

HARMONIC SCRUBBER FOR DETECTED MODULATION FREQUENCIES

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In dedication to my parents.

Thanks for all the love, patient, and support you have given me.

In memory of my mother.

Now you can stand by my side all the time.

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## SYMBOLS

$[i]$	the nearest integer of value $i$ (round)
$ i $	the absolute value of $i$
$\lceil i \rceil$	the smallest integer greater than or equal to $i$ (ceil)
$f_s$	sampling rate of the sound record
$t_s$	length of sound record in second
$PSD_{dB}$	PSD signal of audio signal in dB
$PSD_{avg}$	PSD signal filtered with moving average filter
$PSD_{nol}$	normalized $PSD_{avg}$ signal
$PSD_{thr}$	dynamic threshold for PSD signal
$thr_{st}$	fixed threshold value for the PSD signal
$thr_{width}$	fixed threshold value for the strong tone signal width
$f_{st}$	strong tone frequency
$amp_{abs}$	absolute amplitude of strong tone frequency
$amp_r$	relative amplitude of strong tone frequency
$pk_{width}$	strong tone width in number of point
$MPSD_{dB}$	PSD of modulation frequency in dB
$f_m$	modulation frequency of a strong tone signal
$amp_m$	modulation depth of a modulation frequency
$\Delta f$	resolution of modulation frequencies
$\Omega$	the modulation frequency set, and the frequencies are sorted in increasing order
Amp	the set of modulation depth for each modulation frequency in $\Omega$
$\Omega'$	the modified whole modulation frequency set
$\omega'_i$	the subset of $\Omega'$ , which contained elements from $f'_1$ to $f'_{i-1}$

$f'_0$	the potential fundamental frequency
$\hat{f}_0$	the modified potential fundamental frequency
$HG(f_i)$	the set of all possible harmonic groups that generated from $f_i$
$\Phi$	the set of all $HG(f_i)$
$\hat{f}_k^i$	the largest $f'_0$ in $HG(f_i)$ that formed a harmonic group with k elements
$thr_{f_0}$	the threshold for potential fundamental frequency
$thr_{gcd}$	the threshold for "gcd" function
$f_{mode}$	the most frequent $f'_0$ value in PFF
$f_{0-est}$	the estimated fundamental frequency
$num_{est}$	the number of $f_{0-est}$ needed
$\bar{H}$	the non-harmonic frequencies in $\Omega$
AmpH	the group power of the harmonic group

## ABBREVIATIONS

FT	Fourier Transform
DFT	Discrete Fourier Transform
FFT	Fast Fourier Transform
PSD	Power Spectral Density
PWelch	Welch's method to estimate PSD
FV	Feature Vector
SFV	Strong Tone Feature Vector
MFV	Modulation Feature Vector
MF	Mechanical Feature
HS	Harmonic Scrubber
HG	Harmonic group
PPF	Potential fundamental frequencies
"gcd"	Similar to the gcd function in math. Use for checking if one number is similar to the multiple of the other

## ABSTRACT

Wang, Xihui Master, Purdue University, May 2019. Harmonic Scrubber For Detected Modulation Frequencies. Major Professor: Jan P. Allebach.

Acoustic signals have long been used to monitor the performance of machinery containing mechanical moving parts, especially machinery used in manufacturing. Rotating components generate harmonic signals with a fundamental frequency corresponding to the period of rotation, although the fundamental frequency and some of the harmonics may be missing. In addition, the meshing of the teeth in gears generates higher frequencies corresponding to the period of the gear teeth interaction. We call the former frequencies harmonic frequencies and the latter frequencies strong tone frequencies. Each strong tone frequency typically has associated with it, a set of modulation frequencies.

For each strong tone frequency, it is important to be able to determine which modulation frequencies correspond to a particular harmonic series, since this can help to determine which component in the overall mechanism is failing. In this work, we describe a novel process for selecting from a set of candidate modulation frequencies that comprise one or more harmonic sequences.

We also describe the signal processing pipeline used to extract the frequency components from the raw acoustic signal.

# 1. INTRODUCTION

In modern society, mechanical devices are ubiquitous in people's daily life from home to office. Therefore the noise produced by defective components has become an imperative concern for both customers and the companies.

## 1.1 Motivation

The forerunner that designs, implements and improves the technique to identify noise types will tremendously decrease the time, effort, and resource that the company spends on troubleshooting. As a result, this technique will significantly improve C2C experience.

When a customer called and complained to the customer service department of the company about the loud noise that the machine made, the customer service staff will send a technician immediately to the customer side. After diagnose, the expert locates the defective part of the device and back to the company. Then the technician will back and replace the broken part with a new one from the company after a while. However, even an experienced technician can not guarantee to solve customers' problems in one step. Thus the above procedure could repeat for several times causing the money spent on repairing the device is more than buying a new one.

Thus, there is a huge demand for coming up with a new method to do the diagnosis test remotely before sending a technician to repair the machine.

## 1.2 Relative Work

Accurately extract the fundamental frequency has been a popular research topic for many years. Markel built a SIFT algorithm in his research based on a simplified version of digital inverse filter and then find the fundamental frequency from the autocorrelation signal [1]. Boersma presented a method to accurately detect the fundamental frequency of a periodic signal in the autocorrelation domain, and the harmonics-to-noise ratio represented harmonic level of the sample signal [2]. Later, the YIN method was introduced by Kawahara and De Cheveigne mainly based on the autocorrelation method and the cumulative mean normalized difference function [3]. The idea is to take the interplay between autocorrelation and cancellation into account as the "yin" and "yang" thinking in the oriental philosophy. Based on previous fundamental frequency estimators invented, Klapuri proposed a multiple fundamental frequency estimation method [4].

## 1.3 The Proposed Solution

The updated self-detection method is a further study based on the noise source detection method studied by Xue [5]. The overall procedure is shown in Figure 1 as below.

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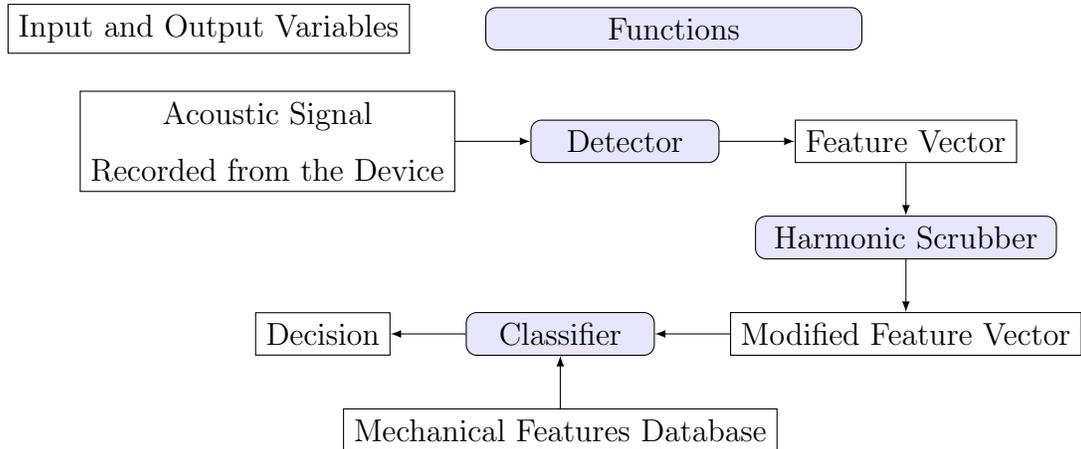


Fig. 1.1. Generalized Comprehensive Process Diagram

As specifically mentioned in Xue’s paper [5], the relationship between the acoustic information extracted from the machine running sound and the rotational speed of device inner components is the basis of the first part of this study (i.e., detector part). It was found that different component will result in different noise tone signal, and the modulation frequency of the tone signal is associated with the rotational speed of the machine’s component.

Arfken and Weber [6] stated in their book that periodical signals causing by rotating machines can be decomposed as a sum of sine and cosine waves with different amplitudes, whose frequencies are harmonically related. The harmonic frequencies are the integral multiple of the fundamental frequency, which is the one contained the most energy and also correlated to the rotating speed. So far the other significant fact was noticed, which is the rotational speed of a device component could associate with the fundamental frequency of a harmonic modulation frequency group instead of one modulation frequency. The second fact funds the rest of the study, which is the harmonic scrubber part.

The idea is to embed a detector in a device, and the detector will analyze the acoustic signal generated by the device once a while, extract acoustic information

from the acoustic signal, and then send this diagnose results (i.e., Feature Vector in the above diagram) to the company's cloud. More detailed information will be discussed in section 2 (i.e., Detector Part) later.

Before matching the diagnose results with the device's inner components, the feature vector needs to be modified through the harmonic scrubber function. The primary purpose of this step is to find the fundamental frequencies of all possible harmonic modulation frequency groups and calculate the amplitude of each harmonic group. More detailed information will be discussed in section 3 (i.e., Harmonic Scrubber Part) later.

