INTERFERENCE MANAGEMENT IN DYNAMIC WIRELESS NETWORKS

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ABSTRACT

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Interference management is necessary to meet the growth in demand for wireless data services. The problem was studied in previous work by assuming a fixed channel connectivity model, while network topologies tend to change frequently in practice.

The associations between cell edge mobile terminals and base stations in a wireless interference network that is backed by cooperative communication schemes is investigated and association decisions are identified that are information-theoretically optimal when taking the uplink-downlink average. Then, linear wireless networks are evaluated from a statistical point of view, where the associations between base stations and mobile terminals are fixed and channel fluctuations exist due to shadow fading. Moreover, the considered fading model is formed by having links in the wireless network, each subject independently to erasure with a known probability.

Throughout the information theoretic analysis, it is assumed that the network topology is known to the cooperating transmitting nodes. This assumption may not hold in practical wireless networks, particularly Ad-Hoc ones, where decentralized mobile nodes form a temporary network. Further, communication in many next generation networks, including cellular, is envisioned to take place over different wireless technologies, similar to the co-existence of Bluetooth, ZigBee, and WiFi in the 2.4 GHz ISM-Band. The competition of these wireless technologies for scarce spectrum resources confines their coexistence. It is hence elementary for collaborative interference management strategies to identify the channel type and index of a wireless signal, that is received, to promote intelligent use of available frequency bands. It is shown that deep learning based approaches can be used to identify interference between the wireless technologies of the 2.4 GHz ISM-Band effectively, which is compulsory for identifying the channel topology. The value of using deep neural network architectures such as CNN, CLDNN, LSTM, ResNet and DenseNet for this problem of Wireless Channel Identification is investigated. Here, the major focus is on minimizing the time, that takes for training, and keeping a high classification accuracy of the different network architectures through band and training SNR selection, Principal Component Analysis (PCA) and different sub-Nyquist sampling techniques. Finally, a number theoretic approach for fast discovery of the network topology is proposed. More precisely, partial results on the simulation of the message passing model are utilized to present a model for discovering the network topology. Specifically, the minimum number of communication rounds needed to discover the network topology is examined. Here, a single-hop network is considered that is restricted to interference-avoidance, i.e., a message is successfully delivered if and only if the transmitting node is the only active transmitter connected to its receiving node. Then, the interference avoidance restriction is relaxed by assuming that receivers can eliminate interference emanating from already discovered transmitters. Finally, it is explored how the network size and the number of interfering transmitters per user adjust the sum of observations.

1. INTRODUCTION

The rise of new technologies push current wireless communication systems to the edge of performance, while researchers worldwide agree, that 5G communications offer the potential to overcome the aggressive demand on high speed data transmission and low latency networks.

With the emerging importance of Cloud Radio Access Networks (C-RAN) ([38]-[43]) the focus is on fundamental frameworks for the fifth generation of cellular communications and hence Cloud computing , as well as cooperative communication (also known as Coordinated Multi-Point or CoMP) will heavily contribute in the design of such networks. The main challenge is to identify information-theoretical limits of these frameworks.

Due to the superposition and broadcasting characteristics of the wireless medium, interfering signals confine the user performance for communication in wireless networks. The focus is on understanding the limits of dynamic networks with interference characteristics and the resulting coding schemes. The term *Dynamic Interference Management* is being introduced, which covers learning the dynamics, diminishing overhead and delay of these networks.

To understand the relations between interference management and the dynamics of the simplified abstraction of linear networks, the problem of interference management is analyzed first in an uplink-downlink scenario in the context of a non-dynamic linear interference network. Then, the Wyner's interference channel [51] is investigated from a statistical viewpoint, that is understanding the influence of long term channel fluctuations due to shadow fading. Finally, the applicability of deep neural networks is discussed for the Channel Identification problem and hence these empirical results are put in context of the information-theoretic framework. The main interest in this last part of the work is to formulate an exemplary approach for discovering the network topology of dynamic networks.

First, the information-theoretic models are targeted by interference networks that consist of K Base Station (BS)-Mobile Terminal (MT) pairs, where each BS is connected to the MT carrying the same index as well as $L \geq 1$ following MTs. The value of the connectivity parameter L is fixed as the per user Degrees of Freedom (puDoF) is analyzed for large networks of size K, as K goes to infinity. A cloud-based controller, that has an overall view of the network, determines the associations of a MT with N_c BSs. It is important to mention, that N_c, the number of asssociations, is subject to be mostly constrained by the backhaul limited capacity. Also, full DoF is achieved by the constraint $N_c > L$ in the uplink, since each MT is associated with the L+1 BSs connected to it, and hence, a simple message passing approach - through the backhaul - can eliminate all interference. A BS can transmit its message in the downlink or have its decoded message in the uplink only if it is associated with a BS. Relations between cell edge mobile terminals and base stations are analyzed. Here, these relations maximize the average rate across uplink and downlink sessions, respectively, while it is allowed that one mobile terminal is associated with more than one base station and cooperative transmission and reception schemes are used between base stations in the downlink and uplink sessions, correspondingly. Optimal decisions for these associations consider the whole network topology due to a cloud-based controller, while the purpose is to maximize a sum rate function.

Cloud-based CoMP communication is an established technology for collaborative interference management, that in essence could elevate the rates of cell edge users (see [37] and [44] for an overview of CoMP) with transmission and reception schemes that are custom tailored to the network topology in the downlink and uplink of cellular networks. The effect of cooperation between transmitters, as well as between receivers (CoMP transmission and reception) on an information-theoretical model was studied in [45]. Using tools from Algebraic Geometry, it was shown for a network with K pairs of transmitters and receivers (users), that full Degree of Freedom (DoF) can be achieved when M_t transmitters carrying a specific message and M_r received signals are used to decode this message, satisfy the lower bound condition $M_t + M_r \geq K + 1$.

Alternative concepts based on massage passing between base stations were applied to cooperation in both downlink and uplink in [46]. In the downlink, base station transmitters share quantized analog transmit signals among each other. In the uplink however, sharing of decoded messages occurs from one base station receiver to another. Here, the objective is that sharing information about multiple messages occurs from one transmitter to another with the expense of distributing only one whole message, if only information is shared to eliminate interference, which is produced by the messages at undesignated receivers, by applying dirty paper coding as in [48]. Furthermore, the same backhaul infrastructure can be used supportively for schemes in the downlink and uplink scenario due to the duality proposed in [46].

While [12] describes the reception scheme of CoMP, which demands on distributing analog received signals over the backhaul, the cooperative transmission through dirty paper coding introduced in [46] demands on distributing quantized analog signals by a backhaul, where a delay is experienced that scales with the network size.

Regarding the uplink and downlink scenario, only sharing of digital message information over the backhaul is taken into account. Due to sharing whole messages over the backhaul, the CoMP transmission archetype of [12] is obtained and the message passing decoding archetype of [46] as special cases of the setting.

In the uplink, the puDoF of message passing decoding of locally connected interference networks is characterized. Moreover, for both, the uplink and downlink, the obstacle is discussed of jointly optimizing the assignment of messages over the backhaul to maximize the average puDoF. Given there exist an association of each base station with N_c mobile terminals, while an association has to exist, if in in either the downlink or the uplink, a BS *uses* a MT's message. The objective of using a message is either for message delivering in downlink, message decoding in uplink, or for cancelling interference.

It will be demonstrated how the results for the uplink scenario settle the average puDoF problem when $N_c \leq \frac{L}{2}$. Then the uplink scheme to the optimal uplink-only scheme is fixed when $N_c > L$, such that an association exists between each mobile terminal and its connected $L+1$ base stations, and with respect to this restriction, the optimal downlink scheme is described.

Finally, the optimality of the proposed schemes are proved for the linear interference network introduced by Wyner [51] (when L=1) from an information-theoretical perspective.

The second part of the research is on developing an information-theoretic framework for analyzing interference networks with link block erasures that capture deep fading conditions. Statistical knowledge about the changing network connectivity has to be considered for decision-making of transmitter and receiver selection, fractional reuse, CoMP transmission and reception schemes, and interference alignment and zero-forcing as these choices can differ significantly from decisions regarding any specific realization of the network. As this is a first step towards developing this framework, the problem of assigning messages to transmitters is adressed ensuring achievability of optimal average performance in a single-hop dynamic linear interference network.

Despite the traditional approach to estimate the channel coefficients at the receivers, that is to transmit known pilot signals and feed them back to the transmitters (see e.g. [16], [17] and [18]), the overhead of the estimation and communication of channel coefficients is neglected, in order to derive insights relevant to the remaining design parameters of the coding scheme.

Literature offers other methods for analyzing the influence of ambiguity of the network topology.

A packet repetition coding scheme is introduced in [19] to profit from previous receptions in a two-user interference channel, which is subject to collisions. A two-user channel that is constrained by a bursty interference model is introduced in [20], where the duty of feedback is analyzed and demonstrated to describe a symmetric deterministic channel's capacity region.

The analyzed problem offers similarities with the model introduced in [20] due to the existence of interference, that is based on Bernoulli random state.

Nonetheless, the studied problem takes large networks and the problem of assigning messages to transmitters to possibly profit from cooperative transmission is taken into account. Given that the duration of erasures expand for the full block of communication. The capacity criterion is computed over one block to bypass the scenario, where feedback or past receptions are used.

It is demonstrated in [23] for linear interference networks where erasures are absent, that the optimal schemes restricted to a maximum amount of transmitters per message can be applied to attain the DoF, such that only the total backhaul load is subject to a limit. Here, the limit of the total backhaul load restricts the size of the average transmit set, such that a few message are assigned to a large number of transmitters, at the expense of having other messages assigned to less transmitters. This study takes the restriction into account, that is invoked by the maximum size of the transmit set, since the complexity of the problem is reduced with respect to its combinatorial approach. Furthermore, this is the first effort to obtain results from this problem in the context of dynamic interference networks.

The literature summarizes the analysis of interference networks with respect to the DoF as the Topological Interference Management (TIM) problem. Here, weak interference links are ignored and the only CSIT accessible to the transmitters is the given network topology. In [24], the TIM problem was analyzed in detail for networks, that considered fixed channel coefficients for the total communication duration. In contrast, the TIM problem with time-sensitive connectivity was studied in [25]. Furthermore, the TIM problem was used in [26] to analyze networks with multiple antennas, which is known as the MIMO TIM problem. It is demonstrated, that networks, which consist of more transmitters than antennas, offer no advantage, which emphasizes the elementary constraints of MIMO TIM.

In [27], the TIM problem is applied to two-dimensional topologies. Here, transmitters of a hexagonal cellular network in a downlink setting have only access to network topology information, while adjecent inter-cell-interference is taken into account. Further, the relation of the dropping gain of the optimal solution over basic frequency reuse with the growing number of interfering cells was demonstrated. In [28], the essential conditions for achieving the optimal orthogonal access schemes for cellular networks with respect to the TIM problem were outlined. The TIM problem in context of transmitter cooperation is analyzed in [29] while in [30], the benefit of applying transmitter cooperation with absent CSIT at a subset of the transmitters is examined.

In this work, the presence of perfect CSIT is considered for outlining the transmission scheme after assigning messages to transmitters through a backhaul. Here, the considered network has a topology, that has a sparse characteristics, and hence, the achieved optimal solution for the cell association problem, where transmitters that can only carry one message, depends on a Time Division Multiple Access (TDMA) scheme, where no information about the channel coefficients is needed, such that the problem for this special case is linked to preceding studies regarding the TIM problem. Section 4.1 demonstrates that a result in [28] builds the foundation of the illustration of the per user DoF for the cell association problem.

In the preceding paragraphs, the research was illuminated more from the perspective of fixed channel topologies and transmitter corporation. Despite the fact that conventional wireless communication models assume a fixed channel topology for longer duration, wireless Ad hoc networks appear to gain more significance with the rising industrial interest in Internet of Things (IoT). Here, the dynamic feature lies in the decentralized mobile nodes that form a temporarily network. Therefore, the information-theoretical part of the research is round off with the problem of wireless network discovery. It is essential to control the transmitting nodes and thus coordinate data traffic in a network to ensure optimal throughput at the receiving nodes. Hence, techniques that determine the topology of a network fast will back interference-free communication. Previous work discusses approaches to discover the network topologies between mobile devices in communication networks. In [74], device-to-device (D2D) communications, that allow devices to communicate directly without using base stations, was studied and a hybrid approach that combines network-assisted and direct D2D was proposed. While conventional approaches (see [75]) investigated the scenario where each node discovers the entire network topology, [76] takes advantage of the fact that transmitting nodes in hybrid cellular networks only need to have knowledge of an approximated network topology of receiving nodes they are providing service to, which is addressed as the Compact Topology Graph (CTG) discovery problem. Here, a fast scheme is introduced, which allows the receiving nodes in the cellular network to deduce the topology among the transmitting nodes with a supplementary radio in its cell.

A tree type network topology was considered in [77], where a node cannot transmit and receive at the same time. Furthermore transmitter/receiver patterns are demonstrated that permit network nodes to form a tree type network and discover each other directly over the air using half-duplex TDD technology and it was shown that a discovery pattern exists for each communication node, which requires bi-directional communication and supports large network sizes.

The interest lies in the network discovery for cellular networks and hence focus on bipartite graphs, where each receiving node is connected to a fixed number of transmitting nodes. Progress was made by [73] when the network topology of environments with interference characteristic was analyzed from a graph-theoretical perspective. The objective is to recover the interference graph of the n access points (APs) of a WLAN while minimizing the number of required measurements. Conflict between APs that are within each other's carrier sensing range was analyzed and referred as *direct* interference. Given d, the maximum number of interfering APs per AP, it was shown that the number of measurements, that is needed to establish the interference graph, is proportional to $d^2 \log(n)$.

The contribution of the research is to develop an algorithm for fast topology discovery in wireless networks with interference characteristics. The designed algorithm allocates communication rounds to each transmitting node and each transmitting node can only transmit during the assigned round. Moreover, the effect of fading conditions to the algorithm are investigated, where each link in the network is independently subject to erasure with probability p. The performance of the algorithm is analyzed when each transmitting node is restricted to be locally connected to a set of neighboring transmitting nodes and interference cancellation is allowed, where a link that was discovered in a previous communication round can be used to cancel interference.

In the last part of the research, the focus is on the empirical analysis of Channel Identification through Deep Learning techniques. As illustrated in the previous paragraphs, the focus is on interference management. For applicability of the proposed schemes, it is of interest to guarantee that MTs can precisely discriminate between wireless channels, which originate from different wireless technologies, that coexist in the same frequency band. [61] and [62] demonstrate the applicability of deep learning algorithms for signal analysis at the receiver side in wireless communication systems by classifying the modulation type. More precisely, Convolutional Long Short-term Deep Neural Networks (CLDNN) [63], Long Short-Term Memory neural networks (LSTM) [64], and deep Residual Networks (ResNet) [65] are used to achieve a classification accuracy of 90% for 10 modulation types at high SNR. [66] proposes a 5-layer CNN that achieves a classification accuracy above 90% at high SNR for recognizing 802.x standards operating in the entire 80 MHz wide 2.4 GHz ISM band

A CNN architecture was suggested in [67] with respect to the Channel Identification problem, where each snapshot for a data point was subject to a duration limitation of 12.8 μs and bandwidth limitation of 10 MHz. The analyzed dataset contains 15 classes each corresponding to packet transmissions with overlapping frequency channels within the 2.4 GHz ISM band of IEEE 802.11 b/g, IEEE 802.15.4 or IEEE 802.15.1. Here, the proposed CNN architecture outperforms state-of-the-art solutions for the Channel Identification problem with a classification accuracy above 95% for signal-to-noise ratios beyond -5 dB.

The work in [67] is extened by analyzing the same data set with focus on training time reduction while maintaining a high classification accuracy. An average accuracy of around 89.5% using deep neural network architectures such as CNN, ResNet, CLDNN, and LSTM was achieved, while maintaining the very high accuracy in [67] at moderately high SNR values. Furthermore, the suggested CNN architecture is tuned and select an architecture such that a fair comparison is obtained with the results from [67], which relies only on one CNN architecture. The proposed CNN architecture requires 60% of the training time than the CNN architecture of [67] for an average of 10 simulations. Here, three techniques are considered for further training time reduction: Band selection by studying the two lower and upper 2 MHz frequency bands, Training SNR selection by choosing only a subset of the training set corresponding to one SNR value, and PCA and various sub-Nyquist sampling techniques.

2. SYSTEM MODEL AND NOTATION

The standard model for the $K-$user interference channel is borrowed with single-antenna transmitters and receivers for each of the downlink and uplink sessions and the dynamic interference management problem from [69],

$$Y_i(t) = \sum_{j=1}^{K} H_{i,j}(t)X_j(t) + Z_i(t), \tag{2.1}$$

with time index t, transmitted signal $X_j(t)$ of transmitter j, received signal $Y_i(t)$ at receiver i, zero mean unit variance Gaussian noise $Z_i(t)$ at receiver i, and the channel coefficient $H_{i,j}(t)$ from transmitter j to receiver i over time slot t. The time index is left out in the succeeding paragraphs due to conciseness. Moreover, Y_i and X_i denote the receive and transmit signals at base station and mobile terminal i^{th} in the uplink, respectively, and mobile terminal and base station i^{th} in the downlink, respectively. Furthermore, $H_{i,j}$ denotes the channel coefficient between mobile terminal i and base station j. Mobile terminal i and base station j are denoted as MT i and BS j, correspondingly, and set $\{1, 2, \ldots, K\}$ is represented by $[K]$. Additionally, the abstraction $X_{\mathcal{A}}$, $Y_{\mathcal{A}}$, and $Z_{\mathcal{A}}$ is used for any set $\mathcal{A} \subseteq [K]$ to describe the sets $\{X_i, i \in \mathcal{A}\}$, $\{Y_i, i \in \mathcal{A}\}$, and $\{Z_i, i \in \mathcal{A}\}$, respectively. Since only the downlink setting is taken into account for the dynamic interference management problem in Chapter 4, BS and MT are referred in this chapter as transmitters and receivers, correspondingly.

2.1 Channel Model

Consider an interference network that is subject to locally conceitedness, i.e. each mobile terminal with index i is connected to base stations $\{i, i-1, \cdots, i-L\}$. Note, that the first L mobile terminals are connected only to all the base stations with a similar or lower index, i.e. the notation is borrowed from [69],

$$H_{i,j} = 0 \text{ iff } i \notin \{j, j+1, \cdots, j+L\}, \forall i, j \in [K], \tag{2.2}$$

with non-zero channel coefficients, which are all drawn from a continuous joint distribution for the problem studied in Chapter 3. Also, for the problem studied in Chapter 4, communications occurs in blocks of time slots, and each non-zero link can be erased independently in any given block with probability p. It is important to mention that all mobile terminals and base stations have access to global channel state information.

In the following, the **interference set** is elaborated from [69], that is the set of receivers a transmitter is connected to.

Definition 1 *In the uplink, the interference set of MT ν is the set of base stations with indices in the set $\{\nu, \nu-1, \cdots, \nu-L\}$. In the downlink, the interference set of BS μ is the set of mobile terminals with indices in the set $\{\mu, \mu+1, \cdots, \mu+L\}$*

According to [68], as Wyner's asymmetric interference network is obtained with L=1, the channel model becomes

$$H_{i,j} \text{ is identically 0 iff } i \notin \{j, j+1\}, \forall i, j \in [K]. \tag{2.3}$$

Consider the dynamic Wyner's interference network studied in 4 and assume that communication takes place over blocks of time slots to concede the effects of long-term fluctuations, whereas p denotes the probability of block erasure. For each j, and each $i \in \{j, j+1\}$ during a time slot, it is $H_{i,j} = 0$ with probability p. Similarly, $H_{i,j}$ does not equal to 0 with probability $\bar{p} \triangleq 1-p$. Also, the events of link erasure are mutually independent and in each time slot, all non-zero channel coefficients are drawn independently from a continuous distribution due to short-term channel fluctuations.

2.2 Uplink Cell Association

Let $\mathcal{C}_i \subseteq [K] \; \forall i \in [K]$ denote the set of base stations mobile terminal i is associated to, more precisely, those base stations at which the terminal's message are available in the downlink and can have its decoded message for the uplink. In the downlink, transmission of message (word) W_i to mobile terminal i takes place simultaneously through any subset of the transmitters in $\mathcal{C}_i$. In the uplink, W_i is decoded by any of the base station receivers in $\mathcal{C}_i$ and passed to other receivers in the set. Moreover, each cell association is bounded by the cardinality of the set $\mathcal{C}_i$ by a number N_c. The purpose of this constraint is to abbreviate a limited backhaul capacity constraint, where exchanging messages over the backhaul is limited as described in [69].

$$|\mathcal{C}_i| \leq N_c, \forall i \in [K]. \tag{2.4}$$

Further, **only full messages can be shared over the backhaul**, i.e. it is not allowed to divide messages into parts and share them as in [52], or to share quantized signals as in [46]. The following definition is borrowed from [69] for cell associations that *cover* each mobile terminal with all base stations connected to it.

Definition 2 *Assume that the cell association scheme is a* ***Full coverage association*** *if each mobile terminal is associated with all the base stations connected to it. More precisely,* $\forall i \in [K], \{i, i-1, \cdots i-L\} \subseteq \mathcal{C}_i$.

It will be observed in the uplink, that complete interference cancellation comes with full coverage associations.

2.3 Uplink Cell Association

Let W_i denote the message destined for receiver i and call $\mathcal{T}_i \subseteq [K]$ the transmit set of receiver i, i.e., those transmitters with the knowledge of W_i, for each $i \in [K]$, where transmission of message W_i to the receiver i occurs simultaneously through all transmitters in $\mathcal{T}_i$. Messages $\{W_i\}$ are assume to be independent of each other.

The *cooperation order* M is characterized in [69] as the maximum transmit set size:

$$M = \max_i |\mathcal{T}_i|. \tag{2.5}$$

2.4 Message Assignment Strategy

Any sequence of transmit sets $(\mathcal{T}_{i,K}), i \in [K], K \in \{1, 2, \ldots\}$ characterizes a message assignment strategy. i Moreover, it is $\mathcal{T}_{i,K} \subseteq [K], |\mathcal{T}_{i,K}| \leq M$ for each positive integer K and $\forall i \in [K]$. The purpose of message assignment strategies is to describe the transmit sets for a sequence of $K-$user channels. The k^{th} channel in the sequence has k users, and the transmit sets for this channel are defined as follows. The transmit set of receiver i in the k^{th} channel in the sequence is the transmit set $\mathcal{T}_{i,k}$ of the message assignment strategy.

2.5 Degrees of Freedom

Consider the average transmit power constraint P at each transmitter with the alphabet for message W_i, that is $\mathcal{W}_i$. Consequently, for the case that the decoding error probabilities of all messages can be jointly made arbitrarily small for a large enough coding block length n, the rates $R_i(P) = \frac{\log|\mathcal{W}_i|}{n}$ are attainable for almost all channel realizations. Here, $d_i = \lim_{P\to\infty} \frac{R_i(P)}{\log P}$ denotes the degrees of freedom $d_i, i \in [K]$m, where the DoF region $\mathcal{D}$ describes the closure of the set of all achievable DoF tuples. Moreover, the total number of degrees of freedom is denoted by ι, that is the maximum value of the sum of the achievable degrees of freedom, i.e. $\iota = \max_{\mathcal{D}} \sum_{i\in[K]} d_i$.

Consider a K-user locally connected network with connectivity parameter L. Denote $\iota(K, L, N_c)$ as the optimal achievable ι on average taken over both downlink and uplink sessions over all decisions of cell associations fulfilling the backhaul load constraint in (2.4). For simplification, denote the asymptotic per user DoF (puDoF) as $\tau(L, N_c)$, which is used to measure how $\iota(K, L, N_c)$ scales with K for fixed L and N_c,

$$\tau(L, N_c) = \lim_{K\to\infty} \frac{\iota(K, L, N_c)}{K}. \tag{2.6}$$

Denote $\tau_D(L, N_c)$ and $\tau_U(L, N_c)$ as the puDoF when only for the downlink and uplink session are optimized, correspondingly.

Denote ι_p as the average value of ι over potential decisions of non-zero channel coefficients for p, which is the probability of block erasure.

Moreover, denote $\iota_p(K, M)$ as the optimal attainable ι_p over all decisions of transmit sets fulfilling the cooperation order constraint in (2.5) for a K-user channel. For further simplification, the asymptotic average per user DoF $\tau_p(M)$ is defined to measure how $\iota_p(K, M)$ scale with K,

$$\tau_p(M) = \lim_{K\to\infty} \frac{\iota_p(K, M)}{K}. \tag{2.7}$$

If a sequence of coding schemes, that attain $\tau_p(M)$ by applying the transmit sets characterized by the message assignment strategy, exist, then this message assignment strategy is *optimal* for a given probability of erasure p. Moreover, message assignment strategies, that are optimal for all values of p, are *universally optimal.*

2.6 Interference Avoidance Schemes

Consider the set of *interference avoidance* schemes, where receivers are classified as either active or inactive. Active receivers are able to detect their desired signal interference-free. For the downlink session, cooperative zero-forcing transmission is considered where interference caused by a messages is eliminated *over the air* by cooperating transmitters. In particular,

$$X_j = \sum_{i: j\in\mathcal{C}_i} X_{j,i}, \tag{2.8}$$

denotes the transmit signal at the j^{th} transmitter where $X_{j,i}$ relies only on W_i. Moreover, each message is either not sent or assigned 1 DoF. Assume $\tilde{Y}_j = Y_j - Z_j, \forall j \in [K]$. It follows additional to the restriction in (2.8), it is that either the mutual information $I(\tilde{Y}_j; W_j) = 0$ or that W_j fully establishes $\tilde{Y}_j$. Observe that $\tilde{Y}_j$ is established from W_j when user j experiences communication free from interference, and $I(\tilde{Y}_j; W_j) = 0$ for where W_j is not sent. The j^{th} receiver is *active* if and only if $I(\tilde{Y}_j; W_j) > 0$. If zero-forcing transmit beamforming is applied for the j^{th} active receiver, then $I(Y_i; W_i) = 0, \forall i \neq j$. Hence, the j^{th} transmitter is *active* if $I\left(X_j; \{W_i : j \in \mathcal{C}_i\}\right) > 0$.

Furthermore, zero-forcing of interference through message passing decoding is taken into account for uplink sessions, where decoded messages are passed through a cooperating receiver to other receivers that aim to eliminate the interference of these messages. In uplink sessions, the j^{th} mobile terminal is active if $I(X_j; W_j) > 0$. Also, each active mobile terminal applies an optimal AWGN point-to-point code (see e.g., [47]) with transmit power P.

Consider any active base station with index i. Say that B_i denotes the set of all received messages at BS i through the backhaul. Then it is j such that there exist an association of BS i with MT j and W_j determines $\tilde{Y}_i$, given perfect estimates of the messages shared in B_i. More precisely, $\exists\ j$ s.t. $i \in \mathcal{C}_j$ and $I(\tilde{Y}_i; W_j | B_i) > 0$. Also, $\forall k \in [K] : k \neq j$, $I(\tilde{Y}_i^n; W_k | B_i) \overset{n \to \infty}{\longrightarrow} 0$, with block length n. Observe that the estimates of the decoded message are *perfect* since the block length grows towards infinity. Consider the definition below.

Definition 3 *For an uplink zero-forcing scheme, the MT-BS pair $(MT\ j, BS\ i)$ is a* ***decoding pair*** *if W_j is decoded at base station i. More precisely, $i \in \mathcal{C}_j$, $I(\tilde{Y}_i; W_j | B_i) > 0$, and $\forall k \in [K] : k \neq j$, $I(\tilde{Y}_i^n; W_k | B_i) \overset{n \to \infty}{\longrightarrow} 0$.*

The superscript **zf** is included to the puDoF symbol when the restriction is imposed that the used coding scheme is identical to a zero-forcing scheme. For instance, $\tau_U^{\text{zf}}(L, N_c)$ describes the puDoF if only the uplink is taken into account and dictate the constraint to schemes that are based on message passing decoding zero-forcing .

2.7 Subnetworks

In the first two parts of the research, the term *subnetworks* is used, that builds the foundation of the achievability and converse proofs. Subnetworks for the uplink-downlink scenario and the dynamic interference management problem are defined separately, as they differ vastly in their logic and objective.

2.7.1 Subnetworks in the Uplink-Downlink Cooperative Interference Management Problem

The network is treated as a group of subnetworks that are of same size and formed by s successive BS-MT pairs. $\mathcal{L}_k$ denotes the k^{th} subnetwork. $\nu_k = s(k-1)+1$ is defined to describe the top index, i.e. the lowest index, of each subnetwork $\mathcal{L}_k$. Hence, $\mathcal{L}_k$ is topologically beneath $\mathcal{L}_{k-1}$. More precisely, connections exist between mobile terminals from $\mathcal{L}_k$ and some base stations in $\mathcal{L}_{k-1}$. Consider the definition below.

Definition 4 *The considered transmission scheme relies on* ***Subnetwork-only decoding*** *if words originating in a subnetwork can only be decoded in the same subnetwork.*

Subnetwork-only downlink decoding and **Subnetwork-only uplink decoding** are characterized to highlight that Subnetwork-only decoding is applied either for downlink or uplink, correspondingly.

Lastly, consider the definitions below for any scheme that is based on zero-forcing message passing decoding during uplink sessions.

Definition 5 *For an uplink zero-forcing scheme, if there exist decoding pairs $(MT\ i, BS\ j)$ such that the mobile terminal MT i is in $\mathcal{L}_k$ and the base station BS j is in $\mathcal{L}_m, m < k$, then $\mathcal{L}_k$* ***borrows*** *base station j from $\mathcal{L}_m$.*

Define θ_k to describe the number of base stations that $\mathcal{L}_k$ borrows from $\mathcal{L}_{k-1}$ to support decode words from $\mathcal{L}_k$.

Definition 6 *For an uplink zero-forcing scheme, if there exist consecutive base stations in $\mathcal{L}_{k-1}$ indexed by $(\nu_k - \mu_k, \nu_k - \mu_k + 1, \cdots \nu_k - 1)$ such that no words can be decoded at these base stations due to the cell association constraint being tightly met in $\mathcal{L}_k$, $\mathcal{L}_k$* ***blocks*** *the μ_k base stations in $\mathcal{L}_{k-1}$.*

2.7.2 Subnetworks in the Dynamic Interference Management Problem

Any realization of a network with some erased links is viewed as a sequence of non-interfering subnetworks. Consider the definition of subnetworks beneath, while the following definitions have to be made first for a given message assignment and network realization.

Definition 7 *For a given network realization and message assignment, a* **message is enabled** *if there exists a transmitter carrying the message and connected to its destined receiver.*

Furthermore, the definition of *irreducible message assignments* is used from [6] when no links are allowed to be erased (see also [37, Chapter 6]). The definition in [6] is extended to the current scenario where each realization allows links to be erased, by substituting the contraint $|x - y| \leq 1$, with the constraint that there has to exist a connection of at least one receiver with both transmitters of indices q and y. In other words, for each user i, a graph G_i of $|\mathcal{T}_i|$ vertices is formed, that is completely indexed in $\mathcal{T}_i$, such that vertices $x, y \in \mathcal{T}_i$ are linked with an edge if transmitters q and y are linked to the same receiver. Also, vertices i and $i-1$ have a *special mark* if $H_{i,i} \neq 0$ and $H_{i,i-1} \neq 0$, correspondingly. The following definition is made.

Definition 8 *An assignment of message W_i to transmitterqis useful if the vertexqis connected to a marked vertex in the graph G_i.*

Definition 9 *A message assignment is irreducible, if for every user $i \in [K]$, the graph G_i has only one component.*

Hence,the result from [6, Lemma 2] follows. More precisely, a message assignment can be *reduced* if at least one element can be excluded from the transmit set such that it is ensured, that this modification does not cause a decline in the sum rate. This applies if it is not allowed for a transmitter, that carries a message, to either deliver this message to the designated receiver or cancel interference.

These kind of message assignments are not *useful*. The message assignment in the considered scenario relies on the network topology statistics and thus it is the case that some message assignments may be scaled down for a given topology realization.

Definition 10 *A message assignment is topology-reduced for a given network realization, if only useful assignments of enabled messages are present, and all other assignments are removed.*

According to [6, Lemma 2], it is that the topology-reduction of the message assignment does not result in a decline of the sum rate.

Definition 11 *For a given network realization and message assignment, a set of* k *users with successive indices* $\{i, i+1, \ldots, i+k-1\}$ *form a* **subnetwork** *if the following two conditions hold:*

1. *The first condition is that* $i = 1$*, or it is the case that for the topology-reduced version of the message assignment, either* W_i *is not enabled or it is the case that there exists no message* W_x*,* $x < i$ *that is available at a transmitter connected to receiver* i*, and* W_i *is not available at a transmitter connected to any receiver with an index* $x < i$*.*

2. *Secondly,* $i + k - 1 = K$ *or the first condition holds for* $i + k$*, i.e.,* $i + k$ *is the first user in a new subnetwork.*

Definition 12 *The* **subnetwork is atomic** *if it does not contain smaller subnetworks.*

For an atomic subnetwork, observe that the transmitters, that have messages available for users in the subnetwork, are indexed successively, and for any of these transmitters t and receiver r with $r \in \{t, t+1\}$ in the subnetwork, the channel coefficient $H_{r,t} \neq 0$ (is not erased).

