A MOTIVATIONAL FRAMEWORK FOR THE DESIGN AND EVALUATION OF LEARNING ENVIROMENTS IN UNDERGRADUATE MAINSTREAM CALCULUS

by

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To learners of mathematics and those entrusted to facilitate their education.

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ABSTRACT

This study provides a framework to guide educators and researchers within departments of mathematics at institutions of higher education involved in reform efforts for undergraduate mainstream calculus. It does so by using motivational constructs from Self-Determination Theory to define and measure student-centeredness within both traditional and reformed calculus learning environments within a large-scale, quasi-experimental study. Motivational inventories assessing students' perceived satisfaction of basic psychological needs and self-determined motivation were analyzed together with demographic variables, course outcomes, and prior math knowledge within traditional and reformed conditions in both Calculus I and Calculus II courses. Results include 1) positive correlates among students' perceptions of satisfaction of basic psychological needs, intrinsic motivation, and achievement; 2) overall increased student perception of BPN-satisfaction in the reformed condition; and 3) directional variation in achievement and perception of BPN-satisfaction between conditions across subpopulations. The results demonstrate how student-centered calculus learning environments operate through motivational processes to improve academic outcomes and how learning environments may differentially affect demographic categories at institutions of higher education. Specifically, learning environments are not culturally or socially neutral and may, despite good intentions, be centered about privileged populations to the detriment of historically disenfranchised groups.

CHAPTER 1: INTRODUCTION

1.1 The Echoing Call for Mainstream Undergraduate Calculus Reform

Completion of undergraduate Calculus courses that serve as prerequisites for further study in mathematics (so-called 'mainstream' Calculus courses) is a requirement of virtually all undergraduate degree programs in mathematics, engineering, and the physical sciences (ACS, 2015; ABET, 2016). Each fall, more than 230,000 US college students enroll in Calculus I alone at four-year institutions (CBMS, 2010, p. 18). Regrettably, the average success rate is approximately 78% (Apkarian et al., 2017) producing roughly 50,000 failed students now at-risk for attrition from STEM degrees and careers (IES, 2013). Given that math placement based on ability is generally managed at most institutions, the problem may lie with a more systemic product of non-cognitive student factors and ineffective learning environments. While such matters have been well-known for decades, little seems to have changed since the 1987 large-scale colloquium entitled "Calculus for a New Century: A Pump, Not a Filter," aimed at addressing the concern that only 25% of the over 1 million students who take Calculus annually "survive to enter the science and engineering pipeline. And that those who do survive are poorly motivated for advanced study and too uniformly white, male, and middle class" (Steen, 1988, p. 12). Simply stated, Calculus is a stalwart gatekeeper to further study in mathematics and careers in STEM. Calculus reform efforts aimed at all aspects of the nature and role of Calculus in higher education to increase equity, access, and success towards STEM careers, surged more than thirty years ago. Yet, recent evidence of student outcomes shows there remains ample opportunity for improvement and reform. An obvious indicator of the lack of progress is the continued reliance on antiquated pedagogical methods in Calculus on which many institutions of higher education still rely where 46% of students, nationally, are taught in a lecture/recitation learning environments and 33% of students are taught in large lecture-only learning environments (CBMS, 2010, pg 18). Encouragingly, when recently surveyed, many departments of mathematics at four-year institutions report specific desires to implement changes and improve how Calculus is taught; however, they were unsure how to envision and execute that change (Rasmussen, 2016) despite the existence of comparative research on the efficacy of non-lecture-based pedagogical methods (Freeman, 2014) and long-standing exemplar pockets of excellence in student-centered mainstream Calculus instruction (Carreon et al., 2018).

Despite the issues surrounding mainstream undergraduate Calculus being well-known to the mathematics community, there remained a dearth of action plans and insufficient holistic, actionable knowledge about who is taking Calculus and its instructional landscape at colleges and universities. Consequently, the Mathematical Association of America (MAA) orchestrated a 5-year study (2010-14) of Characteristics of Successful Programs in College Calculus (Bressoud, 2015) using nationwide surveys and onsite evaluations of departments of mathematics to elucidate the instructional landscape and student profile of undergraduate Calculus I (Brossoud, & Rasmussen, 2015). Following an extensive mixed-methods analysis and acknowledging extensive research on the efficacy of student-centered instruction to increase achievement and improve STEM retention, the CSPCC report lists the use of active-learning and student-centered pedagogies, as opposed to traditional lecture-based approaches, as one of seven final recommendations to improve the success of Calculus programs along with proper placement, proactive student support services, challenging and engaging courses, communities of practice, graduate teaching assistant training, and use of local data (Brossoud et al, 2015). Subsequently the MAA commenced a broader 5-year study (2015-19), inclusive of pre-calculus through Calculus II environments and programs, called Progress through Calculus (Apkarian et al., 2017). In this study, when asked about pedagogical methods, 44% of departments of mathematics surveyed (N=219) reported that active learning strategies (defined as inquiry-based methods, group work, flipped classes, and the use of clickers) are ‘very important’. Yet, only 15% of departments report that they are ‘very successful’ in implementing active learning strategies. Finally, less than 3% of departments report that they implement active learning strategies as the primary learning technique (Apkarian et al., 2017). However, despite persistent use within higher education, the phrases “student-centered” and “active learning” have been described as part of a “cacophony” of poorly defined educational terminology related to the Calculus instruction whereby false dichotomies are made between teacher-centered and student-centered approaches with respective passive and active roles and where wholesale distinctions ignore the complex dynamics, interactions, and fluidity of classroom instruction (White & Mesa, 2014). Amid such lack of clarity, it is not surprising that decades of reform efforts which rely upon implementing improved pedagogies have fallen short and that the overwhelming majority of departments of mathematics have been unable to cultivate the student success they seek.

1.2 Towards a Student-Centered Definition of Student-Centeredness in Calculus

The objectives of the present study include addressing this lack of common operational understanding of student-centered pedagogies in mainstream undergraduate Calculus as well as the opacity of the mechanisms by which student-centered pedagogies facilitate achievement. Knowledge of these mechanisms and related measures may be used to establish a framework for the design and implementation of pedagogical reform in undergraduate Calculus. This may be accomplished defining and measuring student-centeredness within Calculus learning environments using students’ perception of satisfaction of the empirically-established, minimal set of Basic Psychological Needs (BPN) (autonomy, competence and relatedness) articulated by Self-

Determination Theory (SDT) (Ryan & Deci, 2000). Under SDT, autonomy is the willful, proactive engagement in activities or tasks within an environment, competence refers to the perceived efficacy of an individual as a causal agent towards their objectives, and relatedness describes an individual's ability to foster reciprocal understanding and engagement with others. Research in SDT asserts that individuals will inherently pursue environments and objectives that support their BPN when available and that BPN-supportive environments will facilitate goal achievement relative to those objectives. Unlike physiological needs (e.g., hunger), inadequate satisfaction of BPN may lead to maladaptive or compensatory strategies as substitutes that can lead to negative consequences and diminished autonomous regulation of behavior (Deci & Ryan, 2000).

While teachers may not be able to control all aspects of the motivational climate in the classroom, Reeve et al. (1999) documented teachers' motivating styles and found autonomy-supportive teachers (as measured by the Problems in Schools questionnaire (Deci et al., 1981) to behave with particular characteristics including listening to students, diminished use of directives, perspective-taking statements, asking about students' wants, and answering student-generated questions. Autonomy-enhancing teacher behaviors (fostering relevance, allowing criticism, and providing choice) as well as autonomy-suppressing teacher behaviors (forcing meaningless activities, suppressing criticism, and intruding) have also been delineated by Assor et al (2002). In their study of affect and engagement, they found these subtypes of autonomy-related teacher behaviors to be reliable and that positive affect correlated most strongly ($r=.39$) with fostering relevance, negative affect correlated most strongly ($r=.38$) with Intruding, and engagement correlated most strongly with Suppressing Criticism ($r=-.20$) and Fostering Relevance ($r=.25$). Reeve & Jang (2006) also identify specific types of instructional behaviors that support student perception of BPN-satisfaction such as listening to students, allowing students to work their own way, use of open seating arrangements, offering hints and encouragement, and responsiveness to student questions. In striking similarity, these autonomy-supportive instructional behaviors are well-aligned with activities within exemplar student-centered, active learning Calculus classrooms such as having students explaining their thinking, providing rationales, teamwork, and regular instructor feedback (Larsen et al., 2015) as well as those techniques shrewdly categorized as 'Ambitious Teaching' (teamwork, explanation of thinking, applied/relevant problems, class discussion, novel problems, student presentations, instructor feedback, and minimal direct instruction) in the CSPCC report (Bressoud, 2015). A motivationally-based measure of successful Calculus learning environments also answers earlier calls from the 1987 national calculus reform symposium conducted by the National Research Council (NRC) and MAA (Steen, 1988) where panel members asserted the needs for "excitement in the classroom" (p. 154), for students to have "a sense of purpose" (p. 67), for "a climate of sensitive awareness (p. 94)", "to create learning environments where instructors and students can get to know each other (p. 87)", and for "the most caring and inspiring" (p. 173) teachers to be assigned Calculus. Furthermore, and perhaps more importantly, perceived BPN-satisfaction of students has been shown to support internally regulated behavior and promote academic outcomes in a wide variety of educational settings (Ryan & Deci, 2000) and is, therefore, a viable explanatory mechanism for the efficacy of student-centered and active learning strategies in the mainstream undergraduate Calculus classroom.

Additionally, with an individualized measure of student-centeredness in the Calculus classroom, we gain the ability to understand about which students' specific pedagogical practices might be "centered" and better understand perceived BPN-satisfaction of minoritized learners within a Calculus learning environment. For instance, a Calculus learning environment that is designed and measured to support a domestic, white, male, student's perceived satisfaction of autonomy, competence, and relatedness is student-centered for that student, and the same learning environment may not be centered about international students, students of color, or women. Of course, some pedagogical techniques and learning environments within specific contexts may increase perceived BPN-satisfaction, on average, for a broad range of students and, thus, be labeled as "student-centered" techniques; however, we should not dismiss the idea that those same techniques, in particular contexts, may thwart BPN-satisfaction, internal regulation of motivation, and achievement for other students in those contexts. While SDT is a universal framework for modeling and affecting motivational processes, the manifestation of perceived BPN-satisfaction may vary across diverse contexts and individuals (Deci & Ryan, 2000). Thus, assessment of student-centeredness by student-perceived BPN-satisfaction, in combination with other measures, may be useful tools for understanding the mechanisms by which pedagogical techniques are effective or ineffective (and for whom). Departments of mathematics may leverage this information to compare, design, and refine undergraduate Calculus learning environments to advance mathematics education reform efforts through the evaluation of complete mathematics instructional systems and their complex dynamics (Cohen et al., 2003; Herbst & Chazan, 2012).

Beyond the parallels between autonomy-supportive instructional practices and active learning pedagogies, Self-Determination Theory, as a motivational framework, is particularly suitable for understanding student behavior and achievement in mainstream undergraduate Calculus because it treats extrinsic motivation along a continuum of perceived locus of causality (Ryan & Deci, 2000). The overwhelming majority of students in Calculus take it because it is required for degrees in Science and Engineering, but the tasks and concepts in Calculus are not the practice of Science nor Engineering. In particular, Science and Engineering are empirical and inductive while mathematics is wholly deductive in nature. It should not be assumed that Science or Engineering students have intrinsic motivation for the study of mathematics or Calculus, however, the relationship between Calculus and Science or Engineering in the presence of students' presumed intrinsic motivation for Science or Engineering should provide a motivational source for engagement with Calculus. Specifically, Self-Determination Theory explicates extrinsic motivation with the following continuum and as depicted in Figure 1: 1) Autonomous Extrinsic Motivation (AEM) as Integrated regulation (e.g. "Because acquiring all kinds of knowledge is fundamental for me.") or Identified regulation (e.g. "Because it allows me to develop skills that are important to me.") and 2) Controlled Extrinsic Motivation (CEM) as Introjected regulation (e.g. "Because I would feel guilty if I didn't.") and External regulation (e.g. "Because that's what I was told to do.") comprises. At best, we might hope students who have goals associated with or Intrinsic motivation for (e.g. "Because I enjoy it.") Science and Engineering to have Identified or Integrated regulation of motivation for the study and practice of Calculus. Given this context,

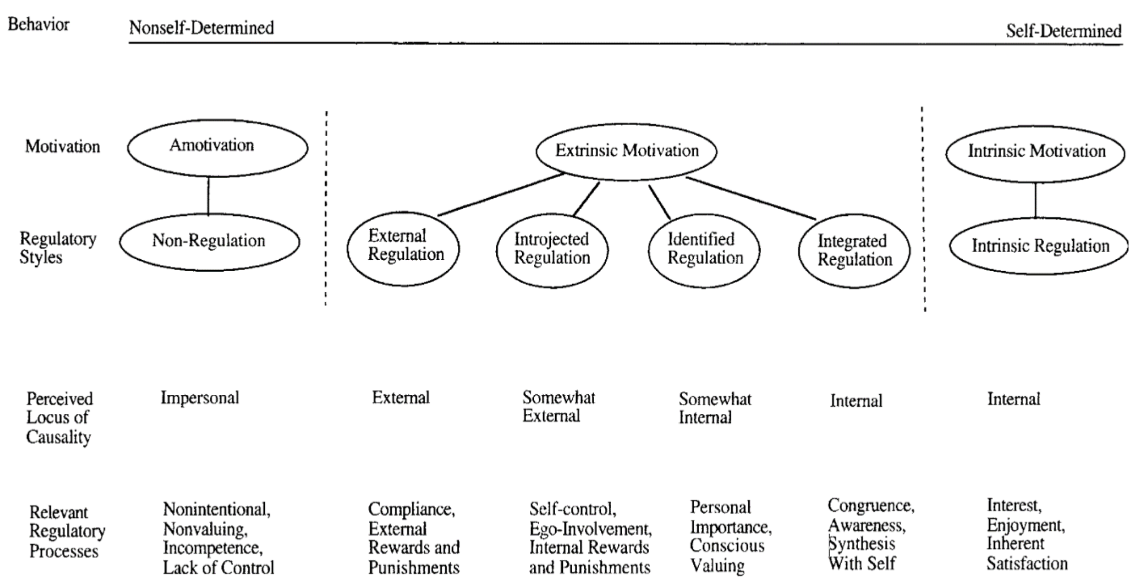

Figure 1: Continuum of Self-Determined Motivation (Ryan & Deci, 2000)

measuring students' AEM is potentially a better way to explain variation in academic performance across groups with equivalent aptitude rather than relying on intrinsic or extrinsic motivation as dichotomous constructs. Furthermore, SDT researchers have demonstrated is numerous studies that AEM provides a similar quality of motivation towards achievement as Intrinsic motivation (Ryan & Deci, 2000).

In summary, an SDT framework may prove useful for advancing undergraduate Calculus reform efforts because of the alignment between autonomy-supportive instructional practices and student-centered pedagogies espoused by the MAA and research related to achievement outcomes in undergraduate Calculus and STEM. Support for student-perceived BPN-satisfaction (which can be manipulated via the learning environment) has been demonstrated to promote student Autonomous Extrinsic Motivation which, in turn, promotes outcomes. It provides a theoretical framework for the necessary conditions for high-quality motivation and student achievement to occur in a manner that is measurable in the classroom. Reformists may use this information to devise holistic, individualized strategies for the refinement of learning environments to support diverse learners. Those strategies are likely to include many ideas canonically associated with student-centered pedagogies and active learning in the Calculus classroom, but extends to all methods-in-context and their ability to affect motivation. While prior mathematics knowledge, skills, and experience are undoubtedly predictors of success in mainstream undergraduate Calculus, the academic profile of Science or Engineering students at selective colleges and universities may be homogeneously high (Data Digest, 2018), yet large variation in academic outcomes persist. Thus, it is reasonable to assume that non-cognitive factors (e.g. motivation) may aid in explaining variation in student achievement in Calculus and that addressing non-cognitive factors within the learning environments may promote student achievement.

1.3 Application of Self-Determination Theory to an Authentic Calculus Reform Project

The present study is situated within the mainstream undergraduate Calculus reform efforts of the Department of Mathematics at a large public, Midwestern research university where extensive investigation of the success of these Calculus students (who incidentally constitute nearly 1% of all mainstream Calculus I students in the United States) in a traditional lecture-recitation learning environment (where students meet in sections of 400+ with a faculty lecturer with interspersed meetings in sections of 40 with a graduate student recitation leader to go over homework and take quizzes) found patterns consistent with or behind national trends. The status quo lecture-recitation learning environment was developed in the 1960s as an effort to reduce teaching loads for faculty and leverage an increasing number of graduate students as teaching assistants. In the early 2010's, amid criticism, the Department of Mathematics agreed to participate in a University initiative called Instructional Matters: Purdue Academic Course Transformation (IMPACT, 2018) to transform the way mainstream Calculus is taught. In the spring of 2013, as Assistant Department Head, I was charged with the project and had the freedom to create an experimental learning environment to best support student learning with some explicit constraints: 1) learning outcomes for the course remain fixed, 2) all students take course-wide common exams, 3) the grading policy remains consistent with the existing traditional learning environment, and 4) economic sustainability, that is, the new learning environment must align with teaching-load standards and student credit-hour production as in the traditional learning environment. A collaborative, problem-based learning environment for Calculus I was developed through a semester-long Faculty Learning Community according to the tenets of instructional design and motivationally-supportive pedagogy led by the IMPACT program (IMPACT, 2018). In the redesigned learning environment, the course met three times per week instead of five and involves no direct instruction, but does provide students access to on-demand instructional videos. Instead of direct instruction, students worked in groups on scaffolded problem sets intended to be challenging but still aligned with the course learning objectives and high-stakes examinations. The problem sessions are held with 120 students and are facilitated by undergraduate teaching assistants, graduate teaching assistants, and a faculty member. Student learning groups are responsible for presenting problem solutions to teaching assistants in smaller sessions with 30-40 students. Table 1 provides specific features of both versions of the course predicted to affect students' perceptions of BPN-satisfaction. Note that the listed features of the student-centered learning environment are predicted to BPN-needs supporting and those of the lecture-based learning environment are predicted to be BPN-needs thwarting.

Table 1: Instructional and Course Features that May Support/Thwart BPNs

Student-Centered Learning Environment	**Lecture-based Learning Environment**
Programmatic interaction with peers, peer mentors, and faculty Routine formative assessment and feedback Presentation of collaborative work Open-ended problem sets Collaboration towards common objectives Required attendance and expected engagement Students control how class time is spent within class structure	Systematic direct instruction of mathematics procedures and concepts by worked examples No opportunity for interaction with others No programmatic engagement or attendance expectations No feedback except on quizzes and exams No student control over how class time is spent

Many motivationally-relevant, BPN-thwarting general features of the course remained intact across both versions such as high stakes multiple-choice assessments, norm-based grading, lack of transparency in grading practices, fixed syllabus topics and rigid calendar not adaptable to students interests or needs, lack of academic support services at scale, and the fact that the course is a curricular requirement for Science and Engineering students. Nonetheless, we predict an increase in the average of students' perception of BPN-satisfaction in the student-centered learning environment over the lecture-based environment. Self-Determination Theory predicts that this rise in students' perception of BPN-satisfaction will coincide with an increase in self-determined motivation and, ultimately, in academic performance. We also predict that the experimental condition may not be universally perceived as BPN-supportive by all students due to differences in experiences, beliefs, abilities, or culture and, therefore, Self-Determination Theory (Deci & Ryan, 2000) predicts that for those students we will not see gains in self-determined motivation or academic performance. Those differences in the perception of BPN-support may manifest specifically among gender, SES, and race/ethnicity categories within each of the learning environments and present the possibility of decreased academic performance for some students within the experimental condition as compared with the control condition.

A pilot of the experimental condition was co-taught by the Associate Department Head and the researcher in spring 2014 and was subsequently assigned to be taught by the faculty at-large using the standard departmental course assignment procedures. The format was replicated for Calculus II and piloted an experimental version of that course in spring 2015. Since the pilot, the Calculus course sequence (Calculus I in fall semesters and Calculus II in spring semesters) has run with two parallel learning environments using common exams and grading policy with roughly 10% (N=200 each semester) of students completing the experimental version. Incidental logistical refinements were made regarding technology, sourcing of undergraduate teaching assistants, and course policy; however, the Department was faced with questions such as 1) Was the experimental version successful? 2) In what ways and for whom was it successful? 3) How might the experimental version be improved? and 4) Should they adopt the experimental version as the primary learning environment for the entirety of the course and/or other courses? Standard departmental decision-making information includes teaching evaluations for faculty and teaching assistants, course grades for the students, and anecdotes from students, faculty and teaching assistants. The lack of structure and inconsistency in this information as well as the lack of clear attributional antecedents of outcomes precluded straightforward decision-making. In search of some clarity, both Calculus I and Calculus II were entered into a 3-year quasi-experimental study as part of an initiative funded by the US Department of Education (STEAM, 2018) and led by the IMPACT program staff to evaluate active learning techniques in STEM disciplines through the lens of Self-Determination Theory. Students (N>8,000) in the control (lecture-based) and the experimental (student-centered, active learning) sections completed motivation inventories assessing perceived satisfaction of each Basic Psychological Need (autonomy, competence, and relatedness) and extent of Self-Determined Motivation (intrinsic, integrated, identified, introjected, external, and amotivation) at the beginning and end of the course.

The present study is the culmination of efforts to evaluate, compare, and improve the design of Calculus learning environments within an authentic reform context in ways that may generalize to mainstream undergraduate Calculus reform efforts at similar institutions of higher education. The study is not necessarily intended to confirm the efficacy of student-centered pedagogies in undergraduate mainstream Calculus, but we predict that the reformed version will outperform the traditional version as measured by course outcomes and students' perception of BPN-satisfaction. Empirical evidence is used to elucidate how achievement in Calculus may be facilitated by student-centered pedagogies via motivational processes, as depicted in Figure 2, as well as how achievement is facilitated differently across diverse learners as contextualized within two disparate types of Calculus learning environments. More broadly, this research aims to extend knowledge of best practices for teaching undergraduate Calculus, including those recommended by the MAA, by defining "active-learning strategies" and "student-centered pedagogies" not necessarily in terms of specific methods, but instead by the ability of methods-in-context to impact individual student's motivational regulatory processes and promote academic achievement.

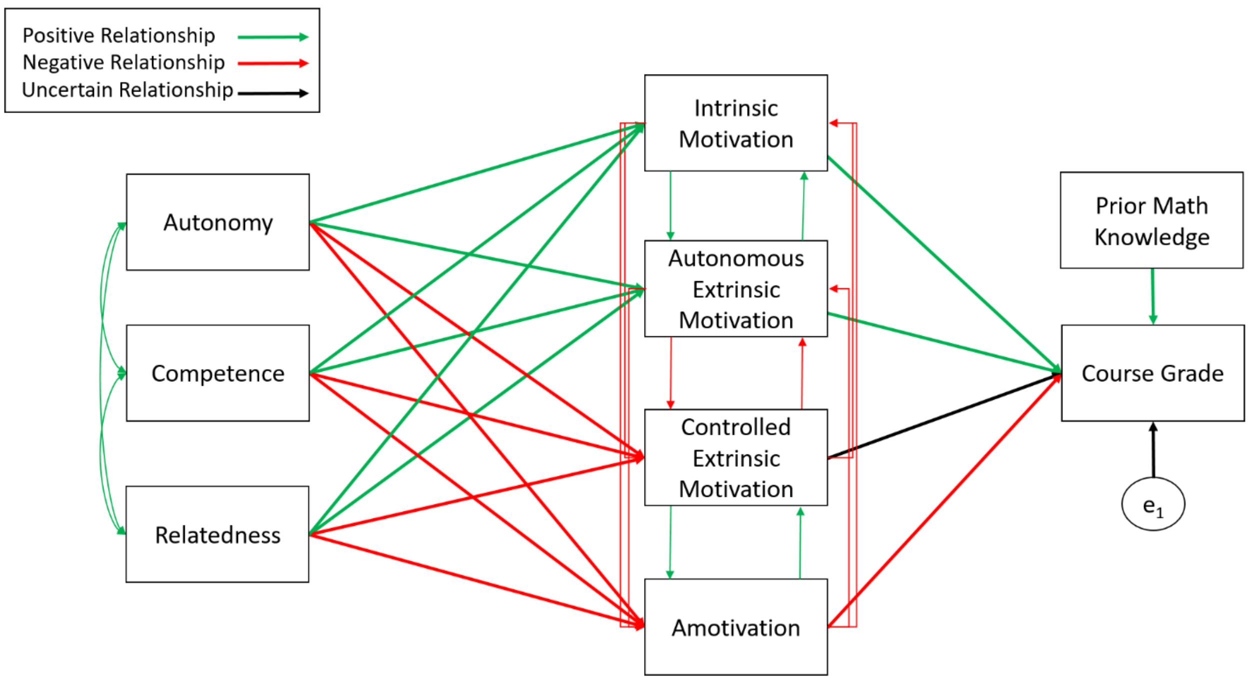

Figure 2: Theoretical relationships among Learning Environment, BPN, Self-Determined Motivation, and Achievements

CHAPTER 2: LITERATURE REVIEW

This literature review is intended to describe the evolution of the role, scope, and implementation of mainstream undergraduate calculus within higher education along with its reform efforts. Research related to understanding non-cognitive factors associated with the success of undergraduate STEM students (who serve as the target population for mainstream undergraduate calculus) will also be reviewed. Detailed attention will be given to research involving the application of Self-Determination Theory to general educational settings with emphasis towards post-secondary and STEM education as well as how Self-Determination Theory has been applied programmatically to support educational reform efforts.

2.1 Reform and Landscape of Mainstream Undergraduate Calculus

2.1.1 Modern History of Undergraduate Mainstream Calculus Instruction

2.1.1.1 Trends in Calculus Instruction from the CBMS 1960-2015.

The Conference Board of Mathematical Sciences (CBMS) has conducted surveys on 5-year intervals (with support from the National Science Foundation (NSF)) since 1960 to assess instruction, enrollments, curricula, courses, faculty, and students in departments of mathematics for the use by academic administrators, professional organizations, and government agencies in decision-making and academic planning. A statistical abstract summarizing the findings of each survey is published, at various times by the CBMS, the Mathematical Association of America (MAA), and American Mathematical Society (AMS), with the most recent volume filling 456 pages. These reports provide critical information about long-term trends in how many students have enrolled in Calculus, how it is taught, and related issues. For instance,

Figure 3 shows that dramatic growth (from nearly 200,000 students to nearly 400,000 students) in Calculus enrollment from 1960 to 1975 and Table 2 reveals the increased prevalence in the Large Lecture learning environments at Universities (which also nearly doubled) at Universities from 1960 to 1970 corresponding to the large growth in Calculus enrollment.

Table 2: Trends in Methods of Instruction, 1960-61 to 1970-01 (Jewett & Phelps, 1972, p. 53)

Method of Instruction	Percent of Departments Using This Method								
	Universities			Public Colleges			Private Colleges		
	60-61	65-66	70-71	60-61	65-66	70-71	60-61	65-66	70-71
Large Lecture Classes With Small Quiz Sections	21%	42%	43%	2%	13%	11%	4%	10%	3%
Large Lecture Classes With Help Sessions	27%	34%	42%	11%	28%	8%	8%	15%	10%
Organized Programs of Independent Study	15%	24%	24%	7%	27%	22%	16%	25%	22%
Courses by Programmed Instruction	0%	6%	9%	0%	11%	7%	1%	11%	10%

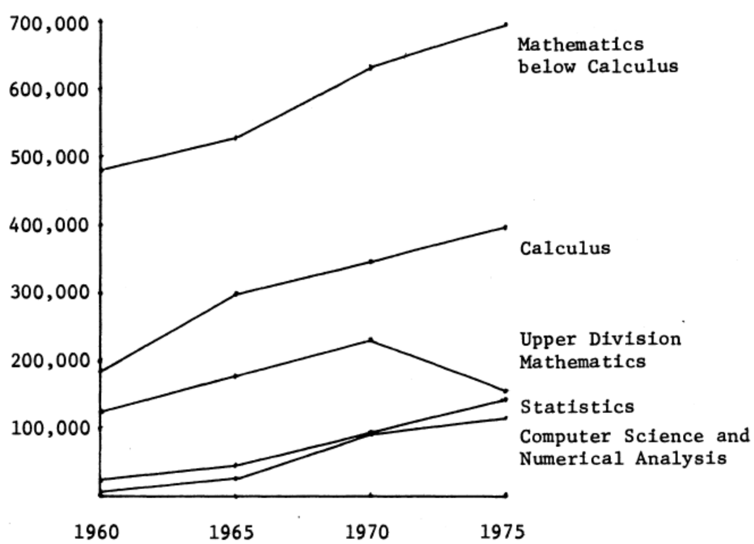

Figure 3: Trends in Calculus Enrollment 1960-75 (Fey, Albers, Jewett, 1976, p. 26)

Table 3 shows the types of non-traditional instructional methods (as opposed to small classroom settings) deployed within departments and their prevalence in 1970 & 1975 showing Large Lecture as the dominant mode of instruction in departments of mathematics. We should note that these percentages represent the percentage of departments who use a particular method, and not the percentage of students or sections taught with this method. In 1980, CBMS began to measure

Table 3: Percentage of Departments using Non-Traditional Methods of Instruction, 1970-75 (Fey et al., 1976, p. 56)

Method of Instruction	University Mathematics 1970	University Mathematics 1975
Large Lectures	56%	56%
Organized Independent Study	24%	12%
Television	6%	10%
Audio-Tutorial	NA	4%
Programmed	9%	6%
Computer Assisted	5%	6%
Computer Managed	NA	-
Self-Paced	NA	22%

NA Not on the 1970 survey questionnaire.

the percentage of actual sections of mathematics taught in various class sizes and formats. In 1980 & 1985 the minority of sections (less than 40%) at Universities were taught in class of less than 40 students compared to those greater than 40 either with or without a 'Quiz Section' (Recitation). From 1980 to 1985, there was a dramatic increase in the number of sections that moved from 40-80 students to greater than 80 students shown in Table 4. Furthermore, it was not until 1990 when CBMS captured instruction methods specifically for Mainstream Calculus. The sections referenced before 1990 represent all mathematics courses taught in mathematics departments.

From 1990-2005, methods of instruction were collected for Mainstream Calculus across various instructional formats as summarized in

Table 5. The CBMS data show a sharp increase and corresponding attenuation of group work and writing assignments before and after the calculus reform movement of the early 1990s as well as a sustained increase (plateauing at about 40%) in the use of graphing calculators. By Fall 2015, 57% (around 145,000) of Mainstream Calculus I

Table 4: Intro Course Section Size, 1980-85 (Albers et al., 1987, p. 68)

Class Format	University		Public 4-Year		Private 4-Year	
	1980	1985	1980	1985	1980	1985
<40	36	38	67	62	79	82
40 - 80	31	20	21	22	13	16
>80, no Quiz Sec.	10	12	2	5	1	-
>80, Quiz Sec.	21	29	9	10	7	2
Self-Paced or Other	1	1	-	1	-	-

students were taught in lecture/recitation instructional modes at 4-year institutions with an average class size of 63 with similar figures for Mainstream Calculus II (Blair et al., 2018, p. 17).

Lecture/Recitation		23	16	5
Regular Section (small)		37	16	2
Regular Section (large)		8	34	5
Total	2	21	19	5
Mainstream Calculus II				
Lecture/Recitation		14	5	4
Regular Section (small)		20	13	6
Regular Section (large)		12	29	1
Total	2	17	12	3
Grand Total	**2**	**20**	**17**	**4**
	% of Sections Using Computer Assignments			
	1990	**1995**	**2000**	**2005**
Mainstream Calculus I				
Lecture/Recitation		18	17	14
Regular Section (small)		17	27	9
Regular Section (large)		8	36	26
Total	5	15	23	18

Table 5 continued

Mainstream Calculus II				
Lecture/Recitation		18	18	8
Regular Section (small)		14	17	17
Regular Section (large)		11	43	15
Total	3	17	23	13
Grand Total	**4**	**20**	**23**	**16**
	% of Sections Using Group Projects			
	1990	**1995**	**2000**	**2005**
Mainstream Calculus I				
Lecture/Recitation		28	5	4
Regular Section (small)		25	11	5
Regular Section (large)		9	25	11
Total	1	21	10	7
Mainstream Calculus II				
Lecture/Recitation		21	2	1
Regular Section (small)		7	10	3
Regular Section (large)		12	16	2
Total	1	15	8	2
Grand Total	**1**	**19**	**9**	**5**

Table 6 shows the continued growth, a 42% increase, in all Calculus enrollments (Mainstream and otherwise) from 2000 to 2015 as well as the enrollments of other categories of mathematics courses.

Table 5: Trends in Methods of Instruction in Mainstream Calculus at PhD-Granting Institutions (Albers, 1992, p. 53; Loftsgaarden et al., 1997, p. 73; Lutzer et al., 2002, p. 112; Maxwell, 2007, p. 119)

Mainstream Calculus at PhD-Granting Institutions	**% of Sections Using Graphing Calculators**			
	1990	**1995**	**2000**	**2005**
Mainstream Calculus I				
Lecture/Recitation		25	35	37
Regular Section (small)		60	46	44
Regular Section (large)		30	47	42
Total	3	33	40	40
Mainstream Calculus II				
Lecture/Recitation		18	41	23
Regular Section (small)		30	52	42
Regular Section (large)		37	28	37
Total	3	27	42	32
Grand Total	**3**	**31**	**40**	**38**
	% of Sections Using Writing Assignments			
	1990	**1995**	**2000**	**2005**
Mainstream Calculus I				
Lecture/Recitation		23	16	5
Regular Section (small)		37	16	2
Regular Section (large)		8	34	5
Total	2	21	19	5
Mainstream Calculus II				
Lecture/Recitation		14	5	4
Regular Section (small)		20	13	6
Regular Section (large)		12	29	1
Total	2	17	12	3
Grand Total	**2**	**20**	**17**	**4**
	% of Sections Using Computer Assignments			
	1990	**1995**	**2000**	**2005**
Mainstream Calculus I				
Lecture/Recitation		18	17	14
Regular Section (small)		17	27	9
Regular Section (large)		8	36	26
Total	5	15	23	18

Table 5 continued

Table 5 continued

Mainstream Calculus II				
Lecture/Recitation		18	18	8
Regular Section (small)		14	17	17
Regular Section (large)		11	43	15
Total	3	17	23	13
Grand Total	**4**	**20**	**23**	**16**
	% of Sections Using Group Projects			
	1990	**1995**	**2000**	**2005**
Mainstream Calculus I				
Lecture/Recitation		28	5	4
Regular Section (small)		25	11	5
Regular Section (large)		9	25	11
Total	1	21	10	7
Mainstream Calculus II				
Lecture/Recitation		21	2	1
Regular Section (small)		7	10	3
Regular Section (large)		12	16	2
Total	1	15	8	2
Grand Total	**1**	**19**	**9**	**5**

Table 6: Calculus Enrollment 2000-15 (Blair, Kirkman, & Maxwell, 2018, p. 7)

	Mathematics Departments			
Course level	2000	2005	2010	2015
Mathematics courses				
Precollege level	219	201	209	253
Introductory level (including Precalculus)	723	706	863	1000
Calculus level	570	587	748	807
Advanced level	102	112	150	154
Other (2-year)	--	--	--	--
Total Mathematics courses	1614	1607	1971	2213
Probability and Statistics courses				
Introductory level	136	148	231	253
Upper level	35	34	32	60
Total Probability and Statistics courses	171	182	262	313
Computer Science courses [1]				
Lower level	90	44	56	45
Middle level	17	8	12	16
Upper level	16	5	10	6
Total Computer Science courses [1]	123	57	77	68
Grand Total	1908	1845	2310	2594

2.1.1.2 Calculus for a New Century: A Pump, Not a Filter.

Steen (1988) served as editor for the 267-page proceedings of a large-scale colloquium conducted by the National Research Council and the Mathematical Association of America in 1987, sponsored by the National Academy of Science and the National Academy of Engineering, aimed at addressing the concern that only 25% of the over 1 million student who take Calculus annually "survive to enter the science and engineering pipeline" and that those who do survive are poorly motivated for advanced study and "too uniformly white, male, and middle class." The scope of the colloquium included all aspects of calculus education, but below are a brief selection of quotes from the proceedings specifically about motivation and the dynamics of instruction (curricular and cognitive matters are not included). The following excerpts have been left in block quotes to directly convey the perspectives, culture, and beliefs of mathematicians and academic administrators related to non-cognitive factors, instructional practices, and institutional change associated with success in calculus.