By comparing the modified diagnose results with mechanical features given device's database, the classifier can tell which part of the device is making the noise and whether the device is running under a good condition or not. Once the result shows any problems, the company will be able to inform the customer and send a technician with a replacement part to the customer side.

## 2. DETECTOR

The revised detector is designed to extract acoustic information from the device running record that helps to identify the defective machine component based on Xue's previous work [5]. The detector will output one feature vector for each input sound record. The structure of the feature vector is shown below.

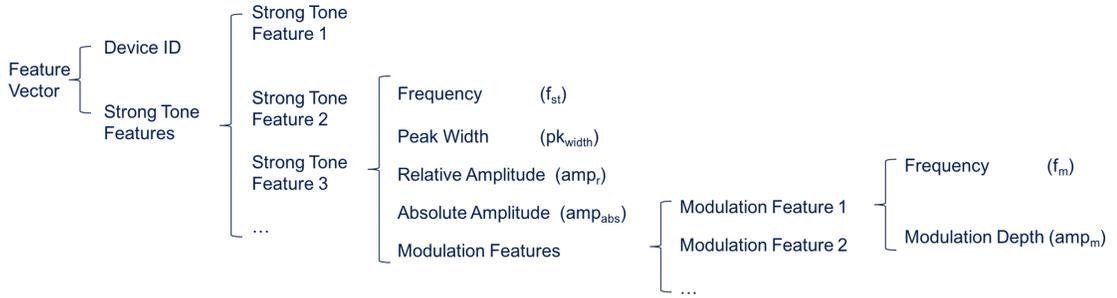


Fig. 2.1. Structure of feature vector

The entire detecting process can be divided into six parts, which are: 1) time-frequency analysis, 2) strong tone selection, 3) isolate the narrow-band strong tone signal, 4) compute signal envelope, 5) calculate the modulation depth, 6) modulation frequency selection.

### 2.1 time-frequency analysis

The time-frequency analysis is aimed to get the power spectral density of the audio signal in frequency domain by applying Welch's method [7]. The benefit of doing that is reducing the number of computation and the noise in PSD in exchange for reducing the frequency resolution.

It takes four major steps to estimate the PSD of a signal, which are: 1) partition the data sequence into  $n$  segments; 2) use FFT to compute the DFT of each windowed sequence; 3) calculate the periodogram value for each segment by equation (1) in below; 4) average all periodogram value to get estimate PSD, represented as  $P$ .

The power for the window applied is represented by  $w$ , the signal after FT is  $X_k$ , then for 2-sided spectrum the periodogram value  $P$  is calculated as below, followed by the PSD in dB.

$$P = \frac{2 * |X_k|^2}{w * f_s} \quad (2.1)$$

$$PSD_{dB} = 10 \log_{10} \left( \frac{P}{4 * 10^{-10}} \right) \quad (2.2)$$

Note:

- 1) hanning window is recommended by Welch [7] in step 2)
- 2) window power for hanning window is  $3/8$

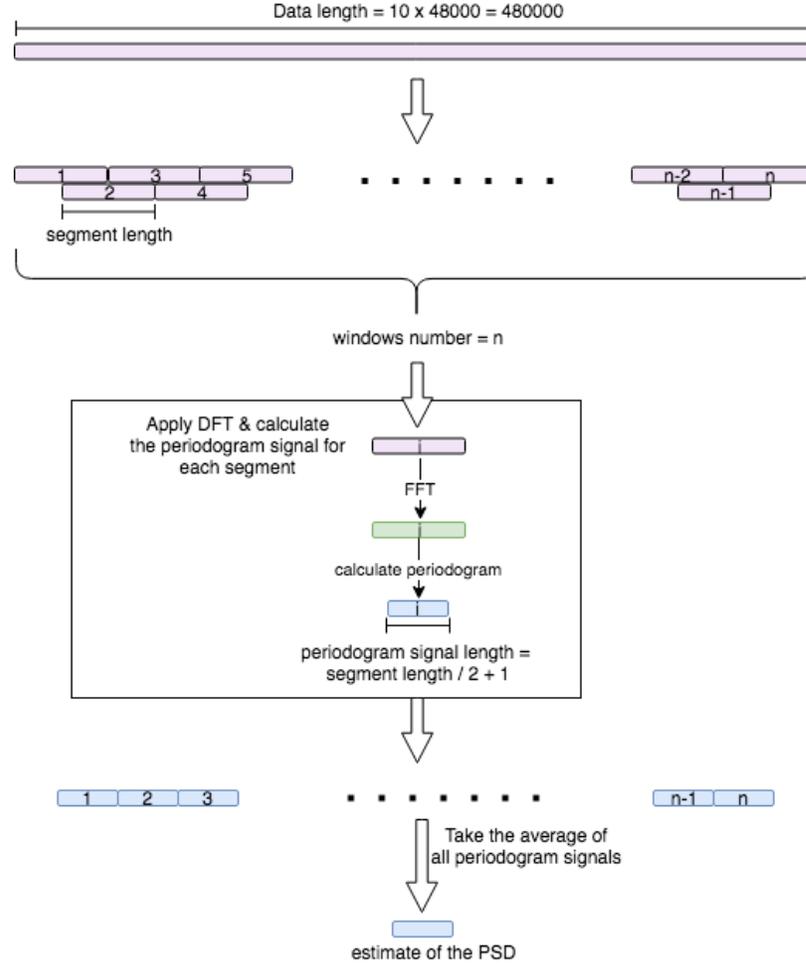


Fig. 2.2. Sample graph of Welch's method

## 2.2 strong tone selection

Next is to find the strong tone frequency among the  $PSD_{dB}$  signal by applying a dynamic threshold to the  $PSD_{dB}$  signal. The strong tone frequencies  $f_{st}$  are the frequency tone peaks that above the dynamic threshold and the tone width is larger or equal to three frequency resolution.

The way to construct a dynamic threshold  $PSD_{thr}$  is to apply a moving average filter [8] to the  $PSD_{dB}$  signal first, then bring the filtered signal  $PSD_{avg}$  down to the

lower level of the original signal by normalizing the filtered signal, finally add a fixed threshold value  $thr_{st}$  to the normalized signal  $PSD_{nol}$ . As shown in the figure below.

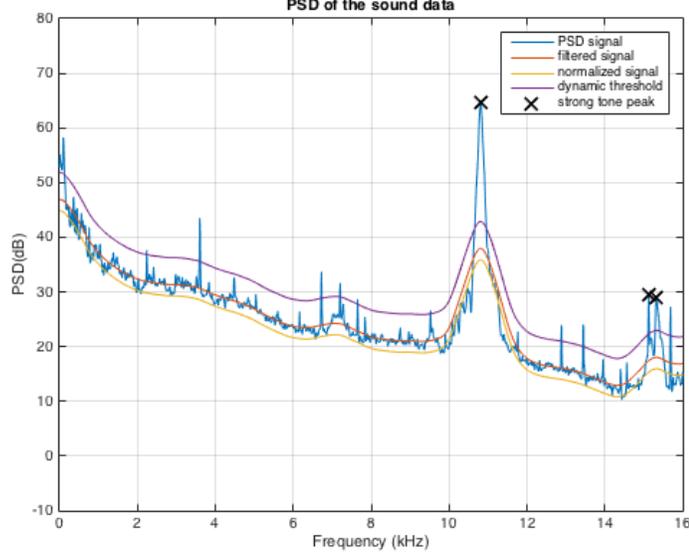


Fig. 2.3. Example of a PSD signal with dynamic threshold

If the window size for the moving average filter is  $M$ , then

$$PSD_{avg}[i] = \frac{1}{M} \sum_{j=0}^{M-1} PSD_{dB}[i + j] \quad (2.3)$$

The difference signal  $PSD_{diff}$  is

$$PSD_{diff} = PSD_{dB} - PSD_{avg} \quad (2.4)$$

Sort the  $PSD_{diff}$  signal in increasing order, and get  $PSD_{sdiff}$ . If the length of the signal is  $L$ , then the normalization factor  $nol$  is calculated as

$$nol = \frac{2}{L} \sum_{i=1}^{L/2} PSD_{sdiff}[i] \quad (2.5)$$

Then the normalized signal and the dynamic threshold are calculated by

$$PSD_{nol} = PSD_{avg} + nol \quad (2.6)$$

$$PSD_{thr} = PSD_{nol} + thr_{st} \quad (2.7)$$

Figure (5) shows an example of a strong tone in Figure(4), where the red and green arrows indicate the relative and absolute amplitudes of the strong tone frequency  $f_{st}$ , and the purple arrow is the tone width. Equations for the relative and absolute amplitudes are