3. JOINT UPLINK-DOWNLINK CELL ASSOCIATIONS

3.1 Prior Work: Downlink-Only Scheme

[49] presents the scenario in context of only downlink transmission, where the puDoF value was described under restriction of only zero-forcing schemes as,

$$\tau_D^{\text{zf}}(L, N_c) = \frac{2N_c}{2N_c + L}. \tag{3.1}$$

The optimal cell association is achieved by splitting the network into subnetworks, each consisting of $2N_c + L$ consecutive pairs of transmitters and receivers. Note, that the last L transmitters in each subnetwork are kept inactive to avoid interference among subnetworks. Here, the goal of the zero-forcing scheme is to decode $2N_c$ messages at the receiver side while securing interference-free communications and hence achieve the puDoF in (3.1). In each subnetwork, two Multiple Input Single Output (MISO) Broadcast Channels (BC), each formed by N_c pairs of transmitters and receivers, are set up. The objective of this concept is to obtain the puDoF value in (3.1) with a cooperation constraint, i.e. by restricting the availability of each message at N_c transmitters, while ensuring that interference is fully eliminated throughout these two BCs. In the following, only the cell association in the first subnetwork is described since the associations in the remaining subnetworks are determined by analogy. The top N_c pairs of transmitters and receivers form the first MISO BC and message W_i is associated with base stations of indices in the set $\mathcal{C}_i = \{i, i+1, \cdots, N_c\}$ for each $i \in \{1, 2, \cdots, N_c\}$. The second MISO BC covers N_c transmitters with indices in $\{N_c + 1, N_c + 2, \cdots, 2N_c\}$ and N_c receivers with indices in $\{N_c + L + 1, N_c + L + 2, \cdots, 2N_c + L\}$. For each $i \in \{N_c + L + 1, N_c + L + 2, \cdots, 2N_c + L\}$, message W_i is associated with transmitters that have indices in the set $\mathcal{C}_i = \{i - L, i - L - 1, \cdots, N_c + 1\}$. It is important to mention, that interference between the two MISO BCs is phased out completely by deactivating the L receivers between them. Confined by the considered cooperation constraint and zero forcing schemes, [49] demonstrates that this scheme leads to the puDoF value of (3.1) and is the optimal value, that is attainable in the downlink.

3.2 Main Results

First, the following average uplink zero-forcing puDoF is elaborated

Theorem 1 *The zero-forcing asymptotic puDoF for the uplink is characterized below:*

$$\tau_U^{zf}(L, N_c) = \begin{cases} 1 & L+1 \le N_c, \\ \frac{N_c+1}{L+2} & \frac{L}{2} \le N_c \le L, \\ \frac{2N_c}{2N_c+L} & 1 \le N_c \le \frac{L}{2} - 1. \end{cases} \tag{3.2}$$

Proof The explicit proof is demonstrated in 3.3. ■

The focus here is on describing the average zero-forcing puDoF through both uplink and downlink.

Theorem 2 *Under the zero-forcing schemes as characterized in Section 2, the obtained inner bounds for the average uplink-downlink puDoF are stated below:*

$$\tau^{zf}(L, N_c) \ge \begin{cases} \frac{1}{2}\left(1 + \gamma_D(N_c, L)\right) & L+1 \le N_c, \\ \frac{2N_c}{2N_c+L} & 1 \le N_c \le L, \end{cases} \tag{3.3}$$

where $\gamma_D(N_c, L)$ is the downlink component of the puDoF when $N_c \ge L+1$, and is given by

$$\gamma_D(L, N_c) = \frac{2\left(\left\lceil \frac{L+1}{2} \right\rceil + N_c - (L+1)\right)}{L + 2\left(\left\lceil \frac{L+1}{2} \right\rceil + N_c - (L+1)\right)}. \tag{3.4}$$

Further, the inner bound in (3.3) *is tight when* $N_c \leq \frac{L}{2}$. *More precisely,*

$$\tau^{zf}(L, N_c) = \frac{2N_c}{2N_c + L}, \quad \forall N_c \leq \frac{L}{2}. \tag{3.5}$$

Proof The detailed proof is demonstrated in Section 3.4. ■

Proceeding with characterizing the zero-forcing optimal downlink scheme when utilizing full coverage associations, it is concluded that these associations lead to a unity uplink puDoF.

Theorem 3 *The optimal zero-forcing downlink puDoF for a full coverage association and* $N_c > L$ *is characterized as,*

$$\gamma_D(L, N_c) = \frac{2\lambda}{2\lambda + L}, \tag{3.6}$$

where $\lambda = \delta + N_c - (L+1)$, *and* $\delta = \lceil \frac{L+1}{2} \rceil$.

Proof The proof is available in Section 3.4.1. ■

Remark 1 *The similarity of the expressions of the optimal downlink puDoF of* $\frac{2\lambda}{2\lambda+L}$ *with the downlink-only optimal puDoF* $\frac{2N_c}{2N_c+L}$ *is explained as follows: Given* N_c, *that is the maximum number of associations of a word with a base station in the downlink,* N_c *lowers to* $\lambda = \lceil \frac{L+1}{2} \rceil + N_c - (L+1)$, *since associations are distributed among uplink and downlink and a full coverage association is applied.*

Finally, the inner bounds in Theorem 2 are proven for Wyner's linear networks, i.e., when $L = 1$, to be information-theoretic optimal.

Theorem 4 *For Wyner's linear network, the average asymptotic puDoF through uplink and downlink, is given by,*

$$\tau(L = 1, N_c) = \begin{cases} \frac{1}{2}\left(1 + \frac{2(N_c-1)}{1+2(N_c-1)}\right) = \frac{4N_c-3}{4N_c-2}, & N_c \geq 2, \\ \frac{2}{3}, & N_c = 1. \end{cases} \tag{3.7}$$

Proof The proof is available in Section 3.4.2. ■

3.3 Uplink-Only Scheme

The proof of Theorem 1 is discussed in the three subsections below.

3.3.1 Proof of Achievability

Consider the puDoF values stated in Theorem 1, which is attained by the following association. It is that each mobile terminal is associated with those $L+1$ base stations, which are connected to it, for $N_c \geq L+1$. Here, the K^{th} base station, that is the last base station in the network, successfully decodes the last message while passing it to the L preceding base stations, which are connected to the mobile terminal of index K. Thus all interference is eliminated, which is produced by the K^{th} mobile terminal. Each base station of lower index than K decodes its messages and passes it on to the preceding base stations such that the interference produced by that message is cancelled successfully and one degree of freedom per user is attained.

If it is $\frac{L}{2} \leq N_c \leq L$, a puDoF value of $\frac{N_c+1}{L+2}$ is attained by a cell association, which requires the network to be separated into subnetworks, each formed by consecutive $L+2$ pairs of transmitters and receivers, and then decode the last N_c+1 words in each subnetwork. For the first subnetwork, the cell association is designed, such that the association of message W_i with base stations $\{i, i-1, \cdots, L+2-N_c+1\} \subseteq \mathcal{C}_i$ applies for each $i \in \{L+2, L+1, \cdots, L+2-N_c+1\}$. It follows that the last N_c words can be decoded while ensuring complete interference elimination among them. Due to interference, that cannot be phased out, from the last transmitter in the subnetwork, the base stations indexed as $\{2, 3, \cdots, L+2-N_c\}$ are deactivated. W_{L+2-N_c} is decoded by the first base station. Each $\mathcal{C}_i, \forall i \in \mathcal{S}$ is extended by the top base station to completely cancel interference produced by the transmitters indexed in $\mathcal{S} = \{L+2-N_c+1, L+2-N_c+2, \cdots, L+1\}$ at the first base station of the subnetwork. At this point $\mu_i = 2+i-(L+2-N_c+1)$ associations have been used for messages indexed in $\mathcal{S}$, where the factor of two is due to the base station resolving

W_i and the first base station of the subnetwork. However, each of the transmitters indexed in $\mathcal{S}\backslash\{L+1\}$ causes interference at the preceding subnetwork.

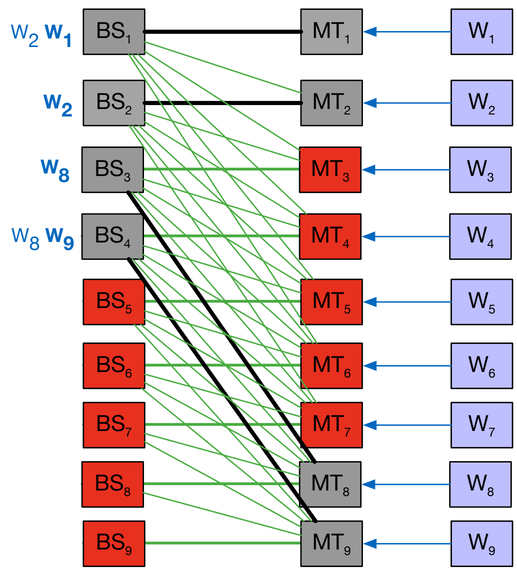
$\forall i \in \mathcal{S}\backslash\{L+1\}$, message W_i causes interference at the last $L+1-i$ base stations of the previous subnetwork, that is identical to the number of associations remaining for the particular message, that is $N_c - \mu_i = L+1-i$, such that interference between subnetworks at those base stations is prevented.

For $1 \leq N_c \leq \frac{L}{2}-1$, the applied cell association to attain the lower bound of $\frac{2N_c}{2N_c+L}$ is derived analogously to the lower bound characterized in Section 3.1 for the downlink scenario. Here, the network is separated into $2N_c+L$ sized dismembered subnetworks of succeeding pairs of transmitters and receivers. In the uplink case, the two sets of transmitters $\mathcal{A}_T = \{1, 2, \cdots, N_c\}$ and $\mathcal{B}_T = \{N_c+L+1, N_c+L+2\cdots, 2N_c+L\}$ and the two sets of receivers $\mathcal{A}_R = \{1, 2, \cdots, N_c\}$ and $\mathcal{B}_R = \{N_c+1, N_c+L+2\cdots, 2N_c\}$ are organized. Furthermore, the association of message W_i with the receivers of indices from $\mathcal{A}_R$ applies for each $i \in \mathcal{A}_T$. Here, receiver of index i decodes W_i while the purpose of the other associations in $\mathcal{C}_i$ is to eliminate interference. Analogous to this, the association of message W_j with the receivers with indices in $\mathcal{B}_R$ applies for each $j \in \mathcal{B}_T$. Note, that receiver $j-L$ decodes W_j and the purpose of the other associations in $\mathcal{C}_j$ is to eliminate interference. The characterized schemes are depicted in Figure 3.1.

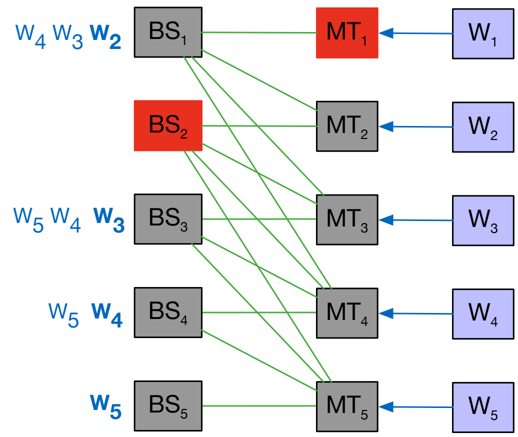
If not constrained by the zero-forcing coding scheme for the last range, $\frac{1}{2}$ puDoF can be achieved by asymptotic interference alignment [53], which is a higher value of that attained by zero-forcing. The following paragraphs conclude the proof Theorem 1.

3.3.2 Converse Proof when $\frac{L}{2} \leq N_c \leq L$

This section outlines the converse proof for the second range of (3.2). Moreover, the expression below is verified.



(a) $N_c < \frac{L}{2}$, $N_c = 2$, $L = 5$



(b) $L + 1 \geq N_c \geq \frac{L}{2}$, $N_c = 4$, $L = 5$

Fig. 3.1. Uplink schemes for $N_c \leq L + 1$

$$\tau_U^{\text{zf}}(L, N_c) = \frac{N_c + 1}{L + 2}, \qquad \frac{L}{2} \leq N_c \leq L. \tag{3.8}$$

The proof starts with the scenario $N_c = L$ and the optimal uplink zero-forcing puDoF denoted as:

$$\tau_U^{\text{zf}}(L, L) = \frac{L + 1}{L + 2}. \tag{3.9}$$

A separation of the network into subnetworks is performed, where each subnetwork is formed by $L + 2$ consecutive pairs of transmitters and receivers. Note that for $N_c + 1 = L + 1$ successive base stations that are active in a subnetwork, then the mobile terminal connected to all these base stations has to be deactivated, since an interference produced by a message can only be eliminated at less than N_c base stations. Consider the set of subnetworks Γ_{BS} with $N_c + 2$ active receivers and the set of subnetworks Φ_{BS} with less or equal N_c receivers, that are active. Additionally, denote Γ_{MT} and Φ_{MT} as the subnetworks with $N_c + 2$ transmitters that are active and less or equal than N_c transmitters that are active, correspondingly. A higher puDoF than (3.9) is achieved, if the following is satisfied altogether: $|\Gamma_{BS}| > |\Phi_{BS}|$ and $|\Gamma_{MT}| > |\Phi_{MT}|$.

Observe that there exist at most N_c active transmitters in each subnetwork that belongs to Γ_{BS}, since the interference produced by any message can only be canceled at less than N_c receivers. It follows $\Gamma_{BS} \subseteq \Phi_{MT}$. In the same way, the number of active transmitters in any subnetwork with $N_c + 1$ active receivers is at most $N_c + 1$ such that $\Gamma_{MT} \subseteq \Phi_{BS}$. Moreover, if $|\Gamma_{BS}| > |\Phi_{BS}|$ applies, then $|\Gamma_{MT}| < |\Phi_{MT}|$, from which the proof of the statement in (3.9) follows.

The two lemmas below lead to the proof of $\tau_U^{\text{zf}}(L, N_c) = \frac{N_c+1}{L+2}$ when $\frac{L}{2} \leq N_c < L$:

Lemma 1 *Given any scheme of zero-forcing, at last one of the statements beneath hold for any two decoding pairs* $(MT\ i_1, BS\ j_1)$ *and* $(MT\ i_2, BS\ j_2)$*:* $j_2 \notin \{i_1, i_1 - 1, \cdots, i_1 - L\}$ *or* $j_1 \notin \{i_2, i_2 - 1, \cdots, i_2 - L\}$.

Proof If the complement to the claim applies, that is $j_2 \in \{i_1, i_1 - 1, \cdots, i_1 - L\}$ and $j_1 \in \{i_2, i_2 - 1, \cdots, i_2 - L\}$, then W_{i_1} and W_{i_2} would interfere with each other such that zero-forcing could not be utilized to decode them, which is implied by the description of zero-forcing message passing decoding, first introduced in [54]. ■

Using Lemma 1, the following corollary applies:

Corollary 1 *For any two decoding pairs* $(MT\ i_1, BS\ j_1)$ *and* $(MT\ i_2, BS\ j_2)$ *in a zero-forcing scheme, if* $i_1 > i_2$ *then* $j_1 > j_2$ *and vice versa.*

Further, the following lemma is stated:

Lemma 2 *For any set* $\mathcal{L} \subseteq [\mathcal{K}]$ *of* $L+1$ *consecutive indices, a maximum of* N_c *mobile terminals with indices in* $\mathcal{L}$ *can be decoded at base stations with indices in* $\mathcal{L}$ *for any zero-forcing scheme.*

Proof Here, this claim is proven by contradiction. Consider the scenario that N_c+1 or more mobile terminals indexed in $\mathcal{L}$ are decoded at base stations indexed in $\mathcal{L}$, then an association of more than N_c base stations with at least one of the mobile terminals would exist, which violates the constraint in (2.4). ■

The following lemma is provided to build a base for the converse argument of this section.

Lemma 3 *If a partitioning of the network users into subnetworks is considered; each of size L+2. For the k^{th} subnetwork $\mathcal{L}_k$, where the largest indexed $p_k \geq 0$ base stations are blocked or borrowed. If $\mathcal{L}_k$ had $N_c + 1 + (q_k - p_k)$ active mobile terminals, where $q_k > 0$ and $q_k \geq p_k$, then $\mathcal{L}_k$ would have to borrow and/or block base stations from the preceding subnetwork $\mathcal{L}_{k-1}$.*

Proof Two cases will be considered here: $p_k = 0$ and $p_k > 0$.

Consider $p_k = 0$. Assume that $N_c + 1 + q_k$ active mobile terminals reside in $\mathcal{L}_k$ with $q_k > 0$. According to Lemma 2, at most N_c words can be decoded by $L+1$ base stations, that have the $L+1$ largest indices. Due to the size of the subnetwork, which is $L + 2$, it follows that at most $N_c + 1$ words can be decoded in $\mathcal{L}_k$. Consequently, borrowing of base stations occurs in order to decode the additional q_k words.

Consider $p_k > 0$. Here, only $L+2-p_k$ base stations exist to decode words from $\mathcal{L}_k$. For the case that all of the p_k mobile terminals, that also have the highest indices, would be active, then according to Lemma 1, all except the two mobile terminals with the largest indices will be decoded in $\mathcal{L}_{k-1}$. For the case when none of the p_k mobile terminals with highest indices is active, then interference occurs between the mobile terminal in $\mathcal{L}_k$, that is active and is of highest index, and all the $L + 2 - p_k$ base stations, that are not used by $\mathcal{L}_{k+1}$. Hence, borrowing of at least $1 + q_k - p_k$ base stations from $\mathcal{L}_{k-1}$ occurs such that all the words originating from $\mathcal{L}_k$ can be decoded. A subset of the p_k mobile terminals, that have the highest indices, can be active. This is true since the following applies. Without loss in generality, assume that there exist only one active mobile terminal from the subset above, that is is MT j_k. If j_k is $\nu_k + L + 1$, then take the active mobile terminal MT m_k into account, which has the second largest index.

It follows by Lemma 1, that m_k is at most $\nu_k + L - p_k$. Therefore, there exist a connection between MT m_k and at least all the base stations in $\mathcal{L}_k$ which are not decoding W_{j_k} or are blocked/borrowed by $\mathcal{L}_{k+1}$. Furthermore, this mobile terminal has a connection with at least p_k base stations in $\mathcal{L}_{k-1}$. Due to the fact that MT m_k is active and has the second highest index, it will meet its association constraint, hence at least p_k base stations in $\mathcal{L}_{k-1}$ will be blocked.

It follows for each p_k, that if there exist $N_c + 1 + (q_k - p_k)$ active mobile terminals in $\mathcal{L}_k$, where $q_k > 0$ and $q_k \geq p_k$, then borrowing and/or blocking of base stations from $\mathcal{L}_{k-1}$ occurs due to $\mathcal{L}_k$. ■

Lemma 3 concludes that *subnetwork-only uplink decoding*, that is when words can only be decoded in the same subnetwork they are originating from, can decode at most $N_c + 1$ words in a subnetwork of size $L + 2$.

The foundation of the proof is that at least one subnetwork of $L + 2$ consecutive pairs of transmitters and receivers has to keep more than $N_c + 1$ mobile terminals active such that the the inner bound characterized in (3.2) is exceeded. Consider this subnetwork as $\mathcal{L}_k$, where subnetworks of same kind must *borrow* base stations from the preceding subnetwork to decode words originating from its own mobile terminals. The *best case* scenario for inter-subnetwork interference is, when the interference originating from mobile terminals of one subnetwork (e.g., $\mathcal{L}_k$) to base stations of another subnetwork (e.g., $\mathcal{L}_{k-1}$) accumulates in the bottom most base stations. The reason to characterize this as the best case scenario arises from Lemma 1, i.e. the fact, that those base stations, which correspond to indices outside the range of the interference caused by the active mobile terminals of $\mathcal{L}_k$, are needed to decode the mobile terminals of $\mathcal{L}_{k-1}$ in $\mathcal{L}_{k-1}$.

The objective is to verify that $\tau_U^{\text{zf}}(L, N_c) \leq \frac{N_c+1}{L+2}$ when $L > N_c \geq \frac{L}{2}$. Due to the pigeonhole principle, there has to exist at least one subnetwork (for example $\mathcal{L}_k$) with $N_c + 1 + q_k$ active mobile terminals, $q_k > 0$, to violate this bound. It follows by Lemma 2, that at least q_k base stations are borrowed from $\mathcal{L}_{k-1}$ by $\mathcal{L}_k$ and hence it is $q_k \leq \theta_k$. The possible scenarios for the value of θ_k are evaluated in the next paragraphs. Furthermore, it is $\theta_k \leq N_c$ due to the network topology and the described cell association constraint.

For $\theta_k = 1$ it is $q_k = 1$, and thus $N_c + 2$ active mobile terminals exist in $\mathcal{L}_k$. If $\mathcal{L}_k$ borrows one base station, assume base station j, it is the case that $N_c + 1$ words have to be decoded previously in $\mathcal{L}_k$. According to Lemma 2, there is at least one decoding pair (MT i, BS n), with $i, n \geq \nu_k$, where BS n is not connected to the mobile terminal in $\mathcal{L}_k$, that is the highest indexed active one. Here, the subnetwork size restricts $n = \nu_k$ and the message originating from $i-th$ mobile terminal decoded at the first base station of $\mathcal{L}_k$.

Considering Lemma 1, this concludes $j \notin \{i, i-1, ...i-L\}$ and $i \leq \nu_k + (L+2-(N_c+1))$, such that $j \leq \nu_k - N_c = \nu_{k-1} + L + 2 - N_c$. Denote θ as the number of remaining base stations in $\mathcal{L}_{k-1}$ that can decode words originating in $\mathcal{L}_{k-1}$. Hence $\theta \leq L + 2 - N_c$ and $L + 2 - \theta \geq N_c$ base stations are blocked or borrowed in $\mathcal{L}_{k-1}$. Due to Lemma 3, $\mathcal{L}_{k-1}$ has at least $N_c + 1$ active mobile terminals and hence base stations from $\mathcal{L}_{k-2}$ would have to be borrowed or blocked. The case of $\mathcal{L}_{k-1}$ having not more than $N_c - 1$ active mobile terminals is neglected since the average number of active mobile terminals through $\mathcal{L}_k$ and $\mathcal{L}_{k-1}$ would be forced not to exceed N_c and hence reinstate the argument from $\mathcal{L}_{k-2}$.

Lemma 3 causes $\mathcal{L}_{k-1}$ to block or borrow at least $L + 2 - \theta$ base stations in $\mathcal{L}_{k-2}$. It follows for $\theta_{k-1} = 1$ that the number of blocked or borrowed base stations in $\mathcal{L}_{k-1}$ is identical to the number of blocked or borrowed base stations in $\mathcal{L}_{k-2}$. The procedure of borrowing/blocking stops if either borrowing at some subnetwork $\mathcal{L}_i$ ends or $\mathcal{L}_1$ is reached. Considering the previous case, it is that the overall average between $\mathcal{L}_k$ and $\mathcal{L}_i$ is $N_c + 1$, since at most N_c active mobile terminals reside in $\mathcal{L}_i$. Also, there is only one additional active mobile terminal over the whole network and this does not change the asymptotic per user DoF. An argument, that follows the same logic as the one for $\theta_k > 1$, applies for $\theta_{k-1} > 1$ and is elaborated below. A similar argument, that was elaborated in the earlier paragraph, applies for $\theta_k > 1$. According to Lemma 1, it is that the borrowed base station in $\mathcal{L}_{k-1}$, that has the highest index, needs to be connected to the mobile terminal of $\mathcal{L}_k$, that is active and of smallest index, while no connection to any other mobile terminal, that is active, in $\mathcal{L}_k$ is allowed. The index of the lowest borrowed base station in $\mathcal{L}_{k-1}$ is at most $\nu_{k-1} + (L + 2 - N_c - q_k)$, because the index of the mobile terminal, which is the smallest indexed among the active ones in $\mathcal{L}_k$, is at most $\nu_k + (L + 2 - (N_c + 1 + q_k)) - 1$. It follows that at most $L + 3 - N_c - q_k$ base stations in $\mathcal{L}_{k-1}$ can decode words originating in $\mathcal{L}_{k-1}$. At least $N_c + 1 + (1 - q_k)$ words have to be decoded by these available base stations to achieve an average not less or equal to $N_c + 1$ active mobile terminals per subnetwork over $\mathcal{L}_k$ and $\mathcal{L}_{k-1}$ without $\mathcal{L}_{k-1}$ borrowing base stations from $\mathcal{L}_{k-2}$. This is not possible for $L + 2 - N_c - q_k < N_c + 1 + 1 - q_k$ corresponding to $N_c \geq \frac{L+1}{2}$. Here, $N_c \geq \frac{L+1}{2}$ requires that at least one base station from $\mathcal{L}_{k-1}$ has to be borrowed from $\mathcal{L}_{k-2}$, resulting in an iterative argument comparable to the argument for $\theta_k = 1$.

According to the best case scenario, the first mobile terminal in $\mathcal{L}_{k-1}$, which is connected to at most $q_k - 2$ base stations that are being borrowed by $\mathcal{L}_k$, but still connected to at least $N_c + 2 - q_k$ available base stations in $\mathcal{L}_{k-1}$, has to be found, such that $\mathcal{L}_{k-1}$ is not forced to borrow base stations from $\mathcal{L}_{k-2}$. Let the index of this mobile terminal be $\nu_{k-1} + \nu$. Then, it is $\nu \leq (L + 2 - N_c - q_k) + (q_k - 2) = L - N_c$. Therefore $N_c + 2 - q_k$ mobile terminals exist, none of which are borrowed from $\mathcal{L}_{k-2}$. Note, that mobile terminal with index $\nu_{k-1} + \nu$ has no further association to use, while at least N_c base stations in $\mathcal{L}_{k-2}$ are connected to it. Thus $N_c + 2 - q_k$ active mobile terminals reside in $\mathcal{L}_{k-1}$ that do not borrow from $\mathcal{L}_{k-2}$. Note, that mobile terminal with index $\nu_{k-1} + \nu$ has no further association to use, while at least N_c base stations in $\mathcal{L}_{k-2}$ are connected to it.

It follows that a maximum of $L + 2 - N_c \leq N_c + 2$ base stations, that are used to decode more words, reside in $\mathcal{L}_{k-2}$, while at least $N_c + 1$ words have to be decoded, which can only occur, if at least two mobile terminals have associations with N_c base stations. Furthermore, the indices of these two mobile terminals are of larger value than $\lambda = \nu_{k-2} + L + 1 - (N_c + 1)$.

Therefore at least N_c of the bottom most L base stations in $\mathcal{L}_{k-3}$ are blocked by $\mathcal{L}_{k-2}$, and the observation here is that a minimum of one base station from the previous subnetwork is blocked by each further subnetwork for the average number of active mobile terminals per subnetwork to still be above $N_c + 1$.

Assume any base stations in $\mathcal{L}_{i-1}$ is not blocked by $\mathcal{L}_i$, then $\mathcal{L}_i$ has no more than N_c mobile terminals, that are active, decoded in $\mathcal{L}_i$. Hence, either $\mathcal{L}_i$ has only N_c active mobile terminals or borrows from $\mathcal{L}_{i-1}$. An analogous iterative as demonstrated above applies when $\mathcal{L}_i$ borrows from $\mathcal{L}_{i-1}$. Otherwise, the average number of mobile terminals that are active accross the $k - i$ subnetworks that are taken into account is $N_c + 1$ per subnetwork since only N_c active mobile terminals reside in $\mathcal{L}_i$. Thus, there exist blocking of base stations in the previous subnetwork by each subnetwork, while the amount of additional mobile terminals that are active in the whole network does not scale. Note that a constant q_k fixes this number, concluding that for every subnetwork of size $L + 2$, $N_c + 1$ is asymptotically approached by the average number of active mobile terminals.

For any subnetwork with not less than or equal $N_c + 1$ mobile terminals that are active and $L \geq N_c \geq \frac{L}{2}$, it has been demonstrated that either the number of additional mobile terminals that are active does not scale with size of the network, or the average across the total network remains bounded by N_c+1 mobile terminals that are active per subnetwork. Consequently, the asymptotic average number of decoded words per subnetwork cannot exceed N_c+1, from which it follows for the uplink with applied zero forcing that the asymptotic puDoF, $\tau_U^{\text{zf}}(L, N_c) \leq \frac{N_c+1}{L+2}$.

Furthermore, it has been demonstrated in Section 3.3 that $\tau_U^{\text{zf}}(L, N_c) \geq \frac{N_c+1}{L+2}$, from which it follows that $\tau_U^{\text{zf}}(L, N_c) = \frac{N_c+1}{L+2}$ whenever $\frac{L}{2} \leq N_c \leq L$, which concludes the proof of (3.8).

3.3.3 Converse Proof when $N_c < \frac{L}{2}$

This section discusses a converse proof for the third range of (3.2). In other words, the validity of the expression below is demonstrated.

$$\tau_U^{\text{zf}}(L, N_c) = \frac{2N_c}{2N_c + L}, \qquad N_c < \frac{L}{2}. \tag{3.10}$$

In an analogous fashion to 3.3.2, the concept of the proof is to show, that at least one subnetwork of $2N_c + L$ consecutive MT-BS pairs must have more than $2N_c$ active mobile terminals to exceed the inner bound described in (3.2). Moreover, subnetworks of this kind have to either *borrow* or *block* base stations from the preceding subnetwork such that words originating in its own mobile terminals can be decoded.

Focus your attention to the first subnetwork which has not less than or equal $2N_c$ mobile terminals that are active and denote it as $\mathcal{L}_k$. Starting with the case when $\theta_k = 0$, i.e no base stations from $\mathcal{L}_{k-1}$ is borrowed by $\mathcal{L}_k$. For further simplification of the proof, three special mobile terminals are marked that belong to $\mathcal{L}_k$, the mobile terminal MT $\nu_k + \nu$, that is active and has the highest index, the mobile terminal MT $\nu_k + \mu$, that is active and has the $(N_c + 1)^{st}$ highest index, and the mobile terminal MT $\nu_k + \gamma$, that is active and has the $(N_c + 2)^{nd}$ highest index. Clearly, $\nu - \mu \geq N_c$, and $\mu - \gamma \geq 1$. Two possible scenarios exist due to Lemma 2. In the first scenario, the N_c words, that have the highest index, are decoded in the $L + 1$ base stations, where each of them has a connection with MT $\nu_k + \nu$, while in the second scenario, only $q < N_c$ words can be decoded for the subnetwork that is formed by the $L + 1$ indices of base stations, where each of them has a connection with MT $\nu_k + \nu$.

At most $2N_c - 1$ base stations are available to decode a minimum of $N_c + 1$ words for the first scenario. Now for MT $\nu_k + \mu$, the upper $2N_c - 1$ base stations in $\mathcal{L}_k$ have to decode not less than or equal to N_c words originating at the upper $2N_c - 1$ mobile terminals in $\mathcal{L}_k$ when $\mu < \nu - (L + 1)$. This is not possible due to Lemma 2. Consider $\mu \geq \nu - (L + 1)$. It is the case that either one of the base stations with indices $\{\nu_k + \mu - L + 1, \cdots, \nu_k + \nu - (L + 1)\}$ decodes $W_{\nu_k+\mu}$ due to Lemma 2 and no other word with indices in $\{\nu_k + \nu - (L+1), \cdots, \nu_k + \mu - 1\}$ is decoded in the set of base stations with indices in $\{\nu_k + \mu - L, \cdots, \nu_k + \nu - (L+1)\}$ due to Lemma 1, or base station BS $\nu_k + \mu - L$ decodes $W_{\nu_k+\mu}$ such that at most $N_c - 1$ base stations are left to decode a minimum of N_c words. This is not possible.

Furthermore, one of the base stations with indices in $\{\nu_k+\mu-L+1, \cdots \nu_k+\nu-(L+1)\}$ decodes $W_{\nu_k+\mu}$, while no mobile terminal with index in $\{\nu_k+\nu-(L+1), \cdots, \nu_k+\mu-1\}$ is decoded at the base stations with index in the set $\{\nu_k+\mu-L, \cdots \nu_k+\nu-(L+1)\}$. Consider BS $\nu_k + \iota$ as the base station that decodes $W_{\nu_k+\mu}$. There exist two cases for the index γ, which are Either $\gamma \leq \iota - 1$ or $\gamma \in \{\iota, \cdots, \mu - 1\}$. If the latter holds, the observation is that a BS with an index value of no more than $\nu_k + \mu - L - 1$ decodes $W_{\nu_k+\gamma}$ due to Lemma 1. However, this implies that at least $N_c - 1$ words are left to be decoded at at most $N_c - 2$ base stations. This is not possible. If the former case holds, that is MT $\nu_k + \gamma$ is not beneath MT $\nu_k + \iota - 1$, as $\nu_k + \iota \leq \nu_k + \nu - L - 1$, then at least N_c words are left to be decoded by at most $2N_c - 2 < L + 1$ base stations. Therefore, Lemma 2 implies no more than $2N_c + 1$ active mobile terminals in $\mathcal{L}_k$, and that if this is true, then base stations in $\mathcal{L}_{k-1}$ are blocked by $\mathcal{L}_k$. More precisely, $\mathcal{L}_k$ blocks $L + 1 - (2N_c - 2) = L - 2N_c + 3$ base stations. The complete result for this scenario is illustrated in Figure 3.2 for $L = 5, N_c = 2$.

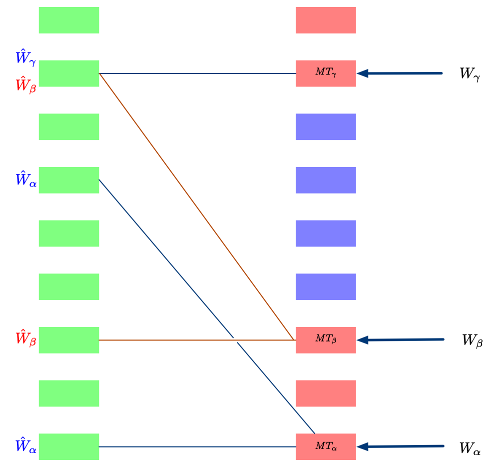


Fig. 3.2. $\mathcal{L}_k$ for $L = 5, N_c = 2$, where active terminals are colored red , and the inactive terminals are colored blue

For the second scenario, another special mobile terminal is marked. More, precisely, the N_c^{th} largest indexed active mobile terminal, which will further be referred as this MT $\nu_k + \psi$. By definition, it is that a base station with an index no more than $\nu_k + \nu - (L+1)$ decodes $W_{\nu_k+\psi}$. It follows, that either one of the base stations with index in $\{\nu_k + \mu - L + 1, \cdots \nu_k + \nu - (L+1) - 1\}$ decodes $W_{\nu_k+\mu}$, while no other mobile terminal with index in $\{\nu_k + \nu - (L+1) - 1, \cdots, \nu_k + \mu - 1\}$ is decoded in the base stations with index in $\{\nu_k + \mu - L, \cdots \nu_k + \nu - (L+1) - 1\}$, or that base station BS $\nu_k + \mu - L$ decodes $W_{\nu_k+\mu}$, such that $N_c - 1$ base stations are left to decode a minimum of N_c words. This is not possible and hence the former case holds. Consequently, MT $\nu_k + \gamma$ is not beneath MT $\nu_k + \nu - (L+1) - 2$, leaving no more than $2N_c - 3 < L + 1$ base stations to decode the left N_c words. Hence, base stations in $\mathcal{L}_{k-1}$ are blocked by $\mathcal{L}_k$, more precisely, more is blocked by $\mathcal{L}_k$ in contrast to the previous scenario. Thus, only the previous scenario is being considered in the following paragraphs.

