INTERACTION AMONG SUPPLY CHAINS:

CONSUMERS, FIRMS AND POLICYMAKERS

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Dedicated to my loving family.

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TABLE OF CONTENTS

Page

LIST OF TABLES . . . viii

LIST OF FIGURES . . . ix

ABSTRACT . . . xi

1 INTRODUCTION . . . 1
- 1.1 Firm-Consumer Interface . . . 1
- 1.2 Supplier-Buyer Interface . . . 2
- 1.3 Firm-Government Interface . . . 3
- 1.4 Organization of the Dissertation . . . 3

2 FIRM-CONSUMER INTERFACE: COMPETITIVE REVENUE MANAGEMENT WITH SEQUENTIAL BARGAINING . . . 5
- 2.1 Synopsis . . . 5
- 2.2 Literature Review . . . 7
- 2.3 The Problem . . . 11
- 2.4 The Benchmark Models . . . 14
  - 2.4.1 A Dynamic Model with a Monopoly Seller . . . 14
  - 2.4.2 A Static Model with Competing Sellers . . . 17
- 2.5 A Dynamic Model with Competing Sellers . . . 21
  - 2.5.1 Characterization of the Sellers' Equilibrium Strategies . . . 23
  - 2.5.2 The Effect of Inventory on Competition Dynamics . . . 32
  - 2.5.3 The Effect of Non-homogeneous Buyer's Valuation . . . 38
- 2.6 Concluding Remarks . . . 40

3 SUPPLIER-BUYER INTERFACE: NEGOTIATIONS IN COMPETITIVE SUPPLY CHAINS: THE KALAI-SMORODINSKY BARGAINING SOLUTION . . . 42
- 3.1 Synopsis . . . 42
- 3.2 Literature Review . . . 46
- 3.3 Preliminaries: The Kalai-Smorodinsky Bargaining Solution . . . 48
- 3.4 The Problem of Multi-Unit Bilateral Bargaining . . . 52
  - 3.4.1 A Benchmark Model: The One-to-One Channel . . . 55
- 3.5 Negotiations in Competing Supply Chains . . . 56
  - 3.5.1 Negotiations without Contingencies . . . 57
  - 3.5.2 Negotiations with Contingencies . . . 64
- 3.6 Discussions . . . 69

Page
3.6.1 The Effect of Negotiation Sequence 69
3.6.2 Alternative Demand Functions 73
3.6.3 Bilaterally Coordinating Contracts 75
3.7 Concluding Remarks 78
4 FIRM-GOVERNMENT INTERFACE: SUBSIDY FOR PRODUCERS IN A FRAGMENTED MARKET 80
4.1 Synopsis 80
4.2 Literature Review 83
4.3 The Problem 86
4.3.1 A Benchmark Model: Without Subsidy 88
4.3.2 Preliminaries: The Majorization Order 91
4.4 The Effect of Subsidies on the Farmers' Incentives 93
4.4.1 Combined Subsidy 93
4.4.2 Selective Subsidy 99
4.4.3 Comparisons 105
4.5 The Government's Objectives and Subsidy Design 106
4.5.1 The Optimal Subsidy Design 106
4.5.2 Comparisons Among Subsidy Programs 109
4.5.3 Implications on Social Welfare 115
4.6 Conclusion 118
5 CONCLUSION AND DIRECTION FOR FUTURE RESEARCH 119
REFERENCES 122
A Proofs of Formal Results in Chapter 2 132
B Proofs of Formal Results in Chapter 3 153
C Proofs of Formal Results in Chapter 4 171
D Derivation of the Sellers' Profit Functions in Section 2.5.1 190
E The Feasible Profit Allocation Set 193
F Derivation of the Bargaining Solutions 195
F.1 One-to-Two Channel 195
F.2 Two-to-One Channel 203
VITA 212

LIST OF TABLES

Table Page

2.1 Trading parties' profits in a single-unit trade under a bargained price s_B. . . 15

2.2 The effect of inventories on the dynamic equilibrium. 36

2.3 The effect of time-dependent buyer's valuation on the dynamic equilibrium. 39

A.1 The effect of inventories on the dynamic equilibrium in the two-period game. 148

A.2 The effect of inventories on the dynamic equilibrium for $t = 3$. 149

LIST OF FIGURES

Figure Page

2.1 Summary of buyer's choice in the static competition, as characterized in (2.4) and (2.6). 18

2.2 Summary of equilibrium regions with respect to (w_j, w_i), as characterized in Proposition 2.5.1 (left panel) and Proposition 2.5.2 (right panel). 30

2.3 Seller j's reservation value and disagreement point with respect to (n_j, n_i). . . 35

2.4 Sellers' reservation values (left panel), disagreement points (middle panel) and buyer's choice (right panel) as functions of (n_j, n_i). 37

3.1 The two-tier supply chain models: one-to-two (left panel) and two-to-one (right panel). 52

3.2 The trade profits in bargaining unit i for a given w_j, as characterized in Lemma 3.5.1. 60

3.3 The negotiated NB profits in bargaining unit i as characterized in Lemma 3.5.1.62

3.4 The negotiated NB profits in bargaining unit i for a given w_j, as characterized in Lemma 3.5.2. 66

3.5 The retailer's equivalent bargaining power implied by the KS solution. . . 68

3.6 The profit comparison in the case with contingency. 72

3.7 The retailer's equivalent bargaining power in the case without contingency. 76

4.1 The equilibrium output levels (left panel) and profits (right panel) under the combined subsidy (s_P, s_H). 96

4.2 The increased output distribution with respect to the increased planting (left) and harvesting (right) subsidies. 98

4.3 Condition for production: selective subsidy. 101

4.4 Equilibrium outcome under the selective subsidy (s_P, s_H): output level (left panel) and profit (right panel). 102

4.5 The increased output distribution with respect to the increased planting (left) and harvesting (right) subsidies. 104

4.6 The budget with respect to the overall output level under the combined subsidy. 108

Figure Page

4.7 The optimal selective subsidy with respect to the target output level. . . 110

4.8 Budget Comparison. 112

4.9 Overall Input Comparison. 113

4.10 The output distribution with respect to the target output level. 115

4.11 The social welfare comparison. 117

D.1 Summary of buyer's choice in the dynamic competition, as characterized in (2.12), (2.13) and (2.14). 190

ABSTRACT

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This study explores the vertical relationships in the supply chain at three different levels, namely, firm-consumer interface, supplier-buyer interface, and firm-government interface. We provide a brief description of the results obtained for the specific problems considered in this study.

The firm-consumer interface is examined in Chapter 2. We explore firms' selling strategy when dynamically competing for a common stream of consumers. In the situation of pure price competition, a commonly studied case, it is known that the seller with a higher stock level can compete more effectively by forcing the seller with a lower stock level to sell out first and enjoy a monopoly power afterward. We show that when the sellers are open to price bargaining as a way of attracting buyers, the competition equilibrium can exhibit different outcomes. When the overall stock held by the sellers is limited, there is a good chance that both sellers deplete the inventories before the end of the selling season. In this case, an incoming buyer would prefer a high inventory seller, with whom he can bargain down the price. Interestingly, such a phenomenon only appears when the length of selling season is long enough. Thus, our study highlights the unique role of bargaining in consumer markets, as well as the importance of time horizon in characterizing equilibrium for dynamic games.

The supplier-buyer interface is studied in Chapter 3. In recent years, an increasing number of studies have applied the Nash bargaining (NB) solution to study channel relationships. However, this solution concept builds on an unrealistic axiom of independence of irrelevant alternatives. We demonstrate that, indeed, the NB solution can produce unreasonable outcomes in vertical negotiations. For example, a supplier

negotiating with a monopoly retailer can end up making a higher profit than the one negotiating with a retailer facing potential competitions. To address this issue, we examine the Kalai-Smorodinsky (KS) solution as an alternative. Our analysis suggests that in competing supply chains, the KS solution appropriately captures the negotiation power shift induced by the decision ownership, the negotiation sequence, the vertical relationship, the competition intensity, the contract contingency, and the contract type. This is the first time the KS solution concept is applied to supply chain negotiations.

The firm-government interface is explored in Chapter 4. From the policymakers' perspective, incentives firms actions toward increasing the product consumption for the needy group or increasing social welfare has a major influence in many supply chains. For example, agricultural products are subsidized by many governments. In this study, we analyze the design of government subsidy programs to induce socially improved firm decisions. We show that subsidizing on production input can lead to a more balanced distribution of market shares and firm profits than subsidizing on production output. Moreover, firms with efficient production technology prefer output subsidy, while those with inefficient production technology favor input subsidy.

1. INTRODUCTION

This study addresses several emerging issues in supply chain management. On the one hand, bargaining is commonly observed in practice. In business-to-business transactions, deals are reached after rounds of negotiations. For consumer products, bargaining is also commonly observed for large ticket items (for example, car and furniture selling). On the other hand, subsidy programs are widely applied in both developing and developed countries to encourage consumption of products that generate positive economical, environmental and social benefits. In this dissertation, we examine interactions among supply chain in different contexts, namely, firm-consumer interface, supplier-buyer interface, and firm-government interface, and understand the role of negotiation and subsidy programs in these interactions.