$$amp_{abs}[f_{st}] = PSD_{dB}[f_{st}] \quad (2.8)$$

$$amp_r[f_{st}] = PSD_{dB}[f_{st}] - PSD_{thr}[f_{st}] \quad (2.9)$$

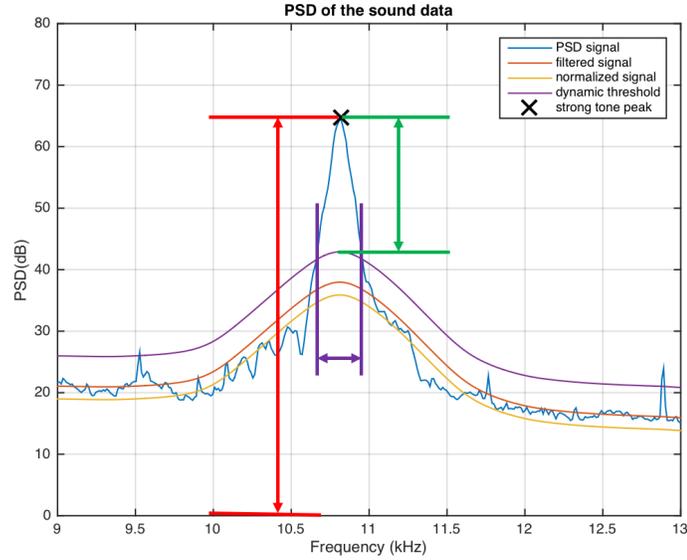


Fig. 2.4. Example a strong tone signal

### 2.3 isolate the narrow-band strong tone signal

A band-pass filter (Butterworth filter) is set in this step in order to isolate the strong tone signal  $f_{st}$  from the original audio signal [5]. The effects of the band-pass filter applied on a audio signal are shown in the figures below.

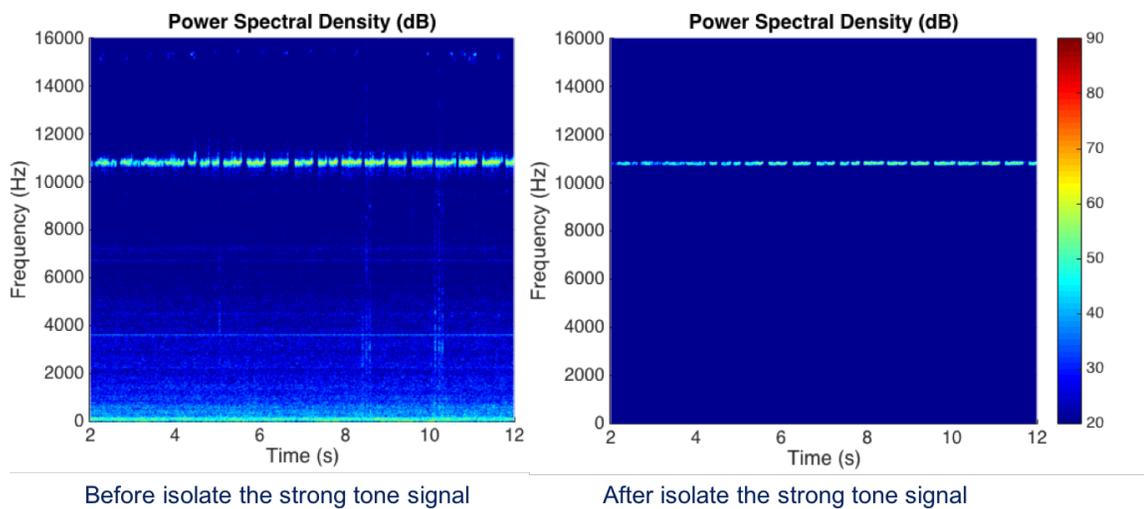


Fig. 2.5. Isolating strong tone signal effect 1

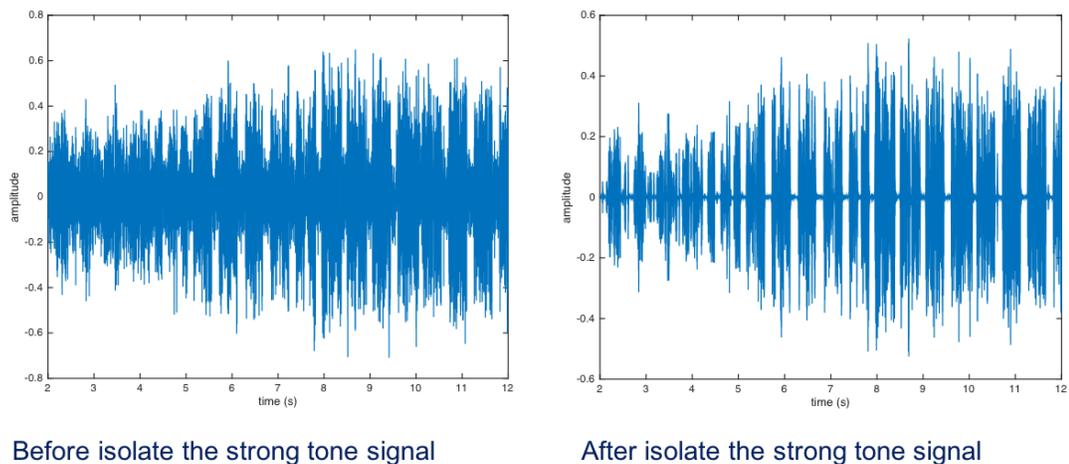


Fig. 2.6. Isolating strong tone signal effect 2

## 2.4 compute signal envelope

Get the envelope signal of the band-pass filtered audio signal by applying a Hilbert filter to the filtered signal and calculating the instantaneous amplitude of the signal [5].

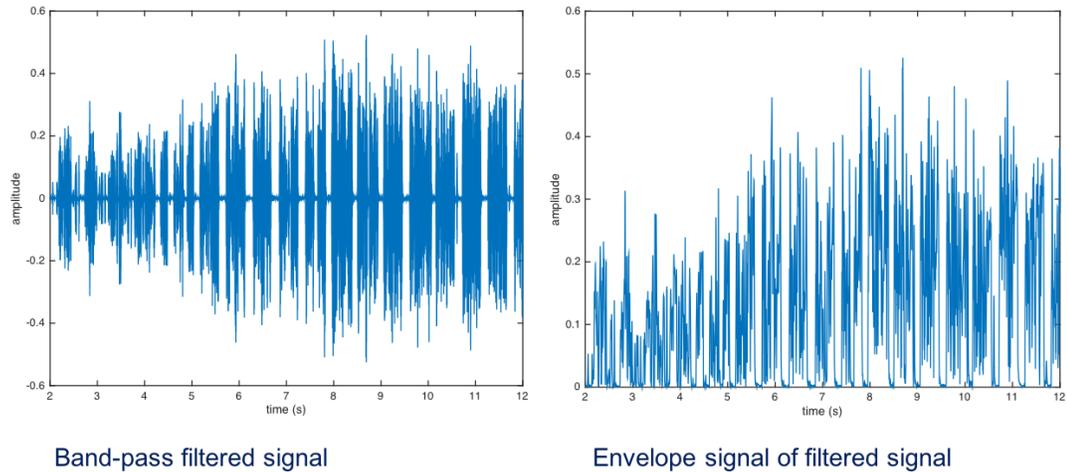


Fig. 2.7. Example of envelope signal

## 2.5 calculate the modulation depth

Apply the FFT on the envelop signal of the strong tone signal to get the PSD of the modulation signal in  $Pa^2/Hz$ , denotes as  $MP$ . Then  $MPSD_{dB}$  is this PSD of the modulation signal in dB calculated by equation (2) showed previously.

The problem of  $MPSD_{dB}$  signal is that the peak values are not obvious enough to find. Therefore, the modulation depth  $Amp_{depth}$  signal is introduced to solve that. The modulation depth signal is the ratio of the modulation amplitude to the level of the carrier frequency's amplitude. It defined as

$$Amp_{depth} = \frac{MP}{amp_{fs}} \quad (2.10)$$

where  $amp_{fs}$  is the absolute amplitude  $amp_{abs}$  of the strong tone frequency in  $Pa^2/Hz$ . Calculate PSD in  $Pa^2/Hz$  is the reverse function of equation (2)

$$amp_{fs} = 4 * 10^{-10} * 10^{amp_{abs}/10} \quad (2.11)$$

The example of PSD signal in dB and the modulation depth signal of the modulation signal is shown in figure below.