In his plenary, W. Dale Compton, Senior Fellow of the National Academy of Engineering, states,

> Calculus should encourage students to proceed to an engineering career—not by being easy, but by being exciting. … It is my guess that students would react positively to a calculus that includes examples that require the exercise of good judgement. (Steen, 1988, p. 33)

John Fulton, Professor of Mathematics at Clemson University, states,

> Only close scrutiny of … classroom discussion of concepts, and student classroom or office presentations can assist in meeting an effective mathematical communication objective for the teaching of calculus. … We should not necessarily call for small sections, but for a sufficient teaching staff to be assigned to calculus to allow for regular feedback for students. (Steen, 1988, p. 24)

Lynn Steen, Professor of Mathematics at St. Olaf College, states,

> …students can go through such a course getting a grade of B, maybe even a grade of A, and never write a complete sentence in the entire semester, and probably never even talk at length about calculus with anyone. …So in order to teach students what

> we want them to learn, we have to understand the interaction that goes on when students construct their own images of mathematics which are quite likely different than the ones we have in our minds. (Steen, 1988, p. 27)

Gilbert Strang, Professor of Mathematics at MIT, states,

> Some of the unspoken or barely spoken needs are fundamental to success in the classroom—and the success of this whole initiative. One is the students' need for a clear sense of purpose, and for a response that encourages more effort. Actually the instructor has the same need. … a recent study that revealed that in more than half of the courses, homeworks are not graded (or even looked at). In other words, the student gets no response. That zero is worse than any grade. To work well without recognition is a lot to ask. (Steen, 1988, p. 67)

Richard Millman, Dean of the College of Science and Mathematics at Wright State University, states,

> Unfortunately, there are many mathematicians who, while they enjoy the discipline tremendously, don't convey the excitement they feel to their students. We do a disservice to both our students and our subject when we regard calculus as a chore and then compound it by communicating that attitude to our students. Enthusiasm is contagious. While it won't substitute for content, we need more excitement in the classroom. (Steen, 1988, p. 154)

Alphonse Buccino, Dean of the College of Education at the University of Georgia, and George Rosenstein, Professor of mathematics at Franklin and Marshall College, facilitated a discussion on 'Objectives, Teaching, and Assessment' and summarized teaching in part as:

> Daily interactions … provide valuable information for the student about the objectives of the courses and the teacher's expectations, and for the teacher about the progress of the class and of individuals… Instructors are often too distant from their students and often are unaware of serious problems or significant successes individuals may be experiencing. Consequently, a climate of sensitive awareness should characterize calculus classes. (Steen, 1988, p. 94)

David Lovelock, Professor of Mathematics at the University of Arizona, and Alan Newell, Head of Mathematics at the University of Arizona, state,

> Above all, we must put our best teachers, the most caring and inspiring, in the first year courses. … One of the primary aims of a mathematical education is to teach students to think logically. … Some material should not be covered in class even though it does lend itself to testing by examination. (Steen, 1988, p. 176-177)

Rhonda Hughes, Chair of Mathematics at Bryn Mawr, discussed women in calculus as follows,

> There is considerable evidence that women are doing well in calculus (… grades and continuations in mathematics or science courses). … A discouraging postscript is that despite their motivation, ability and successful performance, even the brightest women often exhibit a marked lack of self-confidence, and are disproportionately discourage by setback; the lack of encouragement at earlier stages seems to take its tool. For this reason, encouragement and support in calculus are vital elements in counteracting the damage that may have already been done, and may fuel women students for the road ahead. … The reformed calculus would be a streamlined course, more conceptual, more relevant to real-world problems, and taught in a more open-ended, probing fashion than the current versions. These aims are difficult to fault; women students are certainly equal to the challenge of the "new calculus," and should benefit from these changes as much as men students. … While the MAA report frequently mentions the importance of "feedback," there is virtually no mention of offering support to students who need it, perhaps because the latter is far more difficult to "package." However, the difference between "feedback" and "support" is like the difference between "eating" and "dining." … If … the vast untapped resources of women and minorities is to be realized to the fullest extent, we must allow our students to dine on the fruits of mathematics. A "lean and lively" calculus, thoughtfully implemented with the needs of student in mind, could contribute to this goal. (Steen, 1988, p. 138-139)

Rogers Newman, Professor of Mathematics at Southern University at Baton Rouge, and Eileen Poiani, Professor of Mathematics at Saint Peter's College, (Steen, 1988, p. 88) provide the following recommendations for supporting under-prepared minority undergraduates in calculus: 1) Create learning environments where instructors and students can get to know each other, 2) Require attendance to support student motivation. They assert that when well-prepared student do not succeed, it is due to motivational reasons. For these students, they recommend: 1) Relate the value of calculus as a pathway to high-paying careers, 2) Use personal computers in the instructional process, 3) Relate the usefulness of calculus to other discipline via guest speakers, 4) Provide peer mentors, and 5) Implement interactive learning environments.

Shirley Malcom, Head of the Office of Opportunities in Sciences of the American Association for the Advancement of Science, and Uri Treisman, Associate Director of the Professional Development Project at UC-Berkeley, states,

> Hundreds of thousands of capable minority students are felled by the calculus hurdle. … "good" backgrounds in mathematics and "smarts" are necessary but not sufficient conditions for success in calculus courses. (Steen, 1988, p. 141-143)

Moreover, Malcom and Treisman present the following recommendations for promoting success in calculus by women, minority, or disabled students: 1) Set high expectations, 2) Construct an instructional support system, 3) Link mathematics with science and engineering, 4) Eliminate the weed-out mentality of instructors and departments, 5) Use cooperative learning and peer tutoring and 6) Meet the students where they are. (Steen, 1988, p. 144-145)

Towards the future of calculus, Thomas Tucker, Professor of Mathematics at Colgate University, asks,

> The truth of the matter is that our clients have been remarkably tolerant of mainstream calculus. What happens when our calculus clients find we are teaching the moral equivalent of long division while they simply want their students to know how to push buttons intelligently? (Steen, 1988, p. 29)

Ronald Douglas, Dean of the College of Physical Sciences and Mathematics at SUNY Stony Brook, states,

> My interest in calculus was treated as a curiosity. No one ever talked about teaching. Teaching was something we had to do and get over with. …Changing calculus is an enormous and complex undertaking. … Almost everyone has a stake in calculus. … Isolated innovations are not the answer to the problems in mathematics teaching. (Steen, 1988, p. 18-19) When students get the sense that no one is personally interested in what they are doing in the course, they put very little effort in. They put in what they think is the minimum work necessary, and, of course, they often judge wrong. (Steen, 1988, p. 103)

Homer Neal, Chair of the Physics Department at the University of Michigan, states,

> It would be very easy to view present calculus instruction as being an invariant of nature. The way it was taught to us could be thought to be the way it must be taught forever. … It takes unusual insights and courage to challenge such a tradition. (Steen, 1988, p. 24)

Steen (1988, p. 27) predicts five possible futures for calculus at the University: 1) It will disappear completely, 2) It will be relegated to a 'classic,' 3) It will remain unchanged due to inertia, 4) It will double its size due to pressure on client disciplines to bolster mathematics knowledge of their students, 5) It will be taught in a distributed manner by client disciplines.

*2.1.1.3 Characteristics of Successful Programs in College Calculus***.**

Bressoud et al. (2015) served as editors of a report on the instructional landscape for mainstream calculus I culminating from a 5-year study, Characteristics of Successful Programs in College Calculus (CSPCC, 2015), conducted by the Mathematical Association of America (MAA). The editors acknowledge the antecedents and execution of calculus reform efforts since the 1987 national colloquium on calculus (Steen, 1988) and report that little progress has been made. The stated objectives of the study were as follows:

1. To improve our understanding of the demographics of students who enroll in calculus.
2. To measure the impact of the various characteristics of calculus classes that are believed to influence student success.
3. To conduct explanatory case study analysis of exemplary programs in order to identify why and how these programs succeed.
4. To develop a theoretical framework that articulates the factors under which students are likely to succeed in calculus.
5. To use the results of these studies and the influence of the MAA to leverage improvements in calculus instruction across the United States.

Their report summarizes findings from nationwide surveys of students (demographics, academic preparation, beliefs, attitudes, goals, outcomes, and characteristics of learning environment), instructors (experience, instructional practices, curriculum, beliefs, and characteristics of the learning environment), and departments of mathematics (programs, professional development, and placement) on the calculus learning environment. The specific recommendations of the report include:

1. Attention to the effectiveness of placement procedures.
2. Proactive student support services, including the fostering of student academic and social integration.
3. Construction of challenging and engaging courses.
4. Use of student-centered pedagogies and active-learning strategies.
5. Coordination of instruction, including the building of communities of practice.
6. Effective training of graduate teaching assistants.
7. Regular use of local data to guide curricular and structural modifications.

The tables below describe the demographics, experiences, beliefs, and academic abilities of mainstream undergraduate Calculus I students in fall of 2010. Specific research findings related to the study and associated works will be included in Section 2.2.2 Active Learning and Student-Centered Pedagogies in Undergraduate Calculus of the present document.

While the majority of students were white men with career ambitions in the STEM fields, the variation in student demographics is vast. In fact, there is a vast variation in almost all variables included in the survey (time spent working, studying, and in extra-curricular activities; support/self-efficacy for mathematics; preparation for Calculus).

Table 7: Demographics and Attributes of Calculus Students (Bressoud, 2015, p. 2-4).

	Univ (PhD)	**4Y Coll (BA)**	**Univ (MA)**	**2Y Coll (AS)**
Male	55%	53%	53%	66%
White	77%	81%	77%	67%
Black	5%	7%	8%	9%
Asian	15%	10%	9%	12%
Hispanic	9%	10%	7%	16%
Born in US	88%	91%	85%	83%
High School in US	94%	94%	91%	91%
Mean Age (SD)	18.3 (2.4)	18.8 (2.9)	20.5 (5.3)	22.0 (7.4)
Freshmen	83%	73%	50%	25%
Sophomore	10%	16%	27%	40%
Junior/Senior	6%	10%	17%	18%
Full-time Student	99%	98%	91%	76%
Father born in US	77%	83%	80%	67%
Mother born in US	77%	81%	80%	65%
English spoken at home	86%	90%	85%	76%
Father completed college	65%	58%	49%	44%
Mother completed college	62%	56%	47%	40%
Some concern about paying for college	54%	40%	57%	55%
Major concern about paying for college	13%	10%	13%	23%

Additionally career ambitions across gender and ethnicity vary a great deal where Engineering and Computer Science were favored by men, Biological Sciences and Education were favored by women, and Business were favored by Asian students, and STEM majors as a category were favored by Black students over their peers. Of particular note is the 25 percentage point drops in student belief about being ready for Calculus and in being able to solve word problems over the course of the semester show in Table 9 and Table 10.

Table 8: Calculus Students' Career Goals by Demographics (Bressoud, 2015, p. 11)

	All	**Male**	**Female**	**White**	**Black**	**Asian**	**Hispanic**	**Univ (PhD)**	**4Y Coll (BA)**	**Univ (MA)**	**2Y Coll (AS)**
Math	2%	2%	1%	2%	2%	2%	1%	1%	1%	3%	2%
Physical Science	4%	5%	4%	5%	5%	3%	3%	3%	4%	6%	5%
Engineer	31%	38%	14%	29%	26%	19%	26%	35%	20%	22%	29%
CS/IT	5%	10%	2%	6%	9%	6%	7%	4%	7%	7%	10%
Geo Science	2%	2%	3%	3%	2%	1%	2%	2%	3%	5%	2%
Bio/Med Science	30%	19%	43%	27%	36%	34%	33%	31%	30%	27%	28%
Total STEM	74%	76%	67%	72%	80%	65%	72%	76%	65%	70%	76%
Teacher	5%	4%	10%	8%	3%	3%	6%	3%	9%	13%	7%
Social Science	1%	1%	2%	1%	2%	2%	2%	1%	2%	1%	1%
Business	7%	9%	7%	6%	7%	16%	8%	7%	8%	7%	7%
Other	4%	3%	5%	5%	4%	4%	4%	4%	5%	5%	3%
Undecided	8%	7%	9%	8%	5%	9%	7%	8%	10%	4%	6%

Table 9: Calculus Students' Perceived Support and Preparedness for Math and Calculus (Bressoud, 2015, p. 4-8, 14)

	Univ (PhD)	**4Y Coll (BA)**	**Univ (MA)**	**2Y Coll (AS)**
Home supported my studying math	80%	77%	69%	67%
Parents see me as good at math	69%	65%	65%	63%
Teachers see me as good at math	66%	61%	63%	56%
Father encouraged me to study math	44%	39%	32%	31%
Mother encouraged me to study math	43%	38%	33%	31%
Math teacher encouraged me to study math	40%	36%	33%	28%
No one encouraged me to study math	40%	43%	48%	49%
SAT Math (SD)	663 (71)	632 (72)	616 (81)	589 (95)
SAT Critical Reading (SD)	619 (83)	601 (84)	583 (104)	560 (98)
ACT Math (SD)	29.1 (4.0)	27.4 (4.0)	26.2 (4.8)	25.5 (4.5)
ACT Composite (SD)	28.2 (3.4)	26.9 (3.7)	25.4 (3.9)	24.6 (4.2)
Algebra II by end of 10th grade	78%	71%	59%	56%
Precalculus by end of 11th grade	67%	58%	46%	37%
Statistics by end of 12th grade	10%	11%	9%	8%
Calculus by end of 12th grade	67%	50%	40%	22%
Took AB Exam	53%	45%	40%	50%
1 or 2 on AB Examb	36%	46%	50%	63%
3 on AB Examb	32%	29%	40%	17%
4 or 5 on AB Examb	32%	24%	10%	20%
Took BC Exam	11%	7%	6%	11%
1 or 2 on BC Examc	39%	59%	*	*
3 or higher on BC Examc	61%	41%	*	*
Can factor expressions (somewhat)	13%	14%	19%	17%
Can factor expressions (yes)	85%	83%	79%	77%
Can solve inequalities (somewhat)	17%	18%	20%	21%
Can solve inequalities (yes)	80%	80%	78%	74%
Can solve word problems (somewhat)	27%	28%	28%	25%
Can solve word problems (yes)	69%	68%	66%	66%
Understand what I have studied (somewhat)	23%	28%	25%	24%
Understand what I have studied (yes)	75%	69%	72%	73%
Ready for calculus (somewhat)	16%	19%	18%	17%
Ready for calculus (yes)	81%	79%	77%	81%

Table 10: Calculus Students' Abilities and beliefs – end of term (Bressoud, 2015, p. 14-15)

	Univ (PhD)	4Y Coll (BA)	Univ (MA)	2Y Coll (AS)
Was ready for calculus (somewhat)	31%	33%	35%	31%
Was ready for calculus (yes)	56%	54%	51%	57%
Can compute derivatives and integrals (somewhat)	30%	35%	34%	30%
Can compute derivatives and integrals (yes)	66%	60%	61%	66%
Can solve word problems (somewhat)	46%	49%	47%	42%
Can solve word problems (yes)	41%	40%	40%	45%
Increased interest in math (somewhat)	46%	46%	43%	34%
Increased interest in math (yes)	29%	29%	38%	48%

2.1.2 Active Learning and Student-centered Pedagogies in Undergraduate Calculus

2.1.2.1 Initial findings from the Progress through Calculus Project.

The currently active Progress through Calculus Project is a follow-up to the study of Successful Programs in College Calculus (which examined only Calculus I) to investigate programs, structures, and characteristics (including instructional style) of Pre-calculus, Calculus I, and Calculus II at the University. In particular, instructional practices within regular meetings and recitation meetings of mainstream Calculus I and Calculus II were inventoried via surveys to 330 PhD or Master's-granting institutions (response rate=67.6%) (Apkarian et al., 2017). Results in

Table 11 indicate that the typical Calculus I and Calculus II class involves lecturing, answering homework questions, and review. A negligible percentage of respondents reported using active learning technique as a primary activity. Group work is the most used active learning technique followed by clicker surveys and flipped classes. The vast majority of respondents reported that they do not have to plans to change the implementation of mainstream Calculus I or mainstream Calculus II.

Table 11: Prevalence of Active Learning methods in Calculus I and Calculus II (Apkarian et al., 2017)

How important is Active Learning to having a successful precalculus/calculus sequence? (N=219)		
Very		97 (0.443)
Somewhat		102 (0.466)
Not		20 (0.0913)
What best characterizes the current status of the course? (N=296)	**Calculus I (N=322)**	**Calculus II (N=296)**
No significant changes are planned	244 (0.696)	230 (0.777)
Changes have recently/currently being implemented	57 (0.177)	34 (0.115)
Possible changes are being discussed	48 (0.149)	35 (0.118)
What is the primary instructional format during the regular class meeting (not recitation sections)?	**Calculus I (N=322)**	**Calculus II (N=298)**
Lecture and answering student questions	211 (0.653)	219 (0.735)
Lecture incorporating some active learning techniques	55 (0.170)	38 (0.128)
Minimal lecture with mainly active learning techniques	9 (0.028)	3 (0.010)
Lecture plus computer based instruction	7 (0.022)	9 (0.030)
There is too much variation	38 (0.118)	24 (0.081)
Other	3 (0.009)	5 (0.017)
What active learning techniques are used during the regular class meeting? Mark all that apply.	**Calculus I (N=59)**	**Calculus II (N=41)**
Process Oriented Guided Inquiry Learning (POGIL)	3 (0.051)	1 (0.024)
Inquiry-Based Learning (IBL)	10 (0.169)	7 (0.171)
Clicker surveys	13 (0.220)	10 (0.244)
Group work	50 (0.847)	33 (0.805)
Flipped classes	14 (0.237)	6 (0.146)
Other	9 (0.153)	9 (0.220)

Table 11 continued

Which of the following best describes the recitation sections accompanying calculus?	**Calculus I (N=316)**	**Calculus II (N=292)**
Recitation sections are offered for all lecture sections	123 (0.389)	106 (0.363)
Recitation sections are only offered for some lecture sections	17 (0.054)	14 (0.048)
Additional recitation sections are available for all students	6 (0.019)	4 (0.014)
Additional recitation sections are available specifically for students from traditionally underrepresented groups	2 (0.006)	2 (0.007)
Recitation sections are NOT offered for this course	174 (0.551)	167 (0.572)
What is the primary instructional format during the recitation section? (N=138)	**Calculus I (N=138)**	**Calculus II (N=119)**
Mainly homework help, Q&A, and review	101 (0.732)	87 (0.731)
Mainly techniques that incorporate active learning strategies	25 (0.181)	18 (0.151)
Other	12 (0.087)	14 (0.118)
What active learning techniques are used during the recitation section? Mark all that apply.	**Calculus I (N=25)**	**Calculus II (N=18)**
Process Oriented Guided Inquiry Learning (POGIL)	1 (0.040)	0 (0.000)
Inquiry-Based Learning (IBL)	3 (0.120)	1 (0.056)
Clicker surveys	1 (0.040)	0 (0.000)
Group work	24 (0.960)	17 (0.944)
Flipped classes	4 (0.160)	4 (0.222)
Other	6 (0.240)	5 (0.278)

2.1.2.2 Recent Innovations in Pedagogy in Undergraduate Calculus.

A number of studies have been conducted related to the efficacy and impact of instructional methods in Calculus and efficacy of active learning methods has been established across disciplines. In particular, Freeman et al. (2014) performed a meta-analysis on the impact of active learning pedagogies on academic outcomes in undergraduate STEM courses. The authors included 225 studies in which some active learning strategies (collaborative problem-based learning, in-class tutorials, peer instruction, studio environments, and personal technology) were implemented.

The overall effect size (Hedge's g) on examinations and concept inventories was 0.5 and the percentage decrease in failure rates was greater than 10%. Breakdowns by disciplines (through not specifically by course) are included in Figure 4.

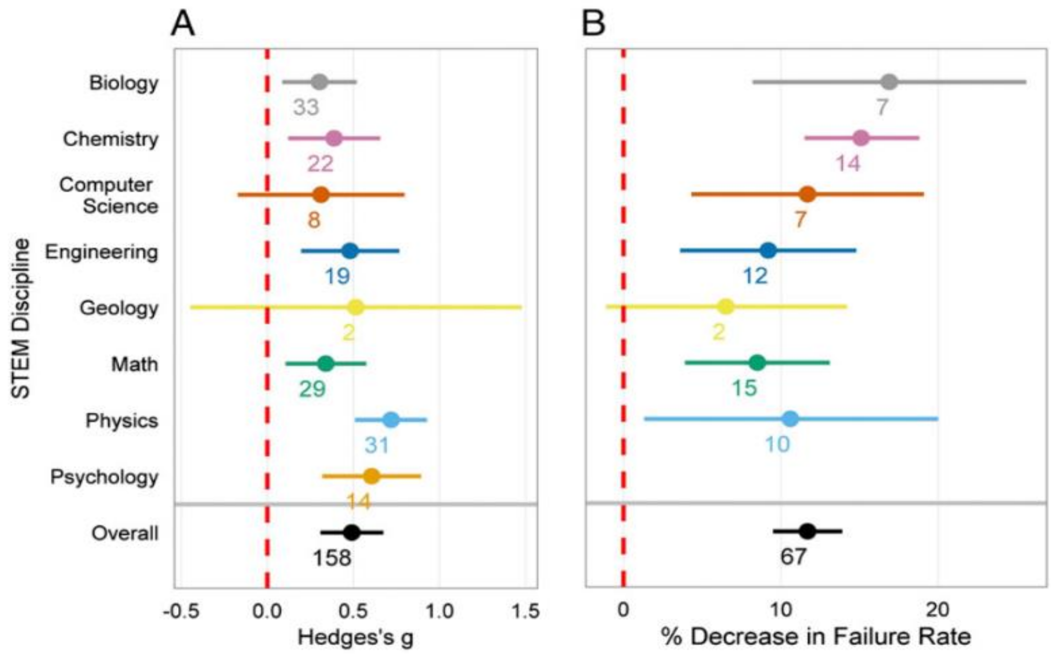

Figure 4: Active Learning Impact by Discipline (Freeman, 2014, p. 8411)

Burn and Mesa (2017) used data from the used data from the Characteristics of Successful Programs in College Calculus study (CSPCC, 2015) to drill down into the instructional method described as 'interactive lectures' in mainstream calculus at two-year colleges. The included data contain faculty interviews, student focus groups, and classroom observations from four colleges selected from the CSPCC survey data as having large gains in students' attitudes towards mathematics and retention in Calculus II. The defining feature of 'interactive lecture' includes substantial time spent lecturing, use of technology for demonstrating concepts, answering homework questions, and providing time in-class for students to work problems. Three instructional methods for motivating students during class were also identified: 1) connecting with students, 2) teaching for understanding, and 3) expression of personal instructor traits. Ellis et al. (2014) used the CSPCC data set to identify instructional methods that promote persistence in Calculus across demographic groups. For STEM students, they show that low frequency of the use of instructional practices (as reported by students) such as class discussion, requiring explanation of student thinking on exams, demonstration of specific problem-types, and use of supplemental materials predict the attrition of students from the Calculus sequence while low frequency of lecture did not. Conversely, Keller et al. (2017) used the same data set, though focusing specifically on questions related to student participatory behavior, found that student participation in discussion and lecture had no impact on success measures. They also report that these findings contradict previous initial studies (Keller et al., 2016; Rasmussen & Ellis, 2013) of participatory behavior with the same data set.

McGivney-Burelle and Xue (2013) examined the process and efficacy of "flipping" a Calculus II course at the University of Hartford whereby students programmatically perform activities out of class preparing for highly engaging in-class learning activities. They note, at the time, there was only one known study (Strayer, 2012) which examines flipped pedagogy in the college mathematics classroom (focused only on statistics students in a flipped setting using the University Classroom Environment Inventory (Fraser et al, 1986)). In their study, two sections of Calculus II were taught using traditional methods for the first chapter of content. A flipped model of instruction was used in the second chapter of content for one section while keeping the traditional methods fixed in the other. Students in both sections were given common chapter exams with results as in Figure 5. They suggest that content had to be covered in the same period of time with similar or better outcomes. The authors (who also designed the course) also note the timeconsuming aspects of preparing the flipped class, but acknowledged that existing open, curricular resources for Calculus II exist that would support scaling and revising the course.

Table 1. Section A Exams (non-flipped class, 29 students) (in percent)

	Exam 1 (Traditional)	Exam 2 (Traditional)
Average	76.24	71.27
Median	82	73

Table 2. Section B Exams (flipped class, 31 students) (in percent)

	Exam 1 (Traditional)	Exam 2 (Flipped)
Average	77.48	76.48
Median	83	80

Table 3. *WebWork* Grades (in percent) Comparison

	Section A	Section B
Chapter 5 (non-flipped)	93.32	92.45
Chapter 6 (section A non-flipped, section B flipped)	77.45	81.48

Figure 5: Academic Outcomes across Flipped and Traditional Calculus
(McGivney-Burelle & Xue, 2013)

There has been increased attention on individual instances, though still few experimental studies, of implementation of active learning and student-centered pedagogies in mainstream undergraduate Calculus since the publication of Insights and Recommendations from the MAA National Study of College Calculus (Bressoud et al., 2015). In a special issue of Problems, Resources, and Issues in Mathematics Undergraduate Studies (PRIMUS) on Improving the Teaching and Learning of Calculus edited by Bressoud (2018), a number of such implementations are highlighted. Crawford et al. (2018) describe small Calculus classrooms at Jacksonville State University of 15-25 students in which pedagogical practices involving collaborative, problem-based learning, writing, technology-enhanced activities, and discovery activities. The authors describe the instructors' approach to teaching as 'Ambitious Teaching' and supportive of building trust and relationships with students while providing an individualized mathematical experience. Carreon et al. (2018) describe the long-standing history of implementation of reformed Calculus at the University of Michigan rooted in the movements of the late 1980s. They emphasize a systematic approach involving uniform coordination across sections, effective placement and advising strategies, instructor training, inquiry-focused, problem-based learning methods and boast a 12% failure rate. Classes are taught in small section with collaborative problem-solving groups required to submit written explanations with their work. Exams are written to be 'highly conceptual'. Procedural skills are developed through individual online homework and gateway skills assessments and students have access to a math help room staffed with graduate students and undergraduate tutors.

Apkarian et al. (2017) describe the addition of an "Active Learning Lab" to the traditional lecture-based environment at San Diego State University. The "bolt-on" approach was predicated in established efficacy of active learning, but under apprehension related to the large-scale deployment along with the required instructor training and buy-in. This approach caused some confusion with student about the relationship between lecture and lab requiring iteration of materials and cross-pollination of instructors between the two environments with the belief that 'active learning' practices will be carried over to the lecture environment.

Bode (2018) describes the implementation of active learning pedagogies and collaborative grading platforms at the University of Illinois at Chicago where failure rates for Calculus I have reached as high as 63% in spring 2014. The implementation of active learning pedagogies (group work and problem sets) attenuated the failure rate to 44% in the following spring term though the grading system was changed from a curved system to a straight scale, leaving direct comparison difficult. The collaborative grading platform provided more consistent, timely, and detailed feedback to students about their work. The University reports sustained improvement in the pass rates of both Calculus I and Calculus II after the changes.

Pilgrim & Gehrtz (2018) describe efforts in Calculus I at Colorado State University, where the failure rate ranges from 27% to 35%, to identify active learning strategies deemed effective, but also that could be readily implemented by the graduate teaching staff. The authors deployed "Write-to-Learn" activities, with applications to Calculus established in Jaafar (2015), as collaborative activities in a pilot section where students consistently scored higher on common exams relative to a comparison group, although no attempt was made to control for extraneous variables. In addition, the coordination of the instructors was more robust and included dedicated meeting time for conversations about pedagogy, evidence-based practices, and opportunity to share ideas and co-teach.

Schroeder et al. (2018) described the ongoing implementation of flipped Calculus at the University of Hartford which began in 2011 (McGivney-Burelle & Xue, 2013) which co-evolved with Departmental and Institution culture shifts towards collaboration and innovation in teaching methods. Of particular note, pedagogical research and the scholarship of teaching and learning were made eligible for inclusion in promotion and tenure.

Adams and Dove (2018) demonstrated substantial achievement gains associated with flipped versions of Calculus I and Calculus II despite no differences in the students' perceptions of learning. The authors collected free responses survey items from students in both condition regarding the positive/negative aspects of the learning environments and recommendations for change. About a quarter of the flipped students recommended more lecturing and frustration with having to teach themselves the material. Other comments included the removal of specific assignment or the use of computational labs.

2.2 Non-cognitive Factors and Success in Mainstream Undergraduate Calculus

Zepke and Leach (2010) provide a review of literature regarding the engagement of undergraduate students in their studies as well as the improvement of outcomes such as graduation and employment rates. They produce ten proposals for action based on the relevant research: 1) enhance students' self-belief, enable students to work autonomously, 2) enjoy learning relationships with others and feel they are competent to achieve their own objectives, 3) recognize that teaching and teachers are central to engagement, 4) create learning that is active, collaborative and fosters learning relationships, 5) create educational experiences for students that are challenging, enriching and extend their academic abilities, 6) ensure institutional cultures are welcoming to students from diverse backgrounds, 7) invest in a variety of support services, 8) adapt to changing student expectations, 9) enable students to become active citizens, and 10) enable students to develop their social and cultural capital. The following literature provides a detailed landscape specific to undergraduate calculus in the context of the ideas presented in these recommendations.

2.2.1 Social and Cultural Factors in Undergraduate Mathematics Education

Researchers have long attempted to understand the impact of white patriarchal culture and differences in gender, race, and ethnicity on the learning of mathematics in the undergraduate classroom. In the early 1980s, Uri Triesman (1983) described the positive effects of the campus-wide efforts of the University of California at Berkeley to close achievement gap for minority students. Despite such efforts, Danny Martin (2018) recently stated the position that:

> Equity for Black learners in mathematics education is a delusion rooted in the fictions of white imaginaries and characterized at best by incremental changes that do little to threaten the maintenance of white supremacy and racial hierarchies inside or outside of mathematics education.

To better understand the role of gender in STEM achievement, Steele, James, and Barnett (2002) studied women and men undergraduate students in either their first year (744 women and 605 men) or final year (333 women and 333 men) at a private university to understand the relationships among perceptions of sex discrimination (in their current major and in their future profession), perceptions of stereotype threat, identification with their major, sex, and whether or not students were in a male-dominated (STEM) or female-dominated (humanities, arts, education) major. Women students in both female-dominated and male-dominated fields reported higher levels of sex discrimination in their major as well as even higher expected levels of sex discrimination in their profession. Women students reported a higher level of stereotype threat in male-dominated areas than in male-dominated areas. Men reported the same level of sex discrimination and stereotype threat across all settings. Dennehy (2017) found in an experimental study (N=150) that women undergraduate engineers with same-gender mentors persisted at higher levels than those with opposite-gender mentors (who actually persisted at lower levels than those no mentors). This is consistent with the finding of Gilbert et al (2015) that women's belief about women (and not about men) and mathematics are predictive of sense of fit in mathematics. Jones, Ruff, and Paretti (2013) examined the relationships among engineering identification, gender identification, gender stereotype endorsement, and student perception of engineering ability in 363 first year engineering students at a public university. Differences between men and women were found in stereotype endorsement and perception of engineering ability with men higher in the former and women higher in the latter. They also found that perception of engineering ability was a predictor of academic performance and correlated with persistence in major for both men and women. Dugan

(2013) used concepts of leadership capacity (Day, 2009), self-efficacy (Bandura, 1997), and leadership efficacy (Hannah, 2008) to examine the perceptions of women STEM majors compared to their non-STEM peers. The results of a survey of 14,698 women from 86 institutions indicate that women in STEM fields report lower levels of leadership efficacy, despite equivalent leadership capacity, than their non-STEM peers paralleling the findings of Steele (2002) with regard to perceptions of discrimination and stereotype threat by women in male-dominated versus female-dominated fields. Vallerand and Bissonnette (1992) showed, in a study of 1042 undergraduates, that, relative to academic work, women undergraduates have higher intrinsic motivation and autonomous extrinsic motivation than men and lower controlled extrinsic motivation and amotivation than men.

Steinberg et al. (2012) manipulated the condition of stereotype threat to understand its effect on mathematics performance in women (moderated by low/average/high calculus GPA and math identification) majoring in STEM fields. The dependent measure was a 15-item mathematics assessment taken from the math GRE. In the stereotype threat condition (where participants were informed that men outperform women on the ensuing mathematics assessment), calculus GPA was negatively correlated with performance for those with high math identification. In general, the control condition (no messaging) outperformed the gender equivalence condition (where participants were informed that there are no gender differences on the assessment) and the stereotype threat condition. Nadler and Komarraju (2016) found that African American undergraduate men with strong academic identification performed worse on Raven's Progressive Matrices items than African American men with weaker academic identification. The results remained true across all combinations of autonomy support and stereotype threat in a 2x2 factorial design (though the finding is significant only in the autonomy supportive condition).While their results around the implications of autonomy support and stereotype threat were inconsistent, the authors acknowledge the support the use of autonomy supportive environments as a means for instructors to improve outcomes in college, particularly in the presence of nuanced influence of stereotype threat and gender roles.

Nieves (2002) examined the use of feminist pedagogies (which emphasize knowledge pursuit, collaboration, personalization, and acceptance) in the college mathematics classroom (in developmental algebra) and their impact on self-confidence and achievement of female Hispanic students. A short-term intervention was designed to promote inference and use of multiple strategies, engagement in authentic activities, support feedback and transparency. Each invention class session consisted of a short lectures (20 minutes) covering new and old material followed by longer (45 minutes) cooperative problem solving activities and student summary of activities. Students kept journals related to the beliefs about themselves as Hispanic females including those related to the utility of mathematics, ability in mathematics, and social acceptance related to success in mathematics. Students also revised instructional content to be framed from their own perspective. Students were also asked to articulate mathematical processes, critique application of processes and summarize key concepts. Although the course was taught in English, extensive language supports were provided to students who primarily spoke Spanish (the majority of participants spoke no English at home). The pedagogical strategies were shown to be effective in promoting achievement outcomes on departmental exams and improving both increased confidence and attitudes (as measured by the Fennema-Sherman Mathematics Attitudes Scale (Fennema & Sherman, 1976)). Collaborative learning strategies are recommended for engaging English language learners (Jao, 2012), however, caution is also recommended as implementation can be challenging and counterproductive when attention is not paid to individuals' social and emotional needs.

Rodriguez et al. (2013) argue that presenting mathematics in college as serving a varied set of objectives can promote a sense of fit and inclusion of diverse populations. Their research indicates that the congruence of self-beliefs were aligned with the presentation of the benefits of learning predicts greater levels of achievement on mathematical tasks. Hernandez et al. (2012) performed a longitudinal study of 1,420 upper division women and underrepresented minorities (African American and Latino) in undergraduate STEM disciplines to identify behaviors and motivational factors, from a goal theory perspective, related to persistence in STEM. Their results show that performance-avoid goals lead to diminished persistence and that in scientific identity increased both task and performance-approach goals. Research experiences were shown to attenuate performance-avoid goals. Chambers et al. (2016) examined longitudinal data from the National Center for Education Statistics finding that Black Women's math self-efficacy declines over the course of their secondary education and is correlated positively with enrollment in a four-year post-secondary institution. They recommend supporting mathematics as part of the identity of Black Women students to promote enrollment and success towards a baccalaureate degree. Wong (2000) showed the control orientation in high-ability high school girls correlated negatively with time studying, academic rank and advanced math/science course-taking behavior after controlling for ability level as measured by PSAT scores. Additionally, Wang (2013) identified "academic interactions" in college (along with a host of precollege factors), including engagement with faculty and students with respect to academic activity to positively influence underrepresented minorities to enroll in STEM majors.

Gutierrez (2018) offers general strategies for “rehumanizing mathematics education” including the use of student-centered pedagogies, but also recommends connecting mathematics to the history and culture of students, providing opportunity for students to “see themselves” in the classroom, emphasizing creativity, inclusion of the body and emotions in the classroom, etc to improve access for all learners. Stipanovic and Woo (2016) recommend also considering factors related encouragement/counseling towards STEM, family expectations, and resource constraints when taking a systems approach to affecting the perceptions and persistence of URM students towards STEM. Additionally, and specifically among ethnic minority first-generation college students, Dennis et al. (2005) find that peer support (along with personal/career motivation) to be a predictor of students’ report of adjustment to college and academic success.

2.2.2 Motivational Factors in Mainstream Undergraduate Calculus

Fraser et al (1986) validated an instrument, College and University Classroom Environment Inventory (CUCEI), to assess learning environments across several disciplines (including mathematics) within higher education. The inventory used dimensions of Personalization, Involvement, Student Cohesiveness, Task Orientation, Innovation, Individualization, and Satisfaction. The researchers measured these seven dimensions for both instructors (preferred and actual) and students (preferred and actual) in 34 classes with 372 students and 20 instructors.

Students’ perception of Satisfaction correlated strongly and positively with student perceptions all other dimensions ($0.46 <= r <= 0.78$). Both Student Cohesiveness and Task Orientation correlated positively ($r=0.40$ and $r=0.48$) with Locus of Control. For all dimensions there was a consistent ordering of levels Instructor-preferred, Student-preferred, Instructor-actual, and Student-actual from highest to lowest. Discrepancies between instructor-actual and student-actual occur only for Involvement, Student Cohesiveness, and Satisfaction. Discrepancies between instructor-preferred and student-preferred occurred for all dimensions except Task Orientation.

Good et al. (2012) published results related to a developed sense of belonging scale within a Calculus course at a highly selective University identifying factors of membership, acceptance, affect, trust, and desire to fade and demonstrated the ability of these factors to predict reported math anxiety, usefulness of math, math confidence, and intent to pursue mathematics. They subsequently conducted a longitudinal study to understand the relationship between sense of belonging, perceptions of environmental stereotyping, perceptions of environmental entity theory, intent to pursue math, interest in math, and grades in calculus finding that, although sense of belonging decreased for all students over time, women reported a significantly lower sense of belonging than men. They also find that sense of belonging predicted grades in calculus and intent to pursue math. Furthermore, they find that an incremental mindset in women reduces the effect of stereotype threat, promoted resilient sense of belonging, and resulted in better outcomes in calculus as well as higher intent to pursue math as shown in Figure 6 and Figure 7. They suggest that women’s perceptions of the learning environment can disenfranchise them for study in advance mathematics courses needed for STEM careers.