We first study competing firms' dynamic pricing strategies considering the possibility of bargaining. Then, we consider the negotiation outcomes in two-tier supply chains by applying different bargaining solution concepts. Furthermore, we consider subsidy programs for producers in a fragmented market. A brief introduction of this dissertation is provided below.

1.1 Firm-Consumer Interface

When firms post prices to sell their products in a competitive market, buyers often seek to bargain down the prices. The existing studies on dynamic competition, however, focus on firms' dynamic pricing strategies without considering the possibility of bargaining. We model a random stream of buyers with heterogeneous valuations who may choose to negotiate for a discount from a posted price. When bargaining is allowed, a seller firm's competing strategy depends not only on her *reservation value* (i.e., her gain from an unsold item when losing the current buyer to the opponent) but also on her *disagreement point* (i.e., the value of an unsold item when the negotiation with an arriving buyer breaks down). We show that the reservation values and the disagreement points can play opposite roles in competition—The seller firms' posted prices are increasing in their reservation values but are decreasing in their disagreement points. In general, a seller becomes more aggressive in price competition when her reservation value becomes lower. However, it is not always the case that a seller with a lower reservation value can successfully win the deal from an arriving buyer, as it is in pure price competition. Because of the possibility of bargaining, the buyer may end up purchasing from a seller who has a higher inventory level and a higher reservation value when the seller's disagreement point is lower than that of her opponent. Interestingly, such an equilibrium outcome can only arise when the selling season is sufficiently long. Our analysis highlights the intriguing role played by negotiation in dynamic competition for sequential selling.

1.2 Supplier-Buyer Interface

Supply chain contract negotiation has gained increasing attention in recent years, and the studies involving negotiations in the operations literature almost exclusively apply the concept of the Nash bargaining (NB) solution. The NB solution, however, is derived based on the axiom of independence of irrelevant alternatives (IIA), an unrealistic assumption widely criticized in economics. Indeed, our analysis suggests that the NB solution can lead to unreasonable negotiation outcomes in competing supply chains. As an alternative, the Kalai-Smorodinsky (KS) solution has been applied in many fields, but has not been introduced to the supply chain contexts. We apply the KS solution to study contract negotiations in competing supply chains, and analyze its connection to and difference from the NB solution. In particular, we show that, compared with the NB solution, the KS solution appropriately captures the negotiation power shift induced by the decision ownership, the negotiation sequence, the vertical relationship, the competition intensity, the contract contingency, and the contract type.

1.3 Firm-Government Interface

Subsidy programs for agricultural products are primarily aimed toward increasing the market output. Two types of aids are commonly used in agricultural industry. Planting subsidy reduces input cost for farmers and harvesting subsidy reduces cost during the output collection and distribution process. The effects of two subsidies on farmers' output decisions as well as their welfare distribution, however, are not well understood. We model a fragmented market in which farmers differ in their productivity levels. The government can offer a *combined subsidy* (i.e., farmers get payments for both plantation and harvesting) or a *selective subsidy* (i.e., farmers each choose either payment). We first observe that under either combined subsidy or selective subsidy, a higher harvesting subsidy widens the gaps among the farmers in both their outputs and profits, while a higher planting subsidy narrows the gap in farmers' outputs when the plantation is not overly subsidized. Moreover, farmers' outputs are more evenly distributed under the combined subsidy than under the selective subsidy. Second, when the government attempts to achieve the target output level with minimum budget, the combined subsidy is always preferred regardless of target output level. However, the combined subsidy requires excessive needed input and induces undesirable social welfare when the target output level is not far from the overall output without subsidy.

1.4 Organization of the Dissertation

In Chapter 2, we consider two firms competing for a stream of incoming buyers with the possibility of bargaining. We formulate a finite-time stochastic dynamic game and characterize the competition equilibrium. Insights are obtained through a detailed discussion of a four-period game.

In Chapter 3, we consider two-tier supply chains consisting of one or two suppliers selling products to one or two retailers. We derive and compare the negotiation outcomes by applying the Nash and Kalai-Smorodinsky bargaining solutions, respectively.

In Chapter 4, we consider a static competition in which farmers vary from one another in their productivity levels. The government, aiming toward increasing the overall market output, initiates a farmer subsidy program. We examine the incentives offered via various subsidy programs on farmers' output decisions and resulting welfare allocation.

Chapter 5 concludes the dissertation and provides suggestions for future research.

Chapter 2 is based on Feng et al. (2020a) and Chapter 3 is based on Feng et al. (2020b). The main results in Chapter 4 are from Feng et al. (2020c). I would like to express my sincere appreciation to my co-authors, Professors Qi Annabelle Feng, J. George Shanthikumar and Yifan Wu for their invaluable contributions.

2. FIRM-CONSUMER INTERFACE: COMPETITIVE REVENUE MANAGEMENT WITH SEQUENTIAL BARGAINING

2.1 Synopsis

Sequential selling in a competitive market is commonly observed in practice. As customers arrive at the market, firms often dynamically adjust their selling strategies to effectively compete against one another and to earn revenue. Understanding firms' decisions of sequentially selling their capacities or inventories is the central theme of revenue management (Bodea and Ferguson 2014, Phillips 2005, Talluri and Ryzin 2004), and there is a growing attention paid to analyzing such decisions in a competitive environment (e.g., Chen and Chen 2015, Gallego and Hu 2014, Martínez-de-Albéniz and Talluri 2011). The studies in this area heavily focus on firms' dynamic pricing strategies.

When firms set public posted prices, buyers often seek to bargain down the prices. In most of the business-to-business transactions, deals are reached through rounds of negotiations between sellers and buyers. In consumer markets, bargaining is also prevalent for large ticket items (e.g., houses, cars, furniture, and expensive electronics) and services (e.g., cleaning, moving, and maintenance). Bargained discounts off posted prices are well documented in automobile, real estate, and retail industries (see, e.g., Gill and Thanassoulis 2013, and references herein). Allowing for bargaining down the posted prices, the seller firms may successfully discriminate the buyers, as buyers with different willingness to pay would settle different negotiated prices (Arnold and Lippman 1998, Feng and Shanthikumar 2018a). The implication of buyer negotiation on seller firms' competition dynamics, however, is not well understood.

When a single firm dynamically sells to a stream of customers, the selling strategy depends critically on the seller firm's *disagreement point* (i.e., her future gain on an unsold item when the negotiation with the buyer breaks down). As suggested by Feng and Shanthikumar (2018a), the monopoly seller firm's disagreement point is lower with a higher stock level and with a shorter selling season left. A lower disagreement point weakens the seller's bargaining position and reduces the negotiated price, likely lowering the final trade price. When there are competing sellers, however, the competitive decisions also depend on the seller firms' *reservation values* (i.e., their future gains when losing the current customer to the competitor). Interestingly, the reservation values and disagreement points affect the sellers' posted prices in the opposite directions—A seller firm posts a higher price when her reservation value becomes larger, or when her disagreement point becomes smaller.

Generally speaking, the intensity of competition between the sellers is determined by their reservation values. The lower the reservation value is, the more aggressive the seller is in competition. A seller with an extremely low reservation value may even price below her disagreement point. In such a situation, an arriving buyer chooses to purchase at the posted price without entering negotiation. In the special case where both sellers have the same disagreement points, the one with a lower reservation value always posts a price that is no higher than that with a higher reservation value. However, posting a price lower than the opponent does not guarantee that the seller wins the competition. Because of the possibility of bargaining, the posted prices become irrelevant to the final trade price if the posted prices are above the arriving buyer's valuation. In this case, the buyer becomes indifferent between the sellers despite the difference in their posted prices.

When the two sellers have different disagreement points, interestingly, the seller with a lower reservation value may post a price higher than her opponent who has a higher reservation value. This happens when the former has a lower disagreement point than her opponent. A lower disagreement point means more trade surplus is available for the buyer when negotiating with that seller. The seller, in turn, can post a higher price and yet stay attractive to an arriving buyer. We also find that, while in some cases the buyer chooses the seller with a lower inventory level as in the case of pure price competition, there are exceptions. Such exceptions occur when both sellers have a good chance to deplete their stocks by the end of the selling season. The competition between the sellers is softened and both sellers post high prices. High posted prices induce negotiation by an arriving buyer, who would choose the seller with more items and thus with a lower disagreement point. It is no longer the case that the seller with a lower initial inventory level can deplete her stocks and then the one with a higher initial inventory level sells at the monopolist prices, as in the situation of pure price competition (Martínez-de-Albéniz and Talluri 2011). These observations highlight that the competition dynamics in a market with bargainers can be significantly different from those in a market without bargainers.

The remainder of the paper is organized as follows. In Section 2.2, we review the related literature and articulate our contributions. The problem of dynamic competition is described in Section 2.3. In Section 2.4, we present two benchmark models, namely, the one-seller case and the one-period case of our problem. In Section 2.5, we develop a detailed formulation of the problem and characterize the competition equilibrium. We conclude in Section 2.6. Proofs of all formal results are relegated to the appendix A.