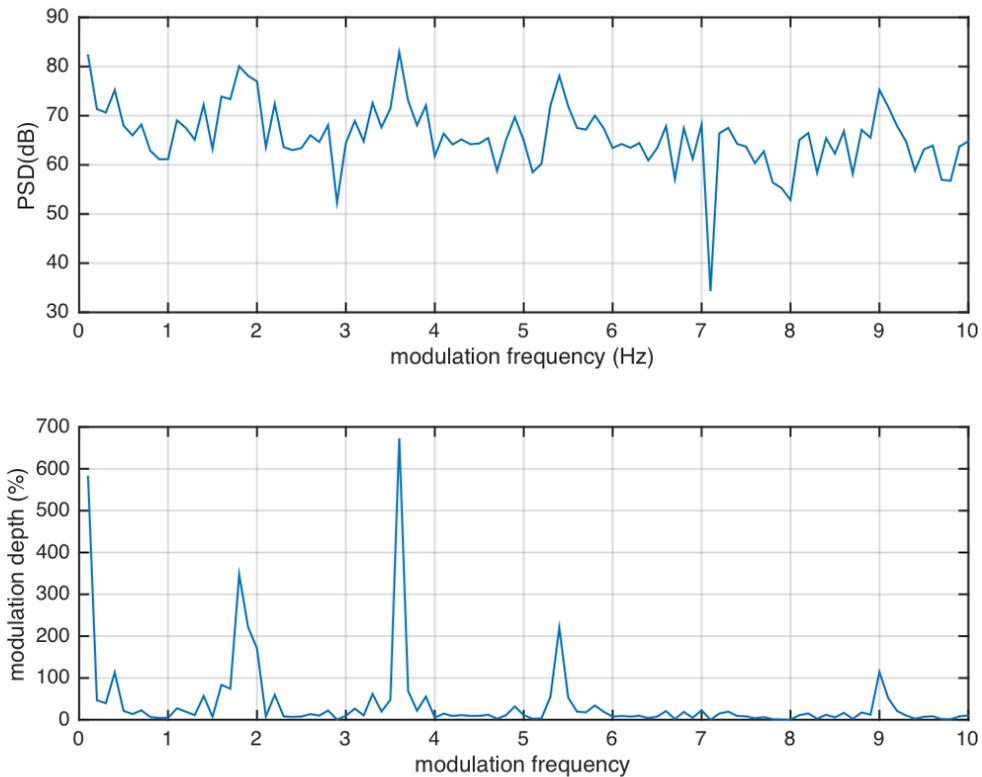


Fig. 2.8. Example of PSD signal of modulation frequency

## 2.6 modulation frequency selection

Each modulation feature includes two parts, the modulation frequency peak and its modulation depth. The method used to extract the modulation features of a strong tone signal here is the same used in Section 2.2 strong tone selection, but

with different averaging filter width and fixed threshold value  $thr_m$ . Example of modulation features of a strong tone signal is shown in the figure below.

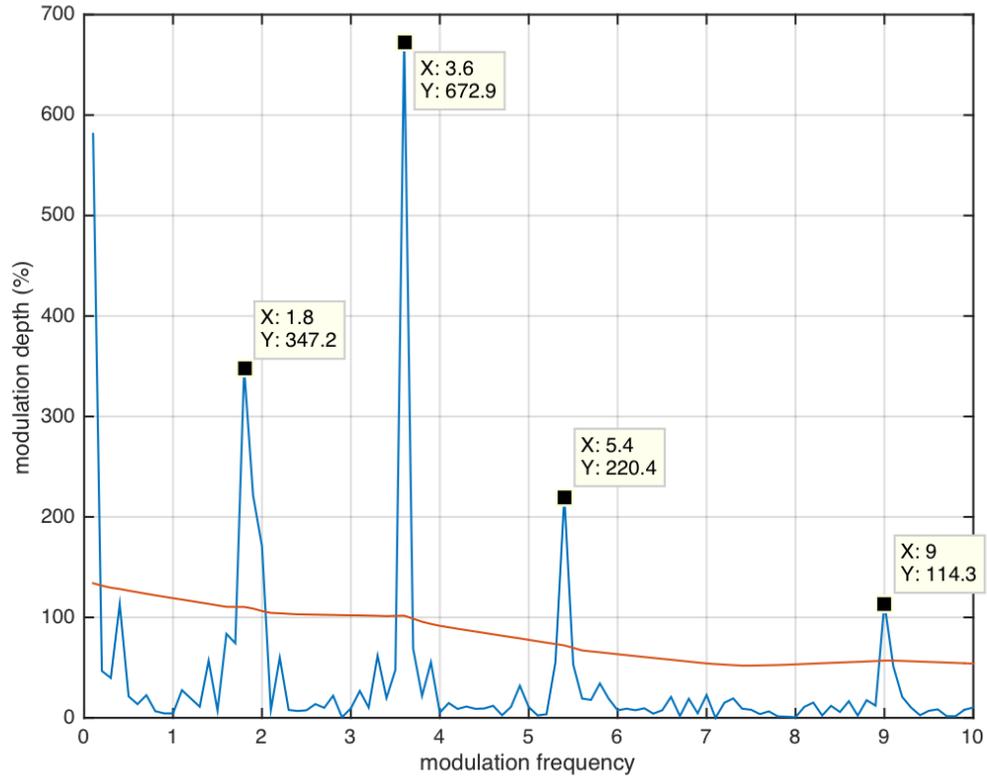


Fig. 2.9. Example of modulation features of a strong tone frequency  $f_{st}$

### 3. HARMONIC SCRUBBER

Fundamental frequency of a harmonic frequency group might hide between two nearby modulation frequencies due to the modulation frequency resolution. Therefore, the harmonic scrubber function is needed before the classifier matching the MFV (modulation features) to the MF (mechanical features) in the database and after the detector extracted FV (feature vectors) from the original sound signal.

The harmonic scrubber is designed to identify all the harmonic frequency groups, find the most convincing fundamental frequencies, and calculate the group power for each harmonic group. The modified feature vector will include the fundamental frequencies and their group powers for each SFV (strong tone feature), and that fundamental frequency is the one actually related to the rotating mechanical part.

The underlying assumption for applying the harmonic scrubber is that the number of MFV of a SFV is larger or equal to 3, otherwise there is no need to apply the harmonic scrubber for the SFV.

#### 3.1 Find all possible harmonic groups

For a harmonic group whose fundamental frequency is  $f_0$ , then the  $i_{th}$  harmonic frequency in that group should be

$$f_i = k * f_0, \quad \text{where } k \in \mathbb{N} \quad (3.1)$$

Given a set of  $n$  modulation frequencies  $\Omega$  with resolution equals to  $\Delta f$ , the idea is to think the above equation reversely in order to find the unknown fundamental frequency  $f_0$ . If the  $i_{th}$  frequency in the modulation frequency set  $\Omega$  belongs to a

harmonic group, then the fundamental frequency  $f_0$  of that harmonic group must be one of the possible fundamental frequencies  $f'_0$  calculated by

$$f'_0 = \frac{f_i}{k}, \quad \text{where } k \in \mathbb{N} \text{ and } k \in [2, \frac{f_i}{\Delta f} - 1] \quad (3.2)$$

Based on the idea above, all possible harmonic groups can be found following the procedures shown in the diagram below.

Color Index:

