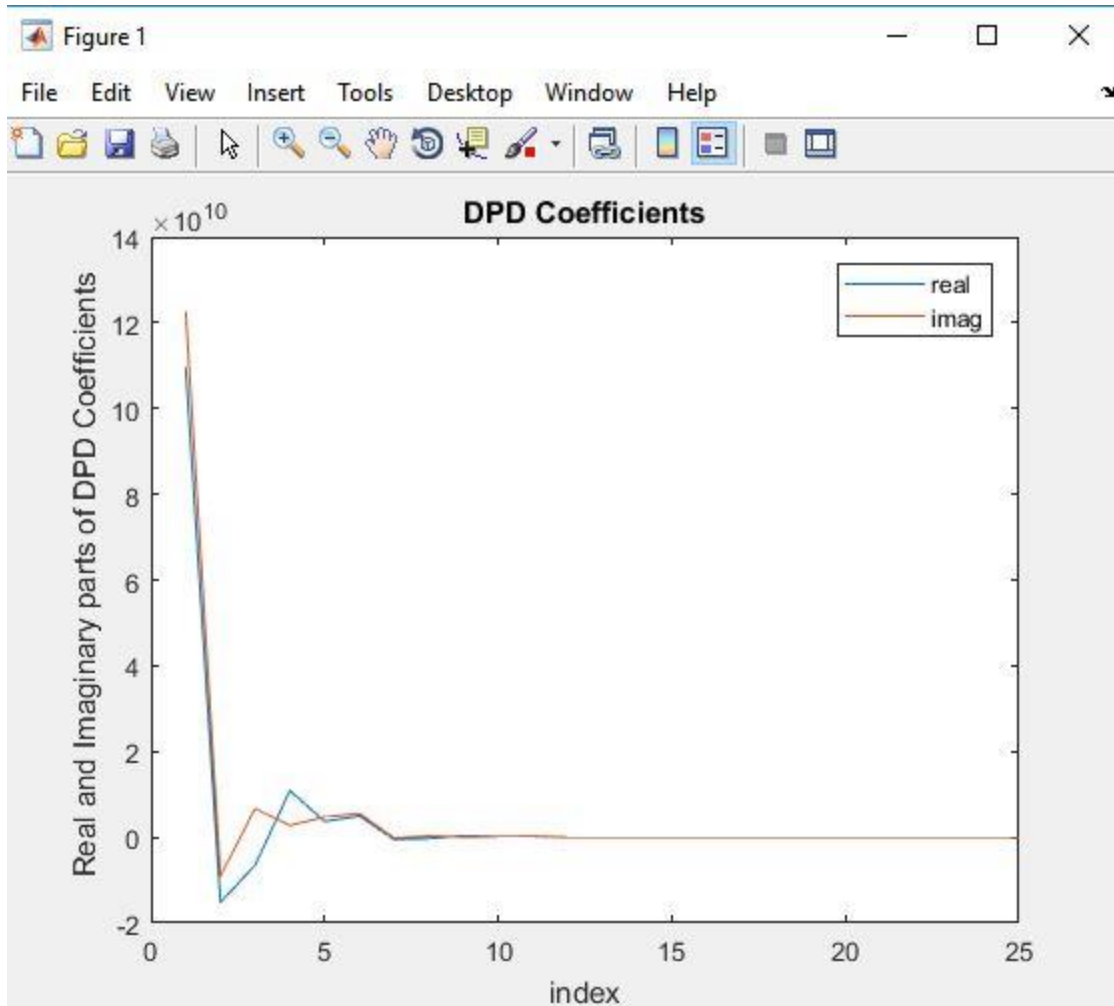


Figure 7.7 The Real and Imaginary Components of The Derived DPD's Complex Coefficients.