2.2 Literature Review

Our research is related to three streams of literature. The first stream concerns the comparison between pricing and bargaining under a static competition, the second stream studies bargaining under a monopoly sequential selling, and the third stream focuses on dynamic price competition.

The comparison between pricing and bargaining as alternative selling mechanisms that firms use to compete for customers has been extensively researched in marketing and economics. For example, Bester (1993) models multiple symmetric sellers, who each decide their own product quality, compete for a single buyer. The buyer only observes the product quality after visiting a seller and incurs a switching cost for visiting each additional seller. Bester shows that all sellers should choose to bargain, as opposed to setting a take-it-or-leave-it price, when the buyer's switching cost is high and the sellers' bargaining power vis-à-vis the buyer is large. Bester (1994) studies a variation of his earlier model by assuming that sellers provide the same product quality but are heterogeneous in their cost to preclude bargaining. In equilibrium, all sellers choose bargaining over pricing when it is not costly for the buyer to switch. Adachi (1999) studies two sellers competing for buyers uniformly located over the Hotelling line and finds sellers choose pricing in equilibrium when the buyers' valuation for the product is sufficiently high. Desai and Purohit (2004) incorporate the fact that not all buyers may like to negotiate in Adachi's model. They show that sellers make more profit by pricing than by bargaining when a significant portion of buyers are bargainers. Gill and Thanassoulis (2009, 2013) consider price competition among multiple sellers who also allow buyers to bargain. They conclude that with an increased portion of bargainers in the market, competing sellers would increase their posted prices, resulting in reduced social welfare.

None of the aforementioned studies consider the effect of inventory on seller competition. The role of inventory in competition has been a focus by many researchers in the area of operations management (see, e.g., Aksoy-Pierson et al. 2013, Bernstein and Federgruen 2004, 2005, Netessine and Shumsky 2005). The operations literature, however, does not consider buyer-seller interaction through bargaining. Moreover, all these studies assume a one-time competition among the sellers. In such settings, it is without loss of generality to assume that a seller's reservation value (i.e., her profit when losing a buyer to her opponent) or her disagreement point (i.e., her profit in the event of negotiation breakdown) is constant and can be normalized to zero. As our discussion unfolds, it will become clear that reservation values and disagreement points play crucial roles in determining sellers' competition strategies in sequential selling processes.

Bilateral bargaining using the Nash bargaining solution (Nash 1950) has gained increasing interests in supply chain research (see, e.g., Chu et al. 2019, Feng and Lu 2013a, Hsu et al. 2016, Wang et al. 2017). Allowing bargaining as an alternative selling strategy in sequential trades has been analyzed for the case of a monopoly seller. Wang (1995) considers a seller with one item to sell over an infinite time horizon. He finds it always profitable for the seller to be open to bargain with arriving buyers. Kuo et al. (2011) extend Wang's model by considering a firm selling multiple units over a finite horizon and conclude that the seller should (not) allow bargaining when she has a large (small) inventory to sell over a short (long) horizon. Arnold and Lippman (1998) provide an alternative formulation of Wang's model by setting the seller's disagreement point for bargaining as her value of keeping the item for a potential future sale. Following the notion of Arnold and Lippman (1998), Feng and Shanthikumar (2018a) study the version of the problem with a finite selling season. They show that when the buyer's valuation is increasing (decreasing) in a certain stochastic order, which they term the *scaled pricing order*, the seller should choose pricing (bargaining) when she has a small (large) stock to sell over a long (short) horizon. All these studies assume complete information and apply the Nash bargaining solution (Nash 1950) to determine the negotiated prices. When the seller and buyers possess private information of their trade values (see, e.g., Ayvaz-Cavdaroglu et al. 2016, Bhandari and Secomandi 2011), the outcome of negotiation is often derived using the direct mechanism proposed by Myerson (1983). In this context, however, there has not been any discussion on how the seller and the buyers may credibly update their beliefs of others' valuations based on common information (e.g., time periods, past sales). It is important to note that a monopoly seller firm's reservation value equals her disagreement point in a sequential selling process. It would not be the case in general when seller competition is introduced.

There is a growing body of literature on inventory-based or capacity-based dynamic price competition (see the survey by Chen and Chen 2015). One common approach is to consider a continuous flow of consumers and formulate a differential game (e.g., Chintagunta and Rao 1996, Currie et al. 2008, Feichtinger and Dockner 1985, Gallego and Hu 2014, Mookherjee and Friesz 2008, Xu and Hopp 2006). An alternative approach to tackle the problem uses robust optimization (e.g., Adida and Perakis 2010, Perakis and Sood 2006). Both methods adopt the concept of open-loop equilibria to preserve tractability. The equilibrium price paths are then specified as functions of time but not those of the system states (e.g., stock levels). Thus, these approaches do not provide much understanding on the role of inventory in dynamic competition. Dudey (1992) studies two sellers with finite stocks competing for a fixed size of homogeneous buyer population. The buyers are assumed to have a constant common valuation of the product. In contrast to Bertrand-Edgeworth's static model, in which a pure equilibrium does not exist in general, Dudey proves the existence of a pure equilibrium under any stock levels. Martínez-de-Albéniz and Talluri (2011) extend Dudey's model by considering an uncertain size of buyer population and conclude that the seller with a lower stock level has a lower reservation value and first depletes her inventory before the opponent can make a sale. Lin and Sibdari (2009) consider multiple sellers with finite stocks competing over a finite horizon. Their numerical analysis suggests that the seller with a higher stock level posts a higher price. The sellers in all these studies post take-it-or-leave-it prices. There is a lack of understanding of how allowing buyer negotiation may change the competition dynamics among the sellers. Analysis from our model reveals that when allowing for bargaining, the seller with a higher stock level may win an arriving buyer. The intricacy of such an equilibrium outcome is due to the fact that a seller's reservation value of losing a buyer is different from her disagreement point for negotiation breakdown in the context of dynamic competition.

2.3 The Problem

We consider two competing seller firms, indexed by 1 and 2, selling a certain product to a stream of incoming buyers over a finite selling season under complete information.

The Buyer Stream. A stream of potential buyers arrives sequentially to purchase the product during the selling season. The selling season is divided into small enough time intervals of equal length so that there can be at most one buyer arrival in each time interval. We use $t \in \{T, T-1, \cdots, 1\}$ to index the time intervals, where T is the length of the selling season. We assume that the probability of a customer arrival is time homogeneous and is denoted by $\lambda \in (0, 1]$. It is, however, straightforward to extend our analysis to time-dependent or Markov-modulated arrivals, with which the key insights obtained from the model remain unchanged though additional Markov state variables appear in the profit functions. The buyers are heterogeneous in their willingness to pay. A potential buyer's valuation of the product is a random variable R. The seller firms observe the value of $R = r$ upon a buyer's arrival. This assumption, though ignores the possibility of information asymmetry and misreporting incentive, is a good approximation in certain applications. For example, in car selling processes, a salesperson normally starts conversations with an arriving buyer about the latter's planned budget, use of the car, and financing options. Such information normally allows an experienced salesperson to have a good idea about the buyer's valuation before price negotiation starts. We assume that $0 \leq \underline{r} \leq R \leq \bar{r}$, allowing the possibility of $\bar{r} = \infty$. Let $F_R(\cdot)$ denote the distribution function and $f_R(\cdot)$ the density function of R. We also use $\bar{F}_R(\cdot) = 1 - F_R(\cdot)$ to denote the survival function and $h_R(\cdot) = f_R(\cdot)/\bar{F}_R(\cdot)$ to denote the hazard rate function of R. An arriving buyer would choose to purchase the product if and only if he obtains a positive trade surplus, i.e., the selling price is below his valuation. Without loss of generality, we assume that the buyer, if choosing not to purchase, obtains zero value. We shall note that the existing literature commonly assumes a constant common buyer's valuation (e.g., Anton et al.

2014, Dasci and Karakul 2009, Martínez-de-Albéniz and Talluri 2011). Alternatively, there are some studies that assume uniformly distributed buyer's valuation (e.g., Liu and Zhang 2013, Mantin et al. 2011). We do not assume any specific valuation distribution for our analysis. All we require is that the distribution has an increasing hazard rate $h_R(\tau)$. This condition, satisfied by most commonly used distributions, allows one to establish the (quasi-)concavity of the sellers' profit functions and thus ensures a unique price as the best response in competition.

The Selling Mechanisms. At the beginning of each period, seller j posts a price s_j, $j \in \{1, 2\}$, and is open to bargain with an arriving buyer. An arriving buyer can choose to purchase from either seller at the posted price s_j, to purchase after negotiating a price s_{Bj} with either seller, or to walk away. We assume that the buyer's bargaining power is $\theta \in (0, 1)$ and each seller's is $(1 - \theta)$. The sellers' marginal costs of offering the product, which may include material and labor costs, are assumed to be zero. Consideration of positive marginal costs does not change the main insights derived from our analysis.