Input and Output

Part 1-1

Part 1-2

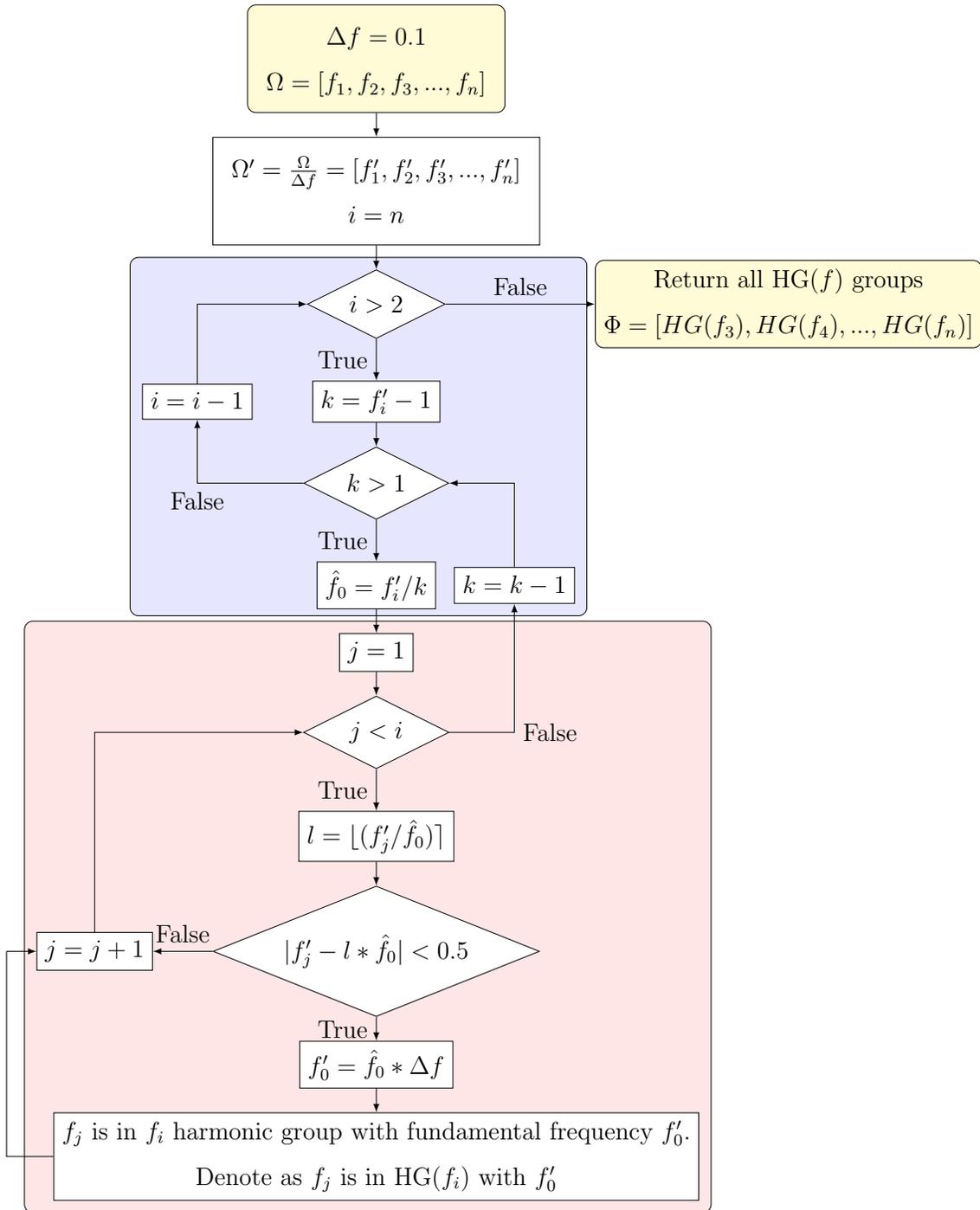


Fig. 3.1. Diagram of Harmonic Scrubber Part 1

### 3.1.1 Find all possible fundamental frequencies

Because of the relationship between the fundamental frequency  $f_0$  and harmonic frequencies is integral multiple, the modulation frequency set  $\Omega$  need to be modified as  $\Omega'$  with resolution to be 1 first. Next step is to calculate all modified possible fundamental frequencies  $\hat{f}_0$  starting from the largest frequency  $f'_n$  in the set  $\Omega'$  by equation (2), and track how many modulation frequencies in the set  $[f'_1, \dots, f'_{i-1}]$  (denote this set as  $\omega'_i$ ) are the harmonics of the possible fundamental  $\hat{f}_0$ .  $\omega'_i$  is a subset of  $\Omega'$ . The relationship between the  $f'_0$  and  $\hat{f}_0$  is

$$f'_0 = \hat{f}_0 * \Delta f \quad (3.3)$$

The equation to calculate the modified possible fundamental frequency  $\hat{f}_0$  is

$$\hat{f}_0 = \frac{f'_0}{\Delta f} = \frac{f_i/k}{\Delta f} = \frac{f_i/\Delta f}{k} = \frac{f'_i}{k}, \quad \text{where } k = 2, 3, \dots, f'_i - 1 \quad (3.4)$$

The modified fundamental frequency  $\hat{f}_0$  calculated outside this k range for the  $i_{th}$  modified modulation frequency  $f'_i$  is meaningless. If  $k=1$ , then  $\hat{f}_0 = \frac{f'_i}{k} = f'_i$ . Since the algorithm is starting from the largest frequency, all elements in the set  $[f'_1, \dots, f'_{i-1}]$  are smaller than  $f'_i$  and can not be the multiple of  $f'_i$ , thus this  $\hat{f}_0$  can not be a fundamental frequency. Additionally, if  $k = f'_i$ , then  $\hat{f}_0 = \frac{f'_i}{k} = 1$ , which is the resolution for the modified modulation frequency set  $\Omega$ , that means the whole set formed one large harmonic group with fundamental frequency equals to the resolution. This is not helpful for identifying the defective machine part.

### 3.1.2 Find the harmonic group for each possible fundamental frequency

After the modified possible fundamental frequency  $\hat{f}_0$  is calculated for  $f'_i$ , the next step is to find the harmonics from the set  $\omega'_i$ , which is  $[f'_1, \dots, f'_{i-1}]$ .

If  $f'_j$  in  $\omega'_i$  is a harmonic of the modified possible fundamental frequency  $\hat{f}_0$ , then there exist an integer  $l$  that satisfied

$$f'_j \cong l * \hat{f}_0 \quad (3.5)$$

Use the equation(5) reversely as before, let  $l = \lfloor f'_j / \hat{f}_0 \rfloor$ , if  $f'_j \in [l * \hat{f}_0 - 0.5, l * \hat{f}_0 + 0.5]$ , then  $f'_j$  belongs to the harmonic group with fundamental frequency equals to  $\hat{f}_0$ . The frequency range is depended on the resolution of the modified frequency set  $\Omega'$ .

Repeat the above process until all the harmonics for  $\hat{f}_0$  are found, and those harmonics formed a harmonic group. Since the modified potential fundamental frequency  $\hat{f}_0$  is calculated based on the  $i_{th}$  element  $f'_i$  in  $\Omega'$ , this harmonic group is denoted as  $HG(f_i)$  with  $f'_i$ , where the relationship of  $f'_i$  and  $\hat{f}_0$  is in equation (3). Here is an example of what  $HG(f_i)$  might looks like

Notes:

Table 3.1.

Example of  $HG(f_i)$

$f'_0$	number of frequencies in the HG	HG
0.758	3	[0.8, 1.5, 3.8]
0.7	4	[0.7, 1.4, 3.5, 7.0]

HG is the abbreviation of the harmonic group.

$\lfloor i \rfloor$  is the nearest integer of value i.

$|i|$  is the absolute value of i.

### 3.2 Derive the estimated fundamental frequencies

Once all possible harmonic groups are stored in set  $\Phi$  with their  $f'_0$ , the next step is to find the estimated fundamental frequencies  $f_{0-est}$ .

If the modulation frequency set  $\Omega$  contained at least one harmonic group, the majority frequencies in  $\Omega$  should belong to a harmonic group. Then  $f'_0$  that close to the actual fundamental frequency  $f_0$  should have the following characteristics: 1) If it shows in  $HG(f_i)$ , then it should generate a large size harmonic group from  $\omega_i$ . Here the large size HG is defined as the groups that contained at least  $\lceil i/2 \rceil$  elements including  $f_i$  itself. 2) It relatively appears more frequent as a possible fundamental frequency in  $\Phi$ . If the  $f'_0$  is close to  $f_0$ , then most of the  $HG(f)$  set should contain this  $f'_0$ , and each  $f'_0$  only able to appear once for each  $HG(f)$ .

The procedure is shown in the diagram below, where the part 2-1 corresponding to the first characteristic of  $f'_0$  that close to  $f_0$ , and the part 2-2 corresponding to the second characteristic.

Note:

PPF is the abbreviation of potential fundamental frequencies.

$\lceil i \rceil$  is the smallest integer greater than or equal to  $i$ .

Color Index:

Input and Output

Part 2-1

Part 2-2

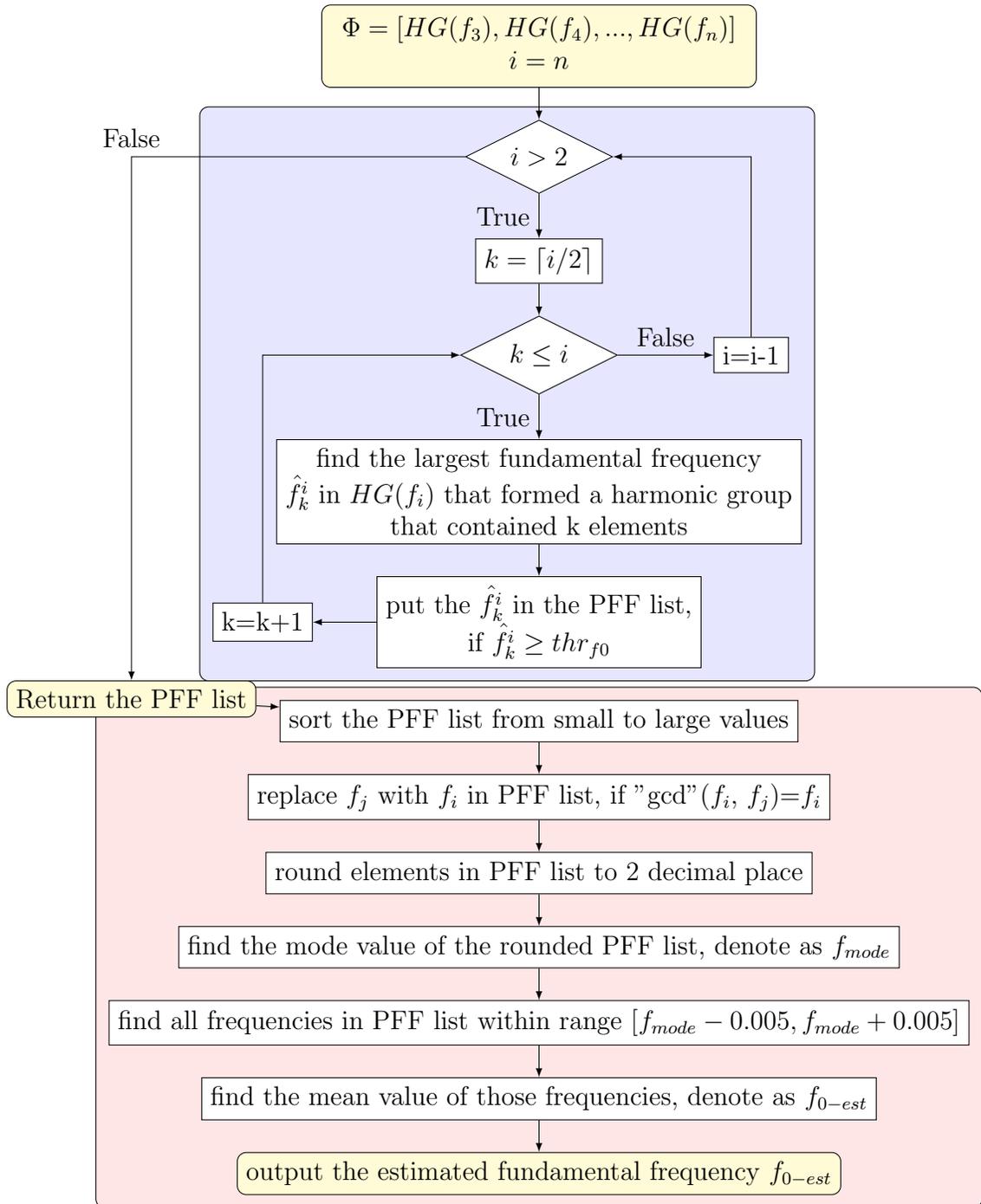


Fig. 3.2. Diagram of Harmonic Scrubber Part 2

### 3.2.1 Form the PFF list

The potential fundamental frequency list (PFF list) contains the largest potential fundamental frequencies  $\hat{f}_k^i$  in each  $HG(f_i)$  that generated a harmonic group with  $k$  elements, where  $k = \lceil i/2 \rceil, \lceil i/2 \rceil + 1, \dots, i$ .

Here is an example for  $HG(f_5)$ . Since  $i = 5$  in this case, a large size harmonic group needs to contain at least  $\lceil i/2 \rceil = \lceil 5/2 \rceil = 3$  elements. The red boxes indicate the potential fundamental frequencies that formed a large size harmonic group, and  $f'_0$  in each red box will form a harmonic group with the same size. The red stars indicate the largest potential fundamental frequencies  $\hat{f}_k^5$  in each box, and those potential frequencies will be stored in the PFF list. In this case,  $\hat{f}_3^5$ ,  $\hat{f}_4^5$ , and  $\hat{f}_5^5$  represented by the three stars will be put in the PFF list.

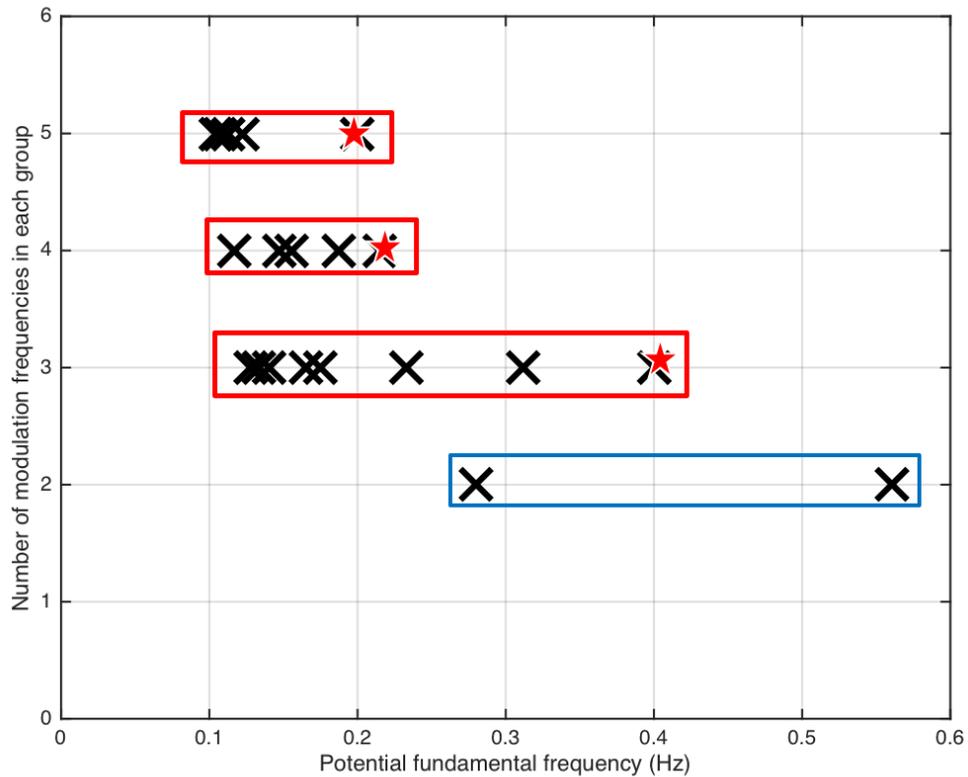


Fig. 3.3. Example of a set of all possible harmonic groups  $HG(f_5)$

### 3.2.2 Estimate the fundamental frequency $f_{0-est}$ from PFF

The PFF list contains the most convincing  $f'_0$  from each  $HG(f)$ , and the scrubber need to find the most frequent one among them and estimate the fundamental frequency in this step.

Remove the harmonics among PFF list is essential before finding the mode possible fundamental frequencies. If one potential fundamental frequency  $f_j$  in the PFF list is similar to the multiple of another  $f_i$  (denote this as "gcd"  $(f_i, f_j) = f_i$ ), then replace the larger one with the smaller one. The  $thr_{gcd}$  is determined by the resolution  $\Delta f$  of  $\Omega$ , that is  $thr_{gcd}$  always has one more decimal place than  $\Delta f$ .

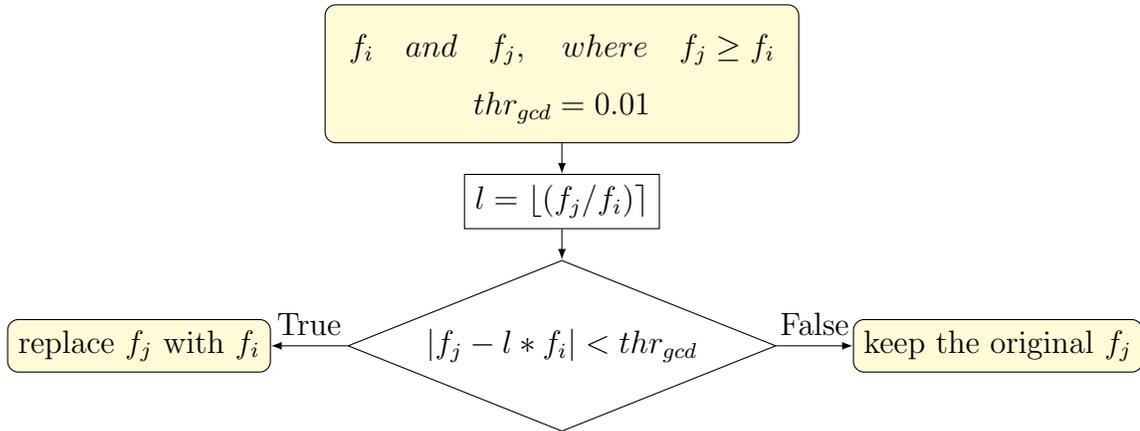


Fig. 3.4. Diagram of "gcd"

Next is to round the modified PFF list and find the mode value  $f_{mode}$ . The accuracy of the estimated fundamental frequency is depends on the frequency resolution  $\Delta f$ , thus elements in PFF list needs to round to one more decimal place than  $\Delta f$ . Then  $f_{0-est}$  is the mean value of all frequencies in PFF that in the range of  $[f_{0-est} - thr_{gcd}/2, f_{0-est} + thr_{gcd}/2]$ . More than one  $f_{0-est}$  can be found by setting the number of the estimated fundamental frequencies  $num_{est}$  wanted, finding the top  $num_{est}$  mode frequencies in rounded PFF, and repeating the procedures to find  $f_{0-est}$ .

### 3.3 Calculate the group power for given $f_{0-est}$

Once all estimated fundamental frequencies are calculated from previous steps, next is to calculate the group power for each harmonic group represented by estimated fundamental frequency. This procedure is shown below, and it is similar to the part 1-2.

