SUPPLY CHAIN RELATIONSHIP FOR QUALITY IMPROVEMENT: EMPIRICAL TESTS ON PRINCIPAL AGENT THEORY

by

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Dedicated to my parents, my wife Xiaoxiao Shi, my daughters Cara and Vera, my advisors and many more who help me through this journey.

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e_M	Manufacturer's quality improvement effort level
$e_M^*(\lambda)$	Manufacturer's optimal quality improvement effort level
e_S	Supplier's quality improvement effort level
$e_S^*(\quad)$	Supplier's optimal quality improvement effort level
$Q(e_M, e_S)$	Quality level of a product measured by defective rate
$Q^*(\quad)$	Optimal quality level
θ_0	Initial quality level of a product
θ_M	Parameter of manufacturer's quality improvement effort
θ_S	Parameter of supplier's quality improvement effort
θ_J	Parameter of manufacturer and supplier's quality improvement joint effort
λ	Percentage of the warranty cost shared to supplier
η_M	Marginal effort cost for the manufacturer
$\check{\eta}_M$	Estimation of parameter η_M in the optimization model from empirical data
η_S	Marginal effort cost for the supplier
$PP100$	Problem Per 100 vehicles in JD Power Initial Quality Studies
$PP100_i$	Problem Per 100 vehicles data in year i
$\widehat{PP100}$	Predicted Problem Per 100 vehicles from model
WRI	Working Relation Index in manufacturer-supplier relationship survey
WRI_i	Working Relation Index data in year i
$\widehat{WRI}$	Predicted Working Relation Index from model
WSR	Warranty Sharing Ratio to measure percentage of warranty costs shared to the supplier
WSR_i	Warranty Sharing Ratio data in year i
$\alpha_1 = \frac{\theta_S}{\eta_S}$	Parameter 1 in the regression model 1
$\hat{\alpha}_1$	Estimation of parameter α_1 in the regression model 1 from empirical data

$\breve{\alpha}_1$	Estimation of parameter α_1 in the optimization model from empirical data
$\alpha_2 = \frac{\theta_J}{\eta_S}$	Parameter 2 in the regression model 1
$\hat{\alpha}_2$	Estimation of parameter α_2 in the regression model 1 from empirical data
$\breve{\alpha}_2$	Estimation of parameter α_2 in the optimization model from empirical data
$\beta_0 = -\frac{\theta_S}{\theta_J}$	Intercept parameter in the regression model 2
$\hat{\beta}_0$	Estimation of parameter β_0 in the regression model 2
$\beta_1 = \sqrt{\frac{\eta_S}{\theta_J \eta_M}}$	Parameter 1 in the regression model 2
$\hat{\beta}_1$	Estimation of parameter β_1 in the regression model 2 from empirical data
$\gamma = \sqrt{\frac{\eta_M \eta_S}{\theta_J}}$	Parameter in the regression model 3
$\hat{\gamma}$	Estimation of parameter γ in the regression model 3
N	Number of data points
ω	Weight parameter to link the multiple objective optimization problem
SSE	Sum of Squared Error

ABSTRACT

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Principal agent theory is widely used to model supply chain relationship, in which a supplier is the agent and a manufacturer is the principal. Both the manufacturer and supplier can influence product quality and consequentially share costs of product failures. Rich theoretical results under the principal agent model framework have been accumulated in the last two decades, but empirical evidence on whether the Stackelberg's leadership game truly imitates practical supply chain relationship remains unfound. We study the domestic automobile industry in the last decade and provides to our best knowledge the first empirical evidence to assess the validity and practicality of principal agent theory and draw the implications of principal agent theory on supply chain relationship costs. Our empirical results suggest that Japanese OEMs behave more like principal agent theory suggests than the US OEMs in general and thus gain significant benefits in terms of marginal effort costs in motivating suppliers' quality improvement behaviors and reducing overall manufacturer's quality costs. Specifically, Toyota behaves closest to the optimal solution in the principal agent theory and therefore has the lowest manufacturer effort costs in improving product quality and achieves the overall lowest manufacturer's quality costs in supply chain. Honda and Nissan are ranked 2nd and 3rd in terms of principal agent behaviors, but their marginal quality improvement effort costs are 33% and 61% higher than Toyota, and their total manufacturer's quality costs are both around 17% higher compared to industrial leader Toyota by our estimate. US OEMs GM, Ford and Chrysler are believed to behave inconsistent to principal agent theory suggest, and consequently suffer a much higher marginal effort cost in motivating supplier's quality improvement than Toyota as well as the overall manufacturer's quality costs. GM and Ford are estimated doubled marginal effort costs than Toyota, and Chrysler is even higher at 1.6 times. GM's overall manufacturer's quality cost is 24% higher than Toyota, Ford is around 31% higher and Chrysler is around 48% higher. Our analysis gives a new perspective from principal agent

theory to explain why Japanese OEMs especially Toyota has a better supply chain quality costs than US OEMs as literature and consensus suggested. In addition, we contribute in literature by linking the principal agent theory with automotive industrial data and first ever empirically validate the legitimacy of principal agent theory in modeling manufacturer-supplier relationship and quantitatively derive practical conclusions on marginal effort costs and manufacturer's total supply chain quality cost implications. To guarantee the robustness of the empirical results, various sensitivity analyses are conducted and our main conclusions remain unchanged.

1. INTRODUCTION

In the past several decades, we see several trends in manufacturing industry. First, companies become more and more specialized and do everything companies are disappearing. When Boeing first builds its commercial aircraft Boeing 707 programs, they almost built everything inhouse from big body and wings of the airplane to small screws and nuts. At that time, vertical integration is what business schools teach us to achieve the overall efficiency and cost advantage by doing everything under one roof. However, when Boeing started to develop Boeing 737 series in the late 60s and early 70s, they outsourced 35%-50% of the components to suppliers mainly in manufacturing and production phases. The figure reached more than 70% in the Boeing's Dreamliner 787 series (Tang C.S., Zimmerman J.D., Nelson J.I., 2009). Approximately 50 tier-1 strategic suppliers, thousands of tier-2, tier-3 suppliers contributed in the Dreamliner 787 programs from designing the parts, developing products and manufacturing the components to testing and validation. Companies doing everything are suffering from not specializing in anything. The last American do everything company General Electric experienced multiple organizational restructuring in the past decade, tumbled in stock price crashes, lost $500 Billion market value in the last 18 years (Clough R., 2018) and was rumored to be broken into multiple independent entities. Second, supply chains become longer, more complicated and more globalized than ever. In Japan only, Toyota is buying about 2 Billion units, 150,000 different kinds of components from more than 200 Japanese tier-1 suppliers in a year. In US, General Motors budgeted $90 billion dollars in US annual purchasing from 20,000 suppliers around the globe (Amend J.M., 2015). The all-new 2019 Chevrolet Silverado pickup truck buys body control modules from Japanese supplier Denso, the prismatic inside mirrors from Canadian supplier Magna, the serpentine belts from German supplier Continental, the front differentials from British supplier GKN (IHS Markit, 2018). In aircraft industry, Boeing 787 Dreamliner purchases wingtips from Korean supplier KAL-ASD, landing gears from British supplier Messier-Dowty, horizontal stabilizer from Italian supplier Alenia, forward fuselage from Japanese supplier Kawasaki, cargo access doors from Swedish supplier Saab and passenger entry doors from French supplier Latecoere (Tinseth R., 2013). Manufacturing is becoming more of a team sports game with team players coming from all around the world. Third, suppliers could become both angels and evils to the manufacturers. Traditionally, suppliers are contract manufacturers who produce certain components or products designed and engineered by the manufacturer and do not hold much intellectual properties. However, as companies in the supply chain become more and more specialized, suppliers have grown their capabilities in the areas like designing, engineering, testing, validation and even research and developments. For example, German automotive supplier Continental spent 3.5 Billion US dollars in research and development in 2017, almost reaching half of General Motors' 7.3 Billion spending. As suppliers become more and more capable, they bring bigger and bigger impacts on the supply chain. The manufacturers could benefit a lot from a good manufacturer-supplier relationship and could count on suppliers to deliver high quality products, help reduce costs, increase efficiency, develop new technologies and drive innovation. However, manufacturers could also suffer significantly from a bad manufacturer-supplier relationship which could lead to low product quality, supply chain disruptions, financial and reputational damages.

1.1 Benefits from a Good Manufacturer-Supplier Relationship

Companies focusing on good manufacturer-supplier relationship benefit significantly from suppliers and could count on suppliers to deliver high quality products, help reduce costs, increase efficiency, develop new technologies and drive innovation.

1.1.1 High Product Quality

A high-quality product such as a modern vehicle or a business aircraft is composed of hundreds and thousands of high-quality components and parts where majority of them are produced by suppliers. A model combustion engine vehicle has more than 1,800 separate components such as engine, transmission, steering wheel and 30,000 parts if counting every part down to the smallest bolts, screws and bearings. All the parts and components use different raw materials and different manufacturing processes. Making good parts requires a lot of efforts from the suppliers. In a good manufacturer-supplier relationship, suppliers are willing to help manufacturer improve supply chain quality from different aspects. (1) Suppliers are willing to spend more money on internal quality and process control to assuring the delivery of high quality components and parts. General Motors initiated a Strategic Supplier Engagement program in 2014 to improve the trust between the manufacturer and its suppliers. They changed their ways of engaging suppliers from traditionally squeezing suppliers to more collaborations, and the change dramatically improved their relationship with its suppliers (Trebilcock B., 2017). They rewarded top suppliers in quality control by providing more business and profitability opportunities and the outcome is a significant progress on GM's quality performance in recent years' JD Power Initial Quality Studies and Consumer Reports Reliability Survey. (2) Suppliers can help manufacturers solve emergent quality issues. As suppliers become more and more specialized, manufacturers sometimes must rely on suppliers' expertise in certain areas to solve emergent issues. For example, infotainment glitches haunted many automakers in recent Consumer Reports Reliability Survey and became the biggest complaints that impacted customers' ownership experience and brand images (Bond Jr.V., 2014). Most of the infotainment systems developed by automakers have sub 50% satisfaction rate and even the best system in the market is full of bugs (Consumer Reports, 2016). Software development is not an area that traditional automotive manufacturers have expertise on and they rely on software powerhouses like Google and Apple to help identify and resolve issues on their vehicles' infotainment systems. (3) Suppliers can help manufacturers set up quality management standards on parts and components. Boeing adopted a new sourcing strategy in the 787 programs to give suppliers more powers to help set up quality standards (Tang C.S., Zimmerman J.D., Nelson J.I., 2009). Due to the lack of expertise in certain areas, manufacturers need to collaborate with suppliers more on quality management system to ensure the right quality standards can be set up for parts and components. (4) Suppliers can share quality data to manufacturers. Under a collaborative and trusty relationship between manufacturers and suppliers, quality data could be shared from downstream suppliers to upstream manufacturers to help assembly process and issue detections. For example, suppliers can share their parts and components design information such as the Failure Modes and Effects Analysis (FMEA) with manufacturers to help improve knowledge sharing and prevent quality defects. In the future of Internet of Things (IoT) era, suppliers can also share their production, manufacturing, testing and validation data to the manufacturers to proactively prevent quality defects, fastened the root-cause analysis and issue detections. Manufacturers maintaining a good relationship with their suppliers could bear the most fruits on product quality.

1.1.2 Cost Saving

Since manufacturers spend majority of their money on purchasing and sourcing, suppliers can help manufacturers to save costs significantly if the manufacturer and the supplier can do it in a trusty and collaborative way. General Motors saved $5.5 billion in purchasing, manufacturing and administration expenses between 2015 and 2018 from its 20,000 global suppliers (Burden M., 2016). In a specific example, GM engineers worked with seating supplier Lear on GM's current full-size Chevrolet Silverado and GMC Sierra truck platform to see if the quality can be improved and costs can be saved. They worked collaboratively together to tore down the competitor vehicles to look at different innovation technology, design ideas and specifications to optimize costs without sacrificing quality. In a manner of an afternoon after working together for 6 hours going through everything together, executives from GM and Lear came up with a $20 million saving plan which could potentially reach $50 million in the life cycle. This shows how the suppliers can help manufacturers save costs in a significantly way if a good manufacturer and supplier relationship can be maintained. Another example in Tesla, the struggling carmaker is counting on Model 3 suppliers to refund a portion of what the electric-car company has spent previously to achieve profitability. This is nothing unseen in the manufacturing industry. If the OEMs make money, the suppliers make money, too. If the suppliers can save costs, the OEMs benefits from that as well. As purchasing parts, components and subsystems from suppliers takes up almost 80% of manufacturers' annual spending, suppliers are good partners for manufactures to take out costs and achieve better financial performance.

1.1.3 Improve Efficiency

Economics theory tells us that specialization shortens learning curve and increase productivity. Suppliers can help manufacturers dramatically improve supply chain efficiencies if manufacturers and suppliers are engaging in a collaborative and trusty relationship. When Boeing first developed Boeing 737 programs, they produced 50%-65% components, parts and subsystems inhouse and need 30 days to complete the final assembly process. Compared to the new Boeing 787 Dreamliner series, Boeing relies on 50 tier-1 suppliers and thousands of tier-2 and tier-3 suppliers to accomplish 70% of the total work. Then they can concentrate on only 30% of the core jobs and that helps reduce the final assembly process to only 3 days (Tang C.S., Zimmerman J.D., Nelson J.I., 2009). On the other hand, as viewed as the disruptor to the century old automotive industry Tesla takes a different path as a much more vertically integrated OEM than Ford, GM and Chrysler. Tesla does not trust suppliers to build key components for them and frequently engaged into disputes with suppliers in terms of quality, price, financial responsibilities (Higgins T., 2018) and delivery time frames (Gene, 2017). Therefore, Tesla chooses to produce components like seats, battery packs inhouse when all other automakers choose outsourcing to suppliers like ZF, Lear, and LG Chem. Consequently, Tesla suffers low productivity and inefficiency to achieve its internal production goal of 5,000 Model 3 vehicles per week or equivalently around 250,000 units per year in its Fremont factory in California which was used to be jointly owned by Toyota and GM producing sedans at a rate of 400,000 units per year, almost twice as more productive as Tesla is producing right now. Specialization and outsourcing can significantly improve the supply chain efficiency if manufacturer can engage in a collaborative and trusty relationship with suppliers.

1.1.4 Drive Innovation

As manufacturing industry is heading towards a more specialized, more intelligent future, innovation is no longer occurring from top to bottom. Suppliers are becoming big force to drive innovations. In 1989, suppliers contributed to less than half of the part designs. However, that figure reached up to 70% in 2011 (Kapadia S., 2018). In 2016, General Motors launched the long-range mass market all electric vehicle Chevrolet Bolt, a revolutionary product which could travel 238 miles per single charge. However, when we looked deep into this innovative product, we found that Korean battery supplier LG Chem was actually the unsung innovative driver behind the scene. LG Chem supplied electric drive motor, on board charger, electric climate control system compressor, power inverter module, high power distribution module, battery heater, accessory power module, battery cells and pack, power line communication module, instrument cluster and infotainment system to Chevrolet Bolt from design to engineering, manufacturing and testing (Ayre J., 2016). Similar trend happens in autonomous vehicle space. Big technology companies become the innovation powerhouse to disrupt the traditional manufacturing business. Google's autonomous vehicle unit Waymo supplied the whole autonomous system to enable Chrysler Pacifica driving autonomously. Chip maker Nvidia provided the Graphical Processing Units (GPU) to empower the onboard computing and sensor provider Mobileye supplied the key sensor systems to Tesla Autopilot self-driving systems. Suppliers in a lot of spaces are leading the technology innovation and changing the landscape of traditional manufacturing business. (Henke Jr. J.W., Zhang C., 2010) found that when manufacturers collaborated with their suppliers to build trust, reduce relational stress and maintain a good relationship, suppliers would increase their innovation-related activities and supplied the manufacturers with their best cutting-edge technologies to help the manufacturer compete in innovation.

1.1.5 Consolidate Supply Chain

A good manufacturer and supplier relationship helps manufacturer build up a solid supply chain network and reduce the supply chain risk. Traditionally, American manufacturers deal with suppliers with competitive bidding on contracts and their relationship is purely economic driven. Suppliers have uncertainties on winning or losing bids of each contract, therefore, they are reluctant to make big investments. On the other side, manufacturers may constantly switch suppliers with lowest price bid in each product life cycle but do not know what they could get from different suppliers. This creates huge fragility on their supply chain. In comparison, Japanese manufacturers do not like using price as the leverage to deal with suppliers and they would like to build up a long-term trusty relationship by rewarding good suppliers with long term contracts and help suppliers expand their business (Taylor C.R., Wiggins S.N., 1997). In return, Japanese manufacturers demonstrate a much higher resilience on supply chain performance than American manufacturers and their suppliers are much more willing to help the manufacturers consolidate their supply chain.

1.2 Harms from a Bad Manufacturer-Supplier Relationship

A bad manufacturer-supplier relationship will create long lasting damages to the manufacturers in a lot of aspects such as product quality, financial and reputational damages and supply chain disruptions. That is the reason why most of the American companies, traditionally handling their suppliers purely based on contracts, are gradually changing their behaviors to encourage long term relationships, create transparent and collaborative cultures, and incentivize suppliers by repeat purchase and help suppliers get new business (Trebilcock B., 2017).

1.2.1 Product Quality Risk

As manufacturers rely more and more on suppliers to produce parts and components, they also face higher and higher supplier quality risk. National Highway Traffic Safety Administration (NHTSA) recall database showed that there was a significantly higher chance that suppliers would be identified in a product recall now than in past years. For example, in NHTSA recall data 70 percent of recalls in 2015 were attributed to suppliers, where only 50 percent were supplier recalls in 2012 and only 15 percent noted in 2008 (Steinkamp, 2016). As suppliers take more and more responsibility in the supply chain, it shoulders higher and higher responsibility in quality control.

Suppliers' misconducts or lack of rigorous quality control practice put manufacturers in huge risk. Takata, the 2^{nd} largest airbag producer with market value \$3.6B in 2006, supplies airbags to 19 global automotive OEMs. It holds 22% market share in automotive airbags in 2014 and 2015 and was a darling of investors delivering 368% return on investment during the 2009-2013 period. However, a catastrophic airbag recall strike hard on Takata and almost all the OEMs who sourced Takata airbags. Since 2014, NHTSA issued multiple recall commands to 19 automakers over 60 million vehicles containing faulty Takata airbags which was linked to 11 deaths and 184 injuries. The recall wiped out almost 99% of Takata's market value from January 2014 (Sharma G., 2018). What even worse is that according to New York Times disclosure Takata first noticed these dangerous defects internally with its airbags in early 2004 but decided not to alert federal safety regulators. Instead, Takata executives concealed the results and deleted testing data and disposed the defective airbag inflators in trash (Tabuchi H., 2014). Because of Takata's misconduct in quality management, totally 17 automakers recalled their products in 2016. Among them, Honda recalled 5.4 million vehicles spreading 15 models in Honda and Acura brands. Toyota recalled 1.9 million vehicles and Ford recalled around 2 million. Almost all GM brands from Cadillac, Chevrolet, GMC, Pontiac, Saab and Saturn suffered in this Takata recall and almost all automakers suffered significant brand damage and stumbled to rescue their customer relationship.

1.2.2 Financial and Reputational Damages

A bad manufacturer-supplier relationship will lead to damages on manufacturer's finance and reputations. In 2007, the federal Consumer Product Safety Commission mandated a recall of more than 25 million unsafe toys and kids' items worth millions of dollars. Many of these toys contained extremely high lead levels and were sourced to Chinese contract manufacturers who cheated on product quality management to save costs, such as changing their business registrations to avoid the required level of inspections, allowing pre-qualified components to be used in final product manufacturing directly (Bapuji, H., Beamish P., 2008a), (Bapuji, H., Beamish P., 2008b). This recall cost Mattel \$30 million for consumer compensation, \$50 million on consumer lawsuits and wiped out 13% of its market value in a year (Tang C.S., 2008). Airbus is a European champion in the aircraft industry and was created in 1969 as a loose consortium of European aerospace companies from multiple European countries like Germany, France and Great Britain. As each company from different country possesses their unique culture, the relationship between Airbus and its suppliers are hindered by their culture difference. Airbus flagship program A380 was a dramatic failure not only because its high costs but also it demonstrated how a bad supply chain relationship could hurt the manufacturer both financially and reputational-wise. The Wall Street Journal reported that at the time of the first A380 test flight in 2005, Airbus managers discovered that French and German designers had used incompatible software in wiring the aircraft which ultimately delayed the delivery of the products to the customers and triggered billions of dollars of delay penalties. Also, blamed to the poor supply chain quality performance, the first group of A380 only served one decade and then got sold for scraps, comparing to a normal large aircraft which could serve for several decades before retiring. Due to quality and delivery concerns, many airlines cancelled their A380 orders or switched to other Airbus models. Because of the lack of demands for A380, Airbus had to terminate this $17 Billion-dollar program in 2021 (Wall R., Michaels D. , 2019). This dramatic failure left Airbus behind its competition with Boeing and badly damaged its reputation as an European aircraft industrial champion.

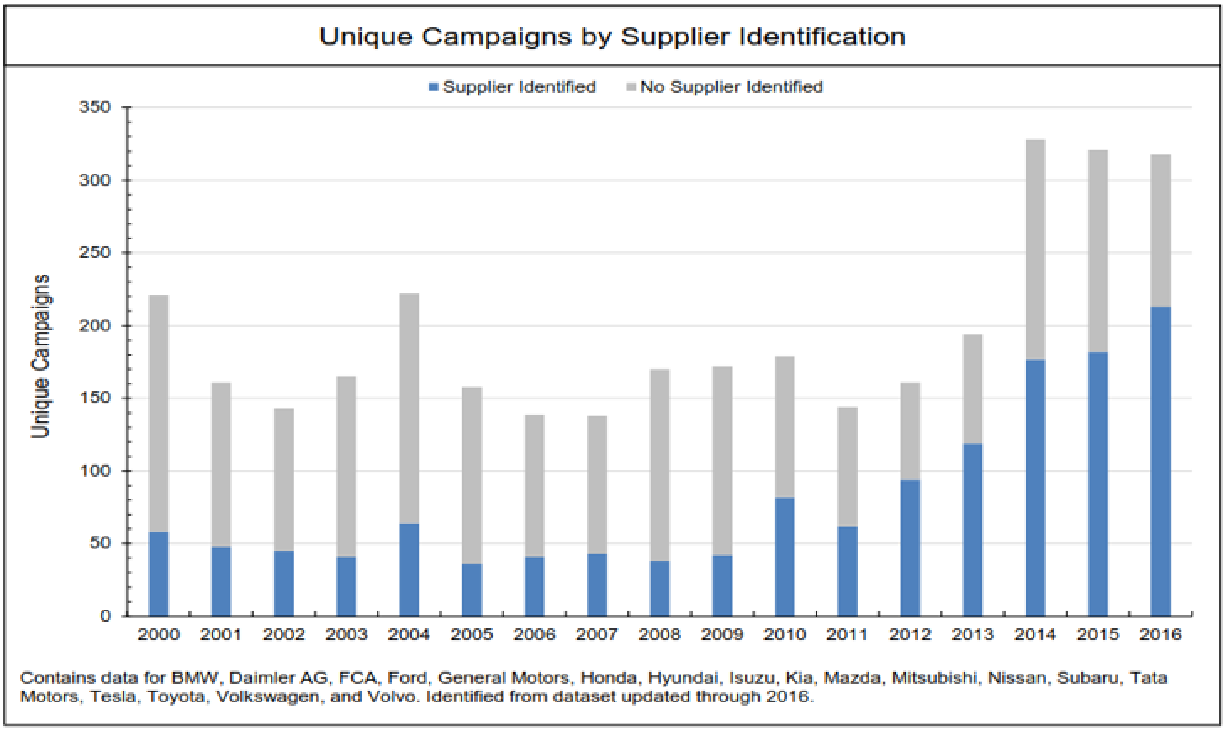
1.2.3 Supply Chain Disruptions

In addition to quality and financial damages, a bad manufacturer-supplier relation will also cause more disputes between manufacturers and suppliers and lead to more supply chain disruptions. Clark-Cutler-McDermott (CCM) was a small Massachusetts-based auto supplier with 115 years history and 45 years of collaboration with General Motors. CCM supplies GM 175 parts and was the exclusive supplier of some interior and acoustic insulation parts. CCM had been named GM’s “Supplier of the Year” four times in the last seven years, and 80 percent of CCM’s revenue came from GM (Burden M., 2016). However, the relationship between CCM and GM started to sour as CCM absorbed losses of $12 million since 2013. Later in 2016, CCM accused GM for aggressive cost cutting which caused CCM losing $30,000 a day to supply GM and filed for bankruptcy protection on July 7. CCM accused GM “doing nothing more than scheming surreptitiously to protect its own interests through a calculated plan to extract the value of CCM for its own gain.” and threatened to shut down GM assembly plants in 19 North America sites which could lead to tens of millions of dollars damage. As GM adopted the Just-In-Time strategy and did not carry significant amount of inventories of the parts that CCM supplied, on July 13, after winning the approval of a federal bankruptcy court judge in Massachusetts GM purchased the supplier’s production tooling, equipment and inventory to keep its North America factories running (Gleason

S., 2016). Having a bad relationship with suppliers will risk the manufacturer with supply chain disruptions.

1.3 Suppliers Key to Product Quality

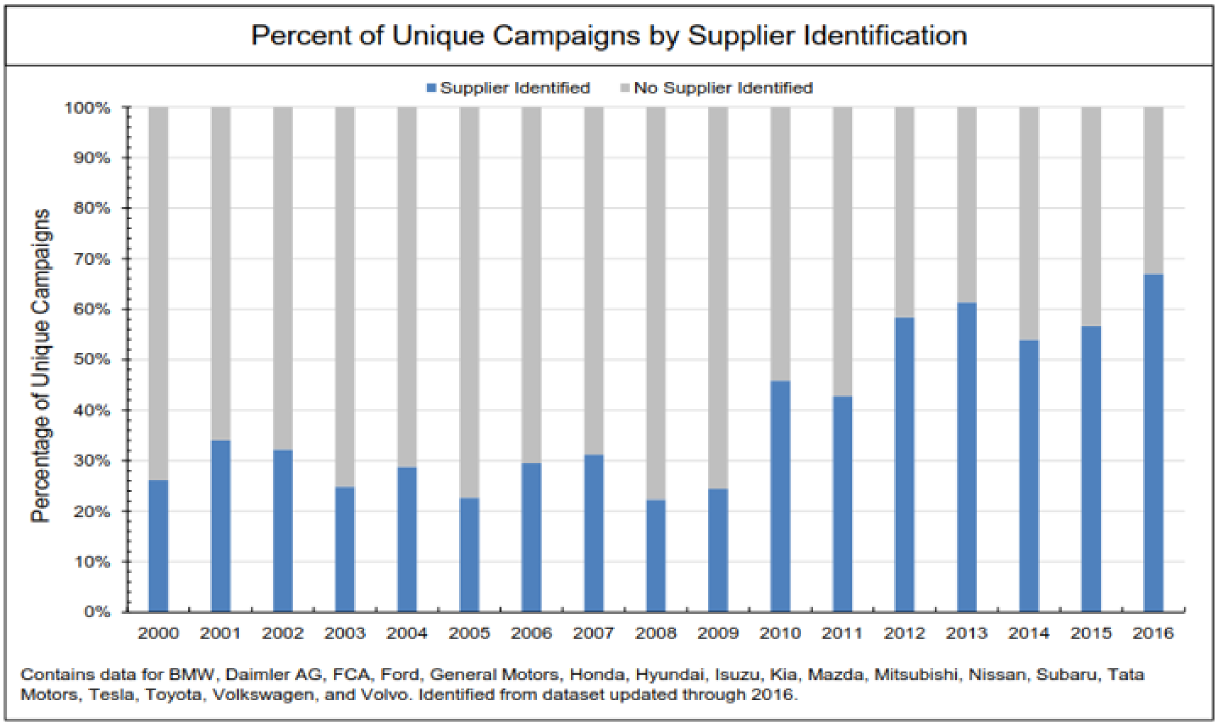
As we have discussed both the benefits and harms that the suppliers could bring to the manufacturer, we can see that suppliers are key to final product quality and the relationship with suppliers will significantly impact the quality of the products that the manufacturers can receive from their suppliers. Financial damages, reputational damages and supply chain disruptions are all subsequent consequences for a bad manufacturer-supplier relationship and poor product quality. (Steinkamp, 2016) studied the U.S. Department of Transportation National Highway Traffic Safety Administration (NHTSA) recall database and the Part 573 Letters that OEMs are required to report during a recall to identify the manufacturer of the defect component. They found that there is a significant increase of recalls with supplier identification in recent years. Before 2009, the recalls with supplier defects are at a level of 50 cases quite stably. However, starting from 2010 the recalls with supplier defects are on a rapid rise to more than 200 cases in 2016.



Source: NHTSA Recall Data and 573 Letters

Figure 1.1 NHTSA Unique Campaigns with Supplier Identification

If we use the unique campaigns by supplier identification divide by total unique campaigns in that year, Figure 1.2 shows that the percentage of unique campaigns by supplier identification is rising from around 20%-30% before 2009 to almost 70% in 2016. This rapid increase of recalls due to supplier defects attracted huge attentions from both academic researchers and industrial practitioners. People looked deeply into the manufacturer-supplier relationship and supply chain management to find out the best way to handle the relationship between manufacturers and the suppliers to improve the incoming product quality.



Source: NHTSA Recall Data and 573 Letters

Figure 1.2 NHTSA Percentage of Unique Campaigns by Supplier Identification

In the next two sections, I will just summarize the main work that has been done in academic society as well as by the industrial practitioners. I will postpone the detailed literature review to the next chapter.

1.4 Manufacturer-Supplier Relationship for Quality Improvement: Theoretical Research

Academic researchers like to simplify the complex supply chain so that they can study the relationship between manufacturer and its suppliers in a nice and elegant mathematical framework. Normally, they make assumptions like followings to derive insights and draw conclusions. However, most of these assumptions are fundamentally flawed and not really representing the reality.

- One manufacturer and one supplier (at most two suppliers): this is not a terrible assumption in an academic world. In academic world, people like to focus on the major objective and ignore the minors. One manufacturer and one supplier assumption will dramatically reduce the complexity of the supply chain but it can still keep the core relationship on the table. However, in a realistic world no manufacturer is living on a one on one relationship and very often manufacturers will deal with hundreds and thousands of suppliers at different tiers, from different countries, with different culture and in different business. Manufacturers behave differently when dealing with different suppliers. Without considering the complexity of relationship that the manufacturer is handling, conclusions drawn from one manufacturer and one supplier setting will be very limiting.

- Both manufacturer and supplier are self-interested profit maximizers: game theory and its derivative models are the most popular approaches to analyze the relationship between the manufacturer and the supplier. However, the fundamental assumption underneath all those game theoretical models are self-interest and profit maximization. It is true that all independent firms, no matter manufacturers or suppliers, are profit maximizers. However, supply chain relationships are not always self-interested and self-interested behaviors are not always maximizing its profits. In a lot time, maintain a good relationship by scarifying some short-term profits might help both manufacturers and suppliers in the future. In many cases collaborations can benefits both parties like cost reductions and technology/data sharing. Game theory models focused too much on modeling conflicts but not the benefits from collaborations.
- Conceptual variables: relationship is a conceptual term that is hard to measure. John Henke and his company Planning Perspective Inc. defined 5 major categories and 16 sub-categories to measure the relationship between automotive OEMs and their suppliers (Zhang C., Henke Jr. J.W., Griffith D.A., 2009). However, in an academic setting, researchers like to use variables with conceptual meanings to capture the essential relationships. For example, (Kim S., Netessine S., 2013) uses variables called collaborative efforts to represent the resources that manufacturer and supplier invest in cost reduction. (Iyer A.V., Schwarz L.B., Stefanos A.Z., 2005) chooses a variable called buyer’s effort to represent the resources that the manufacturer allocates to the supplier. (Zhu K., Zhang R.Q., Tsung F., 2007) defines a term called quality improvement efforts to represent suppliers’ investment and activities on quality control. All these conceptual variables are aimed to simplify the complexity of mathematical modeling, but most of them are lack of concrete

measurement of what it really represents and how industrial practitioners can use real data to measure these metrics.

Academic research on the topic of manufacturer-supplier relationship for quality improvement contributes a lot in discovering managerial insights and providing big picture guidance. However, all the insights and conclusions are derived from models which have fundamental flaws on model assumptions, over-simplification, and used variables that lack of practicality. All these flaws hindered the adaptation of the industrial practitioners and its impacts to decision makers.

1.5 Manufacturer-Supplier Relationship for Quality Improvement: Industrial Practices

Industrial practitioners look at this topic from a bottom to top approach relying on several specific metrics to measure the relationship which is often biased and lack of systematic approach. For example, suppliers may measure the manufacturer-supplier relationship simply based on loyalties.

"Honda is a demanding customer, but it is loyal to us. [American] automakers have us work on drawings, ask other suppliers to bid on them, and give the job to the lowest bidder. Honda never does that." —CEO, industrial fasteners supplier to Ford, GM, Chrysler, and Honda, April 2002 (Liker J., Choi T.Y., 2004)

Suppliers may measure the manufacturer-supplier relationship solely based on cost pressures.

"The Big Three [U.S. automakers] set annual cost-reduction targets [for the parts they purchase]. To realize those targets, they'll do anything. [They've unleashed] a reign of terror, and it gets worse every year. You can't trust anyone [in those companies]." —Director, interior systems supplier to Ford, GM, and Chrysler, October 1999 (Liker J., Choi T.Y., 2004)

Suppliers may evaluate the manufacturer-supplier relationship purely based on attitude.

"In my opinion, [Ford] seems to send its people to 'hate school' so that they learn how to hate suppliers. The company is extremely confrontational. After dealing with Ford, I decided not to buy its cars." —Senior executive, supplier to Ford, October 2002 (Liker J., Choi T.Y., 2004)

Suppliers may judge the manufacturer-supplier relationship simply based on manufacturer's help.

"Toyota helped us dramatically improve our production system. We started by making one component, and as we improved, [Toyota] rewarded us with orders for more components. Toyota is our best customer." —Senior executive, supplier to Ford, GM, Chrysler, and Toyota, July 2001 (Liker J., Choi T.Y., 2004)

We know all of these aspects are quite important to build up a trusty and collaborative manufacturer and supplier relationship and the relationship will impact the product quality that the suppliers delivered to the manufacturer. However, we do not have a systematic approach to study this problem and build solid theory on top of it. Industrial practitioners are still relying on simple metrics to guide suppliers' behaviors and using feedback loops to improve relationships.

1.6 Research Objectives

This research is aimed to bridge the gap between theory and practice to build practical methodologies on top of theoretical models and use theoretical models with industrial data to provide practical insights and guide practical strategies. To achieve that, there are three objectives we need to complete. First, we need to find empirical data to match with theoretical variables to understand which variables are estimable and which variables are not practical so that we can bridge the gap between theoretical research and empirical validation. Second, we need to provide a rigorous framework to validate theoretical models in a systematic way, so we can understand if the empirical data are consistent with theoretical models. Third, we want to use the mathematical model as the solid theoretical foundation to make implications and derive managerial insights so that it is not biased towards any specific measures.

1.6.1 Bridge the Gap between Theoretical Research and Empirical Validations

As I mentioned in Section 1.4 and Section 1.5, there are huge gap between theoretical research in academic society and industrial practices. To bridge the gap, we need to investigate which variables are measurable and can be proxied by empirical data and which models are empirically practical and can be validated with industrial data. To be more specific, we want to exam the game theory models, especially the principal agent models, with empirical data and answer the questions whether principal agent models are the right framework to capture the interactions between manufacturers and suppliers and whether they are companies in the industry who are behaving consistently with what principal agent model described. If the answer is true, we can bridge the gap between theoretical model and empirical validation and then we can proceed to answer the next two research questions.

1.6.2 Propose a Framework to Systematically Validate Principal Agent Models

Although there are no prior work to validate principal agent models in a manufacturer-supplier setting, there are some prior literature validating principal agent relationship in other areas such as insurance, corporate finance, franchising and agriculture. However, there is lack of systematic framework to exam the consistency between empirical data and theoretical principal agent models. Therefore, to bridge the gap, another goal of our research is to establish a systematic framework to validate principal agent models.

1.6.3 Propose a Method to Derive Principal Agent Model Implications

All the prior literature on validating principal agent models stopped at validation stage and no prior work went further to infer what the principal agent model can imply if the empirical data can validate. As deriving managerial insights and providing strategic guidance is the core of Operations Management research, we want to bridge the gap by proposing a framework to derive principal agent model implications.

1.7 Dissertation Outline

The remaining of the dissertation is organized as follows. Chapter Two surveyed literature in streams: the theoretical principal agent model literature and empirical principal agent model validation literature. In addition, we classified the literatures into different categories so that the readers can be very clear of the literature gap that we intend to fill in. Chapter Three introduces a simple principal agent model that is consistent with most of the past literature in model framework but uses variables that are estimable and functional equations that are easy to validate with empirical data. Chapter Four discusses the empirical data that we collect to test the principal agent model and estimate the model parameters. Some descriptive analyses are also proposed to draw intuitions and later can be matched with our implication results. Chapter Five sets up the testing hypotheses and proposes the framework for systematic validation. Chapter Six demonstrates the validation results. Chapter Seven proposes the new framework we first developed to draw principal agent model inferences and validate the results. Chapter Eight discusses various sensitivity analysis on the framework of principal agent model implication and demonstrates the robustness of the results. Chapter Nine includes the discussions on limitations of our analyses on data, model, validation and implication processes. Chapter Ten draws conclusions and proposes potential future work.

2. LITERATURE SURVEY

In this chapter, we will discuss two steams of literature: the theoretical principal agent model literature and empirical principal agent model validation literature. For the theoretical principal agent model literature, we will classify the work by model types such as game theory model, mechanism design model, contract models, and by information type such as symmetric information or asymmetric information. Then readers can have a clear view of the landscape and blueprint of the models that have been proposed in the past two decades. For the empirical principal agent model validation literature, there is no prior art in the area of manufacturer-supplier relationship on quality improvement that we are interested in, so we will be the first to validate principal agent models in this area. However, people since 1980s tried different ways to validate principal agent models in areas such as franchising, insurance market, corporate finance, agriculture. Some of the validation work identified some consistency between empirical data and theoretical models and some failed to build up any connection. However, there is lack of a systematic approach in validation framework that we want to fill the void.

2.1 Theoretical Principal Agent Model Literature

OM researchers start to apply principal agent type of game theoretical models to study manufacturer-supplier supply chain quality management from the early 90s. (Reyniers D.J., Taperio C.S., 1995), (Reyniers D.J., Taperio C.S., 1995) proposed a noncooperative and cooperative framework to study the strategic behaviors in supply chain quality management and their coordination issues. (Taylor C.R., Wiggins S.N., 1997) investigated multi-period quality management of lot sizing and inspection issues under the “Japanese” relational contracting system and “American” competitive bidding system. (Li G., Rajagopalan S., 1998) present a continuous time dynamic programming model to characterize the effects on productivity, quality and cost implications from induced learning. (Iyer A.V., Schwarz L.B., Stefanos A.Z., 2005) analyzed a principal agent model with hidden information and hidden actions to study manufacturer and supplier product specification and production decisions. (Zhu K., Zhang R.Q., Tsung F., 2007) explored the roles of manufacturer and supplier in a supply chain and how the manufacturer’s involvement could make significant impact to suppliers in terms of quality improvement which ultimately benefited both parties and improved the whole supply chain. (Saouma R., 2008) used a principal agent model to analyze various cost, liability and testing conditions to outsource the manufacturer's assembly tasks (second stage) to a pre-established supplier. (Kaya M., Ozer O., 2009) built up a three-stage dynamic model to quantify the effects of quality risk factors and discussed how to use pricing as a strategy to improve product quality and mitigate quality risk. (Babich V., Tang C.S., 2012) investigated deferred payment mechanism, inspection mechanism and the combined mechanism to prevent product adulteration problems in an outsourcing environment. (Wan H., Xu X., Ni T., 2013) studied the acceptance sampling inspection mechanism together with cost sharing to coordinate the supply chain and incentivize the supplier to improve product quality. (Dong Y., Xu K., Xu Y., Wan X., 2016) explored the inspection-based quality management approach and external failure-based quality management approach in a dyadic supply chain and in a multi-level supply chain, and derived the optimal choices under different supply chain conditions to achieve the best quality. (Li C., Wan Z., 2017) researched a supply base design with two potential suppliers competing on cost improvement under various information structures and commitment capabilities.

A special kind of principal agent model - contract theory on manufacturer-supplier contract gets extremely popular in studying the manufacturer-supplier contracting issues to prevent moral hazard behaviors, mitigate supplier's misconduct under information asymmetry, coordinate supply chain interests, select capable suppliers and induce supplier quality improvement efforts. (Baiman, S., Fischer P.E., Rajan M.V., 2000), (Baiman, S., Fischer P.E., Rajan M.V., 2001) investigated the contractibility issues of various failure modes and how the information asymmetry impacts the final product quality. (Lim W.S., 2001) considered a manufacturer-supplier contract design problem in which the uninformed manufacturer used inspection policies, price rebates and warranty cost sharing to assess the private and predetermined quality type of the supplier. (Balachandran K.R., Radhakrishnan S., 2005) examined the warranty/penalty contract between the manufacturer and the supplier based on information from internal inspection failures and external product failures to mitigate moral hazards issues and achieve supply chain coordination. Those early literature normally assume a single manufacturer and a single supplier with one period of interaction. More recent literature extends the analysis to multiple mechanisms, multiple contracting variations, multiple manufacturers and suppliers or multiple periods. (Chao, G., Iravani

S., Savaskan C., 2009) studied two type of root-cause-based cost sharing contracts to coordinate supply chain, decrease information costs, and improve product quality. (Kim S., Netessine S., 2013) considered the collaborative efforts to reduce the cost and the procurement contracting strategies in a two-stage analysis under asymmetric information to screen supplier's private information, encourage supplier's cost reduction efforts and mitigate supplier's moral hazard issues. (Rui H., Lai G., 2015) investigated two kinds of mechanisms, deferred payment mechanism and inspection mechanism to mitigate the product quality adulteration under the contexts that the procurement contract contained multiple units and products had non-negligible lead time to reach customers. (Yan X., Zhao H., Tang K., 2015) analyzed quality contracting of both the manufacturer's first-mover right by posting quality requirement to suppliers and the supplier's first-mover right by promising quality deliverables to the manufacturer. (Lee H., Li C., 2018) studied three strategies cooperation, incentivization, and inspection as well as their combinations that the manufacturer could use to improve the incoming product quality.

The principal agent model and contract theory provided OM researchers a powerful tool to break down the complicated outsourcing quality management problems into a manageable scale and enabled mathematical formulation to single out important variables, strategy, and mechanism to be studied. Normally, first-best solutions are established under a perfect ideal situation without asymmetric information, self-interest, moral hazards and other issues, and second-best solutions will be derived with the focus effects. Comparison of second-best solutions and first-best solutions will provide the important managerial insights. Recent advancement of sophisticated abstract principal agent model and contract theory empowered the understanding of outsourcing, manufacturer-supplier relation, quality management in a more complicated setting like multi-period, multi-suppliers, multi-mechanisms, and so on. However, all the principal agent model and contract theory have very strict restrictions on mathematical formation, and optimal solutions are generally derived under multiple very restrictive assumptions. Therefore, to prove the validity and power of the principal agent models in studying real world outsourcing quality management problems we need to answer several important questions.

Like all the game-theory models, both manufacturer and supplier are assumed to be completely rational and self-interest, performing profit-maximization/cost-minimization calculations on their own. There are a lot of experimental and behavioral economics literature that examined this assumption in laboratory settings or using real-world data and either validated or refuted this most important assumption in game theory. Is the rationality assumption valid or at least close to valid when applied to manufacturer-supplier principal agent problem setting remain questionable and no supply chain literature seems to examine or test it before.

To test the principal agent model in supply chain, we need good proxies with good data for the explanatory variables that can be regressed or examined to validate the theoretical relation between variables. Both (Masten S.E., Saussier S., 2000) and (Chiappori P.A., Salanie B., 2002) in their survey papers concluded that the biggest challenges for testing principal agent and contract theory is to find good proxies on the explanatory variables identified in the theory, and the reason which makes testing principal agent model so hard is usually because of the lack of high quality data on those proxies.

"Data! data! data! I can't make bricks without clay." (Arthur Conan Doyle, The Adventure of the Copper Beeches).

In the theoretical supply chain literature, some explanatory variables like monetary transfer (Lim W.S., 2001), pricing ((Xu X., 2009), (Kaya M., Ozer O., 2009)), penalty (Balachandran K.R., Radhakrishnan S., 2005), quantity (Zhu K., Zhang R.Q., Tsung F., 2007), inspection policy ((Reyniers D.J., Taperio C.S., 1995), (Wan H., Xu X., Ni T., 2013)) are directly measurable with actual data. However, supplier's capability ((Iyer A.V., Schwarz L.B., Stefanos A.Z., 2005), (Li C., Wan Z., 2017)), supplier's quality improvement efforts ((Iyer A.V., Schwarz L.B., Stefanos A.Z., 2005), (Wan H., Xu X., Ni T., 2013), (Zhu K., Zhang R.Q., Tsung F., 2007), (Yan X., Zhao H., Tang K., 2015), (Li C., Wan Z., 2017)), supplier's design effort (Baiman, S., Fischer P.E., Rajan M.V., 2000), manufacturer-supplier collaboration level (Kim S., Netessine S., 2013), manufacturer's inspection effort (Baiman, S., Fischer P.E., Rajan M.V., 2000), manufacturer-internal resource to help the supplier (Iyer A.V., Schwarz L.B., Stefanos A.Z., 2005) are relative abstract concepts and hard to find a good proxy to measure. Product quality measured by quality level/defective rate ((Baiman, S., Fischer P.E., Rajan M.V., 2000), (Balachandran K.R., Radhakrishnan S., 2005)), quality costs (Zhu K., Zhang R.Q., Tsung F., 2007) measured by warranty ((Lim W.S., 2001), (Balachandran K.R., Radhakrishnan S., 2005)) and/or recall costs (Chao, G., Iravani S., Savaskan C., 2009), cost sharing ratio ((Zhu K., Zhang R.Q., Tsung F., 2007), (Chao, G., Iravani S., Savaskan C., 2009), (Wan H., Xu X., Ni T., 2013)) are well defined proxies but normally lack of high quality actual data. For example, defective rates are measured based on an inspection sample that cannot guarantee 100% accuracy. Warranty and recall costs are confidential data to each company and normally won't be reported. Even companies are willing to disclose their warranty costs, it is normally at aggregate and lump sum level and it is hard to track the quality costs by each product failure. Cost sharing ratio between manufacturer and supplier is even more confidential in contracting and the compensation is affected by many factors like root cause, financial health and court judgement. Due to these many reasons, testing the principal agent theory in supply chain becomes extremely challenging and empirical work is almost non-exist.

2.1.1 Complete List

Here we attached the complete list of the theoretical principal agent model literature in supply chain quality improvement area and then we classify the literature in different ways.

Table 2.1 Complete List of Quality Improvement Principal Agent Literature

Authors	Type of Model	Context	Findings
(Reyniers D.J., Taperio C.S., 1995)	Noncooperative and cooperative games	A supplier controls quality, a producer may or may not inspect incoming product quality	Optimal contract design on equilibrium behavior and conditions
(Reyniers D.J., Taperio C.S., 1995)	Noncooperative and cooperative games	A supplier controls quality, a producer inspects quality, the producer could ask for price rebate or share warranty costs with supplier if product fails	Impact of contracting on price rebates and after-sales warranty costs on supplier and producer behaviors and quality performance
(Taylor C.R., Wiggins S.N., 1997)	Incentive contract	A supplier controls quality, a buyer can inspect in American style or not inspect but repeat purchase in Japanese style	Ratio of set-up to inspection costs determines the optimality of American system or Japanese system
(Li G., Rajagopalan S., 1998)	Optimal control model	A monopolist makes efforts on process improvement and quality assurance to improve quality over period of time	Support continuous improvement argument to improve quality over time
(Iyer A.V., Schwarz L.B., Stefanos A.Z., 2005)	Principal agent model with hidden information and hidden actions	A buyer delegates production to a supplier but could commit internal resources to help the supplier, a supplier controls the production process	The optimal use of resources depends on cost relationship (substitutes or complements) between the resources committed by the buyer and its impacts on supplier's ability to cut cost

Table 2.1 continued

(Zhu K., Zhang R.Q., Tsung F., 2007)	Principal agent model	A buyer owns product design and the brand, a supplier controls manufacturing and supplies products to the buyer, the buyer and the supplier share quality and warranty costs	Studied different roles between a buyer and a supplier in a supply chain to improve quality and showed the significant impact of supply chain profitability on buyer's involvement
(Kaya M., Ozer O., 2009)	Principal agent model with hidden information and hidden actions	An OEM outsources functional areas like design, procurement, and manufacturing to Contract Manufacturer and focuses only on sales and pricing	Characterized the effects of multiple quality risk factors on the firm's profits and the resulting product quality; determined how the OEM's pricing strategy would affect the product quality and mitigate the quality risk
(Babich V., Tang C.S., 2012)	Principal agent model	A buyer buys a single unit from a supplier in a decentralized supply chain and could use (a) deferred payment mechanism, (b) inspection mechanism, or (c) the (a), (b) combined mechanism to deal with product adulteration problems	Showed deferred payment mechanism can completely deter the suppliers from product adulterations where inspection mechanism cannot, and also identified four factors that determined the situations that deferred payment mechanism would dominate the inspection mechanism.
(Wan H., Xu X., Ni T., 2013)	Game Theory	A firm procures a product from a supplier and uses incentive inspection to induce supplier's efforts on quality improvement	Inspection serves the purpose of incentive mechanism to improve product quality.
(Dong Y., Xu K., Xu Y., Wan X., 2016)	Principal agent model	A product is outsourced by a brand owner to an independent contract manufacturer in a dyadic supply chain or a multi-level supply chain, the brand owner can choose quality management approaches based on inspection or external failure to improve product quality	Explored the quality management approaches based on inspection and external failure in a dyadic supply chain and a multi-level supply chain, and derived the optimal choices under different supply chain conditions to achieve the best quality
(Li C., Wan Z., 2017)	Principal agent model	One buyer faces two potential suppliers that can compete or exert cost reduction efforts under different information structures and commitment capability	The competition-improvement relation depends on the effort observability of the two suppliers

Table 2.1 continued

(Baiman, S., Fischer P.E., Rajan M.V., 2000)	Principal agent model	A buyer buys one unit of product from a supplier, the supplier incurs product quality improvement costs on failure prevention while the buyer incurs defects identification costs on appraisal	Contractibility issues of various failure modes and the information asymmetry impacts the final product quality
(Lim W.S., 2001)	Game Theory	A producer purchases part from a supplier and uses inspection policies, price rebates and warranty cost sharing to induce product quality	Derived the optimal conditions and optimal compensation themes under different assumptions and various information structure
(Balachandran K.R., Radhakrishnan S., 2005)	Principal agent model with one-sided and two-sided moral hazard	A buyer contracts product quality, warranty/penalty with a supplier based on information from external failures and internal inspection to induce supplier's quality effort choice	Based on information from inspection and external failures, examined the warranty/penalty contract to mitigate single-sided and double-sided moral hazard issues to achieve supply chain coordination
(Chao, G., Iravani S., Savaskan C., 2009)	Principal agent model	Quality improvement efforts can be inserted by both the manufacturer and the supplier to reduce product recall costs. Product recall costs can be shared based on selective root cause analysis or partial cost sharing based on complete root cause analysis	Studied two type of cost sharing contracts to coordinate a supply chain based on root-cause analysis which resulted in buyer's cost reduction and product quality improvement
(Kim S., Netessine S., 2013)	Principal agent model	A manufacturer and a supplier collaborate quality improvement and engaged in either expected margin commitment contract or screening contract to promote collaborations	Derived optimal conditions and optimal contract choice between expected margin commitment contract and screening contract to promote collaboration, eliminate information asymmetry and improve product quality
(Rui H., Lai G., 2015)	Principal agent model	A buyer procures some products from a supplier and uses deferred payment mechanism or inspection mechanism to prevent supplier's effort adulteration with endogenous quantity decision and general process on defect discovery	Derive the equilibrium in both mechanisms and characterized the conditions, compared the performance of the two mechanisms under various conditions.

Table 2.1 continued

(Yan X., Zhao H., Tang K., 2015)	Principal agent model	A buyer procures products from a supplier and could choose either take the first-mover right by posting quality requirements to suppliers or give up the first mover to the suppliers and ask for quality commitment	Analyzed quality contracting of both the buyer's first-mover right by requiring quality and the supplier's first-mover right by promising quality deliverables
(Lee H., Li C., 2018)	Principal agent model	A buyer buys a product from a supplier and could choose cooperation, incentivization and inspections to induce supplier's effort on quality improvement	Studied three strategies cooperation, incentivization, and inspection as well as their combinations that the buyer could use to improve the incoming product quality

2.1.2 Classified by Quality Improvement Mechanisms

There are many mechanisms that researchers proposed in the past to help improve the product quality in the supply chain. Below is a summary of the quality improvement mechanisms and the list of publications.

Table 2.2 Classified by Quality Improvement Mechanisms

Mechanisms	Publications	Findings
Inspection	(Reyniers D.J., Taperio C.S., 1995), (Baiman, S., Fischer P.E., Rajan M.V., 2000), (Wan H., Xu X., Ni T., 2013)	Inspection mechanism could prevent internal failures and incentivize suppliers to improve quality
Rebate/Penalty	(Reyniers D.J., Taperio C.S., 1995), (Lim W.S., 2001)	Price rebate encourages high quality products; Penalty penalize low quality products
Repeat Purchase	(Taylor C.R., Wiggins S.N., 1997)	Repeat purchase a good mechanism to encourage suppliers deliver high quality products
Process Improvement	(Li G., Rajagopalan S., 1998)	Manufacturer should support suppliers' continuous improvement to improve quality
Cost Sharing	(Balachandran K.R., Radhakrishnan S., 2005), (Zhu K., Zhang R.Q., Tsung F., 2007), (Chao, G., Iravani S., Savaskan C., 2009), (Wan H., Xu X., Ni T., 2013)	Warranty cost sharing, recall cost sharing, inspection cost sharing all incentivize suppliers to improve product quality to prevent external failures
Deferred Payment	(Babich V., Tang C.S., 2012), (Rui H., Lai G., 2015)	Deferred payment enables better detection on external product failures, therefore, encourages suppliers' quality improvement behaviors
Commit Resources	(Iyer A.V., Schwarz L.B., Stefanos A.Z., 2005), (Kim S., Netessine S., 2013), (Lee H., Li C., 2018)	Manufacturer can commit resources on process improvement, product design, quality improvement efforts to help supplier improve product quality

2.1.3 Classified by Model Types

The entire supply chain quality improvement literature can also be classified into Game Theory models, Contract Theory Models and Mechanism Design Models by model types under Principal Agent Model umbrella.