The Seller Competition. At the beginning of each period, the seller firms each review their stock levels, denoted by (n_1, n_2), based on which they post selling prices s_1 and s_2, respectively, and both are open for negotiation. An arriving buyer, observing the posted prices, chooses to visit one or neither of the sellers. The buyer may bargain with the chosen seller, expecting to obtain a lower trade price than the posted price. In the event of negotiation breakdown, the buyer can always purchase the product at the price posted by the chosen seller. The buyer has the option to walk away at any time. In situations where a buyer finds himself indifferent between the sellers, we assume that the buyer chooses either seller with equal probability.

The Sequence of Events. At the beginning of each period $t \in \{T, T - 1, \ldots, 1\}$, when there are t periods to the end of the selling season, the following events happen in sequence:

1. Each seller reviews her stock level n_j and decides a price s_j, $j \in \{1, 2\}$.

2. With probability λ, a buyer arrives. Based on the prices posted by the sellers, the buyer may choose to visit one of the sellers or neither.

3. Depending on the buyer's valuation, the negotiated price of the product is determined. The buyer either makes a purchase or walks away.

For ease of exposition, we assume that any leftover item at the end of the selling season, i.e., $t = 0$, has no value to the sellers. Our analysis can be easily extended to the case allowing for positive salvage values.

We would like to point out that the buyer in our model would carefully evaluate his potential options when choosing a seller to interact with. This is suitable for the situation where the sellers' information is easily accessible by the buyer, and the cost of visiting a seller is significant. Thus, a rational buyer would carefully evaluate the potential purchase options, choose one seller to interact with and strike a deal, and avoid the additional cost of switching between the sellers. The high cost of switching can also due to the nature of the transaction. For example, if the purchase of the product requires significant customization based on the buyer's request, it can be costly to go through that process several times. Moreover, if the buyer were to freely negotiate back and forth with both sellers and leverage one bargaining against the other, the negotiated prices would equal the sellers' reservation values (i.e., their values of keeping an item while losing the buyer to their opponent). This is an analytically uninteresting outcome and is rarely observed in reality. The reason that such an outcome is unlikely in practice is two-fold. First, a seller may often restart, rather than resume, a negotiation with a returning buyer because a different seller agent is working with the buyer or the trading conditions may have changed by the time the buyer returns. Second, the buyer always pays time and effort to visit a seller for negotiation. Our way of modeling essentially entails a significant cost of switching sellers, as such the buyer would assess his options before choosing a seller to avoid the switching costs.

We use a superscript $*$ to denote the quantities derived in equilibrium. Like in many models with price competition, a pure strategy equilibrium may not exist when one party can always price lower than the other party. In this case, we use the convention of the ϵ-*equilibrium* (see, e.g., Allon and Gurvich 2010, Dixon 1987, Lu et al. 2009, Radner 1980, Tijs 1981), which allows an equilibrium to arise with one party's price lower than the other's by some small positive value ϵ.

Before analyzing the above dynamic competition problem, we present some key findings from two benchmark models in the next section. These benchmark models become building blocks for formulating the dynamic problem and allow us to obtain clear insights into the competition dynamics in section 2.5.

2.4 The Benchmark Models

In this section, we briefly summarize the observations from two benchmark models, one concerning a monopoly seller in a dynamic setting (section 2.4.1) and the other concerning two competing sellers in a static setting (section 2.4.2).

2.4.1 A Dynamic Model with a Monopoly Seller

When a single seller monopolizes the market, it is intuitive that a seller would never sell an item at a price below the value of that item. This value, denoted by w, is the seller's *reservation value*. Given the dynamic selling process, the seller's reservation value at a given time depends on the number of items the seller has to sell before the end of the selling season, the seller's future decisions and the choices of potential future buyers. By the end of the selling season, there is no future selling opportunity and the seller's reservation value would simply be the salvage value of the remaining items. As we mentioned in section 2.3, we assume this value to be zero and this assumption is not critical to our analysis of the dynamic model. The seller posts a price s and the buyer may choose to purchase at this price, to enter negotiation, or to walk away. If negotiation takes place and a negotiated price s_B is agreed upon, the buyer obtains a value of $r - s_B$ and the seller obtains a value of s_B.

The negotiation outcome critically depends on the trading parties' *disagreement points*. The disagreement points come from the options the parties' have when choosing not to agree on a deal. For the buyer, there are two options, purchasing at the posted price to obtain $r - s$ or walking away to obtain 0. Thus, the buyer's disagreement point is $(r - s)^+$. For the seller, the only option is to walk away from the buyer, as the seller cannot force the buyer to purchase. As a result, the seller's disagreement point is always $v = w$. The trade can then be summarized in Table 2.1.

Table 2.1.
Trading parties' profits in a single-unit trade under a bargained price s_B.

	Buyer	**Seller**
Trade Profit	$r - s_B$	s_B
Disagreement Point	$(r - s)^+$	v
Trade Surplus	$r - s_B - (r - s)^+$	$s_B - v$

By Nash (1950), the negotiated price s_B should maximize the following Nash product:

$$(s_B - v)^{1-\theta}(r - s_B - (r - s)^+)^{\theta}.$$

This leads to a negotiated price of $s_B(r, s, v) = (1 - \theta)(r \wedge s) + \theta v$ and the expected seller's profit from a single-unit trade of

$$\Psi(v) = \max_{s \geq v}\{\mathbb{E}[\mathbb{I}_{\{R<v\}}v] + \mathbb{E}[\mathbb{I}_{\{v \leq R\}}s_B(R, s, v)]\}. \tag{2.1}$$

Lemma 2.4.1 (Price of the Monopoly Seller) *The seller maximizes her single-unit trade profit by posting a price $s^* = \bar{r}$, which makes the buyer's disagreement point zero.*

Lemma 2.4.1 suggests that the monopoly seller would post a price that is unacceptable to (almost) all buyers. This is because in any successful negotiation, the buyer's surplus is no less than what he obtains by purchasing at the posted price. As a result, the buyer always chooses to negotiate. The seller, in turn, posts the highest possible price to keep the buyer's disagreement point as low as possible, so that she can maximize her trade surplus.

To derive the seller's disagreement point and to analyze the seller's dynamic decisions, we follow the development by Feng and Shanthikumar (2018a). Let $V(t,n)$ denote the seller's optimal expected profit with a stock level n when there are t periods left before the end of the selling season. Then the seller's disagreement point in the single-unit trade in period t is essentially her expected profit of carrying that additional unit to the next period, i.e.,

$$v = V(t-1,n) - V(t-1,n-1).$$

With this relation, we can write the recursive equation for the seller's expected profit as

$$V(t,n) = \lambda\Big(\Psi\big(V(t-1,n) - V(t-1,n-1)\big) + V(t-1,n-1)\Big) + (1-\lambda)V(t-1,n).$$

The first term corresponds to the situation when a buyer arrives. In this case, the seller's profit consists of her single-unit trade profit in period t and her future profit of remaining units in period $t-1$. The second term is the seller's profit if no buyer arrives. The terminal conditions of the dynamic program are $V(0,n) = 0$ and $V(t,0) = 0$.

A version of this problem has been analyzed in detail by Feng and Shanthikumar (2018a). The key modeling difference lies in that they do not allow the buyer to purchase at the posted price in the event of negotiation breakdown. Nevertheless, we can obtain very similar observations here. In particular, a unit is worth more to the seller when the seller has a lower stock level and/or when the seller has a longer time to sell the items. This observation is formalized in the next lemma.

Lemma 2.4.2 (Feng and Shanthikumar (2018a)) *$V(t,n) \geq 0$ is increasing in n and t, and $V(t-1,n) - V(t-1,n-1)$ is decreasing in n and increasing in t.*

The results in Lemma 2.4.2, though intuitive, would not always hold once competing sellers are introduced to the market, as we will see from our discussions in section 2.5.

2.4.2 A Static Model with Competing Sellers

A well-studied setting of seller competition is the single-period problem. The two sellers compete to sell to a single buyer. Seller j makes a reservation value w_j if the buyer chooses not to purchase from him. This value reflects seller j's outside option (e.g., selling the item in the secondary market).

Seller j sets a competing price $s_j, j \in \{1, 2\}$. The buyer, knowing the prices, may choose one of the two sellers or neither. When seller j is chosen, the buyer may purchase at the price s_j or enter negotiation with seller j. Following the discussion in section 2.4.1, the buyer's disagreement point in the negotiation with seller j is $(r - s_j)^+$. When walking away from the buyer in the negotiation, seller j can only make a value of the item from the outside option, i.e., $v_j = w_j$. Without loss of generality, suppose $v_j \leq v_i$. Following a similar argument as that for Table 2.1 in the previous subsection, we can see that a buyer would reach a negotiated price of

$$s_{Bj}(r, s_j, v_j) = (1 - \theta)(r \wedge s_j) + \theta v_j \tag{2.2}$$

with seller j. Clearly, seller j should never price s_j below v_j because the seller would be better off not selling the item than selling at a price below v_j. It then follows from the above expression $s_{Bj}(r, s_j, v_j) \leq s_j$ and thus an incoming buyer should always negotiate to obtain a lower trade price.