In the next case, $\mathcal{L}_k$ blocks $L - 2N_c + 3$ base stations in $\mathcal{L}_{k-1}$. Furthermore, at least $2N_c$ words have to be decoded by $\mathcal{L}_{k-1}$ to exceed the average uplink puDoF of $\frac{2N_c}{2N_c+L}$, since $\mathcal{L}_k$ decodes $2N_c + 1$ words. It is that $2N_c + L - (L - 2N_c + 3) = 4N_c - 3$ base stations are provided to decode the $2N_c$ words.

In the same way the argument was outlined above for $\mathcal{L}_k$, three special mobile terminals are marked mobile terminal MT $\nu_{k-1}+\nu'$, that are active and have the highest index, the mobile terminal MT $\nu_{k-1}+\mu'$, that is active and has the $(N_c+1)^{st}$ highest index, and the mobile terminal, that is active and has the $(N_c+1)^{st}$ highest index, MT $\nu_{k-1}+\gamma'$. It is that $L-2N_c+3$ base stations of largest index are not able to contribute to decode any words, and hence a base station with index in $\{\nu_{k-1}+\nu'-(L)+1,\cdots,\nu_{k-1}+4N_c-3-1\}$ has to decode $W_{\nu_{k-1}+\nu'}$, or base station BS $\nu_{k-1}+\nu'-L$ decodes $W_{\nu_{k-1}+\nu'}$.

Assume the former and BS $\nu_{k-1}+\theta'$ decodes $W_{\nu_{k-1}+\nu'}$, then no mobile terminal in $\mathcal{L}_{k-1}$ with index in $\{\nu_{k-1}+\theta',\cdots,\nu_{k-1}+\nu'-1\}$ can be decoded in the interference set of MT $\nu_{k-1}+\nu'$ due to Lemma 1. This implies that the mobile terminal (MT $\nu_{k-1}+\delta'$), that is active and has the second highest index, would have an index value of at most $\nu_{k-1}+4N_c-5$. Since $\nu_{k-1}+\nu'-(L+1)\in\{\nu_{k-1}+\delta'-L,\cdots,\nu_{k-1}+\delta'\}$, either MT $\nu_{k-1}+\delta'$ has an association with the maximum of N_c base stations, or base stations in the interference set of MT $\nu_{k-1}+\delta'$ decode at most N_c-1 words. Consequently, there exist at most $4N_c-4-(L+1)<2N_c-5<L+1$ base stations to decode a minimum of N_c-1 words. Then, by Lemma 1, a base station with index of at most $\nu_{k-1}+2N_c-7$ must decode $W_{\nu_{k-1}+\gamma'}$, and hence the interference set of MT $\nu_{k-1}+\gamma'$ would permit no more than one word to be decoded in $L-2N_c+7$ base stations with highest index in $\mathcal{L}_{k-2}$.

For the latter case, the observation is that exactly $2N_c-1$ base stations have to decode $2N_c-1$ words. Moreover, Lemma 1 states that $\nu_{k-1}+\mu'$ would be exactly $\nu_{k-1}+N_c-1$, forcing MT$\nu_{k-1}+\mu'$ to block exactly $L+1-N_c$ base stations in $\mathcal{L}_{k-2}$. Since $N_c<\frac{L}{2}$, it is $L+1-N_c\geq 3$ and hence a minimum of three base stations are blocked, that is not less than the same number of base stations that $\mathcal{L}_k$ blocked in $\mathcal{L}_{k-1}$. It follows that the argument for $\mathcal{L}_{k-1}$ applies when $\mathcal{L}_{k-2}$ is considered. Again, three special mobile terminals are marked for $\mathcal{L}_{k-2}$, that is MT $\nu_{k-2}+\nu''$ for the mobile terminal, that is active and has the highest index, MT $\nu_{k-2}+\mu''$ for the mobile terminal, that is active and has the third highest index, and MT $\nu_{k-2}+\gamma''$

for the $(N_c+3)^{rd}$ largest indexed active mobile terminal. This leads to two cases for $W_{\nu_{k-2}+\nu''}$, which are as follows. $W_{\nu_{k-2}+\nu''}$ is either decoded in or outside the set of base stations blocked by $\mathcal{L}_{k-1}$. The latter case shows, that the active moblie terminal with secend largest index, denoted as MT $\nu_{k-2}+\theta''$ would be decoded at a base station that has an index, which is no more than MT $\nu_{k-2}+4N_c-7-1$. Lemma 1 forces the index of the mobile terminal, that is active and has the third highest index, not to exceed $\nu_{k-2}+4N_c-7-2$. Therefore, at most $4N_c-9-(L+1)<2N_c-10$ base stations would be left to decode a minimum of N_c-3 words. In contrast to the case above for $\mathcal{L}_{k-1}$, a reduction of the number of available base stations by a minimum of five is observed. Here, the number of words left to decode at these base stations decreased by no more than two. Consequently, the considered propagation pattern of inter-subnetwork interference reaches the first subnetwork, while the average asymptotic puDoF will not be increased by the extra mobile terminal, which is decoded in $\mathcal{L}_k$. Figure 3.3 depicts this inter-subnetwork interference propagation pattern.

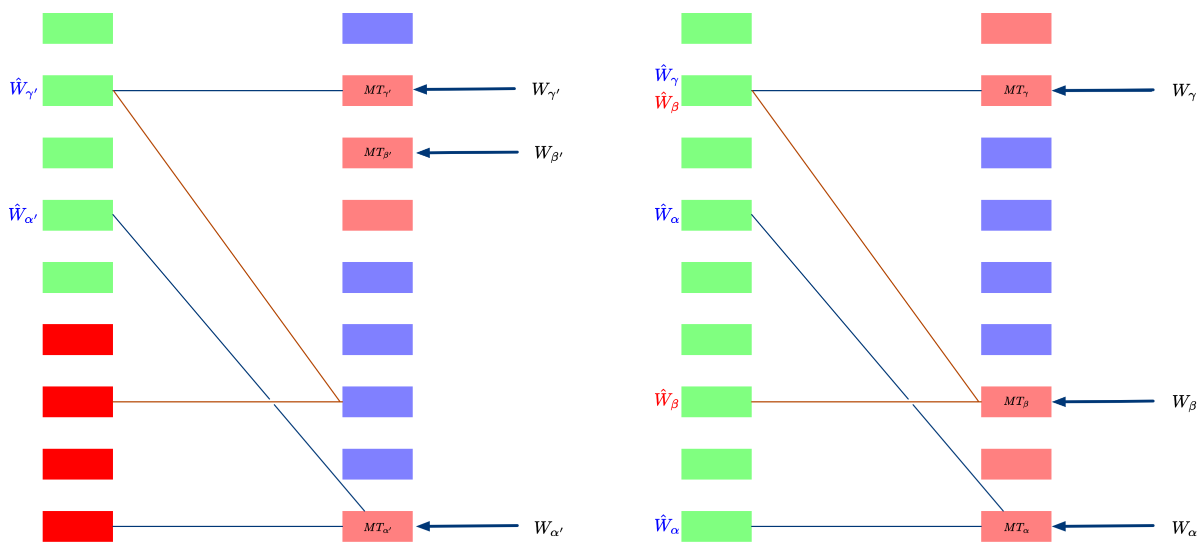


Fig. 3.3. $\mathcal{L}_k$ and $\mathcal{L}_{k-1}$ for $L=5, N_c=2$, where the base stations, that are blocked, are colored red.

Figure 3.3 demonstrates, that the pattern of interference propagation does not allow the number of additional mobile terminals, that are active, obtained in $\mathcal{L}_k$ to increase, as early as $\mathcal{L}_{k-1}$. Clearly, $\mathcal{L}_{k-1}$ is not able to even decode $2N_c$, due to the fact that MT γ' cannot be decoded in $\mathcal{L}_{k-1}$. It follows, that the overall average DoF for the subnetworks $\mathcal{L}_k$ and $\mathcal{L}_{k-1}$ cannot exceed $\frac{2N_c}{2N_c+L}$. This is exactly the proposed upper bound.

It is that at most $4N_c - 9 - (L+1) < 2N_c - 10$ base stations are left to decode $N_c - 2$ words for the former scenario, i.e. the former scenario is more restrictive than the latter one.

When $\theta_k > 0$ for the latter scenario it is that borrowing base stations to decode words originating from $\mathcal{L}_k$ either would block the $L - 2N_c + 3$ base stations, that have the highest index, in $\mathcal{L}_{k-1}$, or, due to Lemma 1, would not allow to decode any word from $\mathcal{L}_{k-1}$. Hence, this situation is as restrictive as the previous scenario. Next, the argument is elaborated in detail.

Here, the same notation is adopted as for the scenario with $\theta_k = 0$. Lemma 2 implies two possible scenarios for the subnetwork $\mathcal{L}_k$, that is the N_c words, that have the highest index, are decoded in the $L+1$ base stations, which have a connection with MT $\nu_k + \nu$, or that for the subnetwork, that is formed by the $L+1$ indices of base stations that have a connection with MT $\nu_k + \nu$, only $q < N_c$ words are decoded.

In the first scenario, the focus is on MT $\nu_k + \mu$. Lemma 2 implies that either $W_{\nu_k+\mu}$ is decoded in one of the base stations with index in $\{\nu_k+\mu-L+1, \cdots \nu_k+\nu-(L+1)\}$. Lemma 1 implies that no other word with index in $\{\nu_k+\nu-(L+1), \cdots, \nu_k+\mu-1\}$ is decoded in the group of base stations with index in $\{\nu_k+\mu-L, \cdots \nu_k+\nu-(L+1)\}$, or that $W_{\nu_k+\mu}$ is decoded at base station BS $\nu_k + \mu - L$.

In the latter case, $N_c - 1$ base stations are left in $\mathcal{L}_k$ to decode any words with an index less than $\nu_k + \mu$ originating in $\mathcal{L}_k$. Here, there exist at least $N_c - 1$ mobile terminals, that are active, with an index that is less than $\nu_k + \mu$. Here, at most $2N_c$ words can be decoded by $\mathcal{L}_k$, since exactly $N_c - 1$ base stations remain to decode $N_c - 1$ words. Therefore, the $L - N_c + 2$ base stations in $\mathcal{L}_{k-1}$, that have the highest index, decode no more than one word. Since at least $2N_c+1$ active mobile terminals reside in $\mathcal{L}_k$, this word has to originate in $\mathcal{L}_k$. Consequently, a minimum of $L-2N_c+1$ of mobile terminals in $\mathcal{L}_{k-1}$, that have the highest index, are not able to decode even a single word originating in $\mathcal{L}_{k-1}$. This is strictly greater than $L-2N_c+3$ and, for $\theta_k=0$, this is also the number of blocked base stations. This means, that less base stations in $\mathcal{L}_{k-1}$ can be used to decode the same number of words. This constraint also applies when $2N_c-q$, $q<N_c-1$ words are decoded in $\mathcal{L}_k$, since $q+1$ words can be decoded in $L-N_c+2$ base stations, that have the highest index, where all words have to originate from $\mathcal{L}_k$, which does not provide any additional decoded words in contrast to the preceding case with $\theta_k=0$.

Focus on the case where one of the base stations with index in $\{\nu_k+\mu-L+1,\cdots\nu_k+\nu-(L+1)\}$ decodes $W_{\nu_k+\mu}$, while no mobile terminal with index $\{\nu_k+\nu-(L+1),\cdots,\nu_k+\mu-1\}$ is decoded at the base stations with index $\{\nu_k+\mu-L,\cdots\nu_k+\nu-(L+1)\}$. For simplicity, denote BS $\nu_k+\iota$ as the base station that decodes $W_{\nu_k+\mu}$ is BS $\nu_k+\iota$. From this, two cases are concluded for MT $\nu_k+\gamma$, that is $\nu_k+\gamma$ is no more than $\nu_k+\iota-1$, or MT $\nu_k+\gamma$ with index in $\{\nu_k+\iota,\cdots,\nu_k+\mu-1\}$. For the latter case, Lemma 1 forces $W_{\nu_k+\gamma}$ to be decoded at a BS with index of no more than $\nu_k+\mu-L-1$. However, this implies that no more than N_c-2 base stations remain to decode a minimum of N_c-1 words. This is not possible. For the former case, that is $\nu_k+\gamma$ is no more than $\nu_k+\iota-1$, as $\nu_k+\iota$ is no more than $\nu_k+\nu-L-1$, then no more than $2N_c-2<L+1$ base stations remain to decode N_c words. However, either the $L-2N_c+3$ base stations in $\mathcal{L}_{k-1}$, that have the highest index, have to decode words originating in $\mathcal{L}_k$, or do not decode any words, such that at least $2N_c+1$ words originating in $\mathcal{L}_k$ can be decoded. Here, at least the same restriction as for $\theta_k=0$ applies. More precisely, for the case when the $L-2N_c+3$ base stations in $\mathcal{L}_{k-1}$, that have the highest index, were blocked, or could not decode any words originating from $\mathcal{L}_{k-1}$.

An additional special mobile terminal is marked for the second scenario, specifically the N_c^{th} largest indexed active mobile terminal, which is denoted as MT $\nu_k+\psi$.

It follows that a base station with an index of no more than $\nu_k + \nu - (L+1)$ decodes $W_{\nu_k+\psi}$. This implies, that either $W_{\nu_k+\mu}$ is decoded at one of the base stations with an index in $\{\nu_k+\mu-L+1, \cdots \nu_k+\nu-(L+1)-1\}$ and no other mobile terminal with an index in $\{\nu_k+\nu-(L+1)-1, \cdots, \nu_k+\mu-1\}$ is decoded at the base stations with an index in $\{\nu_k+\mu-L, \cdots \nu_k+\nu-(L+1)-1\}$, or that $W_{\nu_k+\mu}$ is decoded at base station BS $\nu_k + \mu - L$, such that $N_c - 1$ base stations remain to decode at least N_c words, forcing base stations from $\mathcal{L}_{k-1}$ to borrow. Analogous to the previous scenario, $\mathcal{L}_k$ borrows base stations to decode words but blocks $L - N_c + 1$ base stations in $\mathcal{L}_{k-1}$. But this cannot be less or equal to $L - 2N_c + 3$, which applies for $\theta_k = 0$. Therefore, only the case has to be analyzed, where $W_{\nu_k+\mu}$ is decoded in one of the base stations with an index in $\{\nu_k+\mu-L+1, \cdots \nu_k+\nu-(L+1)-1\}$ and no other mobile terminal with an index in $\{\nu_k+\nu-(L+1)-1, \cdots, \nu_k+\mu-1\}$ is decoded in the base stations with an index in $\{\nu_k + \mu - L, \cdots \nu_k + \nu - (L+1) - 1\}$. Consequently, MT $\nu_k + \gamma$ is no more than MT $\nu_k + \nu - (L+1) - 2$, such that no more than $2N_c - 3 < L+1$ base stations remain to decode the N_c words. Therefore, base stations in $\mathcal{L}_{k-1}$ are blocked and/or borrowed by $\mathcal{L}_k$, that is more than in the previous scenario.

All in all, for the case of allowing borrowing of base stations from neighboring subnetworks by subnetworks, the number of base stations, that are borrowed, forbids to scale the grown number of words, that are active, per subnetwork above $2N_c$. Consequently, the the puDoF is upper bounded by $\frac{2N_c}{2N_c+L}$.

3.4 Average Uplink-Downlink Degrees of Freedom

In this section, the zero-forcing schemes are discussed, which are used to achieve the optimal average rate for the uplink and downlink of a network with arbitrarily connectivity parameter L. In the following, the inner bounds derived in Theorem 2 are verified through proving. Moreover, assuming the inner bound in (3.3), Theorem 1 and the result in [49], which will be elaborated in Section 3.1, complete the proof of (3.5). Consequently, proving the achievability concludes the proof of Theorem 2. Note that the union of the scheme characterized in Section 3.1 and the scheme that attains the third range of (3.2) build the coding scheme that attains the inner bound for the second range of (3.3). The analysis continues by separating the network into dismembered subnetworks, where any subnetwork is formed by $2N_c + L$ subsequent pairs of transmitters and receivers. For further simplification, the two sets of indices $\mathcal{A}_{BS} = \{1, 2, \cdots, N_c\}$ and $\mathcal{B}_{BS} = \{N_c + 1, N_c + 2 \cdots, 2N_c\}$ are defined, which cover N_C consecutive base stations as well as the sets of indices $\mathcal{A}_{MT} = \{1, 2, \cdots, N_c\}$ and $\mathcal{B}_{MT} = \{N_c+L+1, N_c+L+2 \cdots, 2N_c+L\}$ which cover N_C consecutive mobile terminals. It is $\forall\ i \in \mathcal{A}_{MT}$, $\mathcal{C}_i = \mathcal{A}_{BS}$ and $\forall\ j \in \mathcal{B}_{MT}$, $\mathcal{C}_j = \mathcal{B}_{BS}$. Therefore, the optimal puDoF from Sections 3.1 and 3.3 is obtained when $N_c < \frac{L}{2}$. To obtain the inner bound in (3.3) for the region $N_c \geq L + 1$, the coding scheme is elaborated. Here, full puDoF is obtained in the uplink, analogous to the scheme in Section 3.3, by associating each mobile terminal with the $L + 1$ base stations, that have a connection with it. At this point, $\mathcal{C}_i \supseteq \{i, i-1, i-2, \cdots, i-L\} \cap [K], \forall i \in [K]$ holds. Given the set of associations $\mathcal{C}_i^D$, which is additionally needed for MT i in the downlink scheme, it follows that $\mathcal{C}_i = \mathcal{C}_i^D \cup \{i, i-1, \cdots, i-L\}\ \forall i \in [K]$.

In the downlink, a separation of the network into subnetworks occurs, where any subnetwork is formed by $L + 2\left(\lceil \frac{L+1}{2} \rceil + N_c - (L+1)\right)$ subsequent pairs of transmitters and receivers. Given $\delta = \lceil \frac{L+1}{2} \rceil$, and $\lambda = \delta + N_c - (L + 1)$, the cell association is identical for each $2\lambda + L$ BS-MT pairs. Therefore, only the cell association for the top $2\lambda + L$ BS-MT pairs needs to be elaborated. Further simplification is made by considering L as even and odd, respectively. For odd L, the subnetwork is separated into MTs with respect to their indices:

$$
\begin{aligned}
\mathcal{S}_1 &= \{\delta, \delta + 1, \cdots, \delta + \lambda - 1\}, \\
\mathcal{S}_2 &= \{2\delta + \lambda, 2\delta + \lambda + 1 \cdots, 2\delta + 2\lambda - 1\}, \\
\mathcal{S}_3 &= \{1, 2, \cdots, L + 2\lambda\} \setminus (\mathcal{S}_1 \cup \mathcal{S}_2).
\end{aligned}
$$

, while mobile terminals with indices in $\mathcal{S}_3$ are deactivated.

In the downlink, the cell associations are characterized below.

$$\mathcal{C}_i^D = \begin{cases} \{1, 2, \cdots, \lambda - 1\}, & \forall i \in \mathcal{S}_1, \\ \{\delta + \lambda, \delta + \lambda + 1, \cdots, \delta + 2\lambda - 1\}, & \forall i \in \mathcal{S}_2. \end{cases}$$

For even L, a separation of the MT's indices of a subnetwork occurs accordingly:

$$\begin{aligned} \mathcal{S}'_1 &= \{\delta, \delta + 1, \cdots, \delta + \lambda - 1\}, \\ \mathcal{S}'_2 &= \{2\delta + \lambda - 1, 2\delta + \lambda + 1 \cdots, 2\delta + 2\lambda - 2\}, \\ \mathcal{S}'_3 &= \{1, 2, \cdots, L + 2\lambda\} \setminus (\mathcal{S}_1 \cup \mathcal{S}_2). \end{aligned}$$

Here, mobile terminals corresponding to the index set $\mathcal{S}'_3$ are deactivated while the cell associations in the downlink are characterized below: $\mathcal{C}_i^D = \begin{cases} \{1, 2, \cdots, \lambda - 1\}, \\ \{\delta + \lambda, \delta + \lambda + 1, \cdots, \delta + 2\lambda - 1\}, \end{cases}$

A subnetwork size of $L + 2\lambda$ pairs of transmitters and receivers is formed for odd L, while

$$(\delta + \lambda - 1 - \delta + 1) + (2\delta + 2\lambda - 1 - (2\delta + \lambda) + 1) = 2\lambda$$

words are delivered in the downlink and the puDoF expression

$$\frac{2\lambda}{L + 2\lambda} = \frac{2\left(\frac{L+1}{2} + (N_c - (L+1))\right)}{L + 2\left(\frac{L+1}{2} + (N_c - (L+1))\right)}.$$

is achieved. For even L, a subnetwork is formed by $L + 2\lambda$ pairs of transmitters and receivers and

$$(\delta + \lambda - 1 - \delta + 1) + (2\delta + 2\lambda - 2 - (2\delta + \lambda - 1) + 1) = 2\lambda$$

words are delivered successfully in the downlink, that achieves the same inner bounds as for the case of even L, which concludes the proof for the inner bounds in (3.3).

Figures 3.4 and 3.5 exemplary illustrate this section's average uplink-downlink inner bounds.

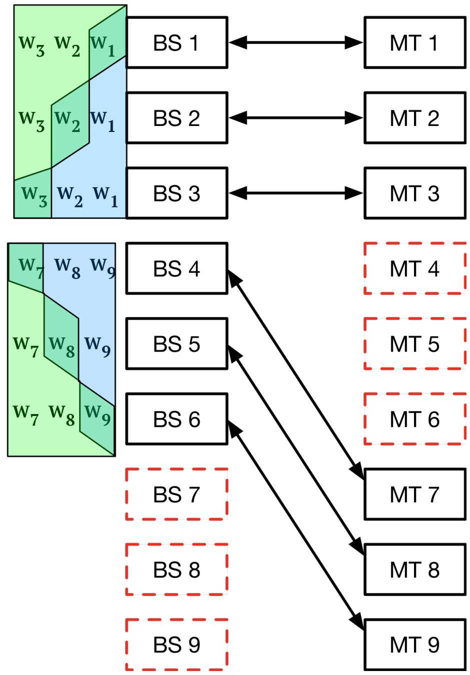


Fig. 3.4. $N_c \leq L$, $N_c = 3$, $L = 3$, where the scheme for average uplink (illustrated with green tones) and downlink (illustrated with blue tones) communication is shown

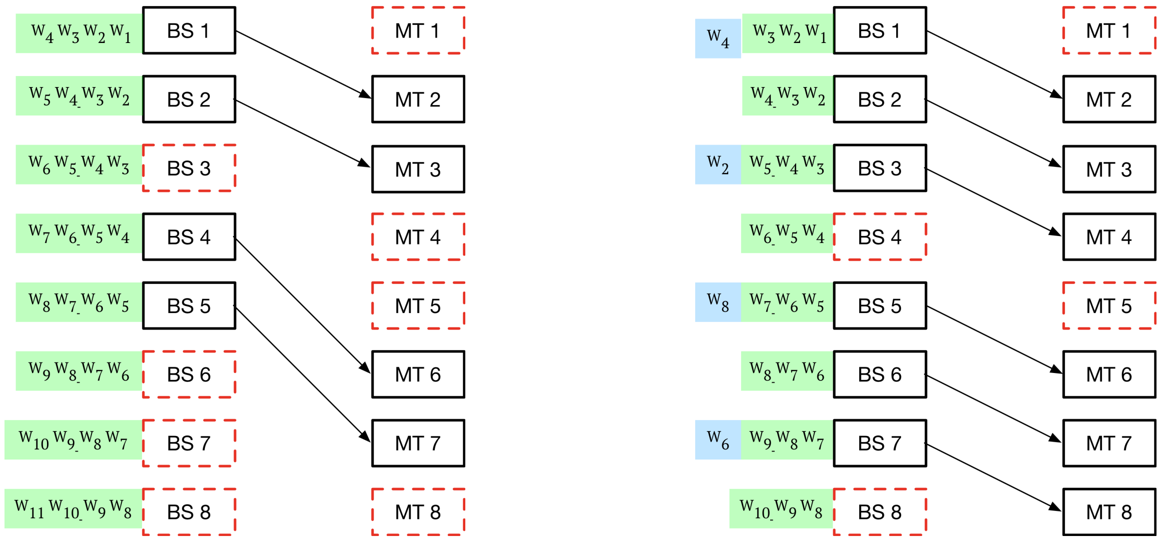
3.4.1 Converse Proof for Full Coverage Associations

In the following, it is demonstrated, that if unity DoF is achieved, i.e. when there exist associations of each mobile terminal with all base stations, that have a connection to it, then optimality of the downlink puDoF as characzerized in Theorem 3 is guaranteed. More precisely, the given scenario is constrained by full coverage cell association schemes.

To proof Theorem 3, the result on downlink cooperative zero-forcing in [50, Lemma 2]is reused.

The definition below, that targets any cooperative zero-forcing scheme, builds the foundation of the proof.

Consider any set $\mathcal{S} \subseteq [K]$. Denote $\mathcal{V}_{\mathcal{S}}$ as the set of receivers, that are active and have a connection to transmitters in $\mathcal{S}$. Further, for any transmitted message W_i,



(a) L odd, $L = 3$, $N_c = 4$

(b) L even $L = 2$, $N_c = 4$

Fig. 3.5. Presented is the downlink scheme with the required associations for optimal uplink, that attains the lower bound described in equation (3.3) when $N_c \geq L + 1$

it is $\tilde{\mathcal{C}}_i \subseteq \mathcal{C}_i$, which is formed by every index of those base stations, that actively transmit W_i. Moreover, a matching between a set of base station transmitters and a set of mobile terminal receivers occurs, whenever a matching between the vertices corresponding to these nodes in the bipartite interference graph occurs.

Lemma 4 ([50]) *Using cooperative zero-forcing in the downlink, it has to be the case that for each transmitted message W_i, there exists a matching between transmitters in $\tilde{\mathcal{C}}_i$ and the set of active receivers connected to them $\mathcal{V}_{\tilde{\mathcal{C}}_i}$.*

In the following, Theorem 3 is proved. Denote n_j as the number of receivers, that are active and indexed, such that it does not exceed j. It will be demonstrated that the statements below for any schemes based on downlink zero-forcing with a full coverage association in a large network,

$$\forall i \in \mathbf{Z}^+, n_{(2\lambda+L)i} \leq 2\lambda i, \tag{3.11}$$

holds. More precisely, the maximum number of receivers, that are active, in any subnetwork is 2λ, if the network is separated into subnetworks with successive $2\lambda + L$ pairs of transmitters and receivers. Without loss of generality, consider L as odd. Furthermore, consider that the first base station (BS 1) is active. Otherwise the argument can be derived from the first base station, that is active. Further, consider $|\mathcal{C}_i| \leq N_c - (L+1) + i, \forall i \in \{1, 2, \cdots, L\}$, while this added restriction does not change the value of the puDoF because this is imposed for a fixed number of mobile terminals, and does not depend on the size of the network.

(3.11) is proved by induction, where

$$n_{2\lambda+L} \leq 2\lambda, \tag{3.12}$$

builds its base, while every receiver in $\{\lambda+L+1, \lambda+L+2, \cdots, 2\lambda+L\}$ is activated when attaining the bound tightly, and no more than δ receivers, that are inactive, indexed in $\{\lambda+1, \lambda+2, \cdots, \lambda+L\}$ exist. Moreover, if the top subnetwork 2λ receivers, that are active, reside in the first subnetwork, then the bottom λ receivers in the same subnetwork are activated, while no more than δ receivers, that are inactive, exist among the L previous receivers.

The induction step is completed, when it is proven, that for the i^{th} subnetwork, if $n_{(2\lambda+L)(i-1)} = 2\lambda(i-1)$, and it is either that there exist no more than δ receivers, that are inactive, among the previous L receivers and the bottom λ receivers in subnetwork $i-1$ are activated, or the previous L receivers have no more than $\delta - 1$ receivers, that are inactive, and the bottom $\lambda - 1$ receivers in subnetwork $i-1$ are active, then $n_{(2\lambda+L)i} \leq 2\lambda i$ holds, and it is either that the previous L receivers have no more than δ receivers, that are inactive, and that the bottom λ receivers in subnetwork i are active, or the previous L receivers have no more than $\delta - 1$ receivers, that are inactive, and that the bottom $\lambda - 1$ receivers in subnetwork i are active for the case that the bound is attained tightly, then either the bottom λ receivers in subnetwork i are activated and the previous L receivers have no more than δ receivers, that are inactive, or the bottom $\lambda - 1$ receivers in subnetwork i are activated and the previous L receivers have no more than $\delta - 1$ receivers, that are inactive. To simplify the proof, the case, where the bound in (3.11) is not attained tightly for any value of i, is neglected, since this case can be derived directly from the induction proof.

Next, the attention is restricted to the case where BS 1 is transmitting to MT 1, that is $I(W_1; X_1) > 0$. Clearly, MT 1 is only allowed to have associations with $N_c - (L+1) = \lambda - \delta$ base stations different than BS 1. Denote q as the highest index of a base station that transmits actively W_1. Due to Lemma 4, it is $n_{q+L} \leq \min(q, \lambda - \delta + 1)$ and therefore $n_{\lambda+\delta} \leq \lambda - \delta + 1 = \lambda + \delta - L$, leading to the base statement. In the following, it is demonstrated, that the base statement is valid even after loosen the constraint that MT 1 is active.

Denote $k \leq \delta$ as the highest index that belongs to an active mobile terminal, and without loss of generality, asssume that BS 1 transmits actively W_k. W_k is only allowed to have associations with $N_c - (L + 1)$ base stations that are higher indexed than k, and due to Lemma 4, it is that $|\mathcal{V}_{\tilde{\mathcal{C}}_k}| \leq N_c - (L + 1) + k$. More precisely, as the top k base stations and other $N_c - (L + 1)$ base stations can only transmit W_k, no more than $N_c - (L + 1) + k = \lambda - \delta + k$ of receivers, that are inactive, have connections with transmitters that actively transmit W_k.

Denote q as the highest index that belongs to a base station actively transmitting W_k, then $n_{q+L} \leq \min(q, \lambda - \delta + k)$ follos from Lemma 4.

The assumption $k \leq \delta$ leads to $n_{\lambda+L} \leq \lambda$, such that the base statement follows.

For the case that the smallest index, that belongs to an mobile terminal $k > \delta$, that is active, only $n_{\lambda+L} > \lambda$ needs to be considered. As analogous to the above cases, the base statement follows otherwise. Clearly, if $n_{\lambda+L} > \lambda$ and the top δ receivers are deactivated, then it follows that $\delta > 1$.

For simplicity, it is assumed that $n_{\lambda+L} = \lambda + 1$, and denote m as the highest index that belongs to a mobile terminal, that is active, among the top $\lambda + L$. Note the scenarios below:

- If all the λ receivers, that are active and indexed below m do not reside in $\mathcal{V}_{\tilde{\mathcal{C}}_m}$, then all the transmitters actively transmitting W_m are indexed at least equal to $m - \delta + 2$. Since only those $\lambda - \delta$ transmitters, which are outside of W_m's interference set, can carry W_m and due to Lemma 4, there are no more than $\lambda - \delta + 1$ receivers, that are active, among the bottom $\lambda + \delta - 1$ receivers in the first subnetwork. More precisely, no more than $\lambda - \delta$ active receivers, that are indexed greater than m, would reside in the first subnetwork, concluding that $n_{2\lambda+L} \leq \lambda + 1 + \lambda - \delta < 2\lambda$.

- If all the λ receivers, that are active and indexed below m reside in $\mathcal{V}_{\tilde{\mathcal{C}}_m}$. Since only those $\lambda - \delta$ transmitters, which are outside of W_m's interference set, can carry W_m and due to Lemma 4, a minimum of $\delta + 1$ transmitters in the interference set of MT m are actively transmitting W_m. Consequently, a minimum of δ receivers succeeding MT m are not active, from which it follows that $n_{2\lambda+L} < 2\lambda$ here as well.

The cases discussed conclude the validity of the base statement. In the following, the induction step is elaborated, while the focus is on the second case ($i = 2$) for further simplification. The same proof as for the base proof applies for the case when no base station exists in the first subnetwork, that actively transmits a message originating in the second subnetwork. Therefore it is sufficient to focus on the case when base stations from the first subnetwork are used. Denote k as the smallest index, that belongs to a mobile terminal, that is active, in the second subnetwork, and denote q as the smallest index that belongs to a base station, that actively transmitts W_k. Note, that the bottom λ receivers in the top subnetwork are active and due to Lemma 4, it means for $q \geq \lambda + L + 1$, that there exist a base station in the second subnetwork, which actively transmits W_k. The rest of the proof is completed in the same manner as for the base case. For $q_k < \lambda + L + 1$, interference occurs at all active bottom λ receivers of the first subnetwork if W_k is transmitted. Therefore, a minimum of $\delta + 1$ base stations in the interference set of MT k would actively transmit

W_k. Consequently, if $k \geq \delta$, W_k is actively transmitted by a minimum of one base station originating in the second subnetwork. The rest of the proof follows the base case. If W_k is not actively transmitted by any transmitter originating in the second subnetwork and $k \leq \delta - 1$, then a minimum of δ receivers that are higher indexed then k are inactive, due to the interfering message W_k. It can be demonstrated by an analogous argument as in the base case, that $n_{4\lambda+2L} \leq 4\lambda$, and $k = 1$ by attaining the bound tightly, while the bottom $\lambda - 1$ receivers originating from the second subnetwork are active, and the L previous receivers have a maximum of $\delta - 1$ receivers, that are inactive. Moreover, proving the induction step for $i \geq 3$ would follow the same manner as for the case with $i = 2$.

This concludes the proof of the induction, as well as the validity of (3.11) for all positive integer values of i, such that the theorem statement is valid.

3.4.2 Converse Proof for Wyner's Linear Network ($L = 1$)

The information-theoretic optimality of the lower bound of Theorem 2 for $L = 1$ is demonstrated in the following paragraphs. In particular, Theorem 4 is proven. Here, the proof for the inner bound in Section 3.4 is used to derive the proof of achievability for Theorem 4. If $N_c = 1$, the upper bound follows since for each of the downlink and uplink sessions, the maximum per user DoF is $\frac{2}{3}$, which also holds for the case when the cell association between the uplink and downlink are changed. [49] discusses the proof of the downlink scenario. As the proof of the uplink scenario is analogous to the downlink scenario, the rest of this section concentrates on the more complex case of $N_c \geq 2$. The focus is first on the supplementary lemmas for determining a converse for the uplink scenario below in order to state the main argument.