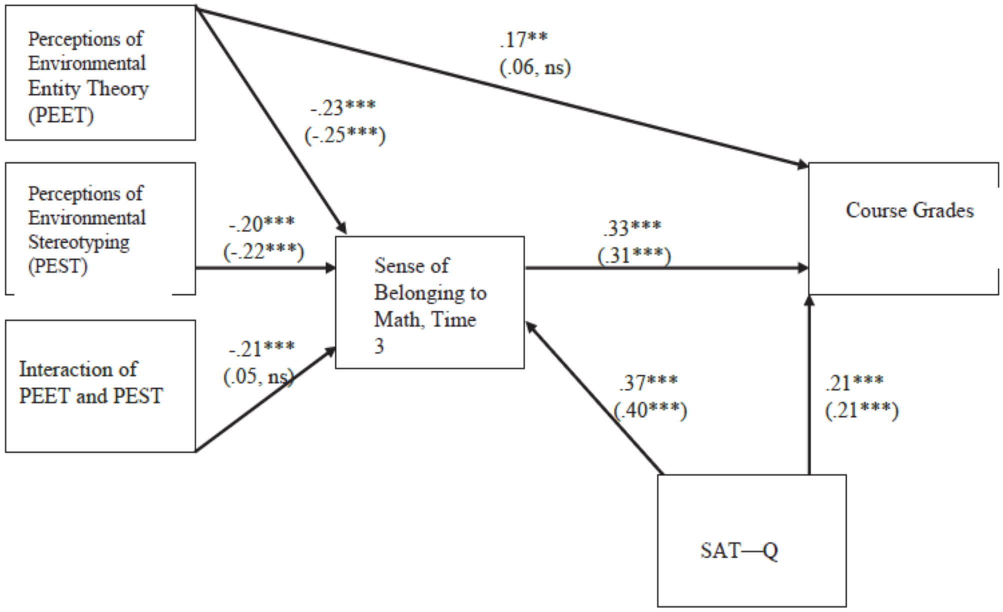

Figure 6: Sense of Belonging to Math and Course Grades (Good et al., 2012, p. 711)

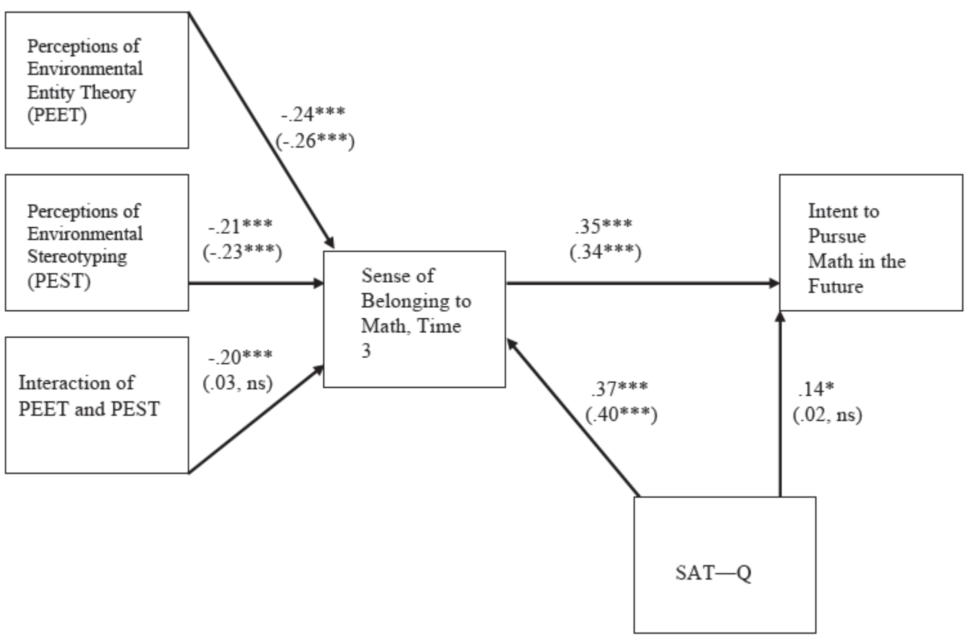

Figure 7: Sense of Belonging to Math and Intent to Pursue Math (Good et al., 2012, p. 712)

Sonnert et al. (2015) used data from the Characteristics of Successful Programs in College Calculus study (CSPCC, NSF DRL #0910240) to perform an exploratory factor analysis with several student variables on attitudes towards mathematics (confidence, enjoyment, choice). Three factors were identified related to classroom practices: “good” teaching (clarity, fairness, availability, etc), “ambitious” teaching (group work, flipped classrooms, elucidate of student reasoning processes, etc), and use of educational technology. They found that, on average, “good” teaching contributes positively with student attitudes and that “ambitious” teaching is slightly negatively correlated with student beliefs. However, “ambitious” teaching was found to have a positive influence on student attitudes in large classroom settings and for student with positive attitude towards mathematics as measured at the beginning of the semester. These findings about student perceptions are consistent with those by Weurlander et al. (2017) who categorized students’ attitudes towards ‘interactive teaching’ in calculus as enthusiastic, skeptical, or averse who hypothesize that students’ attitudes towards innovation in pedagogy are predicated by their beliefs about and experiences with education systems.

2.3 Self-Determination Theory and Education

2.3.1 Self-Determination Theory

Self-Determination Theory (SDT), co-developed by Edward Deci and Richard Ryan over the last 50 years as an advancement of drive and need-satisfaction theories of the mid-1900s (Deci & Ryan, 2000)), is a motivational theory that posits that all individuals have basic psychological needs to volitionally set their own objectives, perceive progress towards those objectives, and to share in those experiences with others. It is predicated on two assumptions: 1) that humans naturally and proactively engage with their environment and 2) that humans internalize phenomena and information from their environment that are then integrated with internal processes. SDT asserts that environments that support an individual's Basic Psychological Needs (BPN) (as opposed to physiological or other organismic needs) for Autonomy, Competence, and Relatedness facilitate internally-regulated extrinsic motivation for activities aligned with an individual's complete set of objectives, values, or beliefs. Environments that fail to support (or thwart) BPN-satisfaction contribute to controlled or externally-regulated extrinsic motivation (as well as amotivation) that can undermine intrinsic proactive engagement, internally-regulated external motivation and, therefore, impedes achievement and well-being in those contexts (Deci & Ryan, 2012, p 85-89).

2.3.1.1 Basic Psychological Needs.

Deci and Ryan (2000) describe how Self-Determination Theory posits the existence of a minimal necessary set of empirically constructed Basic Psychological Needs (Autonomy, Competence, and Relatedness) required for psychological health and growth as part of intrinsically motivated (those performed without regard for external contingency) and internalized (those performed with self-determined external contingencies) activities (Deci & Ryan, 1980). Autonomy is the volitional, proactive engagement in activities within a context and relates to intrinsic motivation by extending the internally-perceived locus of causality (Heider, 1958) to established, internalized phenomena and processes within that context. Competence is the perceived efficacy of an individual as a causal agent with respect to volitional engagement with the environment and relates to intrinsic motivation via feedback mechanism that either reinforce or undermine intrinsic motivation for that engagement or activity. Relatedness describes an individual's ability to internalize other causal agents from their environment and relates to intrinsic motivation in a more distal fashion through autonomous dependence or co-dependence in the orchestration of activities associated with an individual's (perhaps shared) objectives.

SDT predicts that individuals will inherently pursue environments and objectives that support their BPN when available and that these BPN-supportive environments will facilitate goal achievement relative to those objectives. Unlike physiological needs (eg hunger), inadequate satisfaction of BPN may lead to maladaptive or compensatory strategies as substitutes that can lead to negative consequences and diminished autonomous regulation of behavior (Deci & Ryan, 2000). Environmental features interacting with BPN-support may vary across contexts. For instance, Jang et al. (2010) discuss the relationship between autonomy support and structure in educational settings which, at first glance, may appear to be antagonistic, that is, that structure imposes control on individuals. However, in an examination of high school teachers' instructional styles (N=133) and their students perceptions and engagement (N=2,523), they find that structure (as opposed to chaos) correlates positively (r=0.60) with autonomy supportive practices and both contribute to student engagement. They recommend a balance strategy of structured, autonomy-supportive instructional environments for teacher seeking to improve engagement on learning tasks.

2.3.1.2 Continuum of self-determined motivation.

Ryan and Deci (2000) describe how Self-Determination Theory posits the existence of a continuum of self-determined motivation in which extrinsic motivation is delineated into autonomous and controlled types of extrinsic motivation according to perceived locus of causality (Heider, 1958) as positioned within a the larger framework shown in Figure 8. At one end of the broader continuum is Amotivation which involves passive or non-engagement within a context.

At the other end of the continuum is Intrinsic motivation which involves proactive engagement with the environment and natural human tendencies to satisfy Basic Psychological Needs. All other engagement is considered to be supported by non-Intrinsic or Extrinsic motivation, that is, from the integration of external phenomena with internal processes. Sources of Extrinsic motivation are separated into those that are perceived to be internally or externally regulated. Those internally regulated sources of motivation (Autonomous Extrinsic Motivation) are categorized as either wholly Integrated with or Identified to support an individual's activities, beliefs, and values related to BPN-satisfaction associated with an individual's Intrinsic motivation. Those externally regulated sources of motivation are categorized as either entirely disjoint from (External) or associated with negative aspects of (Introjected) an individual's activities, beliefs, and values related to BPN-satisfaction associated with an individual's Intrinsic motivation. External regulation of motivation is sourced from reward or avoidance of punishment. Introjected regulation of motivation is sourced from avoidance of negative emotions such as guilt or regret. Controlled Extrinsic Motivation (CEM) is used to describe both of these external types of regulation.

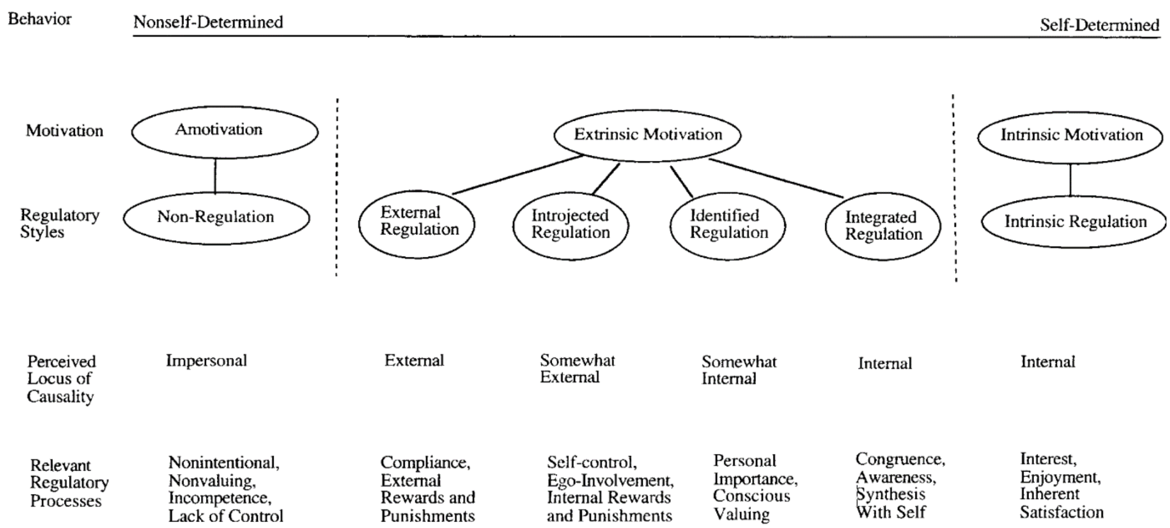

Figure 8: Continuum of Motivational Regulation (Ryan and Deci, 2000)

An early finding related to SDT was the undermining effect of external rewards on intrinsic motivations (Deci & Lanzetta, 1971). In a review of their own meta-analysis (Deci et al., 1999), Deci et al. (2001) argue that extrinsic rewards undermine intrinsic motivation specifically in educational settings. The argument is juxtaposed with a meta-analysis (Cameron & Pierce, 1994) which concluded that the undermining effect was minimal and not relevant to educational systems. These arguments follow a line of discourse related to the question of the undermining effects of extrinsic rewards and the appropriateness the 1994 meta-analysis (Cameron & Pierce, 1996; Lepper et al., 1996; Kohn, 1996; Ryan & Deci, 1996).

2.3.1.3 Universality of Self-determination Theory.

Self-Determination Theory has been cross-cultural validated in multitudes of studies (Chirkov et al., 2011; Soenens & Beyers, 2012; Chirkov et al., 2003; Slemp et al., 2018) dispelling counter-arguments to its universality based on how hierarchy, independence, autonomy, and individualism may function relative to diverse cultural beliefs, values, and practices. As an example, Chirkov et al. (2003) examined the relationship between autonomous motivation and well-being in Russia, Turkey, South Korea, and the United States as examples of nations with diverse perceptions of cultural practices (horizontal collectivism, horizontal individualism, vertical collectivism, and vertical individualism). They found that horizontal practices (those reflecting the interchangeability of individuals) and individualistic practices (those where priority is given to the individuals' objectives over the collective) were more internalized across all nations. Furthermore, they found positive correlation between autonomous motivation and well-being in all cases.

In the context of education, Jang et al. (2009) extensively investigated the SDT framework within educational settings in South Korea. After controlling for cultural factors, self-determined motivation and BPN-support explained considerable variance in achievement outcomes and well-being. Similar findings were established by Vansteenkiste et al. (2005) with respect to adult learners in China and by Zhou et al (2009) with respect to youth learners in China. Kaplan & Madjar (2017) examined the effect of BPN-supportive environments on the perceptions of pre-service teachers in Israel across Muslim and Jewish students. They found that BPN-support positively correlated with student perceptions of BPN-satisfaction (which positively correlated with outcomes) and negatively correlated with students' controlled motivation (which negatively correlated with outcomes) across both Muslim and Jewish student groups.

In the context of the workplace, Slemp et al. (2018) conducted a meta-analysis of 72 studies evaluating the behaviors of management in the workplace in relation to autonomy support and employee outcomes variables (well-being, distress, engagement, positive work behavior, and job satisfaction). They found that Leader Autonomy Support predicted perception of BPN-satisfaction, internalization of motivation regulation, and employee outcomes independent of the individualistic or collectivistic nature of the countries in which the individual studies were conducted.

2.3.2 Applications of Self-Determination Theory to Education and Learning

According to constructivist views, learning is an active process by which an individual engages with the environment to construct knowledge from their experience, thus, motivational support is a necessary condition for successful educational systems (Blumenfeld, 1992). Deci et al (1981) created an instrument to measure teachers' orientation as autonomous or controlled and found that teachers who are more autonomy-oriented tend to have students who are more intrinsically motivated and have higher self-esteem. Relationships among learning environment, educational achievement, and motivational constructs from SDT were subsequently investigated in a small-scale study (Benware & Deci, 1985) involving college undergraduates. The researchers found that students placed into a condition of active orientation for learning (specifically, to teach content to peers) demonstrated increased autonomous motivation, increased conceptual learning, and perception of themselves as more integrated into the learning environment than students placed into a condition of passive orientation towards learning (to be tested themselves). It is interesting to note that there was no difference on rote performance tasks between conditions and that no effect from time spent studying was found across conditions. This finding indicates that passive/active orientation within a learning context can differently facilitate achievement outcomes which might generally be considered to be inseparable in the classroom (ie procedural and conceptual knowledge). Deci et al. (1991) elaborate on the function of BPN for intentional engagement of students within learning contexts towards achievement outcomes. They argue that environments which fail to support BPN alienate students from the learning context citing studies involving the use of positive feedback with and without autonomy support as well as studies linking negative feedback to increased Intrinsic motivation when perception of Competence also increases and to decreased Intrinsic motivation when perception of Competence also decreases. A literature review conducted by Rigby et al. (1992) provides a thorough framework and extensive empirical support for the SDT extrinsic motivation continuum and it relationships with BPN-satisfaction and learning within educational contexts for a broad range of learners. Additionally, the researchers relate consistency with findings related to ego versus task involvement (Nicholls, 1984) and performance versus mastery goal orientation (Dweck, 1986) by which they associate ego involvement and performance goals with Controlled Extrinsic Motivation and likewise associate task involvement and mastery goals with Autonomous Extrinsic Motivation. Niemiec and Ryan (2009) presented a literature review discussing broad applications and long history of SDT in the classroom. They reiterate the global assertion that classrooms that support students' Basic Psychological Needs facilitate academic performance and well-being through the promotion of autonomous regulation of motivation.

In 1993, an Academic Motivation Scale (AMS) was created by Vallerand, et al using tenets of SDT for use with undergraduate students. The instrument was evaluated for concurrent validity by administering to seven subscales (Amotivation, External, Introjected, Identified, Intrinsic-to Know, Intrinsic-Accomplishment, Intrinsic-Stimulation) with four items each to 217 undergraduates alongside subscales from the Children's Academic Intrinsic Motivation Inventory (Gottfried, 1985) and the Personal Goals in School Scale (Nicholls et al., 1985). Correlations between the SDT constructs and the existing scales were deemed adequate. Internal reliability coefficients with the seven subscales ranged from .60 to .86. Additionally, the AMS was analyzed with constructs of motivational antecedents (such as Perceived Competence and Autonomy Support) as well as motivational consequence (such as Grades). In general, Identified and Intrinsic-types of motivation correlated positively (roughly $r=.2$) with Autonomy Support, Perceived Competence, and Grades. External and Introjected regulation of motivation generally showed trivial correlation with the same constructs. In 2006, Grouzet et al. showed longitudinal and gender invariance of AMS constructs in high school students. Smith et al. (2010) examined the validity and reliability of the AMS with undergraduate and MBA students enrolled in business schools. Their analysis showed the seven factor model had best fit across the entire population, within genders, and for undergraduates, but failed for MBA students.

Vansteenkisteen et al. (2006) position SDT relative to locus of goal contents and their relationship to academic motivation arguing that intrinsic goal framing leads to high-quality motivation, increased learning, and increased engagement in learning activities. SDT is compared with alternate motivational theories such as expectancy-value theory (where intrinsic goals are distinctly higher in personal value) (Eccles & Wigfield, 2002) and the matching of causality orientation with the framing of academic goals to support optimal regulation of motivation (Hidi & Harackiewicz, 2000). Additional work on comparing the Match Perspective with Self-Determination Theory by Vansteenkiste et al. (2009) in which intrinsic goal framing facilitated achievement in elementary school students independent of intrinsic or extrinsic personal goal orientation, however, the effects on learning outcomes were only significant for conceptual tasks. Vansteenkiste et al. (2010) examined performance-approach goals positioned within a SDT framework to understand the relationship among academic outcomes, autonomously regulated motivation, goal strength, and adaptive versus maladaptive perfectionism in high school students. Findings indicate that controlled versus autonomous motivation explained maladaptive and adaptive perfectionism and effects of goal strength disappeared when accounting for self-determined motivation.

2.3.2.1 Self-Determination Theory and Post-Secondary Education.

Many studies have been conducted with undergraduate students to understand the relationships between BPN-satisfaction, locus of motivational regulation, achievement, affect, and well-being in higher education. The scope, context, and recommendations are wide ranging, and brief summaries of such studies are presented below to illustrate the mosaic fabric of research involving SDT, motivation, and outcomes in higher education.

Black & Deci (2000) found that perceptions of autonomous motivation predicted course performance in a college-level chemistry course where supplemental active learning strategies were implemented. They recommend the use of active learning strategies to overcome the controlling nature of large-lecture instructional environments with limited opportunity for interpersonal interactions. Jang (2008) demonstrated how non-controlling, external rationales can increase undergraduate students' identified regulation of motivation, engagement and conceptual learning related to an uninteresting activity. Specifically, performance gains in statistical understanding were demonstrated in undergraduate educational psychology students who were informed that studying statistical correlation how been shown to make teachers more reflective in their practice. A large scale (N=1042), non-experimental study (Vallerand & Bissonnette, 1992) of undergraduates within a compulsory foreign language course showed that Autonomous Extrinsic Motivation for academic work predicts can be used to predict dropout rates in the course. Guay et al. (2007) investigated the persistence of undergraduate students across motivational profile (high/low autonomous/controlled motivation and amotivation). In general, autonomous motivation correlated with retention in college and students with high autonomous motivation but low controlled motivation outperformed those with high levels of both types of motivation. Burton et al. (2006) investigated the differential effects of identified versus intrinsic motivation regulatory styles on academic performance and well-being in undergraduate students. Identified motivation was shown to predict final exam grade over intrinsic motivation. Intrinsic motivation was argued to be a protective factor for well-being not contingent upon academic performance.

Bailey and Phillips (2015) found that intrinsic motivation was positively associated with psychological health in first-year undergraduate students, but only contributed to explaining academic performance (grade point average, including mathematics) after considering adaptation to the University, as measured by the Student Adaption to College Questionnaire (Baker & Siryk, (1989)), in the first semester. Guiffrida et al. (2013) found that undergraduates who entered college to satisfy needs for competence and autonomy had greater persistence and academic achievement that those who did not. The need for relatedness had mixed implications depending upon the source (peers, family, and instructors). Kusurkar et al. (2012) investigated at the role of autonomous and controlled motivation in medical students as it related to academic outcomes and study behaviors. They found no correlation between controlled motivation and any outcome measures; however, autonomous motivation correlated with GPA (r=.147) and good study strategies (r=.384), but not with study effort. Although, women reported significantly less controlled motivation and obtained significantly higher GPA. Miquelon et al. (2005) demonstrated relationships between perfectionism orientation (Hewitt & Flett, 1991), self-determined academic motivation (Vallerand et al., 1993) and psychological adjustment (Goldberg & Hillier, 1979) in undergraduate students. Specifically, self-oriented (internally regulated) perfectionism promoted self-determined academic motivation which was negatively correlated with difficulty in psychological adjustment in college. Furthermore, socially-prescribed perfectionism (externally regulated) correlated with non-self-determined academic motivation and was positively correlated with difficulties in psychological adjustment in college. Ciani et al. (2010) demonstrated a relationship between BPN-support, self-determined motivation, and goal orientation in undergraduate preservice teachers. Specifically, students' overall BPN-satisfaction predicted self-determined motivation for course activities and self-determined motivation predicted goal orientation using a 2x2 achievement goal framework (Cury, 2006). They suggest that diminished mastery-goal orientation could be addressed via autonomy-supportive learning environments.

Filak and Sheldon (2003) used students' perception of satisfaction of basic psychological needs (Autonomy, Competence, and Relatedness) to predict course and instructor ratings by students finding that all three dimensions predicted both ratings. A small study of undergraduate psychology students (Boggiano et al., 1993) showed that identical instruction towards solving analytic GRE sample problems produced different results when the solution method was indicated to be the "correct" way versus one of many ways to solve the problems. The "controlling-directive" condition was also shown to have a negative effect on autonomous motivation through it contributed to the students' perception of instructor competence. Chen and Jang (2010) investigated the roles of contextual support (as measured by the Learning Climate Questionnaire, (Williams & Deci, 1996), BPN-satisfaction, self-determined motivation in the education of adults enrolled in an online certificate program. They confirmed position associations between contextual support, BPN-satisfaction, and self-determined motivation; however, no effects of self-determined motivation were found in final grade although BPN-satisfaction and contextual support were strongly positively associated with perceived learning and course satisfaction, respectively. Kanat-Maymon et al. (2015) employed an experimental design creating learning environments that were BPN-thwarting (using game instructions emphasizing the experimenters' control, a lack of interest in the participant, low expectancies, and the role of chance), BPN-supportive (using game instructions emphasizing choice, self-direction, interest in the participants, and positive expectancies), and BPN-neutral (using neutral game instructions) to assess the impact of BPN-satisfaction on academic dishonesty in Israeli undergraduate students. BPN conditions, as confirmed by the Basic Psychological Need Scale (Patrick et al., 2007), resulting in (13.5%, 33.3%, 38.1%) and (8.1%, 11.9%, 28.6) cheating rates across two tasks in the (Support, Neutral, Thwart) conditions. Koh, et al (2010) investigated simulation-based teaching methods for undergraduate engineering students and their effect on BPN-satisfaction, self-determined motivation, and learning outcomes. Although the simulation-based group showed little-to-no increase in BPN-satisfaction or self-determined motivation, there were significant gender effects on learning outcomes across conditions with male students performing higher than female students in the simulation-based condition (no difference were present in the control condition).

2.3.2.2 Self-Determination Theory and K-12 Education.

A number of relevant studies have been conducted in the K-12 educational setting that have implications for the study of motivation in higher education settings. For instance, via a path analysis, Katz et al. (2014) showed that students' self-efficacy for doing homework in elementary school is positively associated with autonomous motivation (r=.42) which is negatively associated with homework procrastination (r=-.41). Self-efficacy itself was also negatively associated with homework procrastination (r=-.41). They suggest that autonomy-supportive learning environments may attenuate maladaptive academic behaviors and should begin at a young age. In an analysis of high-ability 3rd and 4th graders, Miserandino (1996) found that perception of competence and autonomy-support explained variance in academic course performance above and beyond ability as measured by the Stanford Achievement Test. Those students with low perception of competence or autonomy-support were more likely to engage in withdrawal behaviors and report negative emotions.

Tsai et al. (2008) found a positive relationship between the role of perceived autonomy-support in German middle school students and students' interest in the classroom across disciplines and teachers. Liu et al. (2009) examine how middle school students engaged in project-work can be clustered into homogenous groups according to dimensions of high/low controlled motivation and high/low self-determined motivation (using an Intrinsic Motivation Inventory (McAuley et al., 1989)). The high self-determined/low controlled group reported the highest BPN-satisfaction and the most adaptive problem-solving skills. The low self-determined/high controlled group reported the lowest BPN-satisfaction and least adaptive problem-solving skills. Self-determined academic motivation was also shown to be a mediator in relationship between psychological need support from parents and well-being in high school students (Niemiec et al., 2006). Sierens et al. (2009) examine the relationships between structure (communication of expectations, scaffolding of tasks, and relevant feedback), autonomy support, and self-regulated learning (as measured by Motivated Strategies for Learning Questionnaire (Pintrich et al., 1991)) in high school students. They found that structure contributed positively to self-regulated learning with no main effect from autonomy-support; however, a significant interaction between structure and autonomy-support. They theorize that structure provides a framework for students to self-regulate, and autonomy-support provides the motivation to operate within that framework.

Gillet et al. (2011) demonstrated the longitudinal trajectory of academically-oriented (going to school, doing homework, and listening to the teacher) self-determined motivation in children ages 9-17. The found that autonomy support mediating the effect of age on self-determined motivation; however, general trends were identified with self-determined motivation declining in early years followed by an upward trend in high school. Controlled motivation showed a decline in early years with no recovery. Otis et al. (2005) found similar results in students over the period from 8th to 10th grade where both autonomous and controlled motivation decreased. Tian et al. (2014) examined longitudinal (six weeks) relationships in Chinese junior/senior high school students between BPN-satisfaction, as measured by the Adolescent Students' Basic Psychological Needs at School Scale (Tian et al., 2014) and affect/school satisfaction, as measured by the Adolescents' Subjective Well-Being in School Scale (Tian, 2008). They found that autonomy and relatedness predicted subsequent increased school satisfaction and competence predicted subsequent positive affect. Additionally, school satisfaction predicted subsequent increased satisfaction of autonomy, competence, and relatedness; however, positive affect only predicted subsequent increased relatedness and autonomy.

Vansteenkiste et al. (2010) examined performance-approach goals positioned within a SDT framework to understand the relationship among academic outcomes, autonomously regulated motivation, goal strength, and adaptive versus maladaptive perfectionism in high school students. Findings indicate that controlled versus autonomous motivation explained maladaptive and adaptive perfectionism and effects of goal strength disappeared when accounting for self-determined motivation. Katz et al. (2010) investigated how different levels of need support in high school students relates to perceptions of needs-satisfaction and self-determined motivation for homework tasks. They found that students who have a higher level of need (e.g., "I need choice of tasks in homework," "I need to know what homework is good for", "It is important to me that the homework task will be challenging", "I need to feel respected even if I do not succeed in homework") may perceive lower levels of teacher need-support in the same context as compared to students with a lower level of need. In both, cases the perceived of need-support promote self-determined motivation for engaging in homework tasks. In a study of high school students, Reeve and Lee (2014) demonstrate a reciprocal relationship between motivation and engagement. In their analysis, classroom engagement at Time 1 predicted BPN-satisfaction, self-efficacy, goal orientation, and classroom engagement at Time 2. Classroom engagement at Time 2 predicted BPN-satisfaction and self-efficacy at time Time 3 as well as course achievement. They suggest focuses on engagement as a potential antecedent to and creates opportunities for BPN-satisfaction to support autonomous motivation and achievement in the classroom.

2.3.3 Self-Determination Theory and Educational Reform

Self-Determination Theory has be applied not only as a framework for understanding and affecting individuals' behavior and achievement, but also as a framework for understanding and affecting the dynamics of educational systems and as well as the agents within those systems. Deci and Niemiec (2009) address the role of SDT, and, more generally, cognitive or motivational theories at-large, in the formal education of teachers within higher education. They argue that the generalizability of SDT as it relates to basic human nature can contribute to solutions within educational contexts as a tool for understanding and not a prescriptive set of methods to replace qualitative analysis or contextualized knowledge. Deci (2009) advocates for SDT-based large-scale school reform in which all parties (students, parents, teachers, and administrators) participate in BPN-needs supporting communities and activities aligned around developing and implementing policies and procedures for the improvement the local educational system. Assor et al. (2009) propose a four-phase plan for school reform using tenets of SDT. The early phases focus on teacher professional development related to concepts of Basic Psychological Needs and Autonomous versus Controlled motivation. Later phases focus on identifying opportunities in the classroom for improvement and application of need-support as well as including students in the reform process.

In a study of school performance, Fortier et al. (1995) demonstrated a positive relationship with perceived BPN-support from various agents of the institution. Subsequently, Guay and Vallerand (1997) proposed and found evidence for a Motivational Process Model for Academic Achievement shown in Figure 9 in which autonomy support as viewed by various agents of the educational system were related and correlated with student self-determined motivation and

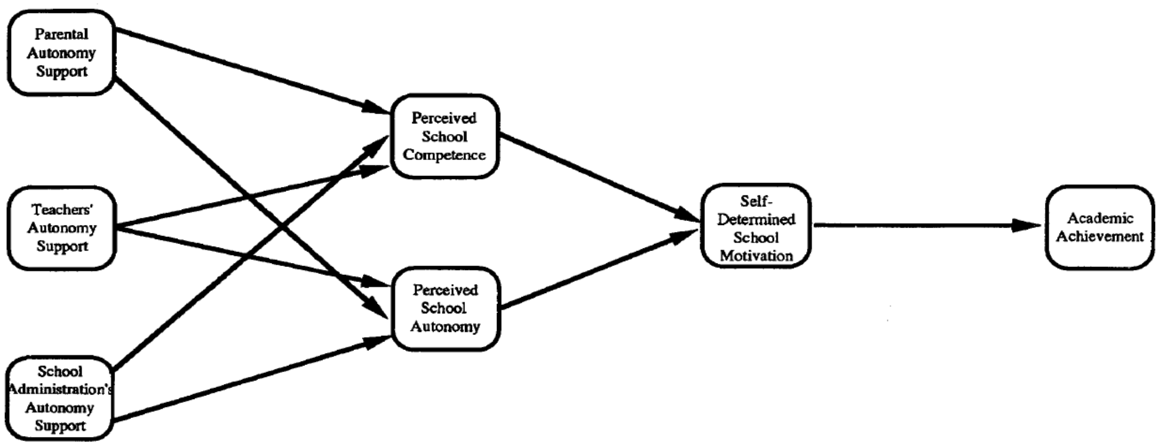

Figure 9: A Motivational Process Model of Academic Achievement (Guay & Vallerand, 1997, p. 213)

achievement. Lee and Reeve (2012) found that teacher and student reports of student engagement (behavioral, emotional, cognitive, and agentic) are relatively aligned; however, teachers are less able to predict student motivation (BPN-satisfaction, self-efficacy, and goal orientation). Deci (2009) underscores that educational reform is most effective when students, instructors, and administrators internalize the value of reform together with it implicit components.

In order to operationalize reform efforts, teaching professionals must be trained and assessment must be conducted to understand the efficacy of the reform and the impact on student achievement. Reeve (1998) tested the hypothesis that teachers can be trained to develop autonomy-supportive interpersonal motiving styles for application in the k-12 classroom. He found that such training to be successful and to result in increased student engagement, however, causality orientation (autonomy, control, or impersonal) of the teachers predicted the extent of the efficacy of the training which potentially limits the population of teachers who would benefit from the training. Roth and Weinstock (2013) explored epistemological orientation of teachers relative to their academic discipline (certainty, source, simplicity, and justification of knowledge (Hofer, 2000)) as a precursor to autonomy-supportive teaching practices. They found that teacher with objectivist orientations tend to be less autonomy-supportive. Results were not analyzed by individual disciplines presenting an opportunity to investigate teacher-related factors that contribute to autonomy-supportive teaching. Reeve et al. (2014) demonstrated, in a small-scale experimental study, that veteran high school teachers incorporated more autonomy-supportive behaviors into their teaching after training in autonomy-supportive methods, consistent with early work done in the field (deCharms, 1976). The increase in autonomy-supportive teacher behaviors produced increased student engagement as measured by task involvement and personal responsibility for learning. McLachlan and Hagger (2010) examine the training of University tutors in autonomy-supportive practices. The efficacy of the training was analyzed by tutor self-reports of autonomy-supportive behaviors and an observational protocol over three phases of implementation (baseline, training 1, and training 2). Significant effects were found in the frequency of listening carefully to students' speech, frequency of student-perspective-acknowledging statement from tutors, time students' spent talking, and frequency of directives issued by tutors. The positive findings indicate that tutors can be instructed to execute autonomy-supportive behaviors while working with students in the University setting.

While some freedom is afforded to teachers, educational systems contain curricular constraints that may thwart autonomy-supportive teaching practices in the classroom. However, Reeve (2006b) asserts that student engagement is highest when both autonomy support and structure are offered. This finding is supported by Katz and Assor (2007) who argue that choice can only be effective in the context of BPN-satisfaction. Without a framework in which to operate towards objectives, choice alone does not support achievement or well-being. For instance, Skinner and Belmot (1993) demonstrated a positive correlation between both autonomy-supportive teacher behavior and optimal structure with student motivation. The reciprocal relationship was also found where student motivation created a positive feedback loop for teacher behavior. Figure 9 shows a proposed model for teacher-as-learning facilitator where the characteristics of attunement (sensing of students' needs and adjusting accordingly), relatedness (making students feel important to the teacher), supportiveness (affirmation of students' ability to operate independently), and gentle discipline (guiding students' thinking about right and wrong behaviors) promote high quality motivation and engagement from students. Bachman and Stewart (2011) convey a theoretical framework for the design and implementation of online templates as a structured tool to promote BPN-satisfaction within learning environments. Specifically, an electronic platform can augment the practices that occur within the face-to-face classroom to give students more choice about how, when, and where to do their work as well as more natural, seamless, and self-regulated means of engaging with peers, the instructor, and course content. The online template provides stable, relevant structure for activities within the course to promote self-regulated learning.

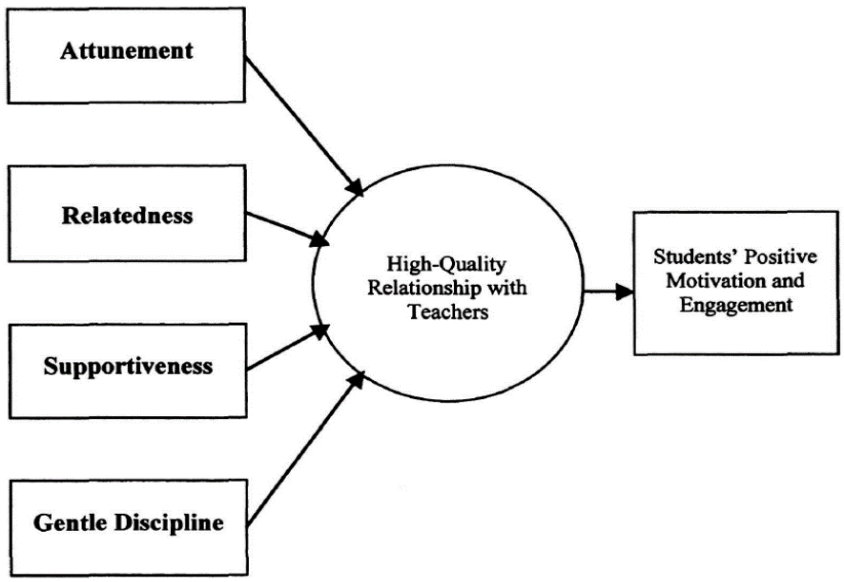

Figure 10: Teacher as Facilitator (Reeve, 2006)

Towards the systematic ability to assess autonomy-supportive instructional practices, Bozack et al. (2008) created the Autonomy Supportive Behavior Instrument (ASBI) and observation protocol for measuring autonomy-supportive behaviors in the classroom using the eight methods from Reeve et al. (2004). Each related question was marked with provided behaviors that address each method to varying degrees. The instrument was validated in five elementary schools participating in Comprehensive School Reform. Interrater reliability was 84% and most constructs had reliability of greater than 90%; however, questions about choice opportunities and manipulations of objects showed definitely lower reliability. To help guide teachers and administrator, Reeve and Halusic (2009) respond to commonly asked questions (eg What is the goal of autonomy-supportive teaching? and How do I know if I provided instruction in an autonomy-supportive way?) from K-12 teachers who have engaged in professional development related to applying principles of autonomy-support in the classrooms. These questions are categorized by broad actions related to tenets of SDT, and followed-up with specific examples, that may help teachers navigate the implementation of autonomy-supportive practices.

*Self-Determination Theory	ASBI Questions
Listening carefully; replying to student-generated questions in a contingent, satisfying way	How does the teacher respond to student-initiated questions or dialogue?
Creating opportunities for students to work in their own ways	What choice opportunities does the teacher provide?
Creating opportunities for students to talk	What kinds of opportunities does the teacher provide for student talk?
Arranging learning materials so students manipulate objects rather than passively watch and listen	How does the teacher allow students to manipulate objects?
Offering encouragement when students show effort & persistence	What kind of encouragement is offered around learning?
Giving hints & praising mastery and progress	How does the teacher engage students during the process of learning?
Acknowledging students' perspectives	How does the teacher make information relevant to students?

* "Self-Determination Theory: A Dialectical Framework for Understanding Sociocultural Influences on Student Motivation," by J. M. Reeve, E. L. Deci, & R. M. Ryan, 2004, in *Big Theories Revisited*, D. M. McInerney & S. Van Etten (Eds.), pp. 31–60. Greenwich, CT: Information Age.
Note: ASBI = Autonomy Supportive Behavior Instrument.

Figure 11: Mapping SDT to ASBI Question Strategies (Bozack et al., 2008)

In an investigation of the motivational dynamics of educational systems, Sarrazin et al. (2010) tested the 'social contagion of motivation' in an experiment where student perception of teacher self-determined motivation (manipulated by telling student groups that a teacher was either a volunteer or a paid instructor) influenced the self-determined motivation of students. Those students then taught the same lesson to peers and passed the levels of self-determined motivation on to the next group. The proposed mechanism for the second-order effects was autonomy-supportive teaching practices spontaneously produced as a consequence of inferred self-determined teacher motivation. The hypothesis was confirmed by analysis of results of an observational protocol developed by Reeve and Jang (2006). In further investigations of the motivational interactions among the agents within educational systems, Roth et al. (2007) established a positive relationship between self-determined motivation of teachers for teaching and the self-determined motivation of students for learning. Pelletier et al. (2002) examined other antecedents to autonomy-supportive teaching. They found that teachers who felt controlled by external forces related to curriculum standards, peers, performance metrics, and their own students (when perceived themselves to be non-self-determined in their motivation) predicts controlling behaviors and attitudes with their students. Reeve (2006) used a 36 pairs of preservice educators in a teacher-student paradigm to measure student perceptions of autonomy to categorize specific teacher behaviors as autonomy supports or thwarts. Eight of eleven (time listening; time allowing student to work in own way; time student talking; praise as informational feedback; offering encouragement; offering hints; being responsive to student-generated questions; and making perspective-acknowledging statements) proposed autonomy supports and six of nine (time holding/monopolizing learning materials, exhibiting solutions/answers, uttering solutions/answers, uttering directives/commands, making should/got to statements, and asking controlling questions) proposed autonomy thwarts were confirmed via correlational analysis.