Table 2.3 Classified by Model Types

Principal Agent Models		
Games Theory	Contract Theory	Mechanism Design
(Reyniers D.J., Taperio C.S., 1995)	(Iyer A.V., Schwarz L.B., Stefanos A.Z., 2005)	(Li G., Rajagopalan S., 1998)
(Reyniers D.J., Taperio C.S., 1995)	(Balachandran K.R., Radhakrishnan S., 2005)	(Baiman, S., Fischer P.E., Rajan M.V., 2000)
(Taylor C.R., Wiggins S.N., 1997)	(Chao, G., Iravani S., Savaskan C., 2009)	(Zhu K., Zhang R.Q., Tsung F., 2007)
(Lim W.S., 2001)	(Kim S., Netessine S., 2013)	(Kaya M., Ozer O., 2009)
(Wan H., Xu X., Ni T., 2013)	(Yan X., Zhao H., Tang K., 2015)	(Babich V., Tang C.S., 2012)
		(Rui H., Lai G., 2015)
		(Dong Y., Xu K., Xu Y., Wan X., 2016)
		(Li C., Wan Z., 2017)
		(Lee H., Li C., 2018)

2.1.4 Classified by Information

According to information type, the supply chain quality improvement literature can be also classified based on whether there is information asymmetric in the model or not. Normally, symmetric information models are easier to empirically validate and asymmetric information models are much harder to validate as some variables in the models are unobservable.

Table 2.4 Classified by Information

Principal Agent Models	
Symmetric Information	Asymmetric Information
(Reyniers D.J., Taperio C.S., 1995)	(Li G., Rajagopalan S., 1998)
(Reyniers D.J., Taperio C.S., 1995)	(Baiman, S., Fischer P.E., Rajan M.V., 2000)
(Taylor C.R., Wiggins S.N., 1997)	(Iyer A.V., Schwarz L.B., Stefanos A.Z., 2005)
(Lim W.S., 2001)	(Balachandran K.R., Radhakrishnan S., 2005)
(Wan H., Xu X., Ni T., 2013)	(Zhu K., Zhang R.Q., Tsung F., 2007)
(Babich V., Tang C.S., 2012)	(Chao, G., Iravani S., Savaskan C., 2009)
(Rui H., Lai G., 2015)	(Kaya M., Ozer O., 2009)
(Yan X., Zhao H., Tang K., 2015)	(Li C., Wan Z., 2017)
(Dong Y., Xu K., Xu Y., Wan X., 2016)	(Lee H., Li C., 2018)
	(Kim S., Netessine S., 2013)

2.1.5 Research Questions

Since our objective is to propose the first empirical validation of the principal agent model in supply chain quality improvement area, we need to find a theoretical model which possess following characteristics that could be potentially validated with empirical data.

- Symmetric Information: symmetric information models are a lot easier to validate compared to asymmetric information models. Therefore, we will start from symmetric information models.
- Cost Sharing and Commitment Resources: if we want to validate principal agent models, we first need to find empirical data on important variables/mechanisms. Inspection data, repeat purchasing data and payment data are very hard to find. Therefore, we should focus on financial data like warranty costs and manufacturer’s commitment resources which are easier to proxy with empirical data.

- A Simple Principal Agent Model: the principal agent models can be very complicated and the relationship between variables can be very hard to capture. For the first empirical work, we will only focus on simple principal agent model with symmetric information.

Therefore, from a theoretical principal agent model perspective the research question is how to find a simple principal agent model with symmetric information and right mechanisms that the model variables and mechanisms can be proxied with empirical data.

2.2 Empirical Principal Agent Model Validation Literature

Although in supply chain manufacturer-supplier studies the empirical work in testing principal agent theory is rare, researchers have made significant progress in proving and validating principal agent theories in other fields.

Insurance Market: classic principal agent theory in insurance market predicts that a more comprehensive coverage will trigger a higher probability on accident rate. (Rothschild-Stiglitz) interprets the prediction based on the adverse selection. A high-risk group will self-select to buy an insurance with better coverage to protect themselves as they know that they have higher accident risk associated, while a low-risk group is willing to get more risk exposure as they perceive themselves as low risk identities. Alternatively, other economists interpret this phenomenon from moral hazard. A high-risk person may become more cautious if she knows that she is only partially protected. On the other hand, a low-risk person may become aggressive if his insurance has more comprehensive coverage. (Dahlby B.G., 1983), (Boyer M., Dionne G., 1989), (Puelz R., Snow A., 1994) and (Chiappori P.A., Salanie B., 1997), (Chiappori P.A., Salanie B., 2000) tested the principal agent model based on automotive insurance market data and found that the choice of liability coverage correlated with the accident and collision frequencies which confirmed the theoretical findings. However, the empirical results showed mix in the life insurance market. (Friedman B.M., Warshawsky M.J., 1990) found a contrast result to a classic life-cycle model on consumption and saving behavior. They empirically showed that rather than buy individual life annuities most elderly individuals in the United States actually preferred a flat age wealth. (Boose, Mary A., 1990) and (Cawley J., Philipson T., 1999) also found many opposite conclusions to the theoretical predicted patterns and those contrasted findings led to new explanations, new theoretical improvements and empirical studies.

Agricultural Economics: In agriculture, theoretical literatures developed transaction cost theories and risk sharing theories to explain the principal agent relationship between farmland owners and tenants. The empirical validations also showed mix results. Early work by (Allen D.W., Lueck D., 1992), (Allen D.W., Lueck D., 1995) validated the transaction cost predictions using farmland contract level data but found little evidence on risk sharing theory. (Laffont J.J., Matoussi M.S., 1995) studied the sharecropping contracts and found that sharecropping contracts would cause productivity decrease while a rental contract could increase production by around 50% which contrasted the theoretical predictions. (Allen D.W., Lueck D., 1999) used detailed case by case agricultural contract data to study the relationship between risk aversion and contract choice. The empirical results failed to validate the relationship that the principal agent model established. However, (Ackerberg D.A., Botticini M., 2002) identified the failures of the early empirical work on risk sharing as the endogenous matching problems in heterogeneous agents. They proposed an endogenous matching technique by controlling the endogenous variables and finally found consistent results with principal agent theory on sharecropping contracts which indeed supports the risk sharing theory in principal agent models.

Labor Economics: In labor economics, principal agent models usually predict a better compensation on a higher commission or higher risk. For example, a more capable manager leading a firm performing better than their peers should get paid more than average manager. A sale personnel who sells more products should receive higher salaries. An entrepreneur who puts more efforts should get more compensations. (Jensen M.C., Murphy K.J., 1990) first tested the relationship between CEOs' pay and firm performance in the period from 1969 to 1983 and found that the manager only gets paid $3.25 more when his firm's value increases by $1,000. (Haubrich J.G., 1994) showed that this empirical result on risk aversion holds even on lower level managers. The intuition from these literatures is that although the sensitivity on CEOs' pay and firm performance is low a large swing on firm's stock price performance will still trigger a large CEOs' payment variations. (Hall B.J., Liebman J.B., 1998) found the low sensitivity was due to low level of stock options in the CEOs' compensation. They showed that if considering the significant increase of stock options in CEOs' compensation package the CEOs' pay could be much more sensitive to firm performance. Their estimate on the change in CEOs' wealth including cash equivalent plus stock and stock options is about $25 in mean and $5.3 in median for $1,000 increase in firm value which is significantly higher than the estimate from (Jensen M.C., Murphy K.J., 1990) due to the inclusion of stocks and stock options. (Eisenhardt K.M., 1985), (Eisenhardt K.M., 1988) focused their empirical validation work on compensation choices in retailing salespeople. They examined the compensation choices between outcome-based commission and behavior-based salary for the sales persons in retailing. They measured the relationship of variables like task, information control, outcome uncertainty on the compensation choice between outcome-based commission and behavior-based salary and found extensive evidences supporting the agency theory predictions on risk compensation. (Conlon E.J., Parks J.M., 1990) set up multiple laboratory experiments to replicate and extend Eisenhardt's work to test predictions from agency theory (Harris M., Raviv A., 1978). They found consistent results with theoretical predictions that the outcome-based compensation is negatively correlated with the principal’s information levels and the institutional predictions can also be validated. (Bitler, M., Moskowitz, T., Vissing-Jorgensen, A., 2005) first developed a theoretical principal agent model between entrepreneur and their private firm and then found unique dataset on variables like entrepreneurial effort and their wealth levels in the firm to test the model’s predictions. They found consistent empirical evidence to support the theoretical principal agent model prediction that both the entrepreneurial effort and the firm’s performance will increase with increased ownership, and the shares of the entrepreneurial ownership is positively correlated with exogenous wealth and negatively correlated with firm’s risk. (Devaro J., Kurtulus F.A., 2010) empirically test a theoretical principal agent theory developed by (Prendergast C., 2002) on risk and incentives tradeoffs, and provided evidences of all the principal agent theoretical results on risk-incentives tradeoff, a positive relationship between incentive pay and the delegation of worker authority, a positive relationship between risk and authority. (Kang Q., Liu Q., 2010) developed a principal agent theoretical model to study the impact of risk–incentive relation from the role of information-based stock trading, and then used real-world executive compensation data to perform the empirical testing. The empirical results showed strong support to the theoretical model prediction.

Franchise: Principal agent model plays a central role in studying franchisor and franchisee and explaining the mechanisms such as risk sharing, one-sided or two-sided moral hazard in franchising relations. (Lafontaine F., 1992) proxied factors like risk, moral hazard, and franchisors' capital need in his empirical model to explain the franchisors' contracting decisions on royalty rates, franchise fees and the contract extent. Lafontaine found the empirical results more consistent with two-sided moral hazard principal agent model but contrasted with many principal agent model predictions on relationship between royalty rates, franchise fees and franchisee's effort. (Lafontaine F., Slade M.E., 1998) first constructed a simple principal agent model to capture all the important factors between salespeople and the franchise, and then derived the theoretical predictions. After examining the empirical evidences, they found on one side with good proxies the empirical findings could be consistent across different industries with the theoretical predictions, on the other side the empirical validation was very fragile on model specifications and assumptions. Therefore, they made an ambiguous conclusion on whether the empirical results can be consistent with agency theory predictions. (Brickley J.A., 1999), (Brickley J.A., 2002) also found similar mixed results in validating the empirical results of the agency theories in franchising contracts. He concluded that the empirical testing based on a large sample of franchise contracts was generally consistent with the multi-task agency theory hypotheses (Brickley J.A., 1999) and supported the predictions of a two-sided moral hazard principal agent model (Brickley J.A., 2002). However, some theoretical results like the externalities could not find support.

Others: (Songer D.R., Segal J.A., Cameron C.M., 1994) tested a principal agent model on supreme court interactions and found some direct and indirect support to the theoretical predictions. (Lord M.D., 2000) studied the constituent-legislator principal agent relationship and analyzed the empirical data to make suggestions. (Berger, A.N., Bonaccorsi di Patti E., 2006) examined the principal agent theory in corporate governance and found that the empirical results using the data on the US banking industry were statistically and economically significant and consistent with the theory. (Schulze W.S., Lubatkin M.H., Dino R.N., Buchholtz A.K., 2001) provided empirical credibility to the principal agent theory in the field of governance of family firms by examining a large dataset on privately-held, family-managed firms. (Saussier S., 1999), (Saussier S., 2000) explored the contracts of coal transportations in France and examined the principal agent theoretical outcomes in the transaction cost theory and contract durations. (Crocker K.J., Reynolds K.J., 1993) studied the incentive effects of incomplete contracts between contractual parties and tested the theoretical predictions using pricing procedure panel data and engine procurement contracts in Air Force. Their paper concluded a support to theoretical predictions that the contract completeness is a reflection of a desire for both parties to minimize the exchange costs. (Shearer

B., 2004) studied the incentive effects of piece rate and fixed wage contracts for workers who are randomly assigned to plants and found 20% productivity gain in an unrestricted statistical method and at least 21.7% gain in structural econometric methods. (Lazear E.P., 2000) tested the agency theory using data from a large automotive company producing auto glasses and found that after changing the workers' compensation from hourly wages to piece rates the workers' productivity dropped significantly. On the other hand, switching compensations from hourly wages to piece rates improved the productivity of output by average 44% per worker. (Shaikh I.A., Peters L., 2018) tested the principal agent theory on board monitoring in incentivizing R&D activities using data from 1997 to 2007 of 1500 S&P US firms and found that the theoretical principal agent model predictions were consistent with empirical results and could be used to help managers make better decisions on R&D investments.

2.2.1 Classified by Areas

Similar to what we did to the theoretical principal agent model in supply chain quality improvement, we classify the empirical principal agent model validation literature based on areas.

Table 2.5 Classified by Areas

Empirical Principal Agent Model Literature				
Insurance Market	Agriculture Econ	Labor Econ	Franchise	Others
(Dahlby B.G., 1983)	(Allen D.W., Lueck D., 1992)	(Jensen M.C., Murphy K.J., 1990)	(Lafontaine F., 1992)	(Songer D.R., Segal J.A., Cameron C.M., 1994)
(Boyer M., Dionne G., 1989)	(Allen D.W., Lueck D., 1995)	(Haubrich J.G., 1994)	(Lafontaine F., Slade M.E., 1998)	(Lord M.D., 2000)
(Puelz R., Snow A., 1994)	(Laffont J.J., Matoussi M.S., 1995)	(Hall B.J., Liebman J.B., 1998)	(Brickley J.A., 1999)	(Berger, A.N., Bonaccorsi di Patti E., 2006)
(Chiappori P.A., Salanie B., 1997)	(Allen D.W., Lueck D., 1999)	(Eisenhardt K.M., 1985)	(Brickley J.A., 2002)	(Schulze W.S., Lubatkin M.H., Dino R.N., Buchholtz A.K., 2001)
(Chiappori P.A., Salanie B., 2000)	(Ackerberg D.A., Botticini M., 2002)	(Eisenhardt K.M., 1988)		(Saussier S., 1999)
(Friedman B.M., Warshawsky M.J., 1990)		(Conlon E.J., Parks J.M., 1990)		(Saussier S., 2000)
(Boose, Mary A., 1990)		(Bitler, M., Moskowitz, T., Vissing-Jorgensen, A., 2005)		(Crocker K.J., Reynolds K.J., 1993)
(Cawley J., Philipson T., 1999)		(Bitler, M., Moskowitz, T., Vissing-Jorgensen, A., 2005)		(Shearer B., 2004)
		(Devaro J., Kurtulus F.A., 2010)		(Lazear E.P., 2000)
		(Kang Q., Liu Q., 2010)		(Shaikh I.A., Peters L., 2018)

We can see that the prior work of empirical validation principal agent models is clustered in four main areas: insurance market, agriculture economics, labor economics and franchising probably because there are more empirical data directly linked to the variables such as accident rates and liability coverage in insurance market, agricultural contracts and productions in agriculture, manager’s pay and company’s stock price performance in labor economics and sale-force compensation and franchising contracts in franchise. With the wide variety of data availability, validating principal agent models in these areas become possible. However, in other research fields that principal agent models are popular such as law, financial planning, manufacturing, there is almost no empirical validation at all due to limited data available.

2.2.2 Classified by Validation Success/Failures

For the prior art on validating the principal agent model, the success rate is varying by areas. Below I classified the empirical principal agent model literature based on the success and failure to show the consistent with the principal agent model theoretical results.

Table 2.6 Classified by Success/Failure in Consistency with Principal Agent Model

	Empirical Principal Agent Model Literature				
	Insurance Market	Agriculture Econ	Labor Econ	Franchise	Others
Consistent	(Dahlby B.G., 1983)	(Allen D.W., Lueck D., 1992)	(Haubrich J.G., 1994)	(Lafontaine F., 1992)	(Songer D.R., Segal J.A., Cameron C.M., 1994)
	(Boyer M., Dionne G., 1989)	(Ackerberg D.A., Botticini M., 2002)	(Hall B.J., Liebman J.B., 1998)	(Lafontaine F., Slade M.E., 1998)	(Lord M.D., 2000)
	(Puelz R., Snow A., 1994)		(Eisenhardt K.M., 1985)	(Brickley J.A., 1999)	(Berger, A.N., Bonaccorsi di Patti E., 2006)
	(Chiappori P.A., Salanie B., 1997)		(Eisenhardt K.M., 1988)	(Brickley J.A., 2002)	(Schulze W.S., Lubatkin M.H., Dino R.N., Buchholtz A.K., 2001)
	(Chiappori P.A., Salanie B., 2000)		(Conlon E.J., Parks J.M., 1990)		(Saussier S., 1999)
			(Bitler, M., Moskowitz, T., Vissing-Jorgensen, A., 2005)		(Saussier S., 2000)
			(Bitler, M., Moskowitz, T., Vissing-Jorgensen, A., 2005)		(Crocker K.J., Reynolds K.J., 1993)
			(Devaro J., Kurtulus F.A., 2010)		(Shearer B., 2004)
			(Kang Q., Liu Q., 2010)		(Lazear E.P., 2000)
					(Shaikh I.A., Peters L., 2018)
Inconsistent	(Friedman B.M., Warshawsky M.J., 1990)	(Allen D.W., Lueck D., 1992)	(Jensen M.C., Murphy K.J., 1990)	(Lafontaine F., Slade M.E., 1998)	
	(Boose, Mary A., 1990)	(Allen D.W., Lueck D., 1995)	(Haubrich J.G., 1994)	(Brickley J.A., 1999)	
	(Cawley J., Philipson T., 1999)	(Laffont J.J., Matoussi M.S., 1995)		(Brickley J.A., 2002)	
		(Allen D.W., Lueck D., 1999)			

From the Table 2.6, we can see that in areas like insurance market and labor economics, there is a higher success rate to empirically show the data consistent with the theoretical principal agent model results. In areas like franchising, the success and failure split. Empirical validation in agriculture economics demonstrates the highest inconsistency with the theoretical principal agent model which shows that principal agent models might not be a proper approach to capture the relationship between farmland owners and tenants. In other areas where there are not many empirical work, the success rate is very high but due to the limitation of publications we cannot make any conclusions on validation.

2.2.3 Research Questions

To summarize our literature survey, the theoretical principal agent models have seen applications in almost all the areas that include a principal and agent relationship. However, the empirical principal agent works are clustered in the fields that only good proxies and good data exist, such as insurance, agriculture, labor contracts and franchising. Success or failure to validate the consistency of empirical data with principal agent model varies by areas.

Our research question is to provide the first empirical evidence in the empirical principal agent literature in the area of supply chain manufacturer-supplier quality management. Then we will assess the consistency of the empirical data with principal agent model and expand the credibility of principal agent models in analyzing manufacturer-supplier relationships in operations management.

3. A SIMPLE PRINCIPAL AGENT MODEL

We start with a simple principal agent model, which captures several important factors for quality improvement in supply chains. Based on this model, we derive causality equations connecting with product quality, warranty cost sharing and quality improvement efforts of suppliers and manufacturers, and form hypotheses that we can test later using empirical data.

3.1 Business Settings

Following conventional settings in the quality improvement literature (see the literature summary in Table 2.1), we consider a supply chain with one manufacturer and one supplier, in which the manufacturer purchases parts/components/subsystems from the supplier. Both the manufacturer and supplier can put efforts and/or commit resources into quality improvement. For example, the manufacturer can put efforts into product design (see, e.g., (Kim S., Netessine S., 2013), (Zhu K., Zhang R.Q., Tsung F., 2007)) to make the component easier to manufacture and/or assemble, which helps improving the product quality. Another way to improve product quality is that the manufacturer can allocate internal resources (e.g., engineering hours) to help the supplier on the quality control task (see, e.g., (Iyer A.V., Schwarz L.B., Stefanos A.Z., 2005), (Zhu K., Zhang R.Q., Tsung F., 2007)). The manufacturer's collaborative effort eases the supplier's investment in failure prevention, which reduces component defects and benefits the manufacturer-supplier relationship (see, e.g., (Baiman, S., Fischer P.E., Rajan M.V., 2000)). Once the product reaches the open market, there is a probability that the product might suffer an external failure such as repairs or recalls. We assume that the manufacturer and the supplier will share the warranty or recall costs (see, e.g., (Balachandran K.R., Radhakrishnan S., 2005), (Chao, G., Iravani S., Savaskan C., 2009), (Zhu K., Zhang R.Q., Tsung F., 2007)). Both the manufacturer and the supplier will incur the quality improvement effort costs and the warranty sharing costs. Both the manufacturer and the supplier are aimed to minimize its own total supply chain costs.

3.2 Mathematical Model

We denote the defective rate of the product as $Q(e_M, e_S)$, where e_S is the supplier's quality improvement effort and e_M is the manufacturer's quality improvement effort. For instances,

$Q(e_M, e_S)$ represents the number of problems experienced per 100 vehicles in the Initial Quality Studies (IQS) conducted by JD Power, which is influenced by vehicle design and the OEM-supplier's collaborative efforts in quality improvement.

For model tractability, we assume $Q(e_M, e_S) = \theta_0 \exp(-\theta_M e_M - \theta_S e_S - \theta_J e_M e_S)$, where all θ parameters $\theta_0, \theta_M, \theta_S, \theta_J > 0$ are positive. Notice that the term of $\theta_J e_M e_S$ captures the interaction effect of quality improvement efforts of the manufacturer and the supplier, which is strategically complementary (see, e.g., (Balachandran K.R., Radhakrishnan S., 2005), (Kim S., Netessine S., 2013), (Lee H., Li C., 2018)). Such an exponential function form is frequently used in the quality improvement literature (see, e.g., (Reyniers D.J., Taperio C.S., 1995), (Chao, G., Iravani S., Savaskan C., 2009)).

Without loss of generality, we normalize the damage cost of one unit defect as one dollar. The damage cost is shared between the manufacturer and supplier. Such a risk-sharing mechanism provides economic incentives for both the manufacturer and supplier to engage in quality improvement and hence is an important focus in the quality improvement literature (see, e.g., (Lim W.S., 2001), (Zhu K., Zhang R.Q., Tsung F., 2007), (Chao, G., Iravani S., Savaskan C., 2009)). We let λ be the percentage of the damage cost that the supplier bears, which implies that the manufacturer pays $1 - \lambda$ percentage of the damage cost and $0 \leq \lambda \leq 1$.

We consider the following principal agent model: (1) As the leader, the manufacturer decides his quality effort and minimizes his cost $(1 - \lambda)Q(e_M, e_S) + \eta_M e_M$, where η_M is the unit effort cost for the manufacturer and $\eta_M > 0$; (2) As the follower, the supplier decides her quality effort and minimizes her cost $\lambda Q(e_M, e_S) + \eta_S e_S$, where η_S is the unit effort cost for the supplier and $\eta_S > 0$. Solving this principal agent model, we obtain the theoretical predictions of the optimal behaviors of both the supplier and the manufacturer, as well as the implied product quality level. These results can be converted into the hypotheses, on which we run empirical tests using the automobile industry data in the next section. First, we solve the theoretical model to derive the propositions below.

3.3 Propositions

Proposition 1: *The optimal quality level is* $Q^*(e_M, e_S) = Q(e_M, e_S^*(\lambda, e_M)) = \frac{\eta_S}{\lambda(\theta_J e_M + \theta_S)}$, *where* $e_S^*(\lambda, e_M) = \left(\ln\left(\frac{\lambda\theta_0(\theta_J e_M + \theta_S)}{\eta_S}\right) - \theta_M e_M\right) \Big/ (\theta_S + \theta_J e_M)$ *is the supplier's optimal quality effort level.*

Proof. Notice that the supplier's cost function is convex in e_S and the FOC equation is $\lambda(\theta_J e_M + \theta_S)Q(e_M, e_S) = \eta_S$.

Next, we solve the manufacturer's problem after deriving the optimal quality level $Q^*(\lambda, e_M)$.

Proposition 2: *The manufacturer's optimal quality effort level is* $e_M^*(\lambda) = \sqrt{\frac{1-\lambda}{\lambda}}\sqrt{\frac{\eta_S}{\theta_J \eta_M}} - \frac{\theta_S}{\theta_J}$ *and the optimal quality level is* $Q^*(\lambda) = \frac{\sqrt{\eta_M \eta_S}}{\sqrt{\lambda(1-\lambda)\theta_J}}$.

Proof. Notice that the manufacturer's cost function is $(1-\lambda)Q^*(\lambda, e_M) + \eta_M e_M$, which is convex in e_M. The FOC equation is $\frac{(1-\lambda)\eta_S\theta_J}{\lambda(\theta_J e_M + \theta_S)^2} = \eta_M$.

Proposition 1 and Proposition 2 set up the relationships that could be converted to hypotheses on which we can use market data to empirically test. In the next sections, we will talk about the data and how to use the data to proxy the variables in the principal agent model for empirical testing.

4. EMPIRICAL DATA

In order to validate the principal agent model, we find real world data from US automotive industry. The reason to use US automotive industry data is because: firstly, the data availability to the public in US automotive industry is much better than other industries. There are all kinds of studies in quality (i.e. JD Power), reliability (i.e. Consumer Reports), OEM-supplier relation-ships (i.e. Planning Perspectives Inc.), service qualities (i.e. JD Power), sales (i.e. Wards Auto), financing (i.e. Experian), customer purchasing behavior (i.e. IHS Markit) are available so that it is much easier for us to find good proxies to the variables in the principal agent models. Secondly, manufacturer-supplier relationship is vital in automotive industry and affects the product quality significantly. GM and Ford spend roughly $90 billion every year in purchasing. Toyota relies on around 170-500 Tier 1 suppliers (Kito T., Brintrup A., New S., Reed-Tsochas F., 2014) to produce key components and subsystems. Many past literatures contribute the success of automotive quality to the supply base and OEMs-supplier relationships (i.e. (Prahinski C., Benton W.C., 2004), (Langfield-Smith K. Greenwood M.R., 1998), (Frazier G.L., Spekman R.E., O’Neal C.R., 1988)). Therefore, based on our model setup we believe the data coming from the automotive industry is more representative to test our principal agent model. Thirdly, there are cross-company data available in automotive industry for us to compare behaviors at company level and draw comparisons. A lot of past literatures also attempt to compare the domestic OEMs and foreign OEMs to build up theories (i.e. (Taylor C.R., Wiggins S.N., 1997)) and best practice (i.e. (Sako M., 2004)) in quality control. Below, we want to discuss about sample selections of automotive OEMs and automotive brands in our study and some characteristics of the data sources. We hope the discussion can convince readers that the proxies we found are the best representations of the variables in our principal agent model as (Chiappori P.A., Salanie B., 2002) mentioned that finding a good proxy with good data is the biggest challenge in validating principal agent models.

4.1 Selected Automotive OEMs, Brands and Data Sources

Six automotive OEMs data is available for our empirical studies including three domestic OEMs: General Motors, Ford and Chrysler (current Fiat-Chrysler Automobile, FCA) and three Japanese OEMs: Toyota, Honda and Nissan (current Renault–Nissan–Mitsubishi Alliance). We decide to choose the most recent decade of data from 2006 to 2017 for multiple reasons: (1) the manufacturer's effort data Working Relation Index is only available since 2003 as well as the Warranty Sharing Ratio data. (2) during the period 2003-2017 JD Power made some methodology changes in the Initial Quality Studies in 2005. Therefore, to maintain the consistency we want to use only the data after the survey change. (3) 12 years captures roughly two product cycles in automotive industry which normally spends 6 years to launch a new product from design to manufacturing, while at the same time the data length provides enough data points for statistical analysis. (4) When we write the dissertation in spring 2019, the 2018 data has not been published yet so 2017 is the most recent data that we can get. To assess the representation of the selected automotive OEMs, we found their market shares in Table 4.1 in the past decade. The six leading automotive OEMs hold market share around 80% in average where the smallest market share was 76.9% in 2012 and the largest market share was 85.6% in 2006.

Table 4.1 US Automotive Market Share by 6 Selected Automotive OEMs

	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
GM	24.5%	23.7%	22.3%	19.9%	19.1%	19.6%	17.9%	17.9%	17.8%	17.6%	17.3%	17.4%
Ford	15.4%	16.2%	16.7%	17.0%	16.9%	16.8%	15.5%	15.9%	15.0%	14.9%	14.8%	14.9%
Chrysler	12.9%	12.9%	11.0%	8.9%	9.4%	10.7%	11.4%	11.6%	12.7%	12.9%	12.9%	12.0%
Toyota	17.5%	15.8%	15.1%	16.1%	15.2%	12.9%	14.4%	14.3%	14.4%	14.3%	14.0%	14.1%
Honda	9.1%	9.6%	10.8%	11.0%	10.6%	9.0%	9.8%	9.8%	9.3%	9.1%	9.3%	9.5%
Nissan	6.2%	6.6%	7.2%	7.4%	7.8%	8.2%	7.9%	8.0%	8.4%	8.5%	8.9%	9.2%
Total Share	85.6%	84.8%	83.1%	80.3%	79.0%	77.2%	76.9%	77.5%	77.6%	77.3%	77.2%	77.3%

We choose brands Chevrolet, Buick, GMC, Cadillac for General Motors, Ford and Lincoln for Ford, Chrysler, Dodge and Jeep for Chrysler, Toyota and Lexus for Toyota, Honda and Acura for Honda, Nissan and Infinite for Nissan. These 15 brands represent majority of the sales volume for the six automotive OEMs and hold US market share around 75% consistently throughout the past decade (see Figure 4.2). Other brands owned by the six OEMs are excluded in our analysis due to various inconsistency reasons like: (1) GM abandoned Pontiac, Hammer, Saab, Saturn in 2009 after GM's bankruptcy, (2) Toyota discontinued Scion brand in 2016, (3) Ford during financial crisis sold Volvo, Land Rover, Jaguar, Mercury, Aston Martin, Mazda to other companies to implement Ford's one Ford strategy, (4) Fiat for Fiat-Chrysler Alliance, Mitsubishi for Renault–

Nissan–Mitsubishi Alliance are alliance brands owned by partner OEMs. (5) Chrysler founded Ram in 2010 as a truck brand and it does not have long enough history.

Table 4.2 US Automotive Market Share by 15 Selected Automotive Brands

	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Ford	14.6%	12.9%	13.9%	13.8%	15.1%	16.1%	14.9%	15.4%	14.4%	14.3%	14.2%	14.3%
Toyota	12.4%	13.4%	12.7%	14.3%	12.8%	10.9%	12.2%	12.2%	12.1%	12.0%	12.0%	12.4%
Chevrolet	14.5%	13.9%	13.5%	12.8%	13.5%	13.9%	12.8%	12.5%	12.3%	12.2%	11.9%	12.0%
Honda	7.9%	8.5%	9.7%	10.0%	9.5%	8.0%	8.7%	8.7%	8.3%	8.1%	8.4%	8.6%
Nissan	5.4%	5.8%	6.3%	6.6%	6.9%	7.4%	7.1%	7.3%	7.7%	7.7%	8.1%	8.4%
Jeep	2.8%	2.9%	3.8%	2.2%	2.5%	3.3%	3.3%	3.1%	4.2%	5.0%	5.3%	4.8%
GMC	2.8%	3.0%	2.7%	2.4%	2.9%	3.1%	2.9%	2.9%	3.0%	3.2%	3.1%	3.3%
Dodge	6.5%	6.6%	2.5%	3.1%	3.3%	3.5%	3.6%	3.8%	3.5%	3.0%	2.9%	2.6%
Lexus	1.9%	2.0%	2.0%	2.1%	2.0%	1.6%	1.7%	1.8%	1.9%	2.0%	1.9%	1.8%
Buick	1.5%	1.2%	2.5%	1.0%	1.3%	1.4%	1.2%	1.3%	1.4%	1.3%	1.3%	1.3%
Chrysler	3.7%	3.4%	1.2%	1.7%	1.7%	1.7%	2.1%	1.9%	1.9%	1.9%	1.3%	1.1%
Cadillac	1.4%	1.3%	1.1%	1.0%	1.3%	1.2%	1.0%	1.2%	1.0%	1.0%	1.0%	0.9%
Acura	1.2%	1.1%	1.0%	1.0%	1.2%	1.0%	1.1%	1.1%	1.0%	1.0%	0.9%	0.9%
Infiniti	0.7%	0.8%	0.8%	0.8%	0.9%	0.8%	0.8%	0.7%	0.7%	0.8%	0.8%	0.9%
Lincoln	0.7%	0.8%	0.9%	0.8%	0.7%	0.7%	0.6%	0.5%	0.6%	0.6%	0.6%	0.6%
Total Share	77.9%	77.6%	74.7%	73.8%	75.6%	74.6%	74.0%	74.4%	74.0%	73.9%	73.8%	73.7%

Under such a selection criterion, for each OEM we have at least one mass-market volume brand like Chevrolet, Buick, GMC for General Motors, Ford for Ford, Chrysler, Dodge for Chrysler, Toyota for Toyota, Honda for Honda, Nissan for Nissan and also one luxury brand for each company like Cadillac for General Motors, Lincoln for Ford, Jeep for Chrysler, Lexus for Toyota, Acura for Honda, Infinite for Nissan. In addition, the total share of selected brands in each OEM represents majority of the sales volumes. The Toyota and Lexus brands for Toyota, Honda and Acura brands for Honda, Nissan and Infiniti brands for Nissan, Ford and Lincoln brands for Ford almost represent 100% of the sales for the four OEMs while Chevrolet, GMC, Buick and Cadillac represent all the volumes after 2009's GM bankruptcy and more than 80% of the sales volumes before 2009. Only Chrysler has relatively lower sale volume representation of the Chrysler, Dodge and Jeep brands due to historical reasons like Ram brand holds large sales volume in truck space but does not have long enough historical data in our analysis. However, these three Chrysler brands still represent almost 70% of total Chrysler sales.

Table 4.3 Selected Brands' Shares within Each Selected Automotive OEM

	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Chevrolet	59.0%	58.9%	60.6%	64.6%	70.7%	70.9%	71.5%	69.8%	69.3%	69.0%	68.9%	68.8%
GMC	11.2%	12.7%	12.2%	12.2%	15.2%	15.8%	16.2%	16.2%	17.1%	18.1%	18.0%	18.7%
Buick	5.9%	4.9%	11.3%	4.9%	6.8%	7.1%	6.7%	7.3%	7.8%	7.2%	7.5%	7.3%
Cadillac	5.6%	5.6%	4.9%	5.3%	6.8%	6.1%	5.6%	6.7%	5.8%	5.7%	5.6%	5.2%
Total GM	81.7%	82.0%	89.1%	87.0%	99.5%	100%	100%	100%	100%	100%	100%	100%
Ford	95.0%	79.6%	83.1%	81.4%	89.3%	95.8%	96.1%	96.9%	96.2%	96.1%	95.7%	95.7%
Lincoln	4.7%	5.0%	5.1%	4.7%	4.1%	4.2%	3.9%	3.1%	3.8%	3.9%	4.3%	4.3%
Total Ford	99.7%	84.7%	88.2%	86.1%	93.5%	100%	100%	100%	100%	100%	100%	100%
Chrysler	28.2%	26.1%	11.1%	19.0%	18.1%	15.9%	18.4%	16.4%	14.7%	14.4%	10.3%	9.1%
Dodge	50.3%	51.0%	23.0%	35.2%	35.1%	32.7%	31.6%	32.8%	27.3%	22.9%	22.5%	21.6%
Jeep	21.5%	22.9%	34.2%	24.9%	26.6%	30.8%	28.9%	26.7%	32.9%	38.3%	41.0%	40.0%
Total Chrysler	100%	100%	68.3%	79.1%	79.8%	79.4%	78.9%	75.9%	74.8%	75.6%	73.8%	70.6%
Toyota	70.5%	84.5%	83.9%	89.2%	84.2%	84.5%	84.7%	85.3%	84.4%	84.0%	86.0%	87.5%
Lexus	11.1%	12.9%	13.0%	12.9%	13.2%	12.4%	11.8%	12.6%	13.1%	13.8%	13.5%	12.5%
Total Toyota	81.6%	97.3%	96.9%	100%	97.4%	96.9%	96.5%	97.9%	97.6%	97.8%	99.5%	100%
Honda	86.7%	88.4%	89.9%	90.8%	89.6%	88.9%	88.8%	88.8%	89.1%	88.8%	90.1%	90.6%
Acura	13.3%	11.6%	9.6%	9.2%	11.3%	11.1%	11.2%	11.2%	10.9%	11.2%	9.9%	9.4%
Total Honda	100%	100%	99.5%	100%	100%	100%	100%	100%	100%	100%	100%	100%
Nissan	88.1%	88.1%	88.1%	89.5%	88.5%	90.2%	89.9%	91.3%	91.5%	91.0%	91.2%	90.4%
Infiniti	11.9%	11.9%	11.3%	10.5%	11.5%	9.8%	10.1%	8.8%	8.5%	9.0%	8.8%	9.6%
Total Nissan	100%	100%	99.4%	100%	100%	100%	100%	100%	100%	100%	100%	100%

This OEM and brand selection enables us to not only study the cross-company effects, but also could compare the performance of OEMs by country such as domestic OEMs versus the Japanese OEMs as well as the luxury brands versus the mass market volume brands.

Table 4.4 Selected OEM, Country of Origin, Volume Brands and Luxury Brands

Selected OEM, Country of Origin, Volume Brands and Luxury Brands			
OEM	Country of Origin	Mass Market Volume Brands	Luxury Brands
General Motors	U.S.	Chevrolet, Buick, GMC	Cadillac
Ford	U.S.	Ford	Lincoln
Chrysler	U.S.	Chrysler, Dodge	Jeep
Toyota	Japan	Toyota	Lexus
Honda	Japan	Honda	Acura
Nissan	Japan	Nissan	Infinite

After deciding the OEMs and brands, we need to find reliable sources for automotive data. We obtained quality data from JD Power Initial Quality Studies which is a determined data source for quality data. We used OEM-Supplier Working Relationship Index from a highly respected purchasing expert John Henke and his consulting company Planning Perspectives Inc. We got the warranty sharing ratio data from Warranty Week which is focusing on warranty analysis. In the next sections, we will discuss each of the three data sets.

4.2 JD Power Initial Quality Studies

J.D. Power publishes two kinds of quality data yearly at the brand level, the Initial Quality Studies (IQS) and Vehicle Durability Studies (VDS). Both studies measure quality at each brand by the number of problems experienced per 100 vehicles, namely PP100. The lower scores on PP100 means fewer problems experienced per 100 vehicles, thus a higher product quality. For example, in 2017 IQS survey, Kia is the best brand with in average 72 problems reported per 100 vehicles, and Fiat is the worst brand with in average 163 problems experienced per 100 vehicles. Further details can be obtained at http://www.jdpower.com/. This quality metric (PP100) is consistent with our definition of the quality metric in our principal agent model, therefore a good proxy for the quality variable $Q(e_M, e_S)$. IQS surveys customers during the first 90 days of ownership and is an index to measure new vehicles coming out of assembly line. On the other side, VDS surveys customers during a 12-month period after an ownership of new vehicles for 3 years. For example, the 2017 JD Power VDS study examines original owners of 2014 model-year vehicles on problems experienced during the past 12 months. Therefore, VDS is focusing more on the long-term quality and the IQS is more focusing on the initial quality. In order to eliminate the potential time lag when examining the relationship between quality and other factors, we choose IQS instead of VDS.

From the time series plot Figure 4.1, we can see that in general Japanese OEMs perform better in terms of initial quality with lower PP100 than the US OEMs historically, but the trend is that everybody is improving and US OEMs are closing the gap.

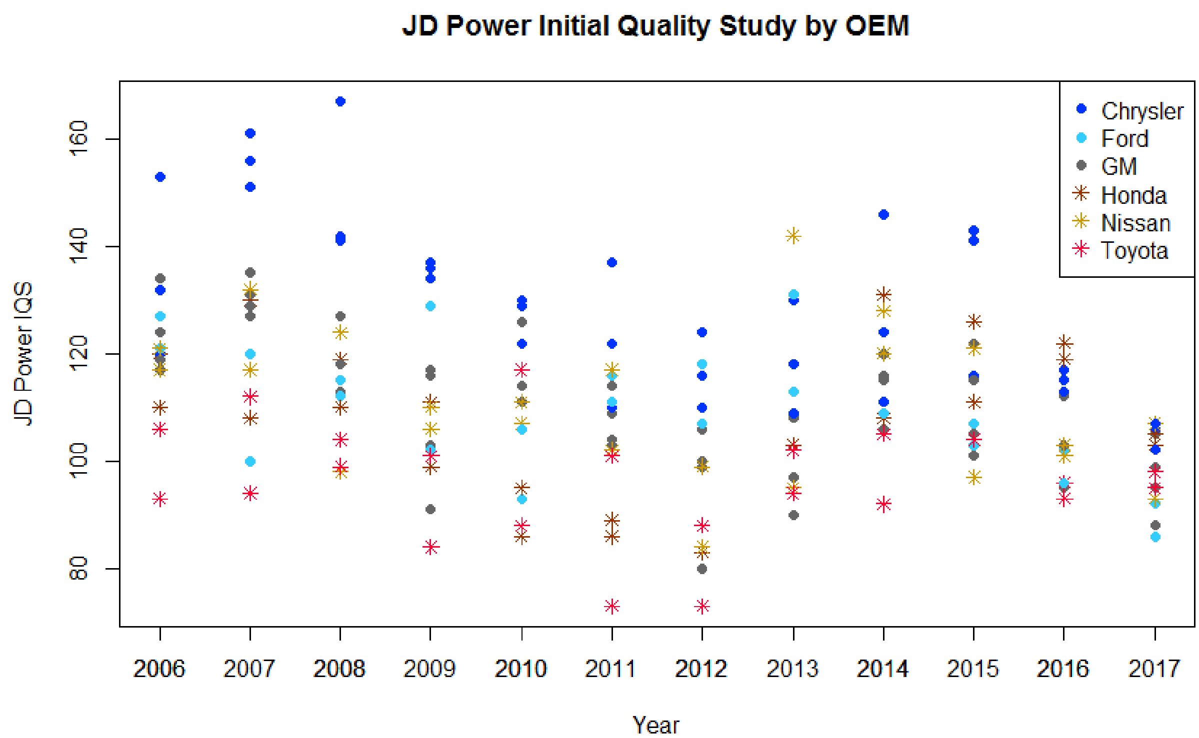


Figure 4.1 JD Power IQS on Selected Brands 2006-2017

4.3 Manufacturer-Supplier Working Relation Index

We choose the Working Relation Index (WRI 2006-2017) published yearly by John Henke and his third-party consulting company Planning Perspectives Inc. (PPI) as the proxy for manufacturer's effort in our principal agent model. WRI is well known in automotive industry and recognized as the benchmark to measure manufacturer's effort of maintaining supplier working relations for the automotive industry and has appeared in several academic journal publications ((Henke Jr. J.W., Zhang C., 2010), (Zhang C., Henke Jr. J.W., Griffith D.A., 2009), (Zhang C., Viswanathan S., Henke Jr. J.W., 2011)). PPI conducts annual surveys starting from 2002 on tier-1 North America suppliers of the six North America automotive OEMs including domestic OEMs GM, Ford, and Chrysler and Japanese OEMs Toyota North America, Honda North America and Nissan North America. The tier-1 suppliers grade the OEMs' efforts on the following five major categories: Buyer-Supplier Relationship, Buyer Communication, Buyer Help, Buyer Hindrance and Supplier Profit Opportunity (and 16 sub-categories see Figure 4.2) in six purchasing areas:

Body-in-White, Chassis, Electronics & Electrical, Exterior, Interior and Powertrain (and 14 commodity areas see Figure 4.3).

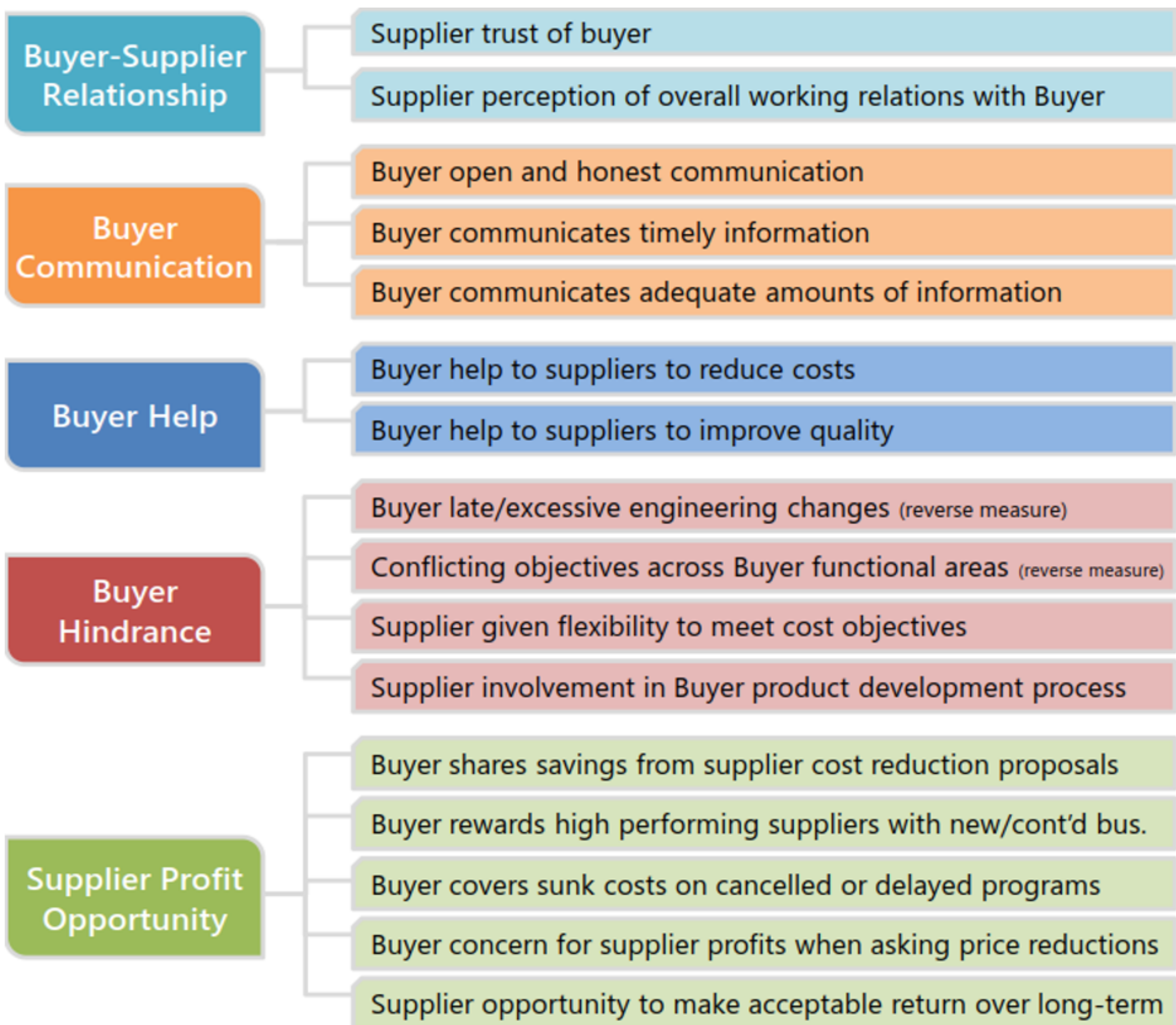


Figure 4.2 Buyer's Efforts on Improving Manufacturer-Supplier Relationship

After collecting questionnaires from each supplier's respondent, scores in each component in each area can be aggregated and weighted to make an overall Working Relation Index ranging from 0 to 500. The higher the score means the higher the manufacturer's efforts on improving the OEM-supplier relationship. For example, if the total Working Relation Index score falls between 0 to 250, it means the manufacturer is doing a poor or very poor job on working with suppliers and the suppliers are very likely showing some adversarial behaviors to fight against the OEM. If the score is between 250 and 350, the OEM is doing an adequate job in maintaining the manufacturer-supplier relationship and the suppliers are likely doing their best to supply the manufacturer. A good or very good relationship, in the range between 350 and 500, requires the OEM to demonstrate great behaviors such as collaboration, open and honest while the supplier will proactively respond to all the manufacturer's needs. Figure 4.4 includes all the details of the Working Relation Index. Further details can be obtained at http://www.ppi1.com/working-relations-index/.

Basis of Participants' Answers

OEM – Commodity Area Buying Situation

OEMs: FCA Nissan GM Ford Honda Toyota

Purchasing Areas	Commodity Areas
Body-in-White	1. Body-in-white, stampings, frames
Chassis	2. Fuel handling, exhaust systems, cooling systems and components 3. Brake systems, steering, suspensions 4. Tires, wheels, and mechanisms
Electronics & Electrical	5. Electronics, ICs, PC boards, ECUs, sensors, wiring 6. Audio systems, safety systems, security systems
Exterior	7. Body panels, exterior ornamentation, fascias, sealing 8. Exterior lighting, mirrors, glass, wiper systems, latches
Interior	9. Seat systems, restraint systems, airbags 10. IPs, consoles, interior trim, headliners, carpeting, matting 11. Heating, ventilation, A/C
Powertrain	12. Engines and engine components 13. Transmissions and transmission components 14. Axles, traction systems and components

Figure 4.3 Purchasing Areas and Commodity Areas in PPI Survey

As an example of the representativeness and comprehensiveness of the dataset, the 2017 WRI survey interviewed 652 sales personnel from 467 Tier 1 suppliers in North America, among which 40 out of 50 are the top 50 North America Tier 1 suppliers, 68 out of 100 are the top 100 North

America Tier 1 suppliers. The survey composed roughly 64% of annual sales of the six OEMs annual buy and represented 1974 OEM-supplier buying situations (Planning Perspectives, Inc., 2017).

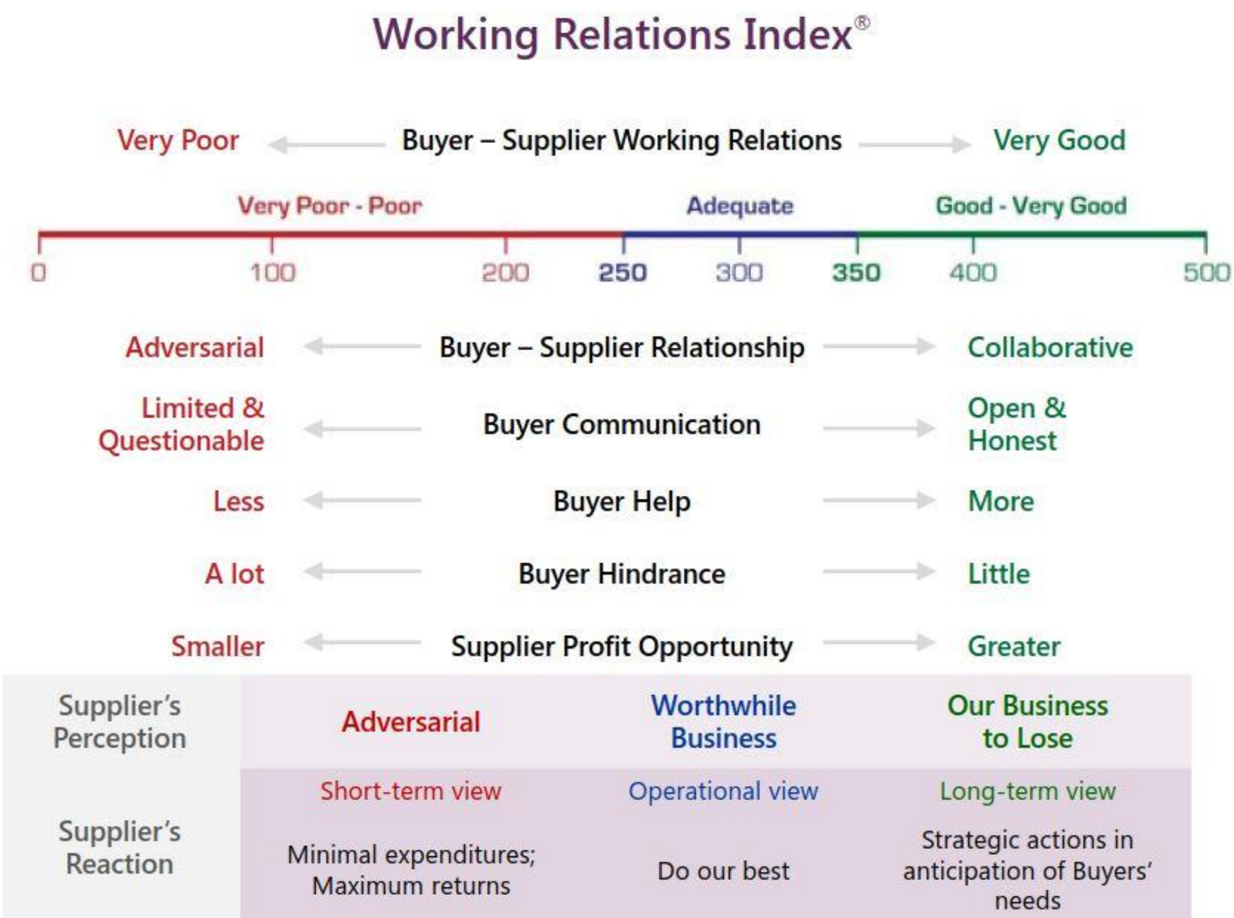


Figure 4.4 Working Relation Index Guideline

WRI provides us an excellent proxy to measure the manufacturer's effort to maintain a good OEM-supplier relationship and could be used to represent the manufacturer's effort level e_M in our principal agent model.