Lemma 5 *Given any cell association and any coding scheme for the uplink, the per user DoF cannot be increased by adding an extra association of mobile terminal i to base station j, where $j \notin \{i, i-1\}$.*

Proof The previous lemma implies that an association of any mobile terminal with a base station, which has no connection to it, is not *useful* for the uplink case

The main scenario, that validates this lemma is that in contrast to the downlink case, information of a message at a base station does not permit for the potential propagation of the interference produced by this message beyond the two original receivers that have a connection with the transmitter that is liable for message-delivering.

More precisely, regardless of which cell association is applied to mobile terminal i, W_i does only interfere at base stations i and $i-1$. Therefore, if this message is located at any other base station, then it does not aid neither in interference-cancellation nor in message-decoding. The detailed discussion of the formal argument can be found below

For any cell association scheme, apply a reliable communication scheme with block length n, where signal $\hat{Y}_k^n = f_k(Y_k^n, \{W_i : k \in \mathcal{C}_i\})$ is used by the decoder at each receiver that is indexed as k to get an estimate of W_k. One can get the signal $\hat{Y}_k^n$ by utilizing a - possibly random - function f_k from the received signal Y_k^n, and side information about all the messages, that have an association with BS k. It is demonstrated that under these conditions, one can always build a reliable communication scheme, where the decoder at each receiver that is indexed as k makes use of a signal $\tilde{Y}_k^n = \tilde{f}_k(Y_k^n, \{W_i : k \in \mathcal{C}_i \cap \{i-1, i\}\})$ to get an estimate of W_k. The signal $\tilde{Y}_k^n$ is obtained by making use of a function $\tilde{f}_k$ from the received signal Y_k^n, and information about all the messages, whose mobile terminal has a connection to BS k and have associations with BS k. Moreover, an independent random variable Q_i is built for any message W_i, which is *stochastically equivalent* to W_i, that is, Q_i has the same alphabet and distribution as W_i. It follows that,

$$\tilde{f}_k(Y_k^n, \{W_i : k \in \mathcal{C}_i \cap \{i-1, i\}\}) = f_k\left(Y_k^n, \{W_i : k \in \mathcal{C}_i \cap \{i-1, i\}\}, \{Q_i : k \in \mathcal{C}_i, k \notin \{i-1, i\}\}\right).$$

Denote R_k as the rate attained for user k in the considered reliable communication scheme. Consider the true statement below.

$$
\begin{aligned}
n \sum_k R_k &= \sum_k \mathsf{H}\left(W_k\right) && (3.13)\\
&\overset{(a)}{\leq} \sum_k I\left(W_k; \{\hat{Y}_i^n : i \in [K], k \in \mathcal{C}_i \cap \{i-1, i\}\right) + o(n) && (3.14)\\
&\overset{(b)}{=} \sum_k I\left(W_k; \{\tilde{Y}_i^n : i \in [K], k \in \mathcal{C}_i \cap \{i-1, i\}\right) + o(n) && (3.15)\\
&= \sum_k \mathsf{H}\left(W_k\right) - \mathsf{H}\left(W_k | \{\tilde{Y}_i^n : i \in [K], k \in \mathcal{C}_i \cap \{i-1, i\}\right) + o(n) && (3.16)
\end{aligned}
$$

with the entropy function for discrete random variables, that is $\mathsf{H}(.)$, and (a) follows from Fano's inequality and the above assumption that the assumed reliable communication scheme makes use of the signals $\hat{Y}_k$ for decoding, and the fact that only received signals corresponding to base stations that have an association and connection with a mobile terminal of a message can be used for message-decoding. Further, (b) is true since for each $i, k \in [K]$ such that $k \in \mathcal{C}_i$ and $k \notin \{i-1, i\}$, the received signal Y_k does not depend on W_i. Therefore substituting W_i with Q_i leaves the joint distribution of the participating random variables in the formulation of the mutual information of (3.15) equal to the one in (3.14). Hence,the following is true,

$$
\sum_k \mathsf{H}\left(W_k | \{\tilde{Y}_i^n : i \in [K], k \in \mathcal{C}_i \cap \{i-1, i\}\right) = o(n). \qquad (3.17)
$$

Therefore, the rates $R_k, k \in [K]$, are attainable in the built scheme. Consequently, the lemma statement holds.

■

Two possible scenarios for selecting the cell association of mobile terminal i can occur due to Lemma 5. It is the case that either mobile terminal i is associated jointly with base stations i and $i-1$ or only one of these base stations. For the latter case, Lemma 7 is used to introduce an upper bound of the degrees of freedom. The abstraction of [49, Lemma 4] below is used for the remaining part of the proof. Define the set of indices of transmitters exclusively carrying messages that are indexed in $\mathcal{A}$ for $\mathcal{A} \subseteq [K]$. Note, that its complement $\bar{\mathcal{U}}_{\mathcal{A}}$ covers the indices of transmitters carrying messages outside $\mathcal{A}$.

Lemma 6 *([49]) In either downlink or uplink sessions, if there exists a set $\mathcal{A}$ of messages that are decodable using a set of received signals $Y_{\mathcal{B}}$, a function f_1, and a function f_2 whose definition does not depend on the transmit power P, and $f_1(Y_{\mathcal{B}}, X_{\mathcal{U}_{\mathcal{A}}}) = X_{\bar{\mathcal{U}}_{\mathcal{A}}} + f_2(Z_{\mathcal{B}})$, then the sum DoF is bounded by the number of received signals in $Y_{\mathcal{B}}$. More precisely, $\iota \leq |\mathcal{B}|$.*

Proof The proof here is similar to the one in [49, Lemma 4], with adequate modification of the variables. Consequently, only an outline is discussed here for conciseness. Constrained by a reliable communication scheme, if the received signals $Y_{\mathcal{B}}$ are known, messages $W_{\mathcal{A}}$ can be decoded reliably, such that the transmit signals $X_{\mathcal{U}_{\mathcal{A}}}$ can be reconstructed. For the case that the remaining transmit signals $X_{\bar{\mathcal{U}}_{\mathcal{A}}}$ can be reconstructed, all messages could be decoded. The hypothesis of the statement of the lemma implies that the ambiguity in reconstructing the remaining transmit signals results from Gaussian noise, which has no influence on the degrees of freedom. Consequently, the bound of the sum DoF is identical to the number of received signals used for decoding all messages $|\mathcal{B}|$. ■

Clearly, the set $\mathcal{B} = \mathcal{A}$, and $\bar{\mathcal{U}}_{\mathcal{A}} = \cup_{i \notin \mathcal{A}} \mathcal{C}_i$ for the downlink case, while $\mathcal{U}_{\mathcal{A}} = \mathcal{A}$ holds for the uplink scenario.

Lemma 7 *If either mobile terminal i or mobile terminal $i+1$ is not associated with base station i, i.e., the following holds,*

$$|\mathcal{C}_i \cap \{i\}| + |\mathcal{C}_{i+1} \cap \{i\}| \leq 1, \tag{3.18}$$

then it is either the case that the received signal Y_i can be ignored in the uplink without affecting the sum rate, or it is the case that the uplink sum DoF for messages W_i and W_{i+1} is at most one, i.e., $d_i + d_{i+1} \leq 1$.

Proof For the case where an association of base station i exists neither with W_i nor with W_{i+1}, it is trivial that Y_i can be neglected for the uplink scenario. Moreover, if only one of the associations of base station i with the two messages exists but is not decodable from Y_i in the uplink, then this received signal can be neglected. In the remainder of this case, the focus is on the scenario when exactly one association of the base station i with W_i or W_{i+1} exists and can be successfully decoded from Y_i in the uplink. Without loss in generality, say that W_i is the message that has an association with base station i. A new network is formed identical to the original with the exception that this occurs by forcing all messages in the network different than W_i and W_{i+1} to be deterministic. It follows that the sum DoF $\iota = d_i + d_{i+1}$. Observe that d_i and d_{i+1} can only grow in this new context, and thus if an upper bound is derived on their sum, it then applies to the original values. Hence, Lemma 6 is used with $\mathcal{A} = \mathcal{B} = \{i\}$ and $d_i + d_{i+1} \leq 1$ is obtained.

■

Starting with the case $N_c = 2$, that is each mobile terminal can have an association with two base stations. For a scheme of a fixed cell association, split the network indices into sets; each formed by three consecutive indices,

Denote q as the fraction of those subnetworks with central base station, that has an association with no more than one of the mobile terminals, that have a connection with the considered base station. It is demonstrated, that the uplink puDoF cannot exceed $(1-q) + \frac{5}{6}q$, and the extra puDoF because of downlink transmission does not exceed $\frac{2}{3}(1-q) + \frac{5}{6}q$. Therefore it is $\tau(L=1, N_c=2) \leq \frac{5}{6}$ as described in (3.7). Starting with the uplink part, for each subnetwork, that has a central mobile terminal, which has no association with the two base stations, that have a connection with the considered mobile terminal, Lemma 7 applies to a minimum of one of these two base stations set as base station i. Denote q_1 as the fraction of these kind of subnetworks, where $d_i + d_{i+1} \leq 1$ holds according to Lemma 7, q_2 is the fraction of such subnetworks, where Y_i can be neglected in the uplink scenario according to Lemma 7.

Furthermore, it is $q = q_1 + q_2$. Here, the uplink puDoF cannot exceed $1 - \frac{1}{3}q_1$, since in each subnetwork counting towards q_1, the maximum achievable DoF is 2 in the uplink for the three users of the subnetwork. By ignoring at least q_2 received signals in the uplink , the uplink puDoF cannot exceed $1 - \frac{1}{3}q_2$. Consequently, the uplink puDoF cannot exceed $1-\frac{1}{3}\max(q_1, q_2)$, demonstrating that it does not exceed $1 - \frac{1}{6}q$. Due to the downlink transmission, the extra puDoF will be bounded in the following paragraphs.

Here, consider any subnetwork, that has a central mobile terminal i, that has an association with the two base stations, that have a connection with the considered mobile terminal. For any of those subnetworks Lemma 5 is applied within the subnetwork with $\mathcal{A} = \{i-1, i+1\}$. It follows that a maximum DoF of 2 can be attained for the three users in the subnetwork (note that $\{i + 1\} \subseteq \mathcal{U}_{\mathcal{A}}$), such that at least $\frac{1}{3}(1-q)$ per user DoF are lost. In addition to that, it only needs to be shown, that at least $\frac{1}{6}q$ per user DoF has to be lost. Denote $\mathcal{S}$ as the superset with elements, each formed by a set of three indices, that portray subnetworks, that have their central mobile terminals associated with no more than one of the base stations, that have a connection to the considered mobile terminal. Here, it only has to be demonstrated, that in a large network, at least a DoF of $\frac{|\mathcal{S}|}{2}$ has to be lost. The following two observations based on upper bounding the downlink DoF build the foundation of the proof:

- **Fact 1:** The achieved DoF for a set of five messages with subsequent indices does not exceed 4. This is implied by using the lemma for irreducible message assignments of [49] to the central message, and then using Lemma 5 with $\mathcal{A}$, that is formed by all five indices but the central one.

- **Fact 2:** A set of three messages with subsequent indices, where the central message has an association with two of the base stations, which have a connection with its mobile terminal, then the attained DoF cannot exceed 2, which is implied by using Lemma 5 with the set $\mathcal{A}$, which is formed by all three indices but the central one.

Assume it is the separation of $\mathcal{S}$, such that every maximal set of subnetworks with successive indices resides in one partition. Any subset $\mathcal{P} \subseteq \mathcal{S}$ describing a partition, that is of an even number of elements or an odd number greater than 3, loses at least $\frac{|\mathcal{P}|}{2}$ DoF according to Fact 1. Hence, it is sufficient to consider only paritions with 1 or 3 elements. For $|\mathcal{P}| = 3$, at least 1 DoF has to be lost among the first five messages included in the first two subnetworks in the partition due to Fact 1. Also, Fact 1 implies that there exist a DoF lost among the five messages, that is the last four messages in $\mathcal{P}$ and the consecutive message which is located at the top of a subnetwork - not in $\mathcal{S}$ - whose uplink DoF was previously upper bounded by 3 and downlink DoF by 2; denote this subnetwork $\tilde{s}$. If the succeeding subnetwork to $\tilde{s}$ is in $\mathcal{S}$, then the DoF of the five messages consisting of the last two in $\tilde{s}$ are bounded and the three of the succeding subnetwork using Fact 1. Consequently, at least an extra DoF has to be lost, such that more than $\frac{|\mathcal{P}|}{2}$ extra DoF, that were not considered before, have to be lost.

For the case that the succeding subnetwork to $\tilde{s}$ is also not in $\mathcal{S}$, then proceed with the set of subsequent subnetworks that is shaped of $\tilde{s}$ and all consecutive subnetworks outside $\mathcal{S}$. The observation is that either each mobile terminal, but the top one, in the considered set of subnetworks has an association with the two base stations, that have a connection to the considered mobile terimals, or at least $\frac{1}{2}$ DoF are lost according to Lemma 7 in the uplink caused by associations in these subnetworks (here, an argument applies, that is analogous to the above uplink upper bound). It follows that a DoF of $\frac{|\mathcal{P}|}{2}$ is lost overall, which were not considered before. If it is the former, then, due to Fact 2, 1 DoF is lost among the three messages formed by the second and third in $\tilde{s}$ and the first in the succeeding subnetwork. Again, Fact 2 is applied among the three messages formed by the second and third in the current subnetwork and the first in the succeeding subnetwork, as long as the succeeding subnetwork is outside $\mathcal{S}$.

If the following subnetwork is in $\mathcal{S}$, then Fact 1 implies that an extra DoF is lost among the five messages formed by the second and third in the current subnetwork, and the three of the succeeding subnetwork. Therefore, $\frac{|\mathcal{P}|+1}{2}$ DoF is lost because of the subnetworks in $\mathcal{P}$ and the first subnetwork in the following set in the partition. Next, the argument is started over again from the second subnetwork in the succeeding set of the partition, instead of the first subnetwork. For brevity, the details are not elaborated, since it followos an analogous argument to the current one. It was demonstrated, that if $|\mathcal{P}| = 3$, then at least $\frac{1}{2}$ puDoF is lost by considering all subnetworks in $\mathcal{P}$. Thus, consider the case for $|\mathcal{P}| = 1$ remains to be elaborated. Here, Fact 1 is used to derive the bound of the DoF for the five messages formed by the three in the subnetwork of $\mathcal{P}$ and the last in the preceding subnetwork and the first in the succeeding subnetwork. The proof follows in an analogous manner to that for the case when $|\mathcal{P}| = 3$, with the exception that both preceding and succeeding subnetworks are considered, instead of only succeeding subnetworks. The main concept here is that a DoF bound that contains a message, different than the central one, in a subnetwork outside $\mathcal{S}$ implies a DoF loss, either in uplink or downlink, of at least $\frac{1}{2}$. For brevity, the extension to the case for $N_c > 2$ for the previous argument is skipped, as it is unambiguous. The main concept considers subnetworks, that are formed by $2N_c - 1$ users, and by reusing the definition of q, it can be demonstrated, that the uplink puDoF does not exceed $(1-q) + \frac{4N_c-3}{4N_c-2}q$, while the additional puDoF does not eceed $\frac{2N_c-2}{2N_c-1}(1-q) + \frac{4N_c-3}{4N_c-2}q$, due to downlink. Therefore, it is $\tau(L=1, N_c) \leq \frac{4N_c-3}{4N_c-2}$ as in (3.7). The network is separated into $2N_c - 1$ sized subnetworks, and the argument for bounding the uplink DoF is identical to the on for $N_c = 2$, due to Lemma 5. The downlink argument follows an analogous pattern as in the case for $N_c = 2$, with the exception to substitute Fact 1 above to involve a bound on the DoF of $2N_c$ for every subsequent $2N_c + 1$ messages, and substituting Fact 2 above to involve a bound of $2N_c - 2$ DoF for every subsequent $2N_c - 1$ messages with a central message that has a full coverage associations.

3.5 Discussion

3.5.1 When Separate Uplink-Downlink Optimization is Sub-optimal

The findings from the previous sections show, that the average zero-forcing puDoF equals the puDof in the uplink or downlink for sufficiently small cell association constraints $\left(N_c \leq \frac{L}{2}\right)$, where L denotes the connectivity parameter. Moreover, managing the cell association decisions with respect to optimizing the uplink or downlink does not decrease the puDoF. From an information theoretic perspective, it is important to mention, that for $N_c < \frac{L}{2}$ zero-forcing is rigorously none-ideal, arising from the fact that a puDoF value of $\frac{1}{2}$ can be attained by non-cooperative asymptotic interference alignment schemes. Also note the trade-off of between uplink and downlink cell association optimization for higher N_c.

3.5.2 Association Strategy for General Network Models

It is shown, that for $\frac{L}{2} < N_c \leq L$, the downlink-optimal zero-forcing puDoF is attained by the suggested scheme for maximizing the average zero-forcing puDoF, whereas the uplink-optimal zero-forcing puDoF is attained for $N_c > L$. As the latter case seems to apply for common network models, it is appealing to study if the previous case would follow the trend. Additionally, does the average optimal zero-forcing scheme guarantee optimal zero-forcing puDoFs for the uplink and downlink, respectively, very small regimes of N_c, while the downlink-optimal zero-forcing puDoF is attained for slightly higher regimes of N_C and the uplink-optimal zero-forcing puDoF is achieved for higher regimes of N_c? These assumptions can be presumed since it is assumed, that the scheme, which is used to maximize the average zero-forcing puDoF, is optimal.

3.5.3 Interference Propagation and its Impact on Converse Proofs

Lastly, it is interesting to spotlight the discrepancy in the reasoning for the zero-forcing puDoF upper bounds in downlink and uplink. The justification introduced in Section 3.4.1 and the justification utilized in [49], to prove the findings in Section 3.1, follow from the fact that the attained puDoF in each subnetwork is confined by the appropriate bound that holds for the entire puDoF. On the other hand, the arguments used in the uplink in Section 3.3 follow a different fashion, since the puDoF bound can be exceeded for some subnetworks, at the expense of not attaining it in adjacent subnetworks, due to the consequencees of borrowing or blocking base stations through subnetworks. Here, the main reason is that distributing a message over the backhaul for cooperative zero-forcing in the downlink produces interference at more mobile terminal receivers.

On the contrary, distributing a message over the backhaul for zero-forcing decoding over the uplink, no interference propagation occurs at other base station receivers. Due to interference propagation, this extra constraint in the downlink shortens the proof for the upper bound zero-forcing by considering only subnetwork-only decoding.

4. DYNAMIC COOPERATIVE INTERFERENCE MANAGEMENT

In Chapter 3, the focus is on a locally connected linear interference network with fixed channel and connectivity parameter L and the problem of finding the optimal cell association decisions is considered from the perspective of the problem of the average uplink-downlink puDoF and taking previous solutions for the downlink into account. The effect of channel fading is analyzed to the network model discussed in the preceding chapter with $L = 1$ and come up with the first solution to maximize the average puDoF in the downlink. Here, each link is independently subject to erasure events with probability p. Each transmitter knows the channel statistic prior to having messages assigned by the backhaul and has information about the channel topology after all messages are assigned. The objective is to maximize the throughput in terms of the per user Degree of Freedom averaged over all possible realizations of the network topology.

4.1 Cell Association

In the cellular downlink, notice the problem of associating MTs with cells, i.e. each transmitter serves only one receiver, where orthogonal schemes are elaborated, which are based on TDMA, and show that they are optimal.

Given the number of messages N_i at transmitter of index i where $i \in [K]$ with $\mathbf{N}^K = (N_1, N_2, \ldots, N_K)$, $\mathbf{N}^K$ is derived from the transmit sets $\mathcal{T}_i, i \in [K]$ and the converse holds due to the lemma below. The main idea is to use the notion of *irreducible* message assignments from [6]. Here, each message is available at exactly one of the transmitters, which are connected to their intended receiver, i.e. an irreducible message assignment with $M = 1$.

Lemma 8 *For any irreducible message assignment where each message is assigned to exactly one transmitter, i.e.,* $|\mathcal{T}_i| = 1, \forall i \in [K]$*, the transmit sets* $\mathcal{T}_i$*,* $i \in [K]$*, are uniquely characterized by the sequence* $\mathbf{N}^K$*.*

Proof Each transmitter, that carries a messages, has to be connected to its intended receiver. In particular, $\mathcal{T}_i \subset \{i-1, i\}, \forall i \in \{2, \ldots, K\}$, and $\mathcal{T}_1 = \{1\}$. Therefore, no more than two messages can be carried by a transmitter, where the first transmitter carries at least W_1, in other words $N_i \in \{0, 1, 2\}, \forall i \in \{2, \ldots, K\}$, and $N_1 \in \{1, 2\}$.

Given $N_i = 1, \forall i \in [K]$, then $\mathcal{T}_i = \{i\}, \forall i \in [K]$. For the case that remains, it is that $i \in \{2, \ldots, K\}$ such that $N_i = 0$, due to $\sum_{i=1}^{K} N_i = K$. This case is discussed in the remaining part of the proof. Consider the transmitters that carry no messages and denote the smallest index of these transmitters as x, i.e., $x = \min\{i : N_i = 0\}$. In the following paragtaphs, it is demonstrated that the transmit sets $\mathcal{T}_i, i \in \{1, \ldots, x\}$ are reconstructed from the sequence $(N_1, N_2, \ldots, N_x)$. Clearly, $\mathcal{T}_i \in [x], \forall i \in [x]$, and because $N_x = 0$, it is that $\mathcal{T}_i \notin [x], \forall i \notin [x]$. Hence, $\sum_{i=1}^{x-1} N_i = x$. Due to $\mathcal{T}_i \subset \{i-1, i\}, \forall i \in \{2, \ldots, x\}$, it follows that no more than one transmitter carries two messages, that is in the first $x-1$ transmitters. It is $\sum_{i=1}^{x-1} N_i = x$, and $N_i \in \{1, 2\}, \forall i \in [x-1]$, and thus is $y \in [x-1]$ such that $N_y = 2$, and $N_i = 1, \forall i \in [x-1] \backslash \{y\}$. Clearly, messages W_y and W_{y+1} are available at transmitter y, and messages W_{j+1} are available at each transmitter with an index $j \in \{y+1, \ldots, x-1\}$, and message W_j is available at each transmitter with an index $j \in \{1, \ldots, y\}$. The following transmit sets are obtained: $\mathcal{T}_i = \{i\}, \forall i \in [y]$ and $\mathcal{T}_i = \{i-1\}, \forall i \in \{y+1, \ldots, x\}$. Consider the network as a series of subnetworks, each having its last transmitter either inactive or it is the last transmitter in the network. The transmit sets in the subnetwork are obtained in an analogous procedure to the transmit sets $\mathcal{T}_i, i \in [x]$ for the case when the last transmitter in a subnetwork is inactive. For the case that transmitter K being the last transmitter in the subnetwork and $N_K = 1$, each message residing in this subnetwork is carried by the transmitter of same index.

■

The objective of Lemma 8 is to elaborate message assignment strategies for networks of larger size by using repeating patterns of short ternary strings. For a ternary string $\mathbf{S} = (S_1, \ldots, S_n)$ of fixed length n such that $\sum_{i=1}^{n} S_i = n$, $\mathbf{N}^K$, $K \geq n$ as characterized below:

- $N_i = S_{i \bmod n}$ if $\quad i \in \left\{1, \ldots, n \left\lfloor \frac{K}{n} \right\rfloor\right\}$,
- $N_i = 1$ if $i \in \left\{n \left\lfloor \frac{K}{n} \right\rfloor + 1, \ldots, K\right\}$.

In the following, all possible message assignment strategies are investigated, which fulfill the cell association constraint, through ternary strings with respect to the previous representation . Here, only irreducible message assignments are considered. More precisely, if two transmitters, each with two messages, of index i and j and $i < j$ exist, then there exist an index k with $i < k < j$, of a third transmitter, which has no messages assigned.

Consequently, any message assignment strategy, that characterizes a string and fulfills the cell association constraint, can only appear as one of the types below:

- $S^{(1)} = (1)$,
- $S^{(2)} = (2, 1, 1, \ldots, 1, 0)$,
- $S^{(3)} = (1, 1, \ldots, 1, 2, 0)$,
- $S^{(4)} = (1, 1, \ldots, 1, 2, 1, 1, \ldots, 1, 0)$.

Here, the main observation is the analogy between the message assignment strategies of $S^{(2)}$ and $S^{(3)}$. Furthermore, all asymptotic per user DoF values, that can be attained by one of the strategies, is attainable by one of the remaining strategies. Henceforward, the three message assignment strategies are established, which are depicted in Figure 4.1, and the per user DoF is outlined that is attained by each of them. Note that the optimal message assignment strategy for any value of p can be derived from one of the three proposed strategies. Consider the definition of greedy TDMA schemes below.

Definition 13 *A TDMA scheme is* greedy*, if it can be designed by scanning the messages in ascending order of index, and delivering each if it is enabled and can be delivered without causing interference at a previously activated receiver.*

Greedy TDMA schemes are optimal due to the following observation.

Proposition 1 *The DoF-optimal TDMA scheme is greedy for any realization of the dynamic linear interference network and any fixed message assignment that respects the cell association constraint $M = 1$.*

Proof Observe, that, due to the definition of convexity in [28], any realization of the dynamic linear network restricted by a cell association is *convex*. Also, note that the definition of Left-to-Right orthogonal schemes in [28] is identical o the definition of greedy TDMA schemes above. It is important to mention, that there does not exist a dependency of TDMA schemes on the availability of the channel state information at the transmitters. Thus, [28, Theorem 1] implies the proposition statement. ■

For each of the candidate message assignment strategies, the per user DoF, which is achieved by the optimal TDMA scheme, is described. Accordingly, the focus is on the massage assignment strategy characterized by the string of the form $S^{(1)} = (1)$, where each message is available at the transmitter of same index.

Lemma 9 *Under the restriction to the message assignment strategy $\mathcal{T}_{i,K} = \{i\}, \forall K \in \mathbf{Z}^+, i \in [K]$, and orthogonal TDMA schemes, the average per user DoF is given by,*

$$\begin{aligned} \tau_p^{(TDMA),(1)} &= \frac{1}{2}\left(\bar{p} + \bar{p}\left(1-\bar{p}^2\right)^2\right) \\ &\quad + \sum_{i=1}^{\infty} \frac{1}{2}\left(1-\bar{p}^2\right)^2 (\bar{p})^{4i+1}. \end{aligned} \tag{4.1}$$

Proof Here, a transmission scheme is elaborated with $\frac{1}{2}\left(\bar{p} + \bar{p}\left(1-\bar{p}^2\right)^2\right)$ achieved average per user DoF, and then changed to demonstrate how $\tau_p^{(TDMA),(1)}$ is achieved. W_i is transmitted if the channel coefficient $H_{i,i} \neq 0$ for any user of odd index i, such

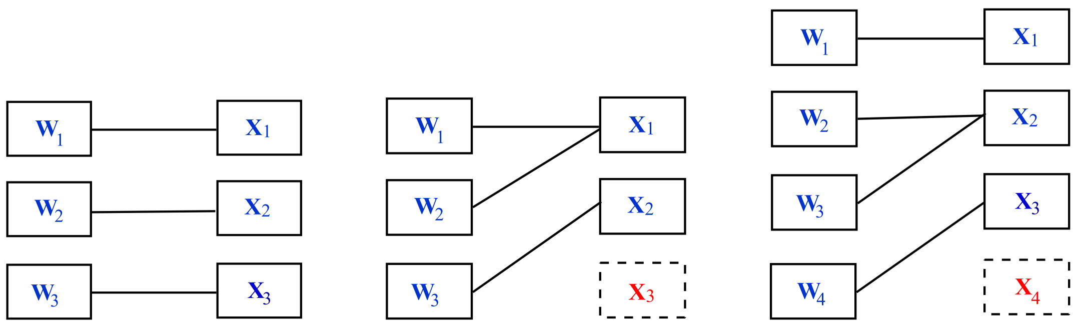


Fig. 4.1. The optimal strategy of message assignments for the cell association problem. Inactive transmit signals are depicted by dashed boxes for all network realizations. Here, (a), (b), and (c) represent the corresponding optimal strategies at large, small, and intermediate regimes of p.

that the attained rate by these users contributes $\frac{1}{2}\bar{p}$ with respect to the average per user DoF. W_i is transmitted for any user of even index i, if the following is true:

$H_{i,i} \neq 0$, W_{i-1} does not produce interference at Y_i, and transmitting W_i does not disturb the communication of W_{i+1} to its intended receiver. Observe, that this occurs if and only if $H_{i,i} \neq 0$ and $(H_{i-1,i-1} = 0$ or $H_{i,i-1} = 0)$ and $(H_{i+1,i} = 0$ or $H_{i+1,i+1} = 0)$. Hence, the attained rate by users with even indices produces $\frac{1}{2}\bar{p}\left(1-\bar{p}^2\right)^2$ with respect to the average per user DoF.

In the following the above scheme is modified to attain $\tau_p^{(TDMA),(1)}$. Again, users of odd index are prioritised. In other words, if their direct links exist, their messages are delivered, and users of even index deliver their messages whenever their direct links exist and the channel connectivity guarantees that no conflict with users, that are prioritised, occurs. But an exception is made here for the priority setting in atomic subnetworks, that are formed by an odd number of users, and the top and bottom users are of even index. These subnetworks contribute one extra DoF by allowing users of even index to be prioritised and deliver their messages. The additional term in the average per user DoF is derived as follows. The probability, that a user of even index is the first user in an atomic subnetwork, that is formed of an odd number of users in a large network, is $\sum_{i=1}^{\infty} \left(1-\bar{p}^2\right)^2 (\bar{p})^{4i+1}$. The sum DoF increases by 1 for any of these cases. Therefore the extra term to the average per user DoF is identical to half this value, because every other user is of even index. The modification above implies, that $\tau_p^{(TDMA),(1)}$ is the per user DoF, taht is attained by the greedy TDMA scheme. Proposition 1 hence implies the DoF optimality within the collection of TDMA schemes. ■

The optimality for high erasure probabilities of the previous scheme is shown in later paragraphs.

[6] elaborates the optimal message assignment for the case of no erasures, where the per user DoF was determined as $\frac{2}{3}$ and obtained by keeping every third transmitter inactive, while each transmitted message contributes a DoF of 1. In the following, the attention is on the expansion of this message assignment described in Figure 4.1(b) and its optimality is shown in the upcommining paragraphs for low probabilities of erasure.

Lemma 10 *Under the restriction to the message assignment strategy defined by the string* $S = (2, 1, 0)$*, and orthogonal TDMA schemes, the average per user DoF is given by,*

$$\tau_p^{(TDMA),(2)} = \frac{2}{3}\bar{p} + \frac{1}{3}p\bar{p}\left(1-\bar{p}^2\right). \tag{4.2}$$

Proof Consider any user of index i with $(i \bmod 3 = 0)$ or $(i \bmod 3 = 1)$. Here, message W_i is transmitted from a transmitter if the link between that transmitter and receiver i exist. Moreover, these users add a factor of $\frac{2}{3}\bar{p}$ to the average per user DoF. Consider any user of index i and $(i \bmod 3 = 2)$. Then, W_i is transmitted through X_{i-1} if the following is true:

$H_{i,i-1} \neq 0$, message W_{i-1} is not transmitted since $H_{i-1,i-1} = 0$, and transmitting W_i does not disturb the communication of W_{i+1} through X_i since $(H_{i,i} = 0)$ or $(H_{i+1,i} = 0)$. Here, these users add a factor of $\frac{1}{3}p\bar{p}\left(1-\bar{p}^2\right)$ to the average per user DoF. By applying the considered message assignment strategy, $\tau_p^{(TDMA),(2)}$ is the per user DoF, that is attained by the greedy TDMA scheme,and therefor it is the optimal average per user DoF achievable by TDMA schemes according to Proposition 1.

■

The message assignment strategy illustrated in Figure 4.1(c) is analyzed, while its optimality is demonstrated in the upcoming paragraphs, that is for a intermediate region of erasure probabilities.

Lemma 11 *Under the restriction to the message assignment strategy defined by the string* $S = (1, 2, 1, 0)$*, and orthogonal TDMA schemes, the average per user DoF is given by,*

$$\tau_p^{(TDMA),(3)} = \frac{1}{2}\bar{p} + \frac{1}{4}\bar{p}\left(1-\bar{p}^2\right)\left(1+p+\bar{p}^3\right). \tag{4.3}$$

Proof The steps of this proof follow a similar fashion as in the proof of Lemma 9

After a transmission scheme is elaborated, that achieves part of the intended rate, it will be modified in order to demonstrate how the added term is achieved, Assume that any message of odd index is delivered if the link connecting the transmitters that carry the message with the intended receiver exist. Here, each of these users add a factor of $\frac{1}{2}\bar{p}$ to the average per user DoF. Consider any user with an index i, such that $i \bmod 4 = 2$. The following has to be true to transmit W_i through X_i: $H_{i,i} \neq 0$, message W_{i+1} is not transmitted through X_i since $H_{i+1,i} = 0$, and transmitting W_i is not disturbed by the communication of W_{i-1} through X_{i-1} since either $H_{i,i-1} = 0$ or $H_{i-1,i-1} = 0$. Here, these users add a factor of $\frac{1}{4}p\bar{p}\left(1-\bar{p}^2\right)$ to the average per user DoF. Consider any user with an index i, such that $i \bmod 4 = 0$. The following has to be true to transmit W_i through X_{i-1}: $H_{i,i-1} \neq 0$ and transmitting W_i does not disturb the communication of W_{i-1} through X_{i-2} since either $H_{i-1,i-1} = 0$ or $H_{i-1,i-2} = 0$. Here, these users add a factor of $\frac{1}{4}\bar{p}\left(1-\bar{p}^2\right)$ to the average per user DoF. The scheme above is modified to demonstrate how $\tau_p^{(TDMA),(3)}$ is attained.