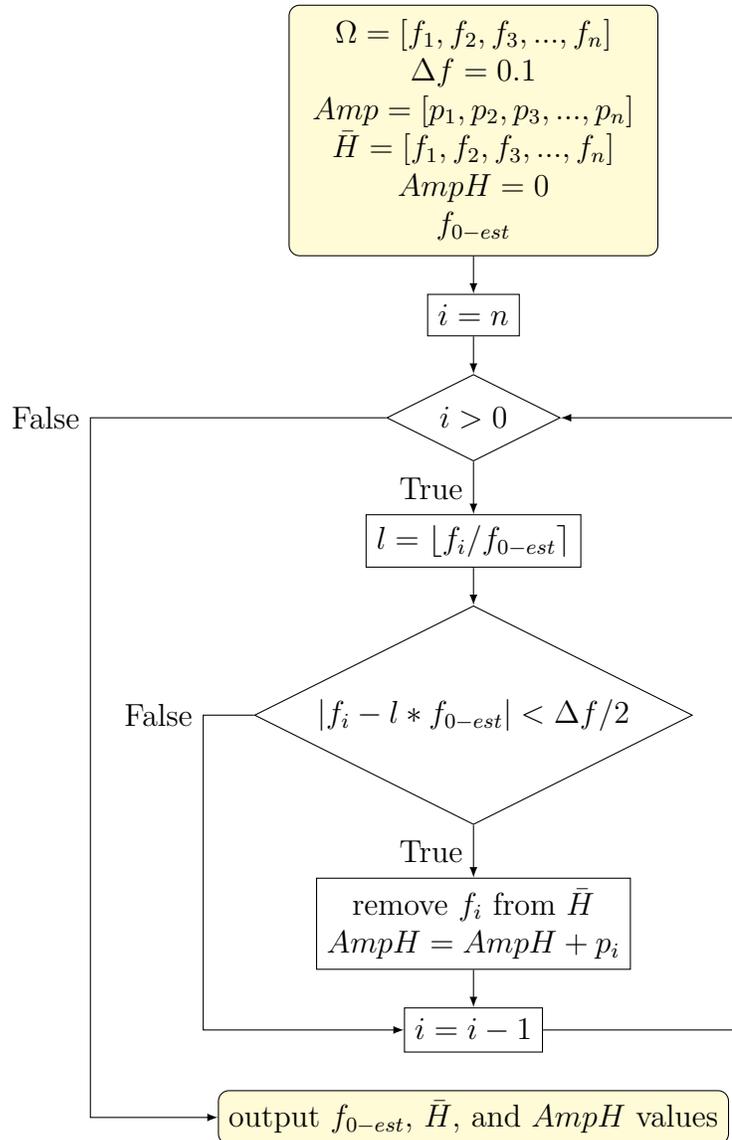


Fig. 3.5. Diagram of Harmonic Scrubber Part 3

Notes:

$\bar{H}$  is the set of all non-harmonic frequencies in  $\Omega$

AmpH is the group power of the harmonic group

## 4. RESULT

The Hewlett-Packard Company provides some audio signals recorded from different broken LaserJet printers to test the detector and harmonic scrubber algorithm. Since Xue [5] already proves the accuracy of the detector in his paper, the main focus in this section will be the reliability of the harmonic scrubber. Here are the results.

The strong tone frequencies and their amplitude for a given sound record are

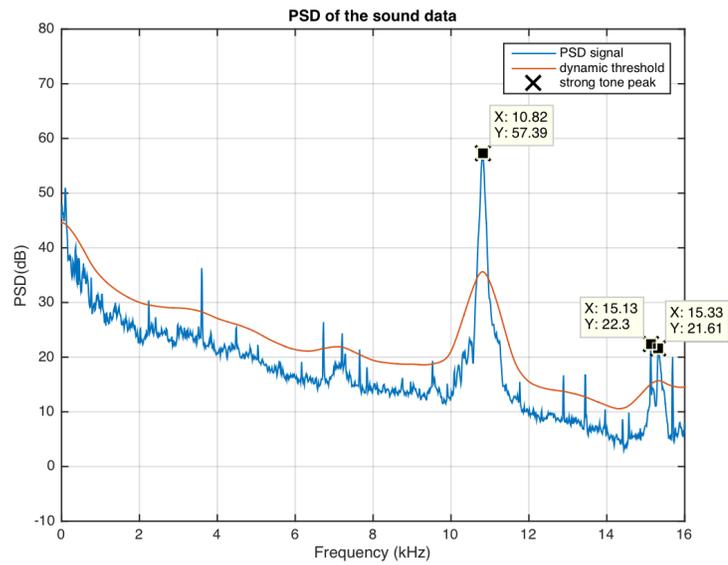


Fig. 4.1. Result of strong tone features' elements

Then the modulation features for each strong tone signal are

Table 4.1.  
Result of strong tone features' elements

$f_{st}$ (Hz)	$amp_r$ (dB)	$amp_{abs}$ (dB)	$pk_{width}$ (pt)
10816	28.7735	57.3915	26
15129	14.2143	22.3004	3
15328	12.9139	21.6073	9

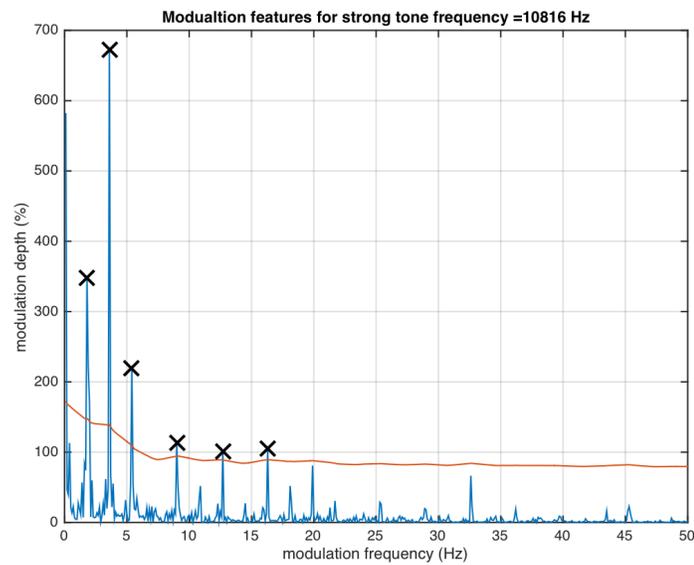


Fig. 4.2. Result of modulation features for strong tone feature 1

Table 4.2.  
Result of modulation features of strong tone feature 1

$f_m$ (Hz)	$amp_m$ (%)
1.8	347.2287
3.6	672.9265
5.4	220.4013
9	114.2705
12.7	100.8557
16.3	105.2616