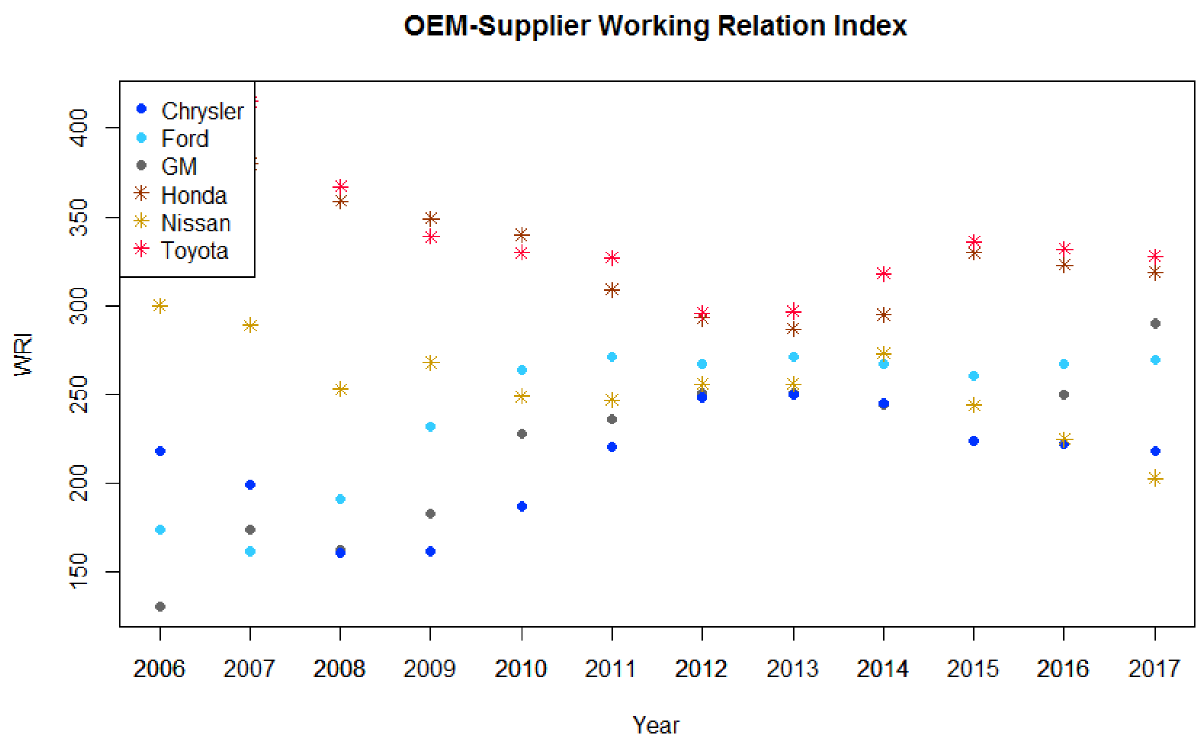


Figure 4.5 OEM-Supplier Working Relation Index Time Series

From the time series plot Figure 4.5, we can see that in general Japanese OEMs maintained a better relationship with their suppliers than the US OEMs historically, but US OEMs closed the gap after the financial crisis during years 2010-2014 and then GM and Ford kept trending up to be close to the top performers Toyota and Honda while Nissan and Chrysler fell to the bottom of the ranks.

4.4 Warranty Week Warranty Sharing Ratio

Like the manufacturer's effort data, cost sharing ratio data is extremely hard to find a good proxy with good data available. We find a unique data source from Warranty Week (Arnum E., 2018). They collected the warranty claims and accruals for 50 automotive US based OEMs and 120 automotive US based suppliers to study the split of warranty payments between OEMs and suppliers. It is of course not going to represent 100% of the U.S. automotive market even this pool is big enough to include most of the major OEMs like Detroit big three, most of the Japanese OEMs' North American units, most of the German and Korean OEMs' North American units and most of the North America based suppliers and most of the North American units of foreign suppliers. International OEMs' units like Toyota, VW, Volvo Trucks and international suppliers such as Magna, Autoliv and Robert Bosch are not included in the pool. In addition, many of these U.S. based companies' export business is not counted in the pool as well. However, since the sample size for both OEMs (50) and suppliers (120) are big and includes most of the major players, we think the warranty sharing ratio calculated from all the OEMs' warranty costs and all the suppliers' warranty costs is close enough to proxy the warranty sharing ratio λ in our simple principal agent model. In Warranty Week's raw data, they summed up the warranty payments by OEMs and by suppliers every quarter, for example in the first quarter of 2017, the 50 automotive OEMs paid in total \$2.508 billion dollars representing 86% of the overall total warranty payments, while the 120 suppliers paid \$408 million dollars representing the remaining 14%. In this way, Warranty Week is able to calculate the percentage of warranty payments paid by OEMs and suppliers by quarters from 1st quarter in 2003 to 4st quarter in 2017. Since our other two data resources are in years, we aggregated the quarterly spending by OEMs and suppliers to yearly level in Figure 4.6.

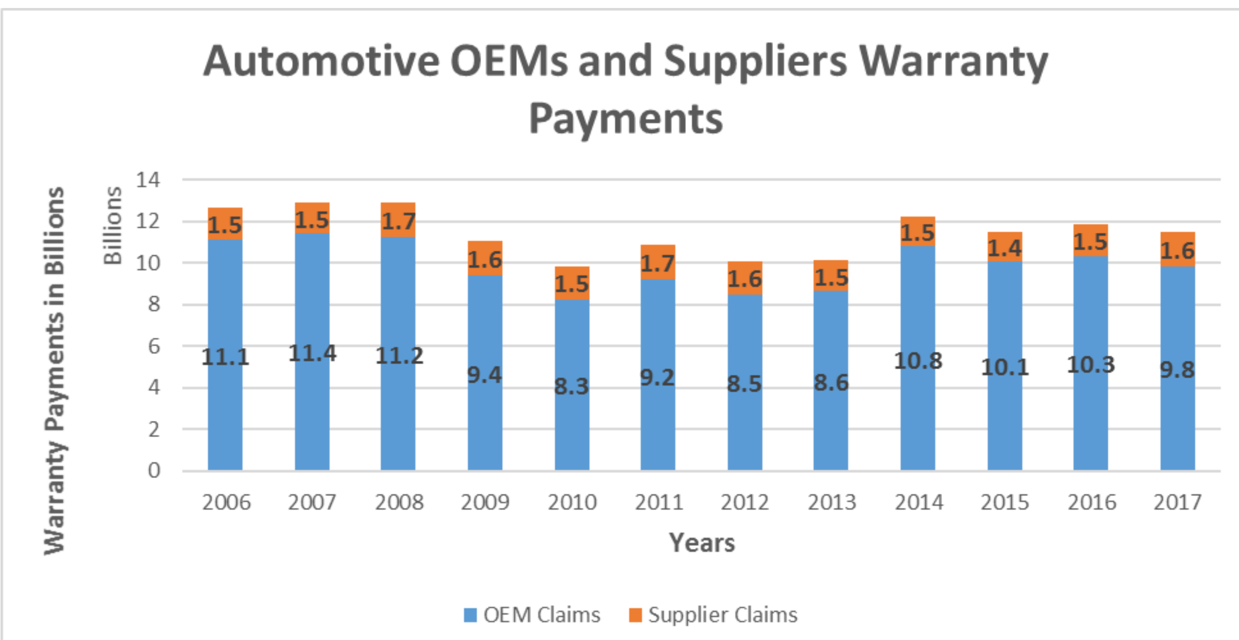


Figure 4.6 Automotive OEMs and Suppliers Warranty Payments Time Series

To assess how representative of the total OEMs and Suppliers warranty payment data from Warranty Week, we compared the total amount of payments to the automotive Original Equipment

Suppliers Association (OESA) data (OESA, 2013) we found in Automotive News (Sedgwick, 2013): "According to the Original Equipment Suppliers Association (OESA), warranty claims reported by publicly traded companies range from $11 billion to $13 billion a year." The lump sum warranty payment of the 50 OEMs and 120 major suppliers is right on the ballpark with 2010 as the lowest around 9.8 Billion and 2007 and 2008 as highest around 12.9 Billion. This validates the representativeness question of the sample that Warranty Week used to calculate the warranty sharing ratio (WSR). Since there is no way to separate supplier's warranty payments to each company and no suppliers report their warranty costs in such details, we decide to use the same warranty sharing ratio (WSR) across different brands and different OEMs. To check the validity of this approximation, we have found two important evidences. Firstly, according to Warranty Week (Arnum E., 2017) "After the recession struck in 2008, OEMs made a concerted effort to recover more of their warranty expenses from their suppliers." For example, General Motors announced a new warranty share program based on 50/50 warranty sharing ratio between GM and its suppliers in 2010, called GM Ordinary Warranty Cost Allocation Terms (Aiello M.A., Spillane T.B., Uetz A.M., 2010). All new or renewed purchase contracts since then would follow this 50/50 warranty share rule. Under this rule, GM could ask suppliers to reimburse up to 50 percent of any "Ordinary Warranty Cost" for the parts sold by that supplier. There are several cases that could be considered as supplier's warranty costs: (1) replacement of the supplier's part are involved in warranty payments, (2) dealer identified the supplier's part in repairing, (3) the dealer submitted the labor code that is linked to the supplier's part during repairing. However, cases such as "Extraordinary Warranty Cost", "Service Parts Mark-up", "Dealer Good Will", or "No Trouble Found" (NTF) will be excluded in Ordinary Warranty Cost. That's the reason why the real warranty cost split by OEMs (80-90%) is a lot higher than 50%. Other OEMs like Ford, Chrysler, Toyota, Honda and Nissan also have similar type of Warranty Cost Allocation Programs with their suppliers (Kohler W.J., Watson L.M., 2012). Secondly, since 1990s suppliers have built R&D capabilities on the key components and subsystems and OEMs are more and more replying on suppliers to purchase the most advanced technologies. Therefore, it becomes more and more common that multiple OEMs are using the same supplier. For example, TRW Automotive Holding Corps supplies occupant-restraint systems, steering, braking & engine components, electronic safety & security systems, fasteners, suspension to almost all North American OEMs, and it generated $6 Billion sales revenues of automotive parts in 2014 and was ranked number 7 in the

Automotive News Top 100 North America suppliers in 2015. According to IHS SupplierBusiness, in the mid-size Sedan market TRW supplies "Foamed Seal Pressure Relief Valve" to 2013 Nissan Altima, "Wiring Clip" to 2013 Ford Fusion, "Molded Seal Pressure Relieve Valve" to 2013 Honda Accord, "Brake Corner Assembly" to 2014 Chevrolet Impala, "Airbag Control Unit" to 2012 Chrysler 200 and "Pressure Relief Valve" to 2012 Toyota Camry. As most of the tier 1 suppliers are supplying parts and components to almost every OEM, we think it makes sense that their warranty sharing contract would be similar across OEMs.

From Figure 4.7, we can see that in the mid-2000, OEMs shared around 10% of their warranty costs with their suppliers and gradually increased suppliers' share during the financial crisis period, and then took more responsibility after the economy recovered.

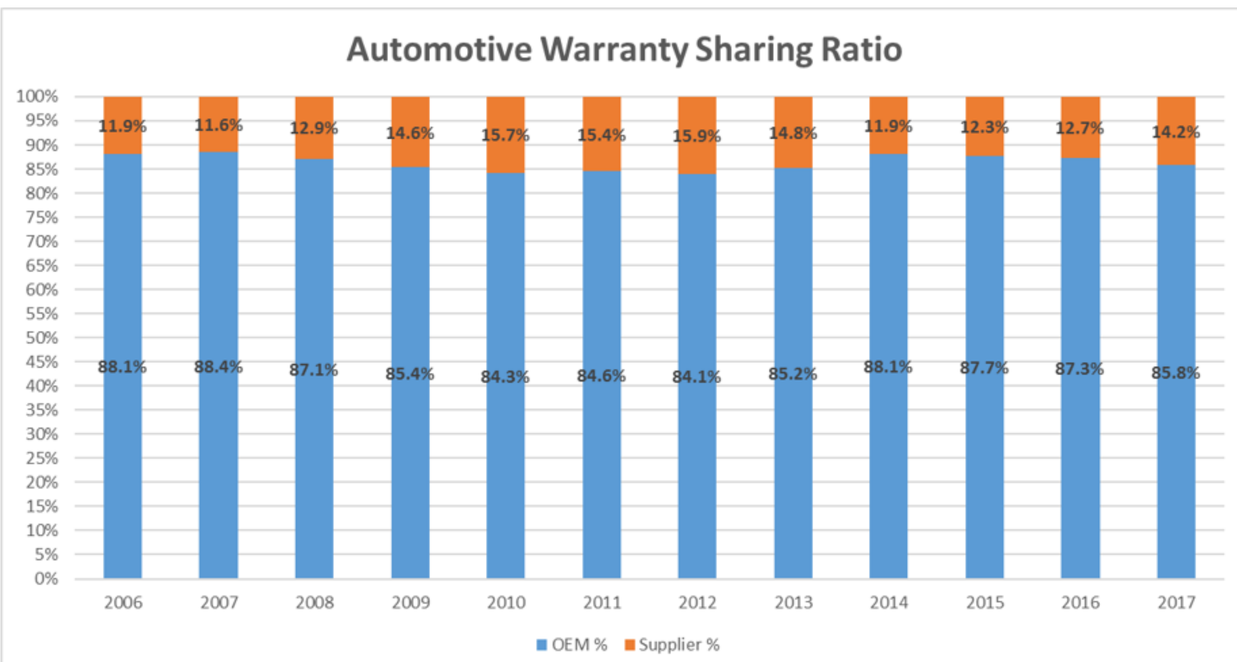


Figure 4.7 Automotive OEMs Warranty Sharing Ratio Time Series

We summarize the descriptive statistics in the Table 2 Summary Statistics of Data for Regression Analysis below at different levels. Japanese OEMs in general have significant higher Working Relation Index compared to US OEMs, which partially explained why the Japanese OEMs also have a lower problem per 100 vehicles (PP100). Also, please notice that the Working Relation Index (WRI) is at OEM level so the same values across brands for a single OEM. The Warranty

Sharing Ratio (WSR) is at industrial level so every OEM every brand uses the same WSR. Only JD Power IQS PP100 is at brand level. This shortfall is due to the data constraints and availabilities, but as we have explained it only affects the accuracy of the modeling but will not overturn the trends we have discovered in the following session.

4.5 Summary Statistics of Empirical Data

Here is a summary of data for further analysis.

Table 4.5 Summary Statistics of Empirical Data

Summary Statistics of Data for Regression Analysis

	Working Relation Index (WRI)			Warranty Sharing Ratio (WSR)			IQS		
Variables	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
Chevrolet	131	219	290	11.6%	13.7%	15.9%	88	106	129
Buick	131	219	290	11.6%	13.7%	15.9%	95	113	134
GMC	131	219	290	11.6%	13.7%	15.9%	90	112	131
Cadillac	131	219	290	11.6%	13.7%	15.9%	80	109	135
OEM-GM	131	219	290	11.6%	13.7%	15.9%	80	110	135
Ford	162	241	271	11.6%	13.7%	15.9%	86	111	131
Lincoln	162	241	271	11.6%	13.7%	15.9%	92	109	129
OEM-Ford	162	241	271	11.6%	13.7%	15.9%	86	110	129
Chrysler	161	213	250	11.6%	13.7%	15.9%	102	123	151
Dodge	161	213	250	11.6%	13.7%	15.9%	106	129	156
Jeep	161	213	250	11.6%	13.7%	15.9%	107	134	167
OEM-Chrysler	161	213	250	11.6%	13.7%	15.9%	102	129	167
US OEMs	131	222	290	11.6%	13.7%	15.9%	80	116	167
Toyota	296	341	415	11.6%	13.7%	15.9%	88	102	117
Lexus	296	341	415	11.6%	13.7%	15.9%	73	91	104
OEM-Toyota	296	341	415	11.6%	13.7%	15.9%	73	97	117
Honda	287	329	380	11.6%	13.7%	15.9%	83	103	119
Acura	287	329	380	11.6%	13.7%	15.9%	84	110	131
OEM-Honda	287	329	380	11.6%	13.7%	15.9%	83	107	131
Nissan	203	255	300	11.6%	13.7%	15.9%	93	116	142
Infiniti	203	255	300	11.6%	13.7%	15.9%	84	105	128
OEM-Nissan	203	255	300	11.6%	13.7%	15.9%	84	111	142
Japanese OEMs	203	309	415	11.6%	13.7%	15.9%	73	105	142
All OEMs	131	257	415	11.6%	13.7%	15.9%	73	112	167

Next, we will investigate the pairwise interactions of the three variables we just discussed.

4.6 Descriptive Analysis I: JD Power IQS vs Warranty Sharing Ratio

Figure 4.8 shows the scatter plot of the warranty sharing ratio data and the quality data JD Power IQS (IQS) as well as the linear trends. All the domestic OEMs are on the top for higher PP100 and the Japanese OEMs are on the bottom for lower PP100. The warranty sharing ratio is assumed to be the same for all the OEMs regardless of domestic or foreign. Looking at the trend lines, both domestic OEMs and Japanese OEMs are improving their quality by sharing more warranty costs to their suppliers, which validates the results or assumptions that a lot of existing literatures were using, such as (Wan H., Xu X., Ni T., 2013), (Lim W.S., 2001), (Chao, G., Iravani S., Savaskan C., 2009) and (Zhu K., Zhang R.Q., Tsung F., 2007).

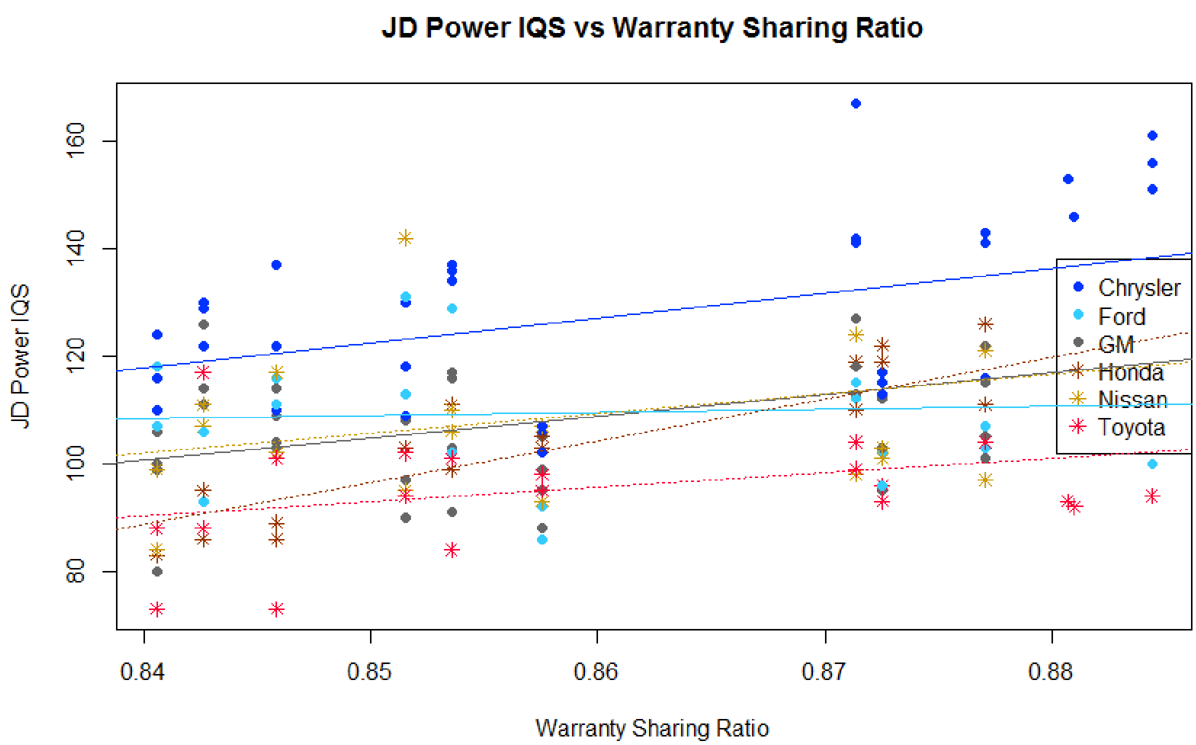


Figure 4.8 JD Power IQS vs Warranty Sharing Ratio

4.7 Descriptive Analysis II: JD Power IQS vs Working Relation Index

Figure 4.9 shows the scatter plot of the manufacturer's effort data OEM-Supplier Working Relation Index (WRI) and the quality data JD Power IQS (IQS) as well as the linear trends. All the Japanese OEMs are on the right bottom part of the figure, meaning a better OEM-Supplier working relation/higher WRI score and a higher quality/lower PP100. All the US OEMs are on the left upper part of the figure, indicating a worse OEM-Supplier working relation/lower WRI score and a lower quality/higher PP100. Looking at the trend lines, there is a clear separation between domestic OEMs and Japanese OEMs. Improving OEM-Supplier working relation will improve initial product quality for all the Detroit big 3 domestic OEMs, but won't necessarily help for Japanese OEMs. It also shows that suppliers may play a bigger role in quality improvement for American OEMs comparing to Japanese OEMs.

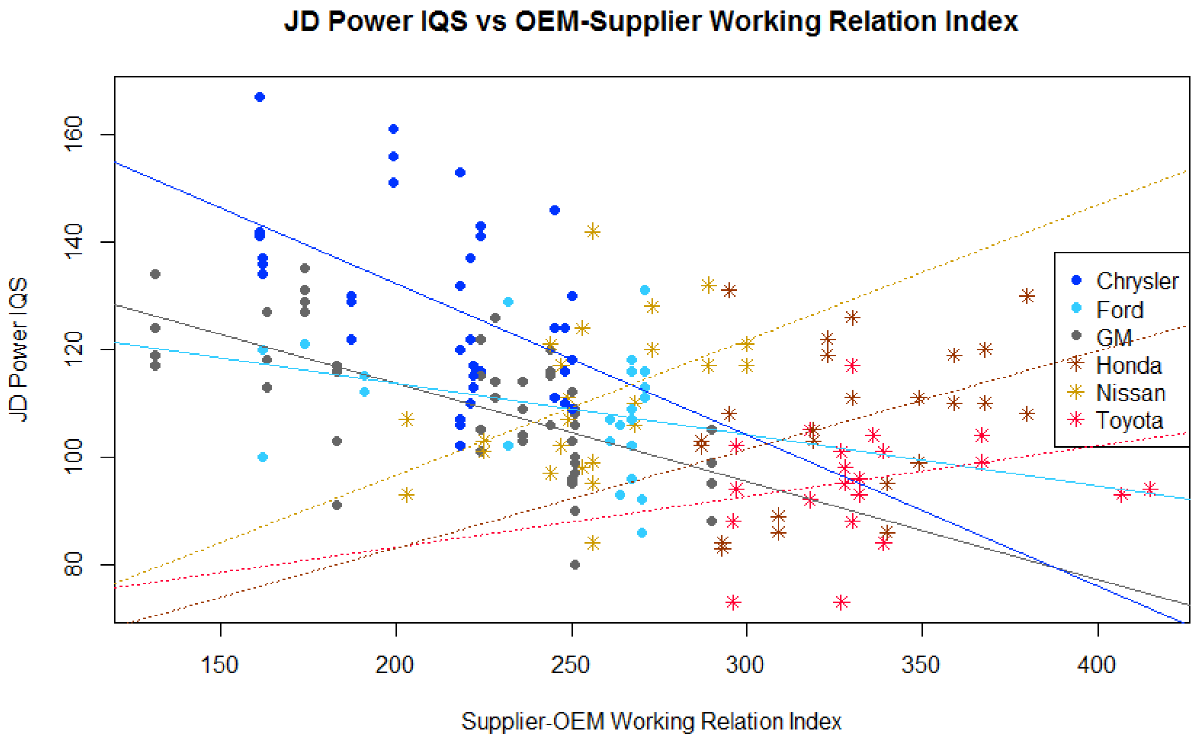


Figure 4.9 JD Power IQS vs OEM-Supplier Working Relation Index

4.8 Descriptive Analysis III: JD Power IQS vs Warranty Sharing Ratio

Figure 4.10 shows the scatter plot of the warranty sharing ratio data and the OEM-Supplier working relation index (WRI) as well as the linear trends. Again, there's a clear separation between Japanese OEMs and US OEMs. All the domestic OEMs are on the bottom because of a low OEM-Supplier working relation index and the Japanese OEMs are on the top because of a better working relation with their suppliers. The warranty sharing ratio is assumed to be the same for all the OEMs regardless of domestic or foreign. Looking at the trend lines, Japanese OEMs are improving their working relations with their suppliers by increasing their own shares of warranty costs. However, sharing less percentage of warranty costs to suppliers won't necessarily improve the working relation index for domestic OEMs.

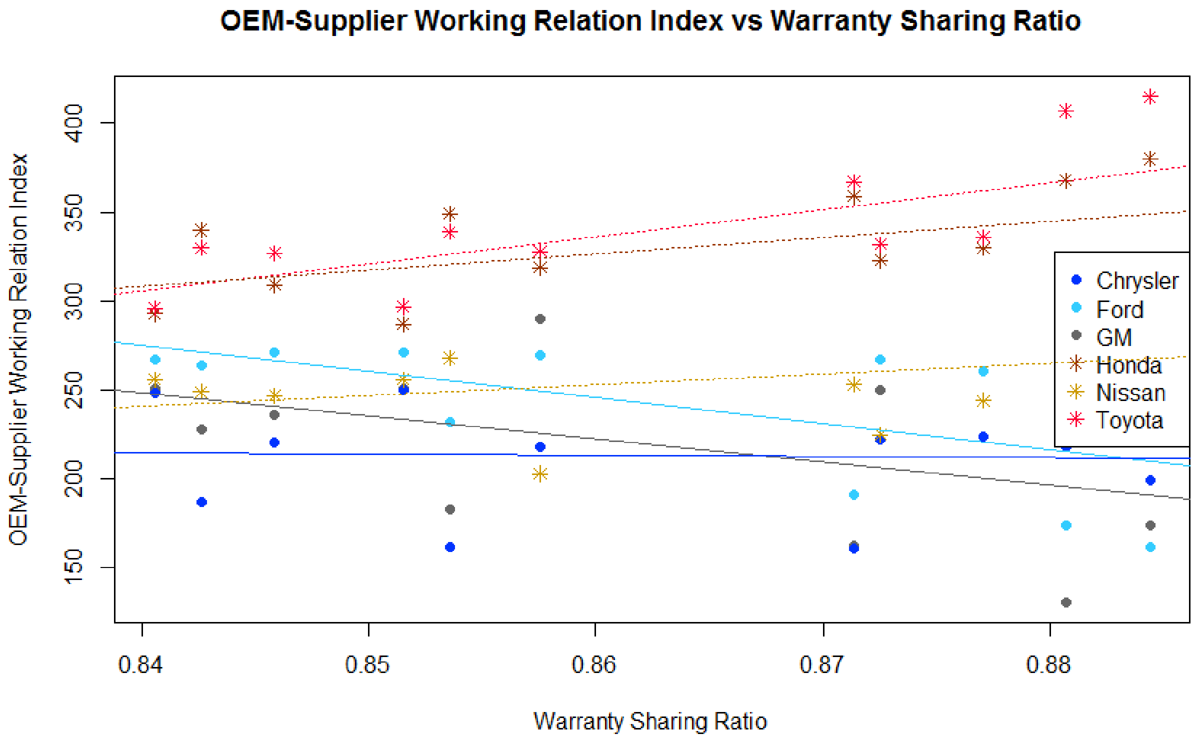


Figure 4.10 Warranty Sharing Ratio vs OEM-Supplier Working Relation Index

In the next Section, we will integrate the three groups of data into our principal agent model to empirically test the principal agent model and validate the Hypotheses that we developed in Section 3.

5. TESTING HYPOTHESES

In this section, we will integrate the three groups of data introduced in Section 4 to the principal agent model where the product quality $Q(e_M, e_S)$ is estimated by PP100 (JD Power IQS), the manufacturer's effort e_M is estimated by the WRI (Working Relation Index), the damage cost sharing ratio that the supplier bears λ is estimated by WSR (Warranty Week warranty cost sharing ratio). Then we can translate the Proposition 1 and Proposition 2 into three regression models for empirical testing.

5.1 First Order Conditions to Regression Models

For the supplier's problem in the principal agent model, Proposition 1 can be translated into regression model (1) below:

$$\frac{1}{PP100} = \alpha_1 WSR + \alpha_2 WSR * WRI \tag{1}$$

Where $\alpha_1 = \frac{\theta_S}{\eta_S} > 0$, $\alpha_2 = \frac{\theta_J}{\eta_S} > 0$ as $\theta_S, \theta_J, \eta_S > 0$.

Similarly, for the manufacturer's problem, Proposition 2 can be translated into regression models (2) and (3) below:

$$WRI = \beta_0 + \beta_1 \sqrt{\frac{1-WSR}{WSR}} \tag{2}$$

Where $\beta_0 = -\frac{\theta_S}{\theta_J} < 0$ and $\beta_1 = \sqrt{\frac{\eta_S}{\theta_J \eta_M}} > 0$ as $\theta_S, \theta_J, \eta_S, \eta_M > 0$.

$$PP100 = \frac{\gamma}{\sqrt{WSR(1-WSR)}} \tag{3}$$

Where $\gamma = \sqrt{\frac{\eta_M \eta_S}{\theta_J}}$.

Unlike most of the empirical principal agent model literature which just focuses on testing the relationships derived from principal agent theory but does not adopt the same functional forms as the principal agent model results when running regression, we will strictly follow the functional forms derived from the simple principal agent model to test the empirical results on top of the principal agent model functional relationships.

In the next section, we will set up the testing hypotheses and discuss our empirical testing strategies.

5.2 Testing Hypotheses: Weak Consistency

By transforming the principal agent model propositions into three regression models in section 5.1, we have already restricted our selection of the functional relationships between variables that we are testing. However, although most of the principal agent model empirical literature does not impose such a restrictive constraint on functional forms, we argue that the principal agent model results captured the real functional relationships between variables so it is vitally important to conduct the empirical testing against the regression models (1), (2), (3) derived from principal agent model propositions.

There are two types of empirical testing that we can conduct. First, we can regress the data on regression models (1), (2), (3) to check the signs of the parameter values against the theoretical results. In this approach, if the signs of the parameter values are consistent with the theoretical results, we call the principal agent model achieved **Weak Consistency**.

Hypothesis 1: *Estimate the parameters $\hat{\alpha}_1$ and $\hat{\alpha}_2$ by regressing regression model (1). If $\hat{\alpha}_1 > 0$ and $\hat{\alpha}_2 > 0$, then the empirical results are consistent with theoretical results in sign for the supplier's problem.*

If Hypothesis 1 is validated, then the supplier's problem in our principal agent model achieves weak consistency.

Hypothesis 2: *Estimate the parameters $\hat{\beta}_0$ and $\hat{\beta}_1$ by regressing regression model (2). If $\hat{\beta}_0 < 0$ and $\hat{\beta}_1 > 0$, then the empirical results are consistent with theoretical results in sign for the manufacturer's problem.*

Hypothesis 3: *Estimate the parameters $\hat{\gamma}$ by regressing regression model (3). If $\hat{\gamma} > 0$, then the empirical results are consistent with theoretical results in sign for the manufacturer's problem.*

If Hypothesis 2 and Hypothesis 3 are both validated, then the manufacturer's problem in our principal agent model achieves weak consistency.

Although we have not found any past literature adopt the functional forms from first order conditions to test the weak consistency, there are some prior arts testing the signs of the parameters to check the consistency (see e.g. (Boose, Mary A., 1990), (Allen D.W., Lueck D., 1999), (Cawley J., Philipson T., 1999)). Here we want to argue that testing principal agent model solely on signs of the parameters is not strong enough to validate the consistency. That is why in the following section, we want to propose a new methodology to test the parameters in value which we call it strong consistency.

5.3 Testing Hypotheses: Strong Consistency

Achieving Hypotheses 1-3 will weakly validate the principal agent model. However, weak consistency only tests the signs of the parameter values against the theoretical principal agent model results. Next Hypotheses 4-6 are testing the consistency of the parameter values estimated from the regression models (1), (2), (3), against the theoretical derivations. If the parameter values estimated from the regression models are consistent with the theoretical results, we call the principal agent model achieved **Strong Consistency**.

Hypothesis 4: *Estimate parameters $\hat{\alpha}_1$ and $\hat{\alpha}_2$ in regression model (1) and $\hat{\beta}_0$ and $\hat{\beta}_1$ in regression model (2). Validate if $\frac{\hat{\alpha}_1}{\hat{\alpha}_2} = -\hat{\beta}_0$.*

Proof. From regression models (1) and (2), we know that $\alpha_1 = \frac{\theta_S}{\eta_S}$, $\alpha_2 = \frac{\theta_J}{\eta_S}$, and $\beta_0 = -\frac{\theta_S}{\theta_J}$.

Therefore, $\frac{\alpha_1}{\alpha_2} = \frac{\frac{\theta_S}{\eta_S}}{\frac{\theta_J}{\eta_S}} = \frac{\theta_S}{\theta_J} = -\beta_0$.

Hypothesis 5: *Estimate parameters $\hat{\alpha}_1$ and $\hat{\alpha}_2$ in regression model (1), $\hat{\beta}_0$ and $\hat{\beta}_1$ in regression model (2) and $\hat{\gamma}$ in regression model (3) independently. Validate if $\frac{\hat{\beta}_0}{\hat{\beta}_1 \hat{\alpha}_1} = -\hat{\gamma}$.*

Proof. From regression models (1), (2), (3), we know that $\alpha_1 = \frac{\theta_S}{\eta_S}$, $\beta_0 = -\frac{\theta_S}{\theta_J}$, $\beta_1 = \sqrt{\frac{\eta_S}{\theta_J \eta_M}}$.

Therefore, $\frac{\beta_0}{\beta_1 \alpha_1} = -\frac{\theta_S}{\theta_J}\sqrt{\frac{\theta_J \eta_M}{\eta_S}}\frac{\eta_S}{\theta_S} = -\theta_S\sqrt{\frac{\eta_M}{\theta_J \eta_S}}\frac{\eta_S}{\theta_S} = -\sqrt{\frac{\eta_M \eta_S}{\theta_J}} = -\gamma$.

Hypothesis 6: *Estimate parameters $\hat{\alpha}_1$ and $\hat{\alpha}_2$ in regression model (1), $\hat{\beta}_0$ and $\hat{\beta}_1$ in regression model (2) and $\hat{\gamma}$ in regression model (3) independently. Validate if $\hat{\beta}_1 \hat{\alpha}_2 \hat{\gamma} = 1$.*

Proof. From Hypothesis 4, we can get $\frac{\alpha_1}{\alpha_2} = -\beta_0$. Plug into Hypothesis 5 to replace β_0, we can get $\frac{-\frac{\alpha_1}{\alpha_2}}{\beta_1 \alpha_1} = -\gamma$. Therefore, $\beta_1 \alpha_2 \gamma = 1$.

Hypotheses 4-6 are testing the consistencies of the parameter values estimated from three independent regression models with the theoretical results. If the estimated parameter values show consistent relationship with the principal agent model theoretical results, then we have achieved the strong consistency.

In this chapter, we define two types of consistency testing for principal agent models. The weak consistency can be achieved by satisfying Hypotheses 1-3 while the strong consistency can be fulfilled by satisfying Hypotheses 4-6. In the next chapter, we are going to test the principal agent model against the empirical data to check if the model can achieve weak consistency and/or strong consistency.

6. PRINCIPAL AGENT MODEL VALIDATION

Chapter 5 described our empirical validation strategies. By strictly following the functional relationships derived from the theoretical principal agent model, we are not only testing the relationships between variables, but also validating the functional relationships that principal agent model described. Given the availability of our empirical data, by aggregating the data in different ways we can perform the empirical testing at four different levels - $\mathbf{Model_A}$: Automotive Industrial level by pooling all data together regardless of country of origin, different OEMs or different brands; $\mathbf{Model_C}$: Country of Origin Level by categorizing data into Domestic OEMs versus Foreign (Japanese) OEMs; $\mathbf{Model_{LM}}$: Luxury brands versus Mass Market volume brands level by combining Cadillac, Lincoln, Jeep, Lexus, Acura, Infiniti into luxury category and the rest of the brands into mass market category; $\mathbf{Model_O}$: OEM levels such as GM, Ford, Chrysler, Toyota, Honda and Nissan.

6.1 Ordinary Least Square Regression Results

As the purpose of the study is to validate the principal agent model with real world automotive data, we choose to run the simplest Ordinary Least Square (OLS) regression on regression models (1) - (3) at the four levels of data aggregation defined above. Then after estimating all the parameter values, Hypotheses 1-6 will be tested against to check if weak consistency and/or strong consistency can be achieved. Table 6.1 summarized all the empirical regression results for regression model (1) – (3). (*) indicated the significant level of the parameters.

Table 6.1 Ordinary Least Square Regression Results

Ordinary Least Square Regression Results							
Models	Levels	Statistics	Regression (1)		Regression (2)		Regression (3)
			α_1	α_2	β_0	β_1	γ
Model_{A}	Industry	Estimate	4.793e-02***	7.238e-05***	290.20***	-13.33	38.167***
		P-Value	< 2e-16	2.26e-09	4.4e-05	0.627	<2e-16
Model_{C}	US	Estimate	5.271e-02***	4.854e-05	442.07***	-87.19***	39.744***
		P-Value	< 2e-16	0.0358	1.45e-12	0.000114	<2e-16
	Japanese	Estimate	5.123e-02***	6.449e-05**	62.38	97.45**	35.801***
		P-Value	2.16e-11	0.00285	0.45585	0.00413	<2e-16
Model_{LM}	Luxury	Estimate	4.664e-02***	8.150e-05***	255.781*	4.222	37.539***
		P-Value	2.14e-13	5.24e-05	0.0243	0.9237	<2e-16
	Mass Market	Estimate	4.969e-02***	6.239e-05***	313.14***	-25.04	38.586***
		P-Value	< 2e-16	4.11e-05	0.000555	0.472679	<2e-16
Model_{O}	GM	Estimate	5.984e-02***	3.123e-05	530.51***	-123.43***	37.706***
		P-Value	9.06e-13	0.254	2.1e-07	0.000803	<2e-16
	Ford	Estimate	8.179e-02***	-6.146e-05	594.48***	-139.79**	37.444***
		P-Value	9.33e-06	0.296	9.33e-06	0.00253	<2e-16
	Chrysler	Estimate	3.519e-02***	1.041e-04*	222.551**	-3.815	43.995***
		P-Value	0.000201	0.012057	0.00575	0.89899	<2e-16
	Toyota	Estimate	4.849e-02*	8.247e-05	-26.14	145.36***	33.000***
		P-Value	0.0189	0.1600	0.764693	0.000315	<2e-16
	Honda	Estimate	7.210e-02***	-6.930e-06	113.07	85.62*	36.598***
		P-Value	4.62e-06	0.851	0.1853	0.0156	<2e-16
	Nissan	Estimate	7.684e-02	-4.004e-05	100.21	61.38*	37.806***
		P-Value	0.000567	0.599124	0.1882	0.0469	<2e-16

From the Table 6.1 above, we can see that most of the parameters in the regression models are significant and the P-values are small.

6.2 Hypotheses Testing Results

With the parameter values estimated for regression models (1) – (3), we can test the weak consistency of the principal agent model by checking the signs of Hypotheses (1) – (3) against the theoretical results. In addition, we can calculate the values to check the parameter relationships against the theoretical outcomes to test the strong consistency. Key parameter signs and values are summarized in the table 4 below.

Table 6.2 Hypotheses Testing Results

Hypotheses Testing Results												
Models	Levels	H1		H2		H3	H4		H5		H6	
		$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\gamma}$	$\frac{\hat{\alpha}_1}{\hat{\alpha}_2}$	$-\hat{\beta}_0$	$\frac{\hat{\beta}_0}{\hat{\beta}_1\hat{\alpha}_1}$	$-\hat{\gamma}$	$\beta_1\alpha_2\gamma$	1
Model_A	Industry	+	+	+	-	+	662.20	-290.20	-454.21	-38.17	-0.04	1
Model_C	US	+	+	+	-	+	1085.91	-442.07	-96.19	-39.74	-0.17	1
	Japanese	+	+	+	+	+	794.39	-62.38	12.50	-35.80	0.22	1
Model_LM	Luxury	+	+	+	+	+	572.27	-255.78	1298.95	-37.54	0.01	1
	Mass Market	+	+	+	-	+	796.44	-313.14	-251.67	-38.59	-0.06	1
Model_O	GM	+	+	+	-	+	1916.11	-530.51	-71.83	-37.71	-0.15	1
	Ford	+	-	+	-	+	-1330.78	-594.48	-51.99	-37.44	0.32	1
	Chrysler	+	+	+	-	+	338.04	-222.55	-1657.74	-44.00	-0.02	1
	Toyota	+	+	-	+	+	587.97	26.14	-3.71	-33.00	0.40	1
	Honda	+	-	+	+	+	-10404.04	-113.07	18.32	-36.60	-0.02	1
	Nissan	+	-	+	+	+	-1919.08	-100.21	21.25	-37.81	-0.09	1

6.3 Validation Summary

Next, we can judge the Hypotheses 1 – 6 below in Table 6.3. For Hypotheses 1 – 3, if the signs are consistent with the theoretical results, we say "Y" stand for "Yes", otherwise, "N" meaning "No". For Hypotheses 4 – 6, if the signs of the two sides of the equations are opposite to each other, then we assign "N", for example the Model_A Hypothesis 4 has positive value on the left-hand side but negative value on the right-hand side. If the two sides of equations have consistent signs but very different in values in scales, we assign "S" standing for "Sort of". For example, Model_O Chrysler Hypothesis 5 has LHS = -1657.74 and RHS = -44.00. Although the LHS and RHS are consistent in sign, the values are different in digits so we classify these cases into "S". Also in these cases, the strong consistency in terms of parameter values are very unlikely to achieve. Finally, if the two sides of the equations have consistent signs and also the values are close enough to each other, for example Model_C US Hypothesis 5 has LHS = -96.19 and RHS = -39.74 or Model_O Toyota Hypothesis 6 has LHS = 0.4 and RHS = 1, we say there is no significant evidence to reject the strong consistency statistically. Therefore, in these cases we assign "Y" to them.

Table 6.3 Hypotheses Validation Results

Table 5: Hypotheses Validations									
Models	Levels	H1		H2		H3	H4	H5	H6
Model_A	Industry	Y	Y	N	N	Y	N	S	N
Model_C	US	Y	Y	N	N	Y	N	Y	N
	Japanese	Y	Y	N	Y	Y	N	N	Y
Model_{LM}	Luxury	Y	Y	N	Y	Y	N	N	S
	Mass Market	Y	Y	N	N	Y	N	S	N
Model_O	GM	Y	Y	N	N	Y	N	Y	N
	Ford	Y	N	N	N	Y	S	Y	Y
	Chrysler	Y	Y	N	N	Y	N	S	N
	Toyota	Y	Y	Y	Y	Y	S	S	Y
	Honda	Y	N	N	Y	Y	S	N	N
	Nissan	Y	N	N	Y	Y	S	N	N

From Table 6.3, we can evaluate the weak consistency and strong consistency of the principal agent model at different levels. Toyota gets 6 “Y” and 2 “S” and is the only company that demonstrates weak consistency over all Hypotheses 1 – 3. Although Toyota does not quite fulfill the Hypotheses 4 and 5, it showed consistency in signs and was the closest to strong consistency company alongside Ford. However, Ford only got 2 “Y” in Hypotheses 1 – 3 and showed little consistency in weak consistency. If we look at the study by country of origin Model_C, Japanese OEMs showed more consistency than US OEMs, especially in terms of weak consistency. US OEMs demonstrated little to none consistency comparing to principal agent model predictions. To summarize, Toyota is the very best principal agent OEM in our empirical validation followed by Honda and Nissan. Japanese OEMs in general demonstrated more consistency than US OEMs, especially in weak consistency test. US OEMs showed least consistent behaviors to principal agent model predictions. Among all US OEMs, Chrysler is the least principal agent OEM followed by Ford and GM.

In the next section, we will make cost comparisons based on the conclusions of our empirical validation and try to answer the questions: what are the benefits of being more principal agent. In other words, is there any cost advantage for behaving more principal agent. If the answer is yes, can we quantify the benefits.

7. PRINCIPAL AGENT MODEL IMPLICATIONS

In this chapter, we will study what are the implications for OEMs behaving principal agent. Specifically, whether Toyota is more cost efficient by behaving closely to what principal agent model suggests on manufacturer-supplier relationship management comparing to other OEMs especially the US counterparties who behave inconsistent to principal agent model suggestions. If there is a cost efficiency advantage by following the principal agent behavior on manufacturer-supplier relationship, can we quantify the differences and make an inference on cost efficiency implications?

7.1 Methodology

In order to estimate the cost efficiency implications for different OEMs deviating from principal agent model predictions, we want to preserve and incorporate the intrinsic relationship between different parameters within principal agent model. The regression model (1) and (2) in Section 5.1 are derived from the Section 3 A Simple Principal Agent Model and regression model (1) and (2) preserve the parameter relationships that $\beta_0 = -\frac{\alpha_1}{\alpha_2}$ and $\beta_1 = \sqrt{\frac{1}{\alpha_2 \eta_M}}$. Therefore, unlike Section 5 that we treated the three regression models separately as independent relations and estimated 5 free parameters $\alpha_1, \alpha_2, \beta_0, \beta_1, \gamma$ to validate the principal agent relations, here after incorporating the intrinsic principal agent model relationships we only have three free parameters $\alpha_1, \alpha_2, \eta_M$ to estimate against the empirical data. After plugging in $\beta_0 = -\frac{\alpha_1}{\alpha_2}$ and $\beta_1 = \sqrt{\frac{1}{\alpha_2 \eta_M}}$ into regression model (2), the regression model (1) and (2) become model (4) and (5) which are linked as below.

$$\frac{1}{PP100} = \alpha_1 WSR + \alpha_2 WSR * WRI \tag{4}$$

$$WRI = -\frac{\alpha_1}{\alpha_2} + \sqrt{\frac{1}{\alpha_2 \eta_M}} \sqrt{\frac{1-WSR}{WSR}} \tag{5}$$

To estimate the three free parameters $\alpha_1, \alpha_2, \eta_M$ in regression models (4) and (5), we need to minimize the sum of square error in model (4) against the empirical data as well as the sum of square error in model (5) against the empirical data at the same time. Technically, this becomes a multiple objective optimization problem and there are many ways to handle multiple objectives optimization problem in literature. Here because our focus is to estimate parameters α_1, α_2, η_M in a consistent way for the 6 OEMs rather than proposing a fancy methodology to solve the multi-objective optimization problem, we decide to pick the easiest approach which is to weight the normalized sum of squared errors between model predictions and real data with a scale factor ω to bridge the two regression functions. With the weight parameter ω, regression models (4) and (5) become

$$\left(\frac{1}{\widehat{PP100}}-\frac{1}{PP100}\right)^2+\omega\left(\widehat{WRI}-WRI\right)^2=\left(\alpha_1 WSR+\alpha_2 WSR*WRI-\frac{1}{PP100}\right)^2+\omega\left(-\frac{\alpha_1}{\alpha_2}+\sqrt{\frac{1}{\alpha_2\eta_M}}\sqrt{\frac{1-WSR}{WSR}}-WRI\right)^2 \tag{6}$$

Treat equation (6) like a regression model to estimate the free parameter $\alpha_1, \alpha_2, \eta_M$ by minimizing the sum of square errors against the empirical data.

7.2 Mathematical Formulation

Equation (6) incorporated the principal agent model intrinsic relationships between parameters and also weighted the two regression models into one optimization problem. However, the structure of equation (6) is nonlinear on free parameters $\alpha_1, \alpha_2, \eta_M$ and by definition $\alpha_1, \alpha_2, \eta_M$ are all positive in values. Therefore, this optimization problem is a linearly constrained nonlinear optimization problem and can be written as following.

$$\min_{\alpha_1,\alpha_2,\eta_M} \frac{\sum_{i=1}^{N}\left\{\left(\alpha_1 WSR_i+\alpha_2 WSR_i*WRI_i-\frac{1}{PP100_i}\right)^2+\omega\left(-\frac{\alpha_1}{\alpha_2}+\sqrt{\frac{1}{\alpha_2\eta_M}}\sqrt{\frac{1-WSR_i}{WSR_i}}-WRI_i\right)^2\right\}}{N}$$

$$\text{s.t.}\quad \alpha_1\geq 0, \alpha_2\geq 0, \eta_M\geq 0$$

Solving nonlinear optimization problems are technically challenging because it involves techniques to search for global optimums and at the same time must avoid local optimums. Adding the constraints to the signs of the parameters add another layer of complications to the problem.

Therefore, we decide to use one section below to discuss about the optimization solver and why we believe the R "constrOptim" package provides the best solution to our problem.

7.3 Optimization Solver

As we know nonlinear constrained optimization problems are hard to solve, but fortunately there are many similar linear constrained nonlinear optimization problems such as maximum likelihood problems in Statistics that shared similar structure, and the Statistics software R has a cutting-edge package called "constrOptim" to help us solve this type of problems. "constrOptim" is designed to use an adaptive barrier algorithm to minimize a function subject to linear inequality constraints. To use the "constrOptim" package, we must pick a good initial starting point in the interior of the feasible regions, for example $\alpha_1 > 0, \alpha_2 > 0, \eta_M > 0$, but the optimal values could be on the boundary. If the algorithm performed correctly and can find the global minimum values, the results should not be too sensitive to the initial value pick. This is something we will check in the Chapter 8 Sensitivity Analysis. Otherwise, "constrOptim" package can handle both optimizations with or without a gradient. If the user does not supply the gradient, a gradient free method named "Nelder-Mead" will be used. Otherwise, if a gradient is provided, a Quasi-Newton method "BFGS" will be the main solution method. In our optimization problem, gradient could be calculated and supplied to the "constrOptim" solver, we will also discuss the impact of with or without gradient in the sensitivity analysis. For other details regarding the barrier algorithm and the General-purpose Optimization package "optim", please refer to R Document: Linearly Constrained Optimization and General-purpose Optimization. Again, our focus for this section is to estimate the three free parameters $\alpha_1, \alpha_2, \eta_M$ in a consistent and effective way and we left most of the technical details to the readers.

To balance the error contributions from the two linked functions, we need to supply a value for the weight ω to make the error contribution from the 1st term $\sum_{i=1}^{N}\left\{\left(\alpha_1 WSR_i + \alpha_2 WSR_i * WRI_i - \frac{1}{PP100_i}\right)^2\right\}$ similar to the error contribution from the 2nd term$\sum_{i=1}^{N}\left\{\omega\left(-\frac{\alpha_1}{\alpha_2} + \sqrt{\frac{1}{\alpha_2\eta_M}}\sqrt{\frac{1-WSR_i}{WSR_i}} - WRI_i\right)^2\right\}$. We know from the historical data that $PP100$ is in the scale around 100 and the manufacturer-supply relation index WRI is usually in the scale of 200-400, so there is scale difference of $\frac{1}{20000}$ between $\frac{1}{PP100}$ and WRI. Also, since we are minimizing the squared error, after some trial and errors we found ω = $\left(\frac{1}{20000}\right)^2 = \frac{1}{4*10^8}$ will balance the error contributions of the two terms to almost the same amount. In the sensitivity analysis, we will change the value of ω to check how sensitive the results depending on the choice of the weight parameter ω and how the unbalanced weight on two error terms will affect the conclusions.

Also since the performance of a numerical optimization method like the "constrOptim" package depends on a good pick of the initial starting value, we want to carefully choose the initial values of $\alpha_1, \alpha_2, \eta_M$ to trigger the nonlinear search algorithm. In the 1st error term, we know $\frac{1}{PP100}$ is in the scale of 0.01, WSR is in the scale of 0.1 and $WSR * WRI$ is in the scale of 10, we could predict that $\alpha_1 < 0.1$ and $\alpha_2 < 0.01$ By trial and error, we found that a random starting value of $0 < \alpha_1 < 0.01, 0 < \alpha_2 < 0.01, 0 < \eta_M < 1$ could be a good initial starting point to our problem and it could lead to a relatively stable outcome.

After estimating the optimal values of the three parameters $\hat{a}_1, \hat{a}_2, \hat{\eta}_M$, we can estimate the values of WRI from equation (5) $\widehat{WRI} = -\frac{\hat{a}_1}{\hat{a}_2} + \sqrt{\frac{1}{\hat{a}_2\hat{\eta}_M}}\sqrt{\frac{1-WSR}{WSR}}$ and then estimate the quality index $PP100$ from equation (4) $\widehat{PP100} = \frac{1}{\hat{a}_1 WSR + \hat{a}_2 WSR * \widehat{WRI}}$. By comparing the estimated values of $\widehat{WRI}$ with the WRI data as well as the $\widehat{PP100}$ with $PP100$ data, we can validate our estimation process and make inferences on cost efficiency implications on different OEMs.

7.4 Parameter Estimations

By solving the optimization problem proposed in the previous section, we can estimate the important parameters $\alpha_1, \alpha_2, \eta_M$ and then infer the $\widehat{WRI}$ and $\widehat{PP100}$ from the regression models (4) and (5). Comparing η_M for different OEMs, we can make implications on the marginal effort cost as well as the total manufacturer's costs. All the empirical implication results are based on a point estimate which is derived from the minimum objective function value of 1000 randomized initial starting values in the parameter range $0 < \alpha_1 < 0.01, 0 < \alpha_2 < 0.01, 0 < \eta_M < 1$. Weight parameter is set to be $\omega = \left(\frac{1}{20000}\right)^2 = \frac{1}{4*10^8}$ and the gradient free method "Nelder-Mead" is the default algorithm. Sensitivity analysis regarding the initial values, weight parameter, optimization methods are delayed to the next section.

From the Table 7.1 below, we can summarize that Japanese OEMs in general have a lower sum of squared error (SSE) comparing to US OEMs which means the Japanese OEMs' empirical data fits the weighted principal agent optimization model better than US OEMs. That also validates our conclusion that Japanese OEMs are more consistent with principal agent model than US OEMs. The marginal effort cost parameter $\hat{\eta}_M$ showed that Toyota has the lowest value followed by the other two Japanese brands Honda and Nissan. US OEMs have much higher marginal effort costs than Japanese OEMs. Using Toyota as the benchmark, we can calculate the percentage difference relative to Toyota's marginal effort cost. In that measurement, Honda is estimated to have a 33% higher marginal effort cost than Toyota and Nissan is 61% higher. GM and Ford almost doubled the figure and Chrysler is trailed at 162% higher comparing to Toyota. This ranking by marginal effort cost parameter $\hat{\eta}_M$ is consistent with our conclusions in the principal agent model empirical validation which in another way validated our results.

Table 7.1 Parameter Estimation from Optimization

Parameters	Japanese OEMs			US OEMs		
	Toyota	Honda	Nissan	GM	Ford	Chrysler
$\hat{a}_1$	1.73E-02	6.53E-03	1.29E-02	1.13E-02	5.74E-03	4.04E-03
$\hat{a}_2$	1.75E-04	1.93E-04	2.11E-04	2.42E-04	2.41E-04	2.48E-04
$\hat{\eta}_M$	0.189	0.252	0.304	0.381	0.384	0.495
$\hat{\eta}_M$% Diff	*	33%	61%	101%	103%	162%
SSE	3.52E-06	2.92E-06	3.54E-06	1.00E-05	1.09E-05	3.95E-06

Compare the values of $\hat{a}_1$ and $\hat{a}_2$, Toyota and Nissan have a relatively higher value in $\hat{a}_1$ which infers that Toyota, Nissan are more capable to reduce defects by its own, while other OEMs are relying more on their suppliers to help them reduce quality issues, indicated by a larger $\hat{a}_2$ value. Cross checking the results in Table 7.1 with Figure 4.9 shows consistent stories. Suppliers may play a bigger role in quality improvement for American OEMs comparing to Japanese OEMs

7.5 Implications on Working Relation

After estimating the parameters $\alpha_1, \alpha_2, \eta_M$, we can derive the Working Relation Index from equation (5) $\widehat{WRI} = -\frac{\hat{a}_1}{\hat{a}_2} + \sqrt{\frac{1}{\hat{a}_2 \hat{\eta}_M}} \sqrt{\frac{1-WSR}{WSR}}$ and compare it to the real data. From Figure 7.1 below we can summarize that overall the estimated WRI all matches with real data relatively well. However, the estimates for the three Japanese OEMs followed the trend of the real data while the three US OEMs had a reversed trend of behavior comparing to the real data. Again, this is a strong evidence that Japanese OEMs are more behaving like what principal agent model suggests while US OEMs are behaving inconsistently to principal agent model results.