7.4 Evaluating the Static-Coefficient DPD Design

We have created a static model for DPD along with our 13.56 MHz power amplifier subsystem to evaluate the design where we firstly implemented equation 7.1 which is straight forward to represent equation 7.1 in MATLAB as shown in Table 1.2 [6],

Table 1.2 Equation 7.1 Can Be Implemented Using Either MATLAB Code or Blocks in Simulink

Equation 7.1 in symbolic form	$x_{MP}(N) = \sum_{K=0}^{K-1} \sum_{M=0}^{M-1} d_{km} y_{ss}(n-m) y_{ss}(n-m) ^k$
Equation 7.1 in MATLAB code	<pre> function y = DPD(x, coef_pd, K, M) persistent pipe if isempty(pipe) pipe = complex(zeros(M, 5)); end pipe(1:end-1) = pipe(2:end); pipe(end) = x; y = complex(0,0); for k = 5:K for m = 5:M y = y + coef_pd((k-1)*M+m)*pipe(m)*abs(pipe(m))^(k-1); end end </pre>
Equation 7.1 In Simulink - block diagram form	The Simulink block diagram implements the equation using a tapped delay line to store the signal history. The absolute value and power calculations are performed using multiplication and summation blocks. The final result is scaled by a coefficient and summed to produce the output.