2.4 Summary

The literature shows tremendous growth in undergraduate mainstream Calculus over the last fifty years with a continuing focus, but unrealized gains, on how to equitably serve students. Departments of mathematics have responded, but have not yet achieved desired change at-scale. Though the MAA recommends student-centered pedagogies in Calculus and research indicates its efficacy, clarity is lacking in how to operationalize and measure student-centeredness. Autonomy-supportive pedagogies align with instructional practices generally considered to be student-centered. Instruments have been developed to measure perceived satisfaction of Basic

Psychological Needs and Self-Determined Motivation and have been used to show that perceptions of BPN-satisfaction facilitate internally regulated motivation and achievement in educational settings. A host of social and cultural factors related to success in undergraduate mainstream Calculus have been established with motivation – as measured through various lenses – associated strongly with achievement across groups. Because Self-Determination Theory is a cross-culturally validated, it may be used to elucidate how different contexts and environments support BPN-satisfaction differentially across groups. The literature lacks experimental or quasi-experimental research across pedagogical methods within undergraduate mainstream Calculus using motivational (or any non-cognitive) measures to understand how those methods function at the individual level. Furthermore, the existing literature fails to delineate how specific student-centered pedagogical methods in undergraduate mainstream Calculus may function differently across demographic subpopulations of interest such as women, underrepresented minorities, students with low socioeconomic status, or even canonically privileged population such as middle-to-upper class, domestic, white men.

2.5 Research Questions

The proposed study applies a universal motivational macro framework, namely Self-Determination Theory (Deci, E. L., & Ryan, R. M. 2000), to capture differences in Calculus students' perceptions of satisfaction of Basic Psychological Needs (BPN) and locus of motivational regulatory processes across lecture-based and collaborative, problem-based learning environments as a means inform and empower mainstream undergraduate calculus reform efforts. Using course grades, a knowledge pre-test, measures of BPN-satisfaction and student motivational regulation, this work analyzes the relationships between motivation, learning environment, and student achievement to extend the recommendations on best practices for teaching Calculus structured in terms of support for students' perceived BPN-satisfaction as the proposed mechanism for existing, pragmatic recommendations for student-centered pedagogies. Practical implications of this research include strategies for faculty development as well as the comparison, design, and evaluation of Calculus learning environments at institutions of higher education. The following are specific questions related to this objective.

Research Question #1

Are there differences in students' perceived BPN-satisfaction and grade outcomes across lecture-based and collaborative, problem-based undergraduate Calculus I and Calculus II learning environments?

Research Question #2

What are the relationships among learning environments, perceived BPN-satisfaction, amotivation, CEM, AEM, intrinsic motivation, and grade outcomes in Calculus?

Research Question #3

Do the relationships and differences among learning environments, perceived BPN-satisfaction, amotivation, CEM, AEM, intrinsic motivation, and grade outcomes in Calculus vary across demographic groups?

CHAPTER 3: METHODS

3.1 Research Questions and Hypotheses

This study has a quasi-experimental design using existing data collected over four years as part of a US Department of Education project entitled 'Success through Transformative Education and Active Mentoring' (STEAM, 2019) completed at large, Midwestern, research university from 2015-19. Through the project, a collaborative, problem-based learning environment (reformed condition) was implemented for Calculus I (fall semesters) and Calculus II (spring semesters). The new learning environment ran in parallel with the existing lecture-based learning environment (traditional condition). Students were able to self-select into either version. Baseline equivalence was established with a course-specific knowledge test administered in class at the beginning of the semester. The reformed condition was designed with features and activities believed to support the Basic Psychological Needs (Autonomy, Competence, and Relatedness) of students. Because the reformed condition involves extensive, programmatic interactions among students,

peer mentors, graduate students, and faculty, it was also hypothesized that social and cultural beliefs, experiences, and characteristics related to demographics (gender, race, socioeconomic status, and country of origin) would influence the theoretical mediating role of motivation as well as interact with effect of the learning environment on the perception of BPN-satisfaction and influence achievement.

Table 12 shows an overview of features of each learning environment and the course at-large that related to satisfaction of students' Basic Psychological Needs. Extensive details of the learning environments are discussed in Section 3.3.4.2 in this document. Because of these features, it was hypothesized that the reformed condition will yield higher perceived satisfaction of students' Basic Psychological Needs in both Calculus I and Calculus II. Additionally, according to the tenets of Self-Determination Theory, it was also hypothesized that the model in

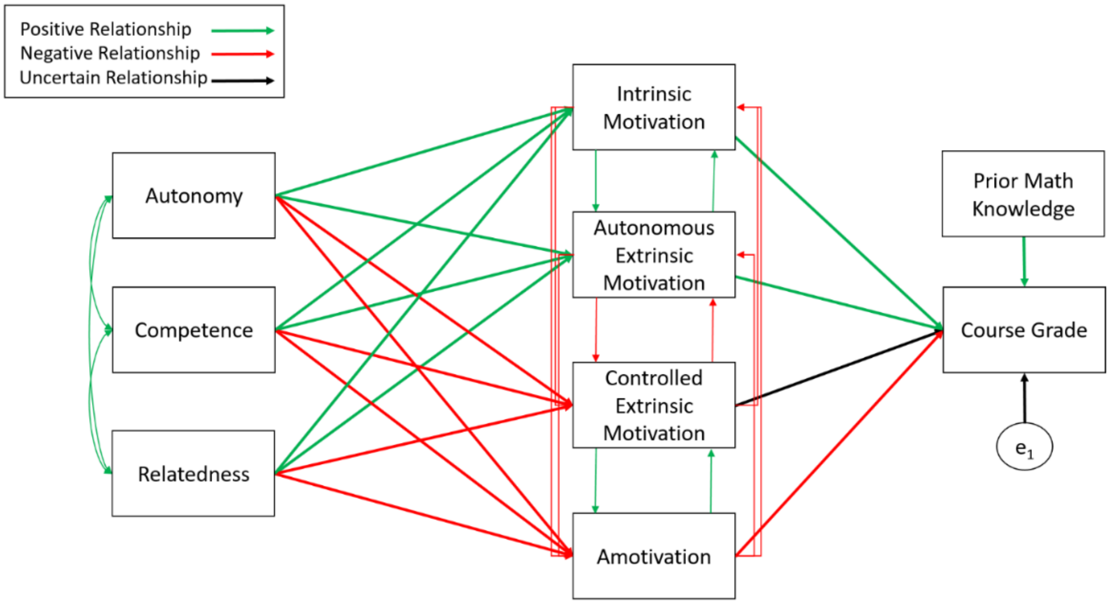

Figure 12 would hold across both learning environments and that the reformed learning environment would produce subsequent effects in self-determined motivation and grades outcomes in the both Calculus I and Calculus II. Because the reformed condition involves extensive, programmatic interactions among students,

peer mentors, graduate students, and faculty, it was also hypothesized that social and cultural beliefs, experiences, and characteristics related to demographics (gender, race, socioeconomic status, and country of origin) would influence the theoretical mediating role of motivation as well as interact with effect of the learning environment on the perception of BPN-satisfaction and influence achievement.

Table 12: Course features that May Support/Thwart BPN-satisfaction

Collaborative, Problem-based Learning Environment	Lecture-based Learning Environment	Common Features of Both Learning Environments
Programmatic interaction with peers, peer mentors, and faculty	Systematic direct instruction of mathematics content by worked examples	High stakes multiple choice assessments Norm-based grading

Routine formative assessment and feedback Collaborative work and goals Open-ended and conceptual problem types Required attendance and expected engagement Students control over how class time is spent Consistent relationships among students, groups, and TAs Student-led presentation of problem solutions	No opportunity for interaction with others No engagement or attendance expectations No feedback except on quizzes and exams Little-to-no student control over how class time is spent	Lack of transparency in determining grades Course is a curricular requirement ("service course") for most students Course topics are fixed and is not adaptive to students' interests Many options for academic support services, but none that scale to the size of the course or long-term needs of individual students

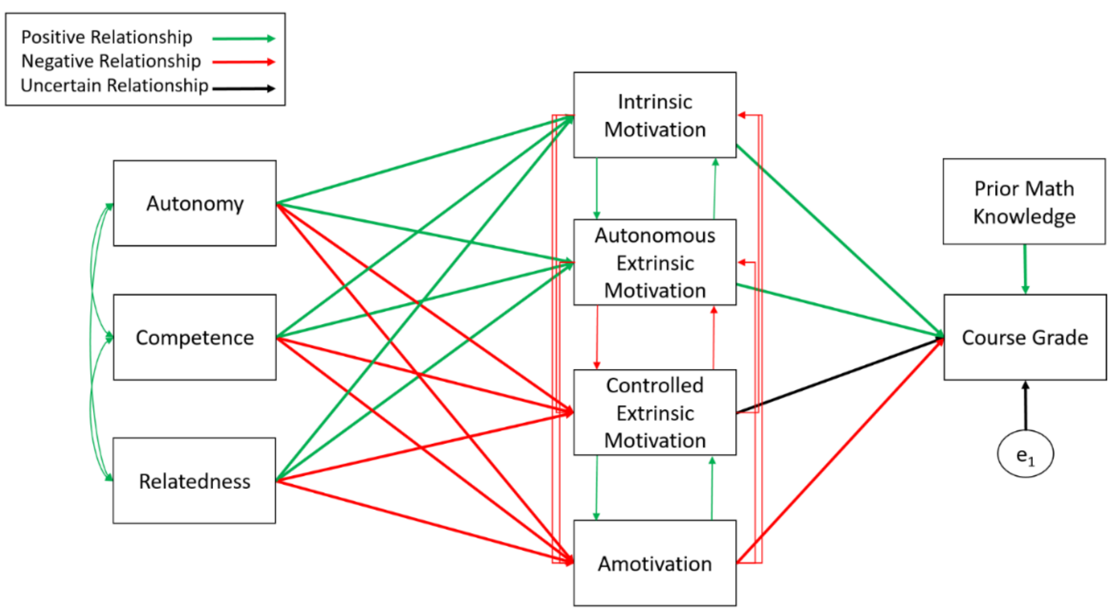

Figure 12: Theoretical Model for Motivation and Achievement

3. 2 Participants

The target population was first-year undergraduate students enrolled in mainstream Calculus sequences at selective, public, research-intensive universities. We used a convenience sample of students enrolled in Calculus I in the fall semesters 2015-18 or enrolled in Calculus II in the spring semesters of 2016-18 at a large public, Midwestern, research university where the vast majority of Calculus students are first-year Science or Engineering majors. All students enrolled in Calculus I met prerequisite scores on departmentally established placement procedures. All students enrolled in Calculus II received credit for Calculus I.

Both Calculus I and Calculus II were offered in the reformed and lecture-based learning environments with course section limits set by the Department of Mathematics. Each semester, the capacities were approximately 200 and 900, respectively, and abiding those limits, students were able to self-select into either learning environment. A number of factors likely influenced students' decision to select one learning environment over the other, such as: 1) availability of seats at times/days suitable to the student, 2) advice of peers, family, and academic advisors, and 3) preference for one learning environment or the other based on prior knowledge and experiences. In the spring term (Calculus II), students had more prior knowledge and experience after having taken Calculus I and may have modified or stronger preferences for one environment or the other than in the fall.

Course grades, demographics, and a pre-knowledge test were collected for 6,866 participants across two the learning environments over seven semesters in both Calculus I (four fall terms, with 687 students in the reformed condition and 3284 in the traditional condition) and Calculus II (three spring terms, with 584 students in the reformed condition and 2,311 in the traditional condition). A total of 3,294 participants electronically completed both the Basic Psychological Needs Scale (Levesque-Bristol et al, 2010) and the Situational Motivation Scale

(Guay et al., 2000) in Calculus I (N=1,720) and Calculus II (N=1,574) yielding response rates of 43% and 54% (41% and 61% in the reformed conditions), respectively.

3.3 Measures and Instrumentation

3.3.1 Knowledge Exam

3.3.1.1 Calculus I.

The knowledge exam for Calculus I was a shortened (ten-item) version of the Calculus Concept Inventory (Epstein, 2013) that could be more readily administered in the confines of the 50-minute class period. Each item was graded as correct or incorrect. The variable KNOW was computed for each student as the percentage correct on the Knowledge Exam. A copy of the knowledge exam for calculus I is included in Appendix B.4. The reliability of the knowledge exam is addressed in the results.

3.3.1.2 Calculus II.

The content of the CCI does not extend to topics in Calculus II. The researcher collaborated with the Department of Mathematics to write a brief (seven-item) pre-Knowledge Exam for Calculus II. Each item was graded as correct or incorrect. The variable KNOW was computed for each student as the percentage correct on the Knowledge Exam. Construct validity was established by a small panel of external, expert reviewers at peer institutions chosen by the researcher. A copy of the knowledge exam for calculus II is included in Appendix B.5. The reliability of the knowledge exam is addressed in the results.

3.3.2 Basic Psychological Needs Scale (BPNS)

The Basic Psychological Needs Scale (Levesque-Bristol et al, 2010) contained 21 items on a 7-point Likert scale (1-strongly disagree, 2-disagree, 3-somewhat disagree, 4-neither agree nor disagree, 5-somewhat agree, 6-agree, 7-strongly agree) and measures perceived satisfaction of Basic Psychological Needs according to Self-Determination Theory. Variables AUT, COMP, and REL were computed from average scores for each participant on the items associated with each basic psychological need. A copy of the BPN survey is included in Appendix A.1. The reliability of the survey was assessed in the analysis.

3.3.3 Situational Motivation Scale (SIMS)

The Situational Motivation Scale (Guay et al., 2000) contained 18 items on a 7-point Likert scale (1-strongly disagree, 2-disagree, 3-somewhat disagree, 4-neither agree nor disagree, 5-somewhat agree, 6-agree, 7-strongly agree) and measures perceived locus of causality according to Self-Determination Theory. Variables INTRIN, INTEG, IDENT, INTRO, EXT, and AMOT were computed from average scores for each participant computed on the items associated with each segment of the perceived locus of causality continuum. CEM (Controlled External Motivation) was computed as the average of INTRO and EXT and AEM (Autonomous External Motivation) was computed as the average of INTEG and IDENT. A copy of the situational motivation survey is included in Appendix A.2. The reliability of the survey were assessed in the analysis.

3.3.4 Course-related Variables

3.3.4.1 Calculus I.

Calculus I is the standard mathematics entry-point for first-year Science and Engineering students. The prerequisite university courses for Calculus I do not count for credit in the colleges of Science and Engineering at this institution. Only students from fall terms will be included for the analysis of Calculus I.

Catalog description:

> 5 credits. Introduction to differential and integral calculus of one variable, with applications.

Learning Outcomes:

1. To compute limits and to apply limit laws.
2. To apply rules of differentiation to compute derivatives of elementary functions.
3. To sketch graphs of functions with the aid of differentiation techniques.
4. To find maxima and minima of functions; optimization problems
5. To compute integrals of some elementary functions and to apply the Fundamental Theorem of Calculus to compute areas of certain planar regions

Advanced Placement Equivalent:

Students who scored a 4 or 5 on the Advanced Placement Calculus AB Exam (College Board, 2019a) were eligible for credit for Calculus I at this institution.

3.3.4.2 Calculus II.

Calculus II continues in the same textbook as Calculus. While several new concepts are introduced, much of the course is directed towards specialized techniques for performing what can be considered as inverse operations for methods in Calculus I. Only students from spring terms will be included for the analysis of Calculus II.

Catalog description:

5 credits. Continuation of Calculus I. Vectors in two and three dimensions, techniques of integration, infinite series, conic sections, polar coordinates, surfaces in three dimensions.

Learning Outcomes:

1. Apply techniques of integration (integration by parts, trigonometric substitution and partial fractions) to compute areas of planar regions, volumes of solids of revolution and areas of surfaces of revolution, work, moments and centers of mass of homogeneous laminas.
2. Apply tests of absolute convergence of series to find the interval of convergence of some power series.
3. Find the Taylor and Maclaurin series of some exponential, rational and trigonometric functions.
4. Use polar coordinates to make it possible to sketch the graphs of some curves.
5. Understand the definition of a Riemann sum, and should be able to apply elementary approximation methods of integration.

Advanced Placement Equivalent:

> Students who scored a 4 or 5 on the Advanced Placement Calculus BC Exam (College Board, 2019b) were eligible for credit for Calculus I and Calculus II at this institution.

3.3.4.3 Lecture-based Learning Environment (REFORMED = 0).

The traditional, lecture-based learning environment consisted of three 50-minutes lectures (Monday, Wednesday, and Friday) and two 50-minute recitations (Tuesday and Thursday) per week for fifteen weeks. New material was presented in a large classroom (up to 480 seats) with little-to-no interaction between the professor and the students or among the students. Attendance was not tracked. Recitations were generally hosted by first-year graduate students in mathematics. Recitation is convened in small classrooms (up to 40 seats) and were used for answering questions on homework problems and taking quizzes. The graduate students were vetted for ability to teach by a panel of mathematics faculty and trained during graduate student orientation to programmatically ask the class for homework problems and then go over as many as time allows. The amount of TA-student interaction in recitation may vary by section, but the structure is rigid and no other activities except answering homework questions and taking quizzes is programmatic.

3.3.4.4 Collaborative, Problem-based Learning Environment (REFORMED = 1).

The collaborative mode consisted of one 75-minute large problem session (Tuesday) and two small problem session/recitations (Wednesday and Friday). The problem session met in a large classroom with reconfigurable crescent-shaped tables (seating 3-4 students) that were generally arranged into circles. Students were placed into groups of three or four. There was no new content presented the problem sessions. Attendance was tracked and mandatory for all students in order to facilitate continual engagement within student groups. The problem session was primarily used to introduce and work on new problem sets with procedural, open-ended, and conceptual items. Examples of problems sets are included in Appendix B. One faculty instructor, three graduate teaching assistants, and three undergraduate teaching assistants facilitate the sessions. The room had a computer with screens throughout the room that was used by the faculty member to present information relevant to the entire class including mathematical solutions to commonly missed problems or other common questions. Each instructional staff member was given basic training in facilitating a class of this nature by the department. In this environment, students generally had control over how class time was spent and were able to get on-demand assistance from their peers, peer-mentors, graduate students, or a faculty member. There were a number of portable whiteboards and easels available for impromptu breakout sessions. The small problem sessions/recitation had components similar to traditional recitation and were convened in small classrooms (up to 40 seats) used for presentation of problem solutions, reviewing homework, and taking quizzes. The student groups (and associated graduate teaching assistant) from the large problem sessions were preserved within the small problem sessions/recitations to provide continuity for the students.

3.3.4.5 Commonalities across Courses and Conditions.

Faculty instructors were required by the Department to host one office hour each week, but did not have the capacity to serve the large number of students enrolled. A Math Help Room staffed by graduate teaching assistants from Calculus (and other courses) was made available to students. There were approximately 30 seats, and it is open Monday-Friday from 9am-4pm with 3-5 graduate students staffing the room at a time. The room served multiple courses with combined enrollment of over 9,000 students and did not have the capacity to serve all students, particularly during high-activity periods before midterm exams. Additionally, the University offered

Supplemental Instruction (Martin & Arendale, 1994) in collaboration with the Department for Calculus I and Calculus II that met Monday-Friday for 50 minutes in the evening led by one undergraduate student. Each session supported approximately 2,000 enrolled students in each course and, again, did not have the capacity to serve all students. All calculus students have access to recorded lectures online, but it was not required to view them.

3.3.4.3 Course Grade (GRADE).

Course grades in Calculus I and Calculus II were determined by departmental and course policy. A grade curving process based on three common, multiple-choice midterm exams and a common, comprehensive, multiple-choice final exam is used to determine the distribution of grades for the course and within each course section. Homework, quizzes, and activities (for the reformed learning environment only) did not change the overall section grade distribution or average, but did affect who in each section receives which grades from that distribution. A complete explanation and example of the grading methodology provided to students is included in Appendix B.3. Letter grades were converted to 'grade points' aligned with institutional regulations (ie. A+=4.0, A=4.0, A-=3.7, B+=3.3, B=3.0, B-=2.7, etc) for analysis.

3.3.4.4 Pass/Fail (PASS).

The variable PASS was computed for each student in the grade sample where PASS = 1 for student with a C- or above and PASS = 0 for students with a grade of D+ or below.

3.3.5 Demographics variables.

Table 13: Demographic Variables

Name	Values	Description
FEMALE	Male (0) or Female (1)	As reported in University's Student Information System.
PELL	Eligible (1) or Ineligible (0)	As established by the University's Office of Financial Aid.
URM	URM (1) or non-URM (0)	URM in includes Hispanic/Latino, Black, African American, and Native Americans. Non-URM includes White, Asian, 2 or more races, or Unknown. This field is recorded for Domestic students only.
INTERNATIONAL	International (1) or Domestic (0)	As established by the University's Office of Admissions.
COLLEGE	Engineering, Science, or Other	College of Primary Major.

3.4 Procedures

3.4.1 Knowledge Exams

Course instructors administered the Knowledge Exam as a quiz at the beginning of the second week of each semester. A small amount of extra credit was provided to students based on completion. An alternate assignment was offered to students who chose not to take the Knowledge Exam. The scores for the Knowledge Exams were provided to the Center for Instructional Excellence. These data were provided to the researcher by the staff of the University's Center for Instructional Excellence in accordance with IRB protocols and data agreements.

3.4.2 Surveys

The Situational Motivation Scale and Basic Psychological Needs Scale surveys were sent electronically to all students by the University's Center for Instructional Excellence during the second week of each term and during the second-to-last week in each term. Reminder emails were sent by the course coordinator and a small amount of extra-credit was provided to students who completed the surveys. An alternate assignment was offered to students who chose not to take the surveys.

3.4.3 Demographic Data and Course Grades

Student demographic data and course grades were extracted from the University's Student Information System by the University's Evaluation and Learning Research Center under data agreements with the Office of the Registrar. Data were provided to researchers by the staff of the Center for Instructional Excellence in accordance with IRB protocols and data agreements.

3.5 Overview of Analysis

3.5.1 Samples of analysis

3.5.1.1 Complete Grade Records and Pre-Knowledge Exam.

A subsample of students (Grade Sample) was created from the entire sample by selecting students who complete Calculus I or Calculus II with the a grade of F, D, D+, C-, C, C+, B-, B, B+, A-, A, or A+. Students who receive other grades related to Incomplete, Withdrawal, Audit, or Not Submitted will be excluded due the varied antecedents that may results in such grades. Students who did not complete the pre-Knowledge Exam will be excluded.

3.5.1.2 Complete Grade Records, Pre-Knowledge Exam and Post-Surveys.

A subsample of students (Survey Sample) will be created from the Grade Sample by selecting students who completed all items on the pre-Knowledge Exam, all items on the Situational Motivation Scale post-survey, and all items on the Basic Psychological Needs Scale post-survey. Within the Survey Sample, SDT construct variables will be presumed to use the post-survey unless otherwise stated.

3.5.2 Descriptive Statistics

3.5.2.1 Grade sample.

Any missing values in the Grade Sample for AUT, COMP, and REL will be imputed from the Survey Sample using linear regression on GRADE, COURSE, CONDITION, GENDER, PELL, URM, and INTERNATIONAL. Mean grades and significance of the effect of the reformed condition on GRADE were computed for each course by learning environment and broken out by demographic variables. Mean values for AUT, COMP, and REL (including imputed values) and significance of the effect of the reformed condition on each BPN were computed for each course by learning environment and broken out by demographic variables.

3.5.2.2 Survey Sample.

Correlation matrices for each course including GRADE, KNOW, AUT, COMP, REL, AMOT, EXT, INTRO, IDENT, INTEG, and INTR were computed using the Survey Sample.

3.5.3 Comparison of Learning Environments

3.5.3.1 Grade sample.

Baseline equivalence between learning environments within each course will be evaluated within the Grade Sample using KNOW. ANCOVA analyses were conducted with GRADE, COMP, AUT, and REL as the dependent measures for each COURSE using CONDITION as the independent variable with KNOW as a covariate. Logistic regression will be used to estimate the impact of demographic variables on likelihood to pass across conditions.

3.5.4 Relationships among Learning Environments, SDT, and Grade Outcomes

3.5.4.1 Survey Sample. For each course, a path analysis will be conducted with the model proposed in

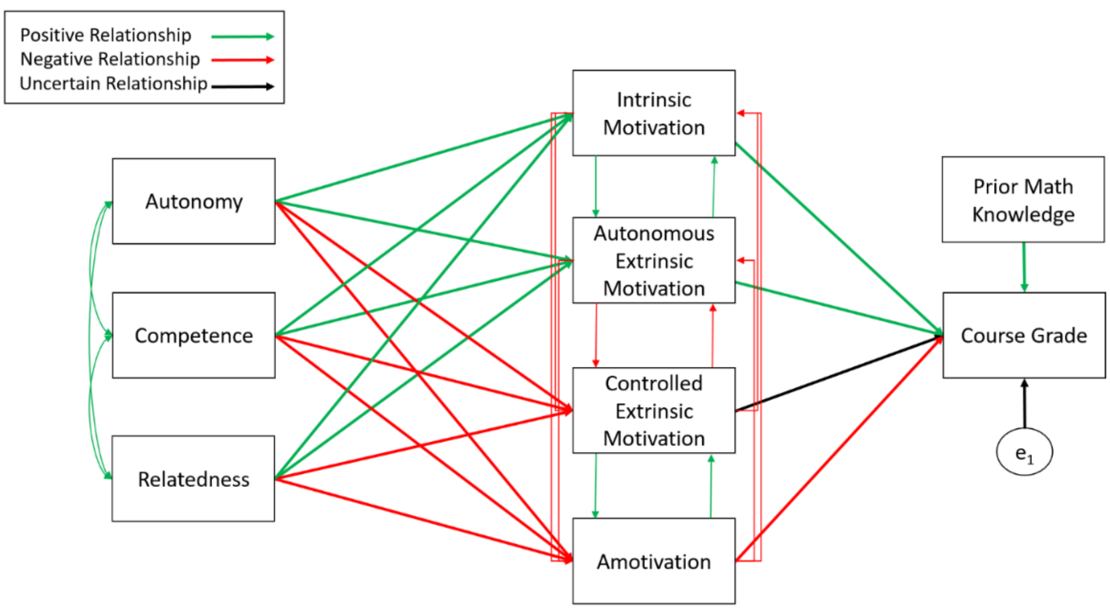

Figure 12.

3.5.5 Variation across Demographic Groups

3.5.5.1 Grade Sample.

Demographic variables were added to the ANCOVA above as well as interaction terms for demographics with CONDITION. Descriptive statistics will be inspected and specific comparison of means may be conducted for targeted subpopulations. Because URM and PELL are relevant for Domestic students only, the comparison group non-URM and non-PELL will exclude International students.

3.5.5.2 Survey Sample.

Correlation matrices were computed for each of WOMEN, URM, PELL, DOMESTIC, and INTERNATIONAL. Path analyses were conducted for subpopulations by GENDER and INTERNATIONAL.

CHAPTER 4: RESULTS

Course grades, demographics, and a pre-knowledge test were collected for 6,866 participants (grade sample) across the two learning environments over seven semesters in both Calculus I (four fall terms, with 687 students in the reformed condition and 3284 in the traditional condition) and Calculus II (three spring terms, with 584 students in the reformed condition and 2,311 in the traditional condition). A total of 3,294 participants (survey sample) also electronically completed both the Basic Psychological Needs Scale (Levesque-Bristol et al, 2010) and the Situational Motivation Scale (Guay et al., 2000) in Calculus I (N=1,720) and Calculus II (N=1,574) which yielded response rates of 43% and 54% (41% and 61% in the REFORMED conditions), respectively. Descriptive statistics for each measure (using the Grade Sample with imputed motivational measures), broken out by populations of interest, are reported in Table 14, Table 15, Table 16, and Table 17. Section 4.1 will address the knowledge test, grade outcomes, and demographics across conditions in both Calculus I and Calculus II for the grade sample. The motivational inventory (limited to the survey sample) will be discussed in the context of grade outcomes and demographics in section 4.2. In section 4.3, imputed basic psychological needs data for the entirety of the grade sample will be presented along with demographics and grade outcomes. In all tables, ^ indicates $p < 0.10$, * indicates $p < 0.05$, ** indicates $p < 0.01$, *** indicates $p < 0.001$, and ns indicates 'not significant'.

Table 14 Descriptive Statistics for BPN and Academic Measures, Calculus I

Cacleulus I mean(standard deviation) All / Traditional / Reformed	N	KNOWLEDGE	GPA	AUTONOMY	COMPETENCE	RELATEDNESS
All	3971 / 3284 / 687	0.5(0.2) / 0.5(0.2) / 0.4(0.2)	2.4(1.1) / 2.4(1.1) / 2.5(1.1)	4.1(0.5) / 4.1(0.5) / 4.3(0.5)	4.4(0.7) / 4.4(0.7) / 4.5(0.7)	4.4(0.5) / 4.3(0.5) / 4.6(0.5)
Men	2837 / 2342 / 495	0.5(0.2) / 0.5(0.2) / 0.5(0.2)	2.4(1.1) / 2.4(1.1) / 2.5(1.1)	4.1(0.5) / 4.1(0.5) / 4.3(0.5)	4.5(0.6) / 4.4(0.6) / 4.5(0.6)	4.4(0.5) / 4.3(0.5) / 4.7(0.5)
Women	1134 / 942 / 192	0.4(0.2) / 0.4(0.2) / 0.4(0.2)	2.4(1.1) / 2.4(1.1) / 2.4(1.1)	4.2(0.5) / 4.1(0.5) / 4.3(0.5)	4.4(0.7) / 4.4(0.7) / 4.4(0.7)	4.4(0.5) / 4.4(0.5) / 4.6(0.5)
Domestic	3007 / 2495 / 512	0.5(0.2) / 0.5(0.2) / 0.4(0.2)	2.2(1.1) / 2.2(1.1) / 2.3(1.1)	4.0(0.5) / 4.0(0.5) / 4.2(0.5)	4.4(0.7) / 4.4(0.7) / 4.4(0.7)	4.4(0.5) / 4.3(0.5) / 4.7(0.5)
International	964 / 789 / 175	0.5(0.2) / 0.5(0.2) / 0.5(0.2)	3.0(1.0) / 3.1(1.0) / 3.0(1.0)	4.5(0.5) / 4.5(0.5) / 4.5(0.5)	4.7(0.5) / 4.7(0.5) / 4.7(0.5)	4.5(0.5) / 4.4(0.5) / 4.6(0.5)
Engineering	1510 / 1290 / 220	0.5(0.2) / 0.5(0.2) / 0.5(0.2)	2.5(1.1) / 2.5(1.1) / 2.5(1.1)	4.1(0.5) / 4.1(0.5) / 4.3(0.5)	4.5(0.7) / 4.5(0.7) / 4.5(0.7)	4.4(0.5) / 4.3(0.5) / 4.7(0.5)
Science	1332 / 1113 / 219	0.5(0.2) / 0.5(0.2) / 0.5(0.2)	2.6(1.0) / 2.5(1.0) / 2.7(1.0)	4.2(0.6) / 4.2(0.6) / 4.3(0.6)	4.5(0.7) / 4.5(0.7) / 4.6(0.7)	4.4(0.5) / 4.3(0.5) / 4.6(0.5)
Other College	1129 / 881 / 248	0.4(0.2) / 0.4(0.2) / 0.4(0.2)	2.1(1.1) / 2.0(1.1) / 2.2(1.1)	4.1(0.5) / 4.1(0.5) / 4.3(0.5)	4.3(0.6) / 4.3(0.6) / 4.4(0.6)	4.4(0.4) / 4.3(0.4) / 4.6(0.4)
Domestic Men	2127 / 1758 / 369	0.5(0.2) / 0.5(0.2) / 0.5(0.2)	2.2(1.1) / 2.2(1.1) / 2.4(1.1)	4.0(0.5) / 4.0(0.5) / 4.2(0.5)	4.4(0.6) / 4.4(0.6) / 4.5(0.6)	4.3(0.5) / 4.3(0.5) / 4.7(0.5)
Domestic Women	880 / 737 / 143	0.4(0.1) / 0.4(0.1) / 0.3(0.1)	2.1(1.0) / 2.1(1.0) / 2.1(1.0)	4.0(0.6) / 4.0(0.6) / 4.2(0.6)	4.3(0.8) / 4.3(0.8) / 4.3(0.8)	4.4(0.6) / 4.3(0.6) / 4.6(0.6)
Domestic Engineering	1278 / 1093 / 185	0.5(0.2) / 0.5(0.2) / 0.4(0.2)	2.4(1.0) / 2.4(1.0) / 2.4(1.0)	4.0(0.5) / 4.0(0.5) / 4.3(0.5)	4.4(0.7) / 4.4(0.7) / 4.5(0.7)	4.4(0.6) / 4.3(0.6) / 4.7(0.6)
Domestic Science	952 / 795 / 157	0.5(0.2) / 0.5(0.2) / 0.5(0.2)	2.3(1.0) / 2.3(1.0) / 2.5(1.0)	4.1(0.6) / 4.0(0.6) / 4.3(0.6)	4.4(0.8) / 4.4(0.8) / 4.5(0.8)	4.3(0.5) / 4.3(0.5) / 4.6(0.5)
Domestic Other College	777 / 607 / 170	0.4(0.2) / 0.4(0.2) / 0.4(0.2)	1.7(1.1) / 1.7(1.1) / 2.0(1.1)	4.0(0.4) / 3.9(0.4) / 4.1(0.4)	4.2(0.6) / 4.2(0.6) / 4.3(0.6)	4.3(0.4) / 4.2(0.4) / 4.6(0.4)
URM	399 / 335 / 64	0.4(0.1) / 0.4(0.1) / 0.4(0.1)	1.9(1.0) / 1.9(1.0) / 1.9(1.0)	4.0(0.5) / 3.9(0.5) / 4.2(0.5)	4.2(0.7) / 4.2(0.7) / 4.2(0.7)	4.3(0.5) / 4.3(0.5) / 4.5(0.5)
Non-URM	2608 / 2160 / 448	0.5(0.2) / 0.5(0.2) / 0.4(0.2)	2.2(1.1) / 2.2(1.1) / 2.4(1.1)	4.0(0.5) / 4.0(0.5) / 4.2(0.5)	4.4(0.7) / 4.4(0.7) / 4.5(0.7)	4.4(0.5) / 4.3(0.5) / 4.7(0.5)
Pell-Eligible	483 / 386 / 97	0.4(0.2) / 0.4(0.2) / 0.4(0.2)	2.0(1.1) / 1.9(1.1) / 2.2(1.1)	4.0(0.6) / 4.0(0.6) / 4.0(0.6)	4.3(0.7) / 4.3(0.7) / 4.4(0.7)	4.2(0.5) / 4.1(0.5) / 4.6(0.5)
Non-Pell-Eligible	2524 / 2109 / 415	0.5(0.2) / 0.5(0.2) / 0.4(0.2)	2.2(1.1) / 2.2(1.1) / 2.3(1.1)	4.0(0.5) / 4.0(0.5) / 4.3(0.5)	4.4(0.7) / 4.4(0.7) / 4.5(0.7)	4.4(0.5) / 4.3(0.5) / 4.7(0.5)

Table 15 Descriptive Statistics for Self-Determined Motivation Measures, Calculus I