The buyer's choice between the sellers essentially depends on the comparison between the anticipated trade prices, which depend on the sellers' disagreement points and the prices posted by the sellers. Specifically, the buyer evaluates the sign of

$$\Delta = s_{Bj}(r, s_j, v_j) - s_{Bi}(r, s_i, v_i) = (1 - \theta)(r \wedge s_j - r \wedge s_i) - \theta(v_i - v_j). \tag{2.3}$$

The buyer chooses seller j (i) if Δ is negative (positive), and is indifferent between the sellers if Δ is zero. Depending on whether the sellers have the same disagreement points, we have two cases to consider, which are depicted in Figure 2.1.

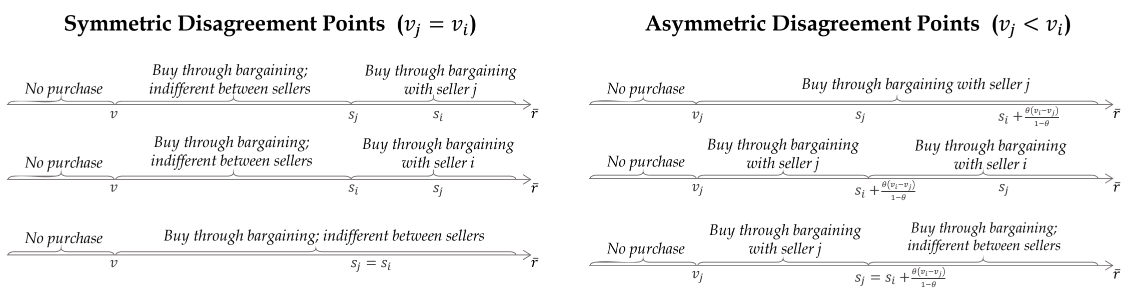


Figure 2.1. Summary of buyer's choice in the static competition, as characterized in (2.4) and (2.6).

If the sellers' disagreement points happen to be the same, i.e., $v_j = v_i = v$, we can easily derive

$$\Delta \begin{cases} > 0 & \text{if } s_i < r \wedge s_j, \\ < 0 & \text{if } s_j < r \wedge s_i, \\ = 0 & \text{if } s_i = s_j \text{ or } r \leq s_i \wedge s_j. \end{cases} \tag{2.4}$$

From the above relation, the buyer would choose seller i in the first case, seller j in the second case, and either seller with equal chance in the last case. Of course, the sellers would not sell an item to a buyer whose valuation is below the disagreement point v. Then, based on the left panel of Figure 2.1, we can write seller j's pricing problem as

$$\begin{aligned} \Psi_j^S(v, s_i) \quad = \quad \max_{s_j \geq v} \Big\{ & \mathbb{E}[\mathbb{I}_{\{R<v\}\cup\{s_i<R\wedge s_j\}} v] + \mathbb{E}[\mathbb{I}_{\{s_j<R\wedge s_i\}} s_{Bj}(R, s_j, v)] \\ & + \mathbb{E}\left[\mathbb{I}_{\{v\leq R\}\cap\{s_i=s_j \text{ or } R\leq s_i\wedge s_j\}} \frac{s_{Bj}(R, s_j, v) + v}{2} \right] \Big\}. \end{aligned} \tag{2.5}$$

Seller i's problem can be written symmetrically.

When the sellers' disagreement points are not the same, i.e., $v_j < v_i$, we can derive from (2.3)

$$\Delta \begin{cases} > 0 & \text{if } r \wedge s_j > s_i + \frac{\theta(v_i - v_j)}{1-\theta}, \\ < 0 & \text{if } r \wedge s_j < s_i + \frac{\theta(v_i - v_j)}{1-\theta}, \\ = 0 & \text{if } r \wedge s_j = s_i + \frac{\theta(v_i - v_j)}{1-\theta}. \end{cases} \tag{2.6}$$

Then, based on the right panel of Figure 2.1, the sellers' pricing problems for $v_j < v_i$ can be written as

$$\begin{aligned} \Psi_j^{A:s}(v_j, v_i, s_i) &= \max_{s_j \geq v_j} \Big\{ \mathbb{E}\big[\mathbb{I}_{\{R < v_j\} \cup \{R \wedge s_j > s_i + \frac{\theta(v_i - v_j)}{1-\theta}\}} v_j\big] \\ &\quad + \mathbb{E}\big[\mathbb{I}_{\{v_j \leq R \wedge s_j < s_i + \frac{\theta(v_i - v_j)}{1-\theta}\}} s_{Bj}(R, s_j, v_j)\big] \\ &\quad + \mathbb{E}\Big[\mathbb{I}_{\{v_j \leq R \wedge s_j = s_i + \frac{\theta(v_i - v_j)}{1-\theta}\}} \frac{s_{Bj}(R, s_j, v_j) + v_j}{2}\Big]\Big\}, \qquad (2.7) \\ \Psi_i^{A:b}(v_i, v_j, s_j) &= \max_{s_i \geq v_i} \Big\{ \mathbb{E}\big[\mathbb{I}_{\{R < v_j\} \cup \{v_j \leq R \wedge s_j < s_i + \frac{\theta(v_i - v_j)}{1-\theta}\}} v_i\big] \\ &\quad + \mathbb{E}\big[\mathbb{I}_{\{R \wedge s_j > s_i + \frac{\theta(v_i - v_j)}{1-\theta}\}} s_{Bi}(R, s_i, v_i)\big] \\ &\quad + \mathbb{E}\Big[\mathbb{I}_{\{v_j \leq R \wedge s_j = s_i + \frac{\theta(v_i - v_j)}{1-\theta}\}} \frac{s_{Bi}(R, s_i, v_i) + v_i}{2}\Big]\Big\}. \qquad (2.8) \end{aligned}$$

Proposition 2.4.1 (Static Competition) *Suppose that $h_R(\tau) = f_R(\tau)/\bar{F}_R(\tau)$ is increasing in τ. The equilibrium satisfies the following.*

i) If $v_j = v_i = v$, the sellers' equilibrium posted prices are $s_j^* = s_i^* = v$. A buyer whose valuation is above v purchases at the negotiated price and is indifferent between the sellers.

ii) If $v_j < v_i < (1-\theta)\bar{r} + \theta v_j$, the sellers' equilibrium posted prices are $s_j^* = \frac{v_i - \theta v_j}{1-\theta} - \epsilon$ and $s_i^* = v_i$. A buyer whose valuation is above v_j purchases from seller j at the negotiated price.

iii) If $v_j < (1-\theta)\bar{r} + \theta v_j \leq v_i$, the sellers' equilibrium posted prices are $s_j^* = \bar{r}$ and s_i^* can be any value within $[v_i, \bar{r}]$. A buyer whose valuation is above v_j purchases from seller j at the negotiated price.

One immediate observation is that the buyer would never trade with a seller whose disagreement point is higher than her opponent's. When they have the same disagreement points (Proposition 2.4.1-i), each seller posts a price equal to her disagreement point. In this case, the buyer randomly chooses one seller to trade, provided that his valuation is above the sellers' disagreement points.

When seller j's disagreement point is slightly lower than that of seller i (Proposition 2.4.1-ii), seller i would choose the lowest possible price that she can post (i.e., $s_i^* = v_i$). This, in turn, pushes seller j to price low enough to attract the buyer. Seller j obtains a trade surplus of $(1-\theta)((\frac{v_i - \theta v_j}{1-\theta} - \epsilon) \wedge r - v_j)$, which exactly reflects her advantage in bargaining position.

When seller j's disagreement point is much lower than that of seller i (Proposition 2.4.1-iii), seller i becomes vacuous in the competition and is indifferent to price at any value above her disagreement point (i.e., $s_i^* \in [v_i, \bar{r}]$). In this case, seller j enjoys monopolist power and chooses the highest possible price that she can post (i.e., $s_j^* = \bar{r}$). As a result, seller j obtains the maximum possible trade surplus of $(1-\theta)(r - v_j)$, which corresponds to its counterpart in the model described in section 2.4.1.

One interesting observation worth highlighting from Proposition 2.4.1 is that the seller's posted price is not necessarily increasing in her own disagreement point. In particular, when seller j's disagreement point is only slightly lower than that of seller i (Proposition 2.4.1-ii), seller j's equilibrium price is decreasing in v_j. Such an equilibrium outcome, very different from its counterpart in the monopoly seller model analyzed in section 2.4.1, is a sole consequence of competition. Though a low disagreement point provides the seller a weak bargaining position, it presents an advantage of attracting an arriving buyer in the face of price competition, as it alleviates the competitive pressure from the opponent seller. With softened competition, the seller would be able to increase her price and yet gain the buyer as her disagreement point becomes significantly lower than her opponent's.