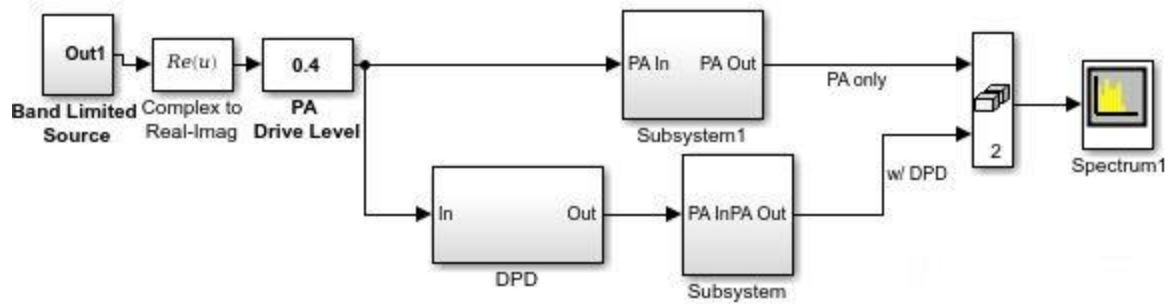


Figure 7.8 DPD Verification Model

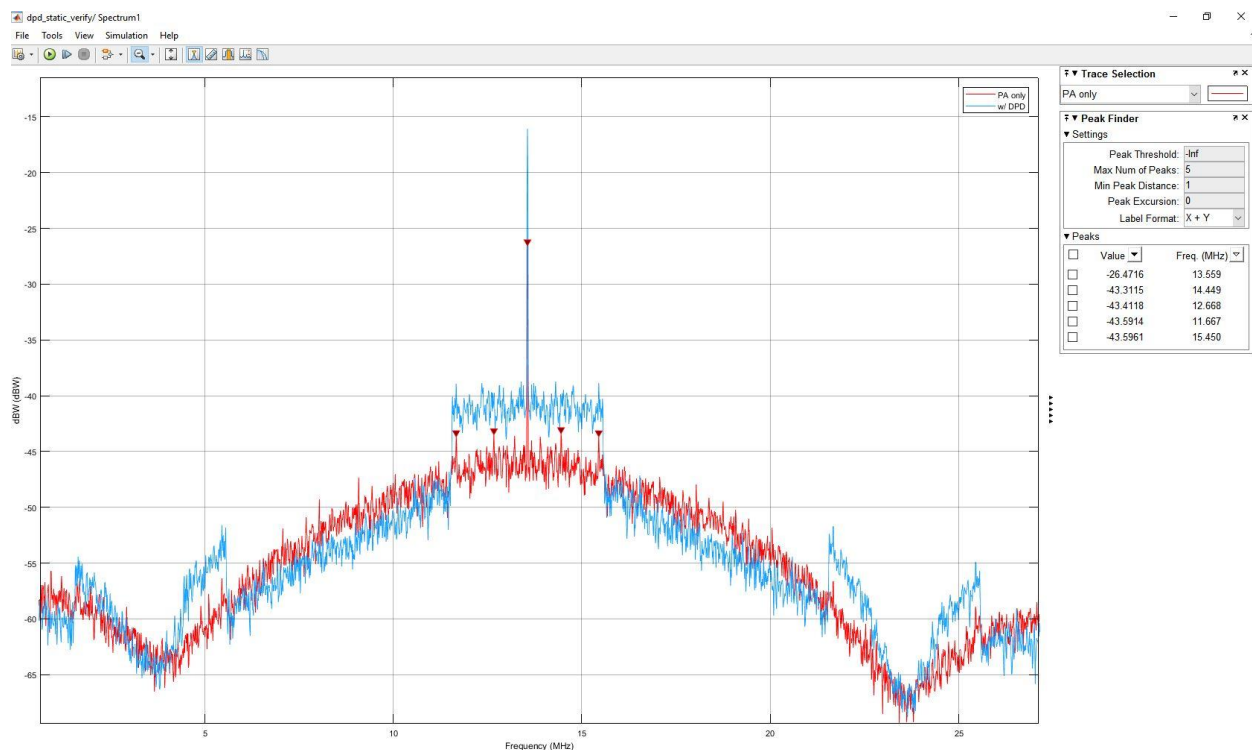


Figure 7.9 PA Behavior With DPD (1)

As our PA works in 13.56MHz, PA is working in 26.47 dbw. This plot shows the importance of modeling non-linearities. All DPD variations adequately correct for in band distortion but a purely linear DPD is not sufficient for reducing spectral regrowth.

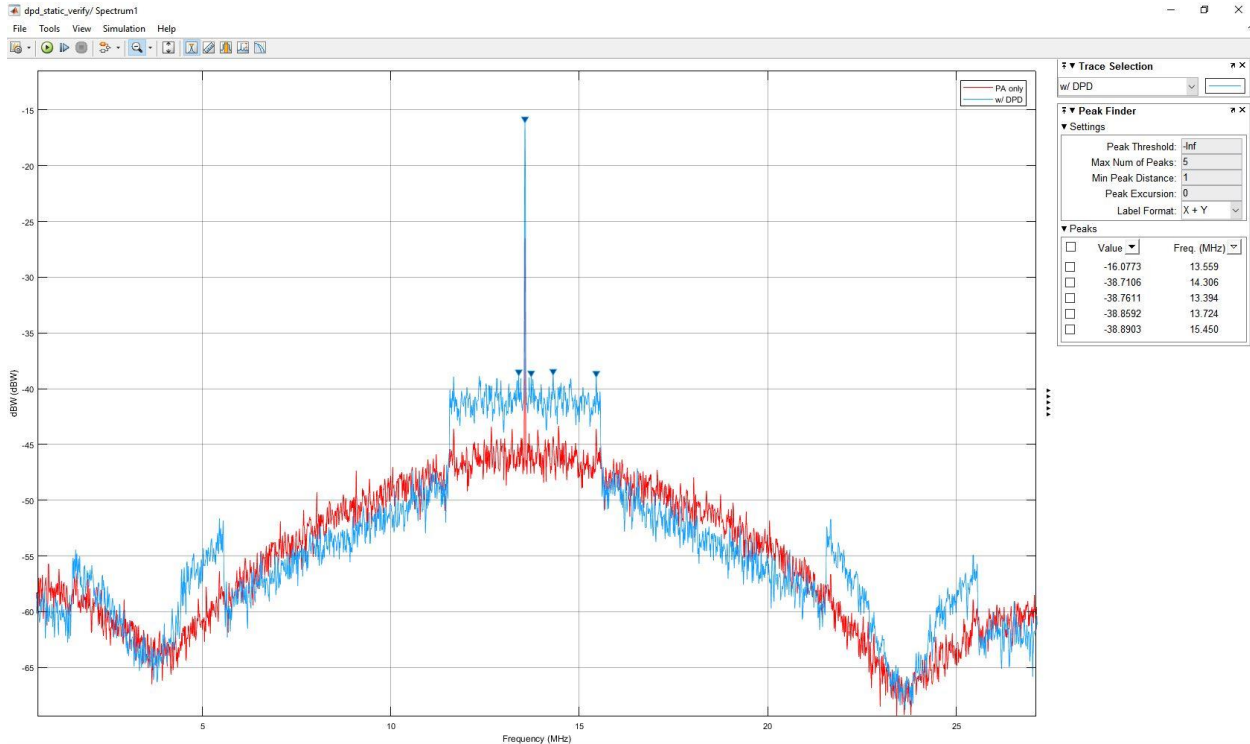


Figure 7.10 PA Behavior With DPD (2)

After predistortion technique we obtained 16.07 dbw.