Cacleulus I mean(standard deviation) All / Traditional / Reformed	AMOTIVATION	EXTERNAL	INTROJECTED	IDENTIFIED	INTEGRATED	INTRINSIC
All	2.5(0.9) / 2.5(0.9) / 2.6(0.9)	4.9(0.9) / 5.0(0.9) / 4.7(0.9)	3.6(1.0) / 3.7(1.0) / 3.5(1.0)	5.1(0.8) / 5.1(0.8) / 5.0(0.8)	5.1(0.8) / 5.1(0.8) / 5.1(0.8)	4.4(1.1) / 4.4(1.1) / 4.4(1.1)
Men	2.6(0.9) / 2.6(0.9) / 2.6(0.9)	4.9(0.9) / 5.0(0.9) / 4.7(0.9)	3.6(1.0) / 3.7(1.0) / 3.4(1.0)	5.1(0.8) / 5.1(0.8) / 5.1(0.8)	5.1(0.8) / 5.1(0.8) / 5.1(0.8)	4.4(1.0) / 4.4(1.0) / 4.5(1.0)
Women	2.4(0.8) / 2.4(0.8) / 2.5(0.8)	4.9(0.9) / 5.0(0.9) / 4.8(0.9)	3.6(1.0) / 3.6(1.0) / 3.6(1.0)	5.1(0.8) / 5.1(0.8) / 4.9(0.8)	5.2(0.7) / 5.2(0.7) / 5.0(0.7)	4.3(1.0) / 4.4(1.0) / 4.1(1.0)
Domestic	2.4(0.9) / 2.4(0.9) / 2.5(0.9)	5.0(0.9) / 5.0(0.9) / 4.8(0.9)	3.5(0.9) / 3.5(0.9) / 3.4(0.9)	5.0(0.8) / 5.0(0.8) / 4.9(0.8)	5.0(0.7) / 5.0(0.7) / 5.1(0.7)	4.1(1.0) / 4.1(1.0) / 4.2(1.0)
International	2.9(1.0) / 2.9(1.0) / 2.7(1.0)	4.8(0.9) / 4.9(0.9) / 4.6(0.9)	4.0(1.1) / 4.1(1.1) / 3.5(1.1)	5.4(0.9) / 5.5(0.9) / 5.3(0.9)	5.4(0.9) / 5.5(0.9) / 5.2(0.9)	5.1(1.0) / 5.2(1.0) / 5.0(1.0)
Engineering	2.4(0.9) / 2.4(0.9) / 2.5(0.9)	5.0(1.0) / 5.0(1.0) / 4.7(1.0)	3.6(0.9) / 3.6(0.9) / 3.5(0.9)	5.1(0.9) / 5.2(0.9) / 5.0(0.9)	5.1(0.8) / 5.1(0.8) / 5.1(0.8)	4.3(1.0) / 4.3(1.0) / 4.3(1.0)
Science	2.5(1.0) / 2.5(1.0) / 2.5(1.0)	4.9(1.0) / 4.9(1.0) / 4.7(1.0)	3.6(1.1) / 3.7(1.1) / 3.4(1.1)	5.1(0.9) / 5.1(0.9) / 5.1(0.9)	5.1(0.9) / 5.1(0.9) / 5.1(0.9)	4.5(1.1) / 4.5(1.1) / 4.5(1.1)
Other College	2.8(0.8) / 2.8(0.8) / 2.7(0.8)	4.9(0.8) / 5.0(0.8) / 4.7(0.8)	3.7(0.9) / 3.8(0.9) / 3.5(0.9)	5.1(0.7) / 5.1(0.7) / 5.0(0.7)	5.1(0.7) / 5.1(0.7) / 5.1(0.7)	4.4(1.0) / 4.4(1.0) / 4.4(1.0)
Domestic Men	2.5(0.9) / 2.5(0.9) / 2.5(0.9)	5.0(0.9) / 5.0(0.9) / 4.7(0.9)	3.5(0.9) / 3.5(0.9) / 3.4(0.9)	5.0(0.8) / 5.0(0.8) / 5.0(0.8)	5.0(0.7) / 5.0(0.7) / 5.1(0.7)	4.2(1.0) / 4.1(1.0) / 4.3(1.0)
Domestic Women	2.4(0.8) / 2.3(0.8) / 2.4(0.8)	5.0(0.9) / 5.0(0.9) / 4.9(0.9)	3.4(0.9) / 3.4(0.9) / 3.5(0.9)	5.0(0.8) / 5.0(0.8) / 4.8(0.8)	5.1(0.7) / 5.1(0.7) / 4.9(0.7)	4.1(1.0) / 4.1(1.0) / 3.9(1.0)
Domestic Engineering	2.3(0.9) / 2.3(0.9) / 2.5(0.9)	5.0(0.9) / 5.0(0.9) / 4.8(0.9)	3.5(0.9) / 3.5(0.9) / 3.5(0.9)	5.1(0.8) / 5.1(0.8) / 5.0(0.8)	5.1(0.7) / 5.1(0.7) / 5.1(0.7)	4.2(0.9) / 4.2(0.9) / 4.2(0.9)
Domestic Science	2.4(0.9) / 2.4(0.9) / 2.4(0.9)	4.9(1.0) / 4.9(1.0) / 4.7(1.0)	3.5(1.0) / 3.5(1.0) / 3.4(1.0)	5.0(1.0) / 5.0(1.0) / 4.9(1.0)	5.0(0.9) / 5.0(0.9) / 5.1(0.9)	4.2(1.1) / 4.2(1.1) / 4.2(1.1)
Domestic Other College	2.6(0.8) / 2.6(0.8) / 2.6(0.8)	5.0(0.7) / 5.0(0.7) / 4.8(0.7)	3.5(0.9) / 3.5(0.9) / 3.5(0.9)	4.9(0.6) / 4.9(0.6) / 4.9(0.6)	5.0(0.6) / 5.0(0.6) / 5.1(0.6)	4.0(0.9) / 4.0(0.9) / 4.1(0.9)
URM	2.7(1.0) / 2.7(1.0) / 2.7(1.0)	5.0(0.9) / 5.1(0.9) / 4.4(0.9)	3.6(1.0) / 3.6(1.0) / 3.4(1.0)	5.0(0.9) / 5.1(0.9) / 4.7(0.9)	5.1(0.7) / 5.2(0.7) / 4.8(0.7)	4.2(1.1) / 4.2(1.1) / 4.1(1.1)
Non-URM	2.4(0.8) / 2.4(0.8) / 2.5(0.8)	4.9(0.9) / 5.0(0.9) / 4.8(0.9)	3.5(0.9) / 3.5(0.9) / 3.5(0.9)	5.0(0.8) / 5.0(0.8) / 5.0(0.8)	5.0(0.7) / 5.0(0.7) / 5.1(0.7)	4.1(1.0) / 4.1(1.0) / 4.2(1.0)
Pell-Eligible	2.5(0.6) / 2.5(0.6) / 2.2(0.6)	4.9(0.9) / 5.0(0.9) / 4.4(0.9)	3.3(0.8) / 3.3(0.8) / 3.1(0.8)	5.0(0.8) / 5.0(0.8) / 5.0(0.8)	5.1(0.6) / 5.1(0.6) / 5.2(0.6)	4.2(1.0) / 4.2(1.0) / 4.2(1.0)
Non-Pell-Eligible	2.4(0.9) / 2.4(0.9) / 2.6(0.9)	5.0(0.8) / 5.0(0.8) / 4.9(0.8)	3.5(0.9) / 3.5(0.9) / 3.5(0.9)	5.0(0.8) / 5.0(0.8) / 4.9(0.8)	5.0(0.8) / 5.0(0.8) / 5.0(0.8)	4.1(1.0) / 4.1(1.0) / 4.2(1.0)

Table 16 Descriptive Statistics for BPN and Academic Measures, Calculus II

Cacldculus II mean(standard deviation) All / Traditional / Reformed	N	KNOWLEDGE	GPA	AUTONOMY	COMPETENCE	RELATEDNESS
All	2895 / 2311 / 584	0.5(0.2) / 0.5(0.2) / 0.6(0.2)	2.4(1.1) / 2.4(1.1) / 2.6(1.1)	4.1(0.7) / 4.0(0.7) / 4.2(0.7)	4.3(0.8) / 4.3(0.8) / 4.4(0.8)	4.4(0.6) / 4.3(0.6) / 4.7(0.6)
Men	2100 / 1682 / 418	0.5(0.2) / 0.5(0.2) / 0.6(0.2)	2.4(1.1) / 2.3(1.1) / 2.6(1.1)	4.0(0.7) / 4.0(0.7) / 4.2(0.7)	4.3(0.8) / 4.3(0.8) / 4.5(0.8)	4.4(0.6) / 4.3(0.6) / 4.6(0.6)
Women	795 / 629 / 166	0.6(0.2) / 0.5(0.2) / 0.6(0.2)	2.4(1.1) / 2.4(1.1) / 2.6(1.1)	4.1(0.7) / 4.0(0.7) / 4.3(0.7)	4.2(0.9) / 4.2(0.9) / 4.4(0.9)	4.4(0.6) / 4.4(0.6) / 4.8(0.6)
Domestic	2080 / 1672 / 408	0.5(0.2) / 0.5(0.2) / 0.6(0.2)	2.2(1.1) / 2.1(1.1) / 2.5(1.1)	3.9(0.7) / 3.9(0.7) / 4.1(0.7)	4.2(0.9) / 4.2(0.9) / 4.4(0.9)	4.3(0.6) / 4.2(0.6) / 4.7(0.6)
International	815 / 639 / 176	0.6(0.2) / 0.6(0.2) / 0.6(0.2)	2.9(1.1) / 3.0(1.1) / 2.9(1.1)	4.3(0.6) / 4.3(0.6) / 4.5(0.6)	4.5(0.8) / 4.5(0.8) / 4.6(0.8)	4.5(0.6) / 4.5(0.6) / 4.7(0.6)
Engineering	1530 / 1252 / 278	0.5(0.2) / 0.5(0.2) / 0.6(0.2)	2.4(1.1) / 2.4(1.1) / 2.6(1.1)	4.0(0.7) / 4.0(0.7) / 4.2(0.7)	4.3(0.8) / 4.2(0.8) / 4.4(0.8)	4.4(0.6) / 4.3(0.6) / 4.7(0.6)
Science	935 / 735 / 200	0.6(0.2) / 0.6(0.2) / 0.6(0.2)	2.5(1.1) / 2.4(1.1) / 2.7(1.1)	4.1(0.7) / 4.1(0.7) / 4.2(0.7)	4.3(0.9) / 4.3(0.9) / 4.5(0.9)	4.4(0.7) / 4.3(0.7) / 4.7(0.7)
Other College	430 / 324 / 106	0.5(0.2) / 0.5(0.2) / 0.6(0.2)	2.2(1.1) / 2.1(1.1) / 2.3(1.1)	4.1(0.6) / 4.0(0.6) / 4.3(0.6)	4.2(0.8) / 4.1(0.8) / 4.4(0.8)	4.3(0.6) / 4.2(0.6) / 4.6(0.6)
Domestic Men	1512 / 1226 / 286	0.5(0.2) / 0.5(0.2) / 0.6(0.2)	2.2(1.1) / 2.1(1.1) / 2.5(1.1)	3.9(0.7) / 3.9(0.7) / 4.1(0.7)	4.2(0.8) / 4.2(0.8) / 4.4(0.8)	4.3(0.6) / 4.2(0.6) / 4.6(0.6)
Domestic Women	568 / 446 / 122	0.5(0.2) / 0.5(0.2) / 0.5(0.2)	2.2(1.0) / 2.1(1.0) / 2.5(1.0)	3.9(0.7) / 3.9(0.7) / 4.2(0.7)	4.1(0.9) / 4.1(0.9) / 4.4(0.9)	4.4(0.7) / 4.3(0.7) / 4.8(0.7)
Domestic Engineering	1170 / 965 / 205	0.5(0.2) / 0.5(0.2) / 0.6(0.2)	2.2(1.1) / 2.2(1.1) / 2.5(1.1)	3.9(0.6) / 3.9(0.6) / 4.1(0.6)	4.2(0.8) / 4.2(0.8) / 4.3(0.8)	4.4(0.6) / 4.3(0.6) / 4.7(0.6)
Domestic Science	640 / 505 / 135	0.5(0.2) / 0.5(0.2) / 0.6(0.2)	2.3(1.1) / 2.2(1.1) / 2.6(1.1)	4.0(0.8) / 3.9(0.8) / 4.1(0.8)	4.2(0.9) / 4.2(0.9) / 4.4(0.9)	4.3(0.7) / 4.2(0.7) / 4.7(0.7)
Domestic Other College	270 / 202 / 68	0.5(0.2) / 0.5(0.2) / 0.5(0.2)	1.9(1.1) / 1.8(1.1) / 2.2(1.1)	3.9(0.6) / 3.8(0.6) / 4.2(0.6)	4.1(0.9) / 4.0(0.9) / 4.4(0.9)	4.3(0.6) / 4.1(0.6) / 4.7(0.6)
URM	240 / 200 / 40	0.5(0.2) / 0.5(0.2) / 0.5(0.2)	1.9(1.1) / 1.9(1.1) / 1.8(1.1)	3.9(0.5) / 3.9(0.5) / 3.9(0.5)	4.2(0.8) / 4.2(0.8) / 4.2(0.8)	4.3(0.7) / 4.3(0.7) / 4.4(0.7)
Non-URM	1840 / 1472 / 368	0.5(0.2) / 0.5(0.2) / 0.6(0.2)	2.2(1.0) / 2.2(1.0) / 2.5(1.0)	3.9(0.7) / 3.9(0.7) / 4.2(0.7)	4.2(0.9) / 4.2(0.9) / 4.4(0.9)	4.3(0.6) / 4.2(0.6) / 4.7(0.6)
Pell-Eligible	323 / 257 / 66	0.5(0.2) / 0.5(0.2) / 0.5(0.2)	2.1(1.1) / 2.0(1.1) / 2.2(1.1)	4.0(0.7) / 4.0(0.7) / 4.0(0.7)	4.2(1.0) / 4.2(1.0) / 4.2(1.0)	4.3(0.7) / 4.2(0.7) / 4.6(0.7)
Non-Pell-Eligible	1757 / 1415 / 342	0.5(0.2) / 0.5(0.2) / 0.6(0.2)	2.2(1.1) / 2.1(1.1) / 2.5(1.1)	3.9(0.7) / 3.9(0.7) / 4.2(0.7)	4.2(0.8) / 4.2(0.8) / 4.4(0.8)	4.3(0.6) / 4.3(0.6) / 4.7(0.6)

Table 17 Descriptive Statistics for Self-Determined Motivation Measures, Calculus II

Cacldculus II mean(standard deviation) All / Traditional / Reformed	AMOTIVATION	EXTERNAL	INTROJECTED	IDENTIFIED	INTEGRATED	INTRINSIC
All	2.6(1.2) / 2.6(1.2) / 2.7(1.2)	5.0(1.1) / 5.1(1.1) / 4.9(1.1)	3.6(1.3) / 3.6(1.3) / 3.7(1.3)	4.8(1.1) / 4.8(1.1) / 4.9(1.1)	4.9(1.0) / 4.9(1.0) / 5.0(1.0)	4.1(1.3) / 4.0(1.3) / 4.2(1.3)
Men	2.7(1.2) / 2.7(1.2) / 2.7(1.2)	5.0(1.0) / 5.0(1.0) / 4.8(1.0)	3.6(1.3) / 3.6(1.3) / 3.7(1.3)	4.8(1.0) / 4.8(1.0) / 4.9(1.0)	4.9(1.0) / 4.9(1.0) / 5.1(1.0)	4.1(1.3) / 4.0(1.3) / 4.3(1.3)
Women	2.6(1.1) / 2.6(1.1) / 2.7(1.1)	5.2(1.1) / 5.3(1.1) / 5.1(1.1)	3.7(1.3) / 3.7(1.3) / 3.6(1.3)	4.9(1.1) / 4.9(1.1) / 4.9(1.1)	5.0(1.1) / 5.0(1.1) / 4.9(1.1)	4.1(1.4) / 4.1(1.4) / 4.0(1.4)
Domestic	2.6(1.0) / 2.6(1.0) / 2.5(1.0)	5.1(1.0) / 5.2(1.0) / 5.0(1.0)	3.5(1.2) / 3.5(1.2) / 3.5(1.2)	4.7(1.1) / 4.7(1.1) / 4.8(1.1)	4.8(1.0) / 4.8(1.0) / 4.9(1.0)	3.8(1.3) / 3.7(1.3) / 3.9(1.3)
International	2.9(1.3) / 2.8(1.3) / 3.1(1.3)	4.8(1.2) / 4.9(1.2) / 4.7(1.2)	3.9(1.3) / 3.9(1.3) / 4.1(1.3)	5.2(1.0) / 5.2(1.0) / 5.3(1.0)	5.3(1.0) / 5.2(1.0) / 5.3(1.0)	4.8(1.1) / 4.8(1.1) / 5.0(1.1)
Engineering	2.6(1.2) / 2.6(1.2) / 2.7(1.2)	5.1(1.1) / 5.1(1.1) / 4.9(1.1)	3.6(1.3) / 3.6(1.3) / 3.7(1.3)	4.8(1.1) / 4.8(1.1) / 4.9(1.1)	4.9(1.0) / 4.9(1.0) / 5.0(1.0)	4.0(1.3) / 4.0(1.3) / 4.2(1.3)
Science	2.6(1.1) / 2.6(1.1) / 2.7(1.1)	5.0(1.0) / 5.1(1.0) / 4.9(1.0)	3.6(1.3) / 3.6(1.3) / 3.7(1.3)	4.9(1.0) / 4.8(1.0) / 5.0(1.0)	5.0(1.0) / 4.9(1.0) / 5.0(1.0)	4.1(1.3) / 4.1(1.3) / 4.3(1.3)
Other College	2.8(1.2) / 2.8(1.2) / 2.8(1.2)	5.0(1.2) / 5.0(1.2) / 4.8(1.2)	3.6(1.3) / 3.7(1.3) / 3.5(1.3)	4.8(1.2) / 4.8(1.2) / 4.9(1.2)	5.0(1.1) / 4.9(1.1) / 5.0(1.1)	4.1(1.4) / 4.0(1.4) / 4.3(1.4)
Domestic Men	2.6(1.1) / 2.6(1.1) / 2.6(1.1)	5.1(1.0) / 5.1(1.0) / 4.9(1.0)	3.5(1.3) / 3.5(1.3) / 3.6(1.3)	4.7(1.0) / 4.7(1.0) / 4.8(1.0)	4.8(1.0) / 4.8(1.0) / 4.9(1.0)	3.8(1.2) / 3.7(1.2) / 4.0(1.2)
Domestic Women	2.4(0.9) / 2.4(0.9) / 2.4(0.9)	5.3(1.1) / 5.4(1.1) / 5.1(1.1)	3.5(1.2) / 3.5(1.2) / 3.3(1.2)	4.7(1.1) / 4.7(1.1) / 4.7(1.1)	4.8(1.1) / 4.9(1.1) / 4.7(1.1)	3.7(1.3) / 3.7(1.3) / 3.7(1.3)
Domestic Engineering	2.5(1.0) / 2.5(1.0) / 2.6(1.0)	5.2(1.1) / 5.2(1.1) / 5.0(1.1)	3.6(1.2) / 3.5(1.2) / 3.6(1.2)	4.7(1.0) / 4.7(1.0) / 4.7(1.0)	4.8(1.0) / 4.8(1.0) / 4.8(1.0)	3.8(1.2) / 3.8(1.2) / 3.9(1.2)
Domestic Science	2.5(1.1) / 2.6(1.1) / 2.5(1.1)	5.1(0.9) / 5.2(0.9) / 5.0(0.9)	3.4(1.3) / 3.4(1.3) / 3.5(1.3)	4.7(1.1) / 4.6(1.1) / 4.8(1.1)	4.8(1.1) / 4.8(1.1) / 4.9(1.1)	3.8(1.3) / 3.7(1.3) / 4.0(1.3)
Domestic Other College	2.6(0.9) / 2.7(0.9) / 2.5(0.9)	5.0(1.1) / 5.1(1.1) / 4.8(1.1)	3.4(1.1) / 3.4(1.1) / 3.2(1.1)	4.6(1.1) / 4.6(1.1) / 4.7(1.1)	4.8(1.1) / 4.8(1.1) / 4.8(1.1)	3.7(1.3) / 3.6(1.3) / 3.9(1.3)
URM	2.6(1.0) / 2.5(1.0) / 2.7(1.0)	5.3(0.9) / 5.4(0.9) / 4.9(0.9)	3.6(1.2) / 3.6(1.2) / 3.6(1.2)	4.9(0.8) / 4.9(0.8) / 5.0(0.8)	5.1(0.8) / 5.1(0.8) / 5.2(0.8)	4.1(1.2) / 4.1(1.2) / 4.2(1.2)
Non-URM	2.6(1.0) / 2.6(1.0) / 2.5(1.0)	5.1(1.0) / 5.1(1.0) / 5.0(1.0)	3.5(1.2) / 3.5(1.2) / 3.5(1.2)	4.7(1.1) / 4.7(1.1) / 4.7(1.1)	4.8(1.1) / 4.8(1.1) / 4.8(1.1)	3.7(1.3) / 3.7(1.3) / 3.9(1.3)
Pell-Eligible	2.5(1.2) / 2.5(1.2) / 2.5(1.2)	5.2(1.1) / 5.2(1.1) / 5.1(1.1)	3.4(1.3) / 3.4(1.3) / 3.4(1.3)	4.7(1.2) / 4.8(1.2) / 4.7(1.2)	5.0(1.1) / 5.0(1.1) / 5.0(1.1)	3.8(1.3) / 3.8(1.3) / 3.6(1.3)
Non-Pell-Eligible	2.6(1.0) / 2.6(1.0) / 2.5(1.0)	5.1(1.0) / 5.2(1.0) / 4.9(1.0)	3.5(1.2) / 3.5(1.2) / 3.5(1.2)	4.7(1.0) / 4.7(1.0) / 4.8(1.0)	4.8(1.0) / 4.8(1.0) / 4.8(1.0)	3.8(1.3) / 3.7(1.3) / 4.0(1.3)

4.1 Grade Outcomes and Demographics across Conditions

Table 18 and

Table 19 present the demographic and academic performance breakdowns of the grade sample in Calculus I and Calculus II. The columns allow filtering for a particular subpopulation. The row provides breakdown relative to that subpopulation. For instance, the column 'of URM' with the row 'Engineering' indicates that 210 (52.6%) of the students from URM were engineering majors. Alternatively, the column 'of Engineering' with the row 'URM' indicates that 210 (8.6%) of the engineering majors were students from URM groups.

Calculus I had an overall pass rate (ABC) of 79.6% (81.2% in the reformed version and 79.3% in the tradition version). Calculus II had an overall pass rate of 80.3% (84.8% in the reformed version and 79.2% in the tradition version). Note that international students passed at a substantially higher rate (around 15 percentage points) in both Calculus I and Calculus II. Other general achievement discrepancies across populations of interest are present and will be discussed throughout the results. Baseline equivalence across conditions was evaluated with the pre-knowledge test. The overall effect size (Cohen's d) of condition on the pre-knowledge test was -0.082 in Calculus I and 0.177 in Calculus II. Among subpopulations of interest (international, domestic, women, Pell-eligible, and under-represented minorities), the absolute value of effect size of condition on the pre-knowledge test varied between 0.024 and 0.244. These values meet the criteria for baseline equivalence (though requiring statistical adjustments) established by the Institute of Education Science (What Works Clearinghouse, 2020, p.13). The reliability of the pre-knowledge tests were examine by computing Cronbach's alpha yielding values of 0.611 and 0.646 for Calculus I and Calculus II, respectively. The low reliability values may be due, in part, to multiple concepts being assessed and variation in the discrimination level of items. Gleason et al

Table 18: Demographics of Calculus I Grade Sample

GS Calc I N = 3971	of Total	of REFORMED	of TRADITIONAL	of ABC	of DF	of Engineering	of Science	of Other	of Domestic	of International	of Men	of Women	of Pell	of Non-Pell	of URM	of non-URM
REFORMED	687	687	0	558	129	220	219	248	512	175	493	192	99	588	64	623
	17.3%	100%	0%	17.6%	15.9%	14.6%	16.4%	22%	17%	18.2%	17.4%	16.9%	20.4%	16.9%	16%	17.4%
TRADITIONAL	3284	0	3284	2604	680	1290	1113	881	2495	789	2340	942	386	2898	335	2949
	82.7%	0%	100%	82.4%	84.1%	85.4%	83.6%	78%	83%	81.8%	82.6%	83.1%	79.6%	83.1%	84%	82.6%
ABC	3162	558	2604	3162	0	1273	1103	786	2281	881	2270	888	334	2828	271	2891
	79.6%	81.2%	79.3%	100%	0%	84.3%	82.8%	69.6%	75.9%	91.4%	80.1%	78.3%	68.9%	81.1%	67.9%	80.9%
DF	809	129	680	0	809	237	229	343	726	83	563	246	151	658	128	681
	20.4%	18.8%	20.7%	0%	100%	15.7%	17.2%	30.4%	24.1%	8.6%	19.9%	21.7%	31.1%	18.9%	32.1%	19.1%
Engineering	1510	220	1290	1273	237	1510	0	0	1278	232	1053	457	155	1355	210	1300
	38%	32%	39.3%	40.3%	29.3%	100%	0%	0%	42.5%	24.1%	37.2%	40.3%	32%	38.9%	52.6%	36.4%
Science	1332	219	1113	1103	229	0	1332	0	952	380	924	407	169	1163	76	1256
	33.5%	31.9%	33.9%	34.9%	28.3%	0%	100%	0%	31.7%	39.4%	32.6%	35.9%	34.8%	33.4%	19%	35.2%
Other	1129	248	881	786	343	0	0	1129	777	352	856	270	161	968	113	1016
	28.4%	36.1%	26.8%	24.9%	42.4%	0%	0%	100%	25.8%	36.5%	30.2%	23.8%	33.2%	27.8%	28.3%	28.4%
Domestic	3007	512	2495	2281	726	1278	952	777	3007	0	2123	880	483	2524	399	2608
	75.7%	74.5%	76%	72.1%	89.7%	84.6%	71.5%	68.8%	100%	0%	74.9%	77.6%	99.6%	72.4%	100%	73%
International	964	175	789	881	83	232	380	352	0	964	710	254	2	962	0	964
	24.3%	25.5%	24%	27.9%	10.3%	15.4%	28.5%	31.2%	0%	100%	25.1%	22.4%	0.4%	27.6%	0%	27%
Men	2833	493	2340	2270	563	1053	924	856	2123	710	2833	0	334	2499	287	2546
	71.3%	71.8%	71.3%	71.8%	69.6%	69.7%	69.4%	75.8%	70.6%	73.7%	100%	0%	68.9%	71.7%	71.9%	71.3%
Women	1134	192	942	888	246	457	407	270	880	254	0	1134	150	984	112	1022
	28.6%	27.9%	28.7%	28.1%	30.4%	30.3%	30.6%	23.9%	29.3%	26.3%	0%	100%	30.9%	28.2%	28.1%	28.6%
Pell	485	99	386	334	151	155	169	161	483	2	334	150	485	0	118	367
	12.2%	14.4%	11.8%	10.6%	18.7%	10.3%	12.7%	14.3%	16.1%	0.2%	11.8%	13.2%	100%	0%	29.6%	10.3%
Non-Pell	3486	588	2898	2828	658	1355	1163	968	2524	962	2499	984	0	3486	281	3205
	87.8%	85.6%	88.2%	89.4%	81.3%	89.7%	87.3%	85.7%	83.9%	99.8%	88.2%	86.8%	0%	100%	70.4%	89.7%
URM	399	64	335	271	128	210	76	113	399	0	287	112	118	281	399	0
	10%	9.3%	10.2%	8.6%	15.8%	13.9%	5.7%	10%	13.3%	0%	10.1%	9.9%	24.3%	8.1%	100%	0%
non-URM	3572	623	2949	2891	681	1300	1256	1016	2608	964	2546	1022	367	3205	0	3572
	90%	90.7%	89.8%	91.4%	84.2%	86.1%	94.3%	90%	86.7%	100%	89.9%	90.1%	75.7%	91.9%	0%	100%

Table 19: Demographics of Calculus II Grade Sample

GS Calc II N = 2895	of Total	of REFORMED	of TRADITIONAL	of ABC	of DF	of Engineering	of Science	of Other	of Domestic	of International	of Men	of Women	of Pell	of Non-Pell	of URM	of non-URM
REFORMED	584 20.2%	584 100%	0 0%	495 21.3%	89 15.6%	278 18.2%	200 21.4%	106 24.7%	408 19.6%	176 21.6%	416 19.8%	166 20.9%	67 20.7%	517 20.1%	40 16.7%	544 20.5%
TRADITIONAL	2311 79.8%	0 0%	2311 100%	1831 78.7%	480 84.4%	1252 81.8%	735 78.6%	324 75.3%	1672 80.4%	639 78.4%	1680 80.2%	629 79.1%	257 79.3%	2054 79.9%	200 83.3%	2111 79.5%
ABC	2326 80.3%	495 84.8%	1831 79.2%	2326 100%	0 0%	1244 81.3%	763 81.6%	319 74.2%	1589 76.4%	737 90.4%	1677 80%	645 81.1%	236 72.8%	2090 81.3%	158 65.8%	2168 81.7%
DF	569 19.7%	89 15.2%	480 20.8%	0 0%	569 100%	286 18.7%	172 18.4%	111 25.8%	491 23.6%	78 9.6%	419 20%	150 18.9%	88 27.2%	481 18.7%	82 34.2%	487 18.3%
Engineering	1530 52.8%	278 47.6%	1252 54.2%	1244 53.5%	286 50.3%	1530 100%	0 0%	0 0%	1170 56.3%	360 44.2%	1114 53.1%	416 52.3%	160 49.4%	1370 53.3%	153 63.8%	1377 51.9%
Science	935 32.3%	200 34.2%	735 31.8%	763 32.8%	172 30.2%	0 0%	935 100%	0 0%	640 30.8%	295 36.2%	663 31.6%	270 34%	109 33.6%	826 32.1%	51 21.3%	884 33.3%
Other	430 14.9%	106 18.2%	324 14%	319 13.7%	111 19.5%	0 0%	0 0%	430 100%	270 13%	160 19.6%	319 15.2%	109 13.7%	55 17%	375 14.6%	36 15%	394 14.8%
Domestic	2080 71.8%	408 69.9%	1672 72.3%	1589 68.3%	491 86.3%	1170 76.5%	640 68.4%	270 62.8%	2080 100%	0 0%	1508 71.9%	568 71.4%	323 99.7%	1757 68.3%	240 100%	1840 69.3%
International	815 28.2%	176 30.1%	639 27.7%	737 31.7%	78 13.7%	360 23.5%	295 31.6%	160 37.2%	0 0%	815 100%	588 28.1%	227 28.6%	1 0.3%	814 31.7%	0 0%	815 30.7%
Men	2096 72.4%	416 71.2%	1680 72.7%	1677 72.1%	419 73.6%	1114 72.8%	663 70.9%	319 74.2%	1508 72.5%	588 72.1%	2096 100%	0 0%	231 71.3%	1865 72.5%	168 70%	1928 72.6%
Women	795 27.5%	166 28.4%	629 27.2%	645 27.7%	150 26.4%	416 27.2%	270 28.9%	109 25.3%	568 27.3%	227 27.9%	0 0%	795 100%	92 28.4%	703 27.3%	72 30%	723 27.2%
Pell	324 11.2%	67 11.5%	257 11.1%	236 10.1%	88 15.5%	160 10.5%	109 11.7%	55 12.8%	323 15.5%	1 0.1%	231 11%	92 11.6%	324 100%	0 0%	78 32.5%	246 9.3%
Non-Pell	2571 88.8%	517 88.5%	2054 88.9%	2090 89.9%	481 84.5%	1370 89.5%	826 88.3%	375 87.2%	1757 84.5%	814 99.9%	1865 89%	703 88.4%	0 0%	2571 100%	162 67.5%	2409 90.7%
URM	240 8.3%	40 6.8%	200 8.7%	158 6.8%	82 14.4%	153 10%	51 5.5%	36 8.4%	240 11.5%	0 0%	168 8%	72 9.1%	78 24.1%	162 6.3%	240 100%	0 0%
non-URM	2655 91.7%	544 93.2%	2111 91.3%	2168 93.2%	487 85.6%	1377 90%	884 94.5%	394 91.6%	1840 88.5%	815 100%	1928 92%	723 90.9%	246 75.9%	2409 93.7%	0 0%	2655 100%

(2019) have expressed concerns with the Calculus Concept Inventory (of which a subset was used for the Calculus I pre-knowledge test in this study) and do not recommend its continued use in educational research; however, in this research, the CCI performed equally well, or, rather, equally poorly, as the traditional measures of mathematics ability, specifically ACT and SAT mathematics scores, in explaining variance in course GPA within this data. ACT and SAT mathematics scores correlated moderately ($r \approx 0.40$) with the Calculus I pre-knowledge test. The reliability of the knowledge tests will be addressed in the context of other results and in the limitations of the study.

Table 20 shows the levels and significance of differences in GPA (A+ = 4.0, A = 4.0, A-=3.7, B+ = 3.5, etc) of specific subpopulations in Calculus I and Calculus II across conditions. Without controlling for knowledge (computed as percent correct on each course-based pre-knowledge test), R^2 values are trivial; nonetheless, these tables are included to show mean values and variation in GPA for each condition across subpopulations of interest (where INT is the average grade for the traditional condition and REFORMED is the difference between the reformed condition and the traditional condition).

Table 21 shows the same breakdown controlling for knowledge. Observe that, controlling for knowledge, the reformed condition improves overall GPA by 0.13 and 0.15 (ie, approximately one eighth of a letter grade) in Calculus I and Calculus II, respectively; however, the gains were not uniformly realized by all demographic groups. Neither international students, women, nor underrepresented minorities saw statistically significant benefit from the reformed condition in either course when examined individually. In Calculus I, Pell-eligible students saw large gains (more than one third of a letter grade). Non-Pell-eligible, domestic, white men benefitted as well, but to half the extent of Pell-eligible students. On average, domestic students attained a 0.16 higher GPA in the reformed condition than in the traditional condition. In Calculus II, effects of the reformed condition were more pronounced. Domestic students averaged a quarter of a letter grade higher in the reformed condition than in the traditional condition; interestingly, in this course, Pell-eligible students saw no benefit. Non-Pell-eligible, domestic, white men (who constituted nearly half of the domestic population and a third of the entire population) in the reformed condition saw an increase nearly one third of a letter grade over their counterparts in the traditional condition.

Table 20: GPA by Condition and Demographic Groups

Calc I GPA	**TOTAL (REFORMED)**	**INT**	**REFORMED**	**R^2**	**pval**
All	3971 (687)	2.43***	0.09^	0.001	0.0778
International	964 (175)	3.11***	-0.09	0.001	0.3180
Domestic	3007 (512)	2.22***	0.13*	0.002	0.0191
Pell	485 (99)	1.97***	0.35**	0.014	0.0092
URM	399 (64)	1.97***	0.01	0	0.9534
Women	1134 (192)	2.41***	0.01	0	0.9032
Non-Pell, Domestic, White, Men	1288 (210)	2.26***	0.17*	0.003	0.0460
Calc II GPA	**Total (REFORMED)**	**INT**	**REFORMED**	**R^2**	**pval**
All	2895 (584)	2.41***	0.24***	0.007	< 0.0001
International	815 (176)	3.00***	-0.07	0.001	0.4568
Domestic	2080 (408)	2.18***	0.34***	0.014	< 0.0001

Pell	324 (67)	2.09***	0.18	0.003	0.2932
URM	240 (40)	1.95***	-0.13	0.002	0.5271
Women	795 (166)	2.46***	0.20*	0.005	0.0491
Non-Pell, Domestic, White Men	942 (164)	2.24***	0.39***	0.019	< 0.0001

Table 21: GPA by Condition and Demographic Groups Controlling for Prior Knowledge

Calc I GPA	**Total (REFORMED)**	**INT**	**KNOW**	**REFORMED**	**R^2**	**pval**
All	3971 (687)	1.39***	2.03***	0.13**	0.152	< 0.0001
International	964 (175)	2.21***	1.69***	-0.05	0.132	< 0.0001
Domestic	3007 (512)	1.19***	2.04***	0.16**	0.164	< 0.0001
Pell	485 (99)	1.08***	1.91***	0.38**	0.137	< 0.0001
URM	399 (64)	0.87***	2.35***	0.13	0.185	< 0.0001
Women	1134 (192)	1.41***	2.22***	0.083	0.149	< 0.0001
Non-Pell, Domestic, White Men	1288 (210)	1.16***	2.02***	0.18*	0.175	< 0.0001
Calc II GPA	**Total (REFORMED)**	**INT**	**KNOW**	**REFORMED**	**R^2**	**pval**
All	2895 (584)	1.33***	1.83***	0.15**	0.183	< 0.0001
International	815 (176)	1.91***	1.61***	-0.11	0.175	< 0.0001
Domestic	2080 (408)	1.26***	1.66***	0.25***	0.156	< 0.0001
Pell	324 (67)	0.99***	2.04***	0.08	0.202	< 0.0001

URM	240 (40)	1.01***	1.64***	-0.21	0.139	< 0.0001
Women	795 (166)	1.35***	1.86***	0.12	0.176	< 0.0001
Non-Pell, Domestic, White Men	942 (164)	1.40***	1.49***	0.31***	0.142	< 0.0001

Table 22 shows the results of a logistic regression to estimate the likelihood of passing (C or better) each of Calculus I and Calculus II for each condition and population of interest. Despite overall improvement in GPA in the reformed condition in Calculus I, only a slight effect

on the likelihood to pass was found. Notably, Pell-eligible Calculus I students in the reformed condition were nearly 1.6 times more likely to pass than their peers in the traditional condition. Pell-eligible Calculus II students in the reformed condition were found to be nearly 1.4 (ns) times more likely to pass than their peers in the traditional condition. The largest effect of the reformed condition was for non-Pell-eligible, domestic, white men who were twice as likely to pass Calculus II as their counterparts in the traditional condition. URM students were three quarters as likely (ns) to pass Calculus II in the reformed condition as opposed to in the traditional condition.

Table 22: Likelihood to Pass by Condition and Demographic Groups

Calc I PASS/FAIL	**Total (REFORMED)**	**INT**	**KNOW**	**Likelihood to Pass (REFORMED)**	**McFadden's Pseudo- R^2**
All	3971 (687)	-0.25*	3.436***	1.21^	0.081
International	964 (175)	0.80**	3.43***	0.90	0.072
Domestic	3007 (512)	-0.47***	3.44***	1.25^	0.083
Pell	485 (99)	-0.77**	3.388***	1.58^	0.083
URM	399 (64)	-0.91**	3.77***	1.23	0.086
Women	1134 (192)	-0.14	3.469***	1.14	0.068
Non-Pell, Domestic, White, Men	1288 (210)	-0.45**	3.33***	1.37	0.085
Calc II PASS/FAIL	**Total (REFORMED)**	**INT**	**KNOW**	**Likelihood to Pass (REFORMED)**	**McFadden's Pseudo- R^2**
All	2895 (584)	-0.14	2.78***	1.32*	0.086
International	815 (176)	0.38	3.35***	0.73	0.126
Domestic	2080 (408)	-0.19	2.47***	1.50**	0.069
Pell	324 (67)	-0.42	2.67***	1.38	0.088
URM	240 (40)	-0.38	2.15***	0.75	0.052
Women	795 (166)	-0.17	2.94***	1.13	0.086
Non-Pell, Domestic, White Men	942 (164)	0.09	2.14***	2.08**	0.058

Table 23 shows the results of a linear model, for each of Calculus I and Calculus II, to predict GPA for domestic students with demographic and condition interaction terms in order to better understand the effect of the reformed condition on subpopulations of interest. Here, we identify an overall performance gap, controlling for knowledge, between URM and non-URM students in both courses and conditions. Women, on average, outperformed men in Calculus I in the traditional condition with the reverse true in reformed condition. There were no gender effects in Calculus II in either condition. Pell-eligible students in the Calculus I traditional condition underperformed by 0.18 grade points compared to non-Pell-eligible peers, however, the Pell-eligible Calculus I students in the reformed condition saw the general benefit of 0.19 grade points as well as a specific benefit of 0.23 grade points which reversed the achievement gap. However, no such effect was found for Pell-eligible Calculus II students. Additionally, URM students in Calculus II saw a detrimental half letter drop in the reformed condition compared to non-URM peers which dramatically increases the achievement gap due to the benefit of the reformed condition for non-URM populations. The models for both courses explained a relatively small, but impactful, amount of the variance (17%) in GPA.

Table 23: Overall Effects of the Reformed Condition on GPA across Domestic Demographics

Domestic Students	**Calc I – GPA (N=3007)**	**Calc I – Likelihood to Pass (N=3007)**	**Calc II – GPA (N=2080)**	**Calc II – Likelihood to Pass (N=2080)**
INTERCEPT	1.20***	0.64***	1.29***	0.88
KNOWLEDGE	2.04***	31.61***	1.64***	11.52***
URM	-0.16**	0.75*	-0.19*	0.69*
FEMALE	0.14**	1.22^	0.02	1.07
PELL	-0.18**	0.71**	-0.04	0.9
REFORMED	0.19**	1.31	0.32***	1.83**
URM x REFORMED	-0.10	0.92	-0.48*	0.43*
FEMALE x REFORMED	-0.18	0.82	0.00	0.82
PELL x REFORMED	0.23^	1.24	-0.19	0.89
R^2	0.1729	0.08 (McFadden’s)	0.1660	0.08 (McFadden’s)
pval	< 0.0001	< 0.0001	< 0.0001	< 0.0001

4.2 Motivational Outcomes and Demographics across Conditions

The survey sample is a subsample of the grade sample and contains a total of 3,294 participants who electronically completed both the Basic Psychological Needs Scale (Levesque-Bristol et al, 2010) and Situational Motivation Scale (Guay et al., 2000) at the end of the semester on a 7-point Likert scale (1-strongly disagree, 2-disagree, 3-somewhat disagree, 4-neither agree nor disagree, 5-somewhat agree, 6-agree, 7-strongly agree). For Calculus I, the survey sample had a higher pass rate of 82.9% (81.4% in the reformed version versus 83.2% in the traditional version) than the grade sample which had a pass rate of 79.6% (81.2% in the reformed version versus 79.3% in the tradition version). Note the traditional version outperformed the reformed version in the survey sample by 1.8% points while the reformed version outperformed the traditional version by 1.9% points in the larger grade sample. This underscores the importance of using imputed motivational measures for the grade sample (addressed in section 4.3) when conducting analysis on level differences in motivation between conditions due to the pervasive correlation between grade outcomes and motivation. For Calculus II, the survey sample had a higher pass rate of 85.5% (87.6% in the reformed version versus 84.9% in the traditional version) than the grade sample which had a pass rate of 80.3% (84.8% in the reformed version versus 79.2% in the tradition version). Despite differences in course outcomes, respondents and non-respondents had equivalent pre-knowledge scores in both courses (|Cohen's d| < 0.02). Women were overrepresented in the survey samples for both Calculus I and Calculus II by 5 percentage points and international students were overrepresented in the survey sample by 4 percentage points. Students from underrepresented minorities were slightly underrepresented in the survey sample. Other groups were equivalently represented in both samples. Table 24 and

Table 25 show breakdowns for the survey sample analogous to

Table 18 and

Table 19 for the grade sample.