Given the i^{th} transmitter is inactive for every i such that $i \bmod 4 = 0$, users $\{i-3, i-2, i-1, i\}$ are isolated from the rest of the network for every i such that $i \bmod 4 = 0$. More precisely, a subnetwork is formed by these users. The modification is elaborated for the first four users, while the modification for each following set of four users follow an analogous fashion. For the case, when no interference is produced at Y_2 by W_1, since $H_{1,1} = 0$ or $H_{2,1} = 0$, and it is $H_{2,2} \neq 0$, $H_{3,2} \neq 0$, $H_{3,3} \neq 0$, and $H_{4,3} \neq 0$. Here, an atomic subnetwork is formed by users $\{2, 3, 4\}$ with probability $(1-\bar{p}^2)\,\bar{p}^4$. Also, messages W_2 and W_4 are prioritised instead of message W_3. Therefore the sum DoF for messages $\{W_1, W_2, W_3, W_4\}$ is raised by 1. Consequently, the average per user DoF experiences an extra term of $\frac{1}{4}\,(1-\bar{p}^2)\,\bar{p}^4$. According to the modification above, it is that $\tau_p^{(TDMA),(3)}$ is the average per user DoF, that is attained by the greedy TDMA scheme for the message assignment strategy that is taken into account here. Hence, according to Proposition 1, $\tau_p^{(TDMA),(3)}$ is the optimal attainable average per user DoF for this message assignment strategy, while contrained by TDMA schemes. ■

The values of $\frac{\tau_p^{(TDMA),(1)}}{\bar{p}}$, $\frac{\tau_p^{(TDMA),(2)}}{\bar{p}}$, and $\frac{\tau_p^{(TDMA),(2)}}{\bar{p}}$ in Figure 4.2 are put into contrast, while the identity of $\max\left\{\tau_p^{(TDMA),(1)}, \tau_p^{(TDMA),(2)}, \tau_p^{(TDMA),(3)}\right\}$ with $\tau_p^{(TDMA),(1)}$ at high probabilities of erasure, and with $\tau_p^{(TDMA),(2)}$ at low probabilities of erasure, and with $\tau_p^{(TDMA),(3)}$ in a middle regime can be observed.

Constrained by TDMA schemes, it is demonstrated that one of the message assignment strategies described in Lemmas 9, 10, and 11 achieves optimality for any value of p.

Theorem 5 *For any value $0 \leq p \leq 1$, the average per user DoF under restriction to orthogonal TDMA schemes is given as follows.*

$$\tau_p^{(TDMA)} = \max\left\{\tau_p^{(TDMA),(1)}, \tau_p^{(TDMA),(2)}, \tau_p^{(TDMA),(3)}\right\}, \tag{4.4}$$

where $\tau_p^{(TDMA),(1)}$, $\tau_p^{(TDMA),(2)}$, and $\tau_p^{(TDMA),(3)}$ are given in (4.1), (4.2), *and* (4.3), *respectively.*

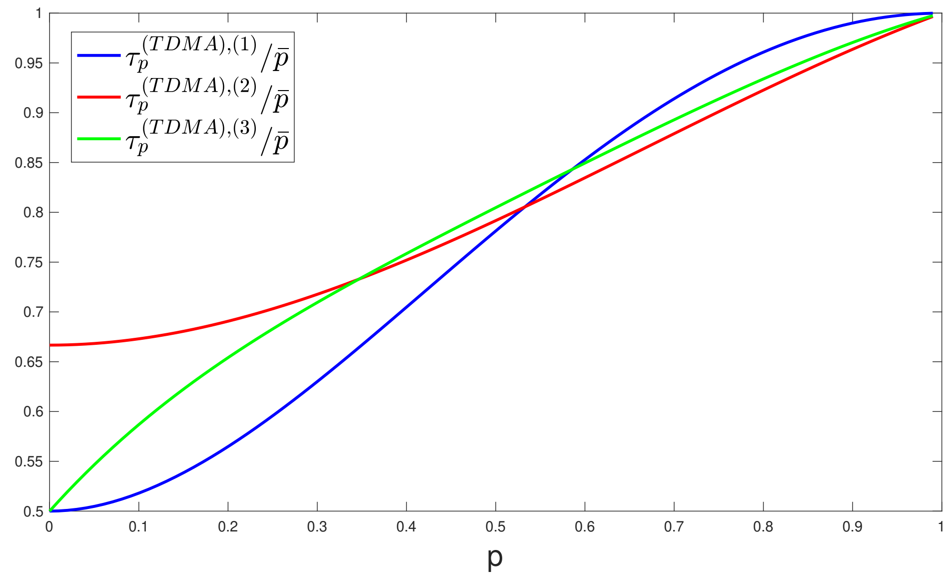


Fig. 4.2. The average puDoF attained by applying the strategies in Lemmas 9, 10, and 11, divided by $\bar{p}$.

Proof Lemmas 9, 10, and 11 build the foundation for the proof of the inner bound. The converse is proved by taking all irreducible message assignment strategies into account, where each transmitter carries only one message.

According to Lemma 9, it is the case that the TDMA average per user DoF, which is attained through the strategy characterized by the string of all ones of form $S^{(1)} = (1)$ is identical to $\tau_p^{(TDMA),(1)}$. Thus the upper bound is valid in this case.

Next, it is demonstrated that the TDMA average per user DoF attained through strategies characterized by strings that have the form $S^{(2)} = (2, 1, \ldots, 1, 0)$ is upper bounded by a convex combination of $\tau_p^{(TDMA),(1)}$ and $\tau_p^{(TDMA),(2)}$. It follows that it is upper bounded by $\max\left\{\tau_p^{(TDMA),(1)}, \tau_p^{(TDMA),(2)}\right\}$.

The message assignment strategy that was taken into account divides each network into subnetworks. Each subnetwork is formed by a transmitter carrying two messages and a number of further consecutive transmitters, that carry one message, while the last transmitter in the subnetwork has no messages assigned. Starting with the case of odd number of transmitters, that only carry single messages, consider the message assignment strategy characterized by the string $(2, 1, 1, 1, 0)$. Moreover, the proof follows an analogous fashion for strategies characterized by strings of the form $(2, 1, 1, \ldots, 1, 0)$, where the number of ones is arbitrary and odd. Here, it only is necessary to demonstrate that the average per user DoF in the top subnetwork is upper bounded by a convex combination of $\tau_p^{(TDMA),(1)}$ and $\tau_p^{(TDMA),(2)}$.

The top subnetwork is formed by the top five users. While W_1 and W_2 can be transmitted through X_1, W_3, W_4 and W_5 can be transmitted through X_2, X_3, and X_4, correspondingly, where the transmit signal X_5 is deactivated.

In the following, the optimal TDMA scheme for the considered subnetwork is elaborated. Starting with a simple scheme, it will be modified in order to obtain the optimal scheme. The delivery of the messages W_1, W_3, and W_5 is successful, if the link connecting its carrying transmitter and its intended receiver exists. Message W_2 is delivered if message W_1 is not transmitted, and message W_3 does not produce interference at Y_2. Message W_4 is transmitted if W_5 does not produce interference at Y_4, and the transmission of W_4 through X_3 does not disturb the communication of W_3. In the following, the modification will be elaborated. For every atomic subnetwork that is formed by users $\{2, 3, 4\}$, then the priority setting is changed within this subnetwork, and hence, messages W_2 and W_4 will be delivered instead of message W_3.The optimality of this TDMA based scheme results from [28, Theorem 1] for each realization of the network. Observe, that the average sum DoF for messages $\{W_1, \ldots, W_5\}$ is identical to their sum DoF in the original scheme but with an additional term because of the modification. Messages $\{W_1, W_2, W_5\}$ in the original have an average sum DoF that is identical to $3\tau_p^{(TDMA),(2)}$, and the sum of the average sum DoF for messages $\{W_3, W_4\}$ and the additional term is upper bounded by $2\tau_p^{(TDMA),(1)}$. Consequently, the upper bound of the average per user DoF is $\frac{2}{5}\tau_p^{(TDMA),(1)} + \frac{3}{5}\tau_p^{(TDMA),(2)}$. A generalized version of the proof can demonstrate that the average TDMA per user DoF for message assignment strategies characterized by strings of form $S^{(2)}$ where the number n of ones is odd , has an upper bound, that is $\frac{n-1}{n+2}\tau_p^{(TDMA),(1)} + \frac{3}{n+2}\tau_p^{(TDMA),(2)}$.

Considering message assignment strategies characterized by a string of the form $S^{(2)}$, where the number n of ones is even, the TDMA average per user DoF is upper bounded by $\frac{n}{n+2}\tau_p^{(TDMA),(1)} + \frac{2}{n+2}\tau_p^{(TDMA),(2)}$ and this can be demonstrated in a analogous fashion as above. Furthermore, if strategies characterized by a string of the form $S^{(3)} = (1, 1, \ldots, 1, 2, 0)$ with a number of ones n are taken into account, the TDMA average per user DoF is identical to that of a strategy characterized by a string of the form $S^{(2)}$ with equal number of ones. Therefore a convex combination of $\tau_p^{(TDMA),(1)}$ and $\tau_p^{(TDMA),(2)}$ forms the upper bound. Lastly, strategies characterized by a string of the form $S^{(4)} = (1, 1, \ldots, 1, 2, 1, 1, \ldots, 1, 0)$ with a number n of ones, have an upper bound for the average per user DoF that is $\frac{n-2}{n+2}\tau_p^{(TDMA),(1)} + \frac{4}{n+2}\tau_p^{(TDMA),(3)}$, where the proof follows the same footsteps as for the case above. ■

In the following paragraphs, the optimality of TDMA schemes are elaborated. Here, all message assignment strategies are considered such that the average per user DoF for the cell association problem is characterized. Consequently, for each realization of a network, Lemma 4 from [6] is used to derive an information theoretic upper bound on the per user DoF as stated below. Let $\mathcal{A} \subseteq [K]$ denote any set of receiver indices, where $U_{\mathcal{A}}$ is the index set of those transmitters, at which only the messages for the receivers in $\mathcal{A}$ are available, and denote its complement as $\bar{U}_{\mathcal{A}}$, i.e. $U_{\mathcal{A}} = [K] \setminus \cup_{i \notin \mathcal{A}} \mathcal{T}_i$.

Lemma 12 *[6, Lemma 4] If there exists a set $\mathcal{A} \subseteq [K]$, a function f_1, and a function f_2 whose definition does not depend on the transmit power constraint P, and $f_1(Y_{\mathcal{A}}, X_{U_{\mathcal{A}}}) = X_{\bar{U}_{\mathcal{A}}} + f_2(Z_{\mathcal{A}})$, then the sum DoF $\iota \leq |\mathcal{A}|$.*

Theorem 6 *The average per user DoF for the cell association problem is given by,*

$$\tau_p(M=1) = \tau_p^{(TDMA)} = \max\left\{\tau_p^{(TDMA),(1)}, \tau_p^{(TDMA),(2)}, \tau_p^{(TDMA),(3)}\right\}, \tag{4.5}$$

where $\tau_p^{(TDMA),(1)}$, $\tau_p^{(TDMA),(2)}$, and $\tau_p^{(TDMA),(3)}$ are given in (4.1), (4.2), and (4.3), respectively.

Proof The statement is proven by demonstrating that $\tau_p(M=1) \leq \tau_p^{(TDMA)}$. This is shown by Lemma 12, where it is used to elaborate that the asymptotic per user DoF is that of the attained one through the optimal TDMA scheme for any irreducible message assignment strategy fulfilling the cell association constraint and any network realization.

Devote your attention to the message assignment strategies characterized by strings having one of the three forms $S^{(1)} = (1)$, $S^{(2)} = (2, 1, 1, \ldots, 1, 0)$, and $S^{(3)} = (1, 1, \ldots, 1, 2, 0)$. Here, each network realization is treated as a series of atomic subnetworks, and it is demonstrated that for each atomic subnetwork, the sum DoF is attained by the optimal TDMA scheme.

Observe for an atomic subnetwork formed by a number n of users, that active $\left\lfloor \frac{n+1}{2} \right\rfloor$ users are located in the optimal TDMA scheme. Next, it is demonstrated for this case by Lemma 12 that the sum DoF for users in the subnetwork is bounded by $\left\lfloor \frac{n+1}{2} \right\rfloor$.

Given the indices $\{i, i+1, \ldots, i+n-1\}$ of the users in the atomic subnetwork, Lemma 12 is applied by considering the set $\mathcal{A} = \left\{i + 2j : j \in \left\{0, 1, 2, \ldots, \left\lfloor \frac{n-1}{2} \right\rfloor\right\}\right\}$, where the cases of message assignment strategies defined by strings having one of the forms $S^{(1)} = (1)$ and $S^{(3)} = (1, 1, \ldots, 1, 2, 0)$ with an even number of ones are excluded, where the set $\mathcal{A} = \left\{i + 1 + 2j : j \in \left\{0, 1, 2, \ldots, \frac{n-2}{2}\right\}\right\}$ is used. Observe that each transmitter carrying a message for a user in the atomic subnetwork and is indexed in $\bar{U}_{\mathcal{A}}$, is connected to a receiver in $\mathcal{A}$, and this receiver is connected to an additional transmitter indexed in $U_{\mathcal{A}}$, and therefore the unknown transmit signals $X_{\bar{U}_{\mathcal{A}}}$ can be restored from $Y_{\mathcal{A}} - Z_{\mathcal{A}}$ and $X_{U_{\mathcal{A}}}$. It follows that the condition of Lemma 12 is fulfilled. Hence it is possible to prove that the upper bound of the sum DoF for users in the atomic subnetwork is $|\mathcal{A}| = \left\lfloor \frac{n+1}{2} \right\rfloor$.

Consider the analogy of this proof and message assignment strategies characterized by strings that have the form $S^{(4)} = \{1, 1, \ldots, 1, 2, 1, 1, \ldots, 1, 0\}$, where it differs in choosing the set $\mathcal{A}$ for atomic subnetworks formed by users indexed in $\{i, i+1, \ldots, i+x, i+x+1, \ldots, i+n-1\}$, where $1 \leq x \leq n-2$, and messages W_{i+x} and W_{i+x+1}

are jointly carried by transmitter $i+x$. Here, Lemma 12 is used with the previously described set $\mathcal{A}$, with the exception to add indices $\{i+x, i+x+1\}$ and remove indices $\{i+x-1, i+x+2\}$. Clearly, the condition in Lemma 12 is fulfilled here, and the proved upper bound on the sum DoF for each atomic subnetwork, is attainable through TDMA. ■

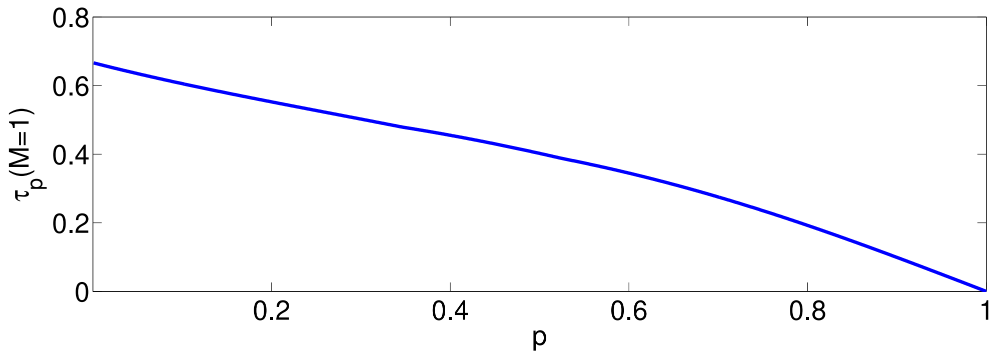


Fig. 4.3. The average puDoF for the cell association problem

The plot of $\tau_p(M=1)$ is shown in Figure 4.3. Theorem 6 indicates for high, low, and middle regimes of p the optimality of those message assignment strategies from Lemmas 9, 10, 11 , correspondingly. Furthermore, for networks, which are connected densely, Figure 4.1(b) depicts the *interference-aware* message assignment strategy, which is for low probabilities of erasure. Throughout this assignment strategy, the amount of links, that can be used for communication that is free from interference, can be maximized when no erasure occurs. For high erasure probabilities, the linearity of the channel connectivity does not influence the optimal message assignment decision.

Since the interference effects are phased out for high erasure probabilities, optimality is achieved if each message is available at a different transmitter, as illustrated by Figure 4.1(a). Figure 4.1(c) illustrates the optimal message assignment strategy for middle regimes of p. More precisely, the network is separated into four user subnetworks. Furthermore, the optimality of the assignment holds for the first subnetwork, since the amount of communication links, which are free of interference, can be maximized for the two cases, with an atomic subnetwork, which is formed by users $\{1, 2, 3\}$ or users $\{2, 3, 4\}$.

4.2 Coordinated Multi-Point Transmission

It is demonstrated that there exist no universally optimal message assignment for all values of p. This insight also applies to the case where each message is available at multiple transmitters and hence applies to all values of the cooperation constraint $M \in \mathbf{Z}^+$. Consequently, any message assignment that facilitates the achievability of $\tau_p(M)$ at high probabilities of erasure, cannot be chosen for achieving $\tau_p(M)$ for low probabilities of erasure. For any irreducible message assignment (see Definition 9) of a K-user channel that follows this strategy, no transmitter with an index in the set $\{i : i \in [K], i = j(2M-1), j \in \mathbf{Z}^+\}$ can carry messages with index $\{i : i \in [K], i = (2M-1)(j-1) + M, j \in \mathbf{Z}^+\}$. According to [6, Lemma 2], any optimal message assignment is irreducible and hence conclude that for any of these message assignments that is optimal at high erasure probabilities, the per user DoF for the case of no erasures is upper bounded by $\frac{2M-2}{2M-1}$, which is suboptimal and follows by applying Lemma 12 for each K-user channel with the set $\mathcal{A}$ defined such that the complement set $\bar{\mathcal{A}} = \{i : i \in [K], i = (2M-1)(j-1) + M, j \in \mathbf{Z}^+\}$.

Moreover, a new role for cooperation in dynamic interference networks is implied by the condition of optimality for high erasure probabilities. Besides cancelling interference at other receivers, a massage being available at more than one transmitter increases the chance of delivering the message to its designed receiver and thus to maximize *coverage*. Three effects have to be considered for the case of high erasure probability. The achieved DoF in the considered linear interference network becomes larger than that of K parallel channels, in particular, $\lim_{p \to 1} \frac{\tau_p(M>1)}{\bar{p}} = 2$. Furthermore, due to the phased out interference for high erasure probabilities , the two transmitters, which are connected to their designed receiver, carry all messages, where a interference avoidance scheme is applied for each realization of a subnetwork and described in the scheme of Theorem 8.

Hence, interference management does not require channel state information at transmitters, while knowledge of slow changes regarding the topology of the network is required to attain the optimal average DoF. In contrast to the optimal scheme of [6, Theorem 4] for the case of no erasures with some transmitters, that are always deactivated, at least one network realization exist, where all transmitters are used, such that the optimal DoF is attained at high erasure probabilities. Consider the case where two transmitters can carry a message and transmit it simultaneously, such that $M = 2$. Two message assignment strategies are elaborated in Theorems 7 and 8, with optimality in $p \to 0$ and $p \to 1$, correspondingly, while closed form expressions are derived for inner bounds on the average puDoF $\tau_p(M = 2)$ in context of the discussed strategies. For the case of no erasures, i.e. $p = 0$, the message assignment of Figure 4.4(a) was proven to be DoF optimal in [6]. Here, the whole network is divided into subnetworks, each consisting of five users in consecutive order, while inter-subnetwork interference is avoided by keeping the last transmitter inactive for each subnetwork. Message W_3 is not transmitted in the first subnetwork, such that each other message is delivered successfully at the intended receiver while interference is eliminated. Moreover, the transmit beams for messages W_1 and W_5 supporting the transmit signals X_2 and X_3 have the purpose of interference cancellation at the corresponding receivers Y_2 and Y_4. The scheme for each following subnetwork follows an analogous pattern. Hence, it is $\tau_p(M = 2) = \frac{4}{5}$ for $p = 0$. The results below are derived by extending the message assignment of Figure 4.4(a), such that the potential existence of block erasures is taken into account.

Theorem 7 *For $M = 2$, the following average per user DoF is achievable using a zero-forcing scheme,*

$$\tau_p^{(ZF)}(M = 2) \geq \frac{1}{5}\bar{p}\,(4 + A \cdot p)\,, \tag{4.6}$$

where

$$A = 2p + \left(1 - \bar{p}^2 + p\bar{p}^3\right)\left(1 + \bar{p}^2\right), \tag{4.7}$$

and

$$\lim_{p \to 0} \tau_p(2) = \frac{4}{5}. \tag{4.8}$$

Proof From [6] it is known that $\lim_{p\to 0} \tau_p(2) = \frac{4}{5}$, and hence, it suffices to show that the inner bound in (4.6) is valid. For each $i \in [K]$, message W_i is assigned as follows,

$$\mathcal{T}_i = \begin{cases} \{i-1, i\}, & \text{if } i \equiv 2 \bmod 5, \text{ or } i \equiv 4 \bmod 5, \\ \{i-1, i-2\}, & \text{if } i \equiv 0 \bmod 5, \\ \{i, i+1\}, & \text{otherwise,} \end{cases}$$

This message assignment is depicted in Figure 4.4(b). Since the transmit signals $\{X_i : i \equiv 0 \bmod 5\}$ are inactive, the network is split into subnetworks of five users, where no interference occurs between successive subnetworks. Observe, that for each of the following subnetworks, a similar scheme applies as for the first subnetwork. Here, any receiver in the subnetwork receives its designed message and no interference occurs, or is deactivated, where regardless of any network realization, any transmitter does not transmit more than one message and thus conclude that for each transmitted message, 1 DoF is obtained.

Messages W_1, W_2, W_4, and W_5 are transmitted through X_1, X_2, X_3, and X_4, correspondingly, for the case when coefficients $H_{1,1} \neq 0$, $H_{2,2} \neq 0$, $H_{4,3} \neq 0$, and $H_{5,4} \neq 0$, correspondingly.

Observe that the transmit beam for message W_1 contributing to X_2 can be build to eliminate the interference it causes at Y_2. In an analogous fashion, the interference produced by W_5 at Y_4 can be eliminated through X_3. An additional scenario occurs where W_4 should be delivered through X_4, if $H_{4,4} \neq 0$ and $H_{4,3} = 0$ and $H_{5,4} = 0$, which occurs with probability $p^2\bar{p}$. Also, an additional case exists for delivering W_2 through X_1 if $H_{2,1} \neq 0$ and $H_{1,1} = 0$ and $H_{2,2} = 0$. Therefore $\bar{p}$ DoF is attained for

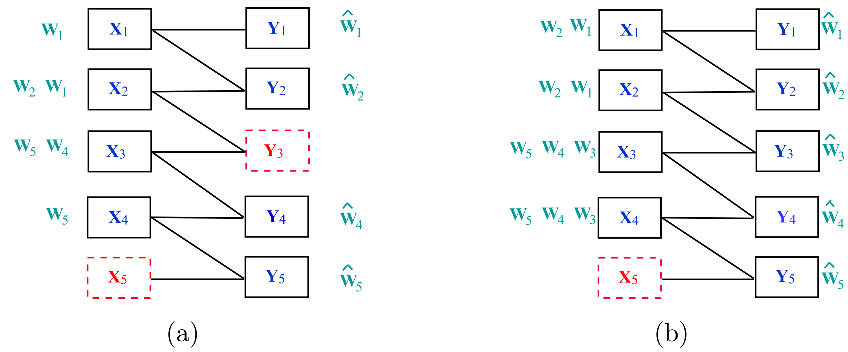


Fig. 4.4. The message assignment in (a) is optimal for a linear network with no erasures ($p = 0$). This message assignment is extended in (b) to consider non-zero erasure probabilities. In both figures, the red dashed boxes correspond to inactive signals.

each of W_1 and W_5 and $\bar{p}(1+p^2)$ DoF is attained for each of W_2 and W_4. It follows that $\tau_p(2) \geq \frac{4}{5}\bar{p} + \frac{2}{5}p^2\bar{p}$.

In this paragraph, the scenarios for transmitting W_3 by considering the previously elaborated four priority links are discussed. Here, message W_3 can only be delivered through X_3 whenever it is either that one of the following is true, that is $H_{2,2} = 0$ and $H_{3,2} = 0$, or $H_{1,1} = 0$ and $H_{2,1}, H_{2,2}, H_{3,2}$ collectively exist. Otherwise, interference is produced at Y_3 by sending W_2 through X_2 and this interference cannot be canceled and occurs with probability $(1 - \bar{p}^2 + p\bar{p}^3)$. Also, $H_{3,3} \neq 0$ with probability $\bar{p}$. Furthermore, it is either $H_{4,3} = 0$ such that W_4 cannot be delivered through X_3 and W_3 would not produce interference at Y_4, or it is $H_{4,3} \neq 0, H_{4,4} \neq 0$ and $H_{5,4} = 0$, such that W_3 can be delivered through X_3, while its interference is eliminated at Y_4, and W_4 is delivered through X_4, while W_5 cannot be delivered through X_4. The first of the previously described scenarios occurs with probability p, while the second scenario occurs with probability $p\bar{p}^2$.

Finally the inner bound in (4.6) is derived by adding up the probabilities of the above transmissions. ■

Despite the optimality of the scheme presented in Theorem 7 for the case of no erasures ($p = 0$), the existence of better schemes at high erasure probabilities is known. Moreover, in a subnetworks of five users with respect to the scheme presented in Theorem 7, only messages of two users are assigned to those two transmitters, which can be connected to the designed receiver, where four users have only one of these transmitters carrying their messages, the asymptotic limit of $\frac{7}{5}$ for the achieved average per user DoF normalized by $\bar{p}$ as $p \to 1$ is obtained. Hence, an alternative message assignment is encountered. Here, the two transmitters assigned with message i are the two transmitters $\{i-1, i\}$ that can be connected to its intended receiver, leading to $\frac{\tau_p(2)}{\bar{p}} \to 2$ as $p \to 1$. Based on this assignment, a transmission scheme is investigated in the theorem below.

Theorem 8 *For $M = 2$, the following average per user DoF is achievable using a zero-forcing scheme,*

$$\tau_p^{(ZF)}(M=2) \geq \frac{1}{3}\bar{p}\left(1 + \bar{p}^3 + B \cdot p\right), \tag{4.9}$$

where

$$\begin{aligned} B &= 3 + \left(1+\bar{p}^3\right)\left(1 - \bar{p}^2 + p\bar{p}^3\right) \\ &+ p\left(1 + \bar{p}^2\right), \end{aligned} \tag{4.10}$$

and

$$\lim_{p \to 1} \frac{\tau_p(2)}{\bar{p}} = 2. \tag{4.11}$$

Proof Observe that no message is transmitted whenever the links from both transmitters carrying the message to its intended receiver do not exist for any message assignment. Therefore, the average DoF that is attained for any message cannot exceed $\bar{p}^2$. Clearly, $\lim_{p\to 1} \frac{\tau_p(2)}{\bar{p}} \leq \lim_{p \to 1} \frac{\bar{p}(1+p)}{\bar{p}} = 2$. Hence, it is sufficient to prove that the inner bound in (4.9) holds. Each message is assigned to the two transmitters that can establish a connection to its intended receiver for the achieving scheme, i.e.

$\mathcal{T}_i = \{i-1, i\}, \forall i \in [K]$. Furthermore, for each network realization, each transmitter sends no more than one message and any sent message is delivered at its intended receiver without interference. Consequently, 1 DoF is gained for any transmitted message. Therefore, the probability of transmission is identical to the average DoF attained for each message.

Devote you attention to message W_i with $i \equiv 0 \bmod 3$, which is sent through X_{i-1} if $H_{i,i-1} \neq 0$, and is sent through X_i if $H_{i,i-1} = 0$ and $H_{i,i} \neq 0$. Hence, d_0 DoF is attained for each of these messages, and,

$$d_0 = \bar{p}(1+p). \tag{4.12}$$

For messages W_i such that $i \equiv 1 \bmod 3$, the transmission occurs through X_{i-1} if $H_{i,i-1} \neq 0$ and $H_{i-1,i-1} = 0$. Observe, that if $H_{i-1,i-1} \neq 0$, message W_i cannot be sent through X_{i-1} since sending W_i through X_{i-1} here prevents W_{i-1} from being sent since either interference occurs at Y_{i-1} or transmitter X_{i-1} is shared. Consequently, $d_1^{(1)} = p\bar{p}$ DoF is attained for sending W_i through X_{i-1}. Further, W_i is sent through X_i if it is not sent through X_{i-1} and $H_{i,i} \neq 0$ and either $H_{i,i-1} = 0$ or W_{i-1} is sent through X_{i-2}. In particular, W_i is sent through X_i if the following conditions are jointly satisfied: $H_{i,i} \neq 0$, and either $H_{i,i-1} = 0$ or it is that $H_{i,i-1} \neq 0$ and $H_{i-1,i-1} \neq 0$ and $H_{i-1,i-2} \neq 0$. Hence, $d_1^{(2)} = p\bar{p} + \bar{p}^4$ is attained for sending W_i through X_i, such that d_1 DoF is attained for each W_i with $i \equiv 1 \bmod 3$, where,

$$d_1 = d_1^{(1)} + d_1^{(2)} = 2p\bar{p} + \bar{p}^4. \tag{4.13}$$

Devote your attention to W_i with $i \equiv 2 \bmod 3$, where any such message is sent through X_{i-1} if the following jointly applies:

- $H_{i,i-1} \neq 0$.
- Either $H_{i-1,i-1} = 0$, or W_{i-1} is not transmitted.
- W_{i+1} is not causing interference at Y_i.

Here, the first condition is fulfilled with probability $\bar{p}$. To mathematically derive the probability of fulfilling the second condition, observe that W_{i-1} is prohibitted from being sent for the case when $H_{i-1,i-1} \neq 0$ only if W_{i-2} is sent through X_{i-2} and producing interference at Y_{i-1}, that is, only if $H_{i-2,i-3} = 0$ and $H_{i-2,i-2} \neq 0$ and $H_{i-1,i-2} \neq 0$.

Hence, the second condition applies with probability $p + p\bar{p}^3$.

Further, the third condition is not fulfilled only if $H_{i,i} \neq 0$ and $H_{i+1,i} \neq 0$. Therefore, it is fulfilled with a probability of at least $1 - \bar{p}^2$. More precisely, even for the case that if $H_{i,i} \neq 0$ and $H_{i+1,i} \neq 0$, the third condition applies if W_{i+1} is sent through X_{i+1} without producing interference at Y_{i+2}, that is, if $H_{i+1,i+1} \neq 0$ and $H_{i+2,i+1} = 0$.

Consequently, the third condition is d with probability $1 - \bar{p}^2 + p\bar{p}^3$, and $d_2^{(1)}$ DoF is attained by sending W_i through X_{i-1}, with

$$d_2^{(1)} = p\bar{p}\left(1 + \bar{p}^3\right)\left(1 - \bar{p}^2 + p\bar{p}^3\right). \tag{4.14}$$

W_i with $i \equiv 2 \bmod 3$ is sent through X_i if $H_{i,i} \neq 0$, and $H_{i+1,i} = 0$, and either $H_{i,i-1} = 0$ or W_{i-1} is sent through X_{i-2}. Hence, sending W_i through X_i attains $d_2^{(2)}$ DoF, with

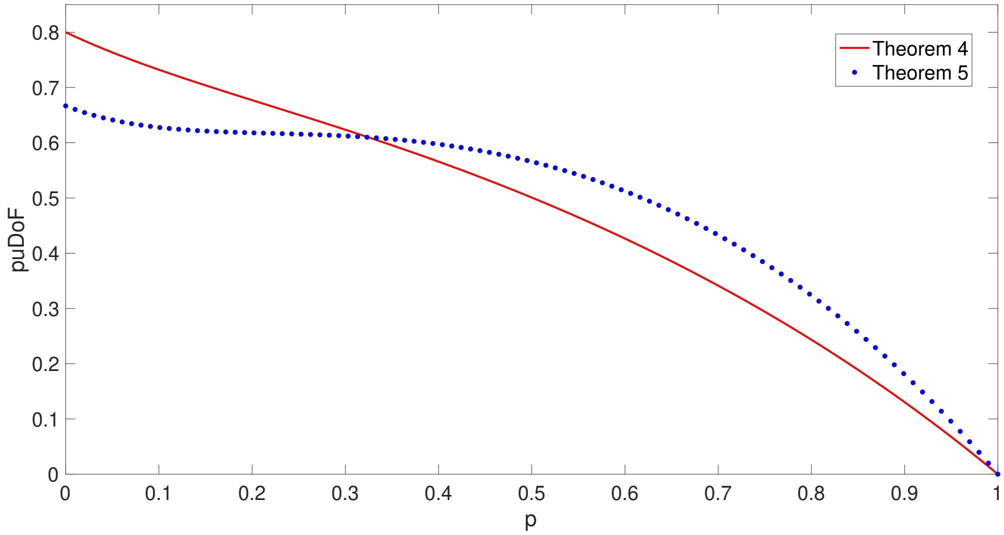
$$\begin{aligned} d_2^{(2)} &= p\bar{p}\left(p + d_1^{(1)}\bar{p}\right) && (4.15)\\ &= p^2\bar{p}\left(1 + \bar{p}^2\right), && (4.16) \end{aligned}$$

It follows that $d_2 = d_2^{(1)} + d_2^{(2)}$ DoF is attained for each W_i with $i \equiv 2 \bmod 3$. Finally,

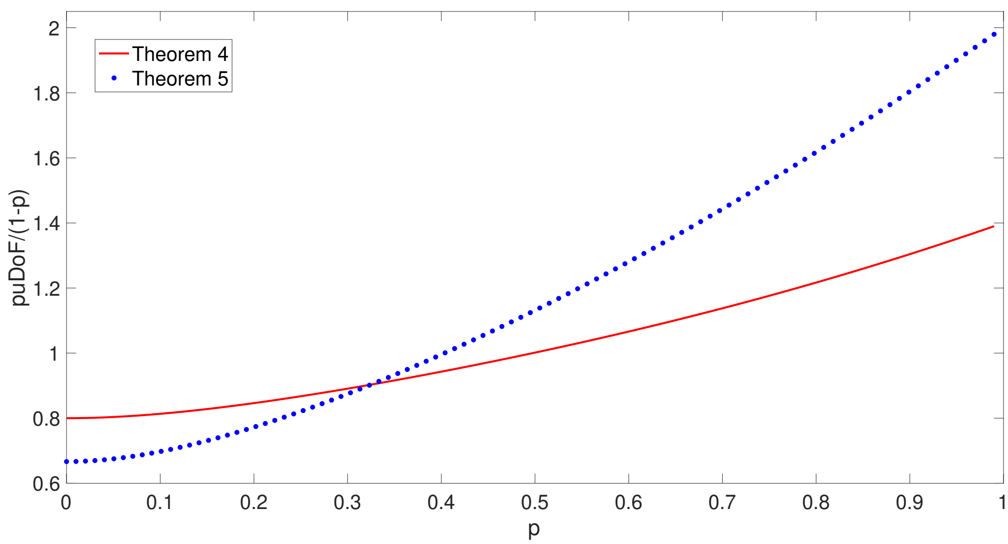
$$\tau_p(2) \geq \frac{d_0 + d_1 + d_2}{3}, \tag{4.17}$$

is obtained which is identical to the inequality in (4.9). ■

As the inner bounds of (4.6) and (4.9) are plotted in Figure 4.5, observe that the scheme of Theorem 7 has better performance for a threshold erasure probability of below $p \approx 0.34$ while the scheme of Theorem 8 is proposed for higher probabilities of



(a)



(b)

Fig. 4.5. Attained inner bounds of Theorems 4 and 5. (a) depicts the attained puDoF. (b) depicts the attained puDoF divided by $\bar{p}$.

erasure. Despite the considered channel model enables the usage of the interference alignment scheme of [53] over multiple channel realizations, the suggested schemes demand on only coding over one channel realization due to the sparse nature of the linear network.