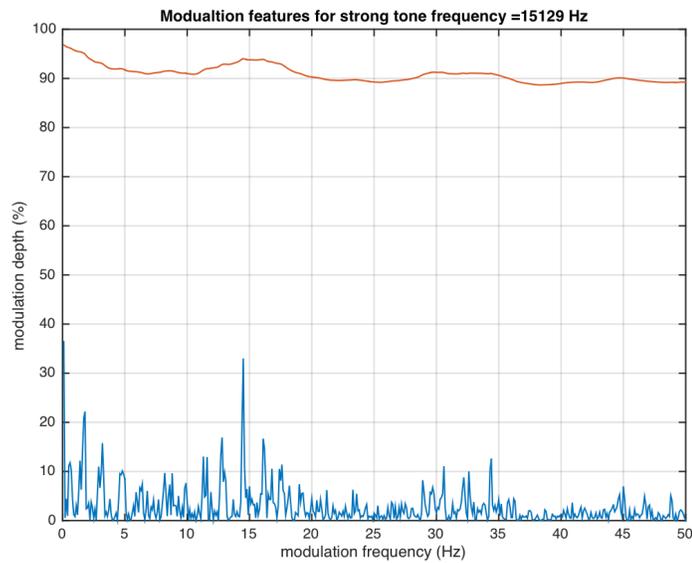


Fig. 4.3. Result of modulation features for strong tone feature 2

Table 4.3.  
Result of modulation features of strong tone feature 2

$f_m$ (Hz)	$amp_m$ (%)
0	0

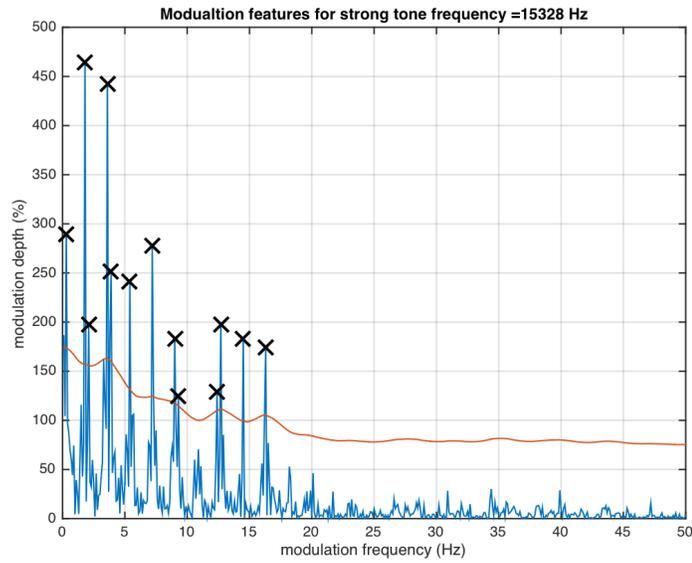


Fig. 4.4. Result of modulation features for strong tone feature 3

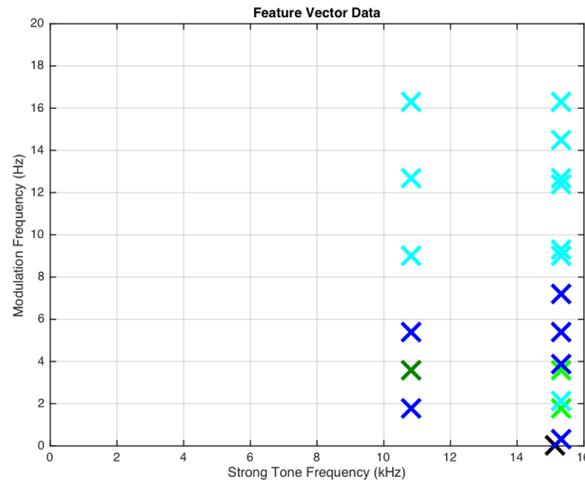
The feature vector detected from the revised detector is shown in the figure below, where the color of "X" marks indicate the modulation depth level and the color index is shown in the bottom table.

Table 4.4.  
Result of modulation features of strong tone feature 3

$f_m$ (Hz)	$amp_m$ (%)
0.3	289.8306
1.8	464.0191
2.1	197.6426
3.6	442.4907
3.9	251.2937
5.4	240.8711
7.2	277.7605
9	183.3588
9.3	124.1874
12.4	128.2945
12.7	197.1082
14.5	183.0358
16.3	173.6813

Table 4.5.  
Result of modified modulation features of strong tone feature 1

$f_m$ (Hz)	$amp_m$ (%)
1.8	1354.8
1.8127	1446.7



Modulation depth range	0	[90, 200)	[200, 400)	[400, 600)	[600, 800)	[800, 1000)	[1000, ∞)
Number of modulation features	1	10	6	2	1	0	0

Fig. 4.5. Result of feature vector before applying HS

Then apply the harmonic scrubber to the modulation frequencies and their depth for each strong tone signal. Modulation frequencies [1.8, 3.6, 5.4, 9] belong to the harmonic group with  $f_0 = 1.8$ , and modulation frequencies [1.8, 3.6, 5.4, 12.7, 16.3] belong to the harmonic group with  $f_0 = 1.8127$ .

Modulation frequencies [0.3, 1.8, 2.1, 3.6, 3.9, 5.4, 7.2, 9, 9.3] belong to the harmonic group with  $f_0 = 0.3$ , and modulation frequencies [0.3, 1.8, 2.1, 3.6, 3.9, 5.4, 7.2, 9, 9.3, 12.4, 12.7, 14.5, 16.3] belong to the harmonic group with  $f_0 = 0.2583$ .

Table 4.6.  
Result of modified modulation features of strong tone feature 3

$f_m$ (Hz)	$amp_m$ (%)
0.3	2471.5
0.2583	3153.6

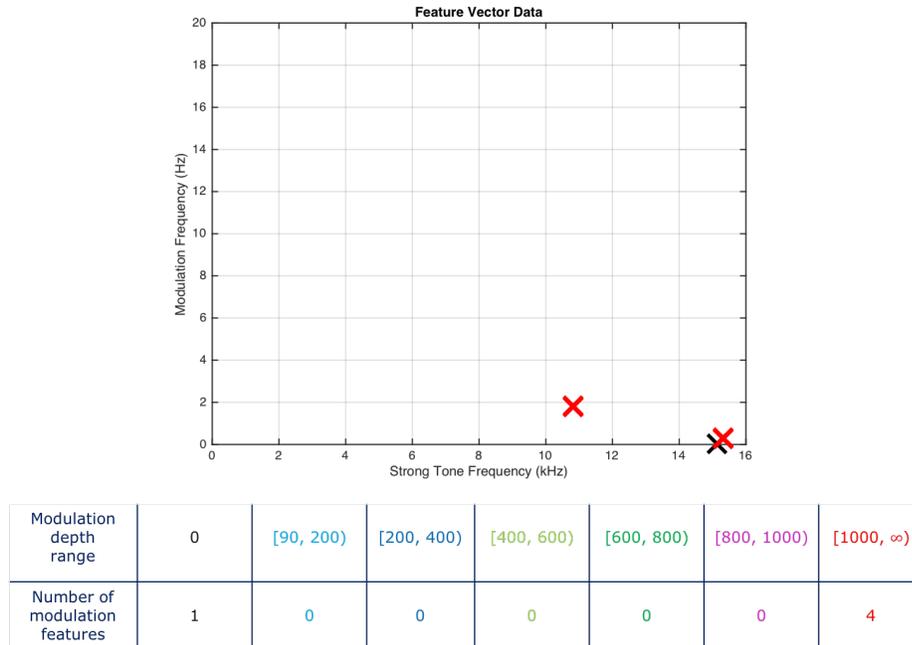
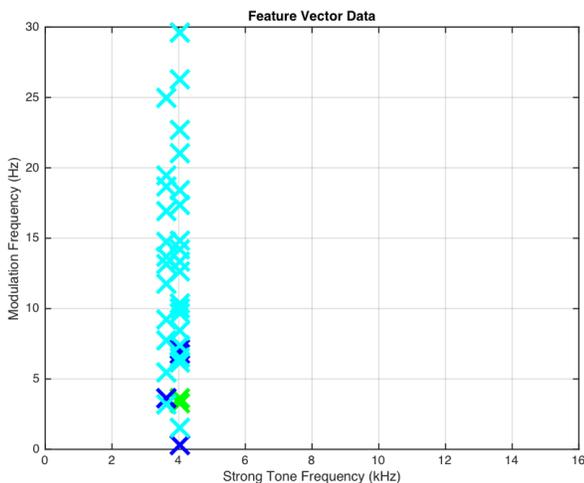


Fig. 4.6. Result of feature vector after applying HS

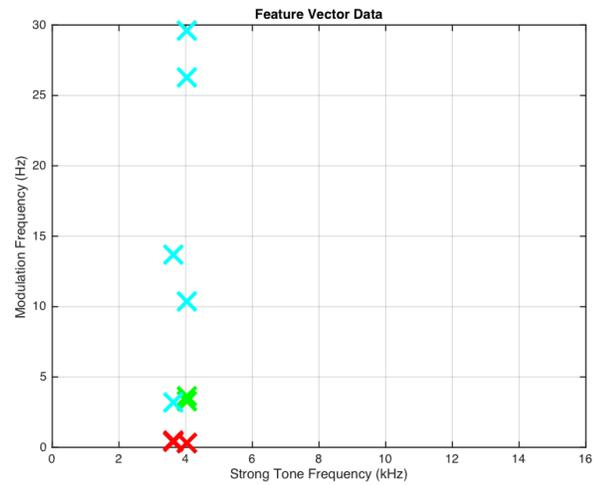
From the example shown above, the total number of modulation features reduced from 20 to 5 and reduced 75% modulation features.

Here is another example of the feature vector extracted from a different sound record before and after applying harmonic scrubber.



Modulation depth range	0	[90, 200)	[200, 400)	[400, 600)	[600, 800)	[800, 1000)	[1000, ∞)
Number of modulation features	0	30	4	2	0	0	0

Fig. 4.7. Result of feature vector before applying HS



Modulation depth range	0	[90, 200)	[200, 400)	[400, 600)	[600, 800)	[800, 1000)	[1000, ∞)
Number of modulation features	0	5	0	2	0	0	4

Fig. 4.8. Result of feature vector after applying HS

The total number of modulation features reduced from 36 to 11 and reduced 69.44% modulation features.

## 5. SUMMARY AND FUTURE WORK

The self-detection method was invented to reduce the resources the company put on the after-sell service by improving the efficiency of diagnosing machinery faults.

The detector embedded in the mechanical device could extract the audio information related to the defective component inside of the device. The current detector is more accurate and reliable on detecting the strong tone frequency peaks and their modulation features comparing to the previous generation. The harmonic scrubber was first implemented to lessen a large number of harmonic modulation frequencies by locating the fundamental frequency of the harmonic group. Thereby it reduces the amount of work to put on matching harmonics with the rotational speed of a machinery component.

Next step for the self-diagnose method could be reducing the computation of the harmonic scrubber or improving the matching efficiency and intelligence of the classifier part.

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## REFERENCES

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