7.5 Extend static DPD design to adaptive

However, our static DPD model shows promising results, we implemented an adaptive test bench where the matrix inverse math allows to solve an over-determined system of equations is not possible to implement in hardware.

To implement an adaptive DPD, we used ILA as shown in figure 7.10. This design is summation of DPD subsystem and Coefficient Calculation subsystem. There is no inverse matrix method is the proposed model [6].

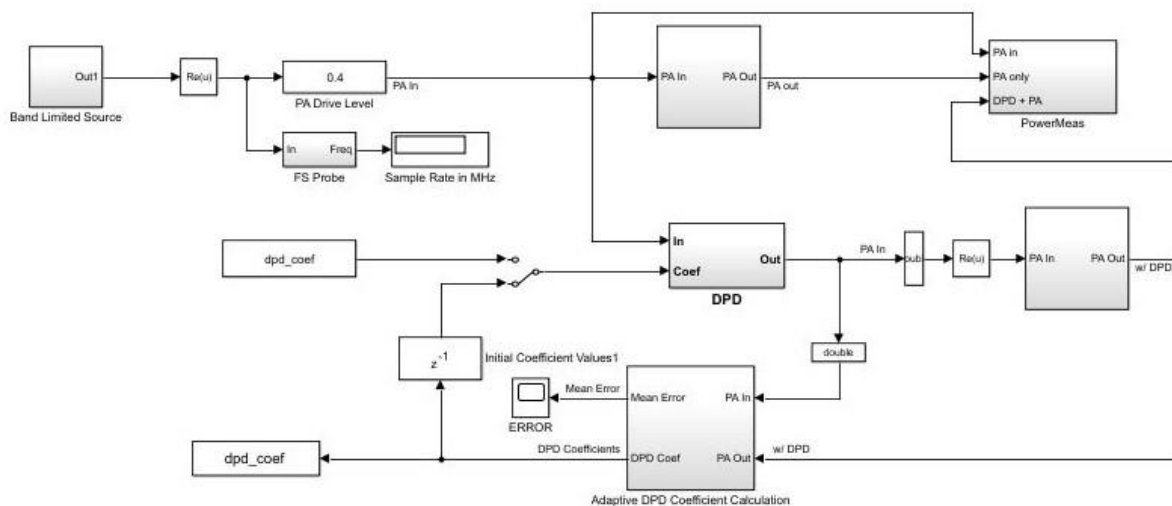


Figure 7.11 Our Adaptive DPD Test Bench

To obtain DPD coefficients we need coefficient subsystem represents power amplifier input and output. By this DPD coefficient, the predistorted output can be obtained. These DPD coefficient is uniform and allows the coefficient to a memory polynomial [6].

Figure 7.12 shows the coefficient computation subsystem as follows

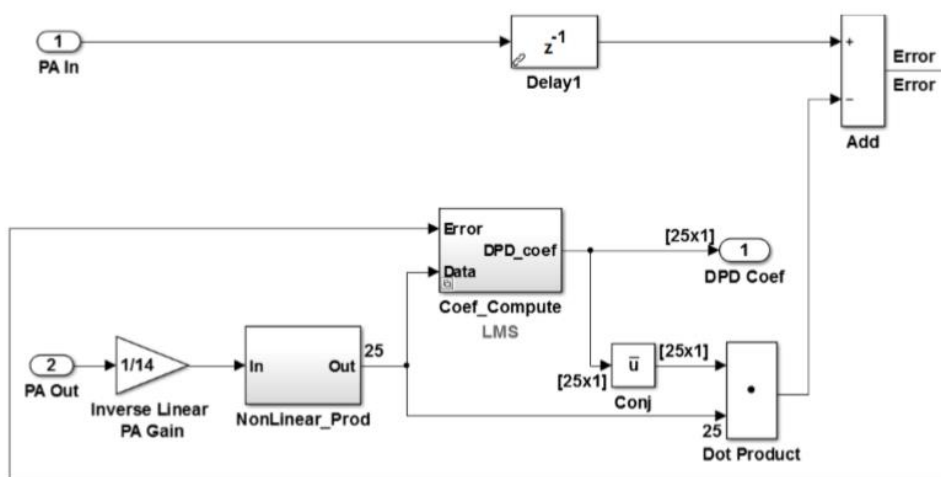


Figure 7.12 Coefficient Computation Subsystem Model [6]

Adaptive DPD model has two types of algorithm-

- For learning the coefficients and
- Other one for implementing them.