2.5 A Dynamic Model with Competing Sellers

Unlike in the models of dynamic monopolist and static competition, a seller's disagreement point is generally different from her reservation value when competing with the other seller dynamically for a stream of buyers. To see this, let $V_j(t, n_j, n_i)$ be seller j's equilibrium expected profit when her own stock level is n_j and her competitor's stock level is n_i, $j \in \{1, 2\}$ and $i = \{1, 2\} \backslash \{j\}$, and there are t periods to the end of selling season. When negotiating with an arriving buyer, seller j's *disagreement point* is her future value of carrying an extra item, as we discuss in section 2.4.1. This value is

$$v_j = V_j(t-1, n_j, n_i) - V_j(t-1, n_j - 1, n_i). \tag{2.9}$$

Given that the buyer has chosen seller j, the negotiation outcome has no impact on seller i's stock level, which stays at n_i.

Because of the competition, seller j's strategy depends not only on her own disagreement point v_j, but also on her competitor's disagreement point v_i, as is in the model discussed in section 2.4.2. However, the pair (v_j, v_i) alone is not sufficient to fully describe the competition dynamics between the two sellers. We note from (2.9) that the disagreement points are computed under the premises that the arriving buyer has chosen the seller to negotiate.

At the stage of price competition, each seller has to evaluate her profit when the buyer ends up choosing the opponent. Thus, seller j's *reservation value* for losing a buyer in price competition is

$$w_j = V_j(t-1, n_j, n_i - 1) - V_j(t-1, n_j - 1, n_i). \tag{2.10}$$

The first term on the right-hand side is the seller j's future profit when the opponent wins the deal with the arriving buyer and the second term is that when seller j succeeds in gaining the arriving buyer. Intuitively, a seller with a higher reservation value has a less incentive to compete than her opponent does because she gains more when losing the arriving buyer in the competition. Note that

$$w_j - v_j = V_j(t-1, n_j, n_i - 1) - V_j(t-1, n_j, n_i) \tag{2.11}$$

is simply the seller's marginal gain of inventory reduction by her opponent. It is thus easy to see that v_j and w_j are in general different. As our discussion unfolds, we will see that both cases $w_j \geq v_j$ and $w_j < v_j$ can happen in the equilibrium of the stochastic dynamic competition game.

By their definitions, v_j, v_i, w_j, w_i are bounded from the above by $\bar{r}$ because each unit of stock can be sold at most at a price of $\bar{r}$. As our discussion unfolds in the next subsection, it becomes clear that (v_j, v_i, w_j, w_i) are critical values to determine the sellers' dynamic competing equilibrium.

To compute seller j's single-unit trade profit, we apply the logic of the problem formulation in section 2.4.1. An immediate observation is that we shall now take into account her reservation value w_j in addition to her disagreement point v_j. Moreover, because of the competition, seller j's single-unit trade profit is affected by the opponent's disagreement point v_i and price decision s_i as in the static competition setting in section 2.4.2. Define $\bar{\Psi}_j(v_j, v_i, w_j, w_i)$ as seller j's single-unit trade profit in equilibrium. Then seller j's equilibrium expected profit in period t with states (n_j, n_i) must satisfy the following recursion

$$V_j(t, n_j, n_i) = \lambda\big(\bar{\Psi}_j(v_j, v_i, w_j, w_i) + V_j(t-1, n_j - 1, n_i)\big) + (1-\lambda)V_j(t-1, n_j, n_i),$$

with w_j defined in (2.10) and v_j defined in (2.9). The terminal conditions are $V_j(0, n_j, n_i) = 0$ and $V_j(t, 0, n_i) = 0$.

To obtain the equilibrium single-unit trade profit $\bar{\Psi}_j$, we need to derive each seller's best response to the other seller's price decision. We denote $\Psi_j(v_j, v_i, w_j, s_i)$ as seller j's optimal single-trade profit when seller i posts a price s_i. As our analysis unfolds, it will become clear that Ψ_j does not depend on w_i. Moreover, following the derivation for the static competition model, we can express Ψ_j as

$$\Psi_j(v_j, v_i, w_j, s_i) = \begin{cases} \Psi_j^S(v_j, w_j, s_i) & \text{if } v_j = v_i, \\ \Psi_j^{A:s}(v_j, v_i, w_j, s_i) & \text{if } v_j < v_i, \\ \Psi_j^{A:b}(v_j, v_i, w_j, s_i) & \text{if } v_j > v_i. \end{cases}$$

In the next subsection, we provide a detailed derivation and analysis of Ψ_j^S, $\Psi_j^{A:s}$ and $\Psi_j^{A:b}$.

2.5.1 Characterization of the Sellers' Equilibrium Strategies

In this subsection, we formulate the seller firms' single-unit trade problems based on the disagreement points and the reservation values. As our discussion unfolds, it becomes evident that because of the reservation values, the single-unit trade problem is in fact much more complex than the static competition model. It turns out that the problem formulations can be quite different depending on whether the two sellers have the same disagreement points.[1]

2.5.1.1 The Case with Symmetric Disagreement Points

We first consider the case where the sellers are symmetric in their disagreement points, i.e., $v_j = v$ for $j \in \{1, 2\}$. Whether or not seller j can make a successful sale depends not only on v, but also on the reservation values (w_j, w_i) and the prices (s_j, s_i) posted.

In the static competition model treated in section 2.4.2, a seller would never post a price lower than her disagreement point v, which is also her reservation value in that

[1]Like in the static model described in Proposition 2.4.1, multiple price equilibria can arise in the dynamic model with the seller losing the competition indifferent over a range of prices. To simplify the exposition for our later analysis, we would choose the price that leads to the highest off-equilibrium profit of the losing seller in case the buyer deviates from his equilibrium strategy (e.g., in Proposition 2.4.1-iii, seller i would pick $s_i^* = \bar{r}$). Such a choice of equilibrium makes the losing seller's decision robust to potentially irrational deviations made by the buyers.

model, because v is exogenously determined. Therefore, an arriving buyer always enters negotiation instead of accepting the posted price. In an essential contrast, a seller facing dynamic competition may choose a price that is *lower* than her disagreement point—Our later discussions show such a case can indeed arise in equilibrium. In particular, when the chosen seller's posted price is below her disagreement point, an arriving buyer would choose to trade at the posted price, provided that his valuation is above the posted price. Thus, we need to modify the formulation in (2.5) to obtain seller j's single-unit trade profit as

$$\begin{aligned}\Psi_j^S(v, w_j, s_i) &= \max_{s_j} \Bigg\{ \mathbb{E}\Bigg[\mathbb{I}_{S_1} v + \mathbb{I}_{S_2} w_j + \mathbb{I}_{S_3} s_{Bj}(R, s_j, v) + \mathbb{I}_{S_4} s_j \\ &\qquad + \mathbb{I}_{S_5} \frac{s_{Bj}(R, s_j, v) + w_j}{2} + \mathbb{I}_{S_6} \frac{s_j + w_j}{2} \Bigg] \Bigg\}, \end{aligned} \tag{2.12}$$

where $S_1 = \{R < s_j \wedge s_i \wedge v\}$, $S_2 = \{s_i < R \wedge s_j\}$, $S_3 = \{v \leq s_j < R \wedge s_i\}$, $S_4 = \{s_j < R \wedge s_i \wedge v\}$, $S_5 = \{v \leq R \wedge s_j \wedge s_i\} \cap \{s_j = s_i \text{ or } R \leq s_j \wedge s_i\}$ and $S_6 = \{s_j = s_i < R \wedge v\}$. These cases are described in the left panel of Figure D.1 in the Appendix B. Specifically, S_1 represents the set of buyers who would not purchase, S_2 those who purchase from seller i, S_3 those who purchase from seller j at the negotiated price, S_4 those who purchase from seller j at the posted price, S_5 those who purchase at the negotiated prices and are indifferent between the sellers, and S_6 those who purchase at the posted prices and are indifferent between the sellers.

A difference between (2.12) and (2.5) is that under dynamic competition, seller j makes an additional surplus of w_j whenever the opponent seller makes a sale. It turns out that the reservation value plays a critical role in determining the competition outcome. We explicitly analyze two cases depending on whether w_j equals w_i.