4.2.1 Optimal zero-forcing scheme with two transmitters per message ($M = 2$)

The two schemes presented above depend on zero-forcing transmit beamforming and achieve the limits for $\tau_p(2)$ as $p \to 0$ and $p \to 1$. An algorithm is proposed that simplifies the problem of defining $\tau_p^{(\mathrm{ZF})}(2)$ to that of determining the optimal message assignment for each value of p.

Algorithm 1 operates on an atomic subnetwork with user indices $[N] = \{1, 2, \cdots, N\}$ and gives the transmit signals $\{X_i, i \in [N]\}$ as an output, employing zero-forcing transmit beamforming to maximize the DoF value for users within the atomic subnetwork. Furthermore, the first receiver can have a connection to a transmitter of preceding index and the index of this transmitter is set as zero.

For each message W_i, a set of binary variables $c_{i,j}, j \in \{i-2,\ i-1,\ i,\ i+1\}$ is initialized to zero. Further, the circumstances under which sending and decoding a message at its designed receiver without interference are elaborated in a consecutive order starting from W_1 to W_N. Moreover, $c_{i,j}$ is set to one, whenever it is decided to transmit message W_i from transmitter j.

Examine the two cases of Algorthm 1 for sending message W_i to its destination by using either transmitter i or $i-1$ and justify decisions that lead to optimality for both cases. Note that the first two users form an exception due to their location at the beginning of the atomic subnetwork and hence are described independently in lines $4-21$

Case 1: The conditions for transmitting W_i from transmitter $i-1$ are elaborated starting from 23 of Algorithm 1. In this case W_i has to be available at transmitter $i-1$. that is $(i-1) \in \mathcal{T}_i$. Devote your attention to the scenario when transmitter $i-1$ does not transmit W_{i-1}. that is, $c_{i-1,i-1} = 0$.

To guarantee that transmitter $i-1$ does not produce interference at receiver $i-1$, the possibility has to be taken into account when receiver $i-1$ cannot decode its desired message anyway, that is W_{i-1} is not transmitted from transmitter $i-2$. If these conditions apply, then $c_{i,i-1} = 1$. Moreover, beginning at line 26, the scenario is considered where W_i would produce interference at receiver $i-1$. However, this interference can be eliminated by sending W_i from transmitter $i-2$. Here, W_i has to be carried by transmitter $i-2$, that is, $(i-2) \in \mathcal{T}_i$. Further, it has to be guaranteed that sending W_i from transmitter $i-2$ would not produce interference at receiver $i-2$, which would occur when W_{i-2} is not being delivered, that is, $c_{i-2,i-2} = 0$ and $c_{i-2,i-3} = 0$. Starting at line 29, the last scenario takes into account the case for delivering W_i and W_{i-1} through X_{i-1}, while ensuring that the interference they produce at receivers $i-1$ and i, correspondingly, is eliminated. For the last case, W_i is sent from transmitters $i-2$ and $i-1$ and W_{i-1} is sent from transmitters i and $i+1$. Observe that for any irreducible message assignment, W_i can be available at transmitter $i-2$ only if it is available at transmitter $i-1$, justifying why $(i-1) \in \mathcal{T}_i$ is not checked at line 29 .

Case 2: W_i is transmitted from transmitter i (lines 32-37). To accomplish this, the trivial conditions imply that message W_i is carried by transmitter i, and W_i is not being delivered through transmitter $i-1$. In this scenario, it has to be guaranteed that receiver i can decode message W_i without experiencing interference, that is when transmitter $i-1$ is inactive. Therefore, $c_{i,i} = 1$ if the conditions above apply. There exist one additional case where the interference from transmitter $i-1$ can be eliminated whenever message W_{i-1} is carried by transmitter i and receiver i only encounters interference by W_{i-1}. Hence, $c_{i,i}$ and $c_{i-1,i}$ are jointly set to 1 in this last case.

Algorithm 1 Algorithm outputs the optimal zero-forcing DoF value within an atomic subnetwork.

for i=1:N **do**
  Define $c_{i,i-2} = c_{i,i-1} = c_{i,i} = c_{i,i+1} = 0$
end for
if $H_{1,0} \neq 0 \ \wedge \ 0 \in \mathcal{T}_1$ **then**
  $c_{1,0} = 1$
else
  $c_{1,1} = 1$
end if
if $1 \in \mathcal{T}_2 \ \wedge \ c_{1,1} = 0$ **then**
  if $0 \in \mathcal{T}_2 \ \wedge \ H_{1,0} \neq 0$ **then**
    $c_{2,1} = 1$; $c_{2,0} = 1$
  end if
else if $0 \in \mathcal{T}_2 \ \wedge \ H_{1,0} \neq 0 \ \wedge \ 2 \in \mathcal{T}_1$ **then**
  $c_{2,1} = 1$, $c_{2,0} = 1$, $c_{1,2} = 1$, $c_{1,1} = 1$
else if $2 \in \mathcal{T}_2$ **then**
  if $c_{1,1} = 0$ **then**
    $c_{2,2} = 1$
  else if $2 \in \mathcal{T}_1$ **then**
    $c_{2,2} = 1$; $c_{1,2} = 1$
  end if
end if
for i=3:N **do**
  if $(i-1) \in \mathcal{T}_i \ \wedge \ c_{i-1,i-1} = 0$ **then**
    if $c_{i-1,i-2} = 0$ **then**
      $c_{i,i-1} = 1$
    else if $(i-2) \in \mathcal{T}_i \ \wedge \ c_{i-2,i-2} = 0 \wedge \ c_{i-2,i-3} = 0$ **then**
      $c_{i,i-1} = 1$; $c_{i,i-2} = 1$
    end if
  else if $(i-2) \in \mathcal{T}_i \ \wedge \ i \in \mathcal{T}_{i-1} \wedge c_{i-2,i-3} = 0 \wedge c_{i-2,i-2} = 0$ **then**
    $c_{i,i-1} = 1$, $c_{i,i-2} = 1$, $c_{i-1,i} = 1$, $c_{i-1,i-1} = 1$
  end if
  if $i \in \mathcal{T}_i \wedge c_{i,i-1} = 0 \wedge c_{i-2,i-1} = 0$ **then**
    if $c_{i-1,i-1} = 0$ **then**
      $c_{i,i} = 1$
    else if $i \in \mathcal{T}_{i-1}$ **then**
      $c_{i,i} = 1$; $c_{i-1,i} = 1$
    end if
  end if
end for

for i=0:N-1 **do**
Set $X_i = 0$
Generate $X_{i,i}$ from W_j using an optimal AWGN channel point-to-point code (see e.g., [47])
    if $i > 0$ **then**
        Generate $X_{i,i-1}$ from W_j using an optimal AWGN channel point-to-point code
        if $c_{i,i} = 1$ **then**
            $X_i \leftarrow X_i + X_{i,i}$
        end if
    end if
    if $c_{i+1,i} = 1$ **then**
        $X_i \leftarrow X_i + X_{i,i+1}$
    end if
end for
if $H_{N,N} \neq 0$ **then**
    Set $X_N = 0$
    Generate $X_{N,j},\ j \in \{N-1, N\}$ from W_N using an optimal AWGN channel point-to-point code.
    if $c_{N,N} = 1$ **then**
        $X_N \leftarrow X_N + X_{N,N}$
    end if
end if
for i = 0:N **do**
    if $i \geq 2 \ \wedge \ c_{i-1,i} = 1$ **then**
        $X_i \leftarrow X_i - \frac{H_{i,i-1}X_{i-1,i-1}}{H_{i,i}}$
    end if
    if $i \leq N-2 \ \wedge \ c_{i+2,i} = 1$ **then**
        $X_i \leftarrow X_i - \frac{H_{i+1,i+1}X_{i+2,i+1}}{H_{i+1,i}}$
    end if
end for

Lemma 13 *For any message assignment such that each message is only assigned to two transmitters ($M = 2$), Algorithm 1 leads to the DoF-optimal zero-forcing transmission scheme for users within the input atomic subnetwork.*

Proof The algorithm considers the messages in ascending order from W_1 to W_N. Hence, it is determined, which transmitter can deliver message W_i such that it can be decoded at its intended receiver without causing interference at any previous interference-free receiver that has so far been interference-free, while the message is sent whenever the above is true. Further, if this applies through any of the transmitters i and $i-1$, then sending W_i from transmitter $i-1$ is prioritised. It is demonstrated by induction in the following paragraphs that this method achieves the optimal transmission scheme. Here, the focus is on the base case, that is, it is verified that sending W_1 from transmitter 0 is always optimal if it is available and the link $H_{1,0} \neq 0$, while otherwise sending W_1 from transmitter 1 is optimal.

Given the feasible set $\Omega \triangleq \{H_{i,j} : i \in [N], j \in \mathcal{T}_i, H_{i,j} \neq 0\}$ that summarizes the subset of all links $H_{i,j}$ through which a message W_i can be transmitted and decoded at its intended receiver. Assume an arbitrary set of links $\mathcal{S} \subset \Omega \backslash H_{1,0}$, such that all links in $\mathcal{S}$ can be used *simultaneously* to deliver messages to their intended receivers while cancelling interference. For $H_{1,0} \in \Omega$, it is demonstrated in the paragraphs below that either $H_{1,0}$ can be included to $\mathcal{S}$, or the first link in $\mathcal{S}$ is substituted by $H_{1,0}$ and it can be shown that this substitution does not lower the DoF. Observe that if $H_{1,0} \in \Omega$, then no receiver in the atomic subnetwork would experience interference because W_1 is sent from transmitter 0, which is due to transmitter 0 being connected to receiver 1. Also, if $H_{2,1}$ is the first link in $\mathcal{S}$, then it is substituted by $H_{1,0}$ and the sum DoF in the atomic subnetwork does not change by delivering W_1 instead of W_2.

If $H_{1,0} \notin \Omega$, the identical technique as in the case before are used by replacing $H_{1,0}$ with $H_{1,1}$. As before, the substitution of the first link in $\mathcal{S}$ by $H_{1,1}$ doe not lower the DoF since transmitting W_1 from the first transmitter produces only interference at the second receiver, and since $H_{2,j},\ j \in \{1,2\}$ is either not in $\mathcal{S}$ or it is the first link in $\mathcal{S}$ that is substituted by $H_{1,1}$, sending W_1 from transmitter 1 does not entail a decline in the number of links in $\mathcal{S}$. Lastly, if W_1 can be delivered through either transmitter 0 or transmitter 1, then selecting transmitter 0 can only cancel the interference produced by W_1 at consecutive receivers. Therefore, sending W_1 from the first transmitter in its transmit set $\mathcal{T}_1$ is always optimal as long as the corresponding link exists.

In the following, the proof is enlarged to all users through induction. Starting with the induction hypothesis in the i^{th} step, it is assumed that transmissions for messages $\{W_k : k < i\}$ are chosen optimally to maximize the sum DoF. Denote $\mathcal{S}_1 \subset \Omega$ as the set of links $H_{k,l}$, through which a subset of $\{W_k, k < i\}$ can be delivered simultaneously to their intended receivers, while interference is canceled completely. Assume that the decisions regarding links in $\mathcal{S}_1$ are taken optimally, that is changing any of these links does not cause the number of delivered messages to be exceeded.

The induction step is as follows. Denote $\mathcal{S}_2 \subset \Omega$ as any set of links $H_{k,l}$, through which a subset of the messages $\{W_k, k > i\}$ can be sent simultaneously such that they can be decoded at their intended receivers. Further, the links in $\mathcal{S}_2$ are selected optimally to maximize the number of delivered messages. W_i can be transmitted through $H_{i,i-1}$ without causing a conflict with any of the messages, that are transmitted through the links in $\mathcal{S}_1$. The same logic applies to $H_{i,i-1}$ as to $H_{1,0}$ in the base case. In particular, if sending W_i through $H_{i,i-1}$ does not produce interference at any preceding, interference-free receiver, and it can be decoded at receiver i while interference can be cancelled due to any message with a preceding index, $H_{i,i-1}$ can be either included to $\mathcal{S}_2$ or the first link in $\mathcal{S}_2$ can be substituted, such that an optimal set of links can be obtained for sending the messages $\{W_k, k \geq i\}$. This applies due to the fact that sending W_i through $H_{i,i-1}$ does not produce interference at any receiver indexed with $k > i$, and any of the links $\{H_{i+1,k}, k \in \{i, i+1\}\}$ is either not in $\mathcal{S}_2$ or it is the link that is substituted by $H_{i,i-1}$. If W_i cannot be transmitted through $H_{i,i-1}$ without being in conflict with any of the messages that are sent through the links in $\mathcal{S}_1$, but this is possible through $H_{i,i}$, then the same argument applies for adding $H_{i,i}$ to $\mathcal{S}_2$. Observe that the priority to transmit W_i through $H_{i,i-1}$ is optimal due to the fact that $H_{i,i-1}$ may only be in conflict with $H_{i+1,i}$ in $\mathcal{S}_2$, while $H_{i,i}$ may be in conflict with any of $H_{i+1,i}$ and $H_{i+1,i+1}$. Hence, as long as the previously mentioned priority rule is applied, transmitting W_i through a link $H_{i,j}$ is always optimal as long as W_i can be decoded at receiver i without producing interference at a preceding, interference-free receiver.

Therefore, it has been demonstrated, that the greedy approach of Algorithm 1 to first examine every possible scenario to deliver W_i through $H_{i,i-1}$, and if not possible, explore all possible scenarios to deliver it through $H_{i,i}$, without causing any conflict with any delivered message with a preceding index, is optimal with respect to the DoF under restriction to zero-forcing schemes. ■

The simplification of the optimal algorithm by the optimality of the greedy approach is two fold. On the one hand, the links can be examined separately and it is checked if a message can be transmitted to its deignated receiver without being in conflict with any of the previous active messages. It will always be chosen to send the message whenever it is possible. On the other hand, choices that already have been made do not have to be adjusted afterwards, since it is ensured that conflicts with previously activated messages are avoided at each step. As depicted below, this procedure is applied in Algorithm 1.

In the following, the decision conditions for the first two messages in the input atomic subnetwork are elaborated. If $H_{1,0} \in \Omega$, sending W_1 is optimal, as shown in the base case of the proof by induction. Hence, set $c_{1,0} = 1$. If not, then it must be the case that $H_{1,1} \in \Omega$, because otherwise receiver 1 would not have belonged to the atomic subnetwork. In that last case, set $c_{1,1} = 1$, since it is optimal then to send W_1 from transmitter 1 as shown in the above proof.

Next, the possibilities for delivering W_2 to its destination through transmitter 1 are considered. If $H_{2,1} \in \Omega$, the following possibilities can occur. If $0 \in \mathcal{T}_2$ and $c_{1,0} = 1$, then send W_2 from transmitter 1 and cancel interference at the first receiver by sending W_2 from transmitter 0. Hence, set $c_{2,0} = 1$ and $c_{2,1} = 1$ in this first case. In the second case, both W_1 and W_2 are delivered to their destinations through transmitter 1, and cancel their interference through transmitters 2 and 0, respectively. This second case is possible when $\mathcal{T}_1 = \{1, 2\}$ and $\mathcal{T}_2 = \{0, 1\}$ and transmitter 0 is in the atomic subnetwork, i.e., $H_{1,0} \neq 0$. The above two possibilities are the only ones that exist for delivering W_2 through transmitter 1.

Consider the cases for delivering W_2 through transmitter 2. If $H_{2,2} \in \Omega$ and W_2 is not sent from the first transmitter, i.e. $c_{2,1} = 0$, then if $c_{1,1} = 0$, W_1 is not causing interference at the second receiver and $c_{2,2} = 1$ is set. The second possible case is when W_1 is causing interference at receiver 2, but this interference can be canceled through transmitter 2, i.e., when $\mathcal{T}_1 = \{1, 2\}$. In this case, $c_{2,2} = 1$ and $c_{1,2} = 1$ are set.

The illustration of the greedy approach for transmitting messages $\{W_i, i \in \{3, 4, \cdots, N\}\}$ follows from the explanation of lines $22 - 39$ of the algorithm, that is provided above before Lemma 13. It is demonstrated in the following that Algorithm 1 can be used to achieve the optimal zero-forcing DoF in a general K-user network.

Theorem 9 *Algorithm* 1 *can be used to achieve the optimal zero-forcing DoF for any message assignment satisfying the cooperation order constraint* $M = 2$*, and any realization of a general* K*-user dynamic linear network.*

Proof Consider any realization of a K-user linear dynamic network. It is demonstrated below how the network can be seperated into atomic subnetworks with no inter-subnetwork interference. According to Lemma 13, it is that Algorithm 1 attains the optimal zero-forcing DoF in each atomic subnetwork. Also, due to absence of interference between the subnetworks, and the fact that no connection between a transmitter in a subnetwork and a receiver in another subnetwork exist, it is that invoking Algorithm 1 for each of the atomic subnetworks in the partition leads to the optimal zero-forcing DoF for the entire network.

First, the non-erased channel links are grouped in a way where each group is formed by a maximal set of successive non-erased links. In particular, all links are checked in ascending order with respect to their index, and check if they are erased, that is, first $H_{1,0}$ is checked, then $H_{1,1}$, then $H_{2,1}$, then $H_{2,2}$, and so forth. Then, links are added to the first group until the first erased link is confronted, and then following non-erased links are added to the second group until an erased link is confronted again, and so forth.

It is then demonstrated how to adjust the above grouping of links to build atomic subnetworks. For each group of links, the receivers connected to any link in the group are analyzed in ascending order of index. The analyzed receivers are then included to a subnetwork if the message corresponding to the analyzed receiver is carried by a transmitter, which is connected to that receiver. Otherwise, the current subnetwork is ended and a new subnetwork is started, and analyzing receivers is continued. Here, the network is partioned into non-interfering subnetworks. Finally, to ensure that the subnetworks are atomic, the transmitters, that are connected to receivers in the subnetwork, are analyzed in ascending order of index in each subnetwork. If the analyzed transmitter does not have a message available for a receiver in its subnetwork, then the subnetwork is separated at the index of that transmitter. More precisely, the two receivers connected to that transmitter will belong to two different subnetworks. This process of analyzing transmitters is resumed until in each subnetwork, each transmitter connected to a receiver in the subnetwork has at least one message available for a receiver in the subnetwork.

This process has demonstrated the considered partitioning of the network into atamoic subnetworks. The optimality of Algorithm 1 then follows from Lemma 13 by applying the algorithm for each atomic subnetwork.

■

4.2.2 Information-theoretic optimality of Algorithm 1

In this section the optimality of Algorithm 1 is elaborated to outline $\tau_p(M = 2)$. Adopting the same procedure of the proof of Theorem 9, it is sufficient to show that the outcome of the proposed algorithm achieves the optimal DoF when the input subnetwork is atomic. This leads to the following conclusion.

Lemma 14 *For any message assignment such that each message is only assigned to two transmitters ($M = 2$), Algorithm 1 leads to the DoF-optimal transmission scheme for users within an input atomic subnetwork whose size $N \in \{1, 2, 3, 4, 5\}$.*

Algorithm 2 Lemma 12 is applied to prove optimality of Algorithm 1 with subnetwork size $N = 5$.

---

if transmitter 0 is in the atomic subnetwork and transmitter 5 is not in the atomic subnetwork **then**
    if $3 \notin \mathcal{T}_5$ **then**
        if $0 \notin \mathcal{T}_1$ **then**
            $\mathcal{A} = \{2, 3, 4\}$
        else if $3 \notin \mathcal{T}_3$ **then**
            $\mathcal{A} = \{1, 2, 4\}$
        else if $\mathcal{T}_2 = \{1, 2\}$ **then**
            $\mathcal{A} = \{1, 3, 4\}$
        else
            $\mathcal{A} = \{1, 2, 4, 5\}$
        end if
    else if $3 \notin \mathcal{T}_4$ **then**
        if $0 \notin \mathcal{T}_1$ **then**
            $\mathcal{A} = \{2, 3, 5\}$
        else if $3 \notin \mathcal{T}_3$ **then**
            $\mathcal{A} = \{1, 2, 5\}$
        else if $\mathcal{T}_2 = \{1, 2\}$ **then**
            $\mathcal{A} = \{1, 3, 5\}$
        else
            $\mathcal{A} = \{1, 2, 4, 5\}$
        end if
    else
        $\mathcal{A} = \{1, 2, 4, 5\}$
    end if
end if
if transmitter 0 is not in the atomic subnetwork and transmitter 5 is in the atomic subnetwork **then**
    if $(2 \notin \mathcal{T}_1)$ **then**
        if $5 \notin \mathcal{T}_5$ **then**
            $\mathcal{A} = \{2, 3, 4\}$
        else if $2 \notin \mathcal{T}_3$ **then**
            $\mathcal{A} = \{2, 4, 5\}$

```
        else if $\mathcal{T}_4 = \{3,4\}$ then
            $\mathcal{A} = \{2,3,5\}$
        else
            $\mathcal{A} = \{1,2,4,5\}$
        end if
    else if $(2 \notin \mathcal{T}_2)$ then
        if $5 \notin \mathcal{T}_5$ then
            $\mathcal{A} = \{1,3,4\}$
        else if $2 \notin \mathcal{T}_3$ then
            $\mathcal{A} = \{1,4,5\}$
        else if $\mathcal{T}_4 = \{3,4\}$ then
            $\mathcal{A} = \{1,3,5\}$
        else
            $\mathcal{A} = \{1,2,4,5\}$
        end if
    else
        $\mathcal{A} = \{1,2,4,5\}$
    end if
end if
if both transmitters 0 and 5 are in the subnetwork then
    $\mathcal{A} = \{1,2,4,5\}$
end if
if both transmitters 0 and 5 are not in the subnetwork then
    if $2 \notin \mathcal{T}_1$ then
        $\mathcal{A} = \{2,3,4\}$
    else if $2 \notin \mathcal{T}_2$ then
        $\mathcal{A} = \{1,3,4\}$
    else if $3 \notin \mathcal{T}_4$ then
        $\mathcal{A} = \{2,3,5\}$
    else if $3 \notin \mathcal{T}_5$ then
        $\mathcal{A} = \{2,3,4\}$
    else
        $\mathcal{A} = \{1,2,4,5\}$
    end if
end if
```

Proof The formal proofs for the information-theoretic optimality of Algorithm 1 for atomic subnetworks that have sizes $N \leq 5$ are provided below. To be concise, only the proof for $N = 5$ will be discussed here, since the optimality for smaller subnetwork sizes are less complex.

The concept of the proof is to examine all possible scenarios for the message assignment, and set up the optimality of Algorithm 1 for each by demonstrating the optimality of the DoF that is attained by the algorithm. This is done by applying Lemma 5 according to the procedure declared in Algorithm 2 to build the set $\mathcal{A}$ of received signals that is sufficient to reconstruct all received signals in the subnetwork, with an ambiguity that does not raises with the transmit power P. Consider that according to [6, Lemma 2], the optimal message assignment is irreducible, and therefore it is sufficient to proof the statement of the lemma below for irreducible message assignments.

In the following paragraphs, the procedure is described, which are executed by Algorithm 2. Observe that **a transmitter is in the atomic subnetwork** if the contitions below apply:

1. The transmitter is connected to a receiver whose index is in the subnetwork.

2. The transmitter is carrying a message whose index is in the subnetwork.

Due to the size of the atomic subnetwork $N = 5$, it is that transmitters with index in $\{1, 2, 3, 4\}$ are in the subnetwork. Therefore, cases on the membership of transmitters 0 and 5 are taken into account.

The case, if transmitter 0 is included to the subnetwork and transmitter 5 is not is stated at the start of the algorithm. It is important to mention for this case that Algorithm 1 always leads to attaining 3 DoF, since W_5 can be delivered through X_4, and W_1 and W_2 can be delivered simultaneously while cancelling interference. The focus here is on the case when W_5 is not carried by transmitter 3:

1. If it is also the case that W_1 is not available at transmitter 0, then all transmit signals can be reconstructed from Y_2, Y_3 and Y_4 as stated in line 4 of the algo-

rithm. This is because for any reliable communication scheme, W_2, W_3 and W_4 can be reconstructed from these received signals, and hence reconstruct X_0 and X_3 from the considered conditions on the message assignment. Following the linear connectivity of the network, X_4, X_2 and X_1 can be reconstructed from Y_4, Y_3 and Y_2, with respect to order.

2. The second case is when W_3 is not available at transmitter 3, then Lemma 12 is applied with the set $\mathcal{A} = \{1, 2, 4\}$ as in line 6 of the algorithm. Since both W_3 and W_5 do not contribute to X_3, this transmit signal can be reconstructed. Then X_4 can be reconstructed from Y_4. Further, since transmitter 0 can only have W_1 and W_2 since it can be focused on irreducible message assignments, and hence X_0 can be reconstructed. Following the connectivity of the network, X_1 and X_2 can be reconstructed from Y_1 and Y_2, respectively.

3. The third case is when W_2 is available at transmitters 1 and 2. In this case, Lemma 12 can be applied with the set $\mathcal{A} = \{1, 3, 4\}$ as in line 8 of Algorithm 2. X_0 can be reconstructed since it is known that W_1, and W_2 is not available at transmitter 0. Also, X_1 can then be reconstructed from Y_1. Since W_2 is not available at transmitters 3, 4, X_3 and X_4 can be reconstructed as well. Finally, X_2 can be reconstructed from Y_3.

4. In the final case, all of the previous three cases do not apply. Here, Lemma 12 holds with the set $\mathcal{A} = \{1, 2, 4, 5\}$ per line 10 of the algorithm. The proof for the upper bound is identical to the one where erasures are absent [37, Chapter 6]. Algorithm 1 leads to attaining 4 DoF within the input subnetwork here as stated beneath. Due to the fact that the first case from above does not hold, W_1 can be delivered through X_0. Due to the fact that the second case does not hold, W_3 can be delivered through X_3.

   Further, transmitter 5 is not in the subnetwork, and W_5 is not carried by transmitter 3, and thus it is that W_5 is carried by transmitter 4, and can be delivered through X_4. Lastly, due to the fact that the second case does not hold, it is either $\mathcal{T}_2 = \{0, 1\}$ and hence W_2 is delivered through X_1 and its interference at Y_1 is eliminated through X_0, or it is $\mathcal{T}_2 = \{2, 3\}$ and hence W_2 is delivered through X_2 and its interference at Y_3 is eliminated through X_3.

Observe that the argument, when W_5 is carried by transmitter 3, while W_4 is not carried by transmitter 3, analyzed in lines $12 - 24$ of Algorithm 2, equals to the previous case where W_5 is carried by transmitter 3, but with exchanging the indices 4 and 5 for the receiver and message. Also, the argument when the subnetwork includes transmitter 5, while transmitter 0 is excluded, that is analyzed in lines $26-50$ of Algorithm 2, equals to the argument where the subnetwork includes transmitter 0, while transmitter 5 is excluded, but with exchanging indices i and $5 - i$ of the transmitters and exchanging the indices i and $6-i$ of the receivers for $i \in \{1, 2, 3, 4, 5\}$. More precisely, the subnetwork is viewed here as a mirrored adaptation of the first case (upside down) and the analogous logic is used. Therefore, the cases when the subnetwork either includes or excludes both transmitters 0 and 5 has to be checked. For the case when the subnetwork includes both transmitters, then Lemma 12 is used with $\mathcal{A} = \{1, 2, 4, 5\}$ and the proof is identical to the case where erasures are absent. Moreover, Algorithm 1 here can be applied to attain 4 DoF, since W_1, W_2, W_4 and W_5 can be delivered interference-free.

Line 54 of Algorithm 2 begins the last scenario, where the subnetwork excludes transmitters 0 and 5. In this case, applying Algorithm 1 results in 3 DoF inside the input-subnetwork, by using X_1 to deliver W_1 and by using X_4 to deliver W_5, and using either X_2 or X_3 to deliver W_3. The converse proof is divided into the scenarios below.

1. In the first case, W_1 is not carried by transmitter 2 and hence Lemma 12 is used with $\mathcal{A} = \{2, 3, 4\}$ and is stated in line 56 of Algorithm 2. Due to no presence of contribution by W_1 and W_5, X_2 is reconstructed. Moreover, it follows from the linear connectivity that X_1, X_3 and X_4 is reconstructed from Y_2, Y_3 and Y_4, correspondingly.

2. In the second case where W_2 is not carried by transmitter 2 and hence Lemma 12 is used with $\mathcal{A} = \{1, 3, 4\}$. Due to no presence of contribution by W_2 and W_5, X_2 is reconstructed. Moreover, X_1, X_3 and X_4 is reconstructed from Y_1, Y_3 and Y_4, correspondingly.

Observe that proving the following two scenarios, where W_3 is not carried by transmitters 5 and 4, follows the exact footsteps as the previous two scenarios, respectively, except with exchanging indices i and $5 - i$ of transmitters and indices i and $6 - i$ of receivers and messages for $i \in \{1, 2, 3, 4\}$. The last scenario starts at line 63 of Algorithm 2, where messages W_1 and W_2 are carried by transmitter 2, while messages W_4 and W_5 are carried by transmitter 3. Here, Algorithm 1 uses X_1 to deliver W_1, while X_2 is used to eliminate the interference caused by W_1 at Y_2. Moreover, X_2 is used to deliver W_2. Also, X_4 is used to deliver W_5, and X_3 is used to eliminate the interference caused by W_5 at Y_4. Finally, W_4 is delivered through X_3. Observe that the argument for the converse in the final scenario is identical to the one for absent erasure. ■

Despite that Lemma 14 only considers realizations of the dynamic linear network with atomic subnetworks of maximum size $N = 5$, it carries significant insights to learn the optimal message assignment and transmission scheme for CoMP transmission as annotated below.

Remark 2 *Due to the presence of a minimum of* 10 *successive channel links an atomic subnetwork of size* $N > 5$ *exists, which occurs with a probability in the regime of to* $\bar{p}^{10}$. *Clearly, these cases are highly uncommon for meaningful probabilities of erasure.*

Remark 3 *A scenario of a message assignment, that fulfills* $M = 2$, *i.e. the co-operation order constraint, and an atomic subnetwork of size* $N > 5$, *where it was not possible to apply Lemma 12 to prove that Algorithm* 1 *is information-theoretic optimal, in an analogous fashion when Lemma 14 was proved by applying Algorithm*

2, *could not be discovered. It can be concluded that Algorithm* 1 *can indeed be applied to describe* $\tau_p(M = 2)$.

4.3 Simulation

In this part, a simulation is performed to identify the best attainable average per user DoF resulting from Algorithm 1. Clearly, if the hypothesis that Lemma 14 can be completed such that it covers atomic subnetworks of arbitrarily large sizes, then the outlined findings describe basically $\tau_p(M = 2)$. Therefore, adequately sizable numbers of n channel realizations are generated to calculate the average puDoF for the considered message assignment and erasure probability p.[1] Here, the network is subdevided into atomic subnetworks, where each link is subject to erasure with probability p, before Algorithm 1 operates. By dividing the average number of decoded messages with the network size K, the average per user DoF is obtained. Note that the last transmitter of the network is kept inactive to assure correctness for the average per user DoF of large networks. More precisely, for a large network that is formed by K sized subnetworks, the calculated average puDoF will be obtained in the large network by reapplying the scheme for each subnetwork due to no present inter-subnetwork interference. The simulation is run for a collection of message assignments that differ in fractions $f(p)$ of messages that are carried by one transmitter, which has a connection with their intended receiver and another transmitter, which can be utilized to eliminated interference. Furthermore, the fraction of messages that are carried by both transmitters that have a connection with their designated receiver

[1]The MATLAB code is provided in https://github.com/toluhatake/Fundamental-Limits-of-Dynamic-Interference-Management-with-Flexible-Message-Assignments

is denoted by $1 - f(p)$. The network size K is modified to take up to 100 users into account, where the considered assignment strategy is presented below.

$$\mathcal{T}_i = \begin{cases} \{0,\ 1\} & i = 1 \text{ and f(p)} = 0.01, \\ \{1,\ 2\} & i = 1 \text{ and f(p)} > 0.01, \\ \{K-2,\ K-1\} & i = K, \\ \{i,\ i+1\} & i = 1 + n \cdot \max\left\{2,\ \left\lfloor \frac{K}{f(p)\cdot K - 1} \right\rfloor\right\}, \\ & n \in \{1, 2, \ldots, \min\left\{f(p)\cdot K - 2,\ \lfloor \frac{K}{2} - 1 \rfloor\right\}, \\ \{i,\ i+1\} & i = 2n,\ n \in \left\{1, 2, \ldots, \lceil (f(p) - \frac{1}{2})K \rceil - 1\right\}, \\ \{i-1,\ i\} & \text{otherwise}, \end{cases}$$

where the notation $\{1, 2, ..., x\}$ is used to denote the set $[x]$ when $x \geq 1$ and the empty set when $x < 1$. The value of $f(p)$ is varied from 0 up to 1 in steps of $\frac{1}{100}$, calculating the average puDoF as a function of p for each of these message assignments.