Table 24: Demographics of Calculus I Survey Sample

SS Calc I N = 1720	of Total	of REFORMED	of TRADITIONAL	of ABC	of DF	of Engineering	of Science	of Other	of Domestic	of International	of Men	of Women	of Pell	of Non-Pell	of URM	of non-URM
REFORMED	285 16.6%	285 100%	0 0%	232 16.3%	53 18%	99 15%	111 17.7%	75 17.3%	202 16.4%	83 17%	190 16.7%	94 16.3%	35 16.5%	250 16.6%	25 16.9%	260 16.5%
TRADITIONAL	1435 83.4%	0 0%	1435 100%	1194 83.7%	241 82%	561 85%	515 82.3%	359 82.7%	1029 83.6%	406 83%	950 83.3%	483 83.7%	177 83.5%	1258 83.4%	123 83.1%	1312 83.5%
ABC	1426 82.9%	232 81.4%	1194 83.2%	1426 100%	0 0%	570 86.4%	532 85%	324 74.7%	968 78.6%	458 93.7%	963 84.5%	460 79.7%	160 75.5%	1266 84%	109 73.6%	1317 83.8%
DF	294 17.1%	53 18.6%	241 16.8%	0 0%	294 100%	90 13.6%	94 15%	110 25.3%	263 21.4%	31 6.3%	177 15.5%	117 20.3%	52 24.5%	242 16%	39 26.4%	255 16.2%
Engineering	660 38.4%	99 34.7%	561 39.1%	570 40%	90 30.6%	660 100%	0 0%	0 0%	540 43.9%	120 24.5%	438 38.4%	222 38.5%	75 35.4%	585 38.8%	68 45.9%	592 37.7%
Science	626 36.4%	111 38.9%	515 35.9%	532 37.3%	94 32%	0 0%	626 100%	0 0%	423 34.4%	203 41.5%	400 35.1%	225 39%	82 38.7%	544 36.1%	36 24.3%	590 37.5%
Other	434 25.2%	75 26.3%	359 25%	324 22.7%	110 37.4%	0 0%	0 0%	434 100%	268 21.8%	166 33.9%	302 26.5%	130 22.5%	55 25.9%	379 25.1%	44 29.7%	390 24.8%
Domestic	1231 71.6%	202 70.9%	1029 71.7%	968 67.9%	263 89.5%	540 81.8%	423 67.6%	268 61.8%	1231 100%	0 0%	795 69.7%	433 75%	212 100%	1019 67.6%	148 100%	1083 68.9%
International	489 28.4%	83 29.1%	406 28.3%	458 32.1%	31 10.5%	120 18.2%	203 32.4%	166 38.2%	0 0%	489 100%	345 30.3%	144 25%	0 0%	489 32.4%	0 0%	489 31.1%
Men	1140 66.3%	190 66.7%	950 66.2%	963 67.5%	177 60.2%	438 66.4%	400 63.9%	302 69.6%	795 64.6%	345 70.6%	1140 100%	0 0%	122 57.5%	1018 67.5%	91 61.5%	1049 66.7%
Women	577 33.5%	94 33%	483 33.7%	460 32.3%	117 39.8%	222 33.6%	225 35.9%	130 30%	433 35.2%	144 29.4%	0 0%	577 100%	89 42%	488 32.4%	57 38.5%	520 33.1%
Pell	212 12.3%	35 12.3%	177 12.3%	160 11.2%	52 17.7%	75 11.4%	82 13.1%	55 12.7%	212 17.2%	0 0%	122 10.7%	89 15.4%	212 100%	0 0%	52 35.1%	160 10.2%
Non-Pell	1508 87.7%	250 87.7%	1258 87.7%	1266 88.8%	242 82.3%	585 88.6%	544 86.9%	379 87.3%	1019 82.8%	489 100%	1018 89.3%	488 84.6%	0 0%	1508 100%	96 64.9%	1412 89.8%
URM	148 8.6%	25 8.8%	123 8.6%	109 7.6%	39 13.3%	68 10.3%	36 5.8%	44 10.1%	148 12%	0 0%	91 8%	57 9.9%	52 24.5%	96 6.4%	148 100%	0 0%
non-URM	1572 91.4%	260 91.2%	1312 91.4%	1317 92.4%	255 86.7%	592 89.7%	590 94.2%	390 89.9%	1083 88%	489 100%	1049 92%	520 90.1%	160 75.5%	1412 93.6%	0 0%	1572 100%

Table 25: Demographics of Calculus II Survey Sample

SS Calc II N = 1574	of Total	of REFORMED	of TRADITIONAL	of ABC	of DF	of Engineering	of Science	of Other	of Domestic	of International	of Men	of Women	of Pell	of Non-Pell	of URM	of non-URM
REFORMED	354 22.5%	354 100%	0 0%	310 23%	44 19.3%	176 20.9%	119 22.9%	59 28%	255 22.5%	99 22.3%	242 22.7%	110 21.8%	39 23.1%	315 22.4%	25 20.5%	329 22.7%
TRADITIONAL	1220 77.5%	0 0%	1220 100%	1036 77%	184 80.7%	668 79.1%	400 77.1%	152 72%	876 77.5%	344 77.7%	825 77.3%	394 78.2%	130 76.9%	1090 77.6%	97 79.5%	1123 77.3%
ABC	1346 85.5%	310 87.6%	1036 84.9%	1346 100%	0 0%	713 84.5%	464 89.4%	169 80.1%	924 81.7%	422 95.3%	915 85.8%	428 84.9%	132 78.1%	1214 86.4%	91 74.6%	1255 86.4%
DF	228 14.5%	44 12.4%	184 15.1%	0 0%	228 100%	131 15.5%	55 10.6%	42 19.9%	207 18.3%	21 4.7%	152 14.2%	76 15.1%	37 21.9%	191 13.6%	31 25.4%	197 13.6%
Engineering	844 53.6%	176 49.7%	668 54.8%	713 53%	131 57.5%	844 100%	0 0%	0 0%	645 57%	199 44.9%	587 55%	257 51%	87 51.5%	757 53.9%	78 63.9%	766 52.8%
Science	519 33%	119 33.6%	400 32.8%	464 34.5%	55 24.1%	0 0%	519 100%	0 0%	356 31.5%	163 36.8%	333 31.2%	185 36.7%	63 37.3%	456 32.5%	27 22.1%	492 33.9%
Other	211 13.4%	59 16.7%	152 12.5%	169 12.6%	42 18.4%	0 0%	0 0%	211 100%	130 11.5%	81 18.3%	147 13.8%	62 12.3%	19 11.2%	192 13.7%	17 13.9%	194 13.4%
Domestic	1131 71.9%	255 72%	876 71.8%	924 68.6%	207 90.8%	645 76.4%	356 68.6%	130 61.6%	1131 100%	0 0%	754 70.7%	374 74.2%	169 100%	962 68.5%	122 100%	1009 69.5%
International	443 28.1%	99 28%	344 28.2%	422 31.4%	21 9.2%	199 23.6%	163 31.4%	81 38.4%	0 0%	443 100%	313 29.3%	130 25.8%	0 0%	443 31.5%	0 0%	443 30.5%
Men	1067 67.8%	242 68.4%	825 67.6%	915 68%	152 66.7%	587 69.5%	333 64.2%	147 69.7%	754 66.7%	313 70.7%	1067 100%	0 0%	106 62.7%	961 68.4%	76 62.3%	991 68.3%
Women	504 32%	110 31.1%	394 32.3%	428 31.8%	76 33.3%	257 30.5%	185 35.6%	62 29.4%	374 33.1%	130 29.3%	0 0%	504 100%	62 36.7%	442 31.5%	46 37.7%	458 31.5%
Pell	169 10.7%	39 11%	130 10.7%	132 9.8%	37 16.2%	87 10.3%	63 12.1%	19 9%	169 14.9%	0 0%	106 9.9%	62 12.3%	169 100%	0 0%	41 33.6%	128 8.8%
Non-Pell	1405 89.3%	315 89%	1090 89.3%	1214 90.2%	191 83.8%	757 89.7%	456 87.9%	192 91%	962 85.1%	443 100%	961 90.1%	442 87.7%	0 0%	1405 100%	81 66.4%	1324 91.2%
URM	122 7.8%	25 7.1%	97 8%	91 6.8%	31 13.6%	78 9.2%	27 5.2%	17 8.1%	122 10.8%	0 0%	76 7.1%	46 9.1%	41 24.3%	81 5.8%	122 100%	0 0%
non-URM	1452 92.2%	329 92.9%	1123 92%	1255 93.2%	197 86.4%	766 90.8%	492 94.8%	194 91.9%	1009 89.2%	443 100%	991 92.9%	458 90.9%	128 75.7%	1324 94.2%	0 0%	1452 100%

Motivation was measured using constructs from Self-Determination Theory including satisfaction of basic psychological needs (autonomy, competence, and relatedness) and the continuum of self-determined motivation (amotivation, extrinsic, introjection, identification, integration, and intrinsic). Table 26 shows the reliability each construct as measured in this study. The reliability for autonomy, competence, and relatedness are lower than is desired and will be discussed in the limitations of the study. However, slight improvement in the reliability of the BPN measures (0.70, 0.77, and 0.83) occurred when analyzing only domestic students (note that the reliability of other motivational measures remained more-or-less fixed).

Table 26: Reliability of Motivational Constructs

Subscale	Cronbach's Alpha
Amotivation	0.85
Extrinsic	0.83
Introjection	0.85
Identification	0.86
Integration	0.86
Intrinsic	0.95
Autonomy	0.65
Competence	0.72
Relatedness	0.79

Table 27 and Table 28 show the overall correlation matrices for the grade, pre-knowledge, and motivational constructs from Self-Determination Theory. For the general population, in both Calculus I and Calculus II, internally regulated motivation were positively correlated with grade outcomes ($r \approx 0.35$, 0.20, and 0.25 for intrinsic, integrated, and identified motivation, respectively). Introjected motivation did not correlate with grade outcomes in either course; however, extrinsic motivation negatively correlated with grade outcomes ($r \approx -0.12$) in both courses. Amotivation negatively correlated with grade outcomes in both courses ($r \approx -0.14$ and -0.22, respectively). Perceived satisfaction of BPN were positively correlated with grade outcomes in both courses ($r \approx$

0.30, 0.40, and 0.15 for autonomy, competence and relatedness). Measures of internally regulated motivation are highly inter-correlated (between r ≈ 0.60 and r ≈ 0.80) as are measures of perceived satisfaction of BPN (between r ≈ 0.45 and r ≈ 0.70). Amotivation correlates most strongly with competence (r ≈ -0.50) as well as introjection (r ≈ 0.40) and extrinsic (r ≈ 0.20) motivation. Amotivation is only weakly correlated (between r ≈ -0.20 and r ≈ -0.05) with internally regulated motivation for the general population. Interestingly, knowledge (measured at the beginning of the semester) correlates, albeit weakly between r ≈ 0.10 and r ≈ 0.30, with perceived satisfaction of BPN as well as internally regulated motivation (measured at the end of the semester).

Correlation matrices for international students in Table 29 and Table 30 show some similar patterns, but with much attenuated correlations between motivational constructs and grade outcomes, particularly in Calculus I. Correlations between amotivation and externally regulated motivation are stronger for international students as are the inter-correlations among internally regulated motivation (r ≈ 0.75) and externally regulated motivation (r ≈ 0.60). Amotivation is correlated more strongly with internally regulated motivation for domestic students (values between r ≈ -0.15 and r ≈ -0.30) than international students (non-significant values between r ≈ -0.03 and r ≈ 0.10). For domestic students (shown in Table 31 and Table 32), intrinsic and extrinsic motivation are negatively correlated (r ≈ -0.25) while they are positively correlated for international students (r ≈ 0.15). In fact, for international students, internally regulated motivation is, in general, positively correlated with externally regulated motivation (between r ≈ 0.10 and r ≈ 0.35). Correlation matrices for other subpopulations of interest (not shown) does not result in any major deviations from the patterns or levels of the correlations of the general population. However, because of the negative impact of the reformed condition on URM students in Calculus II, separate correlation matrices were produced for URM in the reformed condition and the traditional condition shown in Table 33 and Table 34. While correlation patterns for URM students in the traditional condition matched the patterns for domestic students at-large – in particular, with a correlation between grade and intrinsic motivation of $r \approx 0.45$ ($p < 0.001$) – intrinsic motivation was not found to correlate with grade ($r \approx 0.025$ (ns)) in the reformed conditions. Instead, competence held a much stronger relationship with grade in the reformed condition ($r \approx 0.64$) versus the traditional condition ($r \approx 0.37$). Additionally, CEM components correlated (negatively) with grade more strongly in the reformed condition.

The key components of the theoretical path models with standardized coefficients for Calculus I and Calculus II are shown in Figure 13 and Figure 14. As expected, moderate covariance among exogenous variables was present and is reported for both Calculus I and Calculus in Table 35. The standardized path coefficients among endogenous variables related to self-determined motivation are similar for both Calculus I and Calculus II and are reported in Table 36. Fit statistics for Calculus I include Comparative Fit Index (CFI) = 0.98, Root Mean Square Error of Approximation (RMSEA) = 0.230, and Standardized Root Mean Square Residual (SRMR) = 0.022, respectively. Both CFI and SRMR statistic meet general standards, CFI > 0.94, RMSEA < 0.09, and SRMR < 0.09 (O'Rourke, Hatcher, 2013, pp. 142-146), however, RMSEA is notably high, likely due to the complexity of the model and some variation among subpopulations in motivational correlations. The path model for Calculus II has similar values for CFI, RMSEA, and SRMR of 0.98, 0.258, and 0.023, respectively, as well as similar associations to Calculus I within the model itself. In both Calculus I & Calculus II, LaGrange Multiplier test statistics indicate that paths from KNOW to self-determined motivation variables may improve model fit; however, including those path did not alter the nature of the output, nor result in significant paths. Wald test statistics do not identify any paths that would significantly improve fit by being removed from the model (including those non-significant paths). Despite the RMSEA value, the standardized coefficients from the path model provide some insight into the relative importance of components of the model. For instance, contrary to expectations, AEM has a small negative effect (CEM had no effect) on grade outcomes leaving intrinsic motivation as the only positively associated variable. Both autonomy and competence support intrinsic motivation; however, previous analyses found that relatedness most strongly impacted BPN in the reformed condition and was not shown to drive intrinsic motivation. Increases in competence corresponded with decreases in both amotivation and CEM and increases in AEM and intrinsic motivation. Interestingly, autonomy has a slight positive relationship with amotivation. Regarding the paths among self-determined motivation variables, these coefficients aligned broadly with expectation; however, anomalies such as intrinsic motivation predicting amotivation and autonomous extrinsic motivation predicting controlled extrinsic motivation were not consistent with Self-Determination Theory in general.

Given differences in the structure of the correlation matrix for international students, the same model was applied restricted to that subpopulation. Fit statistics for that model were roughly equivalent to the model applied to the general population. Just as in the structure of the correlation matrices, evidence of slight differential functioning of motivation processes was found.

and Figure 16 shows key components of the path model applied to International students with standardized coefficients. The model indicates that international students are more relatedness-driven, specifically as it relates to intrinsic motivation which remained the only driver of course achievement. These patterns were consistent across both Calculus I & Calculus II. It would have been desirable to run the model restricted to URM students, for the same reasons as noted in the correlational analyses above, however, sample size made this prohibitive.

Table 27: Correlation Matrix for Calculus I, All Students

Calc I - All (N = 1720)	**GPA**	**KNOW**	**INTRIN**	**INTEG**	**IDENT**	**INTRO**	**EXTRIN**	**AMOT**	**AUT**	**COMP**	**REL**
GPA	1										
KNOW	0.409 ***	1									
INTRIN	0.364 ***	0.113 ***	1								
INTEG	0.18 ***	0.028	0.634 ***	1							
IDENT	0.234 ***	0.083 ***	0.749 ***	0.778 ***	1						
INTRO	0.03	0.013	0.168 ***	0.247 ***	0.2 ***	1					
EXTRIN	-0.112 ***	-0.007	-0.138 ***	0.053 *	-0.003	0.444 ***	1				
AMOT	-0.138 ***	-0.096 ***	-0.081 ***	-0.093 ***	-0.193 ***	0.444 ***	0.239 ***	1			
AUT	0.288 ***	0.124 ***	0.519 ***	0.365 ***	0.466 ***	-0.052 *	-0.246 ***	-0.251 ***	1		
COMP	0.399 ***	0.178 ***	0.578 ***	0.407 ***	0.553 ***	-0.135 ***	-0.227 ***	-0.484 ***	0.652 ***	1	
REL	0.128 ***	0.01	0.326 ***	0.345 ***	0.355 ***	-0.045 ^	-0.064 **	-0.208 ***	0.489 ***	0.459 ***	1

Table 28: Correlation Matrix for Calculus II, All Students

Calc II - All (N = 1574)	**GPA**	**KNOW**	**INTRIN**	**INTEG**	**IDENT**	**INTRO**	**EXTRIN**	**AMOT**	**AUT**	**COMP**	**REL**
GPA	1										
KNOW	0.45 ***	1									
INTRIN	0.369 ***	0.256 ***	1								
INTEG	0.199 ***	0.135 ***	0.646 ***	1							
IDENT	0.283 ***	0.184 ***	0.771 ***	0.78 ***	1						
INTRO	0.01	-0.024	0.242 ***	0.272 ***	0.26 ***	1					
EXTRIN	-0.129 ***	-0.056 *	-0.169 ***	-0.017	-0.068 **	0.38 ***	1				
AMOT	-0.218 ***	-0.127 ***	-0.072 **	-0.109 ***	-0.183 ***	0.404 ***	0.214 ***	1			
AUT	0.313 ***	0.207 ***	0.529 ***	0.418 ***	0.486 ***	-0.039	-0.285 ***	-0.237 ***	1		
COMP	0.431 ***	0.262 ***	0.595 ***	0.455 ***	0.567 ***	-0.104 ***	-0.247 ***	-0.488 ***	0.677 ***	1	
REL	0.153 ***	0.115 ***	0.333 ***	0.35 ***	0.37 ***	-0.022	-0.073 **	-0.232 ***	0.525 ***	0.5 ***	1

Table 29: Correlation Matrix for Calculus I, International Students

Calc I - International (N = 489)	GPA	KNOW	INTRIN	INTEG	IDENT	INTRO	EXTRIN	AMOT	AUT	COMP	REL
GPA	1										
KNOW	0.36 ***	1									
INTRIN	0.151 ***	0.11 *	1								
INTEG	0.087 ^	0.084 ^	0.713 ***	1							
IDENT	0.081 ^	0.077 ^	0.802 ***	0.834 ***	1						
INTRO	-0.035	0.108 *	0.25 ***	0.337 ***	0.279 ***	1					
EXTRIN	-0.062	0.072	0.2 ***	0.344 ***	0.259 ***	0.608 ***	1				
AMOT	-0.076 ^	-0.001	0.07	0.017	-0.025	0.561 ***	0.336 ***	1			
AUT	0.185 ***	0.071	0.489 ***	0.382 ***	0.47 ***	-0.142 **	-0.198 ***	-0.319 ***	1		
COMP	0.198 ***	0.077 ^	0.372 ***	0.306 ***	0.403 ***	-0.223 ***	-0.143 **	-0.545 ***	0.623 ***	1	
REL	0.057	0.036	0.436 ***	0.39 ***	0.428 ***	-0.065	-0.003	-0.176 ***	0.514 ***	0.462 ***	1

Table 30: Correlation Matrix for Calculus II, International Students

Calc II - International (N = 443)	GPA	KNOW	INTRIN	INTEG	IDENT	INTRO	EXTRIN	AMOT	AUT	COMP	REL
GPA	1										
KNOW	0.407 ***	1									
INTRIN	0.229 ***	0.262 ***	1								
INTEG	0.135 **	0.151 **	0.705 ***	1							
IDENT	0.17 ***	0.204 ***	0.819 ***	0.816 ***	1						
INTRO	0.013	-0.002	0.268 ***	0.319 ***	0.298 ***	1					
EXTRIN	-0.051	-0.038	0.107 *	0.306 ***	0.235 ***	0.583 ***	1				
AMOT	-0.198 ***	-0.144 **	0.083 ^	0.078	0.02	0.565 ***	0.371 ***	1			
AUT	0.277 ***	0.238 ***	0.591 ***	0.447 ***	0.529 ***	-0.045	-0.2 ***	-0.248 ***	1		
COMP	0.347 ***	0.262 ***	0.49 ***	0.355 ***	0.463 ***	-0.23 ***	-0.225 ***	-0.537 ***	0.7 ***	1	
REL	0.151 **	0.188 ***	0.512 ***	0.458 ***	0.498 ***	-0.067	-0.014	-0.213 ***	0.579 ***	0.581 ***	1

Table 31: Correlation Matrix for Calculus I, Domestic Students

Calc I - Domestic (N = 1231)	GPA	KNOW	INTRIN	INTEG	IDENT	INTRO	EXTRIN	AMOT	AUT	COMP	REL
GPA	1										
KNOW	0.418 ***	1									
INTRIN	0.326 ***	0.079 **	1								
INTEG	0.154 ***	-0.017	0.597 ***	1							
IDENT	0.228 ***	0.064 *	0.727 ***	0.749 ***	1						
INTRO	-0.02	-0.052 ^	0.089 **	0.183 ***	0.141 ***	1					
EXTRIN	-0.115 ***	-0.034	-0.241 ***	-0.055 ^	-0.092 **	0.391 ***	1				
AMOT	-0.255 ***	-0.168 ***	-0.212 ***	-0.182 ***	-0.314 ***	0.365 ***	0.204 ***	1			
AUT	0.229 ***	0.113 ***	0.478 ***	0.328 ***	0.438 ***	-0.074 **	-0.257 ***	-0.289 ***	1		
COMP	0.422 ***	0.197 ***	0.617 ***	0.424 ***	0.587 ***	-0.14 ***	-0.249 ***	-0.514 ***	0.646 ***	1	
REL	0.128 ***	-0.008	0.288 ***	0.323 ***	0.325 ***	-0.052 ^	-0.081 **	-0.243 ***	0.481 ***	0.453 ***	1

Table 32: Correlation Matrix for Calculus II, Domestic Students

Calc II - Domestic (N = 1131)	GPA	KNOW	INTRIN	INTEG	IDENT	INTRO	EXTRIN	AMOT	AUT	COMP	REL
GPA	1										
KNOW	0.415 ***	1									
INTRIN	0.321 ***	0.183 ***	1								
INTEG	0.165 ***	0.085 **	0.611 ***	1							
IDENT	0.259 ***	0.127 ***	0.743 ***	0.758 ***	1						
INTRO	-0.048	-0.075 *	0.198 ***	0.232 ***	0.221 ***	1					
EXTRIN	-0.12 ***	-0.031	-0.242 ***	-0.128 ***	-0.168 ***	0.308 ***	1				
AMOT	-0.299 ***	-0.158 ***	-0.194 ***	-0.227 ***	-0.318 ***	0.305 ***	0.147 ***	1			
AUT	0.261 ***	0.143 ***	0.469 ***	0.38 ***	0.442 ***	-0.076 *	-0.3 ***	-0.275 ***	1		
COMP	0.429 ***	0.232 ***	0.613 ***	0.471 ***	0.586 ***	-0.086 **	-0.242 ***	-0.51 ***	0.661 ***	1	
REL	0.124 ***	0.065 *	0.262 ***	0.301 ***	0.314 ***	-0.021	-0.084 **	-0.261 ***	0.5 ***	0.468 ***	1

Table 33: Correlation Matrix for Calculus II, Traditional URM Students

Traditional Calc II - URM (N = 97)	GPA	KNOW	INTRIN	INTEG	IDENT	INTRO	EXTRIN	AMOT	AUT	COMP	REL
GPA	1										
KNOW	0.434 ***	1									
INTRIN	0.431 ***	0.151	1								
INTEG	0.221 *	0.045	0.623 ***	1							
IDENT	0.34 ***	0.052	0.82 ***	0.759 ***	1						
INTRO	0.046	0.025	0.34 ***	0.241 *	0.272 **	1					
EXTRIN	-0.177 ^	-0.111	-0.219 *	0.009	-0.143	0.115	1				
AMOT	-0.177 ^	-0.119	-0.031	-0.089	-0.108	0.375 ***	0.247 *	1			
AUT	0.21 *	0.134	0.413 ***	0.551 ***	0.521 ***	0.045	-0.154	-0.143	1		
COMP	0.374 ***	0.186 ^	0.575 ***	0.561 ***	0.661 ***	0.006	-0.297 **	-0.42 ***	0.61 ***	1	
REL	0.1	0.015	0.097	0.313 **	0.197 ^	-0.13	-0.108	-0.186 ^	0.567 ***	0.415 ***	1

Table 34: Correlation Matrix for Calculus II, Reformed URM Students

Reformed Calc II - URM (N = 25)	GPA	KNOW	INTRIN	INTEG	IDENT	INTRO	EXTRIN	AMOT	AUT	COMP	REL
GPA	1										
KNOW	0.421 *	1									
INTRIN	0.025	-0.029	1								
INTEG	-0.003	-0.171	0.536 **	1							
IDENT	0.172	0.254	0.504 *	0.733 ***	1						
INTRO	-0.237	0.023	0.268	0.365 ^	0.144	1					
EXTRIN	-0.327	-0.377 ^	-0.064	0.174	0.06	0.269	1				
AMOT	-0.387 ^	-0.102	0.111	-0.061	-0.234	0.601 **	0.222	1			
AUT	0.353 ^	0.012	0.325	0.602 **	0.655 ***	-0.12	-0.058	-0.284	1		
COMP	0.636 ***	-0.138	0.346 ^	0.472 *	0.492 *	-0.167	0.059	-0.483 *	0.677 ***	1	
REL	0.234	0.058	0.034	0.338 ^	0.491 *	-0.275	0.196	-0.256	0.559 **	0.479 *	1

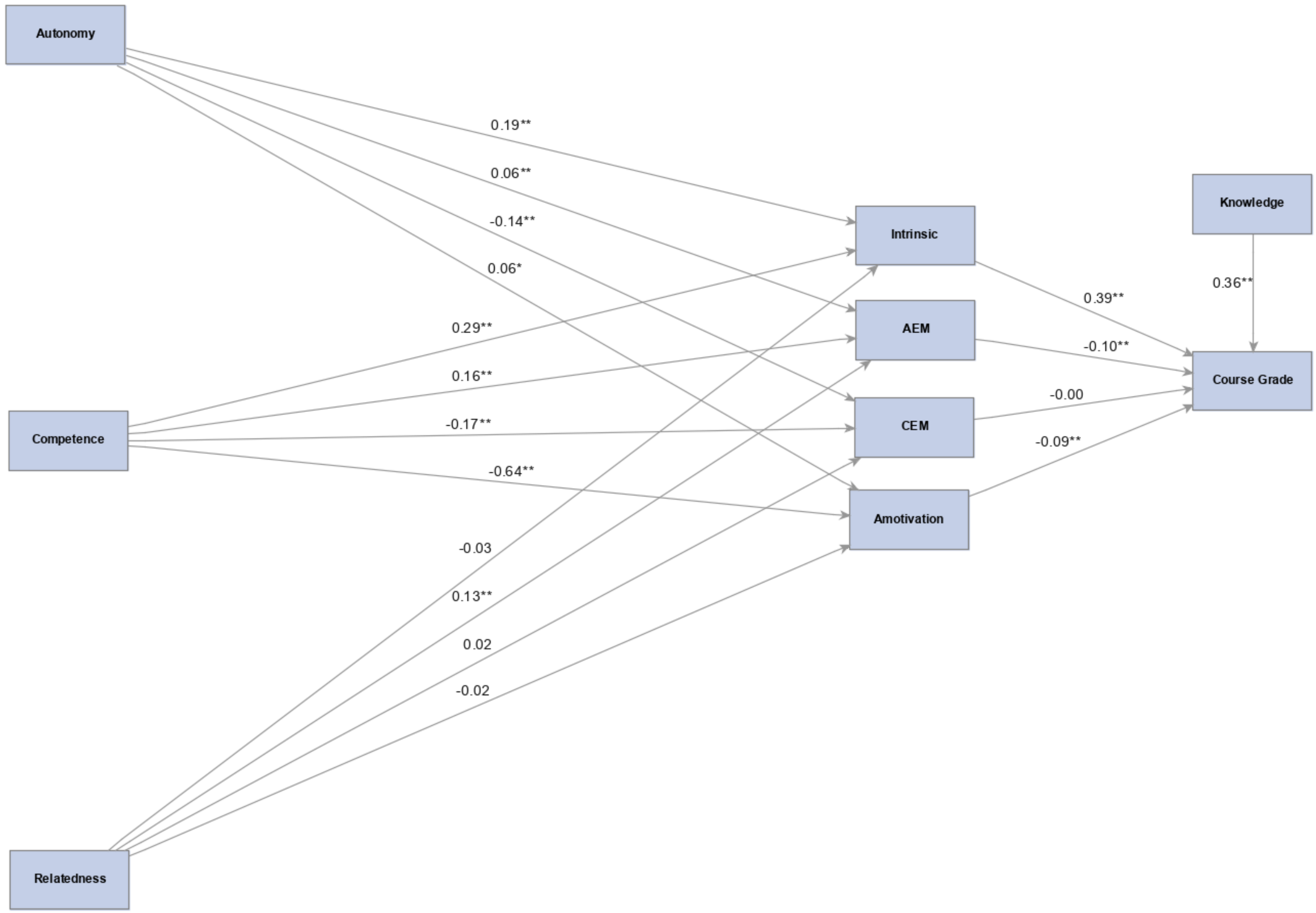

Figure 13: Path Model with Standardized Coefficients for Calculus I, All Students

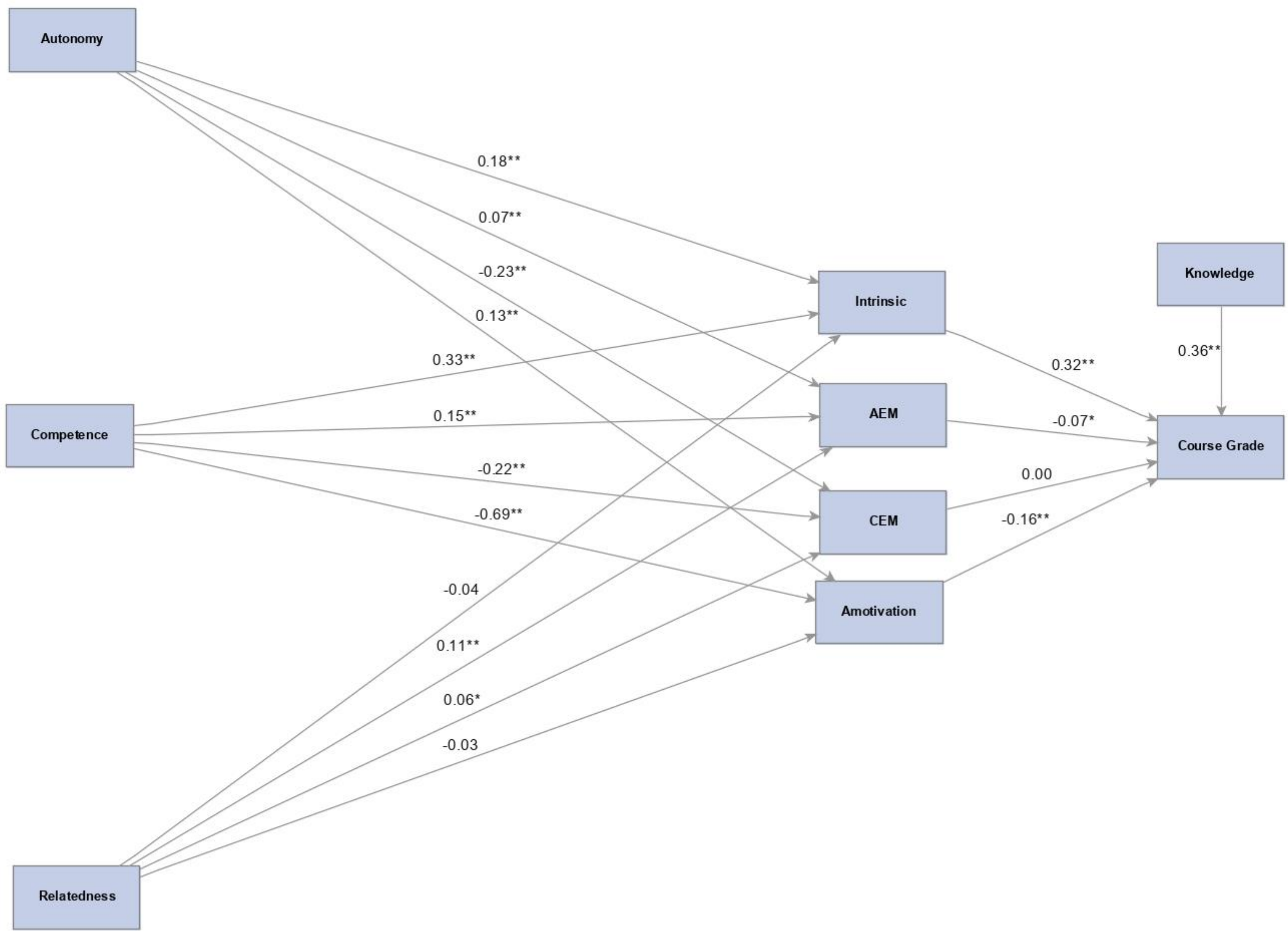

Figure 14: Path Model with Standardized Coefficients for Calculus II, All Students

Table 35: Standardized Covariance for Exogenous Variables in Calculus I & Calculus II

Variable 1	Variable 2	Calculus I	Calculus II
AUT	**REL**	0.49***	0.53***
AUT	**COMP**	0.65***	0.68***
REL	**COMP**	0.46***	0.50***

Table 36: Standardized Coefficients for Self-Determined Motivation Variables

Variable	Predictor	Calculus I	Calculus II
INTRIN	**AEM**	0.29***	0.32***
INTRIN	**CEM**	-0.05***	-0.02
INTRIN	**AMOT**	0.02	-0.01
AEM	**INTRIN**	0.45***	0.39***
AEM	**CEM**	-0.02	0.05**
AEM	**AMOT**	-0.04*	-0.07***
CEM	**INTRIN**	0.08***	-0.01
CEM	**AEM**	0.38***	0.32***
CEM	**AMOT**	0.22***	0.23***
AMOT	**INTRIN**	0.32***	0.32***
AMOT	**AEM**	-0.07***	-0.06***
AMOT	**CEM**	0.12***	0.15***

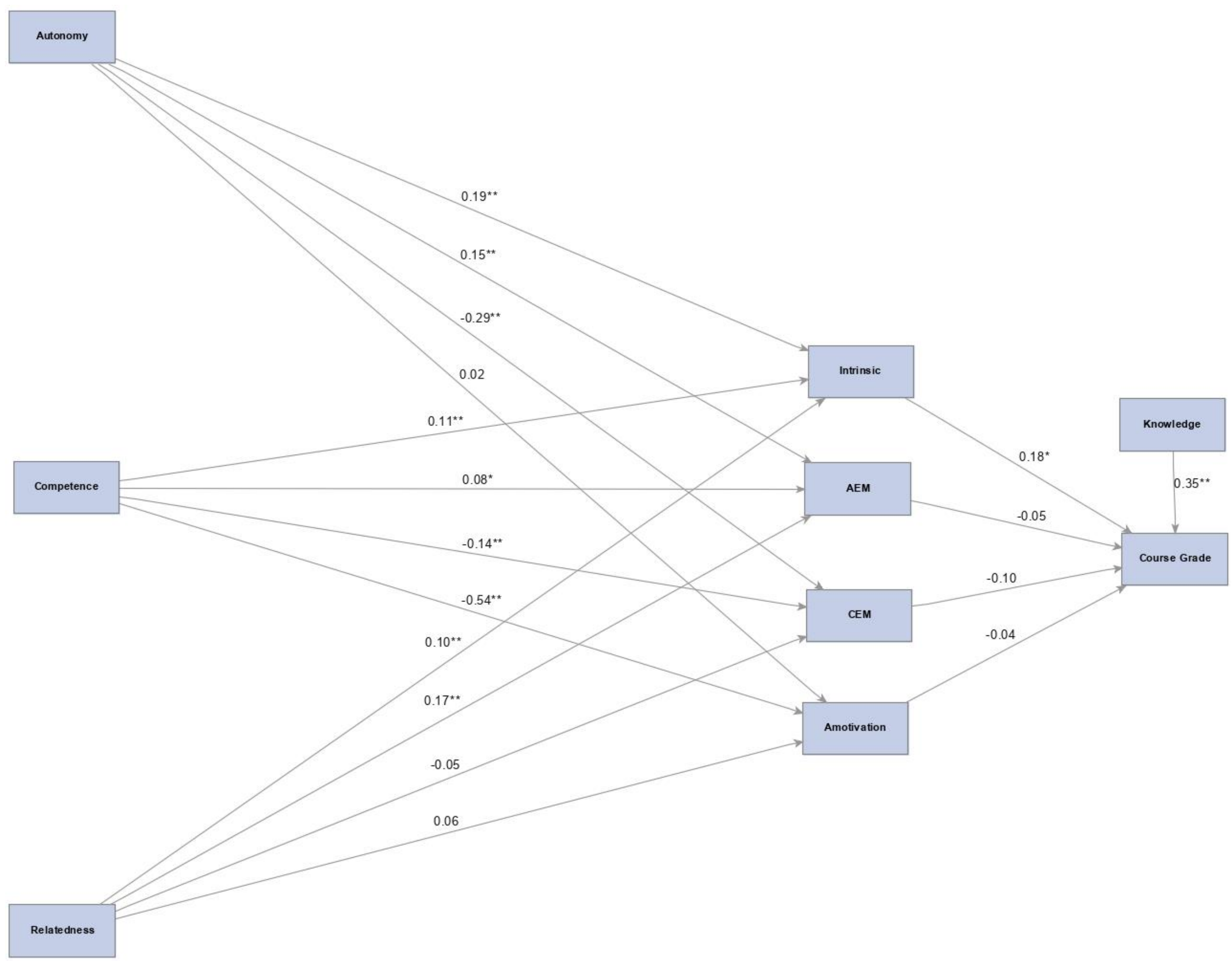

Figure 15: Path Model with Standardized Coefficients for Calculus I, International Students

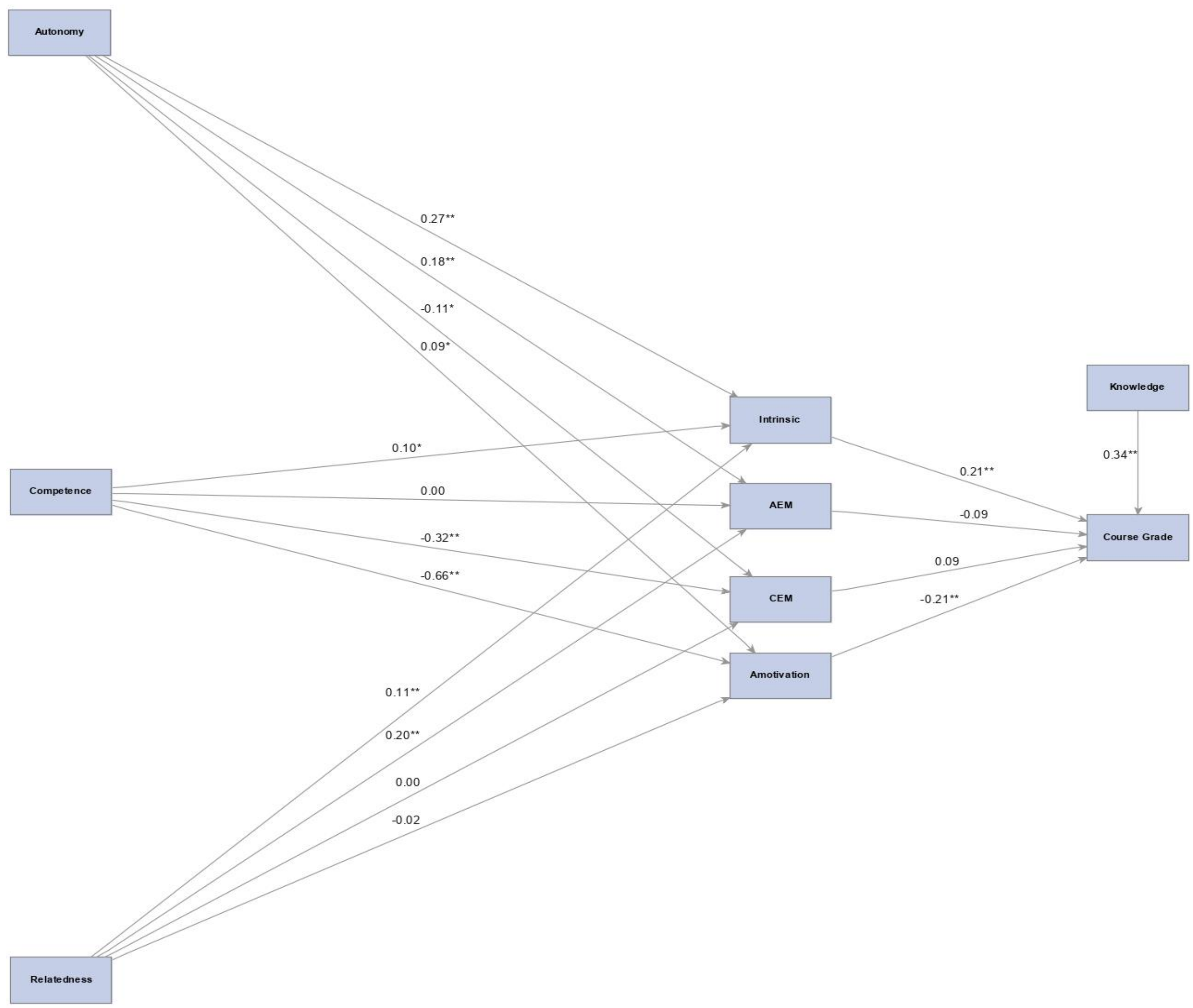

Figure 16: Path Model with Standardized Coefficients for Calculus II, International Students

4. 3 Basic Psychological Needs and Demographics across Conditions

In order to correct for response bias in the survey sample and more accurately estimate the level of students' perception of BPN support across conditions, missing values for averages of BPN measures (autonomy, competence, and relatedness) in the grade sample were imputed using linear regression on grade, demographics, course, and condition.