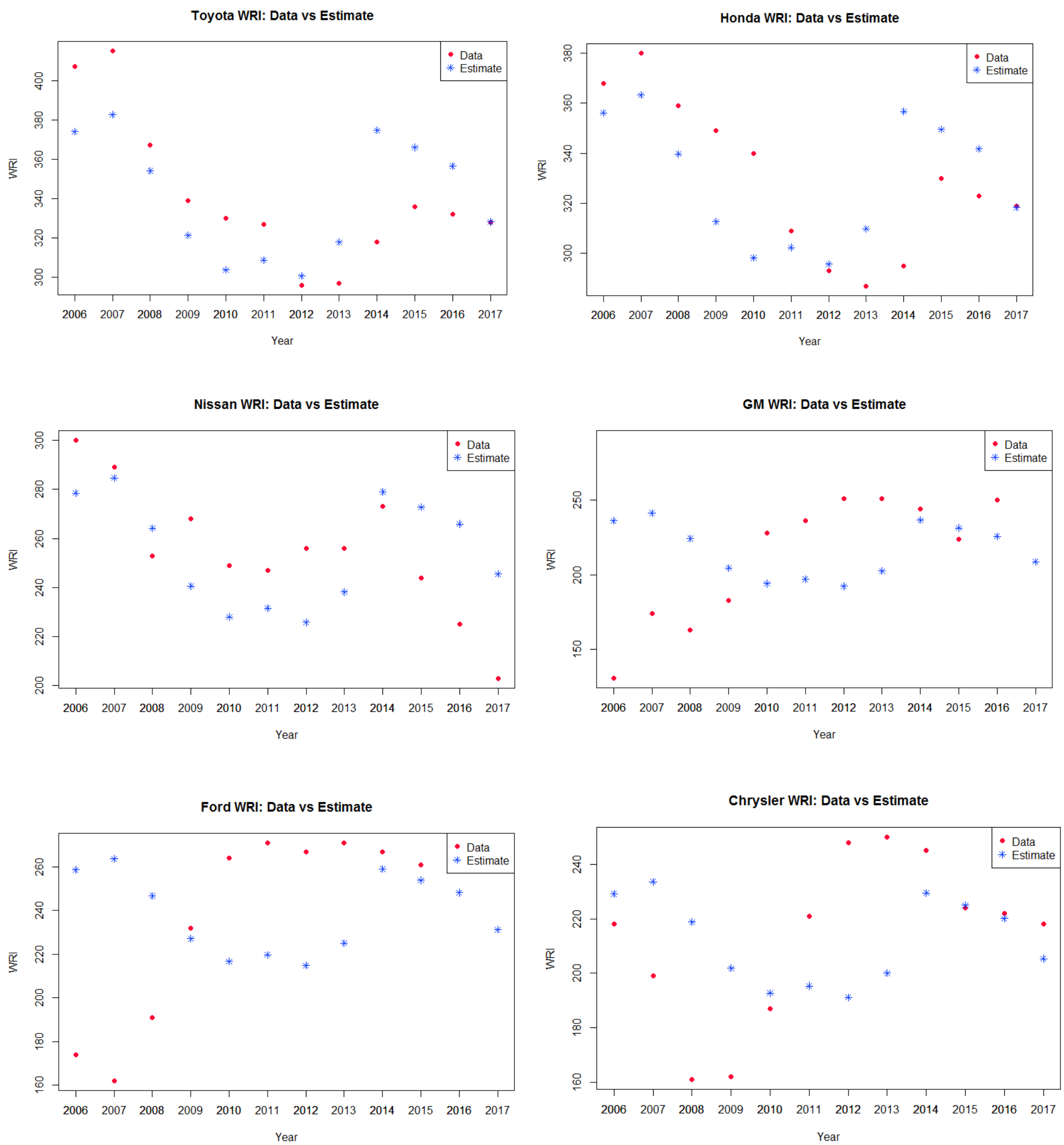


Figure 7.1 Estimated Working Relation Index vs Working Relation Index Data

7.6 Implications on Quality

After estimating the optimal values of the three parameters $\hat{a}_1, \hat{a}_2, \hat{\eta}_M$, and the working relation index $\widehat{WRI}$, we can go on to estimate the quality index $PP100$ from equation (4) $\widehat{PP10}0 = \frac{1}{\hat{a}_1 WSR + \hat{a}_2 WSR * \widehat{WRI}}$. From Figure 7.2 below we can summarize that the estimated quality metric PP100 all approximated the real data very well for each OEM. Just like what the real data suggests, the estimated PP100 from principal agent model also predicts that Toyota and Japanese OEMs in general have a lower defective rate comparing to US OEMs. For example, Toyota's PP100 number is ranged between 90 to 100, Honda is around 100-110 and Nissan is around 110-120; while for US OEMs GM is around 100-110, Ford is around 120-130 and Chrysler is around 130-140. There's a clear separation in quality for OEMs behaving more principal agent than OEMs who does not.

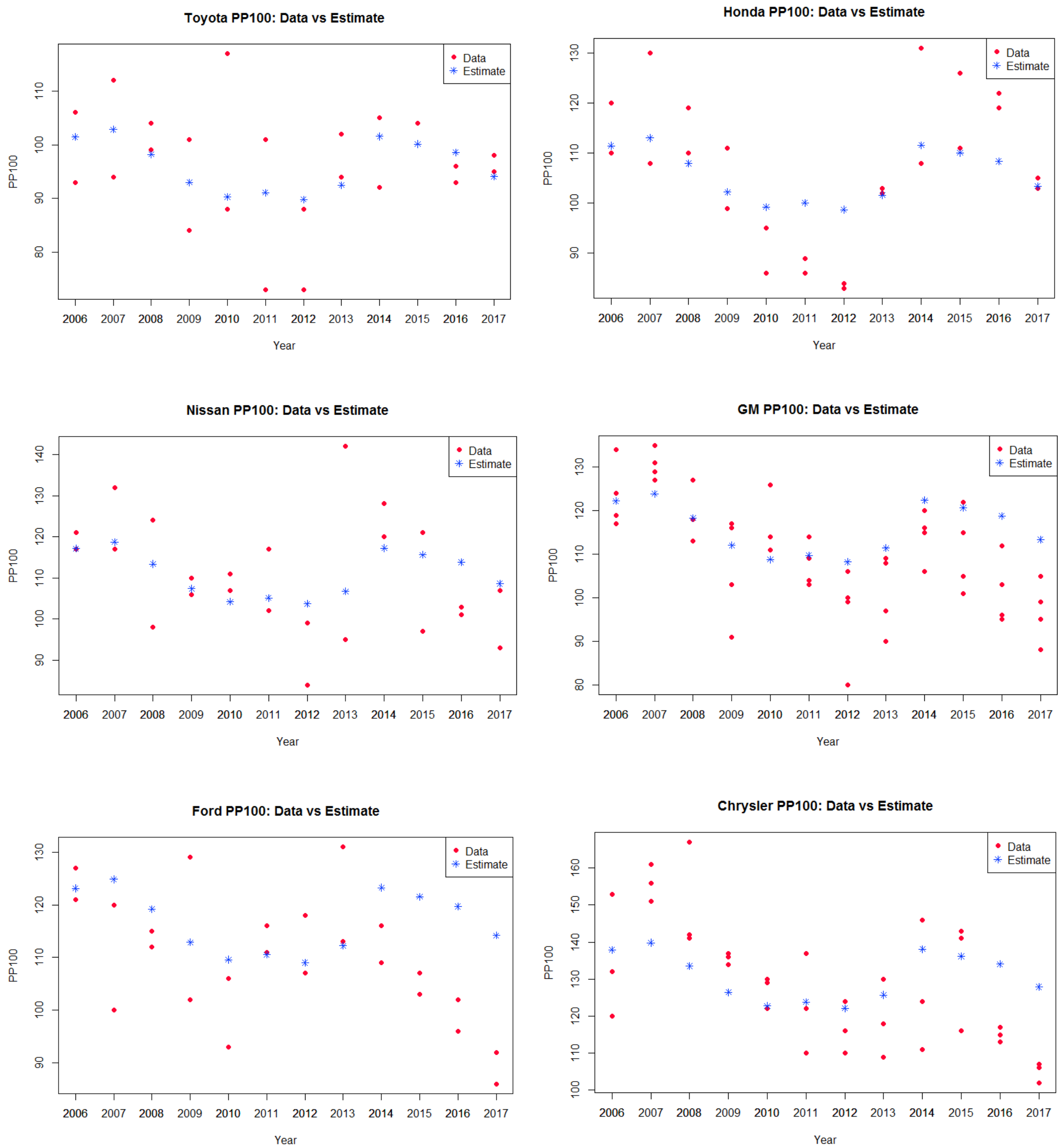


Figure 7.2 Estimated PP100 vs PP100 Data

7.7 Implications on Total Manufacturer's Costs

With parameter $\hat{\eta}_M$, $\widehat{WRI}$ and $\widehat{PP100}$ estimated from the optimization and regression models. The total manufacturer's cost can be calculated via the total manufacture's cost function $(1 - WSR)\widehat{PP100} + \hat{\eta}_M\widehat{WRI}$. From Figure 7.3 below we can summarize that the behavior of the total manufacturer cost function is similar to the WRI Figure 7.1. The Japanese OEMs followed the trend with the real data very well, while US OEMs do not follow the trend behavior. This gives another validation that Japanese OEMs are behaving consistently with principal agent models but US OEMs are behaving differently.

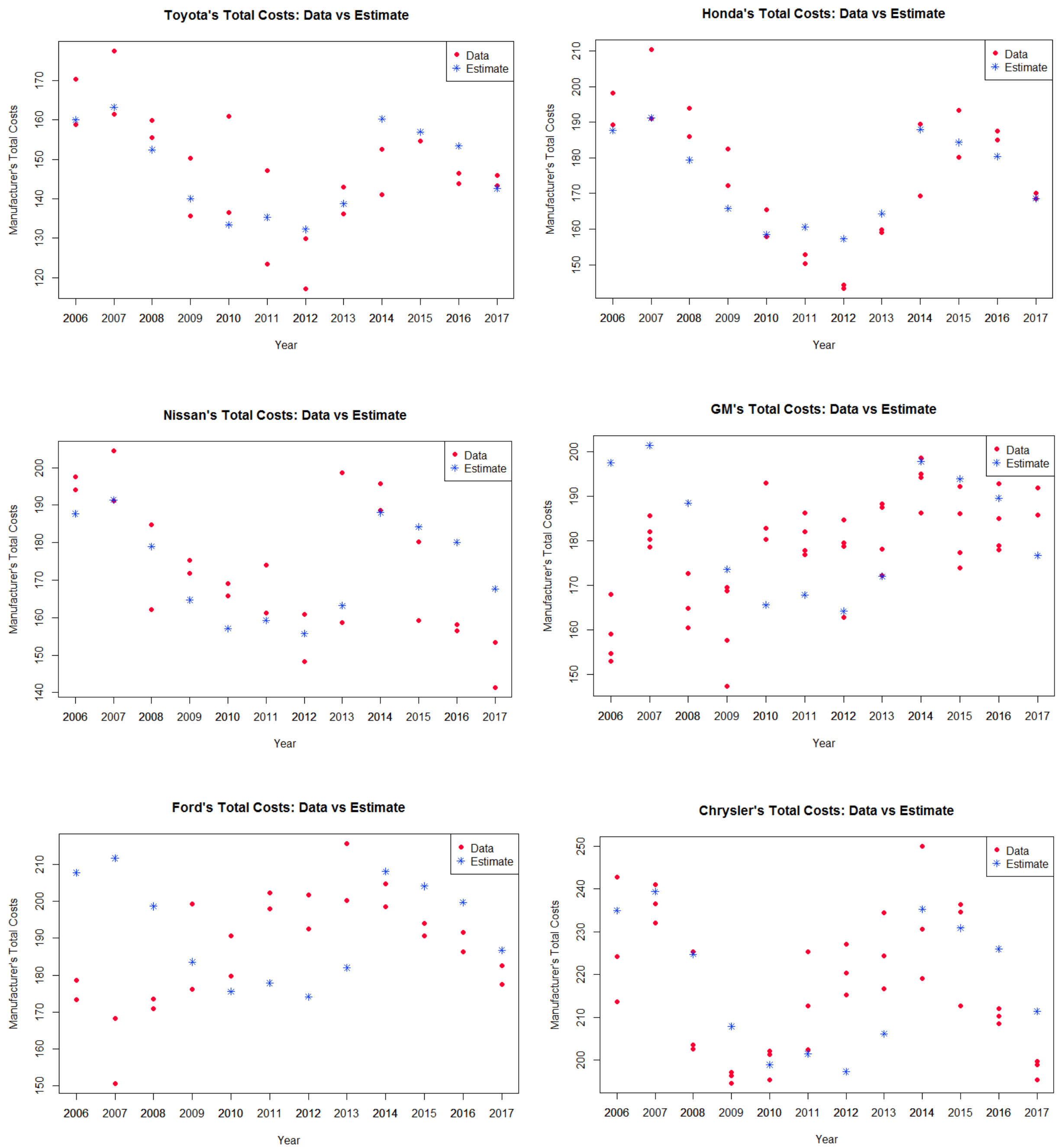


Figure 7.3 Estimated Manufacturer's Total Costs vs Data

When we plot all the estimated manufacturer's costs together in one figure (see Figure 7.4 below), we can clearly see that Toyota is leading all OEMs in total manufacturer's costs while Honda and Nissan are following. US OEMs have a higher total manufacturer costs than Japanese competitors and Chrysler is the worst. Linking the consistency testing of the OEMs to principal agent model, we can see that behaving more consistent with principal agent model suggests benefits the OEM with lower total supply chain costs while OEMs, behaving inconsistent with principal agent models, suffer from a much higher cost in supply chain quality.

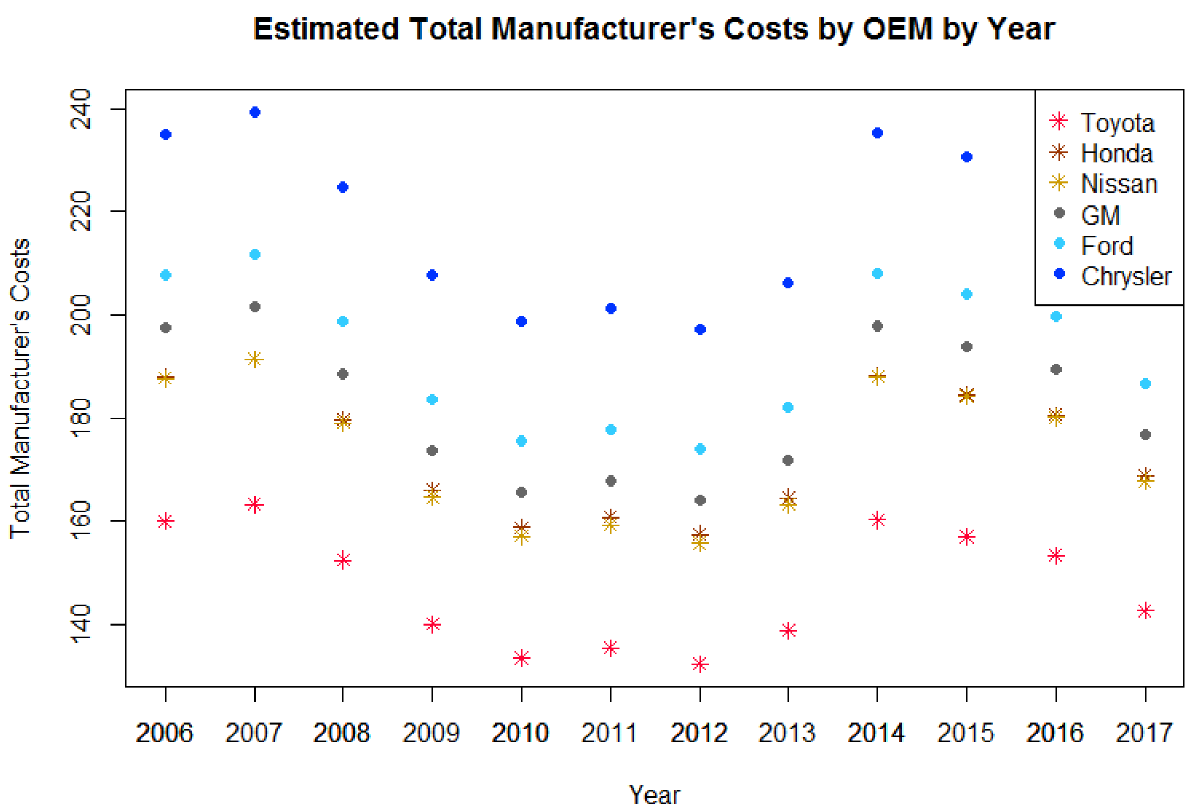


Figure 7.4 Estimated Total Manufacturer's Costs by OEM by Year

From a managerial perspective, when we compare the numbers and use Toyota as the industrial benchmark, Honda and Nissan in average have 17% higher total manufacturer's cost in supply chain quality comparing to Toyota, while GM is the best US OEM and has 24% higher total manufacturer's cost. The number for Ford is 31% and for Chrysler is 48%.

Table 7.2 Estimated Percentage Difference on Total Manufacturer's Costs

Estimated Total Manufacturer Costs	Japanese OEMs			US OEMs		
	Toyota	Honda	Nissan	GM	Ford	Chrysler
2006	160	188	188	198	208	235
2007	163	191	191	201	212	239
2008	152	180	179	188	199	225
2009	140	166	165	174	184	208
2010	133	159	157	166	176	199
2011	135	161	159	168	178	201
2012	132	157	156	164	174	197
2013	139	165	163	172	182	206
2014	160	188	188	198	208	235
2015	157	185	184	194	204	231
2016	153	181	180	190	200	226
2017	143	169	168	177	187	211
Average	147	174	173	182	192	218
% Diff	*	17%	17%	24%	31%	48%

Table 7.2 provides a quantitative assessment of the total manufacturer's costs on supply chain quality which validates the consensus that Toyota and Japanese OEMs are the leaders in supply chain quality costs but fills the void in the literature that lacks concrete quantitative analysis.

7.8 Principal Agent Model Implication Summary

In Chapter 6, we treated the three regression equations derived from principal agent model as independent relations and regress the regression models against the empirical data to test the principal agent relations. In this section, we preserve the linkage between manufacturer and supplier problems and use the multiple objective optimization methodology to measure the cost parameters in the principal agent model and derive implications. There are three main insights we can summarize in this Chapter: (1) the multiple objective optimization approach validates the conclusions that Toyota is the most principal agent OEM and Japanese OEMs are more principal agent than US competitors. The sum of square error rates, trend of WRI, total manufacturer's cost curves are good evidences of it. (2) By behaving more principal agent, OEMs are benefit from a lower marginal effort cost to motivating suppliers improve product quality. Toyota enjoyed a 33% lower marginal effort cost than Honda and 61% benefit than Nissan by being more principal agent, while US OEMs suffers for not behaving in the principal agent way. GM and Ford are both estimated to almost double the marginal effort costs and Chrysler is the worst at 162% higher than Toyota. (3) Being more principal agent benefits OEMs in total manufacture's quality costs along the supply chain. As the most principal agent OEM, Toyota has a 17% lower total manufacture's quality cost comparing to the other two Japanese OEMs Honda and Nissan. US OEMs are much less cost efficient for being less principal agent. GM is estimated to have 24% higher total manufacturer's quality cost comparing to Toyota and the number for Ford and Chrysler is around 31% and 48%.

8. SENSITIVITY ANALYSIS

In the previous Chapter, all the conclusions are derived under one set of parameter setting. However, the results may be sensitive to parameters. Therefore, in this Chapter, we want to test the sensitivity of the results against different settings to make sure the techniques we used in the previous section are robust and the conclusions can stand.

8.1 Sensitivity Analysis on Initial Values

We all know in solving the nonlinear optimization problem, the initial value pick will potentially influence the final results. Therefore, it is important to understand if our linearly constrained nonlinear optimization problem will be impact by initial value pick and if so how big the impact is and how sensitive the results are compared to different initial value pick.

To quantify the sensitivity, we perform 1000 times of randomized initial starting value and then run the optimization algorithm to report the quantile values on parameters $\hat{\alpha}_1, \hat{\alpha}_2, \hat{\eta}_M$, as well as the objective function and percentage of convergences. To enable the cross comparison of the algorithm performance on different OEMs, we choose to use the same initial value intervals for $\alpha_1, \alpha_2, \eta_M$. From the results reported in Table 7.1, we could know that $0 < \alpha_1 < 0.01$, $0 < \alpha_2 < 0.01$, $0 < \eta_M < 1$ should be a reasonable range of initial values to pick. Therefore, we will implement a random draw for each of the initial values $\alpha_1, \alpha_2, \eta_M$ in the defined range above.

For each optimization run, we check the convergence of the algorithm. If the algorithm is not converged, we will not include the results in our analysis. Toyota, Honda, Nissan, GM and Ford all have 100% convergence rate which indicates that the range of the initial value pick is reasonable and the algorithm is robust to converge to an optimal solution either locally or globally every single time. Chrysler has 5 cases that the optimization runs did not converge so we got rid of the solutions that did not converge and report the results based on 995 converged cases. We present the boxplot of the three estimated parameters in the sensitivity analysis on initial value pick below in Figure 8.1.

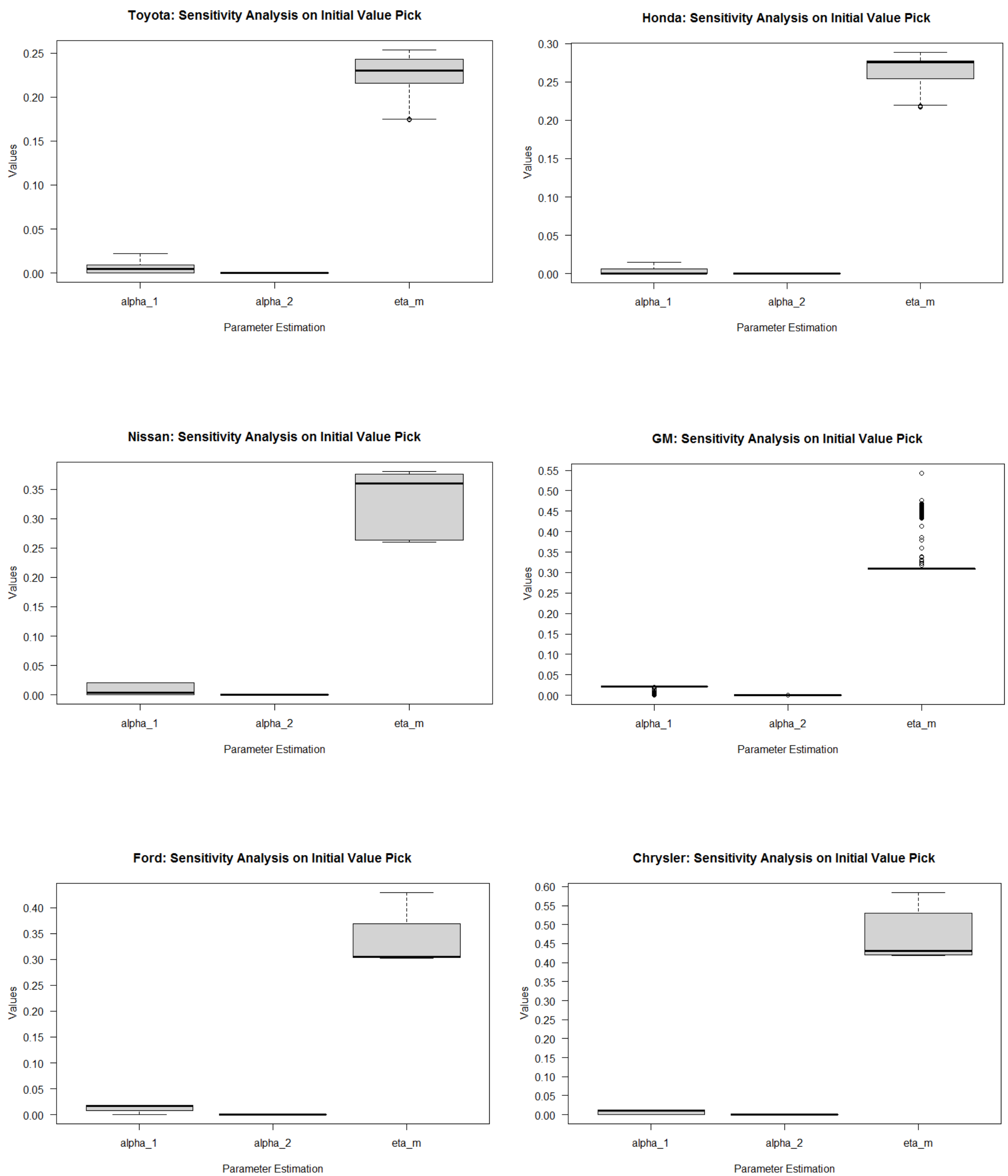


Figure 8.1 Box Plots Estimated Parameters of Sensitivity Analysis on Initial Value

The boxplots show a very narrow bar on each of the three parameters that we estimated, indicating that the algorithm has very stable convergence and little variation on initial values.

Table 8.1 summarizes the quantile statistics of sensitivity analysis on initial values.

Table 8.1 Quantile Statistics of Sensitivity Analysis on Initial Value

Quantile Statistics of Sensitivity Analysis on Initial Value Pick			
Toyota	$\hat{a}_1$	$\hat{a}_2$	$\hat{\eta}_M$
Min	3.82E-09	1.61E-04	0.174
25%	1.17E-04	1.98E-04	0.216
Median	4.43E-03	2.12E-04	0.230
75%	9.09E-03	2.24E-04	0.243
Max	2.20E-02	2.25E-04	0.254
Honda	$\hat{a}_1$	$\hat{a}_2$	$\hat{\eta}_M$
Min	2.92E-10	1.61E-04	0.174
25%	5.56E-05	1.98E-04	0.216
Median	5.20E-04	2.12E-04	0.230
75%	6.09E-03	2.24E-04	0.243
Max	1.52E-02	2.25E-04	0.254
Nissan	$\hat{a}_1$	$\hat{a}_2$	$\hat{\eta}_M$
Min	1.16E-07	1.80E-04	0.260
25%	2.20E-04	1.83E-04	0.263
Median	3.43E-03	2.47E-04	0.360
75%	2.01E-02	2.60E-04	0.376
Max	2.07E-02	2.61E-04	0.381
GM	$\hat{a}_1$	$\hat{a}_2$	$\hat{\eta}_M$
Min	5.31E-06	2.00E-04	0.307
25%	2.09E-02	2.01E-04	0.308
Median	2.09E-02	2.01E-04	0.310
75%	2.11E-02	2.02E-04	0.310
Max	2.12E-02	2.89E-04	0.543
Ford	$\hat{a}_1$	$\hat{a}_2$	$\hat{\eta}_M$
Min	3.59E-06	1.95E-04	0.302
25%	9.25E-03	1.95E-04	0.303
Median	1.73E-02	1.96E-04	0.305
75%	1.76E-02	2.26E-04	0.364
Max	1.77E-02	2.63E-04	0.429
Chrysler	$\hat{a}_1$	$\hat{a}_2$	$\hat{\eta}_M$
Min	1.32E-09	2.12E-04	0.419
25%	7.04E-04	2.13E-04	0.421
Median	1.07E-02	2.17E-04	0.431
75%	1.17E-02	2.62E-04	0.529
Max	1.19E-02	2.66E-04	0.585

The quantile statistics showed a consistent conclusion with the boxplot. Except extreme cases the algorithm might be trapped in a local optimum such as the min and max cases in Table 8.1, most of the other runs converged to almost exactly the same optimal solutions which hopefully is the global optimum.

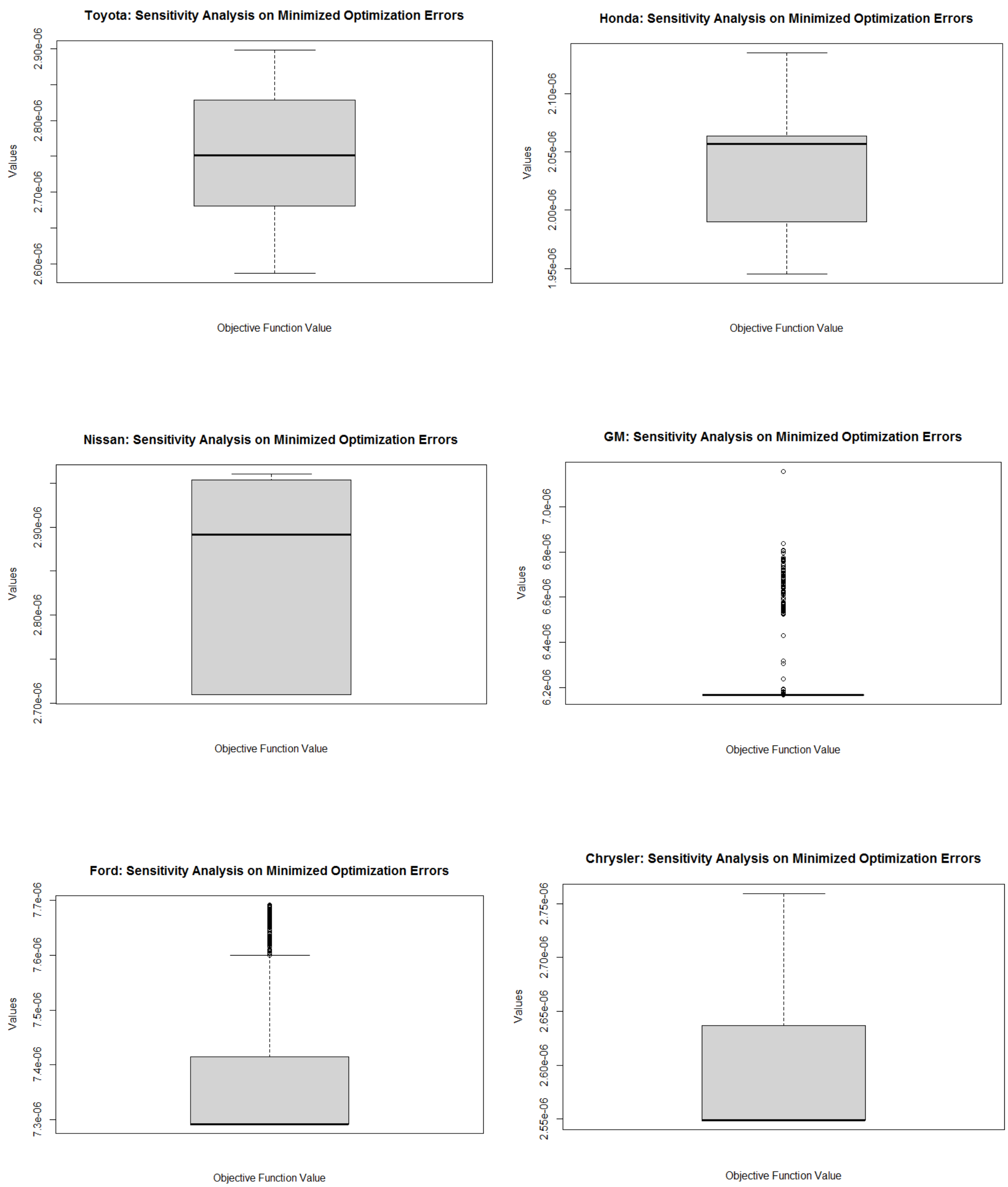


Figure 8.2 Box Plots Errors of Sensitivity Analysis on Initial Value

Figure 8.2 plots the boxplots of the optimization errors. We can see that except several extreme cases which might converge to local optimum and lead to high error rates, the bound is generally very tight (Toyota: 2.6e-06 to 2.9-06, Honda: 1.95e-06 to 2.10e-06, Nissan: 2.70e-06 to 2.90e-06, GM: 6.2e-06 to 6.8e-06, Ford: 7.3e-06 to 7.7e-06, Chrysler: 2.55e-06 to 2.75e-06) and shows a consistent convergence of the algorithm.

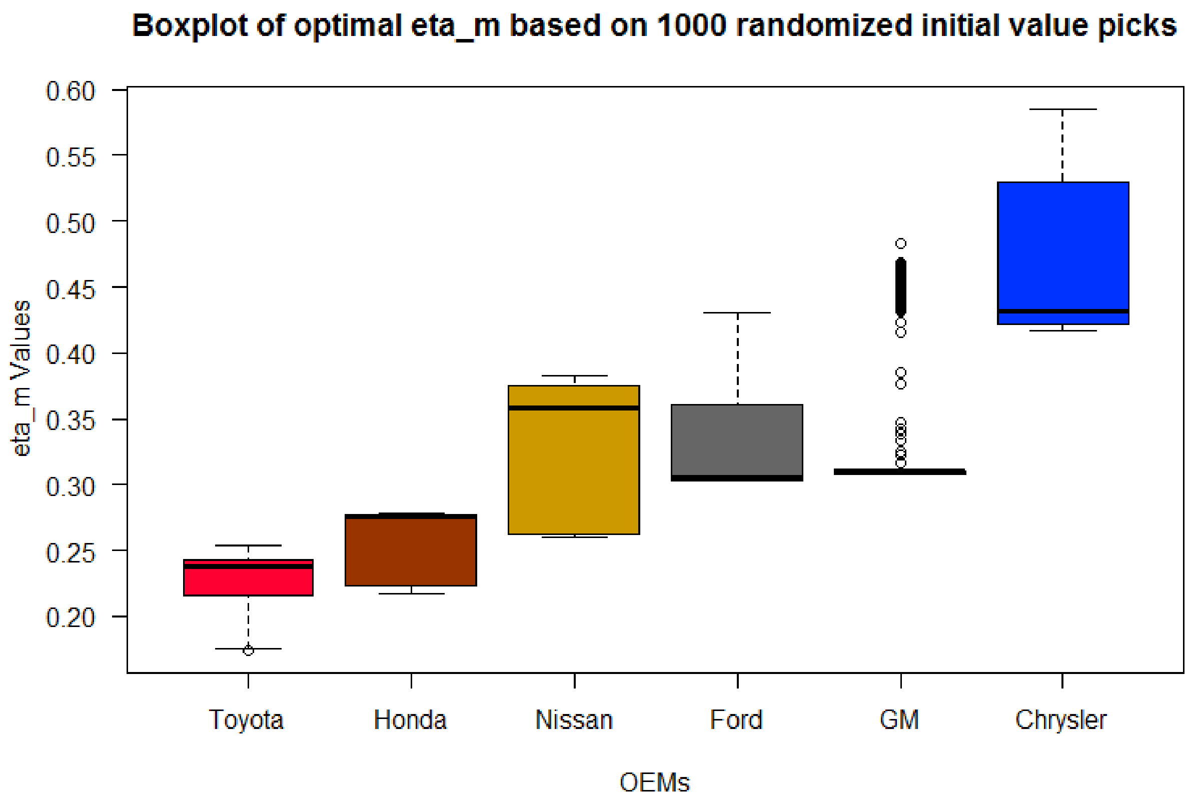


Figure 8.3 Boxplot of η_M on Sensitivity Analysis on Initial Value

We plotted the boxplots of the 6 OEMs together in the Figure 8.3 above. We can see that the conclusion is consistent with Chapter 7: Toyota is the best OEM with lowest marginal manufacturer effort cost η_m while Chrysler is still the worst OEM with highest marginal manufacturer effort cost. The rank orders from lowest manufacturer marginal cost to the highest preserves the order: Toyota, Honda, Nissan, GM, Ford and Chrysler as what we discussed in Chapter 7.4.

8.2 Sensitivity Analysis on Weight Parameter

In this section, we will perform sensitivity analysis on the weight parameter ω to check how sensitive the results are responding to the pick of weight parameter value ω. In the analysis of Chapter 7, we used weight parameter value $\omega = \frac{1}{4*10^8}$ or $\frac{1}{\omega} = 4 * 10^8$ to balance the error contributions of the two first order conditions in the principal agent model of Toyota to get all the results. To be consistent, we will use the same methodology with 1000 randomized initial value pick in the range of $0 < \alpha_1 < 0.01$, $0 < \alpha_2 < 0.01$, $0 < \eta_M < 1$ to trigger the optimization algorithm and report the optimal solution with minimum objective function value. In this case, we will just perform a design of experience for $\frac{1}{\omega}$ = $1 * 10^8$ to $9 * 10^8$ with a 10^8 as increment to check the sensitivity on final results.

Table 8.2 Sensitivity Analysis on Weight Parameter ω

Toyota: Sensitivity Analysis on Weight ω							
$\frac{1}{\omega}$	$\hat{a}_1$	$\hat{a}_2$	$\hat{\eta}_M$	SSE (PP100)	SSE (WRI)	Objective Function	Average Costs
$1 * 10^8$	1.04E-02	1.95E-04	0.210	1.77E-06	7.14E-06	8.91E-06	154.7
$2 * 10^8$	1.34E-02	1.86E-04	0.201	1.73E-06	3.60E-06	5.33E-06	151.5
$3 * 10^8$	1.56E-02	1.80E-04	0.194	1.70E-06	2.42E-06	4.13E-06	149.2
$4 * 10^8$	1.73E-02	1.75E-04	0.189	1.68E-06	1.84E-06	3.52E-06	147.4
$5 * 10^8$	1.86E-02	1.71E-04	0.185	1.67E-06	1.48E-06	3.15E-06	146.0
$6 * 10^8$	1.98E-02	1.67E-04	0.181	1.65E-06	1.25E-06	2.90E-06	144.7
$7 * 10^8$	2.07E-02	1.65E-04	0.178	1.64E-06	1.08E-06	2.72E-06	143.7
$8 * 10^8$	2.17E-02	1.62E-04	0.175	1.63E-06	9.52E-07	2.59E-06	142.7
$9 * 10^8$	2.23E-02	1.60E-04	0.173	1.63E-06	8.52E-07	2.48E-06	142.1
Honda: Sensitivity Analysis on Weight ω							
$\frac{1}{\omega}$	$\hat{a}_1$	$\hat{a}_2$	$\hat{\eta}_M$	SSE (PP100)	SSE (WRI)	Objective Function	Average Costs
$1 * 10^8$	6.97E-08	2.12E-04	0.278	1.18E-06	7.09E-06	8.27E-06	182.6
$2 * 10^8$	8.62E-06	2.12E-04	0.278	1.18E-06	3.55E-06	4.72E-06	182.6
$3 * 10^8$	3.25E-03	2.03E-04	0.265	1.11E-06	2.42E-06	3.53E-06	178.3
$4 * 10^8$	6.53E-03	1.93E-04	0.252	1.05E-06	1.87E-06	2.92E-06	174.0
$5 * 10^8$	9.10E-03	1.85E-04	0.242	1.00E-06	1.54E-06	2.54E-06	170.7
$6 * 10^8$	1.14E-02	1.78E-04	0.233	9.65E-07	1.32E-06	2.28E-06	167.7
$7 * 10^8$	1.31E-02	1.73E-04	0.226	9.35E-07	1.16E-06	2.09E-06	165.4
$8 * 10^8$	1.46E-02	1.68E-04	0.220	9.12E-07	1.03E-06	1.95E-06	163.5
$9 * 10^8$	1.59E-02	1.65E-04	0.215	8.91E-07	9.38E-07	1.83E-06	161.9
Nissan: Sensitivity Analysis on Weight ω							
$\frac{1}{\omega}$	$\hat{a}_1$	$\hat{a}_2$	$\hat{\eta}_M$	SSE (PP100)	SSE (WRI)	Objective Function	Average Costs
$1 * 10^8$	9.22E-07	2.61E-04	0.377	2.25E-06	5.64E-06	7.89E-06	192.0
$2 * 10^8$	4.88E-03	2.42E-04	0.349	2.14E-06	2.92E-06	5.05E-06	184.8
$3 * 10^8$	9.70E-03	2.23E-04	0.322	2.03E-06	2.03E-06	4.06E-06	177.8
$4 * 10^8$	1.29E-02	2.11E-04	0.304	1.96E-06	1.58E-06	3.54E-06	173.2
$5 * 10^8$	1.54E-02	2.01E-04	0.289	1.91E-06	1.31E-06	3.22E-06	169.5
$6 * 10^8$	1.74E-02	1.93E-04	0.278	1.87E-06	1.13E-06	3.00E-06	166.6
$7 * 10^8$	1.90E-02	1.87E-04	0.270	1.84E-06	9.96E-07	2.84E-06	164.4
$8 * 10^8$	2.04E-02	1.81E-04	0.261	1.81E-06	8.97E-07	2.71E-06	162.2
$9 * 10^8$	2.15E-02	1.77E-04	0.255	1.79E-06	8.15E-07	2.61E-06	160.7

Table 8.2 continued

GM: Sensitivity Analysis on Weight ω							
$\frac{1}{\omega}$	$\hat{a}_1$	$\hat{a}_2$	$\hat{\eta}_M$	SSE (PP100)	SSE (WRI)	Objective Function	Average Costs
$1*10^8$	1.80E-06	2.90E-04	0.469	3.41E-06	2.74E-05	3.08E-05	203.0
$2*10^8$	1.93E-05	2.90E-04	0.469	3.41E-06	1.37E-05	1.71E-05	203.1
$3*10^8$	6.69E-03	2.62E-04	0.416	2.93E-06	9.53E-06	1.25E-05	190.7
$4*10^8$	1.13E-02	2.42E-04	0.381	2.62E-06	7.41E-06	1.00E-05	182.4
$5*10^8$	1.47E-02	2.28E-04	0.355	2.42E-06	6.11E-06	8.53E-06	176.4
$6*10^8$	1.73E-02	2.17E-04	0.336	2.28E-06	5.22E-06	7.50E-06	172.0
$7*10^8$	1.93E-02	2.08E-04	0.321	2.17E-06	4.58E-06	6.75E-06	168.5
$8*10^8$	2.10E-02	2.01E-04	0.309	2.08E-06	4.08E-06	6.17E-06	165.8
$9*10^8$	2.25E-02	1.95E-04	0.298	2.01E-06	3.70E-06	5.71E-06	163.3
Ford: Sensitivity Analysis on Weight ω							
$\frac{1}{\omega}$	$\hat{a}_1$	$\hat{a}_2$	$\hat{\eta}_M$	SSE (PP100)	SSE (WRI)	Objective Function	Average Costs
$1*10^8$	1.01E-06	2.63E-04	0.425	4.48E-06	2.57E-05	3.02E-05	202.9
$2*10^8$	4.08E-06	2.63E-04	0.425	4.48E-06	1.28E-05	1.73E-05	203.0
$3*10^8$	5.50E-05	2.63E-04	0.424	4.48E-06	8.56E-06	1.30E-05	202.8
$4*10^8$	5.74E-03	2.41E-04	0.384	4.12E-06	6.73E-06	1.09E-05	192.5
$5*10^8$	9.79E-03	2.25E-04	0.356	3.88E-06	5.60E-06	9.48E-06	185.3
$6*10^8$	1.29E-02	2.13E-04	0.335	3.70E-06	4.83E-06	8.53E-06	179.9
$7*10^8$	1.54E-02	2.03E-04	0.317	3.57E-06	4.26E-06	7.83E-06	175.5
$8*10^8$	1.75E-02	1.95E-04	0.303	3.46E-06	3.83E-06	7.29E-06	171.9
$9*10^8$	1.93E-02	1.88E-04	0.292	3.37E-06	3.49E-06	6.86E-06	169.0
Chrysler: Sensitivity Analysis on Weight ω							
$\frac{1}{\omega}$	$\hat{a}_1$	$\hat{a}_2$	$\hat{\eta}_M$	SSE (PP100)	SSE (WRI)	Objective Function	Average Costs
$1*10^8$	2.92E-06	2.66E-04	0.535	1.33E-06	1.06E-05	1.19E-05	226.6
$2*10^8$	1.98E-05	2.66E-04	0.534	1.33E-06	5.29E-06	6.61E-06	226.4
$3*10^8$	6.26E-04	2.63E-04	0.528	1.31E-06	3.54E-06	4.85E-06	225.0
$4*10^8$	4.04E-03	2.48E-04	0.495	1.23E-06	2.73E-06	3.95E-06	217.8
$5*10^8$	6.49E-03	2.36E-04	0.471	1.18E-06	2.23E-06	3.40E-06	212.6
$6*10^8$	8.47E-03	2.27E-04	0.452	1.14E-06	1.89E-06	3.03E-06	208.5
$7*10^8$	1.00E-02	2.20E-04	0.437	1.11E-06	1.65E-06	2.76E-06	205.2
$8*10^8$	1.13E-02	2.15E-04	0.426	1.09E-06	1.46E-06	2.55E-06	202.7
$9*10^8$	1.24E-02	2.09E-04	0.415	1.07E-06	1.32E-06	2.39E-06	200.4

We can see from the Table 8.2 that the optimal solutions vary as we perturb the weight parameter ω which is not surprising. However, although the change of weight parameter ω will impact the numerical values of the optimal solutions, the ranks order of our conclusions and managerial insights we derived are preserved and consistent. The following figures on the values of $\alpha_1, \alpha_2, \eta_M$, objective function and total costs responding to the change of weight parameter ω will demonstrate that.

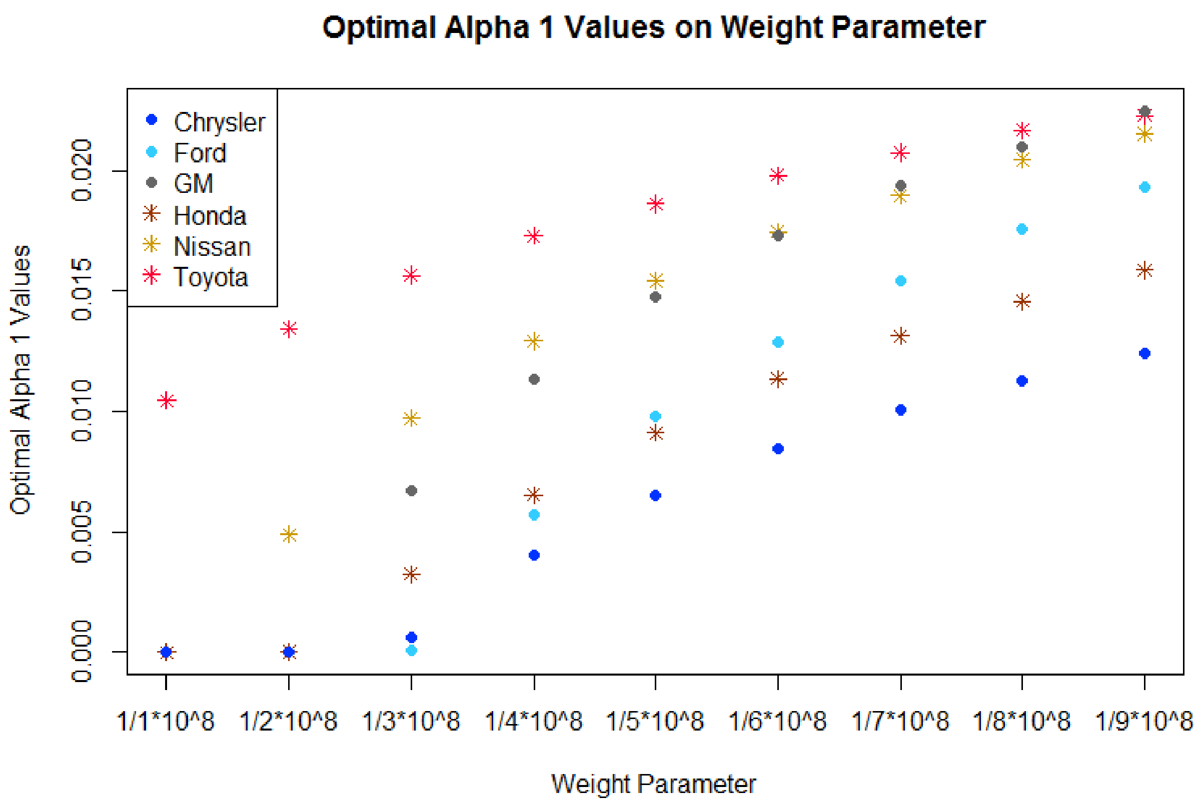


Figure 8.4 Change of α_1 Corresponding to Change of Weight Parameter ω

Figure 8.4 shows that the change of α_1 preserves the rank order and trends while the weight parameter ω changes. Toyota and Nissan have a larger value of α_1 comparing to the other OEMs, indicating that they have a better capability to reduce defects on their own. On the other hand, GM, Ford and Chrysler showed more reliance on their suppliers to collaboratively reduce defects, indicated by a larger α_2 value in Figure 8.5 below.

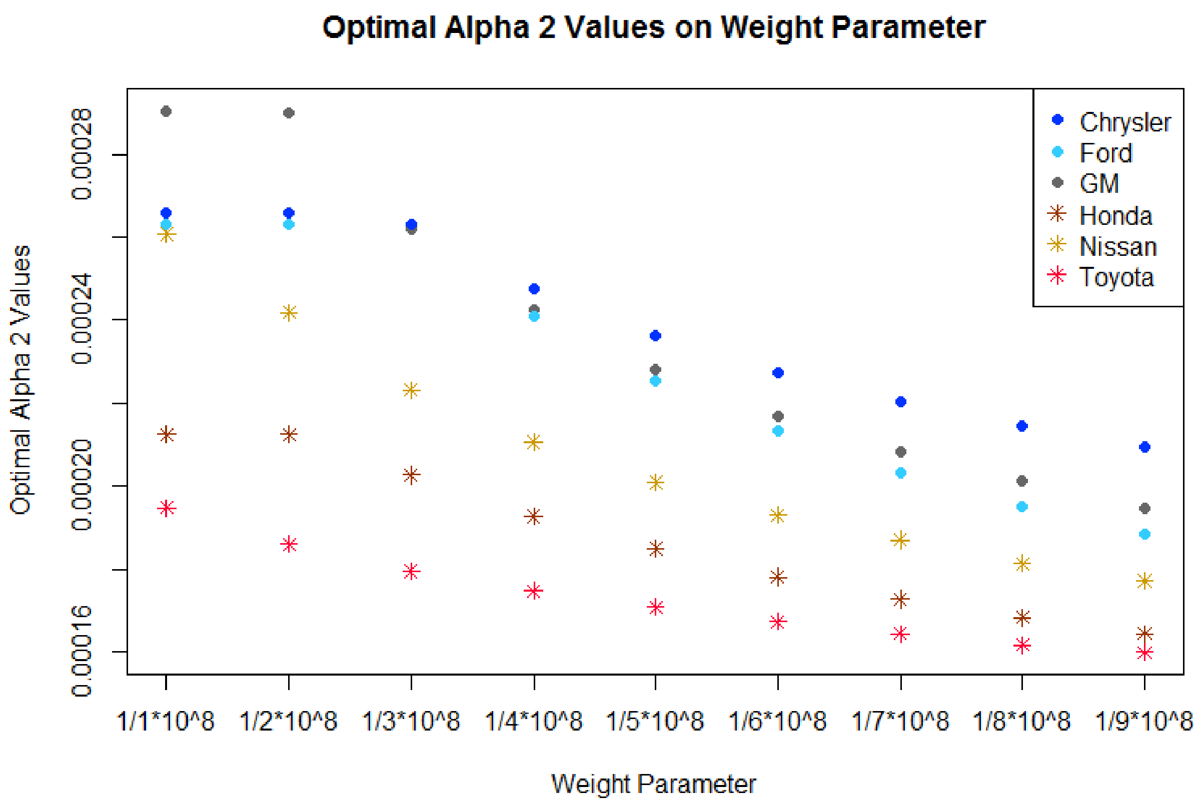


Figure 8.5 Change of α_2 Corresponding to Change of Weight Parameter ω

Also from both Figure 8.4 and Figure 8.5 we can see that the optimal weight parameter ω should take values between $\frac{1}{3*10^8}$ and $\frac{1}{8*10^8}$, as when ω takes small values like $\frac{1}{1*10^8}$ and $\frac{1}{2*10^8}$ or large value like $\frac{1}{9*10^8}$ it breaks the smoothness of the trends and the results might not be as reliable. Therefore, it makes sense for us to draw all the conclusions based on a selection of weight parameter $\omega = \frac{1}{4*10^8}$.

The η_M values also preserve the consistency in rankings by OEMs. No matter how we perturb the weight parameter ω, Toyota still has a significant lead in marginal effort cost, followed by Honda and Nissan. American three OEMs still trail by a significant gap, especially Chrysler.