Consequently, Figure 4.6 depicts the achievable maximum puDoF for the previously characterized set of message assignment.

In contrast to the suggested schemes in Theorems 7 and 8, some message assignments exist, that perform better on middle regimes of p and are tabulated in Table 4.3. Consider that an assignment with $f(p) = \frac{2}{5}$ is optimal for $p \to 0$ and was demonstrated in [1]. Moreover, the assignment illustrated in Theorem 7 for $f(p) = \frac{3}{5}$ attains the identical puDoF for $p = 0$. However, it shows a better performance on the range $(0, 0.15]$. The results listed in Table 4.3 demonstrate a monotonic decrease of the optimal fraction $f(p)$ from $\frac{3}{5}$ to 0 as p grows from 0 to 1. It can be concluded intuitively, that a shifting for the role of cooperation occurs, that is from managing interference, where a large value of $f(p)$ is required, to increasing the probability of coverage for each message, where a small value of $f(p)$ is required, with an increasing probability of erasure.

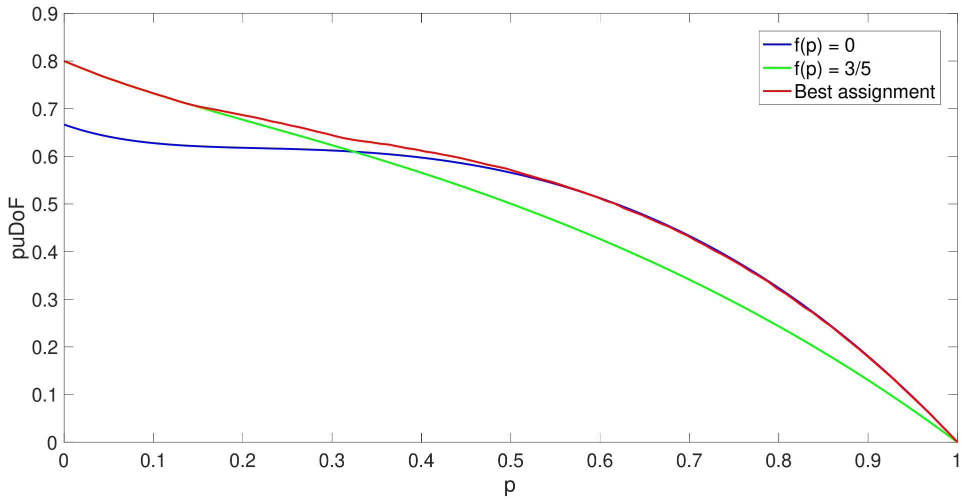


Fig. 4.6. Depicted is the puDoF as a function of the probability of erasure p obtained by using Algorithm 1 for 6000 randomly generated channel realizations for $p \in \{0, 0.01, 0.02, \cdots, 1\}$. The message assignment and transmission strategy described in Theorems 7 and 8, that are optimal as $p \to 0$, and $p \to 1$, are depicted by the green and blue curves, with respect to order. The maximum puDoF, which is obtained by applying the message assignments from the simulation section, is depicted by the red curve.

Table 4.1.
Message assignments with the best performance out of the set of assignments that was simulated.

Range of p	Value of $f(p)$ for best performing message assignment
0 to 0.15	$\frac{3}{5}$ (as in Theorem 7)
0.16 to 0.29	$\frac{1}{2}$
0.3	$\frac{49}{100}$
0.31 to 0.32	$\frac{12}{25}$
0.33 to 0.58	$\frac{1}{50}$
0.59 to 1	0 (as in Theorem 8)

Figure 4.7 depicts τ_p(M=1) from Section 4.1 against the optimal value of the average puDoF, which is the outcome of this simulation for $M = 2$. Here, the value

for the additional backhaul budget for each value of the probability of erasure can be observed from the simulation. Consequently, the additional value for cooperation is of importance up to large values for p. Therefore, independent of the question if cooperation is favorable due to interference management or increasing coverage, it can be examined that important scalable earnings of the degrees of freedom is obtained that also apply for settings with severe shadow fading.

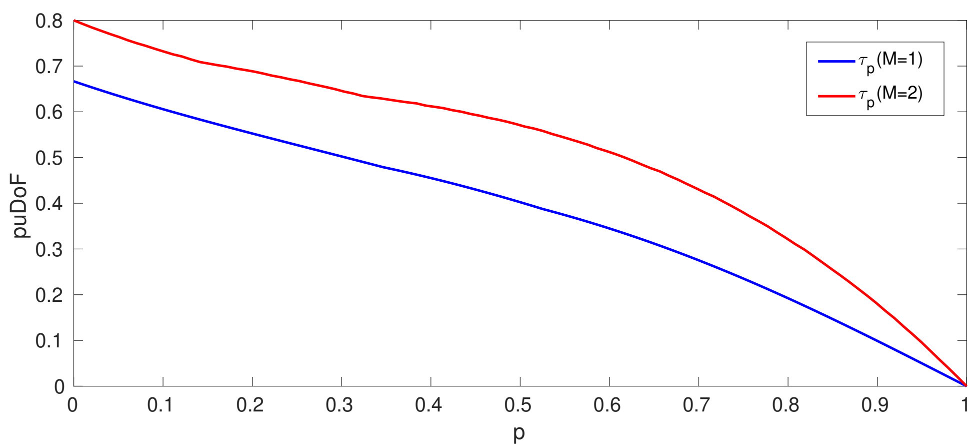


Fig. 4.7. Illustrated are $\tau_p(M = 1)$ from (4.5) and the maximum value for the average puDoF attained for $M = 2$ by Algorithm 1 as obtained by the simulation (Conjectured to be $\tau_p(M = 2)$).

4.4 Discussion: Application in Dynamic Cellular Networks

In this section, the applicability of the presented results for evaluating cellular network models is debated. A K user hexagonal cellular network, where every cell is built by three regions and each region is defined by one user and one base station, is shown in Figure 4.8. Devote your attention to a local interference model with receivers that are exposed to interference from base stations inside adjacent regions of neighboring cells.

The interference between regions that belong to the same cell are neglected due to the high discrepancy in interference powers between it and the other users placed in the region's line of sight. A simplified version of the cellular model is illustrated as an undirected connected graph in Figure 4.9; each vertex describes a pair consisting of one transmitter and one receiver and each edge among two vertices u, v represents a channel placed between the transmitter at u and the receiver at v. Additionally, intra-cell interference is ignored and represented by dotted edges. Consider the scenario where each link is subject to potential erasure probability p in each communication block, that is in analogous fashion to the system model of this work. Furrther, if distinct nodes are kept inactive and two frequency bands, that do not overlap, are utilized for neighbouring transmitters (or receivers), then the network dissolves into a group of linear interference subnetworks with absent inter-subnetwork interference, which is depicted in Figure 4.10. The optimal scheme discussed in Section 4.1 can be applied to the scenario where $M = 1$ for one frequency band, and apply the optimal scheme for zero-forcing discussed in Section 4.2 for the scenario where $M = 2$ for the other frequency band[2]. It follows the attained average per user DoF by $\tau_p^{(\text{cellular})} = \frac{2}{3}\frac{\tau_p(M=1)+\tau_p^{(\text{ZF})}(M=2)}{2}$, which is illustrated in Figure 4.11. Observe that a factor of $\frac{2}{3}$ is present since $\frac{1}{3}$ of the nodes are deactivated as depicted in Figure 4.10. Further, the average backhaul load, or the average number of messages downloaded per transmitter, equals $\frac{2}{3}\frac{1+2}{2} = 1$. More precisley, the average per user DoF illustrated in Figure 4.11 is achieved while each transmitter downloads an average of one message from the backhaul.

The average puDoF, that is attainable by the characterized schemes from this section and Section 4.1 for networks with dynamic cellular and linear characteristics, while it is permitted by the backhaul to associate each transmitter with one message on average, is depicted in Figure 4.11

Overall, the evaluation of dynamic linear networks, which rely on the examined system model above, is extendable to cellular network models with dynamic characteristics similarly as previously demonstrated.

[2]here, its is assumed to select distinct message assignments, or cell associations, in distinct frequency bands.

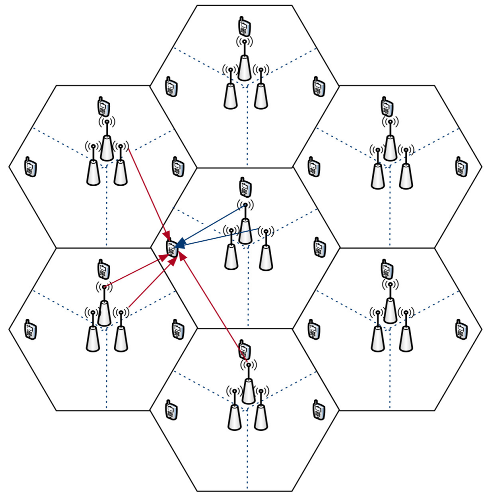


Fig. 4.8. Cellular network model. Intra-cell interference is shown by blue arrows and the inter-cell interference is shown by red arrows.

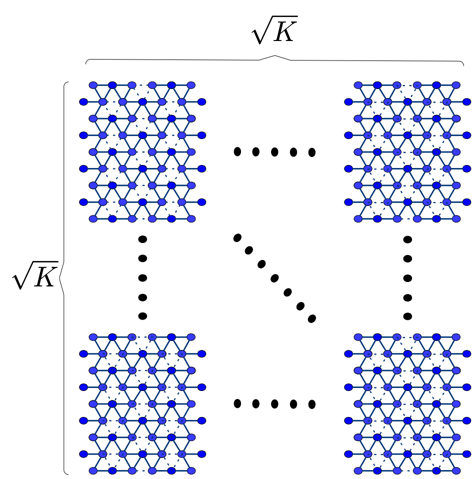


Fig. 4.9. Connectivity graph for the cellular network model.

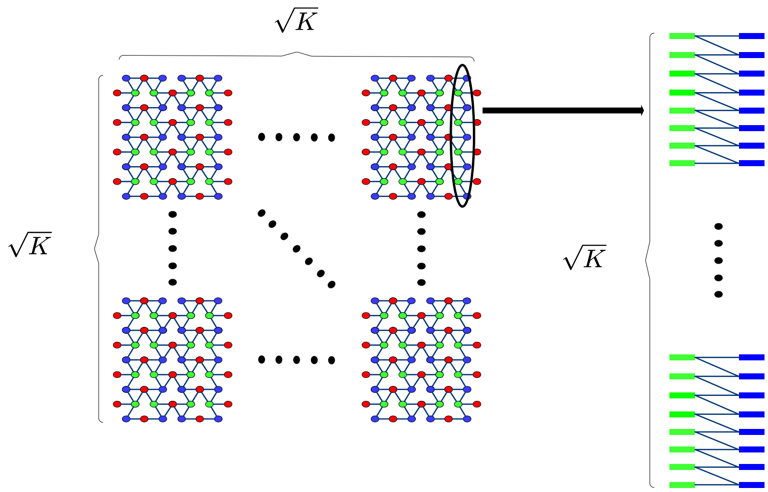


Fig. 4.10. Decomposition of the cellular network into linear subnetworks, by keeping the red nodes inactive. The coloring of the other nodes by blue and green corresponds to the linear subnetwork depicted on the right side, which is obtained by using the two frequency bands. Here, the green and blue nodes correspond to transmitting and receiving nodes and vice versa for the other frequency band.

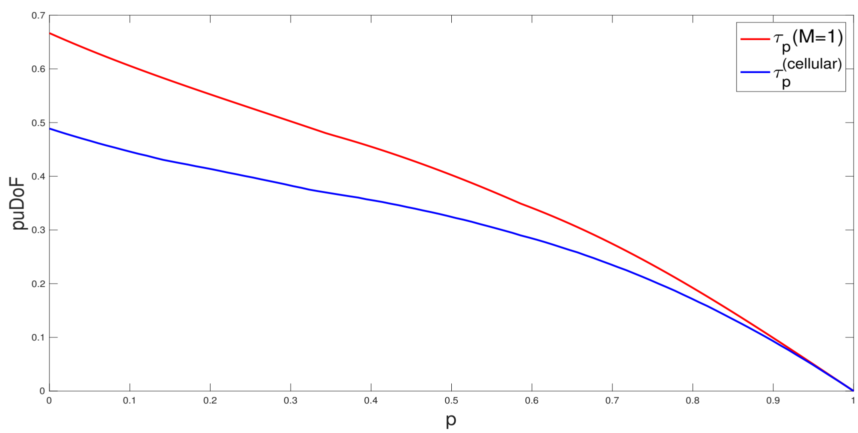


Fig. 4.11. The attained average per user DoF for the dynamic cellular and linear networks for unity backhaul load.

5. NUMBER THEORETIC APPROACH FOR FAST NETWORK DISCOVERY

In Chapters 3 and 4, the locally connected interference network is investigated in detail and demonstrated optimal cell associations and coding schemes to achieve the maximum throughput. However, it is essential for the transmitters in a network to obtain frequent updates about the channel topology. The state-of-the-art approaches for transmission scheduling are based on the ALOHA and CSMA/CA protocols. The benefit of the ALOHA protocol is its straightforward approach. Here, the foundation is to independently authorize transmission from each source and randomly set a back-off time until the next transmission phase occurs. However, collision cannot be fully eliminated by ALOHA which will lead to retransmissions. The Carrier-Sense Multiple Access/Collision Avoidance (CSMA/CA) protocol is an efficient communication protocol which incorporates elementary collision avoidance techniques. The core concept of CSMA/CA is to allow a node to observe if a channel is already used, and then backs-off for another transmission attempt, i.e. transmission occurs over idle channels. However, CSMA/CA does not provide collision-free operation but minimizes retransmission attempts by mitigating the ratio of failed transmissions. ALOHA and CSMA/CA can be optimized by magnifying the fraction of successful transmissions, which is achieved by fine-tuning the mean back-off time in Aloha and the techniques CSMA/CA uses to check if the channel is idle. The main issue is that both approaches may be far from optimal in the context of next generation wireless networks, as these networks are expected to show natural dynamic and complex characteristics, as in autonomous vehicular networks and the fifth generation of cellular communications (5G New Radio). Since ALOHA and CSMA do not have total knowledge of large and complex networks while operating, it is necessary to design new algorithms for transmission scheduling that focus on perfect collision-avoidance by exploiting possible global coordination through the cloud.

For this part of the research, devote your attention to a bipartite network with K transmitting and receiving nodes. Here, each transmitting node is connected to $L < K$ arbitrarily receiving nodes and thus differs from the topology presented in Chapter 3 regarding its local connectedness. Note that each transmitting node only knows about its own index. It is important to mention, that a transmitted message carries knowledge of the index of the transmitting node it is originating from. A message is delivered to a receiving node in each communication round, if and only if no other transmitting node connected to that receiver transmits a message. Otherwise, collision occurs over the air. Each receiving node, that detects a message, learns the index of the transmitting node. The receiving node then reports the learned index to a central decoder at the end of each round and the corresponding link is being discovered. The objective is the minimum number of communication rounds needed for discovering the network topology. Furthermore, the effect of interference cancellation on the network discovery process is discussed, where interference carried over already discovered links can be removed. In the following, an overview of the theoretical results is presented that will build the foundation of the proposed transmission strategy which is referred as the *deterministic distributed algorithm*.

5.1 Theoretical Result

A technique for discovering the whole network in $\mathcal{O}(L^2 \log^2(K))$ rounds was presented in [73] and the lemma below is the starting point.

Lemma 15 *Given the s distinct integers $1 \leq \{x_1, ..., x_s \leq n\}$, then there exist a prime $p \leq s \log_2 n$ for each $1 \leq i \leq s$ s.t. $x_i \neq x_j (\mod p), \ \forall j \neq i$.*

Only one receiver is chosen by each transmitter respectively for transmitting its message, a round for each transmitter-receiver pair and no other transmitter that is connected to the same receiver except the transmitter selected to transmit its message to that receiver transmits. Consequently, Theorem 10 is stated below:

Theorem 10 *For any bipartite network that is formed by K transmitting and K receiving nodes, where each receiving node is only connected to L transmitting nodes, the number of communication rounds to determine the network topology is bounded by $\mathcal{O}(L^2 \log^2(K))$*

Proof Consider the set of all potential communication phases, i.e. all primes $\mathcal{P} = \{p_1, ..., p_s\}$ in the range $\{2, 3, ..., (L+1)\log_2(K)\}$. Each phase consists of s sub-phases and p_i rounds per sub-phase i. In each sub-phase, transmitter j sends its message in the round number $j \mod p_i$, i.e transmitter j is only active in the ($j \mod p_i$)-th round. According to Lemma 15, there exist a prime number $p_i \in \mathcal{P}$ for each transmitter j, such that transmitter j can successfully deliver its message to the targeted receiver. Since there are at most $\mathcal{O}(L\log(K))$ sub-phases, each consisting of at most $\mathcal{O}(L\log(K))$ rounds, the number of rounds to discover the network topology is bounded by $\mathcal{O}(L^2 \log^2(K))$. ■

The simulation results regarding the deterministic distributed algorithm are evaluated in the following section. Furthermore the results are put into contrast to a randomized algorithm with random transmission schedule and interference cancellation. In this case, each transmitter targets one receiver and transmits its message in every round with probability $p = 1/L$. Additionally, a receiver can cancel interference by making use of information about previously discovered transmitters. Also, the impact of shadow fading for $L = 23$ is analyzed. Here, each link in the network is subject independently to erasure with probability $p = \frac{1}{2}$. In the last experiment, the influence of local connectedness to the network discovery problem is investigated.

5.2 Results

5.2.1 Simulation Setup

A Monte-Carlo simulation was executed on a network, where connectivity parameter L and sizes of $K = 2^n, n \in \{3, 4, .., 13\}$ were modified. Here, each network size is subject to 100 network realizations. The algorithm performs in three phases, the *synchronization phase*, the *discovery phase*, and the *termination phase* for each network realization. In the synchronisation phase, all primes $\mathcal{P} = \{p_1, ..., p_s\}$ in the range of $\{2, 3, ..., (L+1)\log_2(K)\}$ are generated. i.e. the set of all potential communication phases, each consisting of s sub-phases and the i-th sub-phase consists of p_i rounds. During each sub-phase, transmitter j is only active in the $(j \mod p_i)$-th round and transmits its message. Then, in the discovery phase, the algorithm operates on the reduced network topology, that consists only of the links associated with the active transmitters, and marks a link, if and only if the attached receiver is connected to only one of the active transmitters. If interference cancellation is enabled, i.e. each discovered link can be used for canceling interference, then the information about the discovered links per sub-phase is shared over time and hence the topology, that is considered at the beginning of each following sub-phase, is reduced by all the links that have already been discovered in the previous sub-phases. In the termination phase, when the network topology for the current channel realization is fully discovered, the total number of communication rounds is

$$\sum_{l=1}^{x-1}(p_l) + (j \mod p_x) + 1, \tag{5.1}$$

that is the sum of all previous sub-phases and the round $(j \mod p_x)$, when the last unknown link attached to transmitter j is being discovered in sub-phase p_x. Note that 1 is added since the lowest mapped sub-phase is of value 0. The randomized algorithm operates in the same three phases as the deterministic distributed approach. Here, only the effect with interference cancellation during the discovery phase is considered and in the synchronization phase, a number in the range of $[0, 1]$ is randomly assigned, which is generated from a uniformly distribution, to each transmitting node and compared to $1/L$. Each transmitting node, that has a number assigned greater than the reciprocal of L is considered as an active transmitter. To simulate the fading model as described in 5.1, a number in the interval of $[0,1]$ is assigned, that is derived from a joint Gaussian distribution, to each existing link, where each link with a number assigned less than $p = \frac{1}{2}$ is erased. In the last experiment, each transmitting node has a radial constraint $r > L$ considering the receiving nodes it is connected to, that is each transmitting node with index $i \in \{1, ..., K-(r-1)\}$ is connected to L receiving nodes with index $j \in \{i, ..., i+(r-1)\}$ and each transmitting node with index $i \in \{K-(r-2), ..., K\}$ is connected to L receiving nodes with index $j \in \{K-(r-1), ..., i, ..., i+(r-2)\}$. These transmitting nodes are referred as *locally connected*.

5.2.2 Simulation Results: Deterministic Distributed vs. Randomized Approach

In this section, the results of the simulation for the network discovery problem with and without interference cancellation for different connectivity parameter L is discussed. Figure 5.1 depicts the required communication rounds to discover the network topology over different network sizes $K = 2^n, n \in \{3, 4, .., 13\}$ for different values of L with and without interference cancellation. Note that the axis for the network sizes is scaled by $\log_2(K)$. To keep the computation times low, only 100 network realizations are considered for each simulation. For the cases with and without interference cancellation, it is observed, that the number of rounds increases almost linear for $\log_2(K)$ respectively and hence show a logarithmic relation between communication rounds and network sizes. Also, for any value of L, the number of rounds increases slower when interference cancellation is allowed than without interference cancellation, for example, for $K = 32$ and $L = 7$, an average of 55.48 rounds is needed without interference cancellation and 32.24 rounds with interference cancellation while 180.27 rounds are needed without interference cancellation and 76.23 rounds are needed with interference cancellation for a network of $K = 8192$. Figure 5.4, Figure 5.3, and Figures 5.2 depict the histogram of the sub-phases, i.e. primes

p_i, before the algorithm terminates for different network sizes with and without interference cancellation. For each of the three simulated connectivity parameter, it is observed that the highest prime per network size when interference cancellation is allowed is less or equal to the highest prime per network when interference cancellation is not allowed. The random algorithm approach for $L = \{3, 5\}$ was simulated. While the randomized algorithm discovers the network with $L = 3$ and $K = 8196$ in 43.65 rounds and thus is comparable to the deterministic distributed approach with and without interference cancellation, the rounds needed for discovering a network with $L = 5$ and $K = 8196$ is 1546 and thus the randomized algorithm is not an approach to discover network topologies fast.

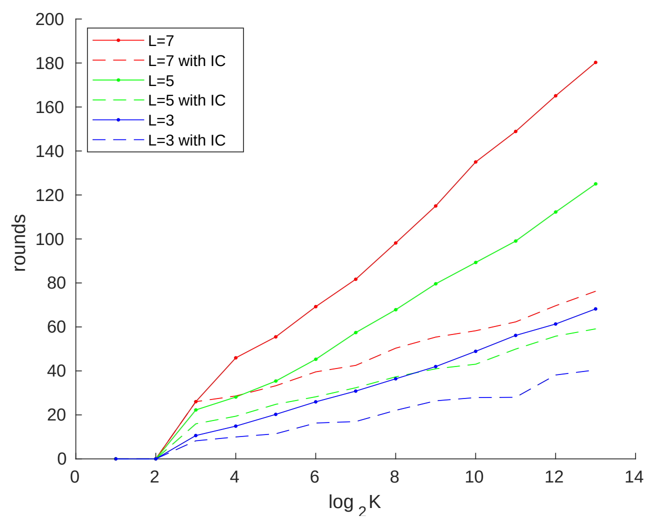
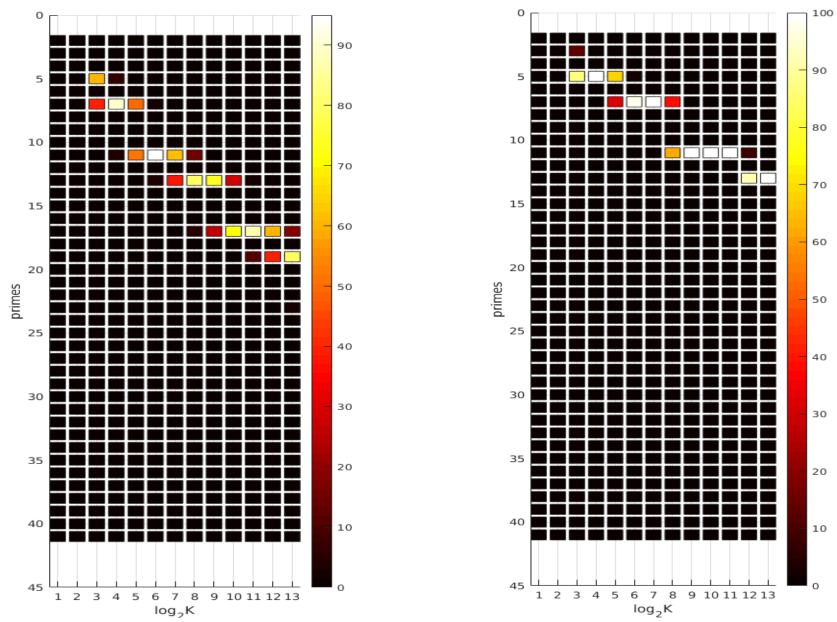


Fig. 5.1. Deterministic distributed algorithm: Number of communication rounds per $\log_2(K)$

5.2.3 Fading and Local Connectedness

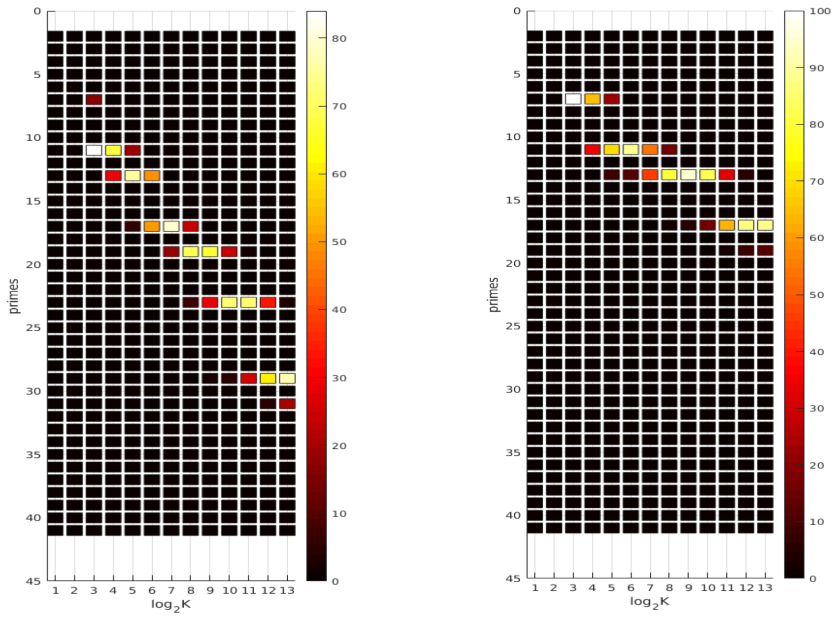
As depicted in Fig. 5.5 and 5.6, applying the fading model has significant impact on the network discovery. In both cases, with and without interference cancellation, the deterministic distributed approach benefits from fading for larger L significantly.



(a) (b)

Fig. 5.2. L=3: Histogram for the primes per $\log_2(K)$ with (a) and without (b) interference cancellation.

For example, while a network of size $K = 8196$ with $L = 3$ and no interference cancellation discovers the network topology in 68.18 rounds and in 54.26 rounds with fading, the same network with $L = 7$ discovers the topology in 180.27 rounds and in 121.64 rounds with fading. Compared to the same experiment, where interference cancellation is allowed, the topology of a network with $K = 8196$ is fully discovered in 76.23 rounds while 57.99 rounds are needed with fading. Interestingly, Figure 5.6 demonstrates for the case with interference cancellation, that fading only effects the network discovery process by a negative offset, while fading decreases the slope of the network discovery process for the case of prohibited interference cancellation. For both cases however, the logarithmic characteristics of the network discovery process is maintained. The observations above hence emphasize the potential of interference cancellation for discovering the topology of a communication network.



(a)

(b)

Fig. 5.3. L=5: Histogram for the primes per $\log_2(K)$ (a) and without (b) interference cancellation.

Comparing locally connected networks with and without interference cancellation as shown in Figure 5.8 and 5.7, it can be concluded that locally connected transmitting nodes have no effect on the network discovery process since no significant change in the needed communication rounds occur.

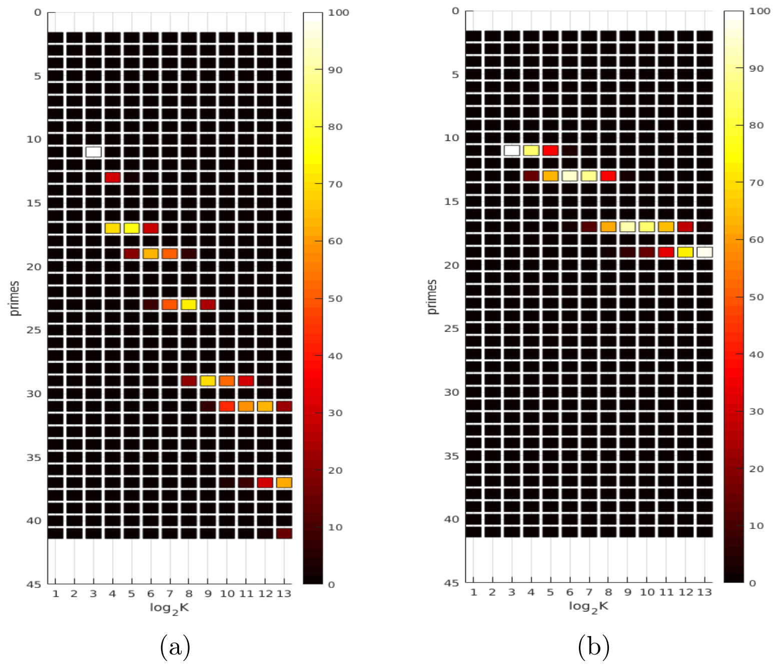


Fig. 5.4. L=7: Histogram for the primes per $\log_2(K)$ (a) and without (b) interference cancellation.

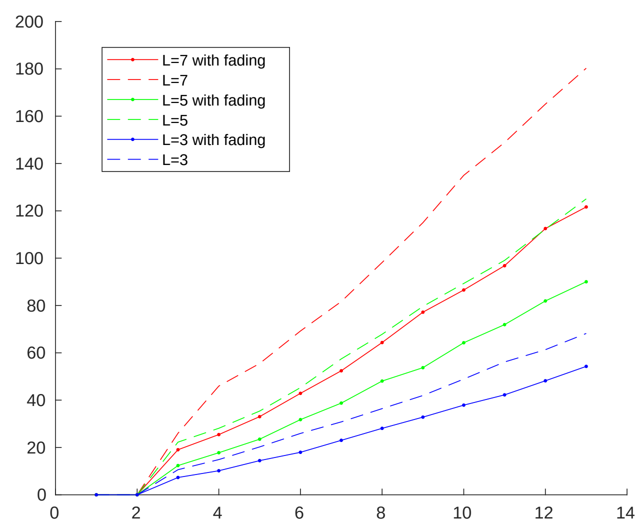


Fig. 5.5. Deterministic distributed algorithm with fading model vs. no fading: Number of communication rounds per $\log_2(K)$

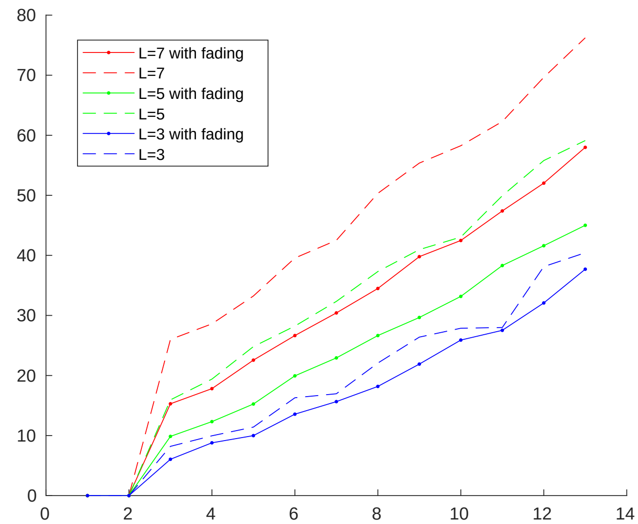


Fig. 5.6. Deterministic distributed algorithm with fading model and interference cancellation vs. no fading with interference cancellation: Number of communication rounds per $\log_2(K)$

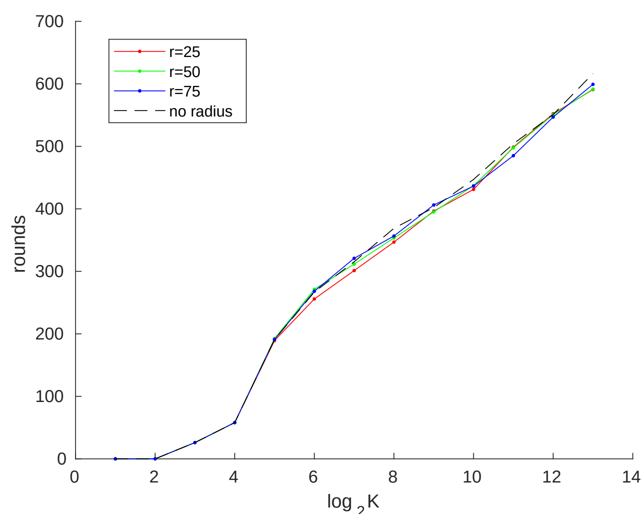


Fig. 5.7. Deterministic distributed algorithm with locally connected transmitting nodes: Number of communication rounds per $\log_2(K)$

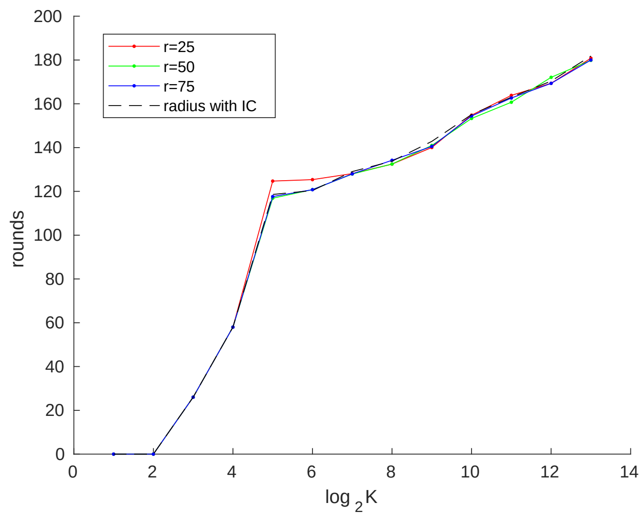


Fig. 5.8. Deterministic distributed algorithm with locally connected transmitting nodes and interference cancellation: Number of communication rounds per $\log_2(K)$

6. DEEP LEARNING FOR CHANNEL IDENTIFICATION

From Chapter 3 through Chapter 5, the problem of Interference Management was viewed from an information-theoretic angle and all simulations rely on algorithms that are backed by asymptotic theoretical guarantees. In this chapter, the impeding problem of Wireless Channel Identification will be discussed, as the potential of Deep Learning based approaches was recently investigated for this problem by [67]. By Channel Identification, the task of a receiving node to identify the channel of a received signal in the 2.4 GHz ISM-Band is addressed, that consists of 15 wireless technologies, grouped into Bluetooth, Zigbee, and WiFi, through capturing only a 10 Mhz narrow-band. Note that this is directly related to the problem of discovering the network topology, that is addressed in Chapter 5.