Lemma 2.5.1 (Symmetric Disagreement Points and $w_j = w_i$) *Suppose that $v_j = v_i = v$, $w_j = w_i = w$ and $h_R(\tau) = f_R(\tau)/\bar{F}_R(\tau)$ is increasing in τ. The equilibrium satisfies the following.*

i) If $w \geq (1-\theta)\bar{r} + \theta v$, then the equilibrium posted prices are $(s_j^*, s_i^*) = (\bar{r}, \bar{r})$. A buyer whose valuation is above v purchases at the negotiated price and is indifferent between the sellers.

ii) If $v \leq w < (1-\theta)\bar{r} + \theta v$*, then the equilibrium posted prices are* $(s_j^*, s_i^*) = \left(\frac{w-\theta v}{1-\theta}, \frac{w-\theta v}{1-\theta}\right)$*. A buyer whose valuation is above* v *purchases at the negotiated price and is indifferent between the sellers.*

iii) If $w < v$*, then the equilibrium posted prices are* $(s_j^*, s_i^*) = (w, w)$*. A buyer whose valuation is above* w *purchases at the posted price and is indifferent between the sellers.*

Lemma 2.5.1-i describes an extreme situation that a seller's reservation value w (her value of that unsold item when the arriving buyer purchases from the opponent) is so high that it even exceeds her maximum possible trade profit $(1-\theta)\bar{r} + \theta v$. In this case, neither seller has an incentive to compete for the buyer as such they both post the highest possible price (i.e., $s_j^* = s_i^* = \bar{r}$), which prevents the buyer from purchasing at the posted price and makes the buyer's disagreement point the lowest. The buyer, in turn, is indifferent between the two sellers and would randomly choose one to negotiate and to trade. The chosen seller in turn obtains a trade profit of $(1-\theta)r + \theta v$. The other extreme case described in Lemma 3-iii arises when the seller's reservation value w is even lower than her disagreement point v for bargaining. In this case, the seller has a strong incentive to compete for the buyer because losing the buyer to the opponent leads to the lowest profit. Consequently, the sellers both lower the price to w, making themselves indifferent between selling to the buyer and losing the buyer to the opponent. An arriving buyer with valuation above w is indifferent between trading with either seller and would purchase at the posted price without entering negotiation. When the seller's reservation value is in the intermediate range specified in Lemma 3-ii, the seller would post a price between w and $\bar{r}$. The buyer is indifferent between the sellers. He would choose a seller to negotiate and make the purchase provided that his valuation is above the seller's disagreement point v.

The next proposition describes the equilibrium when two sellers have the same disagreement points, but different reservation values, i.e., $v_j = v_i = v$ and $w_j \neq w_i$. Such a case does arise in the dynamic equilibrium in our later analysis in section

2.5.2. Without loss of generality, we assume $w_j < w_i$, implying that seller j is more likely to be aggressive in competing for the buyer than seller i is.

Proposition 2.5.1 (Symmetric Disagreement Points and $w_j < w_i$) *Suppose that $v_j = v_i = v$, $w_j < w_i$ and $h_R(\tau) = f_R(\tau)/\bar{F}_R(\tau)$ is increasing in τ. Let $\bar{s}_j(v, w_j) = \max\left\{v, \max\{s \in [\frac{w_j-\theta v}{1-\theta} \wedge \bar{r}, \bar{r}] : \left(s - \frac{w_j-\theta v}{1-\theta}\right)h_R(s) \leq 2\}\right\}$. The equilibrium satisfies the following.*

- *i) If $w_i > w_j \geq (1-\theta)\bar{s}_j(v, w_j) + \theta v$, then the equilibrium posted prices are $(s_j^*, s_i^*) = (\bar{r}, \bar{r})$. A buyer whose valuation is above v purchases at the negotiated price and is indifferent between the sellers.*

- *ii) If $w_j < (1-\theta)\bar{s}_j(v, w_j) + \theta v$ and*

  - *ii-a) $w_i \geq (1-\theta)\bar{s}_j(v, w_j) + \theta v$, then the equilibrium posted prices are $(s_j^*, s_i^*) = (\bar{s}_j(v, w_j), \bar{r})$;*

  - *ii-b) $v < w_i < (1-\theta)\bar{s}_j(v, w_j) + \theta v$, then the equilibrium posted prices are $(s_j^*, s_i^*) = \left(\frac{w_i-\theta v}{1-\theta} - \epsilon, \frac{w_i-\theta v}{1-\theta}\right)$.*

  A buyer whose valuation is above v purchases at the negotiated price. The buyer chooses seller j if his valuation is above s_j^*, and chooses either seller if his valuation is between v and s_j^*.

- *iii) If $w_j < w_i \leq v$, then the equilibrium posted prices are $(s_j^*, s_i^*) = (w_i - \epsilon, w_i)$. A buyer whose valuation is above s_j^* purchases from seller j at the posted price.*

Similar to Lemma 2.5.1, the three cases characterized in Proposition 2.5.1 describe situations where the two sellers face least (i), moderate (ii), and most competition (iii), depending on whether both have high, medium and low reservation values, respectively. There are, however, worth noting differences due to the asymmetry in the sellers' reservation values. Generally speaking, the price posted by a seller increases when she has a higher reservation value, because she can still make a significant profit when losing a buyer to the opponent. The seller with a lower reservation value is able to post a lower price to attract the buyer. However, when both sellers' reservation values become high enough, the posted prices are too high to grant the buyer positive disagreement points in negotiation (recall Table 2.1). In this case, the buyer becomes indifferent between the sellers.

In summary, when the two sellers have the same disagreement points, each seller becomes more (less) aggressive in competition when her reservation value becomes lower (higher). This notion continues to hold in the case that sellers become asymmetric in their disagreement points, though the equilibrium outcomes become much more complex, as we will see in the next subsection.

2.5.1.2 The Case with Asymmetric Disagreement Points

Now we turn to the case where the sellers have different disagreement points, i.e., $v_j \neq v_i$. Taking into account of the cases in (2.6), we can modify the formulation in (2.7) and (2.8) to obtain the sellers' single-unit trade profits for $v_j < v_i$ as

$$\begin{aligned} \Psi_j^{A:s}(v_j, v_i, w_j, s_i) &= \max_{s_j} \Bigg\{ \mathbb{E}\Bigg[\mathbb{I}_{A_1} v_j + \mathbb{I}_{A_2 \cup A_3} w_j + \mathbb{I}_{A_4} s_{Bj}(R, s_j, v_j) + \mathbb{I}_{A_5} s_j \\ &\qquad + \mathbb{I}_{A_6 \cup A_7} \frac{s_{Bj}(R, s_j, v_j) + w_j}{2} + \mathbb{I}_{A_8} \frac{s_j + w_j}{2} \Bigg] \Bigg\}, \quad (2.13) \\ \Psi_i^{A:b}(v_i, v_j, w_i, s_j) &= \max_{s_i} \Bigg\{ \mathbb{E}\Bigg[\mathbb{I}_{A_1} v_i + \mathbb{I}_{A_2} s_{Bi}(R, s_i, v_i) + \mathbb{I}_{A_3} s_i + \mathbb{I}_{A_4 \cup A_5} w_i \\ &\qquad + \mathbb{I}_{A_6} \frac{s_{Bi}(R, s_i, v_i) + w_i}{2} + \mathbb{I}_{A_7 \cup A_8} \frac{s_i + w_i}{2} \Bigg] \Bigg\}, \quad (2.14) \end{aligned}$$

where $A_1 = \{R < s_j \wedge s_i \wedge v_j\}$, $A_2 = \{R \wedge s_j > s_i + \frac{\theta(v_i - v_j)}{1-\theta}$ and $s_i \geq v_i\}$, $A_3 = \{R \wedge s_j > \frac{s_i - \theta v_j}{1-\theta}$ and $v_j \leq s_i < v_i\} \cup \{s_i < R \wedge s_j \wedge v_j\}$, $A_4 = \{v_j \leq R \wedge s_j < s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta}\}$, $A_5 = \{s_j < R \wedge s_i \wedge v_j\}$, $A_6 = \{R \wedge s_j = s_i + \frac{\theta(v_i - v_j)}{1-\theta}$ and $s_i \geq v_i\}$, $A_7 = \{R \wedge s_j = \frac{s_i - \theta v_j}{1-\theta}$ and $v_j \leq s_i < v_i\}$ and $A_8 = \{s_i = s_j < R \wedge v_j\}$. These cases are described in the right panel of D.1 in the Appendix B. Specifically, A_1 represents the set of buyers who would not purchase, A_2 those who purchase from seller i at the negotiated price, A_3 those who purchase from seller i at the posted price, A_4 those who purchase from seller j at the negotiated price, A_5 those who purchase from seller j at the posted price, A_6 those who purchase at the negotiated price and are indifferent between the sellers, A_7 those who purchase from seller $j(i)$ at the negotiated (posted) price and are indifferent between the sellers, and A_8 those who purchase at the posted price and are indifferent between the sellers.