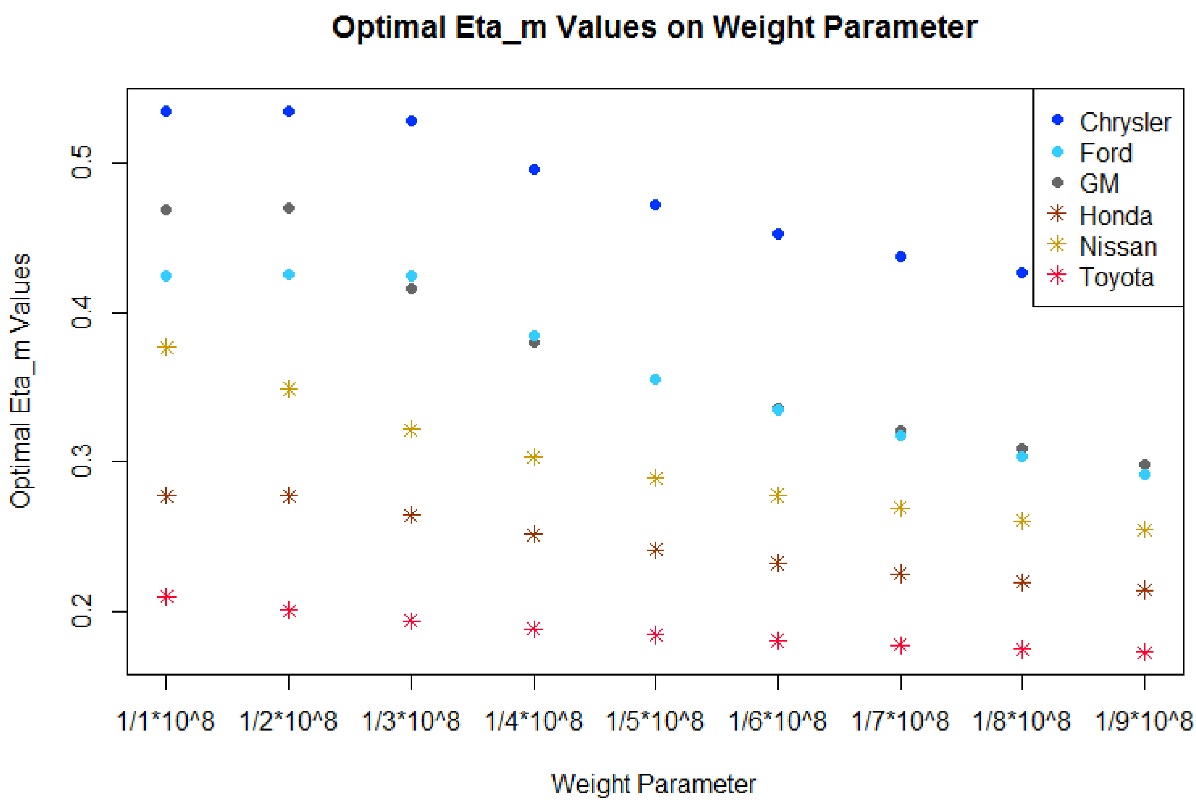


Figure 8.6 Change of η_M Corresponding to Change of Weight Parameter ω

Figure 8.7 below shows a significant drop or change of smoothness when the weight parameter ω changes from $\omega = \frac{1}{2*10^8}$ to $\omega = \frac{1}{3*10^8}$, then the dots smooth out as weight parameter changes from $\omega = \frac{1}{4*10^8}$ to $\omega = \frac{1}{9*10^8}$. The change of objective function value responding to the change of weight parameter ω figure also preserves the rank order of OEMs consistent with principal agent models. American OEMs consistently have larger optimal errors comparing to Japanese OEMs, indicating an inconsistency and misfit from the principal agent models. While Toyota, Honda and Nissan possessed a much smaller model fitting errors and showed consistency with principal agent model fit throughout all the choices of weight parameter ω. Extreme cases like $\omega = \frac{1}{1*10^8}$ showed a significant gap in model fitting between American OEMs and Japanese OEMs which led to consistent observations with what we concluded in previous chapters.

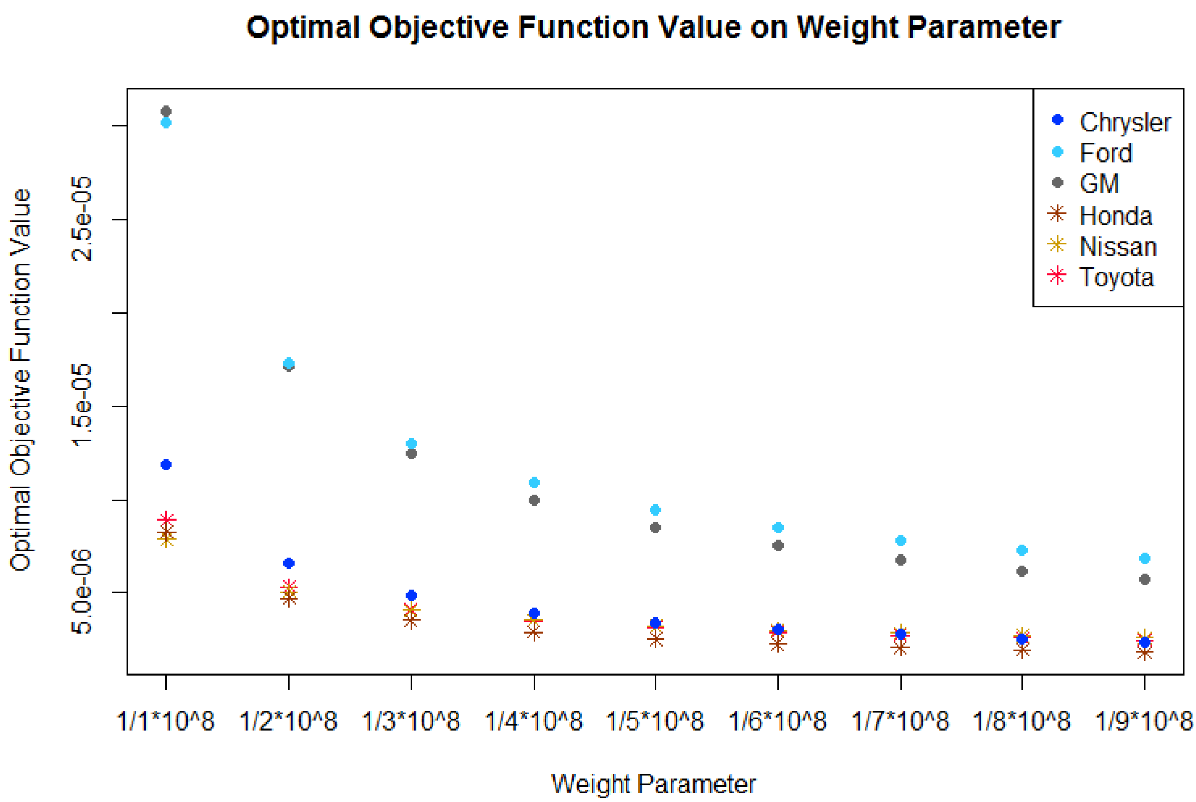


Figure 8.7 Change of Objective Function Corresponding to Change of Weight Parameter ω

Similar to Figure 8.6, Figure 8.8 demonstrates a clear separation of Japanese OEMs to American OEMs on manufacturer's total supply chain quality costs. Toyota is in a significant leading position in total supply chain quality costs which is consistent with the finding in the past literature. While GM, Ford and Chrysler are suffering a significant cost disadvantage which can be validated by their low JD Power IQS rankings. In addition, Chrysler's quality woes are well documented and it hurts their profit margin significantly. (see (Versical D., 2016), (Vellequette L.P., 2017), (Wayland M., 2015))

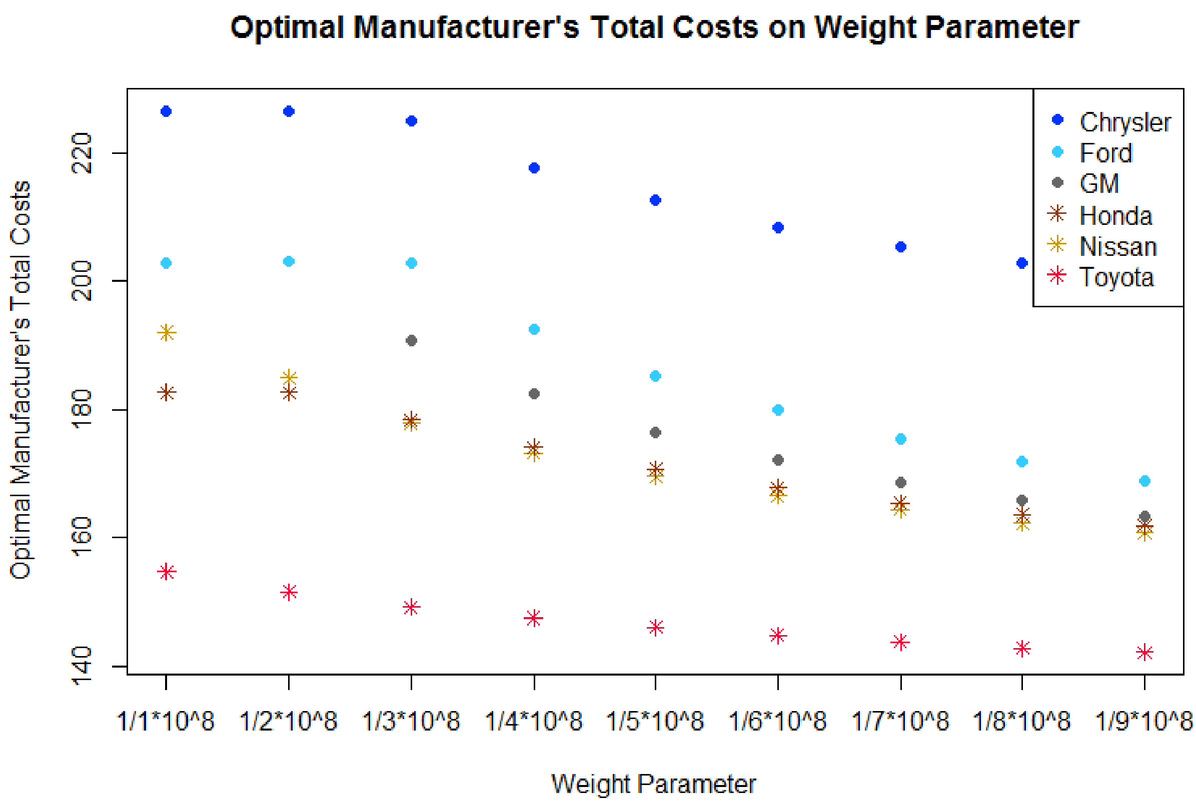


Figure 8.8 Change of Manufacturer's Supply Chain Quality Costs Corresponding to Change of Weight Parameter ω

To summarize, as the weight parameter ω becomes smaller, the objective function of the optimization puts more weight on the PP100 term which leads to the increase of optimal α_1 values and decrease of α_2 and η_M values. The combined objective function value and the manufacturer's total costs will also decrease as increasing the weight parameter ω. To be consistent with our previous analysis, we continue to use the most principal agent OEM Toyota as the benchmark to derive the managerial insights on percentage difference on manufacturer's marginal effort cost η_M in Table 8.3 and total manufacturer's supply chain quality costs in Table 8.4. Data shows the percentage increments over Toyota's number.

Table 8.3 η_M Percentage Difference Relative to Toyota on Change of Weight Parameter ω

η_M Percentage Difference Relative to Toyota on Weight ω						
$\frac{1}{\omega}$	Toyota	Honda	Nissan	GM	Ford	Chrysler
$1 * 10^8$	*	32.0%	79.1%	122.9%	101.9%	154.1%
$2 * 10^8$	*	38.1%	73.6%	133.4%	111.5%	165.6%
$3 * 10^8$	*	36.4%	65.8%	114.5%	118.7%	172.0%
$4 * 10^8$	*	33.3%	60.7%	101.5%	103.2%	162.0%
$5 * 10^8$	*	30.7%	56.7%	92.2%	92.5%	155.1%
$6 * 10^8$	*	28.5%	53.5%	85.6%	84.8%	149.8%
$7 * 10^8$	*	26.7%	51.4%	80.2%	78.3%	145.7%
$8 * 10^8$	*	25.6%	49.1%	76.4%	73.1%	143.2%
$9 * 10^8$	*	24.0%	47.2%	72.2%	68.3%	139.6%

As the Table 8.3 shows, the rank order of the OEMs based on η_M values relative to Toyota won't change as the weight parameter ω changes. Toyota is the best OEM in terms of the marginal effort costs to encourage supplier improve product quality, then followed by the other Japanese OEMs Honda, ranging from 24% to 38.1% more costly, and Nissan, a 47.2% to 79.1% increment over Toyota. Ford and GM are much costlier in encouraging suppliers on quality improvement (72.2% to 133.4% increment over Toyota for GM and 68.3% to 118.7% increment for Ford). Chrysler is always the most inefficient OEM ranked the last and estimated to be 139.6% to 172% more inefficient comparing to Toyota.

When we perform the similar analysis on the total manufacturer's costs relative to Toyota, the conclusion is similar. The rank order of the total manufacturer's costs preserved regardless of the change of the weight parameter. Toyota has a cost advantage of around 13.9% to 20.5% comparing to Honda and 13.1% to 24.1% to Nissan. American OEMs GM, Ford and Chrysler have much higher total manufacturer costs comparing to Toyota, specifically, GM is 14.9% to 34.1% higher, Ford is 18.9% to 36% higher and Chrysler is 41% to 50.8% higher.

Table 8.4 Total Manufacturer's Supply Chain Quality Cost Difference Relative to Toyota on Change of Weight Parameter ω

Total Manufacturer's Cost Difference Relative to Toyota on Weight ω						
$\frac{1}{\omega}$	Toyota	Honda	Nissan	GM	Ford	Chrysler
$1 * 10^8$	*	18.0%	24.1%	31.2%	31.2%	46.5%
$2 * 10^8$	*	20.5%	22.0%	34.1%	34.0%	49.4%
$3 * 10^8$	*	19.6%	19.2%	27.8%	36.0%	50.8%
$4 * 10^8$	*	18.1%	17.5%	23.7%	30.6%	47.7%
$5 * 10^8$	*	16.9%	16.1%	20.8%	26.9%	45.6%
$6 * 10^8$	*	15.9%	15.1%	18.8%	24.3%	44.0%
$7 * 10^8$	*	15.1%	14.4%	17.2%	22.1%	42.8%
$8 * 10^8$	*	14.6%	13.7%	16.1%	20.4%	42.0%
$9 * 10^8$	*	13.9%	13.1%	14.9%	18.9%	41.0%

In this section, we summarized the sensitivity analysis solutions relative to the change of weight parameter ω. Despite some numerical value changes, main conclusions and relative ranking orders are preserved.

8.3 Sensitivity Analysis on Optimization Algorithm

R "ConstrOptim" has two ways to specify the model. One is gradient free method and the algorithm used "Nelder-Mead". The other way is to supply the gradient value and the "BFGS" algorithm will be triggered. In this section, we will first validate that these two algorithms will actually reach similar results. Secondly, we will empirically prove that the gradient free algorithm "Nelder-Mead" will achieve better optimization results than gradient based "BFGS" and is also less sensitive to initial value pick.

8.3.1 Sensitivity Analysis on Optimization Algorithm

Refer to Section 7.2 Mathematical Formulation, the gradient of the objective function can be calculated as follows. Take the objective function as $f(\alpha_1, \alpha_2, \eta_M)$:

$$f(\alpha_1,\alpha_2,\eta_M)=\frac{\sum_{i=1}^{N}\left\{\left(\alpha_1 WSR_i+\alpha_2 WSR_i*WRI_i-\frac{1}{PP100_i}\right)^2+\omega\left(-\frac{\alpha_1}{\alpha_2}+\sqrt{\frac{1}{\alpha_2\eta_M}}\sqrt{\frac{1-WSR_i}{WSR_i}}-WRI_i\right)^2\right\}}{N}$$

The partial derivatives can be calculated as follows:

$$\frac{\partial f(\alpha_1,\alpha_2,\eta_M)}{\partial\alpha_1}=\frac{\sum_{i=1}^{N}\left\{2*WSR_i*\left(\alpha_1 WSR_i+\alpha_2 WSR_i*WRI_i-\frac{1}{PP100_i}\right)-\frac{2\omega}{\alpha_2}\left(-\frac{\alpha_1}{\alpha_2}+\sqrt{\frac{1}{\alpha_2\eta_M}}\sqrt{\frac{1-WSR_i}{WSR_i}}-WRI_i\right)\right\}}{N}$$

$$\frac{\partial f(\alpha_1,\alpha_2,\eta_M)}{\partial\alpha_2}=$$

$$\frac{\sum_{i=1}^{N}\left\{2*WSR_i*WRI_i*\left(\alpha_1 WSR_i+\alpha_2 WSR_i*WRI_i-\frac{1}{PP100_i}\right)+\omega\left(\frac{2\alpha_1}{{\alpha_2}^2}-\sqrt{\frac{1}{\eta_M}}\sqrt{\frac{1}{{\alpha_2}^3}}\sqrt{\frac{1-WSR_i}{WSR_i}}\right)\left(-\frac{\alpha_1}{\alpha_2}+\sqrt{\frac{1}{\alpha_2\eta_M}}\sqrt{\frac{1-WSR_i}{WSR_i}}-WRI_i\right)\right\}}{N}$$

$$\frac{\partial f(\alpha_1,\alpha_2,\eta_M)}{\partial\eta_M}=\frac{\sum_{i=1}^{N}\left\{-\omega\sqrt{\frac{1}{\alpha_2}}\sqrt{\frac{1}{{\eta_M}^3}}\sqrt{\frac{1-WSR_i}{WSR_i}}*\left(-\frac{\alpha_1}{\alpha_2}+\sqrt{\frac{1}{\alpha_2\eta_M}}\sqrt{\frac{1-WSR_i}{WSR_i}}-WRI_i\right)\right\}}{N}$$

Then the gradient is the vector $\left(\frac{\partial f(\alpha_1,\alpha_2,\eta_M)}{\partial\alpha_1},\frac{\partial f(\alpha_1,\alpha_2,\eta_M)}{\partial\alpha_2},\frac{\partial f(\alpha_1,\alpha_2,\eta_M)}{\partial\eta_M}\right)$. Supply this gradient function to the “ConstrOptim” function in R, we can run the optimization using gradient based method like “BFGS”.

8.3.2 Gradient Based “BFGS” vs Gradient Free “Nelder-Mead”

Since most of the results we derived before are based on the gradient free method “Nelder-Mead”. In this section, we want to use the gradient based method “BFGS” to validate the results and to check if they are consistent. First, we want to use the same initial values to run the two optimization algorithms and then compare the result. Second, we want to use the optimal results from gradient free method as the initial values to trigger the gradient based method and check if the optimal results from the gradient based method will differ. Third, we use the optimal results from gradient based method as the initial values to trigger the gradient free method and check the differences.

In order to compare the two optimization algorithms gradient free method “Nelder-Mead” and gradient based method “BFGS”, we use the same initial value range $0 < \alpha_1 < 0.01$, $0 < \alpha_2 < 0.01$, $0 < \eta_M < 1$ to generate 1000 random initial values, then use these 1000 random initial values to trigger the two different optimization algorithms. We saved all the 1000 optimal solutions for the two methods and compare the two methods based on four metrics: (1) convergence rate, (2) cases that each method achieves a lower sum of squared error comparing to the other, (3) spread-out of optimal estimations of the parameters, (4) spread-out of objective function value.

Table 8.5 Convergence Rate of “Nelder-Mead” vs “BFGS” for each OEM

OEMs	Algorithm	Convergence Rate	Lower SSE
Toyota	Nelder-Mead	100%	988/1000
	BFGS	39.3%	12/393
Honda	Nelder-Mead	100%	988/1000
	BFGS	40.5%	12/405
Nissan	Nelder-Mead	100%	994/1000
	BFGS	31.6%	6/316
GM	Nelder-Mead	100%	998/1000
	BFGS	34.9%	2/349
Ford	Nelder-Mead	100%	994/1000
	BFGS	29.6%	6/296
Chrysler	Nelder-Mead	100%	998/1000
	BFGS	22.7%	2/227

We found in Table 8.5 that gradient based method “BFGS” has much lower convergence rate, normally around 20%-40%, comparing to the gradient free method “Nelder-Mead” which converged 100% times. Although this conclusion cannot be generalized to all linearly constraint nonlinear optimization problems, for our specific problem the “Nelder-Mead” seems a better fit than “BFGS” to our model structure. In addition, when comparing the optimal solutions of the two methods, “Nelder-Mead” has overwhelmingly more cases that converged to a lower sum of squared error comparing to “BFGS” method. For example, Toyota only has 12 times out of 393 convergent cases that gradient based “BFGS” achieved lower objective function value than the gradient free “Nelder-Mead” method. In the other 988 cases, “Nelder-Mead” method outperformed the “BFGS” method. Other OEMs demonstrated similar conclusions that the “Nelder-Mead” is a much better method in terms of achieving lower objective function value comparing to the “BFGS” method.

Table 8.6 Variation of Parameters and Objectives of “Nelder-Mead” vs “BFGS”

Toyota								
	α_1				α_2			
	min	median	max	SD	min	median	max	SD
Nelder-Mead	3.97E-08	7.88E-03	1.77E-02	6.48E-03	1.74E-04	2.02E-04	2.26E-04	1.91E-05
BFGS	4.14E-12	1.49E-09	2.02E-01	1.54E-02	5.01E-06	1.58E-04	3.44E-04	4.64E-05
	η_M				Objective Function Value			
	min	Median	max	SD	min	median	max	SD
Nelder-Mead	1.88E-01	2.19E-01	2.44E-01	2.04E-02	3.52E-06	3.59E-06	3.73E-06	8.32E-08
BFGS	6.06E-02	5.08E-01	9.96E-01	2.59E-01	3.52E-06	2.31E-05	1.83E-03	1.04E-04
Honda								
	α_1				α_2			
	min	median	max	SD	min	median	max	SD
Nelder-Mead	2.39E-10	5.12E-04	8.80E-03	3.21E-03	1.86E-04	2.11E-04	2.12E-04	9.61E-06
BFGS	4.95E-13	2.57E-09	1.17E-01	1.34E-02	2.00E-05	1.68E-04	2.87E-04	4.19E-05
	η_M				Objective Function Value			
	min	Median	max	SD	min	median	max	SD
Nelder-Mead	2.43E-01	2.76E-01	2.80E-01	1.29E-02	2.92E-06	2.95E-06	2.96E-06	1.27E-08
BFGS	2.79E-02	3.67E-01	9.95E-01	2.37E-01	2.92E-06	8.22E-06	7.21E-03	3.60E-04
Nissan								
	α_1				α_2			
	min	median	max	SD	min	median	max	SD
Nelder-Mead	4.60E-09	2.38E-03	1.84E-02	6.05E-03	1.91E-04	2.51E-04	2.61E-04	2.34E-05
BFGS	5.36E-13	3.21E-09	7.18E-02	1.31E-02	8.32E-05	2.14E-04	4.71E-04	3.79E-05
	η_M				Objective Function Value			
	min	Median	max	SD	min	median	max	SD
Nelder-Mead	2.70E-01	3.64E-01	3.79E-01	3.46E-02	3.54E-06	3.63E-06	3.66E-06	5.39E-08
BFGS	5.18E-02	5.44E-01	9.71E-01	2.29E-01	3.54E-06	8.67E-06	2.95E-04	2.40E-05
GM								
	α_1				α_2			
	min	median	max	SD	min	median	max	SD
Nelder-Mead	3.84E-07	1.12E-02	2.14E-02	4.56E-03	2.03E-04	2.43E-04	2.90E-04	1.90E-05
BFGS	2.33E-12	3.67E-08	7.15E-02	1.25E-02	1.49E-07	2.60E-04	2.86E-04	4.15E-05
	η_M				Objective Function Value			
	min	Median	max	SD	min	median	max	SD
Nelder-Mead	3.00E-01	3.82E-01	4.93E-01	3.75E-02	1.00E-05	1.00E-05	1.03E-05	8.39E-08
BFGS	9.16E-02	6.03E-01	9.90E-01	2.12E-01	1.00E-05	1.21E-05	1.45E+00	7.74E-02

Table 8.6 continued

Ford								
	α_1				α_2			
	min	median	max	SD	min	median	max	SD
Nelder-Mead	1.30E-08	2.11E-03	6.86E-03	2.74E-03	2.37E-04	2.55E-04	2.63E-04	1.05E-05
BFGS	8.79E-12	3.75E-09	8.61E-02	1.34E-02	1.24E-04	2.29E-04	2.74E-04	2.74E-05
	η_M				Objective Function Value			
	min	Median	max	SD	min	median	max	SD
Nelder-Mead	3.75E-01	4.11E-01	4.45E-01	2.00E-02	1.09E-05	1.09E-05	1.10E-05	2.22E-08
BFGS	8.64E-02	6.05E-01	9.84E-01	2.10E-01	1.09E-05	1.48E-05	3.30E-04	2.34E-05
Chrysler								
	α_1				α_2			
	min	median	max	SD	min	median	max	SD
Nelder-Mead	3.97E-09	3.95E-03	1.04E-02	2.32E-03	2.20E-04	2.48E-04	2.66E-04	1.02E-05
BFGS	2.25E-11	1.44E-08	9.14E-02	1.79E-02	1.19E-04	2.36E-04	3.02E-04	3.89E-05
	η_M				Objective Function Value			
	min	Median	max	SD	min	median	max	SD
Nelder-Mead	4.31E-01	4.95E-01	5.68E-01	2.50E-02	3.95E-06	3.96E-06	4.05E-06	1.30E-08
BFGS	1.39E-01	6.73E-01	9.97E-01	2.32E-01	3.95E-06	6.16E-06	3.68E-04	4.32E-05

If we calculate the prescriptive statistics (see Table 8.6) of the optimal values of the three parameters α_1, α_2 and η_M as well as the objective function value, we can find that almost in all the cases the “Nelder-Mead” method has a narrower bound on the parameters than “BFGS” which means a larger minimum value and a smaller maximum value. Also “Nelder-Mead” method has a much smaller standard deviation than the “BFGS” method. Although both “Nelder-Mead” and “BFGS” methods can achieve the minimum objective function value (for example the minimum objective function value for Toyota’s case is 3.52E-06 for both methods) and find the true global minimum, the “Nelder-Mead” is much more consistent, proving the advantage of the gradient free method “Nelder-Mead” over gradient based “BFGS” method in our model.

The boxplot of the three parameters α_1, α_2 and η_M as well as the objective function value also demonstrated the same conclusion. The gradient free method “Nelder-Mead” has a much shorter bar over gradient based “BFGS” method, indicating that the solutions we got out from the gradient free method “Nelder-Mead” is much more consistent than gradient based “BFGS” method.

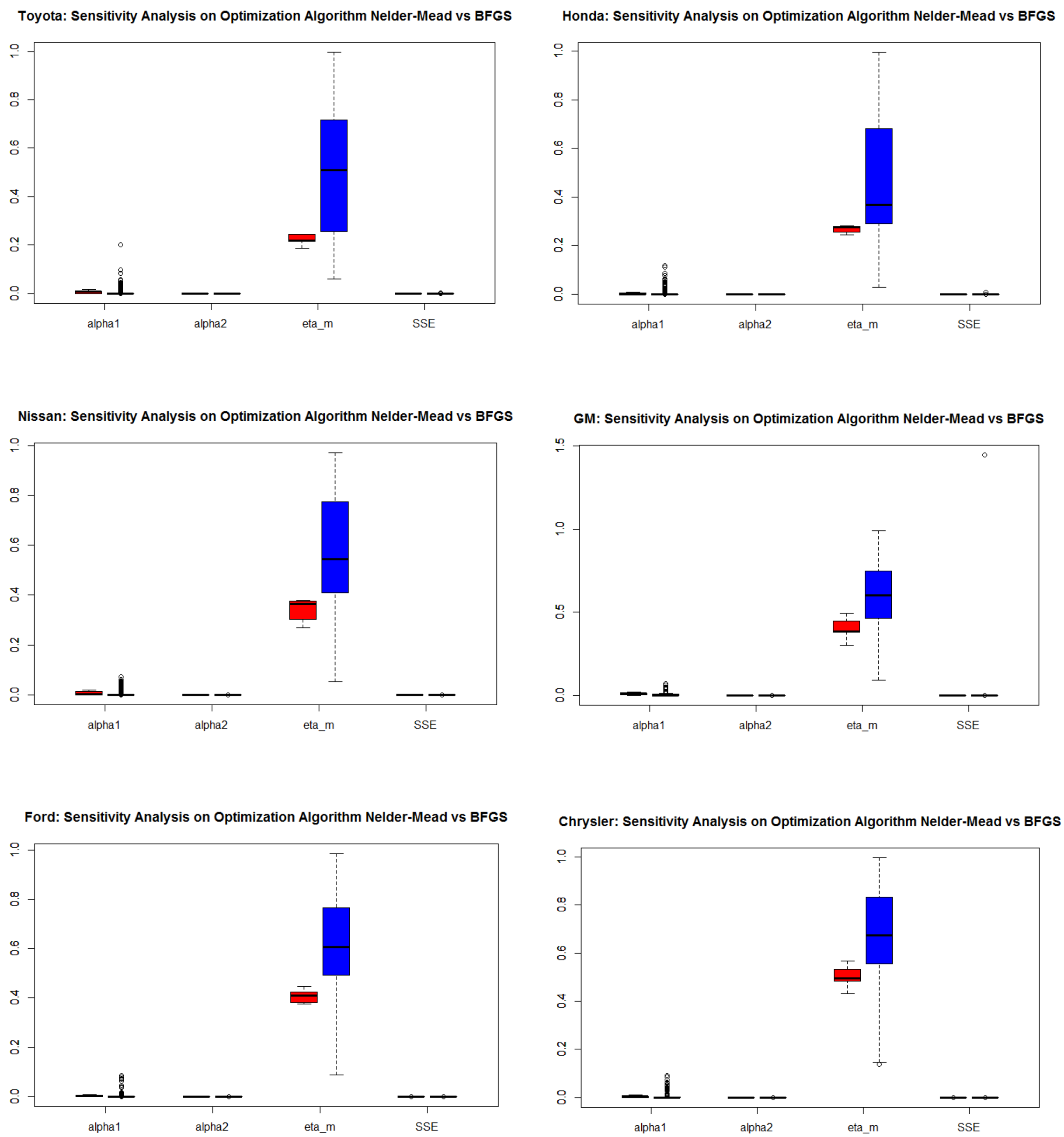


Figure 8.9 Boxplot of Parameters and Objectives of “Nelder-Mead” vs “BFGS”

8.4 Sensitivity Analysis on Global Optimum

Although from the previous section we have proved that the gradient free algorithm “Nelder-Mead” outperformed the gradient based algorithm “BFGS” almost all the time, we can use the gradient based method to validate if the gradient free method “Nelder-Mead” achieved the truly global optimal. To validate the results, we use the optimal solution derived from “Nelder-Mead” as the initial starting point to trigger the gradient based “BFGS” algorithm. Then if the optimal solution from “BFGS” algorithm does not move away from the initial starting point which is also the optimal solution derived from the gradient free “Nelder-Mead” algorithm, we know that the optimal solution of “Nelder-Mead” algorithm is truly a global optimal solution.

For Toyota data, the optimal solution of “Nelder-Mead” as initial starting point for “BFGS” will converge to the same optimal solution as the “Nelder-Mead” found in all cases. Therefore, the gradient based “BFGS” validates the global optimality of the optimal solutions found using gradient free “Nelder-Mead” algorithm.

Table 8.7 Toyota BFGS Method as a Validation for Nelder-Mead Method

Toyota: BFGS Method as a Validation for Nelder-Mead Method				
$\frac{1}{\omega}$	Nelder-Mead			
	α_1	α_2	η_M	SSE
$1 * 10^8$	1.044478E-02	1.948698E-04	2.103756E-01	8.914743E-06
$2 * 10^8$	1.342757E-02	1.860921E-04	2.010250E-01	5.331312E-06
$3 * 10^8$	1.564731E-02	1.795646E-04	1.940615E-01	4.125786E-06
$4 * 10^8$	1.728342E-02	1.747452E-04	1.889189E-01	3.517217E-06
$5 * 10^8$	1.859976E-02	1.708697E-04	1.847864E-01	3.148571E-06
$6 * 10^8$	1.979047E-02	1.673574E-04	1.810443E-01	2.900494E-06
$7 * 10^8$	2.073923E-02	1.645622E-04	1.780708E-01	2.721671E-06
$8 * 10^8$	2.166252E-02	1.618320E-04	1.751620E-01	2.586363E-06
$9 * 10^8$	2.226296E-02	1.600668E-04	1.732883E-01	2.480221E-06
$\frac{1}{\omega}$	BFGS			
	α_1	α_2	η_M	SSE
$1 * 10^8$	1.044478E-02	1.948681E-04	2.103756E-01	8.914743E-06
$2 * 10^8$	1.342757E-02	1.860921E-04	2.010250E-01	5.331312E-06
$3 * 10^8$	1.564731E-02	1.795646E-04	1.940615E-01	4.125786E-06
$4 * 10^8$	1.728342E-02	1.747452E-04	1.889189E-01	3.517217E-06
$5 * 10^8$	1.859976E-02	1.708697E-04	1.847864E-01	3.148571E-06
$6 * 10^8$	1.979047E-02	1.673574E-04	1.810443E-01	2.900494E-06
$7 * 10^8$	2.073923E-02	1.645622E-04	1.780708E-01	2.721671E-06
$8 * 10^8$	2.166252E-02	1.618320E-04	1.751620E-01	2.586363E-06
$9 * 10^8$	2.226296E-02	1.600644E-04	1.732883E-01	2.480221E-06

For Honda data, except the case $\frac{1}{\omega} = 1 * 10^8$ that the "BFGS" method found a slightly better solution than "Nelder-Mead", in all other cases the optimal solution of "Nelder-Mead" as initial starting point for "BFGS" will converge to the same optimal solution as the "Nelder-Mead" found. Therefore, the gradient based "BFGS" validates the global optimality of the optimal solutions found using gradient free "Nelder-Mead" algorithm.

Table 8.8 Honda BFGS Method as a Validation for Nelder-Mead Method

Honda: BFGS Method as a Validation for Nelder-Mead Method				
$\frac{1}{\omega}$	Nelder-Mead			
	α_1	α_2	η_M	SSE
$1 * 10^8$	6.965386E-08	2.124846E-04	2.776677E-01	8.271091E-06
$2 * 10^8$	8.622739E-06	2.124308E-04	2.776841E-01	4.724453E-06
$3 * 10^8$	3.249596E-03	2.026815E-04	2.647441E-01	3.533864E-06
$4 * 10^8$	6.525314E-03	1.927690E-04	2.517721E-01	2.920048E-06
$5 * 10^8$	9.099981E-03	1.849773E-04	2.416067E-01	2.540837E-06
$6 * 10^8$	1.135357E-02	1.781811E-04	2.326566E-01	2.281163E-06
$7 * 10^8$	1.313199E-02	1.727903E-04	2.256347E-01	2.091054E-06
$8 * 10^8$	1.456322E-02	1.684456E-04	2.199923E-01	1.945138E-06
$9 * 10^8$	1.585354E-02	1.645351E-04	2.148877E-01	1.829165E-06
$\frac{1}{\omega}$	BFGS			
	α_1	α_2	η_M	SSE
$1 * 10^8$	5.180570E-08	2.124722E-04	2.776677E-01	8.271089E-06
$2 * 10^8$	8.622528E-06	2.124292E-04	2.776841E-01	4.724453E-06
$3 * 10^8$	3.249596E-03	2.026807E-04	2.647441E-01	3.533864E-06
$4 * 10^8$	6.525314E-03	1.927690E-04	2.517721E-01	2.920048E-06
$5 * 10^8$	9.099981E-03	1.849773E-04	2.416067E-01	2.540837E-06
$6 * 10^8$	1.135357E-02	1.781694E-04	2.326566E-01	2.281163E-06
$7 * 10^8$	1.313199E-02	1.727877E-04	2.256347E-01	2.091054E-06
$8 * 10^8$	1.456322E-02	1.684456E-04	2.199923E-01	1.945138E-06
$9 * 10^8$	1.585354E-02	1.645351E-04	2.148877E-01	1.829165E-06

For Nissan data, the optimal solution of "Nelder-Mead" as initial starting point for "BFGS" will converge to the same optimal solution as the "Nelder-Mead" found in all cases. Therefore, the gradient based "BFGS" validates the global optimality of the optimal solutions found using gradient free "Nelder-Mead" algorithm.

Table 8.9 Nissan BFGS Method as a Validation for Nelder-Mead Method

Nissan: BFGS Method as a Validation for Nelder-Mead Method				
$\frac{1}{\omega}$	Nelder-Mead			
	α_1	α_2	η_M	SSE
$1 * 10^8$	9.221120E-07	2.607943E-04	3.767497E-01	7.893643E-06
$2 * 10^8$	4.878369E-03	2.418887E-04	3.489982E-01	5.052471E-06
$3 * 10^8$	9.704387E-03	2.231316E-04	3.216931E-01	4.060790E-06
$4 * 10^8$	1.290687E-02	2.106702E-04	3.036051E-01	3.543380E-06
$5 * 10^8$	1.541008E-02	2.009357E-04	2.894904E-01	3.221159E-06
$6 * 10^8$	1.744321E-02	1.930319E-04	2.779810E-01	2.999054E-06
$7 * 10^8$	1.895294E-02	1.871343E-04	2.695260E-01	2.835493E-06
$8 * 10^8$	2.044289E-02	1.813620E-04	2.610971E-01	2.709333E-06
$9 * 10^8$	2.151030E-02	1.771952E-04	2.551158E-01	2.608606E-06
$\frac{1}{\omega}$	BFGS			
	α_1	α_2	η_M	SSE
$1 * 10^8$	9.215606E-07	2.607913E-04	3.767497E-01	7.893643E-06
$2 * 10^8$	4.878369E-03	2.418887E-04	3.489982E-01	5.052471E-06
$3 * 10^8$	9.704387E-03	2.231316E-04	3.216931E-01	4.060790E-06
$4 * 10^8$	1.290687E-02	2.106702E-04	3.036051E-01	3.543380E-06
$5 * 10^8$	1.541008E-02	2.009357E-04	2.894904E-01	3.221159E-06
$6 * 10^8$	1.744321E-02	1.930319E-04	2.779810E-01	2.999054E-06
$7 * 10^8$	1.895294E-02	1.871343E-04	2.695260E-01	2.835493E-06
$8 * 10^8$	2.044289E-02	1.813584E-04	2.610971E-01	2.709333E-06
$9 * 10^8$	2.151030E-02	1.771763E-04	2.551158E-01	2.608606E-06

For GM data, except the cases $\frac{1}{\omega} = 1 * 10^8$ and $\frac{1}{\omega} = 9 * 10^8$ that the "BFGS" method found a slightly better solution than "Nelder-Mead", in all other cases the optimal solution of "Nelder-Mead" as initial starting point for "BFGS" will converge to the same optimal solution as the "Nelder-Mead" found. Therefore, the gradient based "BFGS" validates the global optimality of the optimal solutions found using gradient free "Nelder-Mead" algorithm.

Table 8.10 GM BFGS Method as a Validation for Nelder-Mead Method

GM: BFGS Method as a Validation for Nelder-Mead Method				
$\frac{1}{\omega}$	Nelder-Mead			
	α_1	α_2	η_M	SSE
$1 * 10^8$	1.803082E-06	2.904443E-04	4.688258E-01	3.079119E-05
$2 * 10^8$	1.925103E-05	2.900839E-04	4.691607E-01	1.710114E-05
$3 * 10^8$	6.688471E-03	2.620460E-04	4.161984E-01	1.245498E-05
$4 * 10^8$	1.132749E-02	2.424077E-04	3.805855E-01	1.003208E-05
$5 * 10^8$	1.471621E-02	2.279856E-04	3.550948E-01	8.529121E-06
$6 * 10^8$	1.729266E-02	2.170031E-04	3.360188E-01	7.498882E-06
$7 * 10^8$	1.934688E-02	2.082417E-04	3.209655E-01	6.745073E-06
$8 * 10^8$	2.099286E-02	2.011514E-04	3.090483E-01	6.167548E-06
$9 * 10^8$	2.247312E-02	1.948471E-04	2.983706E-01	5.709673E-06
$\frac{1}{\omega}$	BFGS			
	α_1	α_2	η_M	SSE
$1 * 10^8$	5.197813E-07	2.903599E-04	4.688258E-01	3.079106E-05
$2 * 10^8$	1.924938E-05	2.900923E-04	4.691607E-01	1.710114E-05
$3 * 10^8$	6.688471E-03	2.620444E-04	4.161984E-01	1.245498E-05
$4 * 10^8$	1.132749E-02	2.423956E-04	3.805855E-01	1.003208E-05
$5 * 10^8$	1.471621E-02	2.279856E-04	3.550948E-01	8.529121E-06
$6 * 10^8$	1.729266E-02	2.170031E-04	3.360188E-01	7.498882E-06
$7 * 10^8$	1.934688E-02	2.082383E-04	3.209655E-01	6.745073E-06
$8 * 10^8$	2.099286E-02	2.011514E-04	3.090483E-01	6.167548E-06
$9 * 10^8$	2.247312E-02	1.948285E-04	2.983706E-01	5.709672E-06

For Ford data, except the cases $\frac{1}{\omega} = 1 * 10^8$, $\frac{1}{\omega} = 2 * 10^8$ and $\frac{1}{\omega} = 5 * 10^8$ that the "BFGS" method found a slightly better solution than "Nelder-Mead", in all other cases the optimal solution of "Nelder-Mead" as initial starting point for "BFGS" will converge to the same optimal solution as the "Nelder-Mead" found. Therefore, the gradient based "BFGS" validates the global optimality of the optimal solutions found using gradient free "Nelder-Mead" algorithm.

Table 8.11 Ford BFGS Method as a Validation for Nelder-Mead Method

Ford: BFGS Method as a Validation for Nelder-Mead Method				
$\frac{1}{\omega}$	Nelder-Mead			
	α_1	α_2	η_M	SSE
$1 * 10^8$	1.006618E-06	2.632942E-04	4.247615E-01	3.015441E-05
$2 * 10^8$	4.083139E-06	2.632235E-04	4.252058E-01	1.731817E-05
$3 * 10^8$	5.497443E-05	2.630797E-04	4.243786E-01	1.303921E-05
$4 * 10^8$	5.735562E-03	2.410818E-04	3.839685E-01	1.085084E-05
$5 * 10^8$	9.791431E-03	2.253387E-04	3.558006E-01	9.479157E-06
$6 * 10^8$	1.289670E-02	2.132260E-04	3.345215E-01	8.530509E-06
$7 * 10^8$	1.542204E-02	2.033735E-04	3.174566E-01	7.830945E-06
$8 * 10^8$	1.754180E-02	1.951140E-04	3.032106E-01	7.291204E-06
$9 * 10^8$	1.927779E-02	1.883297E-04	2.917274E-01	6.860519E-06
$\frac{1}{\omega}$	BFGS			
	α_1	α_2	η_M	SSE
$1 * 10^8$	4.251143E-07	2.632878E-04	4.247615E-01	3.015433E-05
$2 * 10^8$	2.744610E-06	2.631631E-04	4.252058E-01	1.731812E-05
$3 * 10^8$	5.497443E-05	2.630797E-04	4.243786E-01	1.303921E-05
$4 * 10^8$	5.735562E-03	2.410818E-04	3.839685E-01	1.085084E-05
$5 * 10^8$	9.791431E-03	2.253116E-04	3.558006E-01	9.479156E-06
$6 * 10^8$	1.289670E-02	2.132260E-04	3.345215E-01	8.530509E-06
$7 * 10^8$	1.542204E-02	2.033735E-04	3.174566E-01	7.830945E-06
$8 * 10^8$	1.754180E-02	1.951140E-04	3.032106E-01	7.291204E-06
$9 * 10^8$	1.927779E-02	1.883206E-04	2.917274E-01	6.860519E-06

For Chrysler data, except the cases $\frac{1}{\omega} = 1 * 10^8$, $\frac{1}{\omega} = 2 * 10^8$ that the "BFGS" method found a slightly better solution than "Nelder-Mead", in all other cases the optimal solution of "Nelder-Mead" as initial starting point for "BFGS" will converge to the same optimal solution as the "Nelder-Mead" found. Therefore, the gradient based "BFGS" validates the global optimality of the optimal solutions found using gradient free "Nelder-Mead" algorithm.

Table 8.12 Chrysler BFGS Method as a Validation for Nelder-Mead Method

Chrysler: BFGS Method as a Validation for Nelder-Mead Method				
$\frac{1}{\omega}$	Nelder-Mead			
	α_1	α_2	η_M	SSE
$1 * 10^8$	2.916443E-06	2.656585E-04	5.345113E-01	1.190033E-05
$2 * 10^8$	1.975777E-05	2.657437E-04	5.340015E-01	6.613348E-06
$3 * 10^8$	6.262555E-04	2.631291E-04	5.278487E-01	4.850399E-06
$4 * 10^8$	4.042633E-03	2.475788E-04	4.948821E-01	3.954455E-06
$5 * 10^8$	6.488947E-03	2.364111E-04	4.713580E-01	3.403681E-06
$6 * 10^8$	8.472778E-03	2.273623E-04	4.523357E-01	3.028936E-06
$7 * 10^8$	1.003411E-02	2.202168E-04	4.374867E-01	2.756491E-06
$8 * 10^8$	1.125024E-02	2.146665E-04	4.260028E-01	2.548933E-06
$9 * 10^8$	1.239582E-02	2.093944E-04	4.151475E-01	2.385213E-06
$\frac{1}{\omega}$	BFGS			
	α_1	α_2	η_M	SSE
$1 * 10^8$	1.945926E-06	2.657679E-04	5.345113E-01	1.190025E-05
$2 * 10^8$	1.641983E-05	2.658654E-04	5.340015E-01	6.613297E-06
$3 * 10^8$	6.262555E-04	2.631291E-04	5.278487E-01	4.850399E-06
$4 * 10^8$	4.042633E-03	2.475717E-04	4.948821E-01	3.954455E-06
$5 * 10^8$	6.488947E-03	2.364111E-04	4.713580E-01	3.403681E-06
$6 * 10^8$	8.472778E-03	2.273623E-04	4.523357E-01	3.028936E-06
$7 * 10^8$	1.003411E-02	2.202168E-04	4.374867E-01	2.756491E-06
$8 * 10^8$	1.125024E-02	2.146522E-04	4.260028E-01	2.548933E-06
$9 * 10^8$	1.239582E-02	2.093944E-04	4.151475E-01	2.385213E-06

To Summarize, all the experiments using the six OEM's data demonstrate that the "BFGS" algorithm with the optimal solution of "Nelder-Mead" as initial starting point will converge to the same optimal solution as the "Nelder-Mead" found. Therefore, the gradient based "BFGS" validates the global optimality of the optimal solutions found using gradient free "Nelder-Mead" algorithm.

8.5 Sensitivity Analysis on Robustness

In section 8.1., we studied the sensitivity analysis of optimal solutions based on 1000 randomized initial values and each time we reported the minimum objective function value solution as the optimal solution. In this section, we want to investigate if the minimum objective function value optimal solution based on 1000 randomized initial value pick is robust and stable. To study the stability, we perform 100 iterations of 1000 randomized initial value optimization runs and compare the values of the optimal solution of the minimum objective function value from the 100 iterations. We find that the optimal solution of the 100 iterations are almost the same and thus we can be confident that the optimal solution we get from the 1000 random initials are robust and stable.

Table 8.13 Chrysler BFGS Method as a Validation for Nelder-Mead Method

Toyota Stability Results				
Parameters	min	median	max	SD
α_1	1.70E-02	1.73E-02	1.75E-02	9.25E-05
α_2	1.74E-04	1.75E-04	1.76E-04	2.66E-07
η_M	1.88E-01	1.89E-01	1.90E-01	2.98E-04
Objective Function Value	3.52E-06	3.52E-06	3.52E-06	1.68E-11
Honda Stability Results				
Parameters	min	median	max	SD
α_1	6.55E-03	6.67E-03	6.89E-03	4.90E-05
α_2	1.92E-04	1.92E-04	1.93E-04	1.45E-07
η_M	2.50E-01	2.51E-01	2.52E-01	1.99E-04
Objective Function Value	2.92E-06	2.92E-06	2.92E-06	4.71E-12
Nissan Stability Results				
Parameters	min	median	max	SD
α_1	1.28E-02	1.29E-02	1.31E-02	4.18E-05
α_2	2.10E-04	2.11E-04	2.11E-04	1.58E-07
η_M	3.03E-01	3.03E-01	3.04E-01	2.48E-04
Objective Function Value	3.54E-06	3.54E-06	3.54E-06	3.02E-12
GM Stability Results				
Parameters	min	median	max	SD
α_1	1.12E-02	1.13E-02	1.14E-02	2.92E-05
α_2	2.42E-04	2.42E-04	2.43E-04	1.19E-07
η_M	3.80E-01	3.80E-01	3.81E-01	2.45E-04
Objective Function Value	1.00E-05	1.00E-05	1.00E-05	3.30E-12
Ford Stability Results				
Parameters	min	median	max	SD
α_1	5.61E-03	5.73E-03	5.89E-03	3.40E-05
α_2	2.41E-04	2.41E-04	2.42E-04	1.29E-07
η_M	3.83E-01	3.84E-01	3.85E-01	2.55E-04
Objective Function Value	1.09E-05	1.09E-05	1.09E-05	5.13E-12

Table 8.13 continued

Chrysler Stability Results				
Parameters	min	median	max	SD
α_1	3.97E-03	4.05E-03	4.13E-03	2.61E-05
α_2	2.47E-04	2.48E-04	2.48E-04	1.18E-07
η_M	4.94E-01	4.95E-01	4.95E-01	2.65E-04
Objective Function Value	3.95E-06	3.95E-06	3.95E-06	1.13E-12

Based on the stability results for the 6 OEMs in Table 8.13, the solutions are all robust and stable with a very small standard deviation among iterations. Therefore, we are confident that the optimal solution we demonstrated in the previous section based on the minimum objective function value of 1000 random initial value runs are sufficient and robust.

9. DISCUSSIONS AND LIMITATIONS

Validating principal agent models is extremely hard. Literatures can prove it. Researchers only had very limited success in validating principal agent models in the past and majority of the success is in the areas like labor economics where empirical data are rich. In this work, we contributed another empirical validation of principal agent model in a manufacturer-supplier relationship where no prior work was found. However, we also identified three major challenges that limits our work to further validate our empirical studies.

9.1 Data Limitations

Firstly, the data we used are mainly survey data although they are widely adopted and regarded as the industrial standard. WRI data is a composite index weighted 5 different aspects of manufacturer's behaviors in 6 different purchasing areas based on surveying 600 sales personals. The consistency of the survey and their algorithms to composite the index might limit the objectiveness of the assessment. The JD Power IQS data might be consistent throughout the years as JD Power publishes the IQS studies for almost 30 years. However, things like OEMs got rid of brands, brands changed ownerships all potentially affect the measurement of the quality performance for each OEM. To make things worse, the warranty sharing ratio is based on a pool of US based companies reporting their warranty and recall expenses under the SEC reporting regulation. There is no OEM specific warranty sharing ratio data due to the privacy and also the complication of the supply chain. Due to the availability limitation at company level, we have to take the industrial warranty sharing ratio as the proxy to analyze individual OEM. Many arguments could support that this is a valid approximation as OEMs share suppliers and suppliers produce similar parts to different OEMs. However, there are also many evidences showing that Japanese OEMs and US OEMs may use different contracts to source parts. Therefore, their warranty sharing ratio might be very different. Anyway, since there is no better data available, we have to take this compromise.

9.2 Model Limitations

Secondly, we only discussed the simplest principal agent relationship which is complete information and observable actions. Both the manufacturer and the supplier know everything about each other and could observe each other's action. Literature argued that more interesting and realistic settings might be asymmetric information, hidden actions with even moral hazard. However, these types of principal agent models are too complicated and almost impossible to empirically validate. If there are hidden information or hidden actions, there deem to be no data available on that information or that action, then validating the principal agent model becomes impossible. If there are other constraints in the principal agent model on individual participation or individual compatibility, the First Order Condition will be mathematically complex and impossible to validate against. All these reasons lead us to pick the simplest principal agent relation with symmetric information without constraints. Also, that is the reason why in the world of principal agent models the theoretical work is rich but empirical validation is very rare.

9.3 Validation Limitations

Thirdly, supplier's effort data is missing. We are lucky to find the WRI data as the proxy for manufacturer's effort. However, there is no data available to proxy the supplier's effort, not even any survey data. This is understandable because OEMs outsource hundreds and thousands of parts to suppliers, there is almost impossible to have an aggregate measurement on for example how GM's suppliers behave collectively. However, lack of supplier's effort data limits our analysis at manufacturer's side only and we cannot see how the suppliers react to manufacturer's efforts and whether the suppliers can benefit from a more principal agent like relationship.

10. CONCLUSIONS AND FUTURE WORK

In this paper, we think we made three important contributions in validating principal agent models. First, we made the first attempt to validate a principal agent model in supply chain management literature. We proved with automotive empirical data that Japanese OEMs are behaving more like principal agent model suggests than US OEMs. While most of the manufacturer-supplier relationships are not strictly following what principal agent model describes, there are companies like Toyota who can validate some legitimacy of the principal agent model in studying the supply chain relations. Second, our empirical validation process is strictly based on the first order conditions derived from the principal agent model which captures the structural relations between variables. Almost all the empirical literature in the past only studied the relationship between variables derived from principal agent models. For example, principal agent model may suggest that increasing warranty sharing ratio to suppliers will increase supplier's quality improvement effort, and then people find data on warranty sharing ratio and supplier's quality improvement effort to validate the relations. However, this type of validation is very crude and is not based on the structural relations that the principal agent model inherent. Our regression analysis validates the principal agent model strictly based on the two first order conditions. Therefore, the structural relations between variables are preserved and we argue that this is the strongest way to validate principal agent models. Third, we proposed a way to make principal agent model implications by using a multiple objective optimization approach to bridge the principal's problem with agent's problem. Therefore, we are able to answer two important questions. One is if a company is more principal agent than others, what is the benefit for behaving more principal agent. Two is can we quantify the benefits if a company is more principal agent than others. The multiple objective optimization approach answered these two important questions and established a process to make principal agent model implications.

All our conclusions are based on a simple principal agent model with complete information. There is certainly more research to do to investigate if there are some kinds of asymmetric information or hidden actions are the conclusions still hold, can we find better empirical data to proxy the variables or are there better ways to make implications. Theoretical principal agent models are proved to be a powerful tool to study supply chain relations and it provided us enormous managerial insights in managing supply chain manufacturer-supplier relations. However, we just made the first baby step to try to validate it. Just like most researchers experienced in the past, validating principal agent models are hard but with more data availability and creative algorithms in statistics, optimization and machine learning we should see more and more theoretical results got empirically validated.