6.1 Dataset and Network Architecture

The dataset that was generated by Schmidt et al. [67] is analyzed, which contains 225,225 sample vectors for 15 classes in the SNR range of -20 dB to 20 dB with a stepsize of 2 dB^2 [1].

Any class and any rate of SNR consist of 715 sample vectors, that are partitioned into a training set of 480 vectors and a test set of 235 vectors and each sample vector represents 128 I/Q samples in the time domain, corresponding to 12.8μs. Equivalently, I/Q samples in the frequency domain are used, that are computed by applying the Fast Fourier Transformation (FFT) to the time domain I/Q samples. Also, the I/Q samples are converted into an Amplitude-Phase representation, producing results with better accuracy and training time as demonstrated in Section 6.1.4. To follow a

[1] The dataset is available at `https://crawdad.org/owl/interference/20180925/`

concise terminology, these datasets are addressed as FFT I/Q and FFT Amp-Phase, respectively.

With a sample vector as an input, it is aimed to design deep neural network architectures for a convenient channel type recognition, among the 15 classes shown in Table 6.1. For testing purposes, the ensemble of considered network architectures are shorten to the following: CNN, ResNet, CLDNN, and LSTM. In the training phase, the Adam optimizer and a batch size of 256 are applied to all of the four architectures. A learning rate of 0.0001 is applied for each neural network, that is CNN, ResNet, and CLDNN. For the pure LSTM network the selected learning rate is 0.001. Each layer but the last one encounters the Rectified Linear Unit (ReLU) as the activation function. For the last layer, the Softmax activation function is used. The Categorical Cross Entropy function is used for each layer as the loss function. For all layers but the last dense layer of the CLDNN, a dropout of 60% is chosen, while for each layer but the last convolutional layer and the last dense layer of the CNN, a dropout of 60% is chosen. An Alpha Dropout of 10% is applied to the first and second dense layers of the ResNet. Table 6.2 shows a detailed overview of the four architectures. The experimental setup, that was used to train and test each deep neural network classifier, consist of an GPU server equipped with an Nvidia Tesla P100 GPU and 16 GB of memory. All declared training times are attained by averaging over 10 runs using the reported hardware [2].

[2]The code of this work is available at `https://github.com/dl4amc/dl4wii/`

Table 6.1.
The considered 15 classes of channels.

Class Index	Technology	Center Frequency	Channel Width
1	Bluetooth	2422 MHz	1 MHz
2	Bluetooth	2423 MHz	1 MHz
3	Bluetooth	2424 MHz	1 MHz
4	Bluetooth	2425 MHz	1 MHz
5	Bluetooth	2426 MHz	1 MHz
6	Bluetooth	2427 MHz	1 MHz
7	Bluetooth	2428 MHz	1 MHz
8	Bluetooth	2429 MHz	1 MHz
9	Bluetooth	2430 MHz	1 MHz
10	Bluetooth	2431 MHz	1 MHz
11	WiFi	2422 MHz	20 MHz
12	WiFi	2427 MHz	20 MHz
13	WiFi	2432 MHz	20 MHz
14	Zigbee	2425 MHz	2 MHz
15	Zigbee	2430 MHz	2 MHz

6.1.1 Empirical Findings

The objective is primarily to minimize training time and maintain high classification accuracy. The network architectures that are listed in 6.2 are considered, while all obtained results, except for those in Section 6.1.4, are derived from FFT I/Q data. While the CNN architecture in [67] executes in 180s training time, the proposed CNN architecture achieves a marginally higher accuracy and an average training time of around 108s. Furthermore, similar classification accuracy regarding each wireless technology is achieved for every tested network architecture and the attention is restricted on the CNN architecture since [67] only considers CNN architectures as well. As illustrated in Figure 6.1, it can be observed that the classification accuracy for the 3 classes of WiFi signals are essentially lower than for Bluetooth or Zigbee signals for negative SNR db values. More precisely, the lower performance on WiFi signals results primarily from the confusion among various WiFi channels as

Table 6.2.
Tabulated are all investigated neural network architectures, where the column for convolutional layers shows the number of feature maps and kernel size of each layer. The columns for dense layers show the dimensionality for input and output.

Architecture	Activation Function	Convolutional Layer	Dense Layer	Recurrent Cells	Residual Stacks	Accuracy
CNN [67]	ReLU, Softmax	64 $3*1$, 1024 $3*2$	$126976*128$, $128*15$			0.8941
CNN	ReLU, Softmax	256 $3*1$, 256 $3*2$	$31744*1024$, $1024*15$			0.8962
LSTM	ReLU, Softmax		$512*15$	512, 4		0.8965
ResNet	ReLU, Softmax		$128*128$, $128*128$, 15		5	0.8938
CLDNN	ReLU, Softmax	256 $3*1$, 256 $3*2$	$512, 256$, $256, 15$	256		0.8950

depicted in confusion matrix in Figure 6.2. In addition, it can be abstracted from Table 6.1 that frequency bands occupied by different WiFi channels overlap and are not fully represented in the frequency band of 2421.5-2431.5 MHz. It is believed that these two effects are responsible for the confusion between WiFi signals. Hence, devote your attention on bands close to WiFi center frequencies for the proposed band selection methods as elaborated in the following section.

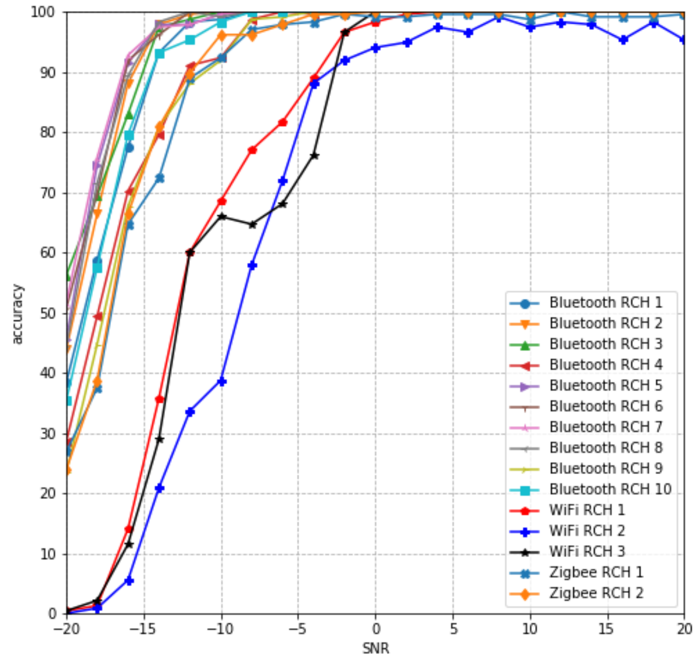


Fig. 6.1. Classification accuracy vs. SNR of CNN on 10 MHz dataset.

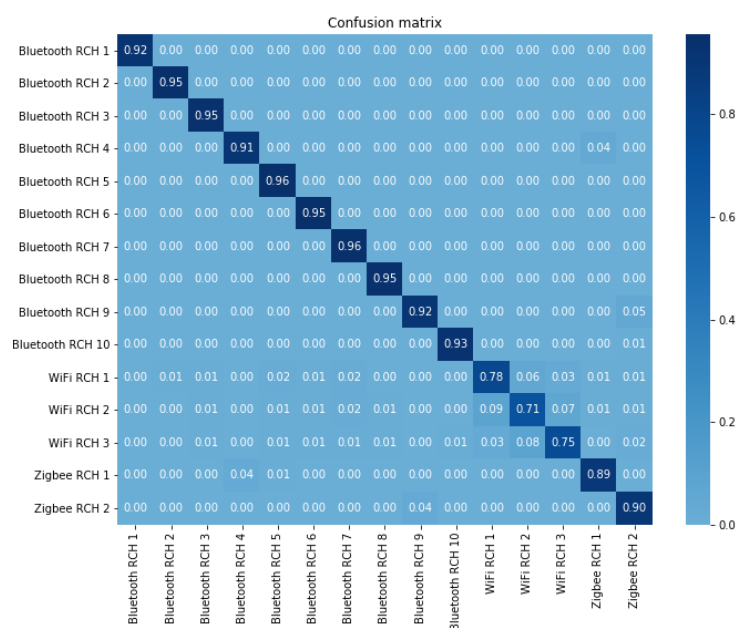


Fig. 6.2. Average confusion matrix of CNN on 10 MHz dataset.

6.1.2 Band Selection

The ambition is to provide fast training of deep neural network classifiers through band selection while maintaining high classification accuracy. Therefore, the term of band selection is established in context of the provided dataset. It is proceeded with comparing the variance of the training time and accuracy prior and post different decisions of band selection. Then, the set of best performing results after band selection are conceptualized in Table 6.3 and unveil important training time contraction with marginally impact on the classification accuracy. Here, band selection implies to utilize only data from a subset of the original 10 MHz frequency range to train and test the neural network classifiers. Band selection is basically attained through keeping segments of each frequency domain sample vector, that is the time-domain dataset transformed with FFT, corresponding to the selected band. It follows that reducing the length of each sample vector compresses the neural network architectures. Since not every class is represented in the narrower range of frequencies, Band Selection leads to fewer classes.

The analysis starts with choosing a narrow band of 2 Mhz width with a frequency range from 2429 to 2431 MHz as the selected sub-band. This procedure leads to only 7 represented classes, which are classes 8, 9, and 10 (Bluetooth), classes 11, 12, and 13 (WiFi), and class 15 (Zigbee). The same neural network architecture is kept as prior band selection, except that a dropout of 60% is included after the first convolutional layer, which has no impact on the classification accuracy but accelerates the training process and is shown in Table 6.3. It can be recognized that the accuracy among WiFi signals is affected the most. Figure 6.3 provides another thought-provoking finding, that is the confusion between class 12 and class 13 is much more serious than that between class 11 and class 12 or between class 11 and class 13 after band selection is performed. Contrarily, in the confusion matrix prior band selection, these numbers are similar. It is assumed that this is due to the selected frequency range, which is just between, and close to, the center frequencies of classes 12 and 13, while the

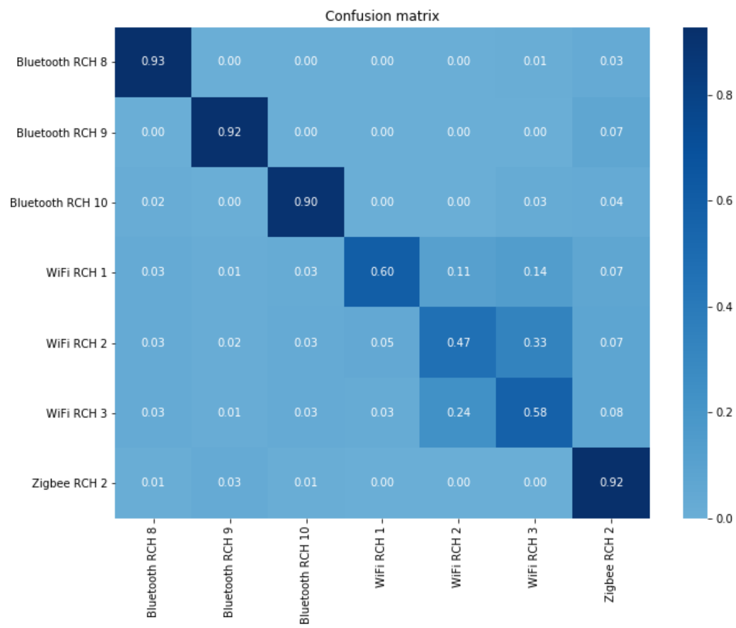


Fig. 6.3. Average confusion matrix of CNN on 2 MHz dataset.

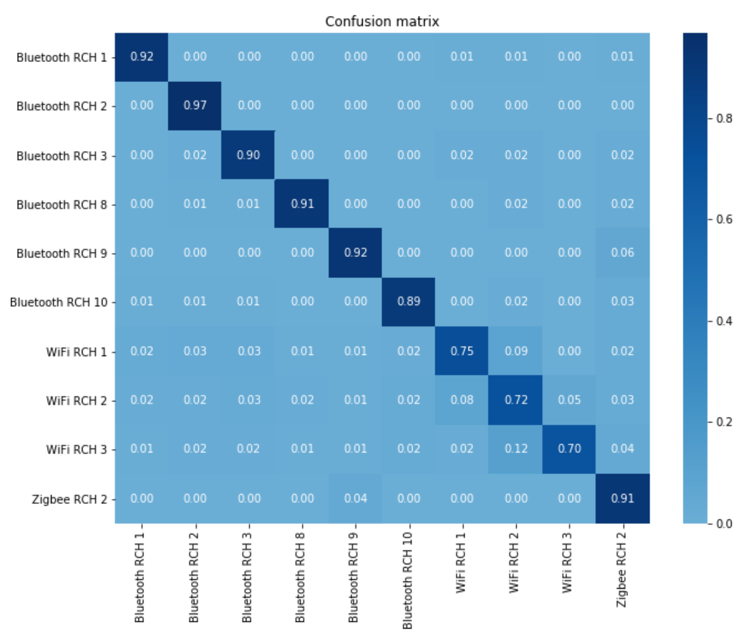


Fig. 6.4. Average confusion matrix of CNN on 4 MHz dataset.

center frequency of class 11 is further from the selected band. Another 2 MHz band between 2422 and 2424 MHz is add to the previously selected band, i.e. a 4 MHz wide frequency range (2422-2424 MHz and 2429-2431 MHz) is analyzed with the objective to enhance the classification accuracy of the WiFi signals. The new 4 MHz band is formed out of 10 classes, with 6 classes of Bluetooth, 3 classes of WiFi, and 1 class of Zigbee. Table 6.3 tabulates the classification accuracy and training time for each wireless technology. The results demonstrate, that expending the frequency band by another band of 2 MHz leads to a major improvement of the classification accuracy for WiFi signals from 0.5288 to 0.7253 and hence a performance is obtained that is only smaller than 3% than the performance prior band selection, while training time decreases by approximately 60%. Additionally, another 4 MHz frequency range from 2424 MHz to 2426 MHz and 2429 MHz to 2431 MHz was analyzed but the classification accuracy for identifying WiFi signals drops from 0.7253 to 0.6363. It can be suspected that the added band from 2424 MHz to 2426 MHz is still too distant to the central frequency of any WiFi signal (2422, 2427, and 2432 MHz).

Table 6.3.
Summary of classification accuracy and training time.

	10 MHz	4 MHz	2 MHz
Bluetooth	0.9402	0.9196	0.9149
WiFi	0.7467	0.7323	0.5255
Zigbee	0.8918	0.8967	0.9286
Total Training Time	108.04s	65.096s	40.745s
Number of Epochs	6.6	15.8	28.1
Training Time per Epoch	16.37s	4.12s	1.45s

6.1.3 Training SNR Selection

Next, the training of the CNN architecture from Section 5.1.1 for FFT I/Q datasets is debated for an individual SNR value for the 10 MHz dataset and the 4 MHz dataset described in Section 6.1.2. Despite significant training time reduction, a high testing accuracy is maintained for high SNR values. The relation between testing accuracy and training SNR values for the 10MHz dataset is depicted in 6.5 and the average testing accuracy for different training SNR values is near to an average accuracy of approximately 75%. In contrast, an average testing accuracy of 90% is obtained for the entire training dataset. The best average classification accuracy is achieved for data corresponding to an SNR value of -10 dB with a total accuracy of slightly over 80%. While training with -10 dB data impairs the classification accuracy for high SNR testing data, an increase in accuracy is obtained for low SNR testing data, compared to training with high SNR data, where the confusions across the WiFi signals are dominant. Figure 6.5 depicts the testing accuracy for each SNR after training with -10 dB. Here, the training time per epoch for training the entire 10 MHz dataset amounts 16.37 seconds and the training time per epoch is reduced to 0.984 seconds considering data at only one SNR value for training. Table 6.4. tabulates the total number of epochs. Furthermore, training the model with only a single SNR value decreases training time by approximately 92.3%. The average testing accuracy for all individual training SNR values for the 4 MHz dataset is depicted in Figure 6.5. An overall average testing accuracy across all training SNR values of around 73% is obtained in contrast to an accuracy of 86% for training data that covers all SNR values. The best performance across all available training SNR values was achieved for the 4 MHz dataset by training with only -2 dB data, that is an accuracy of approximately 77%. The testing accuracy for each SNR value with the model trained only on -2 dB data is illustrated in Figure 6.5. For high SNR values, a classification accuracy is achieved above 90% and training time decreased by 90.9% and is tabulated in Table 6.4. SNR selection in general has the potential for total training time reduction by 18x and above, while achieving a classification accuracy for high SNR testing data of above 90%.

Table 6.4.
Training Time and Number of Epochs for SNR Selection.

	Time per Epoch	Number of Epochs	Accuracy
All SNR 10 MHz	16.37s	6.6	0.8962
-10 dB 10 MHz	0.984s	8.5	0.8022
All SNR 4 MHz	4.12s	15.8	0.8614
-2 dB 4 MHz	0.61s	9.7	0.77

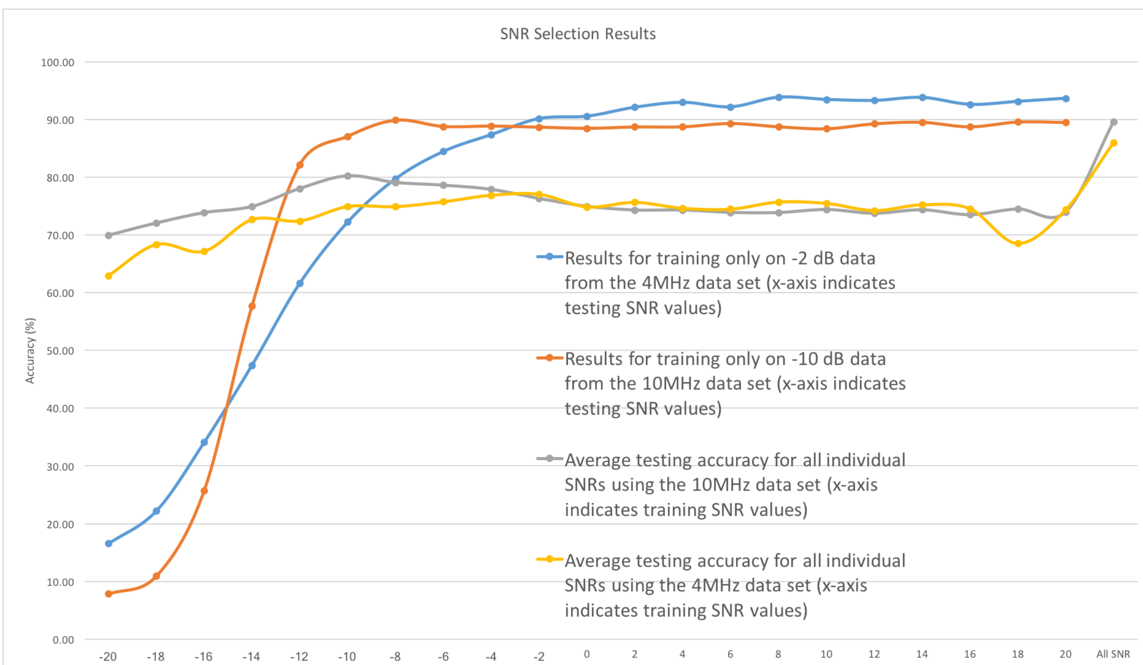


Fig. 6.5. SNR Selection Results

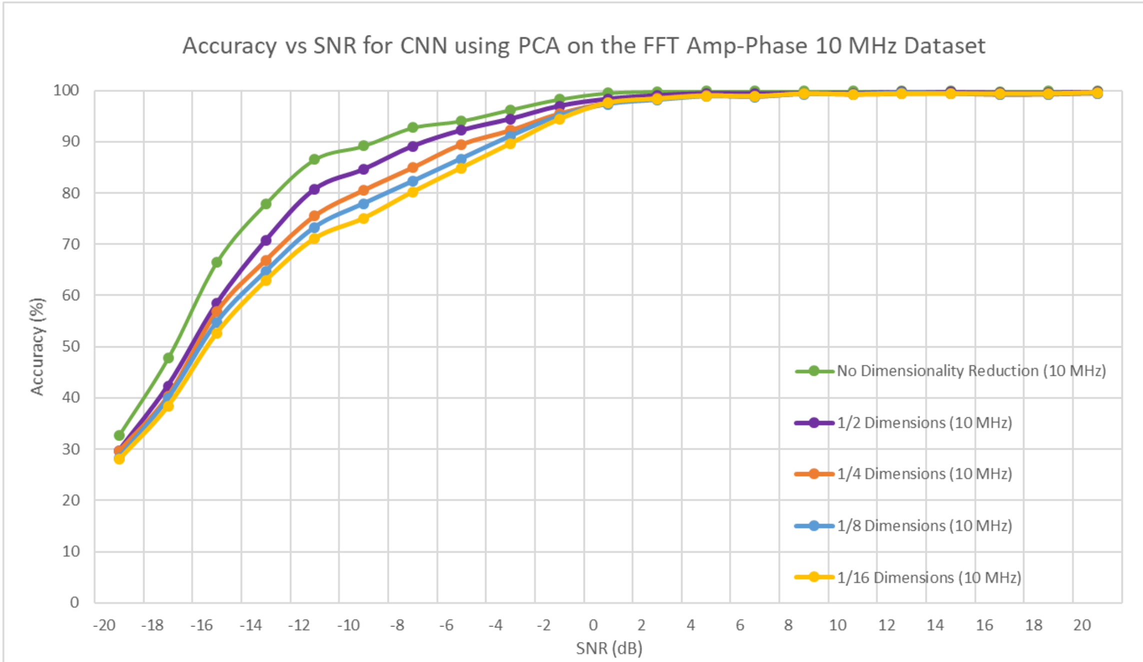


Fig. 6.6. PCA on FFT Amp-Phase data

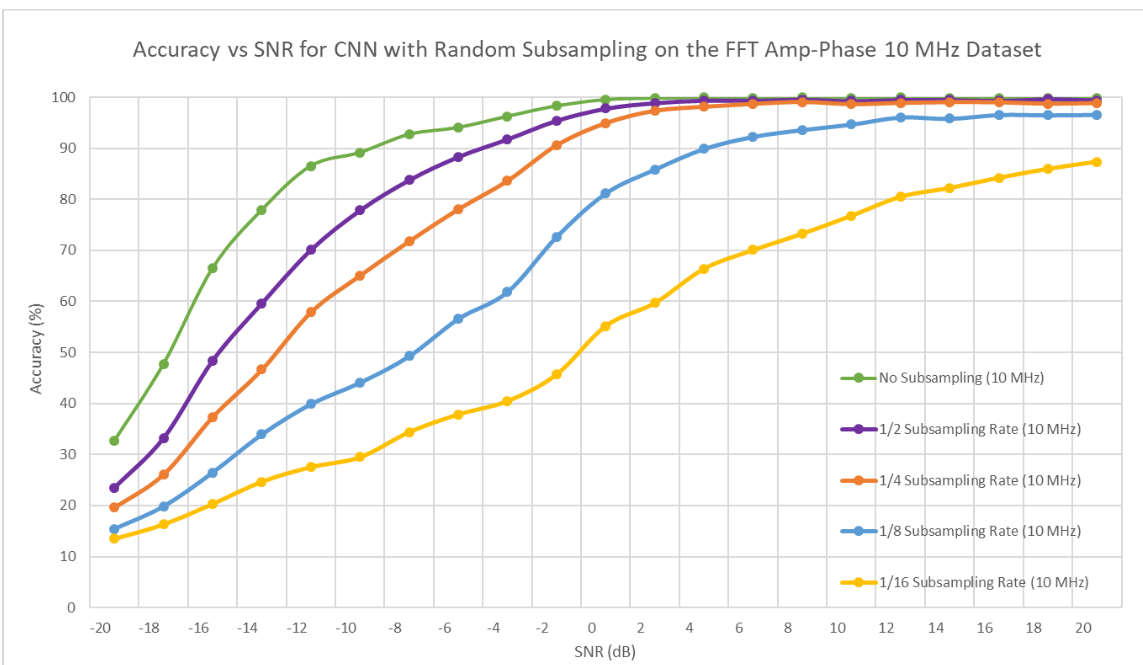


Fig. 6.7. Random Subsampling on FFT Amp-Phase data

Table 6.5.
Comparison of Training Times using PCA

Dimensions/Samples	Time per Epoch	Epochs	Accuracy
All (10 MHz)	16.37s	6.6	0.8962
1/2 (10 MHz)	7.79s	8.6	0.8726
1/4 (10 MHz)	3.86s	8.5	0.8576
1/8 (10 MHz)	2.16s	7.4	0.8487
1/16 (10 MHz)	1.78s	7.3	0.8411
All (4 MHz)	4.12s	15.8	0.8614
1/2 (4 MHz)	2.72s	12.1	0.8358
1/4 (4 MHz)	1.64s	8.6	0.8310
1/8 (4 MHz)	1.33s	8.2	0.8220

6.1.4 PCA and Sample Selection

Next, the influence of applying Principal Component Analysis (PCA) and various subsampling techniques on the training time and classification accuracy of the proposed CNN architecture will be elaborated.

The effect of PCA for the FFT Amp-Phase data is established in Figure 6.6. Here, a confrontation occurs with an imperceptible loss in accuracy for SNR values above 0 dB and a compression rate as high as 16x. Figure 6.7 presents the outcomes of applying Random subsampling on the FFT Amp-Phase data and shows, that, compared to PCA, large drops in accuracy at low SNR values occur. In contrast, significant outcomes are examined at high SNR values with imperceptible loss in accuracy for a subsampling rate as low as $\frac{1}{4}$ and comparable results are achieved with Uniform subsampling. Figure 6.8 shows the outcomes after combining band selection and dimensionality reduction (PCA). For a combination of band selection and reduction rate of up to $\frac{1}{8}$, a stable classification accuracy at moderately high SNR values with corresponding training time reduction can be observed as listed in Table 6.5, that is a proportional decreasing trend when applying PCA to reduce the number of dimensions/samples. Overall, the training times for Random and Uniform subsampling are close to those in Table 6.5.

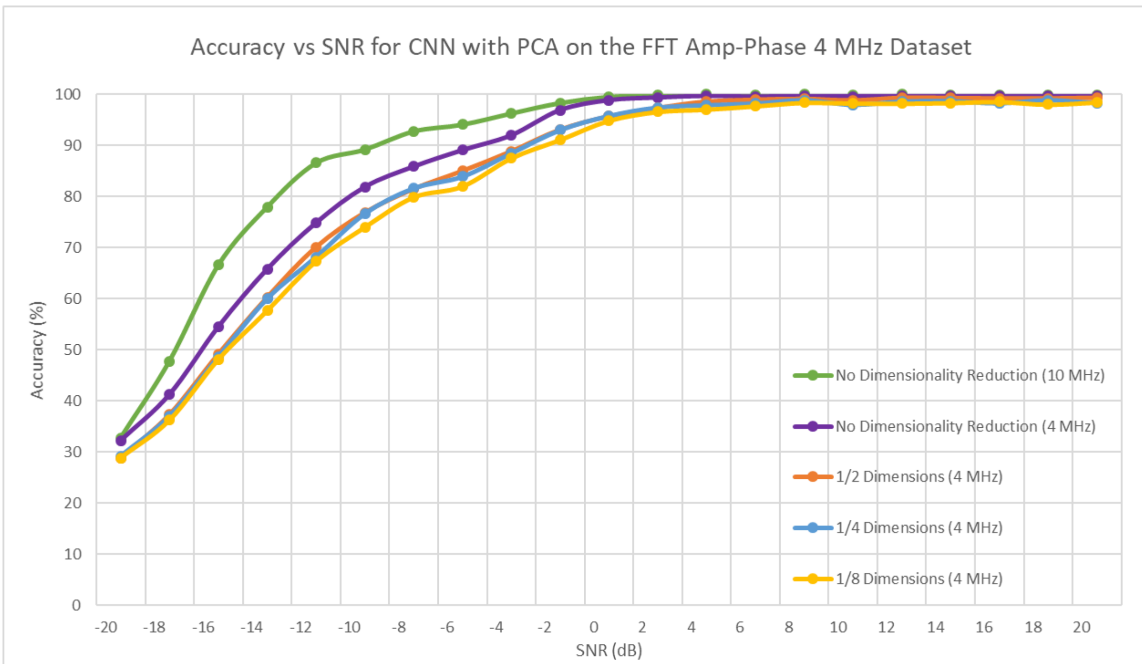


Fig. 6.8. PCA on FFT Amp-Phase 4 MHz data

7. CONCLUSIONS AND FUTURE WORK

The optimal cell association decisions in locally connected interference networks were identified with the objective to optimize for the average uplink-downlink puDoF problem. The considered network was subject to a backhaul constraint such that each mobile terminal is associated with N_c base stations, while each base station in the interference network is connected to a mobile terminal of same index and L consecutively indexed mobile terminals. In addition to outlining the optimal cell association and puDoF for the uplink problem under zero-forcing schemes, it has been demonstrated regarding the average uplink-downlink puDoF, that the description of the optimal association for the case $N_c \leq \frac{L}{2}$ is causally linked to the uplink description and previous work regarding the downlink. Moreover, the optimal zero-forcing downlink scheme was derived by changing the uplink scheme to the uplink-only-optimal scheme when $N_c \geq L + 1$. It is believed that full DoF regarding the uplink when $N_c \geq L + 1$ is optimal and consequently, optimality for the suggested cell association and average puDoF applies. The inner bounds for the zero-forcing average puDoF, which are generalized, jointly across uplink and downlink are demonstrated and its information-theoretic optimality is proved for $L = 1$, that is for Wyner's linear network. In the second part of this dissertation, the problem of linear interference networks was analyzed that are separately subject to link erasure events with probability p over blocks of time slots, with separate coding over different blocks. Here, each message is restricted to be assigned at M transmitters. The optimal message assignment strategies for $M = 1$ at different values of p were examined, and described the average per user DoF $\tau_p(M = 1)$. It was demonstrated for values of $M \geq 1$, that no optimal strategy of message assignments exists for all values of p and proposed strategies of message assignments for $M = 2$, where it was demonstrated inner bounds on $\tau_p(M = 2)$ that are asymptotically optimal as $p \to 0$ and as $p \to 1$. This part of the research was finished with deriving an algorithm for $M = 2$ that leads to the optimal average per user DoF under restriction to cooperative zero-forcing schemes and proved its information-theoretic optimality for a wide class of network realizations. The simulation results confirm the shifting role of cooperative transmission from interference management at low erasure probabilities to increasing coverage at high erasure probabilities. In the third part of this dissertation, partial results on the simulation of the message passing model were used to present a result on discovering the network topology. Morover, the minimum number of communication rounds that is needed to discover the network topology is investigated. Assuming a single-hop network that is restricted to interference-avoidance based schemes, each node of the network is able to deliver a different message to its neighbouring nodes. Furthermore, a transmitting node successfully delivers its message if and only if it is the only active transmitter connected to its receiving node while ignoring the effect of channel noise. In the first case, any collision detection strategy is ignored such that no transmitter can identify neighbouring active transmitters and then the problem is viewed in light of cooperative transmission through backhaul links. In both cases, the effect of fading and local connectedness was analyzed, respectively. The main findings indicate, that the network discovery process is of logarithmic characteristics, i.e. is bounded logarithmically, and that this characteristics is not affected by fading, interference cancellation or local connectedness. However, allowed interference cancellation without any fading effects will accelerate the network discovery process linearly with respect to $\log_2(K)$ while maintaining the logarithmic characteristics. The same behavior can be observed for the presence of fading when no interference cancellation is allowed. Interestingly, allowing interference cancellation in the presence of fading causes only an offset of the network discovery process with respect to $\log_2(K)$, emphasizing the potential of interference cancellation for discovering the network topology. Lastly, it was demonstrated for $L = 23$ that the radial constraint r of a locally connected network has no influence on the network discovery process. In the last part of this dissertation, four deep neural architectures were analyzed that achieve an average accuracy around 89.5% for analyzing a 10 MHz received wireless signal in the 2.4 GHz ISM band, and identifying one of 15 different channels belonging to WiFi, Bluetooth and ZigBee. Moreover, band and training SNR selection techniques were utilized, PCA and sample selection through various sub-Nyquist sampling methods for drastically reducing the training time while maintaining a high classification accuracy were used. In contrast to [67], covering the center frequencies of key wide channels by a band of 4 MHz with a single optimized SNR value for training (-2 dB) achieves an acceleration of the total training time by 30x with an imperceptible loss in classification accuracy for testing SNR values above -4 dB.

Further potential suggests to extend the work presented in Chapter 4 for denser networks with connectivity parameter $L > 1$ as well as allowing messages to be assigned to more transmitters, i.e. $M > 2$. In terms of denser networks, it is interesting to see, how the Channel Identification problem can be adjusted for a multi-class classification problem, i.e. the task of the receiver is to distinguish all the channels contributing to a received superposition signal. Since the attention was restricted to only CNN architectures, analyzing the applicability of RNN, such as Gated Recurrent Units (GRU) and Long Short Term Memory cells (LSTM), for capturing longterm temporal correlations is also an interesting avenue for future work. Here, experimenting with different sequence lengths regrading LSTM and GRU based neural network architectures and diverse voting techniques for evaluation could lead to additional improvement of the overall classification accuracy as well as mitigating individual confusions between different WiFi channel signals. Finally, it is believed that the bounds for discovering the network topology in Chapter 5 can be tightened even for the same proposed algorithm that was considered, as suggested by obtained numerical results.

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