In figure 7.12 shows the summation of non-linear prod and the coefficient computation subsystems.

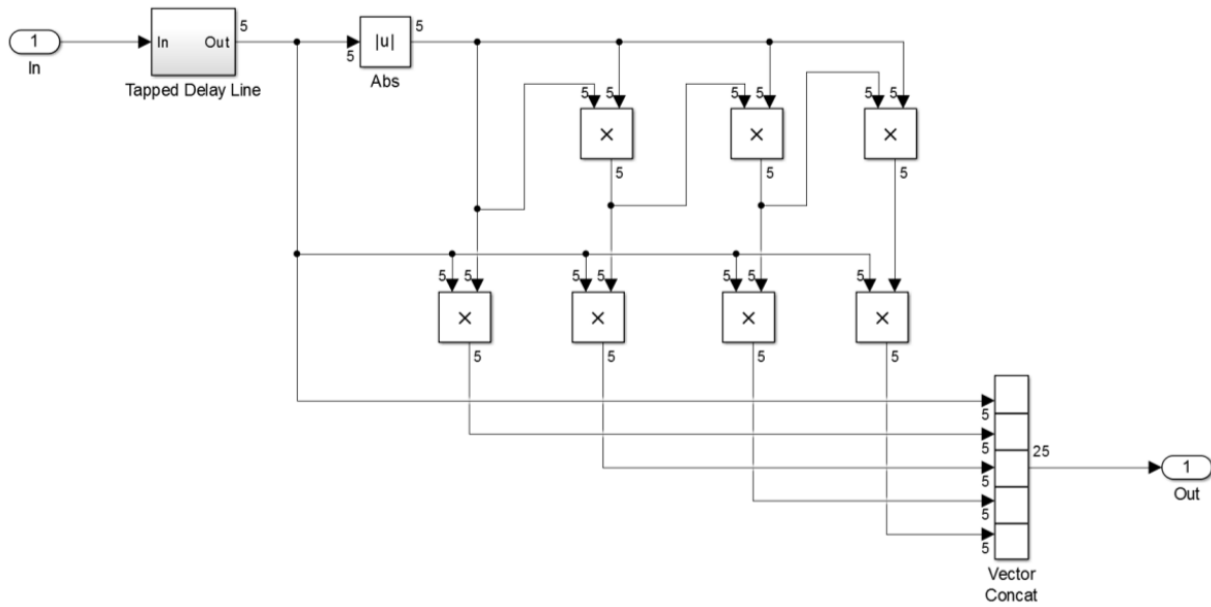


Figure 7.13 The Nonlinear Prod Subsystem Implements The Non-Linear Multiplies [6]

The DPD coefficients, d_{km} , are computed on-the-fly using one of two possible algorithms, the LMS algorithm or Recursive Predictor Error Method (RPEM) algorithm as shown in Figures 7.14 and 7.15 respectively [6].

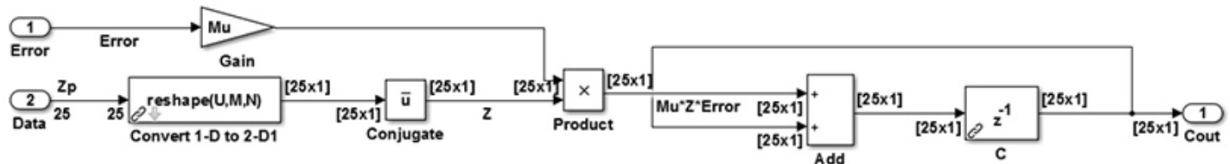


Figure 7.14 The LMS-Based Coefficient Computation is Relatively Simple In Terms of The Required Resources [6]