APPENDIX A: DATA

Make	Company	Country	Segment	Year	PP100	WRI	WSR_S	WSR_M
Acura	Honda	Japan	Luxury	2006	120	368	0.119297	0.880703
Buick	GM	US	Mass	2006	134	131	0.119297	0.880703
Cadillac	GM	US	Luxury	2006	117	131	0.119297	0.880703
Chevrolet	GM	US	Mass	2006	124	131	0.119297	0.880703
Chrysler	Chrysler	US	Mass	2006	120	218	0.119297	0.880703
Dodge	Chrysler	US	Mass	2006	132	218	0.119297	0.880703
Ford	Ford	US	Mass	2006	127	174	0.119297	0.880703
GMC	GM	US	Mass	2006	119	131	0.119297	0.880703
Honda	Honda	Japan	Mass	2006	110	368	0.119297	0.880703
Infiniti	Nissan	Japan	Luxury	2006	117	300	0.119297	0.880703
Jeep	Chrysler	US	Luxury	2006	153	218	0.119297	0.880703
Lexus	Toyota	Japan	Luxury	2006	93	407	0.119297	0.880703
Lincoln	Ford	US	Luxury	2006	121	174	0.119297	0.880703
Nissan	Nissan	Japan	Mass	2006	121	300	0.119297	0.880703
Toyota	Toyota	Japan	Mass	2006	106	407	0.119297	0.880703
Acura	Honda	Japan	Luxury	2007	130	380	0.115576	0.884424
Buick	GM	US	Mass	2007	127	174	0.115576	0.884424
Cadillac	GM	US	Luxury	2007	135	174	0.115576	0.884424
Chevrolet	GM	US	Mass	2007	129	174	0.115576	0.884424
Chrysler	Chrysler	US	Mass	2007	151	199	0.115576	0.884424
Dodge	Chrysler	US	Mass	2007	156	199	0.115576	0.884424
Ford	Ford	US	Mass	2007	120	162	0.115576	0.884424
GMC	GM	US	Mass	2007	131	174	0.115576	0.884424
Honda	Honda	Japan	Mass	2007	108	380	0.115576	0.884424
Infiniti	Nissan	Japan	Luxury	2007	117	289	0.115576	0.884424
Jeep	Chrysler	US	Luxury	2007	161	199	0.115576	0.884424
Lexus	Toyota	Japan	Luxury	2007	94	415	0.115576	0.884424
Lincoln	Ford	US	Luxury	2007	100	162	0.115576	0.884424
Nissan	Nissan	Japan	Mass	2007	132	289	0.115576	0.884424
Toyota	Toyota	Japan	Mass	2007	112	415	0.115576	0.884424
Acura	Honda	Japan	Luxury	2008	119	359	0.128624	0.871376
Buick	GM	US	Mass	2008	118	163	0.128624	0.871376
Cadillac	GM	US	Luxury	2008	113	163	0.128624	0.871376
Chevrolet	GM	US	Mass	2008	113	163	0.128624	0.871376
Chrysler	Chrysler	US	Mass	2008	142	161	0.128624	0.871376
Dodge	Chrysler	US	Mass	2008	141	161	0.128624	0.871376
Ford	Ford	US	Mass	2008	112	191	0.128624	0.871376

GMC	GM	US	Mass	2008	127	163	0.128624	0.871376
Honda	Honda	Japan	Mass	2008	110	359	0.128624	0.871376
Infiniti	Nissan	Japan	Luxury	2008	98	253	0.128624	0.871376
Jeep	Chrysler	US	Luxury	2008	167	161	0.128624	0.871376
Lexus	Toyota	Japan	Luxury	2008	99	367	0.128624	0.871376
Lincoln	Ford	US	Luxury	2008	115	191	0.128624	0.871376
Nissan	Nissan	Japan	Mass	2008	124	253	0.128624	0.871376
Toyota	Toyota	Japan	Mass	2008	104	367	0.128624	0.871376
Acura	Honda	Japan	Luxury	2009	111	349	0.146444	0.853556
Buick	GM	US	Mass	2009	117	183	0.146444	0.853556
Cadillac	GM	US	Luxury	2009	91	183	0.146444	0.853556
Chevrolet	GM	US	Mass	2009	103	183	0.146444	0.853556
Chrysler	Chrysler	US	Mass	2009	136	162	0.146444	0.853556
Dodge	Chrysler	US	Mass	2009	134	162	0.146444	0.853556
Ford	Ford	US	Mass	2009	102	232	0.146444	0.853556
GMC	GM	US	Mass	2009	116	183	0.146444	0.853556
Honda	Honda	Japan	Mass	2009	99	349	0.146444	0.853556
Infiniti	Nissan	Japan	Luxury	2009	106	268	0.146444	0.853556
Jeep	Chrysler	US	Luxury	2009	137	162	0.146444	0.853556
Lexus	Toyota	Japan	Luxury	2009	84	339	0.146444	0.853556
Lincoln	Ford	US	Luxury	2009	129	232	0.146444	0.853556
Nissan	Nissan	Japan	Mass	2009	110	268	0.146444	0.853556
Toyota	Toyota	Japan	Mass	2009	101	339	0.146444	0.853556
Acura	Honda	Japan	Luxury	2010	86	340	0.157404	0.842596
Buick	GM	US	Mass	2010	114	228	0.157404	0.842596
Cadillac	GM	US	Luxury	2010	111	228	0.157404	0.842596
Chevrolet	GM	US	Mass	2010	111	228	0.157404	0.842596
Chrysler	Chrysler	US	Mass	2010	122	187	0.157404	0.842596
Dodge	Chrysler	US	Mass	2010	130	187	0.157404	0.842596
Ford	Ford	US	Mass	2010	93	264	0.157404	0.842596
GMC	GM	US	Mass	2010	126	228	0.157404	0.842596
Honda	Honda	Japan	Mass	2010	95	340	0.157404	0.842596
Infiniti	Nissan	Japan	Luxury	2010	107	249	0.157404	0.842596
Jeep	Chrysler	US	Luxury	2010	129	187	0.157404	0.842596
Lexus	Toyota	Japan	Luxury	2010	88	330	0.157404	0.842596
Lincoln	Ford	US	Luxury	2010	106	264	0.157404	0.842596
Nissan	Nissan	Japan	Mass	2010	111	249	0.157404	0.842596
Toyota	Toyota	Japan	Mass	2010	117	330	0.157404	0.842596
Acura	Honda	Japan	Luxury	2011	89	309	0.154152	0.845848
Buick	GM	US	Mass	2011	114	236	0.154152	0.845848
Cadillac	GM	US	Luxury	2011	103	236	0.154152	0.845848

Chevrolet	GM	US	Mass	2011	109	236	0.154152	0.845848
Chrysler	Chrysler	US	Mass	2011	110	221	0.154152	0.845848
Dodge	Chrysler	US	Mass	2011	137	221	0.154152	0.845848
Ford	Ford	US	Mass	2011	116	271	0.154152	0.845848
GMC	GM	US	Mass	2011	104	236	0.154152	0.845848
Honda	Honda	Japan	Mass	2011	86	309	0.154152	0.845848
Infiniti	Nissan	Japan	Luxury	2011	102	247	0.154152	0.845848
Jeep	Chrysler	US	Luxury	2011	122	221	0.154152	0.845848
Lexus	Toyota	Japan	Luxury	2011	73	327	0.154152	0.845848
Lincoln	Ford	US	Luxury	2011	111	271	0.154152	0.845848
Nissan	Nissan	Japan	Mass	2011	117	247	0.154152	0.845848
Toyota	Toyota	Japan	Mass	2011	101	327	0.154152	0.845848
Acura	Honda	Japan	Luxury	2012	84	293	0.15945	0.84055
Buick	GM	US	Mass	2012	106	251	0.15945	0.84055
Cadillac	GM	US	Luxury	2012	80	251	0.15945	0.84055
Chevrolet	GM	US	Mass	2012	100	251	0.15945	0.84055
Chrysler	Chrysler	US	Mass	2012	116	248	0.15945	0.84055
Dodge	Chrysler	US	Mass	2012	124	248	0.15945	0.84055
Ford	Ford	US	Mass	2012	118	267	0.15945	0.84055
GMC	GM	US	Mass	2012	99	251	0.15945	0.84055
Honda	Honda	Japan	Mass	2012	83	293	0.15945	0.84055
Infiniti	Nissan	Japan	Luxury	2012	84	256	0.15945	0.84055
Jeep	Chrysler	US	Luxury	2012	110	248	0.15945	0.84055
Lexus	Toyota	Japan	Luxury	2012	73	296	0.15945	0.84055
Lincoln	Ford	US	Luxury	2012	107	267	0.15945	0.84055
Nissan	Nissan	Japan	Mass	2012	99	256	0.15945	0.84055
Toyota	Toyota	Japan	Mass	2012	88	296	0.15945	0.84055
Acura	Honda	Japan	Luxury	2013	102	287	0.1485	0.8515
Buick	GM	US	Mass	2013	109	251	0.1485	0.8515
Cadillac	GM	US	Luxury	2013	108	251	0.1485	0.8515
Chevrolet	GM	US	Mass	2013	97	251	0.1485	0.8515
Chrysler	Chrysler	US	Mass	2013	109	250	0.1485	0.8515
Dodge	Chrysler	US	Mass	2013	130	250	0.1485	0.8515
Ford	Ford	US	Mass	2013	131	271	0.1485	0.8515
GMC	GM	US	Mass	2013	90	251	0.1485	0.8515
Honda	Honda	Japan	Mass	2013	103	287	0.1485	0.8515
Infiniti	Nissan	Japan	Luxury	2013	95	256	0.1485	0.8515
Jeep	Chrysler	US	Luxury	2013	118	250	0.1485	0.8515
Lexus	Toyota	Japan	Luxury	2013	94	297	0.1485	0.8515
Lincoln	Ford	US	Luxury	2013	113	271	0.1485	0.8515
Nissan	Nissan	Japan	Mass	2013	142	256	0.1485	0.8515

Toyota	Toyota	Japan	Mass	2013	102	297	0.1485	0.8515
Acura	Honda	Japan	Luxury	2014	131	295	0.118993	0.881007
Buick	GM	US	Mass	2014	120	244	0.118993	0.881007
Cadillac	GM	US	Luxury	2014	115	244	0.118993	0.881007
Chevrolet	GM	US	Mass	2014	106	244	0.118993	0.881007
Chrysler	Chrysler	US	Mass	2014	111	245	0.118993	0.881007
Dodge	Chrysler	US	Mass	2014	124	245	0.118993	0.881007
Ford	Ford	US	Mass	2014	116	267	0.118993	0.881007
GMC	GM	US	Mass	2014	116	244	0.118993	0.881007
Honda	Honda	Japan	Mass	2014	108	295	0.118993	0.881007
Infiniti	Nissan	Japan	Luxury	2014	128	273	0.118993	0.881007
Jeep	Chrysler	US	Luxury	2014	146	245	0.118993	0.881007
Lexus	Toyota	Japan	Luxury	2014	92	318	0.118993	0.881007
Lincoln	Ford	US	Luxury	2014	109	267	0.118993	0.881007
Nissan	Nissan	Japan	Mass	2014	120	273	0.118993	0.881007
Toyota	Toyota	Japan	Mass	2014	105	318	0.118993	0.881007
Acura	Honda	Japan	Luxury	2015	126	330	0.122926	0.877074
Buick	GM	US	Mass	2015	105	224	0.122926	0.877074
Cadillac	GM	US	Luxury	2015	122	224	0.122926	0.877074
Chevrolet	GM	US	Mass	2015	101	224	0.122926	0.877074
Chrysler	Chrysler	US	Mass	2015	143	224	0.122926	0.877074
Dodge	Chrysler	US	Mass	2015	116	224	0.122926	0.877074
Ford	Ford	US	Mass	2015	107	261	0.122926	0.877074
GMC	GM	US	Mass	2015	115	224	0.122926	0.877074
Honda	Honda	Japan	Mass	2015	111	330	0.122926	0.877074
Infiniti	Nissan	Japan	Luxury	2015	97	244	0.122926	0.877074
Jeep	Chrysler	US	Luxury	2015	141	224	0.122926	0.877074
Lexus	Toyota	Japan	Luxury	2015	104	336	0.122926	0.877074
Lincoln	Ford	US	Luxury	2015	103	261	0.122926	0.877074
Nissan	Nissan	Japan	Mass	2015	121	244	0.122926	0.877074
Toyota	Toyota	Japan	Mass	2015	104	336	0.122926	0.877074
Acura	Honda	Japan	Luxury	2016	122	323	0.127451	0.872549
Buick	GM	US	Mass	2016	96	250	0.127451	0.872549
Cadillac	GM	US	Luxury	2016	112	250	0.127451	0.872549
Chevrolet	GM	US	Mass	2016	95	250	0.127451	0.872549
Chrysler	Chrysler	US	Mass	2016	115	222	0.127451	0.872549
Dodge	Chrysler	US	Mass	2016	117	222	0.127451	0.872549
Ford	Ford	US	Mass	2016	102	267	0.127451	0.872549
GMC	GM	US	Mass	2016	103	250	0.127451	0.872549
Honda	Honda	Japan	Mass	2016	119	323	0.127451	0.872549
Infiniti	Nissan	Japan	Luxury	2016	103	225	0.127451	0.872549

Jeep	Chrysler	US	Luxury	2016	113	222	0.127451	0.872549
Lexus	Toyota	Japan	Luxury	2016	96	332	0.127451	0.872549
Lincoln	Ford	US	Luxury	2016	96	267	0.127451	0.872549
Nissan	Nissan	Japan	Mass	2016	101	225	0.127451	0.872549
Toyota	Toyota	Japan	Mass	2016	93	332	0.127451	0.872549
Acura	Honda	Japan	Luxury	2017	103	319	0.142454	0.857546
Buick	GM	US	Mass	2017	95	290	0.142454	0.857546
Cadillac	GM	US	Luxury	2017	105	290	0.142454	0.857546
Chevrolet	GM	US	Mass	2017	88	290	0.142454	0.857546
Chrysler	Chrysler	US	Mass	2017	102	218	0.142454	0.857546
Dodge	Chrysler	US	Mass	2017	106	218	0.142454	0.857546
Ford	Ford	US	Mass	2017	86	270	0.142454	0.857546
GMC	GM	US	Mass	2017	99	290	0.142454	0.857546
Honda	Honda	Japan	Mass	2017	105	319	0.142454	0.857546
Infiniti	Nissan	Japan	Luxury	2017	107	203	0.142454	0.857546
Jeep	Chrysler	US	Luxury	2017	107	218	0.142454	0.857546
Lexus	Toyota	Japan	Luxury	2017	98	328	0.142454	0.857546
Lincoln	Ford	US	Luxury	2017	92	270	0.142454	0.857546
Nissan	Nissan	Japan	Mass	2017	93	203	0.142454	0.857546
Toyota	Toyota	Japan	Mass	2017	95	328	0.142454	0.857546