Table 37 shows, for Calculus I, the average levels in the tradition condition (INT) for each measure across specific subpopulations along with the difference between the reformed and traditional conditions (REFORMED). Controlling for knowledge across conditions had little-to-no effect differences in autonomy, competence or relatedness when examining subpopulations of interest individually. Overall results show significant gains in the reformed condition for all three measures; however, those gains were not uniformly realized by all populations of interest and the magnitude of those gains varied across BPNs. The substantial differences in levels across populations of interest are further analyzed in

Table 38. Despite improvements in GPA for Pell-eligible Calculus I students in the reformed condition, the Pell-eligible students did not see any specific associated gains in autonomy or competence. They did however see gains in relatedness associated with the general population in the reformed condition. On the other hand, the non-Pell-eligible, domestic, white men saw significant gains in autonomy, competence, and relatedness along with GPA. The reformed condition impacted international students' perception of BPN-satisfaction the least. Of the three BPNs, relatedness was most impacted by the reformed condition within all subpopulations of interest; however, results from Section 4.2 indicate that relatedness also had the smallest correlation ($r < 0.15$) with academic outcomes as shown in Table 27 and Figure 13.

Table 37: Student Perception of BPN-satisfaction by Condition and Demographic Groups in Calculus I

Calc I AUTONOMY	**Total (REFORMED)**	**INT**	**REFORMED**	**R^2**	**pval**
All	3971 (687)	4.12***	0.19***	0.0140	< 0.0001
International	964 (175)	4.54***	0.01	0.0000	0.7707
Domestic	3007 (512)	4.04***	0.23***	0.0240	< 0.0001
Pell	485 (99)	4.04***	-0.01	0.0000	0.9367
URM	399 (64)	3.99***	0.21**	0.0230	0.0026
Women	1134 (192)	4.18***	0.15**	0.0070	0.0042
Non-Pell, Domestic, White Men	1288 (210)	4.02***	0.33***	0.0480	< 0.0001
Calc I COMPETENCE	Total (REFORMED)	**INT**	**REFORMED**	**R^2**	**pval**
All	3971 (687)	4.48***	0.08**	0.0020	0.0093
International	964 (175)	4.73***	0.01	0.0000	0.7977
Domestic	3007 (512)	4.40***	0.09**	0.0020	0.0071
Pell	485 (99)	4.34***	0.11	0.0030	0.2244
URM	399 (64)	4.30***	-0.05	0.0010	0.5704
Women	1134 (192)	4.43***	0.04	0.0000	0.5051
Non-Pell, Domestic, White Men	1288 (210)	4.43***	0.18***	0.0110	0.0002
Calc I RELATEDNESS	**Total (REFORMED)**	**INT**	**REFORMED**	**R^2**	**pval**
All	3971 (687)	4.38***	0.32***	0.0450	< 0.0001
International	964 (175)	4.49***	0.17***	0.0150	0.0001
Domestic	3007 (512)	4.34***	0.37***	0.0570	< 0.0001
Pell	485 (99)	4.19***	0.44***	0.0790	< 0.0001
URM	399 (64)	4.35***	0.20**	0.0200	0.0052
Women	1134 (192)	4.41***	0.28***	0.0260	< 0.0001
Non-Pell, Domestic, White Men	1288 (210)	4.33***	0.45***	0.0900	< 0.0001

Table 38: Overall Effects of the Reformed Condition on Students' Perceived BPN-satisfaction across Domestic Demographics in Calculus I

Domestic N = 3971 (408)	Calc I AUTONOMY	Calc I COMPETENCE	Calc I RELATEDNESS
INT	3.83***	4.04***	4.30***
KNOW	0.39***	0.75***	0.10*
URM	-0.04	-0.08*	0.04
FEMALE	0.05*	-0.01	0.06*
PELL	0.02	-0.03	-0.19***
REFORMED	0.30***	0.14**	0.41***
URM x REFORMED	0.02	-0.14	-0.22**
FEMALE x REFORMED	-0.02	-0.09	-0.07
PELL x REFORMED	-0.31***	0.01	0.10
R^2	0.0552	0.0674	0.0732
pval	< 0.0001	< 0.0001	< 0.0001

Table 38 shows the results of similar linear models as Table 23 in section 4.1, however, in this case, to predict each BPN based on domestic subpopulations and condition. Knowledge was a significant predictor of each BPN although to a lesser extent for relatedness. Pell-eligible students were significantly lower than their non-Pell-eligible peers on the relatedness measure (b = -0.19) in the traditional condition and URM students were slightly lower that their non-URM peers on the competence measure (b = -0.08). Women reported slightly higher autonomy-satisfaction and relatedness-satisfaction than men in the tradition condition (b = 0.05 and 0.056, respectively). The reformed condition, in general, improved students' perceptions of autonomy, competence, relatedness (b = 0.30, 0.14, and 0.41); however, those gains were diminished by half in relatedness for students from URM and completely in autonomy for Pell-eligible students.

Table 39 shows, for Calculus II, the average levels of perceived BPN-satisfaction in the tradition condition (INT) for each measure across specific populations along with the difference between the reformed and traditional conditions (REFORMED). Again, the reformed condition resulted in general gains in students' perceptions of BPN-satisfaction (and to a greater extent than in Calculus I), and, again, those gains were not uniformly realized across populations of interest. International students showed diminished gains (as compared domestic students) on all three measures; however, their gains were still more pronounced here than in Calculus I. The reformed condition impacted the perception of BPN-satisfaction in non-Pell-eligible, domestic, white men again showed the greatest extent although the impact of the reformed condition on women was comparable, unlike in Calculus I.

Table 40 shows the results of a linear model predicting students' perceptions of BPN-satisfaction for domestic students in Calculus II. Pre-knowledge predicted students' perception of BPN-satisfaction at similar levels to Calculus I. In the traditional condition, women reported slightly lower competence (b = -0.16) and slightly higher relatedness (b = 0.07) than men. The reformed condition, in general, increased students perceptions of BPN-satisfaction on all three measure with similar levels as in Calculus I (b = 0.20, 0.19, and 0.42 for autonomy, competence, and relatedness respectively). However, those gains were not realized for students from URM and only partially realized (in relatedness) for Pell-eligible students. Small average gains (though not statistically significant) specifically for women in the reformed condition were also present which closed the "gender competence gap" present in the traditional condition.

Table 39: Student Perception of BPN-satisfaction by Condition and Demographic Groups in Calculus II

Calc II AUTONOMY	Total (REFORMED)	INT	REFORMED	R^2	pval
All	2895 (584)	4.06***	0.24***	0.0190	< 0.0001
International	815 (176)	4.36***	0.17**	0.0120	0.0020
Domestic	2080 (408)	3.94***	0.25***	0.0220	< 0.0001
Pell	324 (67)	4.01***	0.04	0.0010	0.6592
URM	240 (40)	3.98***	-0.03	0.0000	0.7575
Women	795 (166)	4.08***	0.24***	0.0180	0.0002
Non-Pell, Domestic, White Men	942 (164)	3.91***	0.28***	0.0280	< 0.0001
Calc II COMPETENCE	**Total (REFORMED)**	**INT**	**REFORMED**	**R^2**	**pval**
All	2895 (584)	4.30***	0.19***	0.0090	< 0.0001
International	815 (176)	4.53***	0.12^	0.0050	0.0506
Domestic	2080 (408)	4.22***	0.21***	0.0100	< 0.0001
Pell	324 (67)	4.30***	-0.03	0.0000	0.7888
URM	240 (40)	4.22***	0.04	0.0000	0.7971
Women	795 (166)	4.21***	0.23**	0.0110	0.0033
Non-Pell, Domestic, White Men	942 (164)	4.27***	0.21**	0.0100	0.0019
Calc II RELATEDNESS	**Total (REFORMED)**	**INT**	**REFORMED**	**R^2**	**pval**
All	2895 (584)	4.36***	0.35***	0.0480	< 0.0001
International	815 (176)	4.52***	0.18***	0.0150	0.0004
Domestic	2080 (408)	4.30***	0.42***	0.0630	< 0.0001
Pell	324 (67)	4.25***	0.37***	0.0470	0.0001
URM	240 (40)	4.36***	0.06	0.0010	0.6036
Women	795 (166)	4.40***	0.40***	0.0540	< 0.0001
Non-Pell, Domestic, White Men	942 (164)	4.28***	0.43***	0.0660	< 0.0001

Table 40: Overall Effects of the Reformed Condition on Students' Perceived BPN-satisfaction across Domestic Demographics in Calculus II

Domestic N = 2080 (512)	Calc II AUTONOMY	Calc II COMPETENCE	Calc II - RELATEDNESS
INT	3.69***	3.79***	4.18***
KNOW	0.41***	0.80***	0.17**
URM	0.05	0.02	0.10^
FEMALE	-0.01	-0.16***	0.07*
PELL	0.09^	0.11*	-0.06
REFORMED	0.27***	0.19**	0.42***
URM x REFORMED	-0.28*	-0.16	-0.40***
FEMALE x REFORMED	0.09	0.16	0.12
PELL x REFORMED	-0.23*	-0.29*	-0.02
R^2	0.0526	0.0772	0.08
pval	< 0.0001	< 0.0001	< 0.0001

4.4 Summary of Results

Table 37 and

Table 39 show that the reformed conditions in both Calculus I and Calculus II improved, on average, student perception of BPN-satisfaction on all three measures with the strongest effect in relatedness.

Table 21 shows that grade outcomes in the reformed conditions for both Calculus I and Calculus II improved by more than an eighth of a letter grade resulting in increases in the pass rates of 2 percentage points and 5 percentage points, respectively. Furthermore, the correlation matrices in Table 27 and Table 28 provide evidence regarding the structure of measures of BPN-satisfaction, self-determined motivation, and achievement. This structure was evaluated via the path diagrams shown in Figure 13 and Figure 14. While fit statistics were not adequate, they do demonstrate the relatively high importance of both competence and intrinsic motivation compared to other constructs in this context. Autonomy was important, but secondary, and all non-intrinsic motivation was either negatively associated with achievement or not associated with achievement at all. The lack of importance – and, in fact, the small negative correlation with achievement - of autonomous external motivation is in direct contradiction with expectations in this context and, more generally, with tenets of Self-Determination Theory. Partial explanation may lie in the conflict between BPN-thwarting features of the overall course (normed grading, high stakes exams, rigid and inauthentic content, and the required nature of the course as a degree requirement) and the BPN-supportive features of the reformed condition. That is, students may have differently interpreted the survey questions as they related to the overarching features of the course and the features contained within each learning environment. These results, in part, demonstrate the effectiveness of the reformed condition and elucidate the motivational mechanisms associated with the effectiveness of the reformed condition. The subsections below discuss results specific to populations of interest and how defining student-centeredness through student-based motivational measures provides insight into the intervention. Individual results regarding the student-centeredness of each learning environment is defined by the extent to which students' perceived BPN-satisfaction in a manner that affected academic outcomes in the course. Relevant results are summarized in Figure 17 and Figure 18. Because of the anomalous findings for URM students in Calculus II, the three terms over which the experiment was conducted are broken out in Figure 19.

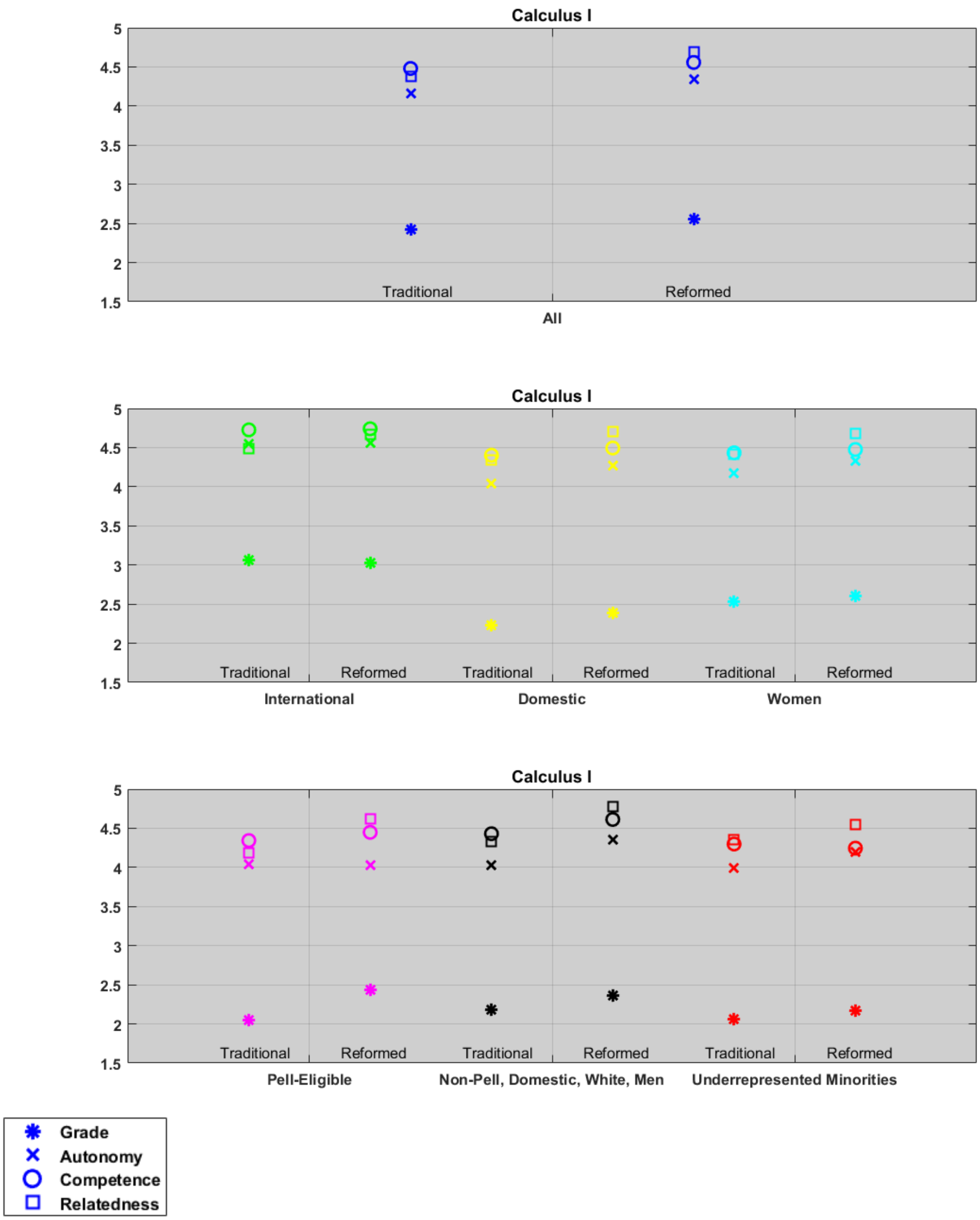

Figure 17: Adjusted Grade Point Averages and Perceived BPN-Satisfaction in Calculus I

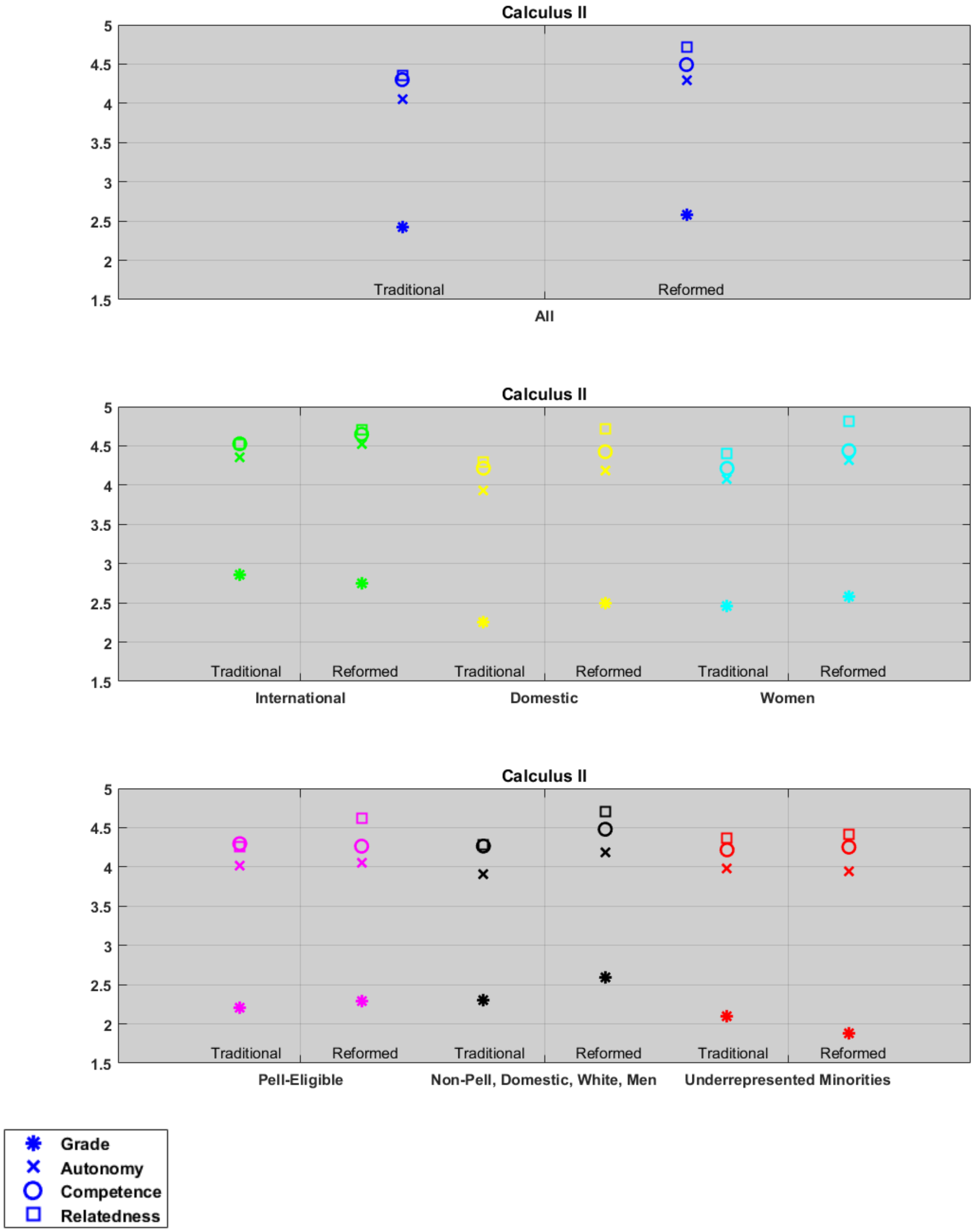

Figure 18: Adjusted Grade Point Averages and Perceived BPN-Satisfaction in Calculus II

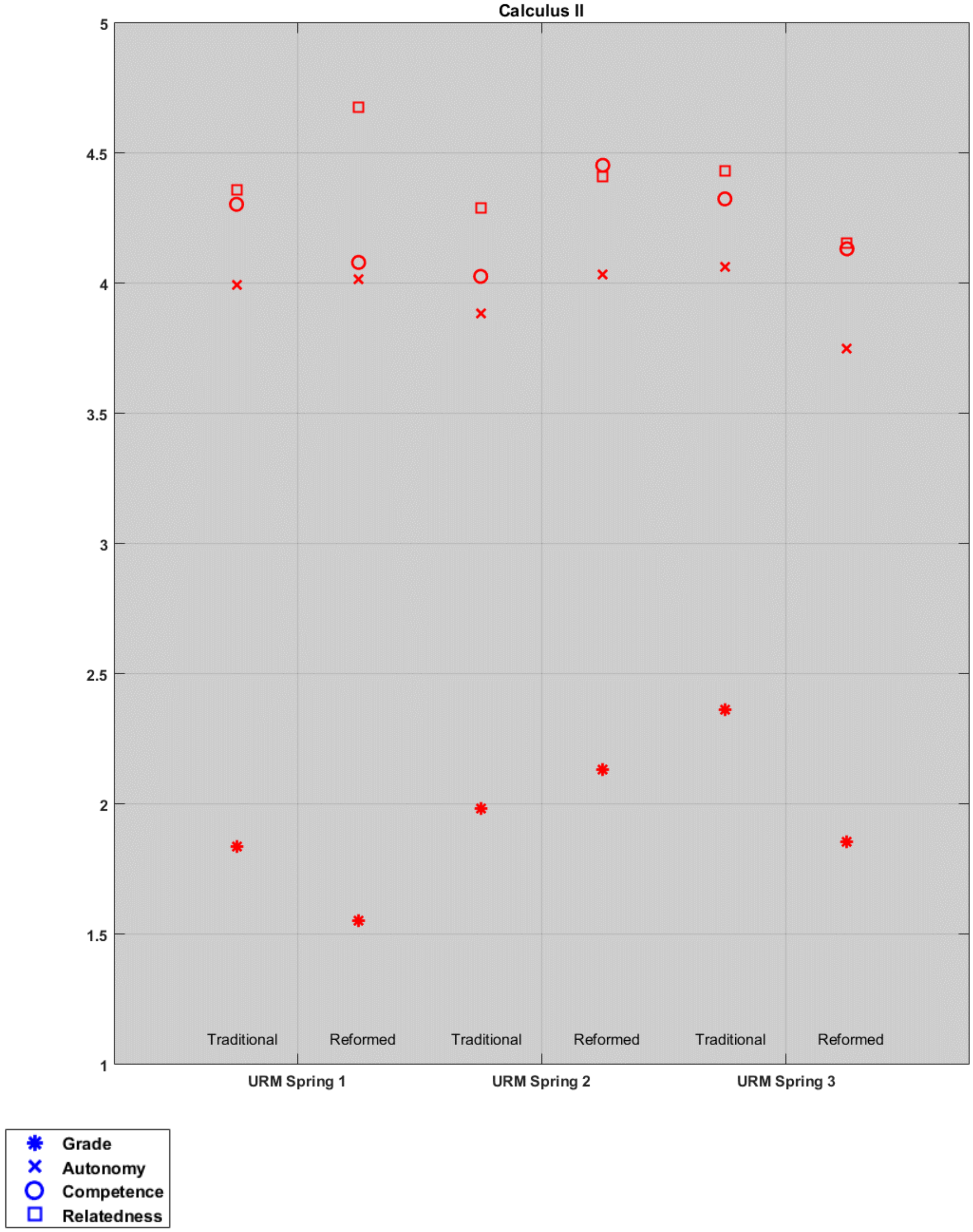

Figure 19: Adjusted Grade Point Averages and Perceived BPN-Satisfaction of URM Students in Calculus II by Term

4.4.1 Non-Pell-eligible, Domestic, White Men

Steen (1988) lamented that "those who do survive (Calculus) are poorly motivated for advanced study and too uniformly white, male, and middle class." Partially consistent with Steen's perception, the present study finds that, in the traditional condition, non-Pell-eligible, domestic, white men (NPDWM), 32% of the total population, performed (controlling for prior knowledge) at or above the levels Pell-eligible or URM students, slightly below the level of women in Calculus I and equally in Calculus II, but exceedingly below the level of international students. However, within the reformed condition, NPDWM reported the highest level of gains in perception of BPN-satisfaction as well as gains in achievement. In particular, controlling for knowledge, NPDWM in the reformed condition are more than twice as likely to pass as NPDWM in the traditional condition. The motivational correlates and paths for achievement discussed above and in the results remained intact for NPDWM. While relatedness was impacted for NPDWM, it appeared to have little effect on achievement. In the context of this research, we would conclude that the reformed condition was more student-centered for NPDWM than was the traditional condition and that the achievement of women was most strongly driven by increased competence, increased autonomy, and, subsequently, intrinsic motivation.

4.4.2 International Students

The international students, 28% of the total population, in this study achieved, on average, a full letter grade above their domestic counter parts boasting failures rates of only 5-6% in both courses and conditions. Additionally, the international students in both conditions reported higher levels of perceived BPN-satisfaction than their domestic peers consistent with the achievement gap. However, there were no significant grade differences between international students in the traditional and reformed conditions in either of Calculus or Calculus II. International students also had no significant differences across conditions in perceived satisfaction of autonomy or competence in Calculus I and small differences in Calculus II relative to other subpopulations. However, international students saw modest gains in perceived satisfaction of relatedness in the reformed condition in both courses. Interestingly, path models indicate international students were less competence-driven and more relatedness-driven with respect to intrinsic motivation and subsequently achievement. However, the path model indicates the relationship between intrinsic motivation and achievement was weaker for international students. In the context of this research, the pedagogical methods in the traditional and reformed condition were comparably student-centered for international students although there were some benefits in relatedness. Furthermore, the motivational mechanisms by which achievement was facilitated were slightly variant to the general population. This could be due to differences in culture, expectations, ability level, and language skills.

4.4.3 Women

Domestic women were the highest performing domestic subpopulation in the traditional Calculus I condition and on par with their male counterparts in the traditional Calculus II condition. However, when examined alone, women at-large saw no statistically significant benefit in achievement from the reformed condition. When including other demographic variables in a single model, we found no women-specific achievement benefit of the intervention. However, domestic women in the reformed condition in Calculus II benefitted by a third of a letter grade due to an overall effect for domestic students along with similar increases in perceived BPN-satisfaction. While there was an overall effect for domestic students in Calculus I, women specifically, on average, did not receive that benefit. Correspondingly, domestic women saw diminished (though not statistically significant) gains compared to their male counterparts in perceived BPN-satisfaction in Calculus I reformed condition. The general structure of correlation matrices for women were equivalent to the general population suggesting the motivational mechanisms in this context did not vary for women. In the context of this research, we would conclude that the Calculus I reformed and traditional were equally centered about women and that in Calculus II, the reformed condition was more centered about women than the traditional condition, though not specifically so, and that the achievement of women was most strongly driven by increased competence, increased autonomy, and, subsequently, intrinsic motivation.

4.4.4 Underrepresented Minorities

Underrepresented minorities (URM) in this study predominantly included African-American or Latino students and constituted roughly 10% of the total population. Controlling for knowledge and other demographic factors, URM students achieved around 0.15-0.20 grade points lower than their non-URM peers in both traditional Calculus I and Calculus II. URM students saw an alarming 0.48 grade point drop compared to their non-URM peers in the reformed condition. This effect resulted in URM students in the reformed condition being three quarters as likely to pass Calculus II as compared to URM students in the traditional condition. Said more directly, the reformed condition exacerbated the existing achievement gap due diminishing the achievement of URM students and enhancing the achievement of non-URM students. The achievement gap in Calculus I was accompanied by a small corresponding gap in perception of BPN-satisfaction. The URM students in the traditional Calculus I condition saw comparable BPN-satisfaction as their non-URM peers; however, URM students, specifically, in the reformed condition saw diminished perceived BPN-satisfaction entirely eliminating the benefit seen by domestic students at large. These differences, however, do not explain the drop in achievement for URM students in the Calculus II reformed condition compared to URM students in the Calculus II traditional condition.

When examining only URM students in the Calculus II reformed condition, we found no correlation between intrinsic motivation and achievement which was the largest driving factor for other subpopulations. The small sample size (N=25) of this subpopulation prohibits running the path model; however, comparison of correlation matrices for the traditional URM Calculus II subpopulation (which has structure similar to the overall population) with the reformed URM Calculus II (which contains anomalies like increased impact of perceived BPN-satisfaction and diminished role of self-determined motivation) demonstrates how motivation operated in the reformed condition for URM students. The drop in achievement in the Calculus II reformed condition may be attributable to a number of factors such as stereotype threat becoming more salient through a more interactive environment, lack of URM instructors, or by enhancing social or cultural failings of the general learning environment centered on non-URM students that already served to disenfranchise URM students, including misalignment between instructional practices and assessment methods and grading procedures. The motivational mechanism that facilitate achievement in the reformed condition appear to diverge dramatically from those of the general population. Figure 19 shows URM data broken out by term. Although the reformed condition within each term contained only 12, 17, and 11 students each, clear deviations from the average pattern emerge. In the second term, URM students' results appear similar to overall results for domestic students. However, the first and third terms drive the overall average effects for URM students. Interestingly, perceived BPN-satisfaction tracks with grade in the second and third terms, but not in the first. These results demonstrate the fragility of the intervention within the reformed condition as it relates to support for the BPNs of URM students. In the context of this research, we would conclude that the reformed condition was less student-centered for URM students than was the traditional condition and that both conditions were less centered about URM students than non-URM students.

4.4.5 Pell-eligible Students

Pell-eligible students constituted around 11-12% of the total population and were almost completely categorized as domestic students. Controlling for knowledge, Calculus I Pell-eligible students in the traditional condition achieved at similar levels as their non-Pell-eligible peers; however, traditional Calculus II Pell-eligible students achieved around a quarter of a letter grade lower than their non-Pell-eligible peers. Pell-eligible Calculus I students in the reformed condition saw significant achievement increases compared to Pell-eligible Calculus I students in the traditional condition resulting in a 50% increase in the likelihood to pass. Interestingly, those achievement increases were not accompanied by increases in the perceived autonomy-support than accompanied such increases in the general domestic population although the general increases in competence and relatedness remained. However, in Calculus II, Pell-eligible students saw no significant benefit from the reformed condition unlike their non-Pell-eligible, domestic counterparts. Similarly, Pell-eligible saw no benefit in perceived BPN-support, except in relatedness (which was shown not to affect achievement), while their non-Pell-eligible counterpart saw substantial gains. In the context of this research, we would conclude that the reformed condition was equally student-centered for Pell-eligible students as the traditional condition although certain features supported achievement in Calculus I, perhaps associated with overcoming obstacles related to first generation status.

CHAPTER 5: DISCUSION

The main objective of this research was to apply motivational theory to the context of mainstream undergraduate calculus in order to inform long-standing reform efforts within the discipline by providing insight about how student-centered pedagogies function and may differentially affect subpopulations. This study examined the use of Self-Determination Theory (SDT) to provide such a theoretical framework in a quasi-experimental study of a traditional, lectured-based learning environment and a reformed, student-centered learning environment as student-centered pedagogies have been identified as important to reform efforts in a recent report of the Mathematical Association of America. SDT posits, universally, that increased satisfaction of Basic Psychological Needs (BPN) leads to increased self-determined motivation which, in turn, leads to increased achievement. Although certain motivationally-relevant overall course features remained intact (high-stakes curved grading, learning outcomes, and curricular role), the reformed condition was designed to support BPN-satisfaction by providing opportunities for student collaboration, choice on how class time is used, and personal, timely feedback. While there was no reason to believe that either condition adequately supported the social and cultural needs of individual students nor that it was finely attuned to the diverse beliefs and experiences of individual students, it was expected that an environment designed as BPN-supportive would benefit all students equally and, perhaps, more so for those who have been systematically disenfranchised from predominantly white, male, middle-to-upper class-oriented educational systems. That is, the expectation was that this BPN-supportive environment would lead to increased self-determined motivation and improved course grades for all populations, though to varying extents. Given these hypotheses, subpopulations of interest were examined including international students (26%), women (28%), Pell-eligible students (12%), and underrepresented minorities (9%) as well as domestic, non-Pell-eligible, white, men (32%) who alone constitute a larger portion of the population at-hand than the primary demographic categories listed. The results indicate that learning environments in undergraduate mainstream calculus matter. They also provide insight as to why they matter and for whom.

5.1 Broad Responses to Research Questions

5.1.1 Research Question #1: Grade Outcomes and Perceived BPN-satisfaction across Conditions

Are there differences in students' perceived BPN-satisfaction and grade outcomes across lecture-based and collaborative, problem-based undergraduate Calculus I and Calculus II learning environments?

Grade outcomes differed across conditions in each course. Controlling for prior knowledge, students in the reformed condition averaged 0.13 grade points higher in Calculus I and 0.15 grade points higher in Calculus II than students in the traditional condition. While the effect sizes of the reformed condition on grade, controlling for knowledge, were quite small (with partial correlation coefficients of 0.04 and 0.06) due to the sizeable variation in student grade outcomes, these increases resulted in students in the reformed condition being 21% and 32% more likely to pass Calculus I and Calculus II, respectively, than students in the traditional condition. Students' perception of BPN-satisfaction also differed across the two conditions. The reformed condition produced higher perception of BPN-satisfaction in each course for all three BPNs. Effects were stronger in Calculus II than in Calculus I and were strong for relatedness than for competence or autonomy in each course. Effect sizes of condition on students' perceived BPN-satisfaction were small with correlation coefficients of 0.12, 0.05, and 0.21 in Calculus I and 0.14, 0.09, and 0.22 in Calculus II for autonomy, competence, and relatedness.

5.1.2 Research Question #2: Perceived BPN-satisfaction, Self-determined Motivation, and Grade Outcomes

What are the relationships among learning environments, perceived BPN-satisfaction, amotivation, CEM, AEM, intrinsic motivation, and grade outcomes in Calculus?