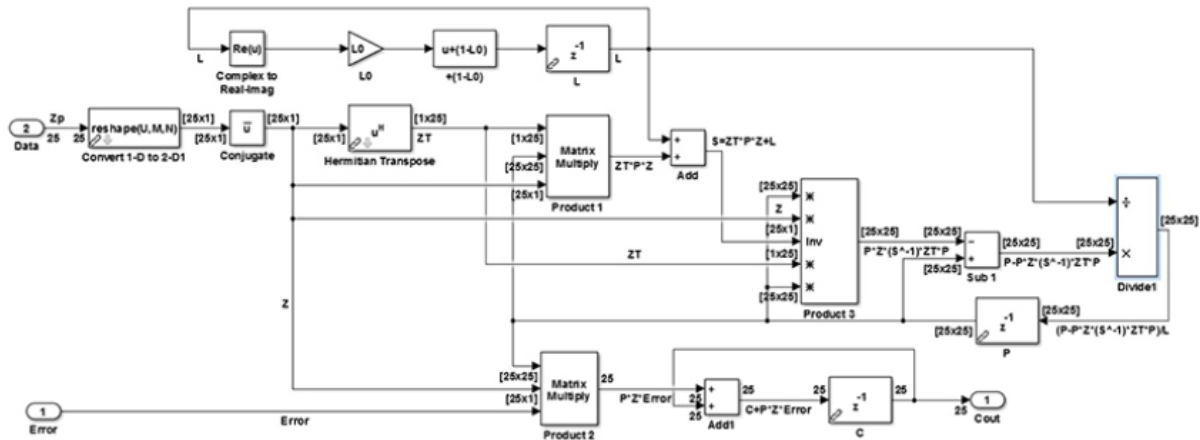


Figure 7.15 The RPEM-Based Coefficient Computation is Far More Complex Than The LMS [6]

To form an error signal, both LMS and RPEM based coefficient computation algorithm is required. The difference of the error signal is all about the measured power amplifier input value and estimated power amplifier input value. Both algorithm tends to push error to zero and makes the possible convergence on the DPD coefficient. The output behavior of adaptive DPD with PA output and only PA output have been shown in Figure 7.16. The RPEM method shows remarkably better results in both spectral growth reduction and rate of convergence.