APPENDIX B: R CODE

```
mydata <- read.csv("C:/…/.csv")
attach(mydata)

##############################################################################
################################################
#                                           Part I: Descriptive Analysis
##############################################################################
################################################

####################################################### 1D Plot: IQS
PP100####################################################
shapes = c(19, 19, 19, 8, 8, 8)
shapes <- shapes[as.numeric(mydata$Company)]
colors <- c("#0033FF", "#33CCFF", "#666666","#993300","#CC9900","#FF0033")
colors <- colors[as.numeric(mydata$Company)]
IQSPP100Figure <- plot(Year, PP100, main = "JD Power Initial Quality Study by
OEM",
                 xlab = "Year", ylab = "JD Power IQS", pch = shapes, col =
colors)
legend("topright", legend = levels(mydata$Company),
       col =  c("#0033FF", "#33CCFF",
"#666666","#993300","#CC9900","#FF0033"), pch = c(19, 19, 19, 8, 8, 8) )
axis(1,at=2006:2017,labels=2006:2017);


####################################################### 1D Plot: WRI
###############################################
shapes = c(19, 19, 19, 8, 8, 8)
shapes <- shapes[as.numeric(mydata$Company)]
colors <- c("#0033FF", "#33CCFF", "#666666","#993300","#CC9900","#FF0033")
colors <- colors[as.numeric(mydata$Company)]
PPIWRIFigure <- plot(Year, WRI, main = "OEM-Supplier Working Relation Index",
                 xlab = "Year", ylab = "WRI", pch = shapes, col = colors)
legend("topleft", legend = levels(mydata$Company),
       col =  c("#0033FF", "#33CCFF",
"#666666","#993300","#CC9900","#FF0033"), pch = c(19, 19, 19, 8, 8, 8))
axis(1,at=2006:2017,labels=2006:2017);


####################################################### 1D Plot: WSR
###############################################
WRIWSR2D <- plot(Year, WSR_S, main = "Warranty Week - Warranty Sharing
Ratio",
                 xlab = "Year", ylab = "WSR", type = 'o', pch = 19, col =
'blue')
axis(1,at=2006:2017,labels=2006:2017);


####################################################### 2D Plot: WRI vs
WSR##################################################
shapes = c(19, 19, 19, 8, 8, 8)
shapes <- shapes[as.numeric(mydata$Company)]
```

```
colors <- c("#0033FF", "#33CCFF", "#666666","#993300","#CC9900","#FF0033")
colors <- colors[as.numeric(mydata$Company)]
WRIWSR2D <- plot(WSR_M, WRI, main = "OEM-Supplier Working Relation Index vs
Warranty Sharing Ratio",
                 xlab = "Warranty Sharing Ratio", ylab = "OEM-Supplier
Working Relation Index", pch = shapes, col = colors)
legend("right", legend = levels(mydata$Company),
       col =  c("#0033FF", "#33CCFF",
"#666666","#993300","#CC9900","#FF0033"), pch = c(19, 19, 19, 8, 8, 8) )
abline(lm(WRI ~ WSR_M, data = mydata[mydata$Company=='GM',]), col="#666666")
abline(lm(WRI ~ WSR_M, data = mydata[mydata$Company=='Ford',]),
col="#33CCFF")
abline(lm(WRI ~ WSR_M, data = mydata[mydata$Company=='Chrysler',]),
col="#0033FF")
abline(lm(WRI ~ WSR_M, data = mydata[mydata$Company=='Toyota',]),
col="#FF0033", lty="dotted")
abline(lm(WRI ~ WSR_M, data = mydata[mydata$Company=='Honda',]),
col="#993300", lty="dotted")
abline(lm(WRI ~ WSR_M, data = mydata[mydata$Company=='Nissan',]),
col="#CC9900", lty="dotted")

regGM <-lm(WRI ~ WSR_M, data = mydata[mydata$Company=='GM',])
summary(regGM)

regFord <-lm(WRI ~ WSR_M, data = mydata[mydata$Company=='Ford',])
summary(regFord)

regChrysler <-lm(WRI ~ WSR_M, data = mydata[mydata$Company=='Chrysler',])
summary(regChrysler)

regToyota <-lm(WRI ~ WSR_M, data = mydata[mydata$Company=='Toyota',])
summary(regToyota)

regHonda <-lm(WRI ~ WSR_M, data = mydata[mydata$Company=='Honda',])
summary(regHonda)

regNissan <-lm(WRI ~ WSR_M, data = mydata[mydata$Company=='Nissan',])
summary(regNissan)


###################################################### 2D Plot: IQS PP100 vs
WSR##################################################
shapes = c(19, 19, 19, 8, 8, 8)
shapes <- shapes[as.numeric(mydata$Company)]
colors <- c("#0033FF", "#33CCFF", "#666666","#993300","#CC9900","#FF0033")
colors <- colors[as.numeric(mydata$Company)]
WRIWSR2D <- plot(WSR_M, PP100, main = "JD Power IQS vs Warranty Sharing
Ratio",
                 xlab = "Warranty Sharing Ratio", ylab = "JD Power IQS", pch
= shapes, col = colors)
legend("right", legend = levels(mydata$Company),
       col =  c("#0033FF", "#33CCFF",
"#666666","#993300","#CC9900","#FF0033"), pch = c(19, 19, 19, 8, 8, 8) )
abline(lm(PP100 ~ WSR_M, data = mydata[mydata$Company=='GM',]),
col="#666666")
abline(lm(PP100 ~ WSR_M, data = mydata[mydata$Company=='Ford',]),
col="#33CCFF")
```

```
abline(lm(PP100 ~ WSR_M, data = mydata[mydata$Company=='Chrysler',]),
col="#0033FF")
abline(lm(PP100 ~ WSR_M, data = mydata[mydata$Company=='Toyota',]),
col="#FF0033", lty="dotted")
abline(lm(PP100 ~ WSR_M, data = mydata[mydata$Company=='Honda',]),
col="#993300", lty="dotted")
abline(lm(PP100 ~ WSR_M, data = mydata[mydata$Company=='Nissan',]),
col="#CC9900", lty="dotted")

regGM <-lm(PP100 ~ WSR_M, data = mydata[mydata$Company=='GM',])
summary(regGM)

regFord <-lm(PP100 ~ WSR_M, data = mydata[mydata$Company=='Ford',])
summary(regFord)

regChrysler <-lm(PP100 ~ WSR_M, data = mydata[mydata$Company=='Chrysler',])
summary(regChrysler)

regToyota <-lm(PP100 ~ WSR_M, data = mydata[mydata$Company=='Toyota',])
summary(regToyota)

regHonda <-lm(PP100 ~ WSR_M, data = mydata[mydata$Company=='Honda',])
summary(regHonda)

regNissan <-lm(PP100 ~ WSR_M, data = mydata[mydata$Company=='Nissan',])
summary(regNissan)


####################################################### 2D Plot: IQS PP100 vs
WRI################################################
shapes = c(19, 19, 19, 8, 8, 8)
shapes <- shapes[as.numeric(mydata$Company)]
colors <- c("#0033FF", "#33CCFF", "#666666","#993300","#CC9900","#FF0033")
colors <- colors[as.numeric(mydata$Company)]
WRIWSR2D <- plot(WRI, PP100, main = "JD Power IQS vs OEM-Supplier Working
Relation Index",
                 xlab = "Supplier-OEM Working Relation Index", ylab = "JD
Power IQS", pch = shapes, col = colors)
legend("right", legend = levels(mydata$Company),
       col =  c("#0033FF", "#33CCFF",
"#666666","#993300","#CC9900","#FF0033"), pch = c(19, 19, 19, 8, 8, 8) )
abline(lm(PP100 ~ WRI, data = mydata[mydata$Company=='GM',]), col="#666666")
abline(lm(PP100 ~ WRI, data = mydata[mydata$Company=='Ford',]),
col="#33CCFF")
abline(lm(PP100 ~ WRI, data = mydata[mydata$Company=='Chrysler',]),
col="#0033FF")
abline(lm(PP100 ~ WRI, data = mydata[mydata$Company=='Toyota',]),
col="#FF0033", lty="dotted")
abline(lm(PP100 ~ WRI, data = mydata[mydata$Company=='Honda',]),
col="#993300", lty="dotted")
abline(lm(PP100 ~ WRI, data = mydata[mydata$Company=='Nissan',]),
col="#CC9900", lty="dotted")

regGM <-lm(PP100 ~ WRI, data = mydata[mydata$Company=='GM',])
summary(regGM)

regFord <-lm(PP100 ~ WRI, data = mydata[mydata$Company=='Ford',])
```

```
summary(regFord)

regChrysler <-lm(PP100 ~ WRI, data = mydata[mydata$Company=='Chrysler',])
summary(regChrysler)

regToyota <-lm(PP100 ~ WRI, data = mydata[mydata$Company=='Toyota',])
summary(regToyota)

regHonda <-lm(PP100 ~ WRI, data = mydata[mydata$Company=='Honda',])
summary(regHonda)

regNissan <-lm(PP100 ~ WRI, data = mydata[mydata$Company=='Nissan',])
summary(regNissan)



########################################################################################################################
#                                            Part II: Regression Models on PA
Model
########################################################################################################################

#######################################################Supplier Problem:
Regression Model 1#################################################

##Industry##
model_A <- lm((1/PP100) ~ -1+WSR_S+WSR_S:WRI, data = mydata)
summary(model_A)

##Country US##
model_C_US <- lm((1/PP100) ~ -1+WSR_S+WSR_S:WRI, data =
mydata[mydata$Country=='US',])
summary(model_C_US)

##Country Japanese##
model_C_Jap <- lm((1/PP100) ~ -1+WSR_S+WSR_S:WRI, data =
mydata[mydata$Country=='Japan',])
summary(model_C_Jap)

##Segment Luxury##
model_LM_Lux <- lm((1/PP100) ~ -1+WSR_S+WSR_S:WRI, data =
mydata[mydata$Segment=='Luxury',])
summary(model_LM_Lux)

##Segment Mass Market##
model_LM_Mass <- lm((1/PP100) ~ -1+WSR_S+WSR_S:WRI, data =
mydata[mydata$Segment=='Mass',])
summary(model_LM_Mass)

##Company GM##
model_O_GM <- lm((1/PP100) ~ -1+WSR_S+WSR_S:WRI, data =
mydata[mydata$Company=='GM',])
summary(model_O_GM)

##Company Ford##
```

```
model_O_Ford <- lm((1/PP100) ~ -1+WSR_S+WSR_S:WRI, data =
mydata[mydata$Company=='Ford',])
summary(model_O_Ford)

##Company Chrysler##
model_O_Chrysler <- lm((1/PP100) ~ -1+WSR_S+WSR_S:WRI, data =
mydata[mydata$Company=='Chrysler',])
summary(model_O_Chrysler)

##Company Toyota##
model_O_Toyota <- lm((1/PP100) ~ -1+WSR_S+WSR_S:WRI, data =
mydata[mydata$Company=='Toyota',])
summary(model_O_Toyota)

##Company Honda##
model_O_Honda <- lm((1/PP100) ~ -1+WSR_S+WSR_S:WRI, data =
mydata[mydata$Company=='Honda',])
summary(model_O_Honda)

##Company Nissan##
model_O_Nissan <- lm((1/PP100) ~ -1+WSR_S+WSR_S:WRI, data =
mydata[mydata$Company=='Nissan',])
summary(model_O_Nissan)


#######################################################Manufacturer's
Problem: Regression Model 2##############################################
mydata$WSR_SQR <- sqrt((1-mydata$WSR_S)/mydata$WSR_S)

##Industry##
Manufacturer_A <- lm(WRI ~ WSR_SQR, data = mydata)
summary(Manufacturer_A)

##Country US##
Manufacturer_C_US <- lm(WRI ~ WSR_SQR, data = mydata[mydata$Country=='US',])
summary(Manufacturer_C_US)

##Country Japan##
Manufacturer_C_Jap <- lm(WRI ~ WSR_SQR, data =
mydata[mydata$Country=='Japan',])
summary(Manufacturer_C_Jap)

##Segment Luxury##
Manufacturer_LM_Lux <- lm(WRI ~ WSR_SQR, data =
mydata[mydata$Segment=='Luxury',])
summary(Manufacturer_LM_Lux)

##Segment Mass Market##
Manufacturer_LM_Mass <- lm(WRI ~ WSR_SQR, data =
mydata[mydata$Segment=='Mass',])
summary(Manufacturer_LM_Mass)

##Company GM##
Manufacturer_O_GM <- lm(WRI ~ WSR_SQR, data = mydata[mydata$Company=='GM',])
summary(Manufacturer_O_GM)

##Company Ford##
```

```
Manufacturer_O_Ford <- lm(WRI ~ WSR_SQR, data =
mydata[mydata$Company=='Ford',])
summary(Manufacturer_O_Ford)

##Company Chrysler##
Manufacturer_O_Chrysler <- lm(WRI ~ WSR_SQR, data =
mydata[mydata$Company=='Chrysler',])
summary(Manufacturer_O_Chrysler)

##Company Toyota##
Manufacturer_O_Toyota <- lm(WRI ~ WSR_SQR, data =
mydata[mydata$Company=='Toyota',])
summary(Manufacturer_O_Toyota)

##Company Honda##
Manufacturer_O_Honda <- lm(WRI ~ WSR_SQR, data =
mydata[mydata$Company=='Honda',])
summary(Manufacturer_O_Honda)

##Company Nissan##
Manufacturer_O_Nissan <- lm(WRI ~ WSR_SQR, data =
mydata[mydata$Company=='Nissan',])
summary(Manufacturer_O_Nissan)

#######################################################Manufacturer's
Problem: Regression Model 3###########################################
mydata$WSR_Demon <- 1/sqrt(mydata$WSR_S*(1-mydata$WSR_S))

##Industry##
Manufacturer_A <- lm(PP100 ~ -1 + WSR_Demon, data = mydata)
summary(Manufacturer_A)

##Country US##
Manufacturer_C_US <- lm(PP100 ~ -1 + WSR_Demon, data =
mydata[mydata$Country=='US',])
summary(Manufacturer_C_US)

##Country Japan##
Manufacturer_C_Jap <- lm(PP100 ~ -1 + WSR_Demon, data =
mydata[mydata$Country=='Japan',])
summary(Manufacturer_C_Jap)

##Segment Luxury##
Manufacturer_LM_Lux <- lm(PP100 ~ -1 + WSR_Demon, data =
mydata[mydata$Segment=='Luxury',])
summary(Manufacturer_LM_Lux)

##Segment Mass Market##
Manufacturer_LM_Mass <- lm(PP100 ~ -1 + WSR_Demon, data =
mydata[mydata$Segment=='Mass',])
summary(Manufacturer_LM_Mass)

##Company GM##
Manufacturer_O_GM <- lm(PP100 ~ -1 + WSR_Demon, data =
mydata[mydata$Company=='GM',])
summary(Manufacturer_O_GM)
```

```
##Company Ford##
Manufacturer_O_Ford <- lm(PP100 ~ -1 + WSR_Demon, data =
mydata[mydata$Company=='Ford',])
summary(Manufacturer_O_Ford)

##Company Chrysler##
Manufacturer_O_Chrysler <- lm(PP100 ~ -1 + WSR_Demon, data =
mydata[mydata$Company=='Chrysler',])
summary(Manufacturer_O_Chrysler)

##Company Toyota##
Manufacturer_O_Toyota <- lm(PP100 ~ -1 + WSR_Demon, data =
mydata[mydata$Company=='Toyota',])
summary(Manufacturer_O_Toyota)

##Company Honda##
Manufacturer_O_Honda <- lm(PP100 ~ -1 + WSR_Demon, data =
mydata[mydata$Company=='Honda',])
summary(Manufacturer_O_Honda)

##Company Nissan##
Manufacturer_O_Nissan <- lm(PP100 ~ -1 + WSR_Demon, data =
mydata[mydata$Company=='Nissan',])
summary(Manufacturer_O_Nissan)

#######################################################The End of Part II:
Regression Analysis##############################################


#############################################################################
################################################
#                                           Part III: Multi-Objective
Optimization
#############################################################################
################################################

## Regression Equation 1: PP100 Function
PP100Function <- function(Lambda, WRI, x)  {
  alpha1 <- x[1]
  alpha2 <- x[2]
  eta_M <- x[3]

  value_equ <- alpha1 * Lambda + alpha2 * Lambda * WRI
  return(value_equ)
}

## Regression Equation 2: WRI Function
WRIFunction <- function(Lambda, WRI, x)  {
  alpha1 <- x[1]
  alpha2 <- x[2]
  eta_M <- x[3]

  value_equ <- -alpha1/alpha2  + sqrt(1/(alpha2 * eta_M))*sqrt((1-
Lambda)/Lambda)
  return(value_equ)
}
```

```
## Multi-Objective Optimization with weight parameter w = 1/400000000
Square_Sum <- function(Lambda, PP100, WRI, x) {
  alpha1 <- x[1]
  alpha2 <- x[2]
  eta_M <- x[3]
  (PP100Function(Lambda, WRI, x) - 1/PP100)^2 + (1/400000000) *
(WRIFunction(Lambda, WRI, x) - WRI)^2
}

## Error Term for PP100 function
Square_Sum_Error1 <- function(Lambda, PP100, WRI, x) {
  alpha1 <- x[1]
  alpha2 <- x[2]
  eta_M <- x[3]
  (PP100Function(Lambda, WRI, x) - 1/PP100)^2
}

## Error Term for WRI function
Square_Sum_Error2 <- function(Lambda, PP100, WRI, x) {
  alpha1 <- x[1]
  alpha2 <- x[2]
  eta_M <- x[3]
  (1/400000000) * (WRIFunction(Lambda, WRI, x) - WRI)^2
}

## Gradient Function: only for gradient-based ConstrOptim(grad = Sum_Grr)
Grr_Sum <- function(Lambda, PP100, WRI, x) {
  alpha1 <- x[1]
  alpha2 <- x[2]
  eta_M <- x[3]
  c(2*Lambda*(alpha1*Lambda+alpha2*Lambda*WRI-1/PP100)-
2*1/400000000/alpha2*(-alpha1/alpha2+sqrt(1/alpha2/eta_M)*sqrt((1-
Lambda)/Lambda)-WRI), 2*Lambda*WRI*(alpha1*Lambda+alpha2*Lambda*WRI-
1/PP100)+1/400000000*(2*alpha1/alpha2-sqrt(1/eta_M)*sqrt(1/alpha2^3)*sqrt((1-
Lambda)/Lambda))*(-alpha1/alpha2+sqrt(1/alpha2/eta_M)*sqrt((1-
Lambda)/Lambda)-WRI), -1/400000000*sqrt(1/alpha2)*sqrt(1/eta_M^3)*sqrt((1-
Lambda)/Lambda)*(-alpha1/alpha2+sqrt(1/alpha2/eta_M)*sqrt((1-Lambda)/Lambda)-
WRI))
}


#######################################################Toyota
Data############################################################
## Normalized Sum of Square Error
Sum_Square <- function(x) {
  (
    Square_Sum(0.119297488, 93, 407, x) + Square_Sum(0.119297488, 106, 407,
x) + #2006
      Square_Sum(0.115575768, 94, 415, x) + Square_Sum(0.115575768, 112, 415,
x) + #2007
      Square_Sum(0.128624275, 99, 367, x) + Square_Sum(0.128624275, 104, 367,
x) + #2008
      Square_Sum(0.14644388, 84, 339, x) +  Square_Sum(0.14644388, 101, 339,
x) + #2009
      Square_Sum(0.1574043, 88, 330, x) +   Square_Sum(0.1574043, 117, 330,
x) + #2010
```

```
      Square_Sum(0.154151942, 73, 327, x) + Square_Sum(0.154151942, 101, 327,
x) + #2011
      Square_Sum(0.159449938, 73, 296, x) + Square_Sum(0.159449938, 88, 296,
x) + #2012
      Square_Sum(0.148499693, 94, 297, x) + Square_Sum(0.148499693, 102, 297,
x) + #2013
      Square_Sum(0.118992575, 92, 318, x) + Square_Sum(0.118992575, 105, 318,
x) + #2014
      Square_Sum(0.12292571, 104, 336, x) + Square_Sum(0.12292571, 104, 336,
x) + #2015
      Square_Sum(0.12745136, 96, 332, x) +  Square_Sum(0.12745136, 93, 332,
x) + #2016
      Square_Sum(0.142454443, 98, 328, x) + Square_Sum(0.142454443, 95, 328,
x)  #2017
  ) / 24

}

## Normalized Sum of Square Error Contributed by PP100 function
Sum_Square_Error1 <- function(x) {
  (
    Square_Sum_Error1(0.119297488, 93, 407, x) +
Square_Sum_Error1(0.119297488, 106, 407, x) + #2006
      Square_Sum_Error1(0.115575768, 94, 415, x) +
Square_Sum_Error1(0.115575768, 112, 415, x) + #2007
      Square_Sum_Error1(0.128624275, 99, 367, x) +
Square_Sum_Error1(0.128624275, 104, 367, x) + #2008
      Square_Sum_Error1(0.14644388, 84, 339, x) +
Square_Sum_Error1(0.14644388, 101, 339, x) + #2009
      Square_Sum_Error1(0.1574043, 88, 330, x) +
Square_Sum_Error1(0.1574043, 117, 330, x) + #2010
      Square_Sum_Error1(0.154151942, 73, 327, x) +
Square_Sum_Error1(0.154151942, 101, 327, x) + #2011
      Square_Sum_Error1(0.159449938, 73, 296, x) +
Square_Sum_Error1(0.159449938, 88, 296, x) + #2012
      Square_Sum_Error1(0.148499693, 94, 297, x) +
Square_Sum_Error1(0.148499693, 102, 297, x) + #2013
      Square_Sum_Error1(0.118992575, 92, 318, x) +
Square_Sum_Error1(0.118992575, 105, 318, x) + #2014
      Square_Sum_Error1(0.12292571, 104, 336, x) +
Square_Sum_Error1(0.12292571, 104, 336, x) + #2015
      Square_Sum_Error1(0.12745136, 96, 332, x) +
Square_Sum_Error1(0.12745136, 93, 332, x) + #2016
      Square_Sum_Error1(0.142454443, 98, 328, x) +
Square_Sum_Error1(0.142454443, 95, 328, x)  #2017
  ) / 24

}

## Normalized Sum of Square Error Contributed by WRI function
Sum_Square_Error2 <- function(x) {
  (
    Square_Sum_Error2(0.119297488, 93, 407, x) +
Square_Sum_Error2(0.119297488, 106, 407, x) + #2006
      Square_Sum_Error2(0.115575768, 94, 415, x) +
Square_Sum_Error2(0.115575768, 112, 415, x) + #2007
```

```
      Square_Sum_Error2(0.128624275, 99, 367, x) +
Square_Sum_Error2(0.128624275, 104, 367, x) + #2008
      Square_Sum_Error2(0.14644388, 84, 339, x) +
Square_Sum_Error2(0.14644388, 101, 339, x) + #2009
      Square_Sum_Error2(0.1574043, 88, 330, x) +
Square_Sum_Error2(0.1574043, 117, 330, x) + #2010
      Square_Sum_Error2(0.154151942, 73, 327, x) +
Square_Sum_Error2(0.154151942, 101, 327, x) + #2011
      Square_Sum_Error2(0.159449938, 73, 296, x) +
Square_Sum_Error2(0.159449938, 88, 296, x) + #2012
      Square_Sum_Error2(0.148499693, 94, 297, x) +
Square_Sum_Error2(0.148499693, 102, 297, x) + #2013
      Square_Sum_Error2(0.118992575, 92, 318, x) +
Square_Sum_Error2(0.118992575, 105, 318, x) + #2014
      Square_Sum_Error2(0.12292571, 104, 336, x) +
Square_Sum_Error2(0.12292571, 104, 336, x) + #2015
      Square_Sum_Error2(0.12745136, 96, 332, x) +
Square_Sum_Error2(0.12745136, 93, 332, x) + #2016
      Square_Sum_Error2(0.142454443, 98, 328, x) +
Square_Sum_Error2(0.142454443, 95, 328, x)  #2017
  ) / 24

}

## Gradient Function
Sum_Grr <- function(x) {

  Grr_Sum(0.119297488, 93, 407, x) + Grr_Sum(0.119297488, 106, 407, x) +
#2006
    Grr_Sum(0.115575768, 94, 415, x) + Grr_Sum(0.115575768, 112, 415, x) +
#2007
    Grr_Sum(0.128624275, 99, 367, x) + Grr_Sum(0.128624275, 104, 367, x) +
#2008
    Grr_Sum(0.14644388, 84, 339, x) +  Grr_Sum(0.14644388, 101, 339, x) +
#2009
    Grr_Sum(0.1574043, 88, 330, x) +   Grr_Sum(0.1574043, 117, 330, x) +
#2010
    Grr_Sum(0.154151942, 73, 327, x) + Grr_Sum(0.154151942, 101, 327, x) +
#2011
    Grr_Sum(0.159449938, 73, 296, x) + Grr_Sum(0.159449938, 88, 296, x) +
#2012
    Grr_Sum(0.148499693, 94, 297, x) + Grr_Sum(0.148499693, 102, 297, x) +
#2013
    Grr_Sum(0.118992575, 92, 318, x) + Grr_Sum(0.118992575, 105, 318, x) +
#2014
    Grr_Sum(0.12292571, 104, 336, x) + Grr_Sum(0.12292571, 104, 336, x) +
#2015
    Grr_Sum(0.12745136, 96, 332, x) +  Grr_Sum(0.12745136, 93, 332, x) +
#2016
    Grr_Sum(0.142454443, 98, 328, x) + Grr_Sum(0.142454443, 95, 328, x)
#2017


}

## Gradient Free Method "Nelder-Mead": Paramter, Objective Function value
(SSE), Convergence (if 0 -> convergent, if 1 -> not convergent)
```

```
GradientFreeParameter <- matrix (nrow = 1000, ncol = 3)
colnames(GradientFreeParameter) <- c("alpha_1", "alpha_2", "eta_m")
GradientFreeObjFuncValue <- matrix (nrow = 1000, ncol = 1)
GradientFreeConvergence <- matrix (nrow = 1000, ncol = 1)

## Generate Random Number 0<alpha1<0.01, 0<alpha2<0.01, 0<eta<1
randomnumber <-
matrix(c(runif(1000,min=0,max=0.01),runif(1000,min=0,max=0.01),runif(1000,min
=0,max=1)),nrow = 1000, ncol = 3)

## Gradient Free method: Error contribution from PP100 and WRI functions
ErrorMatrixGF <- matrix (nrow = 1000, ncol = 2)
colnames(ErrorMatrixGF) <- c("Error_PP100", "Error_WRI")

## Gradient Based method: Error contribution from PP100 and WRI functions
ErrorMatrixGB <- matrix (nrow = 1000, ncol = 2)
colnames(ErrorMatrixGB) <- c("Error_PP100", "Error_WRI")

## Gradient Based Method "BFGS": Paramter, Objective Function value (SSE),
Convergence (if 0 -> convergent, if 1 -> not convergent)
GradientBasedParameter <- matrix (nrow = 1000, ncol = 3)
colnames(GradientBasedParameter) <- c("alpha_1", "alpha_2", "eta_m")
GradientBasedObjFuncValue <- matrix (nrow = 1000, ncol = 1)
GradientBasedConvergence <- matrix (nrow = 1000, ncol = 1)

## 1000 runs of random initial values to trigger both optimization methods
for (i in 1:1000){
  ## Results for Gradient Free method "Nelder-Mead"
  GradientFree <- constrOptim(theta = randomnumber[i,1:3], f = Sum_Square,
grad = NULL, ui = rbind(c(1,0,0),c(0,1,0),c(0,0,1)), ci = c(0,0,0))

  ## Results for Gradient Based method "BFGS"
  GradientBased <- constrOptim(theta = randomnumber[i,1:3], f = Sum_Square,
grad = Sum_Grr, ui = rbind(c(1,0,0),c(0,1,0),c(0,0,1)), ci = c(0,0,0))

  ## Optimal Parameters for Gradient Free method "Nelder-Mead"
  GradientFreeParameter[i,1:3] <- GradientFree$par

  ## Optimal Parameters for Gradient Based method "BFGS"
  GradientBasedParameter[i,1:3] <- GradientBased$par

  ## Objective Function Value for Gradient Free method "Nelder-Mead"
  GradientFreeObjFuncValue[i] <- GradientFree$value

  ## Objective Function Value for Gradient Based method "BFGS"
  GradientBasedObjFuncValue[i] <- GradientBased$value

  ## Convergence for Gradient Free method "Nelder-Mead"
  GradientFreeConvergence[i] <- GradientFree$convergence

  ## Convergence for Gradient Based method "BFGS"
  GradientBasedConvergence[i] <- GradientBased$convergence

  ## Error Contribution from PP100 for Gradient Free method "Nelder-Mead"
  ErrorMatrixGF[i,1] <- Sum_Square_Error1(GradientFree$par)

  ## Error Contribution from WRI for Gradient Free method "Nelder-Mead"
```

```
  ErrorMatrixGF[i,2] <- Sum_Square_Error2(GradientFree$par)

  ## Error Contribution from PP100 for Gradient Based method "BFGS"
  ErrorMatrixGB[i,1] <- Sum_Square_Error1(GradientBased$par)

  ## Error Contribution from WRI for Gradient Based method "BFGS"
  ErrorMatrixGB[i,2] <- Sum_Square_Error2(GradientBased$par)

}

#######################################################Gradient Free "Nelder-
Mead"#############################################################

## Identify Gradient Free non-convergence solutions
IndexMatrixGF <- which(GradientFreeConvergence != 0, arr.ind=TRUE)
IndexMatrixGFRow <- IndexMatrixGF[,1]

## Delete Gradient Free non-convergence solutions
if(length(IndexMatrixGFRow) == 0){
  print(IndexMatrixGFRow)
}else{
  GradientFreeParameter <- GradientFreeParameter[-IndexMatrixGFRow, ]
  GradientFreeObjFuncValue <- GradientFreeObjFuncValue[-IndexMatrixGFRow]
  GradientFreeConvergence <- GradientFreeConvergence[-IndexMatrixGFRow]
  ErrorMatrixGF <- ErrorMatrixGF[-IndexMatrixGFRow,]
}

## Boxplot Optimal Parameters of all the convergent "Nelder-Mead" cases for
the 1000 random initial picks
Toyota_Box <- boxplot.matrix(GradientFreeParameter,main = "Toyota:
Sensitivity Analysis on Initial Value Pick",
                             xlab = "Parameter Estimation",
                             ylab = "Values",
                             yaxt='n', ann=FALSE,
                             col = "lightgray",
                             border = "black")

axis(2, at = seq(0, 0.8, by = 0.05), las=2)

## Qauntile statistics of all the convergent "Nelder-Mead" cases for the 1000
random initial picks
library(matrixStats)
colQuantiles(GradientFreeParameter, probs = seq(from = 0, to = 1, by = 0.25))

## Boxplot Objective Function Value of all the "Nelder-Mead" convergent cases
for the 1000 random initial picks
boxplot(GradientFreeObjFuncValue, main = "Toyota: Sensitivity Analysis on
Minimized Optimization Errors",
        xlab = "Objective Function Value",
        ylab = "Values",
        col = "lightgray",
        border = "black")

## Convergence Rate for "Nelder-Mead"
Percentage_Converge_GF <- sum(GradientFreeConvergence == 0) / 1000
Percentage_Converge_GF
```

```
#######################################################Gradient Based
"BFGS"#######################################################

## Identify Gradient Based non-convergence solutions
IndexMatrixGB <- which(GradientBasedConvergence != 0, arr.ind=TRUE)
IndexMatrixGBRow <- IndexMatrixGB[,1]

## Delete non-convergence solutions
if(length(IndexMatrixGBRow) == 0){
  print(IndexMatrixGBRow)
}else{
  GradientBasedParameter <- GradientBasedParameter[-IndexMatrixGBRow, ]
  GradientBasedObjFuncValue <- GradientBasedObjFuncValue[-IndexMatrixGBRow]
  GradientBasedConvergence <- GradientBasedConvergence[-IndexMatrixGBRow]
  ErrorMatrixGF <- ErrorMatrixGF[-IndexMatrixGBRow,]
}

## Boxplot Optimal Parameters of all the convergent "BFGS" cases for the 1000
random initial picks
Toyota_Box <- boxplot.matrix(GradientBasedParameter,main = "Toyota:
Sensitivity Analysis on Initial Value Pick",
                             xlab = "Parameter Estimation",
                             ylab = "Values",
                             yaxt='n', ann=FALSE,
                             col = "lightgray",
                             border = "black")

axis(2, at = seq(0, 0.8, by = 0.05), las=2)

## Qauntile statistics of all the convergent "BFGS" cases for the 1000 random
initial picks
colQuantiles(GradientBasedParameter, probs = seq(from = 0, to = 1, by =
0.25))

## Boxplot Objective Function Value of all the "BFGS" convergent cases for
the 1000 random initial picks
boxplot(GradientBasedObjFuncValue, main = "Toyota: Sensitivity Analysis on
Minimized Optimization Errors",
        xlab = "Objective Function Value",
        ylab = "Values",
        col = "lightgray",
        border = "black")

## Convergence Rate for "BFGS"
Percentage_Converge_GB <- sum(GradientBasedConvergence == 0) / 1000
Percentage_Converge_GB

#######################################################Comparison between
Gradient Free "Nelder-Mead" vs Gradient Based
"BFGS"#######################################################

## Data frame both gradient based results and gradient free results
df <- data.frame(id = c(rep("Nelder-Mead",length(GradientFreeObjFuncValue)),
rep("BFGS", length(GradientBasedObjFuncValue))),
                 alpha1 = c(GradientFreeParameter[,1],
GradientBasedParameter[,1]),
```

```
                  alpha2 = c(GradientFreeParameter[,2],
GradientBasedParameter[,2]),
                  eta_m = c(GradientFreeParameter[,3],
GradientBasedParameter[,3]),
                  SSE = c(GradientFreeObjFuncValue,
GradientBasedObjFuncValue))

## Boxplot comparison of gradient based results versus gradient free results
boxplot(df[,-1], main = "Toyota: Sensitivity Analysis on Optimization
Algorithm Nelder-Mead vs BFGS", xlim = c(0.5, ncol(df[,-1])+0.5),
        boxfill=rgb(1, 1, 1, alpha=1), border=rgb(1, 1, 1, alpha=1))
#invisible boxes
boxplot(df[which(df$id=="Nelder-Mead"), -1], xaxt = "n", add = TRUE,
boxfill="red", boxwex=0.25,
        at = 1:ncol(df[,-1]) - 0.15) #shift these left by -0.15
boxplot(df[which(df$id=="BFGS"), -1], xaxt = "n", add = TRUE, boxfill="blue",
boxwex=0.25,
        at = 1:ncol(df[,-1]) + 0.15) #shift these right by +0.15


## Boxplot comparison of objective function value of gradient based results
versus gradient free results
df <- data.frame(id = c(rep("Nelder-Mead",length(GradientFreeObjFuncValue)),
rep("BFGS", length(GradientBasedObjFuncValue))),
                  SSE = c(GradientFreeObjFuncValue,
GradientBasedObjFuncValue))

boxplot(df[which(df$id=="Nelder-Mead"), -1], df[which(df$id=="BFGS"), -1],
        col=(c("red","blue")),
        main="Toyota: Sensitivity Analysis on Optimization Algorithm Nelder-
Mead vs BFGS", xlab="Nelder-Mead vs BFGS")


############################## Analysis on Global Optimal Solution (Nelder-
Mead): minimal obj function value out of 1000 optimal runs##############
## Global Optimal Solution: minimal obj function value out of 1000 optimal
runs
MinValueIndexMatrixGFRow <- which.min(GradientFreeObjFuncValue)

## Global optimal parameters, obj function value, errors
etam <- GradientFreeParameter[MinValueIndexMatrixGFRow,3]
alpha1 <- GradientFreeParameter[MinValueIndexMatrixGFRow,1]
alpha2 <- GradientFreeParameter[MinValueIndexMatrixGFRow,2]
obj_func_value <- GradientFreeObjFuncValue[MinValueIndexMatrixGFRow]
errorrate1 <- ErrorMatrixGF[MinValueIndexMatrixGFRow,1]
errorrate2 <- ErrorMatrixGF[MinValueIndexMatrixGFRow,2]

## Actual Data
Toyota_data <- mydata[mydata$Company=='Toyota',]
wsr <- Toyota_data$WSR_S
pp100 <- Toyota_data$PP100
wri <- Toyota_data$WRI
year <- Toyota_data$Year

########################################################## WRI fitted vs WRI
data #############################################3
wri_fitted <- -alpha1/alpha2  + sqrt(1/(alpha2 * etam))*sqrt((1-wsr)/wsr)
```

```
wri_fitted_results <- data.frame(YearWRI = year, WRIWRI = wri_fitted, Type =
"Estimate")
wri_data_results <- data.frame(YearWRI = year, WRIWRI = wri, Type = "Data")
wri_results <- rbind(wri_data_results, wri_fitted_results)

shapes = c(19, 8)
shapes <- shapes[as.numeric(wri_results$Type)]
colors <- c("#FF0033", "#0033FF")
colors <- colors[as.numeric(wri_results$Type)]

plot(wri_results$YearWRI, wri_results$WRIWRI, main = "Toyota WRI: Data vs
Estimate",
     xlab = "Year", ylab = "WRI", pch = shapes, col = colors)
legend("topright", legend = levels(wri_results$Type),
       col =  c("#FF0033", "#0033FF"), pch = c(19, 8)  )
axis(1,at=2006:2017,labels=2006:2017)

######################################################## PP100 fitted vs
PP100 data ##############################################3
pp100_fitted<-1/(alpha1*wsr+alpha2*wsr*wri_fitted)
pp100_fitted_results <- data.frame(YearPP100 = year, PP100PP100 =
pp100_fitted, Type = "Estimate")

pp100_data_results <- data.frame(YearPP100 = year, PP100PP100 = pp100, Type =
"Data")

pp100_results <- rbind(pp100_data_results, pp100_fitted_results)

shapes = c(19, 8)
shapes <- shapes[as.numeric(pp100_results$Type)]
colors <- c("#FF0033", "#0033FF")
colors <- colors[as.numeric(pp100_results$Type)]

plot(pp100_results$YearPP100, pp100_results$PP100PP100, main = "Toyota PP100:
Data vs Estimate",
     xlab = "Year", ylab = "PP100", pch = shapes, col = colors)
legend("topright", legend = levels(pp100_results$Type),
       col =  c("#FF0033", "#0033FF"), pch = c(19, 8)  )
axis(1,at=2006:2017,labels=2006:2017)

######################################################## optimal estimated
cost of manufacturer vs actual data
#############################################3
# Estimated total manufacturer's cost
cost_m_emp<-(1-wsr)*pp100+etam*wri

cost_m_opt<-(1-wsr)*pp100_fitted+etam*wri_fitted

cost_data_results <- data.frame(YearCost = year, CostCost = cost_m_emp, Type
= "Data")

cost_fitted_results <- data.frame(YearCost = year, CostCost = cost_m_opt,
Type = "Estimate")

cost_results <- rbind(cost_data_results, cost_fitted_results)

shapes = c(19, 8)
```

```
shapes <- shapes[as.numeric(pp100_results$Type)]
colors <- c("#FF0033", "#0033FF")
colors <- colors[as.numeric(pp100_results$Type)]

plot(cost_results$YearCost, cost_results$CostCost, main = "Toyota's Total
Costs: Data vs Estimate",
     xlab = "Year", ylab = "Manufacturer's Total Costs", pch = shapes, col =
colors)
legend("topright", legend = levels(cost_results$Type),
       col =  c("#FF0033", "#0033FF"), pch = c(19, 8)  )
axis(1,at=2006:2017,labels=2006:2017)


######################################################## End of Part III
Multi-objective Optimization #############################################3


#######################################################Honda
Data############################################################

Sum_Square <- function(x) {
  (
    Square_Sum(0.119297488, 120, 368, x) + Square_Sum(0.119297488, 110, 368,
x) + #2006
      Square_Sum(0.115575768, 130, 380, x) + Square_Sum(0.115575768, 108,
380, x) + #2007
      Square_Sum(0.128624275, 119, 359, x) + Square_Sum(0.128624275, 110,
359, x) + #2008
      Square_Sum(0.14644388, 111, 349, x) + Square_Sum(0.14644388, 99, 349,
x) + #2009
      Square_Sum(0.1574043, 86, 340, x) + Square_Sum(0.1574043, 95, 340, x) +
#2010
      Square_Sum(0.154151942, 89, 309, x) + Square_Sum(0.154151942, 86, 309,
x) + #2011
      Square_Sum(0.159449938, 84, 293, x) + Square_Sum(0.159449938, 83, 293,
x) + #2012
      Square_Sum(0.148499693, 102, 287, x) + Square_Sum(0.148499693, 103,
287, x) + #2013
      Square_Sum(0.118992575, 131, 295, x) + Square_Sum(0.118992575, 108,
295, x) + #2014
      Square_Sum(0.12292571, 126, 330, x) + Square_Sum(0.12292571, 111, 330,
x) + #2015
      Square_Sum(0.12745136, 122, 323, x) + Square_Sum(0.12745136, 119, 323,
x) + #2016
      Square_Sum(0.142454443, 103, 319, x) + Square_Sum(0.142454443, 105,
319, x) #2017
  )/24
}


Sum_Square_Error1 <- function(x) {

  (
    Square_Sum_Error1(0.119297488, 120, 368, x) +
Square_Sum_Error1(0.119297488, 110, 368, x) + #2006
      Square_Sum_Error1(0.115575768, 130, 380, x) +
Square_Sum_Error1(0.115575768, 108, 380, x) + #2007
```

```
      Square_Sum_Error1(0.128624275, 119, 359, x) +
Square_Sum_Error1(0.128624275, 110, 359, x) + #2008
      Square_Sum_Error1(0.14644388, 111, 349, x) +
Square_Sum_Error1(0.14644388, 99, 349, x) + #2009
      Square_Sum_Error1(0.1574043, 86, 340, x) + Square_Sum_Error1(0.1574043,
95, 340, x) + #2010
      Square_Sum_Error1(0.154151942, 89, 309, x) +
Square_Sum_Error1(0.154151942, 86, 309, x) + #2011
      Square_Sum_Error1(0.159449938, 84, 293, x) +
Square_Sum_Error1(0.159449938, 83, 293, x) + #2012
      Square_Sum_Error1(0.148499693, 102, 287, x) +
Square_Sum_Error1(0.148499693, 103, 287, x) + #2013
      Square_Sum_Error1(0.118992575, 131, 295, x) +
Square_Sum_Error1(0.118992575, 108, 295, x) + #2014
      Square_Sum_Error1(0.12292571, 126, 330, x) +
Square_Sum_Error1(0.12292571, 111, 330, x) + #2015
      Square_Sum_Error1(0.12745136, 122, 323, x) +
Square_Sum_Error1(0.12745136, 119, 323, x) + #2016
      Square_Sum_Error1(0.142454443, 103, 319, x) +
Square_Sum_Error1(0.142454443, 105, 319, x) #2017
  )/24

}

Sum_Square_Error2 <- function(x) {

  (
    Square_Sum_Error2(0.119297488, 120, 368, x) +
Square_Sum_Error2(0.119297488, 110, 368, x) + #2006
      Square_Sum_Error2(0.115575768, 130, 380, x) +
Square_Sum_Error2(0.115575768, 108, 380, x) + #2007
      Square_Sum_Error2(0.128624275, 119, 359, x) +
Square_Sum_Error2(0.128624275, 110, 359, x) + #2008
      Square_Sum_Error2(0.14644388, 111, 349, x) +
Square_Sum_Error2(0.14644388, 99, 349, x) + #2009
      Square_Sum_Error2(0.1574043, 86, 340, x) + Square_Sum_Error2(0.1574043,
95, 340, x) + #2010
      Square_Sum_Error2(0.154151942, 89, 309, x) +
Square_Sum_Error2(0.154151942, 86, 309, x) + #2011
      Square_Sum_Error2(0.159449938, 84, 293, x) +
Square_Sum_Error2(0.159449938, 83, 293, x) + #2012
      Square_Sum_Error2(0.148499693, 102, 287, x) +
Square_Sum_Error2(0.148499693, 103, 287, x) + #2013
      Square_Sum_Error2(0.118992575, 131, 295, x) +
Square_Sum_Error2(0.118992575, 108, 295, x) + #2014
      Square_Sum_Error2(0.12292571, 126, 330, x) +
Square_Sum_Error2(0.12292571, 111, 330, x) + #2015
      Square_Sum_Error2(0.12745136, 122, 323, x) +
Square_Sum_Error2(0.12745136, 119, 323, x) + #2016
      Square_Sum_Error2(0.142454443, 103, 319, x) +
Square_Sum_Error2(0.142454443, 105, 319, x) #2017
  )/24
}


Sum_Grr <- function(x) {
```

```
  Grr_Sum(0.119297488, 120, 368, x) + Grr_Sum(0.119297488, 110, 368, x) +
#2006
    Grr_Sum(0.115575768, 130, 380, x) + Grr_Sum(0.115575768, 108, 380, x) +
#2007
    Grr_Sum(0.128624275, 119, 359, x) + Grr_Sum(0.128624275, 110, 359, x) +
#2008
    Grr_Sum(0.14644388, 111, 349, x) + Grr_Sum(0.14644388, 99, 349, x) +
#2009
    Grr_Sum(0.1574043, 86, 340, x) + Grr_Sum(0.1574043, 95, 340, x) + #2010
    Grr_Sum(0.154151942, 89, 309, x) + Grr_Sum(0.154151942, 86, 309, x) +
#2011
    Grr_Sum(0.159449938, 84, 293, x) + Grr_Sum(0.159449938, 83, 293, x) +
#2012
    Grr_Sum(0.148499693, 102, 287, x) + Grr_Sum(0.148499693, 103, 287, x) +
#2013
    Grr_Sum(0.118992575, 131, 295, x) + Grr_Sum(0.118992575, 108, 295, x) +
#2014
    Grr_Sum(0.12292571, 126, 330, x) + Grr_Sum(0.12292571, 111, 330, x) +
#2015
    Grr_Sum(0.12745136, 122, 323, x) + Grr_Sum(0.12745136, 119, 323, x) +
#2016
    Grr_Sum(0.142454443, 103, 319, x) + Grr_Sum(0.142454443, 105, 319, x)
#2017


}


#######################################################Nissan
Data############################################################

Sum_Square <- function(x) {
  (
    Square_Sum(0.119297488, 117, 300, x) + Square_Sum(0.119297488, 121, 300,
x) + #2006
      Square_Sum(0.115575768, 117, 289, x) + Square_Sum(0.115575768, 132,
289, x) + #2007
      Square_Sum(0.128624275, 98, 253, x) + Square_Sum(0.128624275, 124, 253,
x) + #2008
      Square_Sum(0.14644388, 106, 268, x) + Square_Sum(0.14644388, 110, 268,
x) + #2009
      Square_Sum(0.1574043, 107, 249, x) + Square_Sum(0.1574043, 111, 249, x)
+ #2010
      Square_Sum(0.154151942, 102, 247, x) + Square_Sum(0.154151942, 117,
247, x) + #2011
      Square_Sum(0.159449938, 84, 256, x) + Square_Sum(0.159449938, 99, 256,
x) + #2012
      Square_Sum(0.148499693, 95, 256, x) + Square_Sum(0.148499693, 142, 256,
x) + #2013
      Square_Sum(0.118992575, 128, 273, x) + Square_Sum(0.118992575, 120,
273, x) + #2014
      Square_Sum(0.12292571, 97, 244, x) + Square_Sum(0.12292571, 121, 244,
x) + #2015
      Square_Sum(0.12745136, 103, 225, x) + Square_Sum(0.12745136, 101, 225,
x) + #2016
      Square_Sum(0.142454443, 107, 203, x) + Square_Sum(0.142454443, 93, 203,
x)  #2017
```

```
  )/24

}


Sum_Square_Error1 <- function(x) {

  (
    Square_Sum_Error1(0.119297488, 117, 300, x) +
Square_Sum_Error1(0.119297488, 121, 300, x) + #2006
      Square_Sum_Error1(0.115575768, 117, 289, x) +
Square_Sum_Error1(0.115575768, 132, 289, x) + #2007
      Square_Sum_Error1(0.128624275, 98, 253, x) +
Square_Sum_Error1(0.128624275, 124, 253, x) + #2008
      Square_Sum_Error1(0.14644388, 106, 268, x) +
Square_Sum_Error1(0.14644388, 110, 268, x) + #2009
      Square_Sum_Error1(0.1574043, 107, 249, x) +
Square_Sum_Error1(0.1574043, 111, 249, x) + #2010
      Square_Sum_Error1(0.154151942, 102, 247, x) +
Square_Sum_Error1(0.154151942, 117, 247, x) + #2011
      Square_Sum_Error1(0.159449938, 84, 256, x) +
Square_Sum_Error1(0.159449938, 99, 256, x) + #2012
      Square_Sum_Error1(0.148499693, 95, 256, x) +
Square_Sum_Error1(0.148499693, 142, 256, x) + #2013
      Square_Sum_Error1(0.118992575, 128, 273, x) +
Square_Sum_Error1(0.118992575, 120, 273, x) + #2014
      Square_Sum_Error1(0.12292571, 97, 244, x) +
Square_Sum_Error1(0.12292571, 121, 244, x) + #2015
      Square_Sum_Error1(0.12745136, 103, 225, x) +
Square_Sum_Error1(0.12745136, 101, 225, x) + #2016
      Square_Sum_Error1(0.142454443, 107, 203, x) +
Square_Sum_Error1(0.142454443, 93, 203, x)  #2017
  )/24

}

Sum_Square_Error2 <- function(x) {


  (
    Square_Sum_Error2(0.119297488, 117, 300, x) +
Square_Sum_Error2(0.119297488, 121, 300, x) + #2006
      Square_Sum_Error2(0.115575768, 117, 289, x) +
Square_Sum_Error2(0.115575768, 132, 289, x) + #2007
      Square_Sum_Error2(0.128624275, 98, 253, x) +
Square_Sum_Error2(0.128624275, 124, 253, x) + #2008
      Square_Sum_Error2(0.14644388, 106, 268, x) +
Square_Sum_Error2(0.14644388, 110, 268, x) + #2009
      Square_Sum_Error2(0.1574043, 107, 249, x) +
Square_Sum_Error2(0.1574043, 111, 249, x) + #2010
      Square_Sum_Error2(0.154151942, 102, 247, x) +
Square_Sum_Error2(0.154151942, 117, 247, x) + #2011
      Square_Sum_Error2(0.159449938, 84, 256, x) +
Square_Sum_Error2(0.159449938, 99, 256, x) + #2012
      Square_Sum_Error2(0.148499693, 95, 256, x) +
Square_Sum_Error2(0.148499693, 142, 256, x) + #2013
```

```
      Square_Sum_Error2(0.118992575, 128, 273, x) +
Square_Sum_Error2(0.118992575, 120, 273, x) + #2014
      Square_Sum_Error2(0.12292571, 97, 244, x) +
Square_Sum_Error2(0.12292571, 121, 244, x) + #2015
      Square_Sum_Error2(0.12745136, 103, 225, x) +
Square_Sum_Error2(0.12745136, 101, 225, x) + #2016
      Square_Sum_Error2(0.142454443, 107, 203, x) +
Square_Sum_Error2(0.142454443, 93, 203, x)  #2017
  )/24

}



Sum_Grr <- function(x) {

  Grr_Sum(0.119297488, 117, 300, x) + Grr_Sum(0.119297488, 121, 300, x) +
#2006
    Grr_Sum(0.115575768, 117, 289, x) + Grr_Sum(0.115575768, 132, 289, x) +
#2007
    Grr_Sum(0.128624275, 98, 253, x) + Grr_Sum(0.128624275, 124, 253, x) +
#2008
    Grr_Sum(0.14644388, 106, 268, x) + Grr_Sum(0.14644388, 110, 268, x) +
#2009
    Grr_Sum(0.1574043, 107, 249, x) + Grr_Sum(0.1574043, 111, 249, x) + #2010
    Grr_Sum(0.154151942, 102, 247, x) + Grr_Sum(0.154151942, 117, 247, x) +
#2011
    Grr_Sum(0.159449938, 84, 256, x) + Grr_Sum(0.159449938, 99, 256, x) +
#2012
    Grr_Sum(0.148499693, 95, 256, x) + Grr_Sum(0.148499693, 142, 256, x) +
#2013
    Grr_Sum(0.118992575, 128, 273, x) + Grr_Sum(0.118992575, 120, 273, x) +
#2014
    Grr_Sum(0.12292571, 97, 244, x) + Grr_Sum(0.12292571, 121, 244, x) +
#2015
    Grr_Sum(0.12745136, 103, 225, x) + Grr_Sum(0.12745136, 101, 225, x) +
#2016
    Grr_Sum(0.142454443, 107, 203, x) + Grr_Sum(0.142454443, 93, 203, x)
#2017


}


########################################################GM
Data#############################################################

Sum_Square <- function(x) {
  (
    Square_Sum(0.119297488, 134, 131, x) + Square_Sum(0.119297488, 117, 131,
x) + #2006
      Square_Sum(0.119297488, 124, 131, x) + Square_Sum(0.119297488, 119,
131, x) + #2006
      Square_Sum(0.115575768, 127, 174, x) + Square_Sum(0.115575768, 135,
174, x) + #2007
      Square_Sum(0.115575768, 129, 174, x) + Square_Sum(0.115575768, 131,
174, x) + #2007
```

```
      Square_Sum(0.128624275, 118, 163, x) + Square_Sum(0.128624275, 113,
163, x) + #2008
      Square_Sum(0.128624275, 113, 163, x) + Square_Sum(0.128624275, 127,
163, x) + #2008
      Square_Sum(0.14644388, 117, 183, x) + Square_Sum(0.14644388, 91, 183,
x) + #2009
      Square_Sum(0.14644388, 103, 183, x) + Square_Sum(0.14644388, 116, 183,
x) + #2009
      Square_Sum(0.1574043, 114, 228, x) + Square_Sum(0.1574043, 111, 228, x)
+ #2010
      Square_Sum(0.1574043, 111, 228, x) + Square_Sum(0.1574043, 126, 228, x)
+ #2010
      Square_Sum(0.154151942, 114, 236, x) + Square_Sum(0.154151942, 103,
236, x) + #2011
      Square_Sum(0.154151942, 109, 236, x) + Square_Sum(0.154151942, 104,
236, x) + #2011
      Square_Sum(0.159449938, 106, 251, x) + Square_Sum(0.159449938, 80, 251,
x) + #2012
      Square_Sum(0.159449938, 100, 251, x) + Square_Sum(0.159449938, 99, 251,
x) + #2012
      Square_Sum(0.148499693, 109, 251, x) + Square_Sum(0.148499693, 108,
251, x) + #2013
      Square_Sum(0.148499693, 97, 251, x) + Square_Sum(0.148499693, 90, 251,
x) + #2013
      Square_Sum(0.118992575, 120, 244, x) + Square_Sum(0.118992575, 115,
244, x) + #2014
      Square_Sum(0.118992575, 106, 244, x) + Square_Sum(0.118992575, 116,
244, x) + #2014
      Square_Sum(0.12292571, 105, 224, x) + Square_Sum(0.12292571, 122, 224,
x) + #2015
      Square_Sum(0.12292571, 101, 224, x) + Square_Sum(0.12292571, 115, 224,
x) + #2015
      Square_Sum(0.12745136, 96, 250, x) + Square_Sum(0.12745136, 112, 250,
x) + #2016
      Square_Sum(0.12745136, 95, 250, x) + Square_Sum(0.12745136, 103, 250,
x) + #2016
      Square_Sum(0.142454443, 95, 290, x) + Square_Sum(0.142454443, 105, 290,
x) + #2017
      Square_Sum(0.142454443, 88, 290, x) + Square_Sum(0.142454443, 99, 290,
x)  #2017
  )/48
}


Sum_Square_Error1 <- function(x) {
  (
    Square_Sum_Error1(0.119297488, 134, 131, x) +
Square_Sum_Error1(0.119297488, 117, 131, x) + #2006
      Square_Sum_Error1(0.119297488, 124, 131, x) +
Square_Sum_Error1(0.119297488, 119, 131, x) + #2006
      Square_Sum_Error1(0.115575768, 127, 174, x) +
Square_Sum_Error1(0.115575768, 135, 174, x) + #2007
      Square_Sum_Error1(0.115575768, 129, 174, x) +
Square_Sum_Error1(0.115575768, 131, 174, x) + #2007
      Square_Sum_Error1(0.128624275, 118, 163, x) +
Square_Sum_Error1(0.128624275, 113, 163, x) + #2008
```

```
      Square_Sum_Error1(0.128624275, 113, 163, x) +
Square_Sum_Error1(0.128624275, 127, 163, x) + #2008
      Square_Sum_Error1(0.14644388, 117, 183, x) +
Square_Sum_Error1(0.14644388, 91, 183, x) + #2009
      Square_Sum_Error1(0.14644388, 103, 183, x) +
Square_Sum_Error1(0.14644388, 116, 183, x) + #2009
      Square_Sum_Error1(0.1574043, 114, 228, x) +
Square_Sum_Error1(0.1574043, 111, 228, x) + #2010
      Square_Sum_Error1(0.1574043, 111, 228, x) +
Square_Sum_Error1(0.1574043, 126, 228, x) + #2010
      Square_Sum_Error1(0.154151942, 114, 236, x) +
Square_Sum_Error1(0.154151942, 103, 236, x) + #2011
      Square_Sum_Error1(0.154151942, 109, 236, x) +
Square_Sum_Error1(0.154151942, 104, 236, x) + #2011
      Square_Sum_Error1(0.159449938, 106, 251, x) +
Square_Sum_Error1(0.159449938, 80, 251, x) + #2012
      Square_Sum_Error1(0.159449938, 100, 251, x) +
Square_Sum_Error1(0.159449938, 99, 251, x) + #2012
      Square_Sum_Error1(0.148499693, 109, 251, x) +
Square_Sum_Error1(0.148499693, 108, 251, x) + #2013
      Square_Sum_Error1(0.148499693, 97, 251, x) +
Square_Sum_Error1(0.148499693, 90, 251, x) + #2013
      Square_Sum_Error1(0.118992575, 120, 244, x) +
Square_Sum_Error1(0.118992575, 115, 244, x) + #2014
      Square_Sum_Error1(0.118992575, 106, 244, x) +
Square_Sum_Error1(0.118992575, 116, 244, x) + #2014
      Square_Sum_Error1(0.12292571, 105, 224, x) +
Square_Sum_Error1(0.12292571, 122, 224, x) + #2015
      Square_Sum_Error1(0.12292571, 101, 224, x) +
Square_Sum_Error1(0.12292571, 115, 224, x) + #2015
      Square_Sum_Error1(0.12745136, 96, 250, x) +
Square_Sum_Error1(0.12745136, 112, 250, x) + #2016
      Square_Sum_Error1(0.12745136, 95, 250, x) +
Square_Sum_Error1(0.12745136, 103, 250, x) + #2016
      Square_Sum_Error1(0.142454443, 95, 290, x) +
Square_Sum_Error1(0.142454443, 105, 290, x) + #2017
      Square_Sum_Error1(0.142454443, 88, 290, x) +
Square_Sum_Error1(0.142454443, 99, 290, x)  #2017
  )/48
}

Sum_Square_Error2 <- function(x) {

  (
    Square_Sum_Error2(0.119297488, 134, 131, x) +
Square_Sum_Error2(0.119297488, 117, 131, x) + #2006
      Square_Sum_Error2(0.119297488, 124, 131, x) +
Square_Sum_Error2(0.119297488, 119, 131, x) + #2006
      Square_Sum_Error2(0.115575768, 127, 174, x) +
Square_Sum_Error2(0.115575768, 135, 174, x) + #2007
      Square_Sum_Error2(0.115575768, 129, 174, x) +
Square_Sum_Error2(0.115575768, 131, 174, x) + #2007
      Square_Sum_Error2(0.128624275, 118, 163, x) +
Square_Sum_Error2(0.128624275, 113, 163, x) + #2008
      Square_Sum_Error2(0.128624275, 113, 163, x) +
Square_Sum_Error2(0.128624275, 127, 163, x) + #2008
```

```
      Square_Sum_Error2(0.14644388, 117, 183, x) +
Square_Sum_Error2(0.14644388, 91, 183, x) + #2009
      Square_Sum_Error2(0.14644388, 103, 183, x) +
Square_Sum_Error2(0.14644388, 116, 183, x) + #2009
      Square_Sum_Error2(0.1574043, 114, 228, x) +
Square_Sum_Error2(0.1574043, 111, 228, x) + #2010
      Square_Sum_Error2(0.1574043, 111, 228, x) +
Square_Sum_Error2(0.1574043, 126, 228, x) + #2010
      Square_Sum_Error2(0.154151942, 114, 236, x) +
Square_Sum_Error2(0.154151942, 103, 236, x) + #2011
      Square_Sum_Error2(0.154151942, 109, 236, x) +
Square_Sum_Error2(0.154151942, 104, 236, x) + #2011
      Square_Sum_Error2(0.159449938, 106, 251, x) +
Square_Sum_Error2(0.159449938, 80, 251, x) + #2012
      Square_Sum_Error2(0.159449938, 100, 251, x) +
Square_Sum_Error2(0.159449938, 99, 251, x) + #2012
      Square_Sum_Error2(0.148499693, 109, 251, x) +
Square_Sum_Error2(0.148499693, 108, 251, x) + #2013
      Square_Sum_Error2(0.148499693, 97, 251, x) +
Square_Sum_Error2(0.148499693, 90, 251, x) + #2013
      Square_Sum_Error2(0.118992575, 120, 244, x) +
Square_Sum_Error2(0.118992575, 115, 244, x) + #2014
      Square_Sum_Error2(0.118992575, 106, 244, x) +
Square_Sum_Error2(0.118992575, 116, 244, x) + #2014
      Square_Sum_Error2(0.12292571, 105, 224, x) +
Square_Sum_Error2(0.12292571, 122, 224, x) + #2015
      Square_Sum_Error2(0.12292571, 101, 224, x) +
Square_Sum_Error2(0.12292571, 115, 224, x) + #2015
      Square_Sum_Error2(0.12745136, 96, 250, x) +
Square_Sum_Error2(0.12745136, 112, 250, x) + #2016
      Square_Sum_Error2(0.12745136, 95, 250, x) +
Square_Sum_Error2(0.12745136, 103, 250, x) + #2016
      Square_Sum_Error2(0.142454443, 95, 290, x) +
Square_Sum_Error2(0.142454443, 105, 290, x) + #2017
      Square_Sum_Error2(0.142454443, 88, 290, x) +
Square_Sum_Error2(0.142454443, 99, 290, x)  #2017
  )/48
}


Sum_Grr <- function(x) {

  Grr_Sum(0.119297488, 134, 131, x) + Grr_Sum(0.119297488, 117, 131, x) +
#2006
    Grr_Sum(0.119297488, 124, 131, x) + Grr_Sum(0.119297488, 119, 131, x) +
#2006
    Grr_Sum(0.115575768, 127, 174, x) + Grr_Sum(0.115575768, 135, 174, x) +
#2007
    Grr_Sum(0.115575768, 129, 174, x) + Grr_Sum(0.115575768, 131, 174, x) +
#2007
    Grr_Sum(0.128624275, 118, 163, x) + Grr_Sum(0.128624275, 113, 163, x) +
#2008
    Grr_Sum(0.128624275, 113, 163, x) + Grr_Sum(0.128624275, 127, 163, x) +
#2008
    Grr_Sum(0.14644388, 117, 183, x) + Grr_Sum(0.14644388, 91, 183, x) +
#2009
```

```
    Grr_Sum(0.14644388, 103, 183, x) + Grr_Sum(0.14644388, 116, 183, x) +
#2009
    Grr_Sum(0.1574043, 114, 228, x) + Grr_Sum(0.1574043, 111, 228, x) + #2010
    Grr_Sum(0.1574043, 111, 228, x) + Grr_Sum(0.1574043, 126, 228, x) + #2010
    Grr_Sum(0.154151942, 114, 236, x) + Grr_Sum(0.154151942, 103, 236, x) +
#2011
    Grr_Sum(0.154151942, 109, 236, x) + Grr_Sum(0.154151942, 104, 236, x) +
#2011
    Grr_Sum(0.159449938, 106, 251, x) + Grr_Sum(0.159449938, 80, 251, x) +
#2012
    Grr_Sum(0.159449938, 100, 251, x) + Grr_Sum(0.159449938, 99, 251, x) +
#2012
    Grr_Sum(0.148499693, 109, 251, x) + Grr_Sum(0.148499693, 108, 251, x) +
#2013
    Grr_Sum(0.148499693, 97, 251, x) + Grr_Sum(0.148499693, 90, 251, x) +
#2013
    Grr_Sum(0.118992575, 120, 244, x) + Grr_Sum(0.118992575, 115, 244, x) +
#2014
    Grr_Sum(0.118992575, 106, 244, x) + Grr_Sum(0.118992575, 116, 244, x) +
#2014
    Grr_Sum(0.12292571, 105, 224, x) + Grr_Sum(0.12292571, 122, 224, x) +
#2015
    Grr_Sum(0.12292571, 101, 224, x) + Grr_Sum(0.12292571, 115, 224, x) +
#2015
    Grr_Sum(0.12745136, 96, 250, x) + Grr_Sum(0.12745136, 112, 250, x) +
#2016
    Grr_Sum(0.12745136, 95, 250, x) + Grr_Sum(0.12745136, 103, 250, x) +
#2016
    Grr_Sum(0.142454443, 95, 290, x) + Grr_Sum(0.142454443, 105, 290, x) +
#2017
    Grr_Sum(0.142454443, 88, 290, x) + Grr_Sum(0.142454443, 99, 290, x)
#2017
}


#######################################################Ford
Data############################################################

Sum_Square <- function(x) {
  (
    Square_Sum(0.119297488, 127, 174, x) + Square_Sum(0.119297488, 121, 174,
x) + #2006
      Square_Sum(0.115575768, 120, 162, x) + Square_Sum(0.115575768, 100,
162, x) + #2007
      Square_Sum(0.128624275, 112, 191, x) + Square_Sum(0.128624275, 115,
191, x) + #2008
      Square_Sum(0.14644388, 102, 232, x) + Square_Sum(0.14644388, 129, 232,
x) + #2009
      Square_Sum(0.1574043, 93, 264, x) + Square_Sum(0.1574043, 106, 264, x)
+ #2010
      Square_Sum(0.154151942, 116, 271, x) + Square_Sum(0.154151942, 111,
271, x) + #2011
      Square_Sum(0.159449938, 118, 267, x) + Square_Sum(0.159449938, 107,
267, x) + #2012
      Square_Sum(0.148499693, 131, 271, x) + Square_Sum(0.148499693, 113,
271, x) + #2013
```

```
      Square_Sum(0.118992575, 116, 267, x) + Square_Sum(0.118992575, 109,
267, x) + #2014
      Square_Sum(0.12292571, 107, 261, x) + Square_Sum(0.12292571, 103, 261,
x) + #2015
      Square_Sum(0.12745136, 102, 267, x) + Square_Sum(0.12745136, 96, 267,
x) + #2016
      Square_Sum(0.142454443, 86, 270, x) + Square_Sum(0.142454443, 92, 270,
x)  #2017
  )/24

}



Sum_Square_Error1 <- function(x) {
  (
    Square_Sum_Error1(0.119297488, 127, 174, x) +
Square_Sum_Error1(0.119297488, 121, 174, x) + #2006
      Square_Sum_Error1(0.115575768, 120, 162, x) +
Square_Sum_Error1(0.115575768, 100, 162, x) + #2007
      Square_Sum_Error1(0.128624275, 112, 191, x) +
Square_Sum_Error1(0.128624275, 115, 191, x) + #2008
      Square_Sum_Error1(0.14644388, 102, 232, x) +
Square_Sum_Error1(0.14644388, 129, 232, x) + #2009
      Square_Sum_Error1(0.1574043, 93, 264, x) + Square_Sum_Error1(0.1574043,
106, 264, x) + #2010
      Square_Sum_Error1(0.154151942, 116, 271, x) +
Square_Sum_Error1(0.154151942, 111, 271, x) + #2011
      Square_Sum_Error1(0.159449938, 118, 267, x) +
Square_Sum_Error1(0.159449938, 107, 267, x) + #2012
      Square_Sum_Error1(0.148499693, 131, 271, x) +
Square_Sum_Error1(0.148499693, 113, 271, x) + #2013
      Square_Sum_Error1(0.118992575, 116, 267, x) +
Square_Sum_Error1(0.118992575, 109, 267, x) + #2014
      Square_Sum_Error1(0.12292571, 107, 261, x) +
Square_Sum_Error1(0.12292571, 103, 261, x) + #2015
      Square_Sum_Error1(0.12745136, 102, 267, x) +
Square_Sum_Error1(0.12745136, 96, 267, x) + #2016
      Square_Sum_Error1(0.142454443, 86, 270, x) +
Square_Sum_Error1(0.142454443, 92, 270, x)  #2017
  )/24
}

Sum_Square_Error2 <- function(x) {

  (
    Square_Sum_Error2(0.119297488, 127, 174, x) +
Square_Sum_Error2(0.119297488, 121, 174, x) + #2006
      Square_Sum_Error2(0.115575768, 120, 162, x) +
Square_Sum_Error2(0.115575768, 100, 162, x) + #2007
      Square_Sum_Error2(0.128624275, 112, 191, x) +
Square_Sum_Error2(0.128624275, 115, 191, x) + #2008
      Square_Sum_Error2(0.14644388, 102, 232, x) +
Square_Sum_Error2(0.14644388, 129, 232, x) + #2009
      Square_Sum_Error2(0.1574043, 93, 264, x) + Square_Sum_Error2(0.1574043,
106, 264, x) + #2010
```

```
      Square_Sum_Error2(0.154151942, 116, 271, x) +
Square_Sum_Error2(0.154151942, 111, 271, x) + #2011
      Square_Sum_Error2(0.159449938, 118, 267, x) +
Square_Sum_Error2(0.159449938, 107, 267, x) + #2012
      Square_Sum_Error2(0.148499693, 131, 271, x) +
Square_Sum_Error2(0.148499693, 113, 271, x) + #2013
      Square_Sum_Error2(0.118992575, 116, 267, x) +
Square_Sum_Error2(0.118992575, 109, 267, x) + #2014
      Square_Sum_Error2(0.12292571, 107, 261, x) +
Square_Sum_Error2(0.12292571, 103, 261, x) + #2015
      Square_Sum_Error2(0.12745136, 102, 267, x) +
Square_Sum_Error2(0.12745136, 96, 267, x) + #2016
      Square_Sum_Error2(0.142454443, 86, 270, x) +
Square_Sum_Error2(0.142454443, 92, 270, x)  #2017
  )/24
}



Sum_Grr <- function(x) {

  Grr_Sum(0.119297488, 127, 174, x) + Grr_Sum(0.119297488, 121, 174, x) +
#2006
    Grr_Sum(0.115575768, 120, 162, x) + Grr_Sum(0.115575768, 100, 162, x) +
#2007
    Grr_Sum(0.128624275, 112, 191, x) + Grr_Sum(0.128624275, 115, 191, x) +
#2008
    Grr_Sum(0.14644388, 102, 232, x) + Grr_Sum(0.14644388, 129, 232, x) +
#2009
    Grr_Sum(0.1574043, 93, 264, x) + Grr_Sum(0.1574043, 106, 264, x) + #2010
    Grr_Sum(0.154151942, 116, 271, x) + Grr_Sum(0.154151942, 111, 271, x) +
#2011
    Grr_Sum(0.159449938, 118, 267, x) + Grr_Sum(0.159449938, 107, 267, x) +
#2012
    Grr_Sum(0.148499693, 131, 271, x) + Grr_Sum(0.148499693, 113, 271, x) +
#2013
    Grr_Sum(0.118992575, 116, 267, x) + Grr_Sum(0.118992575, 109, 267, x) +
#2014
    Grr_Sum(0.12292571, 107, 261, x) + Grr_Sum(0.12292571, 103, 261, x) +
#2015
    Grr_Sum(0.12745136, 102, 267, x) + Grr_Sum(0.12745136, 96, 267, x) +
#2016
    Grr_Sum(0.142454443, 86, 270, x) + Grr_Sum(0.142454443, 92, 270, x)
#2017

}



#######################################################Chrysler
Data############################################################

Sum_Square <- function(x) {
  (
    Square_Sum(0.119297488, 120, 218, x) + Square_Sum(0.119297488, 132, 218,
x) + Square_Sum(0.119297488, 153, 218, x) + #2006
```

```
      Square_Sum(0.115575768, 151, 199, x) + Square_Sum(0.115575768, 156,
199, x) + Square_Sum(0.115575768, 161, 199, x) + #2007
      Square_Sum(0.128624275, 142, 161, x) + Square_Sum(0.128624275, 141,
161, x) + Square_Sum(0.128624275, 167, 161, x) + #2008
      Square_Sum(0.14644388, 136, 162, x) + Square_Sum(0.14644388, 134, 162,
x) + Square_Sum(0.14644388, 137, 162, x) + #2009
      Square_Sum(0.1574043, 122, 187, x) + Square_Sum(0.1574043, 130, 187, x)
+ Square_Sum(0.1574043, 129, 187, x) + #2010
      Square_Sum(0.154151942, 110, 221, x) + Square_Sum(0.154151942, 137,
221, x) + Square_Sum(0.154151942, 122, 221, x) + #2011
      Square_Sum(0.159449938, 116, 248, x) + Square_Sum(0.159449938, 124,
248, x) + Square_Sum(0.159449938, 110, 248, x) + #2012
      Square_Sum(0.148499693, 109, 250, x) + Square_Sum(0.148499693, 130,
250, x) + Square_Sum(0.148499693, 118, 250, x) + #2013
      Square_Sum(0.118992575, 111, 245, x) + Square_Sum(0.118992575, 124,
245, x) + Square_Sum(0.118992575, 146, 245, x) + #2014
      Square_Sum(0.12292571, 143, 224, x) + Square_Sum(0.12292571, 116, 224,
x) + Square_Sum(0.12292571, 141, 224, x) + #2015
      Square_Sum(0.12745136, 115, 222, x) + Square_Sum(0.12745136, 117, 222,
x) + Square_Sum(0.12745136, 113, 222, x) + #2016
      Square_Sum(0.142454443, 102, 218, x) + Square_Sum(0.142454443, 106,
218, x) + Square_Sum(0.142454443, 107, 218, x)  #2017
  )/36

}


Sum_Square_Error1 <- function(x) {
  (
    Square_Sum_Error1(0.119297488, 120, 218, x) +
Square_Sum_Error1(0.119297488, 132, 218, x) + Square_Sum_Error1(0.119297488,
153, 218, x) + #2006
      Square_Sum_Error1(0.115575768, 151, 199, x) +
Square_Sum_Error1(0.115575768, 156, 199, x) + Square_Sum_Error1(0.115575768,
161, 199, x) + #2007
      Square_Sum_Error1(0.128624275, 142, 161, x) +
Square_Sum_Error1(0.128624275, 141, 161, x) + Square_Sum_Error1(0.128624275,
167, 161, x) + #2008
      Square_Sum_Error1(0.14644388, 136, 162, x) +
Square_Sum_Error1(0.14644388, 134, 162, x) + Square_Sum_Error1(0.14644388,
137, 162, x) + #2009
      Square_Sum_Error1(0.1574043, 122, 187, x) +
Square_Sum_Error1(0.1574043, 130, 187, x) + Square_Sum_Error1(0.1574043, 129,
187, x) + #2010
      Square_Sum_Error1(0.154151942, 110, 221, x) +
Square_Sum_Error1(0.154151942, 137, 221, x) + Square_Sum_Error1(0.154151942,
122, 221, x) + #2011
      Square_Sum_Error1(0.159449938, 116, 248, x) +
Square_Sum_Error1(0.159449938, 124, 248, x) + Square_Sum_Error1(0.159449938,
110, 248, x) + #2012
      Square_Sum_Error1(0.148499693, 109, 250, x) +
Square_Sum_Error1(0.148499693, 130, 250, x) + Square_Sum_Error1(0.148499693,
118, 250, x) + #2013
      Square_Sum_Error1(0.118992575, 111, 245, x) +
Square_Sum_Error1(0.118992575, 124, 245, x) + Square_Sum_Error1(0.118992575,
146, 245, x) + #2014
```

```
      Square_Sum_Error1(0.12292571, 143, 224, x) +
Square_Sum_Error1(0.12292571, 116, 224, x) + Square_Sum_Error1(0.12292571,
141, 224, x) + #2015
      Square_Sum_Error1(0.12745136, 115, 222, x) +
Square_Sum_Error1(0.12745136, 117, 222, x) + Square_Sum_Error1(0.12745136,
113, 222, x) + #2016
      Square_Sum_Error1(0.142454443, 102, 218, x) +
Square_Sum_Error1(0.142454443, 106, 218, x) + Square_Sum_Error1(0.142454443,
107, 218, x)  #2017
  )/36

}

Sum_Square_Error2 <- function(x) {
  (
    Square_Sum_Error2(0.119297488, 120, 218, x) +
Square_Sum_Error2(0.119297488, 132, 218, x) + Square_Sum_Error2(0.119297488,
153, 218, x) + #2006
      Square_Sum_Error2(0.115575768, 151, 199, x) +
Square_Sum_Error2(0.115575768, 156, 199, x) + Square_Sum_Error2(0.115575768,
161, 199, x) + #2007
      Square_Sum_Error2(0.128624275, 142, 161, x) +
Square_Sum_Error2(0.128624275, 141, 161, x) + Square_Sum_Error2(0.128624275,
167, 161, x) + #2008
      Square_Sum_Error2(0.14644388, 136, 162, x) +
Square_Sum_Error2(0.14644388, 134, 162, x) + Square_Sum_Error2(0.14644388,
137, 162, x) + #2009
      Square_Sum_Error2(0.1574043, 122, 187, x) +
Square_Sum_Error2(0.1574043, 130, 187, x) + Square_Sum_Error2(0.1574043, 129,
187, x) + #2010
      Square_Sum_Error2(0.154151942, 110, 221, x) +
Square_Sum_Error2(0.154151942, 137, 221, x) + Square_Sum_Error2(0.154151942,
122, 221, x) + #2011
      Square_Sum_Error2(0.159449938, 116, 248, x) +
Square_Sum_Error2(0.159449938, 124, 248, x) + Square_Sum_Error2(0.159449938,
110, 248, x) + #2012
      Square_Sum_Error2(0.148499693, 109, 250, x) +
Square_Sum_Error2(0.148499693, 130, 250, x) + Square_Sum_Error2(0.148499693,
118, 250, x) + #2013
      Square_Sum_Error2(0.118992575, 111, 245, x) +
Square_Sum_Error2(0.118992575, 124, 245, x) + Square_Sum_Error2(0.118992575,
146, 245, x) + #2014
      Square_Sum_Error2(0.12292571, 143, 224, x) +
Square_Sum_Error2(0.12292571, 116, 224, x) + Square_Sum_Error2(0.12292571,
141, 224, x) + #2015
      Square_Sum_Error2(0.12745136, 115, 222, x) +
Square_Sum_Error2(0.12745136, 117, 222, x) + Square_Sum_Error2(0.12745136,
113, 222, x) + #2016
      Square_Sum_Error2(0.142454443, 102, 218, x) +
Square_Sum_Error2(0.142454443, 106, 218, x) + Square_Sum_Error2(0.142454443,
107, 218, x)  #2017
  )/36

}


Sum_Grr <- function(x) {
```

```
  Grr_Sum(0.119297488, 120, 218, x) + Grr_Sum(0.119297488, 132, 218, x) +
Grr_Sum(0.119297488, 153, 218, x) + #2006
    Grr_Sum(0.115575768, 151, 199, x) + Grr_Sum(0.115575768, 156, 199, x) +
Grr_Sum(0.115575768, 161, 199, x) + #2007
    Grr_Sum(0.128624275, 142, 161, x) + Grr_Sum(0.128624275, 141, 161, x) +
Grr_Sum(0.128624275, 167, 161, x) + #2008
    Grr_Sum(0.14644388, 136, 162, x) + Grr_Sum(0.14644388, 134, 162, x) +
Grr_Sum(0.14644388, 137, 162, x) + #2009
    Grr_Sum(0.1574043, 122, 187, x) + Grr_Sum(0.1574043, 130, 187, x) +
Grr_Sum(0.1574043, 129, 187, x) + #2010
    Grr_Sum(0.154151942, 110, 221, x) + Grr_Sum(0.154151942, 137, 221, x) +
Grr_Sum(0.154151942, 122, 221, x) + #2011
    Grr_Sum(0.159449938, 116, 248, x) + Grr_Sum(0.159449938, 124, 248, x) +
Grr_Sum(0.159449938, 110, 248, x) + #2012
    Grr_Sum(0.148499693, 109, 250, x) + Grr_Sum(0.148499693, 130, 250, x) +
Grr_Sum(0.148499693, 118, 250, x) + #2013
    Grr_Sum(0.118992575, 111, 245, x) + Grr_Sum(0.118992575, 124, 245, x) +
Grr_Sum(0.118992575, 146, 245, x) + #2014
    Grr_Sum(0.12292571, 143, 224, x) + Grr_Sum(0.12292571, 116, 224, x) +
Grr_Sum(0.12292571, 141, 224, x) + #2015
    Grr_Sum(0.12745136, 115, 222, x) + Grr_Sum(0.12745136, 117, 222, x) +
Grr_Sum(0.12745136, 113, 222, x) + #2016
    Grr_Sum(0.142454443, 102, 218, x) + Grr_Sum(0.142454443, 106, 218, x) +
Grr_Sum(0.142454443, 107, 218, x)  #2017

}



#######################################################End of
Data############################################################
```

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PUBLICATION

The Incentive Effect of Acceptance Sampling Plans in a Supply Chain with Endogenous Product Quality

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Abstract: This article studies a firm that procures a product from a supplier. The quality of each product unit is measured by a continuous variable that follows a normal distribution and is correlated within a batch. The firm conducts an inspection and pays the supplier only if the product batch passes the inspection. The inspection not only serves the purpose of preventing a bad batch from reaching customers but also offers the supplier an incentive to improve product quality. The firm determines the acceptance sampling plan, and the supplier determines the quality effort level in either a simultaneous game or a Stackelberg leadership game, in which both parties share inspection cost and recall loss caused by low product quality. In the simultaneous game, we identify the Nash equilibrium form, provide sufficient conditions that guarantee the existence of a pure strategy Nash equilibrium, and find parameter settings under which the decentralized and centralized supply chains achieve the same outcome. By numerical experiments, we show that the firm's acceptance sampling plan and the supplier's quality effort level are sensitive to both the recall loss sharing ratio and the game format (i.e., the precommitment assumption of the inspection policy). © 2013 Wiley Periodicals, Inc. Naval Research Logistics 60: 111–124, 2013

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1. INTRODUCTION AND LITERATURE REVIEW

In 2007, the federal Consumer Product Safety Commission recalled more than 25 million toys and children's items that were unsafe for children [27]. Many contained excessive levels of lead and were made by Chinese contract manufacturers for U.S. toy makers (e.g., Mattel) and retailers (e.g., Toys "R" Us and Wal-Mart). To ensure customers the product safety they expect and avoid legal suits and enormous fines, toy makers and retailers adopted inspection tests and/or tightened the safety standard of their products. For instance, Disney's inspection plan involves random spot checks at the manufacturing, shipping, and retail levels of more than 65,000 children's products [25]. Moreover, a similar example involves the new safety rules adopted by the Toys "R" Us company. The guidelines include third-party testing of toys imported into the United States and a restrictive new standard of 90 parts per million for lead in surface coating versus the old standard of 600 parts per million [4].

Inspection is one of the oldest quality control tools and is discussed in all major quality control textbooks (see, e.g., [14, 24]). In an inspection process, an inspector takes a sample from a product batch and estimates the overall batch quality. Based on testing results, the inspector then recommends accepting or rejecting the batch. This methodology is known as acceptance sampling [24].

The existing acceptance sampling literature often assumes that product quality is controlled by an exogenous random factor and is not affected by inspection plans. This assumption holds if the inspector is the manufacturer of the product and hence has direct control of product quality. The manufacturer can choose a product quality level for her best interest and then pick an appropriate inspection plan. Under this scenario, acceptance sampling plans are designed to prevent bad product batches from reaching customers. However, if the inspector is the buyer of a product manufactured by a contract manufacturer, then tightening the inspection plan may also provide the contract manufacturer an incentive to improve

Additional Supporting Information may be found in the online version of this article

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product quality (see, e.g., [10]) and protect the buyer from fraud (see, e.g., [17]). For instance, it has been reported that some Chinese contract manufacturers substitute low-quality ingredients/components to save product costs [26]. Using a restrictive inspection plan may prevent the substitution practice.

This article studies the incentive effect of acceptance sampling plans (i.e., offering a supplier an incentive to improve product quality) besides its traditional functionality. We consider a firm that purchases a product batch from a supplier, who can put effort into product quality improvement. In this decentralized supply chain, the firm conducts an inspection and pays the supplier only if the product batch passes the firm's inspection. If the product batch is accepted, then the firm sells the batch in a market during a holiday season. If the product batch has to be recalled from the market due to low product quality, then the firm shares the recall loss with the supplier. The firm may also share the inspection cost with the supplier.

We assume that product quality can be measured by a continuous variable that follows a normal distribution and the quality of each product unit is correlated within the batch. A high variable value means that product quality is good. Hence, a product batch with low variable values is bad and should be rejected after inspection, as the bad batch is likely to be recalled if sold in the market. The normality assumption is common for variables sampling plans (see, e.g., [24]).

As the supplier's quality effort decision depends on the firm's sampling plan and the firm's sampling plan depends on the batch quality and the supplier's quality effort level, there exists a strategic interaction between the firm's acceptance sampling plan and the supplier's quality effort decision. As the inspection is conducted after the product batch is produced, the firm always has incentive to adjust the inspection policy after receiving the product batch from the supplier. Hence, it is often not credible and contractible for the firm to commit an inspection plan before the supplier produces the batch. Without the commitment assumption of the firm's inspection policy, it is appropriate to model the strategic interaction between the firm's acceptance sampling plan and the supplier's quality effort decision in a simultaneous game. We provide sufficient conditions that guarantee the existence of a pure strategy Nash equilibrium and identify the equilibrium form of the sampling plan (i.e., the sample size and rejection threshold) and quality effort level when the supplier's product cost function is linear. The Nash equilibrium predicts a stable quality effort level and inspection plan in a supplier–buyer relationship wherein both parties behave strategically. For the business scenario under which the firm can precommit to the inspection policy before the supplier's production run, we consider a Stackelberg leadership game.

We compare the decentralized supply chain with a centralized supply chain, in which the firm fully controls the supplier. The outcomes in the centralized and decentralized supply chains are different due to the double marginalization, recall loss sharing, and inspection cost sharing effects. However, in the simultaneous game, under some parameter settings, the three effects are canceled out, and, hence, the decentralized supply chain achieves the same outcome as the centralized supply chain. No such coordination can be achieved in the Stackelberg leadership game, and the two games may imply very different supply chain outcomes.

By numerical experiments, we conduct the sensitivity analysis of the optimal/equilibrium acceptance sampling plan and quality effort level with respect to the recall loss sharing ratio and game format and obtain the following managerial insights:

- The recall loss sharing effect is substitutable with the inspection policy; that is, if the supplier bears a large proportion of the recall loss, the firm should adopt an easy inspection policy by reducing the sample size and lowering the rejection threshold. Depending on the recall loss sharing ratio, the firm's sampling size, and rejection threshold in the decentralized supply chain may be larger or smaller than those pertaining to the inspection policy in the centralized supply chain;
- Decentralization often causes product quality deterioration. However, if the supplier bears a proportion of the recall loss, then she increases the quality effort level and improves product quality. When the supplier bears a large proportion of the recall loss, the quality effort level in the decentralized supply chain may be even higher than the quality effort level in the centralized supply chain;
- The firm's inspection policy may be sensitive to whether or not the firm can commit to the inspection policy before the supplier's production run. When the supplier bears a large proportion of the recall loss and with the precommitment condition of the inspection policy, the firm may intentionally use a very loose inspection policy and solely rely on the recall loss sharing effect that provides enough incentive to the supplier to improve the product quality. However, this phenomenon does not happen when the firm cannot precommit to the inspection policy and/or when the supplier bears a small proportion of the recall loss.

Acceptance sampling plans are covered in standard quality control textbooks (see, e.g., [14, 24]). [6, 17] discussed the practical application of acceptance sampling plans. See Ref. [29] for a literature review of acceptance sampling plans from an economic perspective. Lorenzen [12] and Thyregod [21] studied Bayesian acceptance sampling plans, in which a prior distribution of product quality is assumed. They provided sufficient conditions that guarantee the optimality of $(n, \bar{t})$-policies, by which an inspector recommends a rejection if a sample statistic falls below the threshold $\bar{t}$. Moskowitz and Tang [15] assumed a normal prior distribution and derived an optimal Bayesian acceptance sampling plan that minimizes the expected cost of a manufacturer. They provided a computational procedure of the optimal sample size and rejection threshold. The above literature assumes exogenous product quality that is not affected by acceptance sampling plans, and, hence, the results cannot be directly applied in an outsourcing environment.

Quality endogeneity in a supplier–buyer relationship has been studied in the quality coordination literature (see, e.g., [1, 2, 9, 16]). The literature shares some common features. First, the supplier produces a batch of products and determines the defective probability. Second, the product has a zero–one quality attribute (i.e., either good or bad), and the quality of each product unit in the batch is assumed to be i.i.d. (independently and identically distributed). Third, the buyer inspects all units of the batch and may penalize the supplier for detected defects. The quality coordination literature focuses on finding an optimal economic contract (e.g., damage cost sharing) to coordinate the supply chain.

Assuming exogenous product quality, Vander Wiel and Vardeman [22] showed that an All-or-None inspection policy (i.e., inspecting either all units or none of a batch) is optimal, if the quality of each product unit in the batch is i.i.d. and the cost structure is additive. However, Wan and Xu [28] studied a supply chain with endogenous product quality and showed that the All-or-None inspection policy may be suboptimal even if the quality of each product unit in the batch is i.i.d. and the cost structure is additive. Their result suggests that the assumption of the All-inspection policy in the quality coordination literature is not trivial. The traditional understanding of inspection cannot be directly carried into an outsourcing environment with endogenous product quality. See Ref. [23] for more discussions of the All-or-None inspection policy.

Starbird [18–20] studied the impact of a buyer's acceptance sampling plan on a supplier's product quality and production decisions. He considered attribute sampling plans for a batch, in which individual items are i.i.d. and may have continuous quality characteristics. By numerical experiments, Starbird demonstrated that tightening an attribute acceptance sampling plan may inspire the supplier to enhance product quality.

In practice, the quality of each product unit is often correlated within a batch, because the batch may be produced from the same chunk of raw materials and product quality in the batch is heavily affected by the raw material quality. This article allows the dependence of product quality in a batch and differs from Starbird's work on two other important perspectives. First, we assume that product quality takes a continuous measurement (e.g., density of an ingredient in the product) and consider variable sampling plans. See Refs. [14, 17] for advantages and applicability of attribute and variable sampling plans. Second, we derive not only the supplier's optimal quality effort decision but also the buyer's optimal acceptance sampling plan and capture their strategic interaction in both the simultaneous and the Stackelberg leadership games.

Finally, we notice that inspection can be viewed as a mechanism of preventing a supplier from shirking. Shirking behavior widely exists in a principal–agent relationship and has been extensively studied in the contract literature (see, e.g., [3, 13]). An acceptance sampling plan has two elements, the sample size and rejection threshold. Drawing a sample is a monitoring device, which incurs a monitoring/inspection cost. Raising the rejection threshold is an alternative strategy to reduce the chance of accepting a bad product batch and hence provide the supplier an incentive not to shirk. This incentive functionality of acceptance sampling plans was recognized in [7] but has not been formally studied in a game theoretic framework. It is the focus of this article, and the challenge relies on how to balance the game theoretic analysis with the statistical analysis.

In the remainder of the article, we set up the model in Section 2 and study the centralized supply chain in Section 3 and the decentralized supply chain in Section 4. Numerical studies are presented in Section 5, and conclusions and future research are discussed in Section 6. Some technical detailed are covered in Sections A and B of the online appendix, Supporting Information. All proofs and equation derivations can be found in Section C of the online appendix, Supporting Information.

2. THE MODEL

We consider a firm that orders a batch of products (with size B) from an independent supplier and sells the product in a market during a holiday season. We assume that product quality is measured by a continuous variable and follows a hierarchical linear model:

$$Y_i = \Theta + \epsilon_i, \text{ where } \epsilon_i \sim N(0, \sigma_p^2), i = 1, 2, \ldots, B,$$
$$\Theta = \mu + \varepsilon_b, \text{ where } \varepsilon_b \sim N(0, \tau^2).$$

Product quality in the batch $\{Y_i\}_{i=1}^B$ vary around a quality index Θ, which represents the overall batch quality. The batch quality becomes better, as Θ increases. The random disturbances $\{\epsilon_i\}_{i=1}^B$ are independent with each other and represent the variation of the manufacturing process of the product. The supplier can put effort into improving the batch quality, but cannot fully control the batch quality. Hence, there is a positive causal relationship between the quality effort level μ and the batch quality index Θ. Here, ε_b represents random factors that the supplier cannot control and is assumed to be independent of $\{\epsilon_i\}_{i=1}^B$. Notice that the quality of product units is conditionally independent given the batch quality index Θ. However, the unconditional covariance of Y_i and Y_j is τ^2, where $1 \leq i \neq j \leq B$. Hence, the quality of each product unit is correlated in the batch. Finally, we let $c_s(\mu)$ be the unit product cost when the supplier chooses the quality effort level μ.

To demonstrate the above modeling settings, we consider a firm that procures a batch of toys from a foreign supplier and sells the toys during the Christmas season. Product quality is measured by the lead intensity in the product. Notice that product quality increases as the lead intensity drops. The lead intensity in the product is mainly determined by the lead intensity in raw materials, but may change through the manufacturing process. The supplier can reduce the lead intensity level in the batch by purchasing high-grade raw materials (e.g., lead-free paint) and hence incurring high material costs. Although a high price of raw materials often signals high quality, the relationship between the price and quality of raw materials is far from perfect. When the raw material quality is not easy to observe, the quality may vary significantly given a price level. This scenario likely occurs if the supplier does not have the technology and/or management skills to monitor the quality of raw materials.

We assume that the firm is able to conduct an inspection after receiving the product batch. The firm takes a random sample of n units from the batch with unit inspection cost k and performs a nondestructive inspection. Hence, the total inspection cost of the sample is kn. We denote the observations of the random sample as $(X_1, X_2, \ldots, X_n)$, where $X_i = Y_i + \xi_i$, $\xi_i \sim N(0, \sigma_0^2)$ and $i = 1, 2, \ldots, n$. The random disturbances $\{\xi_i\}_{i=1}^n$ represent measurement errors, which are independent of each other and of $\{\epsilon_i\}_{i=1}^n$ and ε_b. As $X_i = \Theta + \epsilon_i + \xi_i$, the observations $\{X_i\}_{i=1}^n$ are independent of each other given the batch quality index Θ and follow a normal distribution $N(\Theta, \sigma^2)$, where $\sigma^2 = \sigma_p^2 + \sigma_0^2$. We let the sample mean be $\bar{X} = \frac{1}{n}\sum_{i=1}^n X_i$, which follows a normal distribution $N(\Theta, \frac{\sigma^2}{n})$ given the batch quality index Θ. Notice that the batch quality index Θ follows $N(\mu, \tau^2)$ ex-ante. After observing the random sample $(X_1, X_2, \ldots, X_n)$, the firm updates its belief of the batch quality index Θ. By the Bayesian theory (see, e.g., Theorem 1 on page 167 of [5]), the posterior distribution of Θ is $N(\frac{\sigma^2\mu+\tau^2 n\bar{X}}{\sigma^2+\tau^2 n}, \frac{\sigma^2\tau^2}{\sigma^2+\tau^2 n})$.