A path model indicated that - of amotivation, CEM, AEM, and intrinsic motivation - only intrinsic motivation produced positive outcomes in grade in each course with standardized path coefficients of 0.38 and 0.31 in Calculus I and Calculus II, respectively. Both, amotivation and AEM produced small negative outcomes in course grade in each course. The relationship between AEM and grade outcomes in the path model is counter to the expectations of this research and with fundamental ideas within SDT. The complex motivational dynamics of this context (including use of high-stakes, multiple-choice, procedurally-focused, norm-graded, timed exams juxtaposed with theoretically-based exposition in the lecture condition and conceptually-based tasks in the reformed condition) were in conflict BPN-support and, therefore, with design features of the intervention resulting in an attenuated role of AEM in students' motivational processes. Students who report taking the course "because acquiring all kinds of knowledge is fundamental for me" may have experienced dissonance between course assessment practice and their AEM. Specifically, high achievement on exams may have been possible without robust understanding of content due to the exams' procedural nature (whereby certain problem types and solutions can be memorized) or due to the effects of norm-based grading which can provide passing grades to students with very low levels of performance. Conversely, low achievement on exams may have been possible in cases where student wholly embraced and encapsulated the broad conceptual nature of the subject and particular topics covered due to failure to perform rapidly have very specific problem types presented on the exam. Many students likely experienced both states making AEM ill-suited to predict achievement in this context. It is possible that including AEM-type items related to STEM-identity could better elucidate expected motivational relationships; however, this would need to be treated as a separate source of AEM. CEM had no relationship with course grade in either course. Autonomy was positively associated with amotivation, AEM, and intrinsic motivation in each course and negatively associated with CEM. Competence was positively associated with AEM and intrinsic motivation in each course and negatively associated with amotivation and CEM. Relatedness was positively associated with CEM and AEM in each course and not associated with amotivation or intrinsic motivation. As stated in the previous section, the reformed condition produced slightly increased student perception of BPN-satisfaction with the strongest effect on relatedness. Generally speaking, the most impactful motivation processes involved increased autonomy and competence in the reformed condition contributing to intrinsic motivation and, subsequently, improved grade outcomes.

5.1.3 Research Question #3: Demographic Differences in Motivational Outcomes, Motivational Processes, and Grade Outcomes

Do the relationships and differences among learning environments, perceived BPN-satisfaction, amotivation, CEM, AEM, intrinsic motivation, and grade outcomes in Calculus vary across demographic groups?

There were no differences in grade outcomes or perception of BPN-satisfaction for international students in either course. Motivational processes for international students involved autonomy, competence, and relatedness (as opposed to only autonomy and competence) driving intrinsic motivation and, subsequently, grade outcomes. Within the domestic population, the general effects stated above were most pronounced for non-Pell-eligible, white men though domestic women saw similar benefit of the reformed condition. URM students in Calculus I saw attenuated benefit; however, URM students in the reformed Calculus II condition saw diminished perception of BPN-satisfaction and grade outcomes. Furthermore, the path model indicated relatedness was negatively correlated with intrinsic motivation for these students and, within the reformed Calculus II condition, there was no correlation between intrinsic motivation and grade outcomes. Explaining these effects would likely require additional data; however, it is plausible that the reformed condition created in-groups and out-groups that thwarted engagement, exacerbated stereotype threat among URM students, failed to provide mentorship from international or URM populations, and that the non-condition-specific features of the course interacted with demographic subgroups in ways that were not captured. Furthermore, the generally observed lack of positive association between AEM and academic outcomes (hypothesized in Section 5.1.2 to be due to incongruent assessment and instructional practices) may have taken a pronounced toll on groups that have historically been disenfranchised from educational systems or other societal structures. These students may have had previous experiences where there are two sets of rules exist (one set explicitly stated and another covert set used to determine achievement). Given that intrinsic motivation did not correlate with achievement for URM students in the Calculus II reformed condition, it is plausible that students', who had a history of high achievement and success, adhered to and thrived under the explicit rules, but were disproportionately, relative to non-URM peers, motivationally diminished when another set of rules were applied to measure their achievement.

Pell-eligible students in the reformed Calculus I condition saw a grade outcomes increase over the traditional Calculus I condition at disproportionately high levels compared to perceived BPN-satisfaction. These effects were not present in Calculus II. Understanding the effect of the reformed condition for Pell-eligible would likely require additional data to be collected; however, a plausible explanation may lie in the high prevalence of first-generation college students who are

Pell-eligible. Features of the reformed environment may have increased transparency in to the socio-academic environment in Calculus I and at the University at-large in ways that facilitated their transition to college. In the spring semester (Calculus II), these transition-supporting features were not as relevant as only those students who were successful in Calculus I were enrolled in Calculus II.

5.2 Implications for Practice

5.2.1 Measuring Motivation-in-Context

This study provided evidence of the importance and efficacy of measuring motivational factors and their relationship to academic outcomes to better understand reform efforts in undergraduate mainstream calculus. The motivational measures applied in this study explained statistically significant variance in achievement, but there is a clear need to identify additional factors, improve these measures, and augment with other methods so that more substantial variance might be explained and a more complete view of motivation – and non-cognitive factors at large – in the context of undergraduate mainstream Calculus can be constructed. Implicitly related to the importance of the present research is the final recommendation of the MAA CSPCC study: "regular use of local data to guide curricular and structural modifications" in support of change within a department (Bressoud, 2015, p. viii). While multitudes of constructs can be measured and metrics evaluated, the findings of this study explicitly warrant the strategic and systematic measurement of student motivation under a cohesive theoretical framework to support reform efforts as part of such an evaluation plan. Measuring students' perceptions related to motivation is not new for departments, as evidenced below from a brief selection of questions from my own departmental teaching evaluations across three departments and two universities.

- The course increased my interest in mathematics.
- My instructor shows respect for me and other students in this class.
- My instructor encourages an atmosphere where ideas can be exchanged freely and easily.
- Was the instructor enthusiastic about teaching?
- Did the instructor motivate students to perform well?
- The feedback I received on assignments and tests gave me the opportunity to improve my performance.

While I do not aim to make any general statements here about the efficacy, use, or governance of teaching evaluations in higher education, it is worth noting that some departments routinely collect students' perceptions related to motivation, and those data could be intentionally aligned with a more comprehensive framework of non-cognitive factors and integrated with other relevant data such as demographics, instructional practice, and students' academic achievement attuned to the specific needs of students, faculty, or administrators. This information could be operationalized into curricular reviews and governance processes supplementing (or replacing) to overall instructor/course ratings, course pass/fail rates, or canonical assessment of learning outcomes.

5.2.2 Measuring Effects by Populations of Interest

There are countless reasons why educational interventions in undergraduate mainstream calculus may be successful for some populations and not others. In settings that have been historically predominated by affluent, white men, it is reasonable to assume that the learning environment has become biased over time, intentionally or unintentionally, by adopting practices aligned with the culture, beliefs and attitudes of affluent, white men. The overrepresentation of white men as the ‘human norm’ has long been noted in other disciplines such as medical research and practice (Dressler, 1992). Furthermore, any incremental or systematic changes that might threaten the success of this predominant population may be more likely to be thwarted than similar changes that would threaten the success of non-predominant populations. Additionally, potential research findings that demonstrate a threat to non-predominant groups may be left uncovered by an overall “average” benefit driven by disproportionate positive benefit for predominant groups. The combination of these effects may readily result in the systematic disenfranchisement of non-predominant groups. All of this, together with findings from this study, establish an imperative to evaluate reform efforts, in practice or in research settings, with respect to populations who are potentially or historically underserved by an educational institution. The findings of this study also issue an imperative to intentionally design for support of social and cultural diversity and the diversity in beliefs and experiences of individuals in ways established in the literature (Dennehy, 2017; Dennis et al, 2015; Gutierrez, 2018; Steele et al., 2002; Steen, 1988, pp. 138-139, 144-145; Stipanovic & Woo, 2016). Furthermore, it is also an imperative to establish whether overall results are based solely by the success of the predominant culture or population.

5.2.3 Moving Beyond Best Practices

This study advocates for a student-centered definition of student-centered pedagogies in the undergraduate mainstream calculus classroom based on students’ perceived satisfaction of basic psychological needs. While recommendations on best practices provide initial direction and guiderails for the design of calculus learning environments, it is difficult to believe that any practice is ‘best’ for all circumstances or than these practices can be naively implemented without considering the dynamics or objectives within a particular instructional context. There is a host of research (Chambers et al., 2016; Dennehy, 2017; Good et al., 2012; Gilbert et al, 2015, Hernandez et al., 2012; Jones et al., 2013; Sonnert et al., 2015; Steele et al., 2002; Steinberg et al., 2012; Weurlander et al., 2017) regarding non-cognitive factors and their important role in the undergraduate mainstream calculus classroom. However, these measures must be operationalized within the curriculum and pedagogical practice to facilitate reform efforts. While the MAA offers valuable recommendations including "Proactive student support services, including the fostering of student academic and social integration", it may be even more constructive to have broader themes integrated with all recommendation such as "Define, measure, and attend to non-cognitive factors throughout all aspects of your calculus program." Best pedagogical practices will follow if we design for and systematically evaluate both cognitive and non-cognitive factors on an individual student basis.

5.3 Limitations and Future Research

5.3.1 Limitations

The data collected in this study were designed through the general lens of Self-determination Theory and applied in the context of mainstream undergraduate calculus at a large, selective, public research university. The strengths of SDT for this research were in its universality, its fundamental nature, its treatment of Extrinsic motivation as a continuum separating AEM from CEM as constructs for specifically understanding the motivation of science and engineering students in calculus tasks, the applicability of BPN-satisfaction for defining student-centered pedagogies, and the ability to influence BPN through learning environments to promote academic outcomes. However, the moderate reliability of students' perception of BPN-satisfaction creates concerns about how the inventory was interpreted by students and how well the results can be situated within the SDT framework. The path model did not meet all expectations in terms of fit and resulted in surprising effects of self-determined motivation on achievement, including AEM being negatively associated with grade outcomes. Higher-level motivational constructs (self-concept, mindset, goal orientation), task specific or fine-grain data (daily, weekly, etc), and cultural/social contexts were not measured. The limitations imposed SDT as a low-level theory may, in part, be overcome by realigning some basic building blocks, supplementing with additional data and theoretical frameworks, and adding a time dimension to the data itself to build higher-level constructs. Other limitations of SDT within the proposed research design were related to its quantitative implementation. A qualitative SDT protocol would provide an opportunity to better contextualize how and why the satisfaction of basic psychological needs occurs as well as what values and beliefs drive the internalization of extrinsic motivation. Clearly, within any attempt at a qualitative component, we would need to supplement or, in fact, design the research with higher motivational constructs in mind including the cultural, social, and emotional context of the individual within the learning environment.

At institutions of higher education, there is a great deal of variation across courses with respect to pedagogy, learning environment, grading and participation policy, and role of a course within the structure of students' plans of study. Even courses with very similar learning outcomes, various instantiations of "Calculus", say, can have structures that are quite different depending upon the department or institution offering the course as well as the intended audience for the course. These aspects of *Course Structure & Policy* may interact with student motivation associated with academic achievement. Examining various dimensions of *Course Structure & Policy*, identifying differences between courses, and mapping those dimensions to applicable motivational constructs may help us understand how why and how interventions may operate differently in different courses. In addition, there may be a great deal of variation in *Student*

Populations enrolled in two similar courses. Certain dimensions of *Student Populations* (experience, aptitude, and academic goals outside of demographic, social, and cultural context) may also interact with student motivation associated with academic achievement and should be understood along with *Course Structure & Policy*. Thus, these results may not be immediately generalizable to other institutions and the broader context of the course offering should be considered carefully.

5.3.2 Threats to validity

Course grade was the fundamental outcome measure in this research. Course grade distributions in the study were entirely determined by multiple-choice, common midterms and final exam. The exams were constructed at the direction of the course coordinator who varies term-to-term. Components of each exam were delegated out to the various faculty lecturers. There was no programmatic mechanism to design reliable or valid exams. Because the exams were machine-graded, inter-rater reliability is presumed high. However, test-retest and internal consistency reliability should not be assumed. There was typically only one form of each exam (up to permutation), but new midterms and finals are constructed each term. At the end of each term, the coordinator decided what grade cutoffs would be for the course based on the performance of the students in an attempt approximate equivalent forms reliability across terms relative to final course grade. This process was not managed departmentally or longitudinally, leaving it up to each coordinator to navigate. While the exams and grading scheme were common to students in both conditions in a given term, the exams and grading standards are likely to vary from term-to-term.

Course grade was used for the purpose of measuring success in the course and, ultimately, determining successful progress towards a STEM degree at the University. The exams were multiple-choice and largely skills-based. Even for students who earned an ‘A’, it is not clear that success on exams measures transferable knowledge about Calculus to Science or Engineering tasks. For students who earn a ‘B’ or ‘C’, they could have mastered 100% of topics at 50-70% proficiency or mastered 50-70% of topics at 100% proficiency leading to very different abilities to apply knowledge outside the course. Part of the intention of reformed learning environment was to mitigate potential losses of transferability of knowledge by providing open-ended and conceptual problems, however, the students were still assessed on the same procedural, multiple-choice exams with normed-based grading which were inherently not compatible with the philosophy of the reformed learning environment.

This study examines the relationship between motivational constructs and grade outcomes in both traditional and reformed learning environments. While the type/configuration of classrooms and time patterns were different for each learning environment, there were no observations to confirm that the activities of the students and instructors in each learning environment conformed to expectations. While highly unlikely due to constraints of size, physical space, and personnel, there could have been an instructor in the lecture-based learning environment who uses the lecture time for group work, collaborative presentations, etc. Likewise, there could have been an instructor in the reformed learning environment who uses the problem session to simply lecture to the students. There was oversight of the course by a coordinator and the Associate Department Head, and, furthermore, teaching assignments at both the course-level and within the learning environments are managed by aligning instructor preferences with available duties. That is to say, instructors were likely teaching in accordance with their assigned learning environment because they requested it. The reasons for requesting the reformed learning environment introduce other threats to validity. Instructors might prefer this mode because it matches their skills and philosophy of teaching; however, other instructors might prefer it simply because it meets once per week instead of three times per week. Most faculty do not tend to request to teach Calculus at all.

Graduate teaching assistants were used in both learning environments. The Department of Mathematics had an administrative process for determining which TAs are qualified to perform various types of instructional assignments based on experience, interest, and past performance. Particularly at the beginning of the study, the Department was hesitant to use inexperienced TAs, which are commonly used for the lecture-based learning environment, to run the more freeform version of recitation component of the reformed learning environment. Generally, the TAs assigned to the reformed learning environment were perceived to be higher-quality instructors by the Department than those assigned to the lecture-based learning environment. Anecdotally, when personally speaking with faculty in the department, they often attributed the better performance of the reformed learning environment to the higher quality of the TAs.

Students were able to self-select into either instructional mode (up to limitations imposed by room capacity). We are able to control for mathematics ability with course-specific knowledge exams, but the students may select an instructional mode based on unobserved characteristics that could correlate with academic success. For instance, students who are oriented towards engagement with others in learning might select the environment perceived to provide more opportunity for peer engagement. Those students might have performed just as well (and better than average) in the traditional learning environment, but might be overrepresented in the reformed learning environment. This effect would be minimal in the fall term (Calculus I) for new students, but returning students in spring were likely to be more aware of the characteristics of the available learning environment as well as their own corresponding preferences and characteristics. Future work should consider this ‘matching’ effect of student with learning environment. Students who made a volitional choice for one learning environment or the other might also report a higher sense of BPN-satisfaction and self-determined motivation not only due to a matching of environment with preferences, but also simply because they chose it for themselves.

Data collected for this study were taken from the “on” semesters for Calculus. That is, Calculus I is generally offered in the spring semesters, but it nearly entirely populated with students who failed Calculus I in the fall and upper-classmen looking to raise their GPA before graduation. Generalization of the results of this study to the ‘off’ semester populations should not be assumed.

5.3.3 Future Research

This research builds on existing studies related to the efficacy of active learning environments and student-centered pedagogies in undergraduate mainstream calculus. Additionally, it builds upon existing studies related to non-cognitive factors associated with success in undergraduate mainstream calculus and STEM at-large. While this is the first large scale, quasi-experimental study to investigate the motivational processes by which pedagogical methods function across learning environments in the undergraduate mainstream calculus classroom, refinement is needed to more acutely measure motivation in this context and to better align those motivational processes with non-cognitive factors in general. For instance, we measured clear differences in levels of perceived BPN-satisfaction between conditions and among different subpopulations as well as distinct differences in how condition effected perceived BPN-satisfaction differently across subpopulations; however, future research should attempt to remove, or control for, some of the motivation-thwarting aspects of the course such as high stakes multiple choice exams with curved grading in order to understand how and if those features interacted with the effects of each learning environment. The grading policy itself may be of particular importance given the recent finding of Canning et al. (2019) regarding the disproportionate effects that introductory STEM learning environments, perceived to be highly competitive, have on first generation students. They found that such environments contribute to imposter syndrome and diminish student engagement and achievement in the classroom. Furthermore, a more robust set of learning environments (with and without group work, with and without undergraduate teaching assistants, with and without diverse instructors or mentors) might also help understand what aspects of the learning environment contributed to motivation or achievement and for whom.

Motivational measures developed and applied by Good et al. (2012) in an undergraduate mainstream calculus setting - including sense of belonging, mindset, and stereotype threat - could be of great value in understanding why motivational and achievement differences occurred and how to design learning environments in light of those differences. The situational motivation inventory applied in this study may need calibration to the context of mainstream undergraduate calculus. It was surprising that autonomous external motivation played such a minor role, and it should be investigated if statement such as, “Because learning all I can about academic work is really essential for me” are representative of the full spectrum of autonomous external motivation for this population in this context. For instance, it is plausible that a statement such as, “Because learning all the knowledge and skills needed to be a successful engineer” might garner different results. The inventory itself might be informed by a qualitative study of free responses to the question of why students are taking the course, or more specifically, why students engage in learning activities within the course from a phenomenological perspective. The results might inform aspects of self-determination theory as well as other related motivational or non-cognitive factors. Additionally, observation protocols such as the Interactive-Constructive-Active-Passive (ICAP) framework (Chi & Wylie, 2014) have been applied in higher education STEM classroom (Henderson, 2019) to better capture what is happening in active learning beyond superficial nomenclature. Such a protocol would supplement the motivationally-based definition of student-centeredness as suggested in the current research.

Lastly, a longitudinal approach including collecting data about both students' prior and future experiences, beliefs, and outcomes could help identify antecedent to success in the classroom as well as a broader definition of success in the classroom beyond grade outcome. The CSPCC study (Bressoud, 2015) asked students about the skills preparation and mathematics grades from high school as well as their experiences with pre-calculus and calculus in high school including delivery mode. They also asked about students' beliefs and attitudes regarding difficulty or success with mathematics, study/learning habits, the nature of mathematics, the role of the instructor, and desire to take mathematics (if it weren't required by their major). These types of data could be helpful in explaining broader perceptions of mathematics as well as motivational processes and achievement in calculus as well as how to design for specific needs. Additionally, students were asked about how class time was spent and how they engaged both inside and outside the classroom. There may, in fact, be sufficient data within the CSCCP study itself to create a holistic analysis of non-cognitive factors and how they interact with learning environments across demographic subpopulation in a way that extends the ideas from the present study to a nationally-representative sample, albeit without a complete SDT (or other motivational) framework, but with additional student-level data related to motivation and other non-cognitive factors. Analysis of data regarding retention in major, future course taking and performance, graduate, and future perceptions of learning mathematics would also be a beneficial supplement to course grade in defining and understanding success in the undergraduate mainstream calculus classroom.

5.4 Conclusion

Reform is hard. Al-Ubaydli et al. (2017) delineate three major barriers to the scalability of positive experimental results: statistical inference, representativeness of the population, and representativeness of the situation - with the latter two being most insoluble. K-12 educators and reformists may envy the academic freedom as well as relative student academic homogeneity of higher education, particularly within selective institutions. However, this autonomy without structure could lead to frustration of long-term, strategic objectives. Theobald et al. (2020) recently advocated for a "heads-and-hearts" approach to improving undergraduate STEM education through intentional, inclusive, high-intensity active learning pedagogies. While University departments generally have academic administrative structures to govern the curriculum and execute logistics, calculus reform efforts have historically been championed and executed by coalitions of the willing who may not have a comprehensive understanding, institutional line-of-sight, or the ability to affect the complexities involved with educational reform including learning theory, motivational theory, curricular theory, change management, academic planning, resource management, faculty development, social and cultural theory, and assessment. The calculus reform community and broader community of educational researchers play a critical role in assisting these champions by providing tools to help navigate these complexities and address scalability threats, particularly as they relate to closing achievement gaps and to promoting equity in learnings. The present research demonstrates the potential of using a motivational framework as a tool for designing and evaluating undergraduate mainstream calculus reform efforts. Frameworks of this nature could be refined, generalized, and adapted into commonplace practice to support diverse student populations across various types of learning environments.

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APPENDIX A. SURVEYS

A.1 Basic Psychological Needs

Please indicate how true each of the following statements is for you given your specific experiences in the course thus far.

Autonomy

I feel like I can make a lot of inputs in deciding how my coursework gets done.
I feel pressured in this course.
I am free to express my ideas and opinions in this course.
When I am in this course, I have to do what I am told.
My feelings are taken into consideration in this course.
I feel like I can pretty much be myself in this course.
There is not much opportunity for me to decide for myself how to go about my coursework.

Competence

I do not feel very competent in this course.
People in this course tell me I am good at what I do.
I have been able to learn interesting new skills in this course.
Most days I feel a sense of accomplishment from this course.
In this course I do not get much of a chance to show how capable I am.
I often do not feel very capable in this course.

Relatedness

I really like the people in this course.
I get along with people in this course.
I pretty much keep to myself when in this course.
I consider the people in this course to be my friends.
People in this course care about me.
There are not many people in this course that I am close to.
The people in this course do not seem to like me much.
People in this course are pretty friendly towards me.

A.2 Situational Motivation Scale (SIMS)

Students have different motivations for taking different courses, and we are interested in your motivations for taking the course thus far.

Intrinsic

Because I really enjoy it.
Because I really like it.
Because it's really fun.

Integration

Because learning all I can about academic work is really essential for me.
Because acquiring all kinds of knowledge is fundamental for me.
Because experiencing new things is a part of who I am.

Identification

Because it allows me to develop skills that are important to me.
Because it’s a sensible way to get a meaningful experience.
Because it’s a practical way to acquire new knowledge.

Introjection

Because I would feel bad if I didn't.
Because I would feel guilty if I didn’t.
Because I would feel awful about myself if I didn’t.

Extrinsic

Because I feel I have to.
Because that’s what I’m supposed to do.
Because that’s what I was told to do.

Amotivation

I don't know. I have the impression I’m wasting my time.
I’m not sure anymore. I think that maybe I should quit (drop the class).
I don’t know. I wonder if I should continue.

B.1 Sample Problem Set from Reformed Calculus I

Problems for Week 4. Due Tuesday September 19.

1. (a) Use the unit circle picture to explain why $\sin^2 t + \cos^2 t = 1$. (Hint: use the equation of the circle or the distance formula.)

   (b) Use the unit circle picture to explain why $\sin t$ is always between -1 and 1.

2. (a) Use the unit circle picture to say what happens to $\sin t$ when t goes from 0 to $\pi/2$. That is, $\sin t$ goes from what to what?

   (b) What happens to $\sin t$ when t goes from $\pi/2$ to π, then from π to $3\pi/2$, then from $3\pi/2$ to 2π?

   (c) Use your answer to (b) and (c) to draw the graph of $\sin t$ for $0 \le t \le 2\pi$.

   (d) Use a similar method to draw the graph of $\cos t$ for $-\pi \le t \le \pi$.

3. Use the unit circle picture, and the fact that $\sin \frac{\pi}{6} = \frac{1}{2}$, to find $\sin \frac{5\pi}{6}$, $\sin \frac{7\pi}{6}$, and $\sin \frac{11\pi}{6}$.

4. Find the limit:
$$\lim_{x \to -1} \frac{2x^2 + 3x + 1}{x^2 - 2x - 3}$$

5. Find the limit:
$$\lim_{x \to 0} \frac{x}{\sqrt{1 + 3x} - 1}$$
   (Hint: $(a - b)(a + b) = a^2 - b^2$)

6. (This is a homework problem from Lesson 8.) Find the limit:
$$\lim_{x \to -4} \frac{\sqrt{x^2 + 9} - 5}{x + 4}$$

7. Find the limit:
$$\lim_{t \to 0^-} \frac{t + 4}{t^2 + 2t}$$

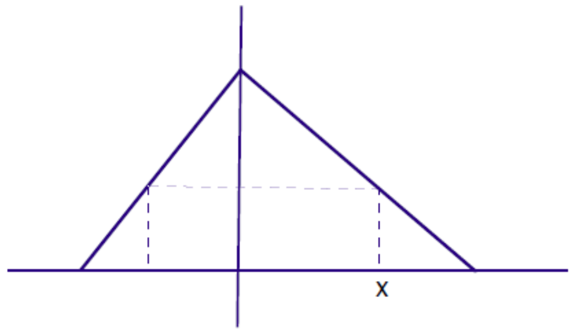
8. (a) Find the limit:
$$\lim_{t \to 0^+} \left(\frac{1}{t} - \frac{1}{t^2 + t}\right)$$
   (Hint: simplify first. The answer is not 0.)

   (b) Is $\infty - \infty$ equal to 0? (Hint: think about part (a))

9. Use common sense to find
$$\lim_{x \to 0^+} e^{-\frac{1}{x}}$$

The rest of today's problems are practice with algebra.

10. The two lines are $y = 4 - 2x$ and $y = 4 + 3x$. Assuming x is between 0 and 2, find a formula for the area of the rectangle in terms of x.



11. Simplify $\sqrt{x^4 + 2x^2 + 1}$

12. Show that

$$\left(\frac{1}{2x-2} - \frac{1}{x}\right) \cdot \left(\frac{x}{2} + \frac{\frac{1}{2}x}{x-2}\right)$$

simplifies to $-\frac{1}{4}$

B.2 Sample Problem Set from Reformed Calculus II

Problems for Week 1. Due Tuesday January 16.

1. Find $\int (2x^3 + 3x^{-1/2} + 4x^{-1})\, dx$

2. Find $\int x^2 e^{x^3}\, dx$

3. Find $\displaystyle\int \frac{\sin(\ln x)}{x}\, dx$

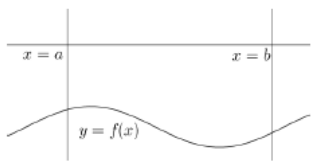
4. This problem and the next are warmups for Lesson 5, which will be Friday January 19. The reason for these problems is to help you understand why the main formula in Lesson 5 works the way it does. This will help you to avoid some common misunderstandings about this formula.

   (a) Find the definite integral $\int_0^2 (-3x)\, dx$

   (b) Find the area between the graph of $y = -3x$, the x-axis, and the line $x = 2$. Do not use calculus for this one. (Hint: the region is a triangle.) **NOTE:** areas are always positive.

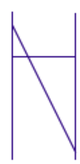
   (c) What is the relation between your answers to (a) and (b)?

   (d) (See picture below.) If $f(x)$ is a function which is negative for $a \leq x \leq b$, and if A is the area of the region between the graph of $f(x)$, the x-axis, and the lines $x = a$ and $x = b$, what do you think is the equation that relates A and $\int_a^b f(x)\, dx$? (Hint: think about your answer to part (c).)



5. (a) Find the definite integral $\int_0^2 (1 - 2x)\, dx$

   (b) In the picture below, the slanted line is $y = 1 - 2x$, the horizontal line is the x axis, and the vertical lines are $x = 0$ and $x = 2$. Find the areas of the two triangles.



   (c) How is your answer to (a) related to your answer to (b)?

6. You know that $\int \frac{1}{x}\,dx$ is $\ln|x| + C$. In this problem we assume x is positive so we have the simpler formula $\int \frac{1}{x}\,dx = \ln x + C$. Now you might expect that $\int \frac{1}{x^2+2x+2}\,dx$ would be $\ln(x^2 + 2x + 2) + C$.

   (a) Explain why this doesn't work. (Hint: what is the derivative of $\ln(x^2 + 2x + 2)$?)

   (b) Find $\int \frac{1}{x^2+2x+2}\,dx$ by using the fact that $x^2 + 2x + 2 = 1 + (x + 1)^2$

7. Two of these integrals can be done by the method of substitution. Do those two. (You'll learn how to do the other later this semester, by using power series.) **Check** your answers by differentiating.

   (a) $\int \frac{x}{1+x^4}\,dx$

   (b) $\int \frac{x^2}{1+x^4}\,dx$

   (c) $\int \frac{x^3}{1+x^4}\,dx$

8. One of these integrals can be done by the method of substitution. Do that one. **Check** your answer by differentiating.

   (a) $\int \frac{e^{1/x}}{x}\,dx$

   (b) $\int \frac{e^{1/x}}{x^2}\,dx$

9. Find $\int \frac{1}{e^{2x}}\,dx$

10. Find the definite integral $\int_0^3 x\sqrt{x^2+16}\,dx$, using the substitution $u = x^2+16$, in two different ways:

    (a) Find the antiderivative as a function of u, plug in the u values, and subtract.

    (b) Find the antiderivative as a function of x, plug in the x values, and subtract. This should give the same answer as part (a).

B.3 Sample Grading Policy for Calculus

Grading Methodology for MA 161

Exam 1	100	pts
+ Exam 2	100	pts
+ Exam 3	100	pts
+ Final Exam	200	pts
= **Common Exam Total**	500	pts
Common Exam Total	500	pts
+ Homework	100	pts
+ Quizzes	100	pts
+ IMPACT Sections Only	(90)	pts
= **Overall Score**	700 (or 790)	pts

A student's course letter grade will be determined from his/her **Overall Score**.
The grade distribution will be made uniform across different sections based on the **Common Exam Totals**.

Cutoffs will be applied uniformly across all sections to the **Common Exam Totals**. The number of each course letter grade A, B, C, D (with ±) and F in each section will be based on the number of each grade on the **Common Exam Totals** for the students in that particular section. By ranking students within each section by **Overall Score**, the course-wide cutoffs for **Common Exam Total** are translated into section-specific cutoffs for **Overall Score**. In this way, the number of allocated grades from a common scale will determine cutoffs on the students' **Overall Scores** within each section, and grades will be assigned accordingly.

EXAMPLE: Suppose the course-wide **Common Exam Total** cutoffs were 450 for A, 385 for B, 310 for C, and 250 for D. Also, suppose that one particular recitation section has ten students each with a **Common Exam Total** of 450 or more, and twenty-five students each with a **Common Exam Total** between 385 and 450, etc. To assign course grades, students in that section are ranked based on **Overall Score**; the instructor then counts from the top and draws a cutoff line after the tenth student, another cutoff line after the thirty-fifth student, etc.

Remarks

- The use of the common exam total in assigning final grades is designed to provide uniformity across different sections under the supervisions of different recitation instructors.
- The ± within category are determined (approximately) by the top 30% getting +, the bottom 30% getting −, and the middle no change. Students within a few points of a cutoff line may be bumped up.
- Although advisory letter grades are announced for each midterm, the **Common Exam Total** is a sum of **raw** numerical scores unadjusted in any way. The advisory cutoffs are designed to help students anticipate where they stand before the **Common Exam Total** cutoffs are assigned.
- Homework and quizzes constitute an extremely important part of the course. A poor score on either one of these categories can jeopardize students' effort towards a good grade, especially in borderline cases. On the other hand, homework and quizzes only contribute 200 points to the **Overall Score** while the exams contribute 500 points. Obviously a poor performance on the exams most likely will not be compensated by good scores on homework and quizzes.

B.4 Knowledge Exam for Calculus I

1. If a function is always positive, then what must be true about its derivative?

   (a) The derivative is always positive.

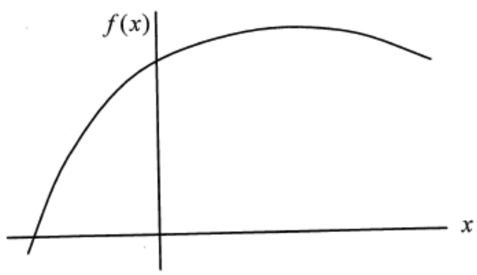
   (b) The derivative is never negative.

   (c) The derivative is increasing.

   (d) The derivative is decreasing.

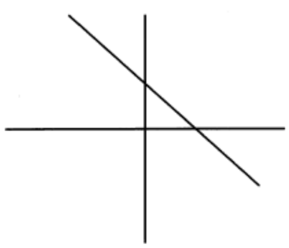
   (e) You can't conclude anything about the derivative.

2. Below is the graph of a function $f(x)$.

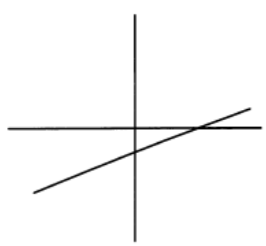


Which choice a) to e) could be a graph of the first derivative, $f'(x)$?

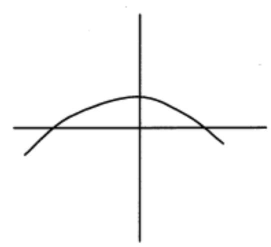
a)



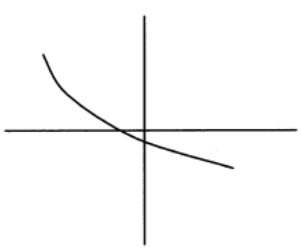
b)



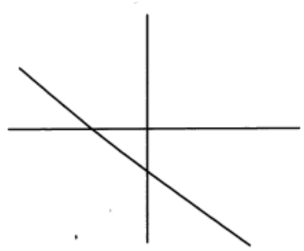
c)



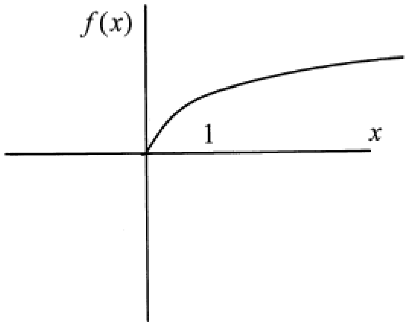
d)



e)



3. The tangent line to this graph at $x = 1$ is given by $y = \frac{1}{2}x + \frac{1}{2}$. Which of the following statements is true everywhere on the graph?



(a) $\frac{1}{2}x + \frac{1}{2} = f(x)$

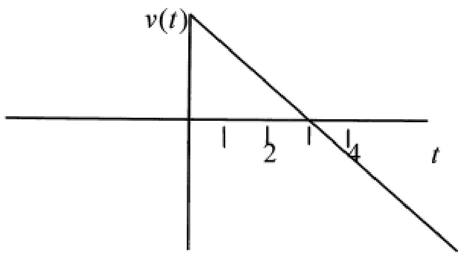
(b) $\frac{1}{2}x + \frac{1}{2} \leq f(x)$

(c) $\frac{1}{2}x + \frac{1}{2} \geq f(x)$

(d) $\frac{1}{2}x = \frac{1}{2}f(x)$

(e) None of these.

4. A graph of a moving object's velocity against time is shown.



The acceleration is the rate of change of the velocity with time. The acceleration of the object is which of the following?

(a) Positive everywhere.

(b) Negative everywhere.

(c) Zero everywhere.

(d) Zero at some times but not others.

(e) It cannot be determined from the given information.

5. Yesterday's temperature at t hours past midnight was $T(t)$ degrees Fahrenheit. At noon the temperature was 50 degrees. The first derivative $T'(t)$ reached a low of $+2$ degrees/hour at 6:00 a.m., then increased for the rest of the day. Which of the following is correct?

   (a) Temperature fell from midnight to noon and rose after that.

   (b) At 1 p.m. the temperature was 52 degrees.

   (c) At 1 p.m. the temperature was 48 degrees.

   (d) The temperature was lower at 6:00 a.m. than at any other time.

   (e) The temperature rose during the whole 24 hour period.

6. The derivative, $f'(x)$, of a function $f(x)$, is negative everywhere. We also know that $f(0) = 0$. What must be true about $f(-1)$?

   (a) $f(-1)$ is negative.

   (b) $f(-1)$ is positive.

   (c) $f(-1)$ is zero.

   (d) Not enough information to conclude anything about $f(-1)$.

7. Let f be the function defined by $f(x) = x + 2$ and let g be the function defined by $g(u) = u + 2$, for all real numbers x and u. Which of the following is true?

   (a) f and g are exactly the same function.

   (b) f and g are different functions because x and u are different variables.

   (c) f and g are different functions whenever x and u are different numbers.

   (d) Not enough information is given to determine if f and g are the same function.

8. The derivative of a function f is given by $f'(x) = ax^2 + b$. What is required of the values of a and b so that the slope of the tangent line to f will be positive at $x = 0$?

   (a) a and b must both be positive numbers.

   (b) a must be positive, while b can be any real number.

   (c) a can be any real number, while b must be positive.

   (d) a and b can be any real numbers.

9. Suppose that from 1980 until 2000, the rate at which a certain redwood tree grew was constant. Let $h(t)$ represent the height of the redwood in feet t years after 1980. Which function could realistically represent $h(t)$ from 1980 to 2000?

   (a) $h(t) = 50$

   (b) $h(t) = 50 + \frac{1}{2}t$

   (c) $h(t) = 1980 + \frac{1}{2}t$

   (d) $h(t) = 50(\frac{3}{2})^t$

   (e) None of these.

10. The derivative of a function is negative everywhere on the interval $x = 2$ to $x = 3$. Where on this interval does the function have its maximum value?

   (a) At $x = 2$.

   (b) At $x = 3$.

   (c) Somewhere between $x = 2$ and $x = 3$.

   (d) It does not have a maximum, since the derivative is never zero.

   (e) We cannot tell if it has a maximum since we don't know where the second derivative is negative.

B.5 Knowledge Exam for Calculus II

1. Find $\int_1^8 x^{-1/3}\,dx$
   (a) 1/2
   (b) 3/2
   (c) 5/2
   (d) 7/2
   (e) 9/2
2. Find $\int_1^e \frac{\ln x}{x}\,dx$
   (a) 1/4
   (b) 1/2
   (c) 1
   (d) 2
   (e) 4
3. Find $\int_1^2 \frac{(x-1)^2}{x}\,dx$
   (a) $\ln 2 - 3/2$
   (b) $\ln 2 - 1/2$
   (c) $\ln 2 + 1/2$
   (d) $\ln 2 + 3/2$
   (e) $\ln 2 + 5/2$
4. Find $\int_0^{\pi/4} \sec^2 x\,dx$
   (a) 1/4
   (b) 1/3
   (c) 1/2
   (d) 2/3
   (e) 1

5. Find $\int_0^2 x^2\sqrt{x^3+1}\,dx$

(a) 50/9
(b) 52/9
(c) 53/9
(d) 55/9
(e) 56/9

6. Find $\int_0^1 \frac{1}{(1+2x)^2}\,dx$

(a) 1/4
(b) 1/3
(c) 1/2
(d) 2/3
(e) 1

7. Find $\int_0^{\pi/4} \sin^3 x \cos x\,dx$

(a) 0
(b) 1/16
(c) 1/8
(d) 1/4
(e) 1/2