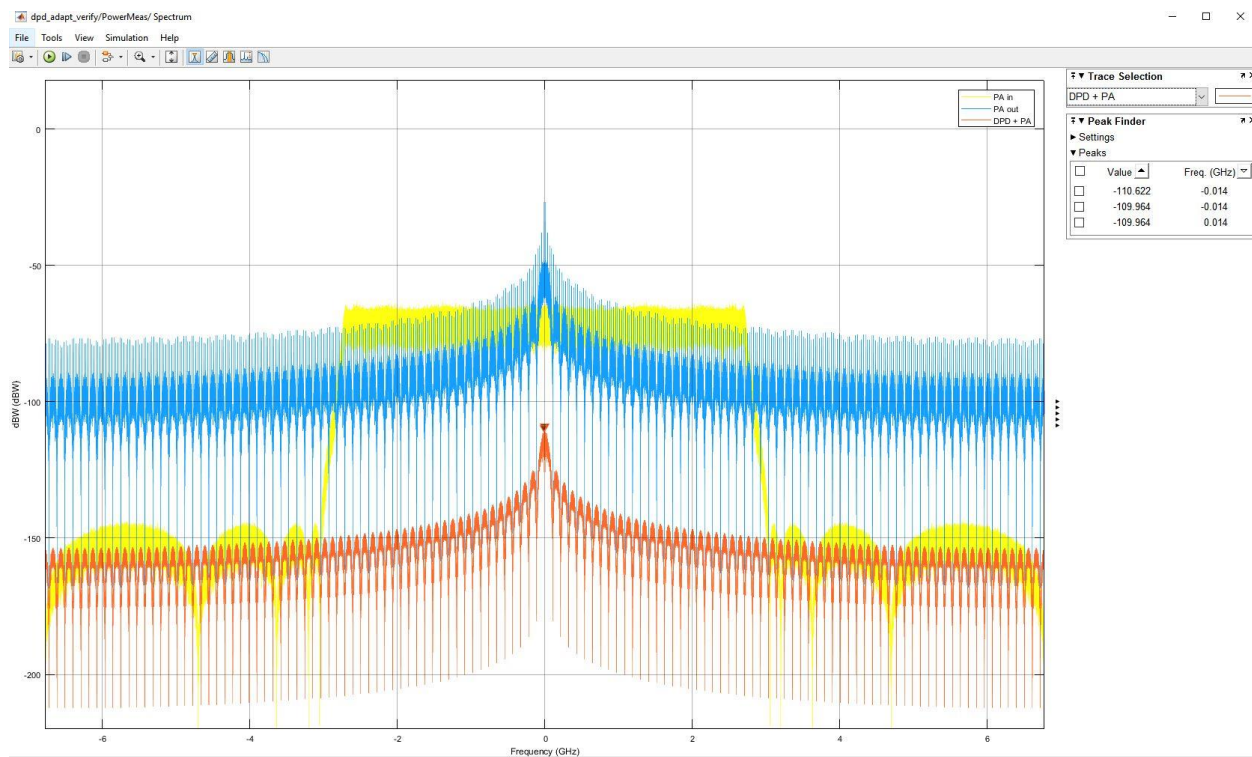


Figure 7.16 Adaptive DPD Design Output Behavior

For Adaptive DPD Design PA gain is 110 dbW at the expense of significant spectral regrowth and in-band distortion.

CHAPTER 8. CONCLUSION

High efficiency and linear PAs are essential components in wireless communications system. There is however a tradeoff between efficiency and linearity: high efficiency PAs behave nonlinearly and linear PA present low efficiency. Due to fulfill the efficiency and linearity requirements, DPD is often used. This thesis has contributed to different aspects of the linearization of PAs using DPD.

The synthesis of a predistorter consists mainly in two tasks: the selection or design of models to be used in the predistorter and the identification of the parameters of the model. For the parameter identification, techniques such as pth-order inverse, ILA and DLA have been proposed in the literature. Among them, ILA is the most widely used because it reduces the identification process to an iterative inverse-modeling process. But the adoption of ILA introduces a critical issue known as gain normalization. In the literature, there is not a clear consensus on how the normalization gain must be computed. We have investigated this issue and have shown that normalization gain affects the linearized PA's output power. Consequently, it computes an extra degree of freedom which should be selected carefully. To overcome this issue, we have proposed a variation of ILA that terminate the normalization gain block and uses the desired output response as input to the predistorter. Experiments showed that the proposed ILA does a better control of the output signal from the linearized PA, since this signal depends on the input signal to the predistorter, not on the normalization gain's accurate selection.

An important issue encountered in DPD has been that the output signal from the predistorter that linearizes the PA is unknown. For this reason, the parameters of the predistorter are identified using techniques such as ILA and DLA. To overcome this issue, we have proposed a novel parameter identification technique based on ILC. To this end, we have designed an ILC scheme for PA linearization. Experimental results showed that the proposed ILC scheme can identify the best linearizing input signal. The experiments also showed that proposed calculating technique can obtain better linearization performance than ILA and DLA when high levels of measurement noise are present and when the PA is in high condensation. The ILC scheme and the identification technique presented here have the capable to enable the design of model structures for digital predistorters.

The introduction of dual-input transmitter architectures has introduced new challenges to digital predistortion. In this work, we have studied linearization aspects of dual-input Doherty PAs and identified some of the associated challenges. We have experimentally shown that the large bandwidth of the control signals and the limited sampling rates used in the linearization are two the main factors that limit their linearity performance. We have addressed this issue with a novel linearization scheme that gives a better linearity compared to the existing schemes for a given sampling rate.

In summary, we have developed new concepts and methods for DPD linearization of PAs. These contributions are expected to make a broad impact on the realization of energy efficient and linear radio transmitters for emerging and future wireless systems.

CHAPTER 9. FUTURE WORK

We implemented the software part in this thesis. However, the noise output is a bit higher, in future it is implementable the hardware part. We have a PXIe 1075 chassis in our RF/Microwave lab. Firstly, we must measure the strength and give a physical form of our E-class PA and then PXIe 1075 chassis and spectrum analyzer could help us to compute the AM/AM and AM/PM. Besides we could get the linearize behavior of our PA after Digital Pre-Distortion.

Moreover, Linearized E-class PA's are very suitable for 5G communications. They can be designed in an extreme dense size to minimize the cost. Moreover, mm-wave power amplifiers have high efficiency at peak and the amount of power withdraw that are relative to those while operates at low frequencies. The greater number of RF signal bandwidth in 5G Frontends mean that DPD (Digital Predistortion) will be completely different from their 4G predecessors.

CHAPTER 10. REFERENCES

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