Next, the firm determines whether to accept the batch or reject it. If the firm rejects the batch, it chooses not to pay the supplier and the latter salvages the batch and recovers v_s per unit product. As the selling season is short, we assume that there is no opportunity for the supplier to make another batch. If the firm accepts the batch, it pays the supplier w per unit and sells the product in the market during the season, where $w > v_s$.

After the product batch reaches the market, there is a probability that the whole product batch has to be recalled due to quality issues (e.g., the massive recalls of lead contaminated toys in 2007). The recall may be triggered by product-related injuries and deaths and/or an investigation conducted by a product safety agency (e.g., the U.S. Consumer Product Safety Commission). In practice, it is usually intractable to explicitly calculate the recall probability, because a recall occurrence depends on many business, environmental, legal, and political factors, besides the quality factor. For mathematical tractability, we focus on the batch quality factor and assume that the recall probability is equal to $\Pr(\Theta < \bar{\theta} | \mathcal{I})$, where $\bar{\theta}$ is a quality threshold and $\mathcal{I}$ represents any available information about the batch quality Θ (e.g., the sample mean $\bar{X}$). This implies that the recall is triggered if the batch quality Θ does not reach the quality threshold $\bar{\theta}$, which is a simple approximate of a complex product recall process in practice. For instance, a batch of toys may be recalled, because its overall material quality does not reach a product safety standard issued by the U.S. Consumer Product Safety Commission (e.g., the average lead level contained in toys is too high).

By selling the product batch in the market, the firm receives a random payoff depending on the recall occurrence. For simplicity, we consider a stepwise payoff function. We assume that selling a product unit gives the firm revenue p. Hence, the total revenue of the product batch is pB. However, if a recall occurs, then the firm loses l per unit. Hence, the total loss is lB. Notice that l may be lower or higher than p. For instance, recalling lead-contaminated toys not only causes the firm to lose revenue p but also incur penalty costs. This implies $l > p$. Another example is that an automobile company recalls cars with defective parts. After the defective part is replaced, cars are returned to their owners. Hence, the company keeps the revenue p when the recall occurs, but incurs a repair cost l for replacing the defective part, which is often much lower than p (i.e., $l < p$). For other form reward/loss functions, see Chapter 11 of [5].

Finally, we assume that the firm shares the recall loss with the supplier. To be specific, the firm bears $100\rho\%$ of the total recall loss and the supplier holds the rest $100(1-\rho)\%$, where $\rho \in [0, 1]$. We also assume that the firm may share the inspection cost with the supplier. To be specific, the firm bears $100\eta\%$ of the inspection cost, and the supplier holds the rest $100(1-\eta)\%$, where $\eta \in [0, 1]$. We assume that the supplier has enough economic incentive (i.e., non-negative expected profit) to engage in the trade with the firm. Such an incentive can be achieved if the firm pays the supplier a lump-sum money beforehand that partially covers the supplier's production and contingent costs. The notation used in this section and the following sections are summarized in Table 1. Notice that all parameters in Table 1 (except μ, n, and $\bar{l}$) are exogenous.

Table 1. The notation.

B	The batch size
Θ	The batch quality index
τ^2	The variance of the batch quality
μ	The quality effort level (decision variable)
σ_p^2	The variance of the individual product in the manufacturing process
σ_S^2	The sampling variance in the inspection process
$\sigma^2 = \sigma_p^2 + \sigma_\alpha^2$	The sum of the variances
$\bar{\theta}$	The quality threshold
k	The unit inspection cost
n	The sample size (decision variable)
$\bar{t}$	The rejection threshold (decision variable)
p	The unit product price
l	The unit recall loss
$c_f(\mu)$ $(c_s(\mu))$	The unit product cost that the firm (supplier) incurs
v_f (v_s)	The unit product salvage value that the firm (supplier) receives
w	The unit purchase price that the firm pays the supplier
ρ	The proportion of the recall loss that the firm bears
η	The proportion of the inspection cost that the firm bears

In this outsourcing environment, the supplier may put insufficient effort into quality improvement due to the lack of three types of economic incentives. First, the supplier only receives a part of the total profit margin of the product and the rest goes into the firm, which reduces the supplier's incentive to invest in quality improvement. This phenomenon is called the double marginalization effect (see, e.g., [11, 30]). Second, the supplier shares the recall loss with the firm, which reduces the supplier's quality risk and hence incentive to invest in quality improvement. This risk sharing phenomenon has been studied in the quality coordination literature (see, e.g., [1, 2]).

Lastly, the supplier does not fully bear the inspection cost and hence can ignore the impact of product quality on the inspection procedure. As far as we know, this new element has not been studied in the previous literature. The emphasis of this article is to study how acceptance sampling interacts with the double marginalization and risk sharing effects and provides additional economic incentive for the supplier to improve product quality in the outsourcing environment. The challenge of this task is to balance the game theoretical analysis with the statistical analysis.

Next, we study a benchmark case, in which the firm fully controls the supplier and maximizes the total supply chain profit. In this centralized supply chain, the double marginalization, risk sharing, and inspection cost sharing effects do not exist. Then, we study how decentralization/outsourcing changes the optimal acceptance sampling plan for the firm, and quality effort level for the supplier and supply chain coordination may arise under some parameter settings in Section 4.

3. THE CENTRALIZED SUPPLY CHAIN

We assume that the firm fully controls the supplier and jointly determines the optimal sampling plan and quality effort level to maximize the total supply chain profit. Because centralization may alter the cost structure, we let $c_f(\mu)$ be the unit product cost that the firm incurs and v_f be the unit product salvage value that the firm receives if the product batch is rejected. As we will see in Section 4, for certain settings of the exogenous economic parameters, the double marginalization, recall loss sharing and inspection cost sharing, effects are canceled, and hence, the centralized and decentralized supply chains achieve the same performance.

There are scenarios that the firm can control the supplier. For instance, the firm appoints a supply source that the supplier has to purchase raw materials from. Under this scenario, the supplier becomes a manufacturing facility provider, and the firm handles all the rest business activities. Hence, the supplier only plays a passive and ignorable role in the supply chain. If the firm has different negotiation power against raw material providers and a different reverse logistic system from the supplier's, then the parameters of $c_f(\mu)$ and v_f in the centralized supply chain case are likely to be different from $c_s(\mu)$ and v_s in the decentralized supply chain case.

Another possibility of the centralized supply chain is simply that the firm produces the product in-house. For instance, a toy company decides to move the manufacturing business back to U.S. and produce in-house. This alternative of moving the production in house likely incurs higher labor, environmental, and hence total production costs (i.e., $c_f(\mu) > c_s(\mu)$) than procuring the product from an independent Chinese supplier.

First, we derive the optimal sampling plan given the quality effort level μ.

3.1. The Optimal Sampling Plan

For an exogenous product quality level, Lorenzen [12] and Thyregod [21] showed that the optimal sampling plan is a $(n, \bar{t})$-policy, by which the firm recommends a rejection if the sufficient statistic $\bar{X}$ falls below the threshold $\bar{t}$. We let the firm's optimal rejection threshold be $\bar{t}^{o}(n;\mu)$ given the sample size n and quality effort level μ.

By selling the product batch, the firm receives expected payoff $pB - lB\Pr(\Theta < \bar{\theta}|\bar{X} = \bar{x})$. If the product batch is rejected, then the firm receives the total salvage value $v_f B$. Hence, the firm should reject the batch iff $p - l\Pr(\Theta < \bar{\theta}|\bar{X} = \bar{x}) \le v_f$, or equivalently, $\Pr(\Theta \le \bar{\theta}|\bar{X} = \bar{x}) \ge \beta$, where $\beta = \frac{p-v_f}{l}$. Let $z_\beta = \Phi^{-1}(\beta)$ be the z-score, where $\Phi(\cdot)$ represents the probability distribution of the standard normal random variable. As the conditional distribution of Θ is $N(\frac{\sigma^2\mu+\tau^2 n\bar{x}}{\sigma^2+\tau^2 n}, \frac{\sigma^2\tau^2}{\sigma^2+\tau^2 n})$ given $\bar{X} = \bar{x}$, the firm should reject the batch iff $\bar{X} \le \bar{t}^o(n;\mu)$, where the rejection threshold is

$$\bar{t}^o(n;\mu) = \bar{\theta} + \frac{\sigma^2}{\tau^2 n}(\bar{\theta} - \mu) - \frac{\sigma z_\beta}{\tau n}\sqrt{\sigma^2 + \tau^2 n}. \quad (1)$$

Notice that the rejection threshold is increasing in $\bar{\theta}$, l, and υ_Γ, but decreasing in μ and p. Hence, the firm should raise the rejection threshold $\bar{t}^o(n;\mu)$ and tighten the inspection, if the quality threshold $\bar{\theta}$ becomes high, the loss of recalling unsafe products (l) becomes large, and/or the product salvage value becomes large. However, the firm should lower the rejection threshold $\bar{t}^o(n;\mu)$ and ease the inspection if a high-quality batch is expected and/or the revenue of selling the product (p) becomes high. Finally, we notice that the rejection threshold is not monotonic in the sample size n. The relationship between the rejection threshold and the sample size depends on the sign of z_β and the difference between the quality threshold $\bar{\theta}$ and quality effort level μ.

Next, we determine the optimal sample size $n^o(\mu)$. Given $\bar{X} = \bar{x}$, the firm's conditional profit is

$$\Pi_\Gamma(\bar{x}, n;\mu) = \begin{cases} pB - lB\Pr(\Theta < \bar{\theta} \mid \bar{X} = \bar{x}) & \\ \quad -kn - Bc_\Gamma(\mu), & \text{if} \quad \bar{x} > \bar{t}^o(n;\mu), \\ \upsilon_\Gamma B - kn - Bc_\Gamma(\mu), & \text{if} \quad \bar{x} \le \bar{t}^o(n;\mu). \end{cases}$$

The firm's ex-ante profit is

$$\begin{aligned} \Pi_\Gamma(n;\mu) &= E[\Pi_\Gamma(\bar{X}, n;\mu)] \\ &= (p - \upsilon_\Gamma)B\Pr(\bar{X} > \bar{t}^o(n;\mu)) \\ &\quad - lBE\left[\Phi\left(\frac{\bar{\theta} - \frac{\sigma^2\mu + \tau^2 n\bar{X}}{\sigma^2 + \tau^2 n}}{\sqrt{\frac{\sigma^2\tau^2}{\sigma^2 + \tau^2 n}}}\right) I_{\{\bar{X} > \bar{t}^o(n;\mu)\}}\right] \\ &\quad - kn - Bc_\Gamma(\mu) + \upsilon_\Gamma B, \end{aligned}$$

where $\bar{X}$ follows $N(\mu, \tau^2 + \frac{1}{n}\sigma^2)$, $\Phi(\cdot)$ represents the cumulative distribution of the standard normal and $I_{\{\cdot\}}$ is an indicator function. By some algebra, the firm's ex-ante profit can be simplified as $\Pi_\Gamma(n;\mu) = pB - C_\Gamma(n;\mu)$, where

$$\begin{aligned} C_\Gamma(n;\mu) = lB\int_{-\infty}^{z_\beta} \Phi\left(\sqrt{\frac{\sigma^2}{n\tau^4} + \frac{1}{\tau^2}}(\bar{\theta} - \mu) - \sqrt{\frac{\sigma^2}{n\tau^2}}y\right) \\ \times \phi(y)dy + kn + Bc_\Gamma(\mu) \quad (2) \end{aligned}$$

and $\phi(y)$ is the density function of the standard normal random variable. See the derivation of Eq. (2) in Section C of the online appendix, Supporting Information.

To maximize the firm's ex-ante profit, we only need to minimize $C_\Gamma(n;\mu)$. We assume that the sample size n is a continuous variable. This is a good approximate if the batch size B is large (e.g., an order of hundreds of toys). Letting $\frac{\partial}{\partial n}C_\Gamma(n;\mu) = 0$, by some algebra, we can show that the optimal sample size $n^o(\mu)$ satisfies the first-order condition

$$\begin{aligned} \frac{k}{Bl} &= \frac{\tau\sigma}{4\pi\sqrt{n}(\sigma^2 + n\tau^2)}\exp\left(-\frac{1}{2}\left(\frac{\bar{\theta} - \mu}{\tau}\right)^2\right) \\ &\quad \times \exp\left(-\frac{\left(\sqrt{\frac{\sigma^2}{n\tau^2} + 1}z_\beta - \sqrt{\frac{\sigma^2}{n\tau^2}}\frac{\bar{\theta} - \mu}{\tau}\right)^2}{2}\right) \\ &= \frac{\tau\sigma}{4\pi\sqrt{n}(\sigma^2 + n\tau^2)}\exp\left(-\frac{z_\beta^2}{2}\right) \\ &\quad \times \exp\left(-\frac{\left(\sqrt{\frac{\sigma^2}{n\tau^2}}z_\beta - \sqrt{\frac{\sigma^2}{n\tau^2} + 1}\frac{\bar{\theta} - \mu}{\tau}\right)^2}{2}\right). \quad (3) \end{aligned}$$

See the derivation of Eq. (3) in Section C of the online appendix, Supporting Information. Finally, we let the optimal rejection threshold be $\bar{t}^o(\mu) = \bar{t}^o(n^o(\mu);\mu)$ given the quality effort level μ.

Second, we determine the optimal quality effort level $\mu^o(n, \bar{t})$ given the $(n, \bar{t})$-inspection policy.

3.2. The Optimal Quality Effort Level

Given the $(n, \bar{t})$-inspection policy, similarly as derived in Section 3.1, we can show that the firm's ex-ante profit is

$$\begin{aligned} \Pi_\Gamma(\mu; n, \bar{t}) &= (p - \upsilon_\Gamma)B\Pr(\bar{X} > \bar{t}) \\ &\quad - lBE\left[\Phi\left(\frac{\bar{\theta} - \frac{\sigma^2\mu + \tau^2 n\bar{X}}{\sigma^2 + \tau^2 n}}{\sqrt{\frac{\sigma^2\tau^2}{\sigma^2 + \tau^2 n}}}\right) I_{\{\bar{X} > \bar{t}\}}\right] \\ &\quad - kn - Bc_\Gamma(\mu) + \upsilon_\Gamma B, \end{aligned}$$

where $\bar{X}$ follows $N(\mu, \tau^2 + \frac{1}{n}\sigma^2)$. Letting $\frac{\partial}{\partial\mu}\Pi_\Gamma(\mu; n, \bar{t}) = 0$, by some algebra, we can show that the optimal quality effort level $\mu^o(n, \bar{t})$ satisfies the first-order condition

$$\begin{aligned} \frac{dc_\Gamma(\mu)}{d\mu} &= \left(\frac{p - \upsilon_\Gamma}{\sqrt{\tau^2 + \frac{\sigma^2}{n}}} - \frac{l}{\sqrt{\tau^2 + \frac{\sigma^2}{n}}}\Phi\left(\sqrt{\frac{n}{\sigma^2} + \frac{1}{\tau^2}}(\bar{\theta} - \mu)\right.\right. \\ &\quad \left.\left. - \sqrt{\frac{n\tau^2}{\sigma^2}}\frac{\bar{t} - \mu}{\sqrt{\tau^2 + \frac{\sigma^2}{n}}}\right)\right)\phi\left(\frac{\bar{t} - \mu}{\sqrt{\tau^2 + \frac{\sigma^2}{n}}}\right) \\ &\quad + \frac{l}{\sqrt{2\pi}\tau}\exp\left(-\frac{(\bar{\theta} - \mu)^2}{2\tau^2}\right)\bar{\Phi}\left(\frac{\sqrt{n}}{\sigma}(\bar{t} - \bar{\theta})\right), \end{aligned} \quad (4)$$

where $\overline{\Phi}(\cdot) = 1 - \Phi(\cdot)$. See the derivation of Eq. (4) in Section C of the online appendix, Supporting Information.

Third, we jointly determine the optimal sampling plan $(n^o, \bar{t}^o)$ and quality effort level μ^o.

3.3. Joint Optimization

By Eq. (1), we have the optimal rejection threshold $\bar{t}^o = \bar{t}^o(n^o; \mu^o)$. The optimal sample size n^o satisfies Eq. (3) given the optimal quality level μ^o. The optimal quality level μ^o satisfies Eq. (4) given the optimal sampling plan $(n^o, \bar{t}^o)$. Combining these results, we have the following theorem.

THEOREM 1: n^o and μ^o jointly satisfy the first-order conditions:

$$\begin{aligned}\frac{k}{Bl} &= \frac{\tau\sigma}{4\pi\sqrt{n^o}(\sigma^2 + n^o\tau^2)}\exp\left(-\frac{1}{2}\left(\frac{\bar{\theta}-\mu^o}{\tau}\right)^2\right)\\ &\quad \times \exp\left(-\frac{\left(\sqrt{\frac{\sigma^2}{n^o\tau^2}+1}z_\beta - \sqrt{\frac{\sigma^2}{n^o\tau^2}}\frac{\bar{\theta}-\mu^o}{\tau}\right)^2}{2}\right)\\ &= \frac{\tau\sigma}{4\pi\sqrt{n^o}(\sigma^2 + n^o\tau^2)}\exp\left(-\frac{z_\beta^2}{2}\right)\\ &\quad \times \exp\left(-\frac{\left(\sqrt{\frac{\sigma^2}{n^o\tau^2}}z_\beta - \sqrt{\frac{\sigma^2}{n^o\tau^2}+1}\frac{\bar{\theta}-\mu^o}{\tau}\right)^2}{2}\right) \qquad (5)\end{aligned}$$

and

$$\begin{aligned}\frac{dc_I(\mu^o)}{d\mu} &= \frac{l}{\tau\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{\bar{\theta}-\mu^o}{\tau}\right)^2\right)\\ &\quad \Phi\left(\sqrt{\frac{\sigma^2}{n^o\tau^2}+1}z_\beta - \sqrt{\frac{\sigma^2}{n^o\tau^2}}\frac{\bar{\theta}-\mu^o}{\tau}\right). \qquad (6)\end{aligned}$$

Theorem 1 facilitates searching for (n^o, μ^o). If the unit product cost $c_I(\mu)$ is affine in the quality effort level μ, searching for $(n^o, \mu^o, \bar{t}^o)$ becomes easier, which is shown in Section A of the online appendix, Supporting Information.

4. THE DECENTRALIZED SUPPLY CHAIN

We continue to study the model in Section 2. In the decentralized supply chain, the firm determines the optimal sampling plan and the supplier determines the optimal quality improvement effort level. To begin, we derive the firm's optimal sampling plan given the supplier's quality effort level μ.

4.1. The Firm's Optimal Sampling Plan

Similarly as argued in Section 3.1, the optimal sampling plan is a $(n, \bar{t})$-policy, by which the firm recommends a rejection if the sufficient statistic $\bar{X}$ falls below the threshold $\bar{t}$.

Given the sample size n and the supplier's quality effort level μ, we derive the optimal rejection threshold $\bar{t}^*(n; \mu)$. As the recall probability is $\Pr(\Theta < \bar{\theta}|\bar{X} = \bar{x})$, the firm should reject the product batch iff $pB - \rho lB \Pr(\Theta < \bar{\theta}|\bar{X} = \bar{x}) \leq wB$, or equivalently, $\Pr(\Theta \leq \bar{\theta}|\bar{X} = \bar{x}) \geq \alpha$, where $\alpha = \frac{p-w}{\rho l}$. Similarly as argued in Section 3.1, the firm should reject the product batch iff $\bar{X} \leq \bar{t}^*(n; \mu)$, where the rejection threshold is

$$\bar{t}^*(n;\mu) = \bar{\theta} + \frac{\sigma^2}{\tau^2 n}(\bar{\theta}-\mu) - \sqrt{\sigma^2+\tau^2 n}\frac{\sigma z_\alpha}{\tau n} \qquad (7)$$

and $z_\alpha = \Phi^{-1}(\alpha)$.

Next, we determine the optimal sample size $n^*(\mu)$. Given $\bar{X} = \bar{x}$, the firm's conditional profit is

$$\Pi_F(\bar{x}, n; \mu) = \begin{cases} pB - wB - \rho lB \\ \quad \Pr(\Theta < \bar{\theta}|\bar{X} = \bar{x}) - \eta kn, & \text{if } \bar{x} > \bar{t}^*(n;\mu), \\ -\eta kn, & \text{if } \bar{x} \leq \bar{t}^*(n;\mu). \end{cases}$$

The firm's ex-ante profit is

$$\begin{aligned}\Pi_F(n;\mu) &= E[\Pi_F(\bar{X}, n; \mu)]\\ &= (p-w)B\Pr(\bar{X} > \bar{t}^*(n;\mu)) - \rho lBE\\ &\quad \times \left[\Phi\left(\frac{\bar{\theta} - \frac{\sigma^2\mu+\tau^2 n\bar{X}}{\sigma^2+\tau^2 n}}{\sqrt{\frac{\sigma^2\tau^2}{\sigma^2+\tau^2 n}}}\right)I_{\{\bar{X}>\bar{t}^*(n;\mu)\}}\right] - \eta kn,\end{aligned}$$

where $\bar{X}$ follows $N(\mu, \tau^2 + \frac{1}{n}\sigma^2)$. By some algebra, the firm's ex-ante profit can be simplified as $\Pi_F(n;\mu) = (p-w)B - C_F(n;\mu)$, where

$$\begin{aligned}C_F(n;\mu) &= \rho lB\int_{-\infty}^{z_\alpha}\Phi\left(\sqrt{\frac{\sigma^2}{n\tau^4}+\frac{1}{\tau^2}}(\bar{\theta}-\mu) - \sqrt{\frac{\sigma^2}{n\tau^2}}y\right)\\ &\quad \times \phi(y)dy + \eta kn. \qquad (8)\end{aligned}$$

See the derivation of Eq. (8) in Section C of the online appendix, Supporting Information.

Letting $\frac{\partial}{\partial n}C_F(n;\mu) = 0$, similarly as the derivation of Eq. (3), we can show that the optimal sample size $n^*(\mu)$ satisfies the first-order condition

$$\frac{\eta k}{\rho l B} = \frac{\tau\sigma}{4\pi\sqrt{n}(\sigma^2+n\tau^2)}\exp\left(-\frac{(\bar\theta-\mu)^2}{2\tau^2}\right)$$
$$\times\exp\left(-\frac{\left(\sqrt{\frac{\sigma^2}{n\tau^2}+1}z_\alpha-\frac{\sigma(\bar\theta-\mu)}{\tau^2\sqrt{n}}\right)^2}{2}\right)$$
$$=\frac{\tau\sigma}{4\pi\sqrt{n}(\sigma^2+n\tau^2)}\exp\left(-\frac{z_\alpha^2}{2}\right)$$
$$\times\exp\left(-\frac{\left(\sqrt{\frac{\sigma^2}{n\tau^2}}z_\alpha-\sqrt{\frac{\sigma^2}{n\tau^2}+1}\frac{\bar\theta-\mu}{\tau}\right)^2}{2}\right). \quad (9)$$

Finally, we let the optimal rejection threshold be $\bar{t}^*(\mu) = \bar{t}^*(n^*(\mu);\mu)$. Notice that if $\eta = \rho = \frac{p-w}{p-v_r}$, then $\alpha = \beta$ and Eqs. (3) and (9) coincide with each other. This implies that $n^*(\mu) = n^o(\mu)$ and $\bar{t}^*(\mu) = \bar{t}^o(\mu)$. The condition of $\rho = \frac{p-w}{p-v_r}$ implies $\frac{p-w}{\rho l} = \frac{p-v_r}{l}$. Hence, the firm has the same gain/loss ratio of selling the product in the market in the centralized and decentralized supply chains and should adopt the same inspection criterion in both business scenarios. The condition of $\rho = \eta$ implies that the ratio of the recall loss over the inspection cost born on the firm stays the same in the decentralized and centralized supply chains, and hence, the incentive of the firm to conduct inspection is unchanged. If both conditions hold, then the firm has the same incentive and risk of selling the product and conducting inspection in the centralized and decentralized supply chains and hence should choose the same sampling plan when facing a fixed quality effort level.

4.2. The Supplier's Optimal Quality Effort Level

Next, we derive the supplier's optimal quality effort level, which depends on the firm's sampling plan. As a $(n,\bar{t})$-inspection policy is optimal for the firm (in Section 4.1), the supplier should expect that the firm uses the $(n,\bar{t})$-policy. Recall that the firm accepts the batch iff $\bar{X} > \bar{t}$, where $\bar{X}$ follows $N(\mu, \tau^2 + \frac{1}{n}\sigma^2)$. If the product batch is accepted, then the supplier receives payment w per unit, from the firm. However, the supplier has to cover $100(1-\rho)\%$ of the recall loss l per unit, if the product batch is recalled. Also, the supplier bears $100(1-\eta)\%$ of the inspection cost. If the product batch is rejected by the firm, then the supplier salvages the batch and receives v_s per unit.

Given $\bar{X} = x$, the supplier's conditional profit is

$$\Pi_S(\bar{x},\mu;n,\bar{t}) = \begin{cases} wB - (1-\rho)lB\Pr(\Theta < \bar\theta \mid \bar{X} = \bar{x}) \\ \quad -(1-\eta)nk - Bc_s(\mu), & \text{if } \bar{x} > \bar{t}, \\ Bv_s - (1-\eta)nk - Bc_s(\mu), & \text{if } \bar{x} \le \bar{t}. \end{cases}$$

The supplier's ex-ante profit is

$$\Pi_S(\mu;n,\bar{t}) = E[\Pi_S(\bar{X},\mu;n,\bar{t})]$$
$$= (w-v_s)B\Pr(\bar{X} > \bar{t})$$
$$-(1-\rho)lBE\left[\Phi\left(\frac{\bar\theta-\frac{\sigma^2\mu+\tau^2n\bar{X}}{\sigma^2+\tau^2n}}{\sqrt{\frac{\sigma^2\tau^2}{\sigma^2+\tau^2n}}}\right)I_{\{\bar{X}>\bar{t}\}}\right]$$
$$+Bv_s-(1-\eta)nk-Bc_s(\mu),$$

where $\bar{X}$ follows $N(\mu,\tau^2+\frac{1}{n}\sigma^2)$. Letting $\frac{\partial}{\partial\mu}\Pi_S(\mu;n,\bar{t}) = 0$, similarly as derived in Eq. (4), we can show that the optimal quality effort level $\mu^*(n,\bar{t})$ satisfies the first-order condition

$$\frac{dc_s(\mu)}{d\mu} = \left(\frac{w-v_s}{\sqrt{\tau^2+\frac{\sigma^2}{n}}} - \frac{(1-\rho)l}{\sqrt{\tau^2+\frac{\sigma^2}{n}}}\Phi\left(\sqrt{\frac{n}{\sigma^2}+\frac{1}{\tau^2}}(\bar\theta-\mu)\right.\right.$$
$$\left.\left.-\sqrt{\frac{n\tau^2}{\sigma^2}}\frac{\bar{t}-\mu}{\sqrt{\tau^2+\frac{\sigma^2}{n}}}\right)\right)\phi\left(\frac{\bar{t}-\mu}{\sqrt{\tau^2+\frac{\sigma^2}{n}}}\right)$$
$$+\frac{(1-\rho)l}{\sqrt{2\pi}\tau}\exp\left(-\frac{(\bar\theta-\mu)^2}{2\tau^2}\right)\bar\Phi\left(\frac{\sqrt{n}}{\sigma}(\bar{t}-\bar\theta)\right). \quad (10)$$

Notice that if $c_s(\mu) = (1-\rho)c_r(\mu)$ and $\rho = 1 - \frac{w-v_s}{p-v_r}$, then Eqs. (4) and (10) are only different with each other by a constant scale $(1-\rho)$. This implies that $\mu^*(n,\bar{t}) = \mu^o(n,\bar{t})$. The conditions imply that the product cost functions in the centralized and decentralized supply chains are different by the ratio of the revenues of selling the product ($\frac{w-v_s}{p-v_r}$). Moreover, as $\frac{p-v_r}{l} = \frac{w-v_s}{(1-\rho)l}$, the firm in the centralized supply chain has the same gain/loss ratio of selling the product as the supplier in the decentralized supply chain. Hence, if the conditions hold, then the firm and the supplier have the same quality improvement incentive and should choose the same quality effort level in the centralized and decentralized supply chains when facing the same sampling plan. Also, the conditions imply that $1-\rho > 0$, and hence, the supplier must bear a share of the recall loss.

4.3. The Simultaneous Game

The firm's optimal sampling plan depends on the batch quality and the supplier's quality effort level (as shown in Section 4.1), and the supplier's optimal quality effort level depends on the firm's sampling plan (as shown in Section 4.2). To model the strategic interaction between the firm's sampling plan and the supplier's quality effort decision, we consider a simultaneous game, in which the firm determines its sampling plan and the supplier determines her quality effort level.

The simultaneous game is appropriate if the firm is not able to commit the inspection plan before the supplier produces the product batch. Because inspection is conducted after the production is done, the firm always wants to cheat the supplier, change the ex-post inspection plan and break the commitment. For instance, the firm can threaten the supplier to use a tight inspection policy, but skip the inspection and save inspection cost after the supplier produces a high-quality batch. Obviously, if the supplier can foresee such an outcome, then the firm's threat becomes empty and ineffective and the supplier makes production adjustment accordingly.

In practice, inspection commitment is hardly to be convincing, because many suppliers have much weaker bargaining power than buyers and are lack of capability to monitor the inspection process conducted by buyers who always have incentive to make inspection changes after suppliers produce the product batch (e.g., adjusting the sample size). If the sampling plan is committable, then we can model the strategic interaction between the firm's sampling plan and the supplier's quality effort decision as a Stackelberg leadership game, which will be studied in Section 4.4.

A pure strategy Nash equilibrium $\{(n^*, \bar{i}^*); \mu^*\}$ is defined as $\mu^* \in \arg\max \Pi_S(\mu; n^*, \bar{i}^*)$, $n^* \in \arg\max \Pi_F(n; \mu^*)$ and $\bar{i}^* = \bar{i}^*(n^*; \mu^*)$. As the firm (supplier) does not want to change the sampling plan (quality effort level) at the equilibrium, the equilibrium outcome is stable in the decentralized supply chain and can be used to study the performance of the decentralized supply chain.

THEOREM 2: If there exists a pure strategy Nash equilibrium $\{(n^*, \bar{i}^*); \mu^*\}$, where $n^* > 0$ and $\mu^* > 0$, then n^* and μ^* must satisfy

$$\begin{aligned}\frac{\eta k}{\rho l B} &= \frac{\tau\sigma}{4\pi\sqrt{n^*}(\sigma^2 + n^*\tau^2)} \exp\left(-\frac{(\bar{\theta} - \mu^*)^2}{2\tau^2}\right) \\ &\quad \times \exp\left(-\frac{\left(\sqrt{\frac{\sigma^2}{n^*\tau^2} + 1}z_\alpha - \frac{\sigma(\bar{\theta}-\mu^*)}{\tau^2\sqrt{n^*}}\right)^2}{2}\right) \\ &= \frac{\tau\sigma}{4\pi\sqrt{n^*}(\sigma^2 + n^*\tau^2)} \exp\left(-\frac{z_\alpha^2}{2}\right) \\ &\quad \times \exp\left(-\frac{\left(\sqrt{\frac{\sigma^2}{n^*\tau^2}}z_\alpha - \sqrt{\frac{\sigma^2}{n^*\tau^2} + 1}\frac{\bar{\theta}-\mu^*}{\tau}\right)^2}{2}\right) \qquad (11)\end{aligned}$$

and

$$\begin{aligned}\frac{dc_s(\mu^*)}{d\mu} &= \frac{\sqrt{n^*}}{\sqrt{2\pi(\tau^2 n^* + \sigma^2)}} \\ &\quad \times \left(w - v_s - \left(\frac{1}{\rho} - 1\right)(p - w)\right) \\ &\quad \times \exp\left(-\frac{\left(\sqrt{\frac{\sigma^2}{n^*\tau^2}}z_\alpha - \sqrt{\frac{\sigma^2}{n^*\tau^2} + 1}\frac{\bar{\theta}-\mu^*}{\tau}\right)^2}{2}\right) \\ &\quad + \frac{(1-\rho)l}{\sqrt{2\pi}\tau} \exp\left(-\frac{(\bar{\theta} - \mu^*)^2}{2\tau^2}\right) \\ &\quad \times \Phi\left(\sqrt{\frac{\sigma^2}{n^*\tau^2} + 1}z_\alpha - \sqrt{\frac{\sigma^2}{n^*\tau^2}}\frac{\bar{\theta} - \mu^*}{\tau}\right). \qquad (12)\end{aligned}$$

Theorem 2 facilitates searching for (n^*, μ^*). If the unit product cost $c_s(\mu)$ is affine in the quality effort level μ, searching for $(n^*, \mu^*, \bar{i}^*)$ becomes easier (see Section A of the online appendix, Supporting Information). Next, we identify parameter settings such that the decentralized supply chain achieves the same outcome as the centralized supply chain.

THEOREM 3: If $\eta = \rho = \frac{p-w}{p-v_f}$, $v_s = v_f$, and $c_s(\mu) = (1-\rho)c_f(\mu)$, then $n^* = n^o$, $\bar{i}^* = \bar{i}^o$, and $\mu^* = \mu^o$.

The parameter settings in Theorem 3 imply that the gain/loss ratio of selling the product and the ratio of the recall loss over the inspection cost for the firm or the supplier are unchanged in the centralized and decentralized supply chains. The joint effects of double marginalization, recall loss sharing, and inspection cost sharing cancel with each other. Hence, the decentralized supply chain achieves the same outcome as the centralized supply chain.

Next, we study how the optimal sampling plan changes with respect to the firm's outsourcing decision. For deriving comparative statics, we assume linear product cost functions (see Section A of the online appendix, Supporting Information). First, we consider the business scenario in which the firm bears all recall loss ($\rho = 1$). This case often holds if the supplier is located in a foreign country and contract enforcement is prohibitively expensive (see [8] for more discussion of legal risks of using foreign suppliers).

THEOREM 4: If $c_f(\mu) = d_f\mu$, $c_s(\mu) = d_s\mu$ and $\rho = 1$, then:

a. n^* is increasing in $l, \tau, \sigma, d_s, v_s$, and B, but decreasing in k and η;
b. $n^o < n^*$ when $\frac{(w-v_s)\eta}{\sqrt{2\pi}d_s\tau}\exp\left(\frac{z_\alpha^2 - z_\beta^2}{2}\right) < 1$;
c. $n^o < n^*$ and $\bar{i}^o < \bar{i}^*$ when $(w - v_s)\eta < \sqrt{2\pi}d_s\tau$, $w = v_f$ and $2p > 2w + l$.

Theorem 4 suggests that the firm should increase the sample size (n^*) to collect information about the batch quality, if the product quality and sampling variations (τ and σ) are high. As an accurate estimation of the batch quality helps the firm make the right decision, the firm should increase the sample size (n^*) if the recall cost (l) is high. Moreover, as an accurate estimation of the batch quality reduces the chance of accepting a bad batch and hence provides the supplier an incentive to improve the batch quality, the firm should increase the sample size (n^*) if the supplier's product cost coefficient (d_s) is high, and/or she can salvage the product at a high price (v_s). Finally, the firm should reduce the sample size (n^*), if the inspection cost (k and η) born on the firm is high and/or the batch size (B) is small.

As discussed in Section 4.1, if $w = v_f$ and $\eta = \rho = 1$, then the firm should choose the same sampling plan in the decentralized and centralized cases when facing a fixed quality effort level. However, as the supplier does not bear any recall loss and hence has less incentive to improve product quality, the firm may choose to provide the supplier extra quality improvement incentive by taking more samples (i.e., $n^o < n^*$), raising the rejection threshold (i.e., $\bar{r}^o < \bar{r}^*$) and tightening the inspection plan when outsourcing the production.

Second, we consider the business scenario in which the firm only bears a proportion of the recall loss ($\rho < 1$).

THEOREM 5: If $c_f(\mu) = d_f\mu$, $c_s(\mu) = d_s\mu$, $\rho = \frac{p-w}{p-v_f}$, $v_s = v_f$ and $\frac{d_s}{d_f} = (1-\rho)\frac{\eta}{\rho}$, then:

a. $n^o > n^*$ and $\bar{r}^o > \bar{r}^*$ when $\rho < \eta$;
b. $n^o < n^*$ and $\bar{r}^o < \bar{r}^*$ when $\rho > \eta$.

Notice that the difference between the parameter settings in Theorems 3 and 5 is $\rho \neq \eta$. Hence, the product revenue is proportional to the recall cost born on the firm/supplier in the centralized and decentralized supply chains, but the recall loss born on the firm is disproportional to the inspection cost in the centralized and decentralized supply chains. If the ratio of the recall loss over the inspection cost in the decentralized supply chain is less than the one in the centralized supply chain (i.e., $\frac{\rho}{\eta} < 1$), the firm chooses to reduce the sample size (i.e., $n^o > n^*$), lower the rejection threshold (i.e., $\bar{r}^o > \bar{r}^*$), and ease the inspection plan in the decentralized supply chain. This is because the firm holds relatively less recall loss and/or incurs relatively higher inspection cost in the decentralized supply chain, in which the firm shares the recall loss and inspection cost with the supplier. The opposite is true, if the ratio of the recall loss over the inspection cost in the decentralized supply chain is larger than the one in the centralized supply chain.

Finally, we study the existence of a pure strategy Nash equilibrium. We focus on the case in which the firm bears all recall loss ($\rho = 1$), and the supplier has a linear product cost $c_s(\mu) = d_s\mu$ (see details in Section A of the online appendix, Supporting Information). If the supplier is located in a foreign country, in which contract enforcement is prohibitively expensive, the firm may have to bear all recall loss (see, e.g., [8]). This case is popular in practice.

Notice that Theorem 2 provides the necessary conditions that a Nash equilibrium must satisfy, but the conditions may not be sufficient to guarantee the existence of a pure strategy Nash equilibrium (see further discussions in Section B of the online appendix, Supporting Information). Hence, we offer sufficient conditions that guarantee the existence of a pure strategy Nash equilibrium in Theorem 6.

ASSUMPTION 1: $\bar{\theta} > 0$ and $2p - 2w \leq l$.

Assumption 1 requires that the unit recall loss is moderately large.

ASSUMPTION 2: $d_s \leq \frac{w-v_s}{2\tau\sqrt{\pi}} \exp\left(-\frac{1}{2}(\frac{\bar{\theta}}{\tau} - z_\alpha)^2\right)$ and $d_s k\eta \leq \frac{Bl(w-v_s)\tau}{16\sigma^2\pi^{3/2}} \exp\left(-\frac{z_\alpha^2}{2} - (\frac{\bar{\theta}}{\tau} - z_\alpha)^2\right)$.

ASSUMPTION 3: $k\eta \leq B\left(\frac{d_s\tau}{w-v_s}\right)^{4+2\kappa^2} \frac{l\kappa(2\pi)^{1+\kappa^2}}{4\sqrt{2}(\kappa^2+2)} \exp(-\frac{5}{2}z_\alpha^2)$, where $\kappa = \frac{\sigma}{\tau}$.

Assumptions 2 and 3 require that the unit inspection cost k and product cost coefficient d_s are sufficiently small.

THEOREM 6: If $\rho = 1$, $c_s(\mu) = d_s\mu$ and Assumptions 1–3 hold, then there exists a pure strategy Nash equilibrium.

By numerical experiments, we find that if the unit inspection cost k and product cost coefficient d_s are moderately small, which may not even satisfy Assumptions 2 and 3, there exists a pure strategy Nash equilibrium, which is the solution of $\{(n^*, \bar{r}^*); \mu^*\}$ in Theorem 2. However, if the unit inspection cost k and/or product cost coefficient d_s are large, the solution of $\{(n^*, \bar{r}^*); \mu^*\}$ in Theorem 2 may not be a pure strategy Nash equilibrium. See a counterexample and further discussions of the simultaneous game and Nash equilibrium in Section B of the online appendix, Supporting Information.

4.4. The Stackelberg Leadership Game

If the firm has the creditability to commit the inspection plan, then we can model the strategic interaction between the firm's sampling plan and the supplier's quality effort decision as a Stackelberg leadership game. For instance, the inspection is conducted by a third party inspector who is informed of the specific inspection plan before receiving the order from the supplier. Under this scenario, although the firm has the incentive to change the inspection plan after the supplier produces the product batch (e.g., skipping inspection and saving inspection cost), it may be inconvenient to do so.

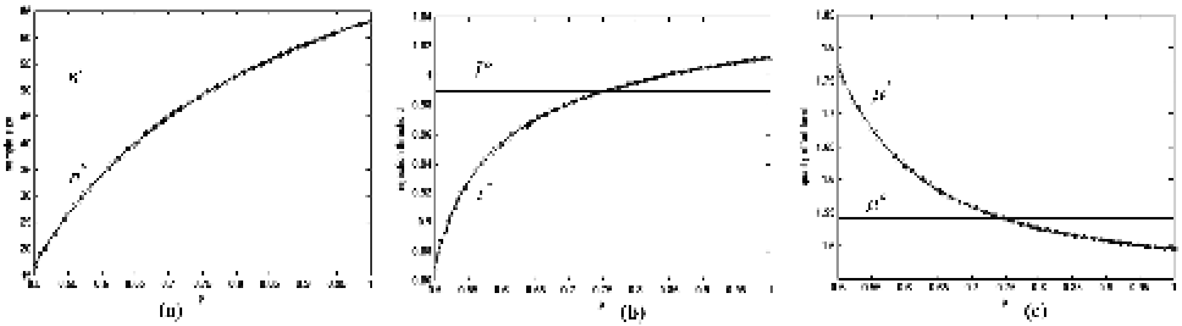

Figure 1. (a) Sample sizes n^* and n^o; (b) Rejection thresholds $\bar{t}^*$ and $\bar{t}^o$; and (c) Quality effort levels μ^* and μ^o.

We assume that the firm commits to an acceptance sampling plan $(n, \bar{t})$. Given the $(n, \bar{t})$-inspection policy, the supplier determines the optimal quality effort level, which is solved in Section 4.2. The optimal quality effort level $\mu^*(n, \bar{t})$ is determined by Eq. (10). Recall that if $c_s(\mu) = (1-\rho)c_r(\mu)$ and $\rho = 1-\frac{w-v_s}{p-v_r}$, then $\mu^*(n, \bar{t}) = \mu^o(n, \bar{t})$. Under these parameter settings, the supplier chooses the same quality effort level as in the centralized supply chain.

In both the simultaneous and Stackelberg leadership games, we assume that the supplier has enough economic incentive (i.e., non-negative expected profit) to engage in the trade with the firm. Such an incentive can be achieved if the firm pays the supplier a lump-sum money before the production occurs. The lump-sum money can take the form of a fixed unit purchase price that is not contingent on the batch quality and inspection. It is not uncommon that the supplier receives a prepayment from the outsourcing firm to cover raw material and labor costs. This type of lump-sum money payment only serves the purpose of alluring the supplier into the trade and does not affect the optimal decisions of the supplier and firm, as it only changes the supplier and firm's profit functions by a constant.

Similarly as derived in Section 4.1, the firm's ex-ante profit is

$$\begin{aligned}\Pi_F(n,\bar{t}) &= (p-w)B\Pr(\bar{X} > \bar{t}) \\ &\quad - \rho l B E\left[\Phi\left(\frac{\bar{\theta} - \frac{\sigma^2\mu^*(n,\bar{t})+\tau^2 n\bar{X}}{\sigma^2+\tau^2 n}}{\sqrt{\frac{\sigma^2\tau^2}{\sigma^2+\tau^2 n}}}\right) I_{\{\bar{X}>\bar{t}\}}\right] - \eta k n \\ &= (p-w)B\bar{\Phi}\left(\frac{\bar{t}-\mu^*(n,\bar{t})}{\sqrt{\tau^2+\frac{1}{n}\sigma^2}}\right) - \rho l B \int_{\frac{\bar{t}-\mu^*(n,\bar{t})}{\sqrt{\tau^2+\frac{1}{n}\sigma^2}}}^{+\infty} \\ &\quad \Phi\left((\bar{\theta}-\mu^*(n,\bar{t}))\sqrt{\frac{1}{\tau^2}+\frac{n}{\sigma^2}} - \frac{\tau}{\sigma}\sqrt{n}z\right)\phi(z)dz \\ &\quad - \eta k n,\end{aligned}$$

where $\bar{X}$ follows $N(\mu^*(n,\bar{t}), \tau^2 + \frac{1}{n}\sigma^2)$. We let $(n^o, \bar{t}^o) \in \arg\max \Pi_F(n, \bar{t})$ be the optimal acceptance sampling plan and $\mu^o = \mu^*(n^o, \bar{t}^o)$ be the optimal quality effort level in the Stackelberg leadership game.

Lastly, we notice that even under the parameter settings of Theorem 3, the firm in the Stackelberg leadership game does not choose the same outcome $(n^o, \bar{t}^o)$ as in the centralized supply chain. Hence, the supply chain is not fully coordinated. This can be seen from the firm's ex-ante profit function $\Pi_F(n, \bar{t})$, which does not include the product cost component and hence is different from its corresponding version in the centralized supply chain. The impact of the acceptance sampling plan on the product cost is carried by the supplier and excluded in the firm's profit function $\Pi_F(n, \bar{t})$.

5. NUMERICAL STUDIES

The difference of the optimal sampling plan and quality effort level in the centralized and decentralized supply chains is caused by three factors, double marginalization, recall loss sharing, and inspection cost sharing effects. In the simultaneous game, the joint effects of these three factors cancel with each other in the parameter settings of Theorem 3. Hence, the decentralized supply chain can achieve the same outcome as the centralized supply chain. This is demonstrated in Example 1.

EXAMPLE 1: We let $p = 3$, $v_r = 1$, $w = 1.5$, $v_s = 1$, $l = 4$, $d_s = 0.25$, $d_r = 1$, $k = 5$, $B = 10{,}000$, $\bar{\theta} = 1$, $\tau = 0.5$, $\sigma = 0.5$, $\eta = 0.75$, and vary $\rho \in [0.5, 1]$. Figure 1 shows the sample sizes n^* and n^o, rejection thresholds $\bar{t}^*$ and $\bar{t}^o$ and quality effort levels μ^* and μ^o.

As ρ increases, the firm (supplier) holds more (less) recall loss. This causes the firm to increase the sample size and raise the rejection threshold and the supplier to reduce the quality improvement effort accordingly (as shown in Fig. 1). Hence, the recall cost sharing mechanism is substitutable with the inspection policy. The decentralized supply chain

Table 2. Simultaneous game versus stackelberg leadership game.

	Simultaneous game						Stackelberg leadership game					
ρ	n^*	$\bar{r}^*$	μ^*	π_F^*	π_S^*	π_T^*	$n^\diamond$	$\bar{r}^\diamond$	$\mu^\diamond$	$\pi_F^\diamond$	$\pi_S^\diamond$	$\pi_T^\diamond$
0.5	16.7997	0.8694	1.7707	13,854	9889	23,743	26.2	0.197	1.9597	14,715	9555	24,270
0.55	26.4311	0.9275	1.678	13,365	10,060	23,425	29	0.195	1.9318	14,665	9613	24,278
0.6	33.9475	0.9539	1.6216	12,996	10,165	23,161	33.5	0.1867	1.8998	14,602	9679	24,281
0.65	40.0347	0.9699	1.5836	12,712	10,236	22,948	38.5	0.18	1.8618	14,518	9756	24,274
0.7	44.9571	0.9808	1.5584	12,503	10,287	22,790	46	0.1733	1.816	14,402	9848	24,250
0.75	49.1176	0.989	1.5398	12,337	10,324	22,661	55.5	0.16	1.7584	14,226	9961	24,187
0.8	52.692	0.9955	1.5259	12,205	10,353	22,558	70.5	0.1467	1.6813	13,929	10,108	24,037
0.85	55.7647	1.0008	1.5156	12,101	10,375	22,476	91	0.135	1.5675	13,323	10,314	23,637
0.9	58.4914	1.0052	1.5075	12,015	10,392	22,407	31	1.15	1.6341	12,306	10,061	22,367
0.95	60.9404	1.009	1.5008	11,940	10,406	22,346	31	1.15	1.6334	12,299	10,062	22,361
1	63.1641	1.0124	1.4951	11,876	10,417	22,293	32	1.15	1.6328	12,293	10,063	22,356

achieves the same outcome as the centralized supply chain at $\rho = 0.75$, where the double marginalization, recall loss sharing, and inspection cost sharing effects cancel out with each other. As shown in Figs. 1a and 1b and suggested by Theorem 5, the sample size and rejection threshold in the decentralized supply chain are less (more) than the ones in the centralized supply chain when ρ is smaller (larger) than $\eta = 0.75$. As shown in Fig. 1c, the supplier's quality effort level is more (less) than the one in the centralized supply chain when ρ is smaller (larger) than 0.75.

Second, we compare the supply chain outcomes in the simultaneous and Stackelberg leadership games with respect to the change of the recall loss sharing ratio ρ.

EXAMPLE 2: We use the same parameter settings as the ones in Example 1 and vary $\rho \in [0.5, 1]$. Table 2 shows the optimal sample sizes n^* and $n^\diamond$, rejection thresholds $\bar{r}^*$ and $\bar{r}^\diamond$, quality effort levels μ^* and $\mu^\diamond$, firm's profits Π_F^* and $\Pi_F^\diamond$, supplier's profits Π_S^* and $\Pi_S^\diamond$, and the supply chain's total profits Π_T^* and $\Pi_T^\diamond$ in both the simultaneous and Stackelberg leadership games.

As demonstrated in Example 1, the recall loss sharing effect is substitutable with the inspection plan. Less recall loss sharing with the supplier (i.e., a high value of ρ) requires the firm to adopt a tighter inspection plan, which reduces the chance that a bad product batch gets into the market. In the simultaneous game that does not require the firm to precommit to an inspection plan, the firm adopts a relatively tight inspection plan (e.g., the rejection threshold is close to the quality threshold) even when the recall loss sharing effect is significant (e.g., $\rho \in [0.5, 0.85]$). Due to the low chance that a bad product batch passes through the tight inspection policy, the recall loss sharing mechanism is unlikely invoked, and hence, the recall loss sharing effect is weakened by the adoption of the tight inspection policy in the simultaneous game. However, this prediction by the Nash equilibrium is a stable outcome of the supply chain in which the firm cannot commit to the inspection policy before the supplier produces the product batch.

In contrast, the Stackelberg leadership game in which the firm commits to the inspection policy before the supplier's production run may imply a different supply chain outcome. Although the firm continues to adopt a tight inspection policy when the recall loss sharing effect is insignificant (e.g., $\rho \in [0.9, 1]$), the firm switches to a very loose inspection policy (e.g., the rejection threshold is far below the quality threshold) when the recall loss sharing effect is significant (e.g., $\rho \in [0.5, 0.85]$). Using a loose inspection policy allows a bad product batch to get into the market and hence increases the chance that the recall loss sharing mechanism is invoked and the supplier is punished. When the recall loss sharing effect is significant (e.g., $\rho \in [0.5, 0.85]$), by intentionally committing to a loose inspection policy, the firm strengthens the recall loss sharing effect and hence enhances the supplier's economic incentive to improve the batch quality. This explains why we see high values of the quality effort level but low values of the rejection threshold when $\rho \in [0.5, 0.85]$ in the Stackelberg leadership game (see the right columns in Table 2).

This surprising outcome of the supply chain critically depends on the precommitment assumption of the firm's inspection policy, because the firm likely prefers raising the rejection threshold up to the quality threshold level after the supplier's production run. Hence, the supply chain outcome predicted in the Stackelberg leadership game is not a stable outcome (i.e., a Nash equilibrium) in the simultaneous game. Finally, we notice that though the inspection policies adopted by the firm are very sensitive to the precommitment assumption and game format, the firm's, supplier's, and total supply chain profits do not change much with respect to the game format (see Table 2).

6. CONCLUSIONS AND FUTURE RESEARCH

This article studies the incentive effect of acceptance sampling plans in a supply chain with endogenous product quality. We consider a firm that purchases a product batch from a supplier, who can put effort into product quality improvement. The quality of each product unit is measured by a continuous variable that follows a normal distribution and is correlated within the batch. The firm determines the acceptance sampling plan, and the supplier determines the quality effort level in either a simultaneous game or a Stackelberg leadership game, in which both parties share recall loss and inspection cost. We provide sufficient conditions that guarantee the existence of a pure strategy Nash equilibrium and identify the equilibrium form of the optimal sample size, rejection threshold, and quality effort level.

In the simultaneous game, we find that the outcomes in the centralized and decentralized supply chains are different due to the double marginalization, recall loss sharing, and inspection cost sharing effects. However, under some parameter settings, the three effects are canceled out, and hence, the decentralized supply chain achieves the same outcome as the centralized supply chain. By numerical studies, we find that as the recall loss sharing effect becomes weak, the firm tightens the inspection policy by increasing the sample size and raising the rejection threshold, but the supplier reduces the quality improvement effort.

In the Stackelberg leadership game, the firm may intentionally commit to a very loose inspection policy that amplifies the recall loss sharing effect and provides the supplier enough incentive to improve the product quality. This strategy is effective when the supplier shares a large proportion of the recall loss with the firm and critically depends on the precommitment assumption of the firm's inspection policy. Without the precommitment assumption (i.e., the simultaneous game) and/or when the supplier shares a small proportion of the recall loss with the firm, the firm should adopt a tight inspection policy (e.g., the rejection threshold is close to the quality threshold).

This article can be extended in the following directions. First, the relationship between a buyer and a supplier may be long-term oriented; that is, the supplier produces a sequence of product batches for the buyer. Under this scenario, the buyer may adopt an inspection policy that varies from period to period. For instance, if inspection results are promising for several batches, then the buyer may want to reduce the sample size of future inspection. On the other hand, if the supplier continues to deliver bad batches, then the buyer may want to increase the sample size of future inspection. This dynamic problem can be modeled as a multiple-stage game. Second, the supplier and buyer may be risk-averse and prefer to avoiding a dramatic recall loss by altering the inspection policy and quality improvement decision. Finally, reducing manufacturing volatility is another important topic of quality control. It is interesting to examine how to induce a supplier reducing manufacturing volatility in a supply chain with inspection and economic contracts.

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