Proposition 2.5.2 (Asymmetric Disagreement Points) *Suppose that $v_j < v_i$ and $h_R(\tau) = f_R(\tau)/\bar{F}_R(\tau)$ is increasing in τ. The equilibrium satisfies the following.*

a) If $w_i \geq (1-\theta)\bar{r} + \theta v_j$, then the equilibrium posted prices are $(s_j^*, s_i^*) = (\bar{r}, \bar{r})$. A buyer whose valuation is above v_j purchases from seller j at the negotiated price.

b) If $w_j < w_i < (1-\theta)\bar{r} + \theta v_j$, then the equilibrium posted prices are $(s_j^*, s_i^*) = \big(\frac{w_i - \theta(w_i \wedge v_j)}{1-\theta} - \epsilon, \frac{w_i - \theta(w_i \wedge v_i)}{1-\theta}\big)$. When $s_j^* \geq (<) v_j$, a buyer whose valuation is above $v_j(s_j^*)$ purchases from seller j at the negotiated (posted) price.

c) If $w_j = w_i < (1-\theta)\bar{r} + \theta v_j$, then the equilibrium posted prices are $(s_j^*, s_i^*) = \big(\frac{w_j - \theta(w_j \wedge v_j)}{1-\theta}, \frac{w_i - \theta(w_i \wedge v_i)}{1-\theta}\big)$. When $s_i^* < v_j$, a buyer whose valuation is above s_j^* purchases at the posted price and is indifferent between the sellers. When $s_i^* \geq v_j$ and $s_i^* \geq (<) v_i$, a buyer purchases from seller j at the negotiated price if his valuation is between v and s_j^*, purchases from seller j at the negotiated price or purchases from seller i at the negotiated (posted) price if his valuation is above s_j^*.

d) If $w_i < w_j < \tilde{s}_i(v_j, w_i)$, then the equilibrium posted prices are $(s_j^*, s_i^*) = \big(\frac{w_j - \theta(w_j \wedge v_j)}{1-\theta}, \frac{w_j - \theta(w_j \wedge v_i)}{1-\theta} - \epsilon\big)$, where

$$\tilde{s}_i(v_j, w_i) = \max\Big\{v_j, \max\Big\{s \in [w_i \wedge ((1-\theta)\bar{r} + \theta v_j), (1-\theta)\bar{r} + \theta v_j] : (\frac{s - w_i}{1-\theta}) h_R(\frac{s - \theta v_j}{1-\theta}) \leq 1\Big\}\Big\}.$$

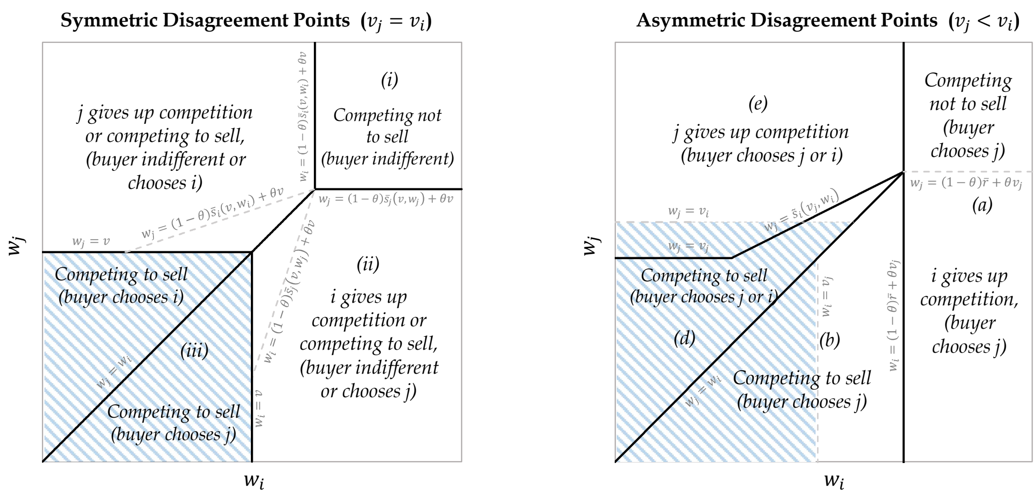
When $s_i^* < v_j$, a buyer whose valuation is above s_i^* purchases from seller i at the posted price. When $s_i^* \geq v_j$ and $s_i^* \geq (<) v_i$, a buyer purchases from seller

j at the negotiated price if his valuation is between v_j and $s_i^* + \frac{\theta(v_i - v_j)}{1-\theta}(\frac{s_i^* - \theta v_j}{1-\theta})$, purchases from seller i at the negotiated (posted) price if his valuation is above $s_i^* + \frac{\theta(v_i - v_j)}{1-\theta}(\frac{s_i^* - \theta v_j}{1-\theta})$.

e) If $w_i < (1-\theta)\bar{r} + \theta v_j$ and $w_j \geq \tilde{s}_i(v_j, w_i)$, then the equilibrium posted prices are $(s_j^*, s_i^*) = \big(\bar{r}, \frac{\tilde{s}_i(v_j,w_i) - \theta(\tilde{s}_i(v_j,w_i) \wedge v_i)}{1-\theta}\big)$. When $s_i^* \geq v_j$ and $s_i^* \geq (<) v_i$, a buyer purchases from seller j at the negotiated price if his valuation is between v_j and $s_i^* + \frac{\theta(v_i - v_j)}{1-\theta}(\frac{s_i^* - \theta v_j}{1-\theta})$, purchases from seller i at the negotiated (posted) price if his valuation is above $s_i^* + \frac{\theta(v_i - v_j)}{1-\theta}(\frac{s_i^* - \theta v_j}{1-\theta})$.

We interpret Proposition 2.5.2 with the help of the right panel of Figure 2.2. Similar to the case with symmetric disagreement points (recall Proposition 2.5.1), the seller with a lower reservation value is more competing because she makes little profit when losing a buyer to the opponent. Of course, both sellers become noncompeting when both have high reservation values, inducing both to set the highest possible price $\bar{r}$. Thus, cases (a) and (b) in Proposition 2.5.2 correspond to the situation where seller i is less competing, while cases (d) and (e) correspond to that where seller j is less competing. When the less competing seller has a very high reservation value, which is the situation for seller i in case (a) and seller j in case (e), she would rather give up the competition and let the opponent make a sale by setting the highest possible price $\bar{r}$. Otherwise, the less competing seller would set a competitive price, which is the situation for seller i in case (b) and seller j in case (d).

Unlike in the situation described in Proposition 2.5.1, the less competing seller is not necessarily less attractive to the buyer. The comparison depends also on her disagreement point relative to that of her opponent. To see that, we first compare cases (a) and (e) where the less competing seller sets the highest possible price $\bar{r}$. With a lower disagreement point (case a), the more competing seller has a weaker bargaining position and offers a lower negotiated price than her opponent does. This allows the more competing seller to set the highest possible price $\bar{r}$. In contrast, when the more competing seller has a higher disagreement point (case e), she is forced to set a price lower than $\bar{r}$ to attract the buyer. This is because if she were to set a price at $\bar{r}$, she would offer a higher negotiated price than her opponent does in view of her stronger bargaining position. As a result, the more competing seller sets a lower price than her opponent to make up for her disadvantage of having a higher disagreement point. From the buyer's perspective, he would generally prefer a seller who posts a lower price and has a lower disagreement point. The difference in the negotiated price induced by the disagreement points, however, is diminishing as the buyer's valuation increases. Therefore, a high-valuation buyer would choose the more competing seller, while a low-valuation one would purchase from the less competing seller.



Note. An arriving buyer purchases *without* bargaining only in the shaded areas.

Figure 2.2. Summary of equilibrium regions with respect to (w_j, w_i), as characterized in Proposition 2.5.1 (left panel) and Proposition 2.5.2 (right panel).

Similar comparisons can be carried out between cases (b) and (d). Generally speaking, a buyer prefers a more competing seller with a lower disagreement point, while only buyers with high valuation would choose a more competing seller with a higher disagreement point.

Another observation from Proposition 2.5.2 is that a buyer may not choose to negotiate when both sellers' reservation values are low (the shaded area in the right panel of Figure 2.2). In this region, both sellers become competing because losing the buyer to the opponent leads to little profit. Consequently, sellers would set prices even lower than their disagreement points. The buyer, in turn, may find it more profitable to purchase at the posted price without entering negotiation.

The observation from cases (d) and (e) makes an interesting contrast to that obtained by Martínez-de-Albéniz and Talluri (2011) in their study of dynamic price competition without negotiation. They find that the more competing seller (i.e., the one with a lower reservation value) always wins over the competition by setting a price equal to the opponent's reservation value. In our model allowing seller-buyer negotiation, it is possible that the seller with a higher reservation value wins the deal from an incoming buyer.

In general, the price posted by a seller is increasing in her reservation value but is decreasing in her disagreement point. This is characterized in the next corollary, where we also show that the slopes of the posted price with respect to the reservation value and the disagreement point are bounded.

Corollary 2.5.1 *Suppose that $v_j < v_i$. In any period other than the last period, the equilibrium prices satisfy the following.*

i) $s_j^*(s_i^*)$ is increasing in both w_j and w_i with a slope not higher than $\frac{1}{1-\theta}$.

ii) $s_j^*(s_i^*)$ is decreasing in $v_j(v_i)$ with a slope not higher than $\frac{\theta}{1-\theta}$; s_j^* is independent of v_i and s_i^* is increasing in v_j with a slope less than $\frac{\theta}{1-\theta}$ or 1.

The observation in Corollary 2.5.1 makes an interesting contrast to that from the static competition model discussed in section 2.4.2. In the static model, each seller's reservation value equals her disagreement point, and we have shown in Proposition 2.4.1-ii that a seller's equilibrium price can increase or decrease in her reservation value or disagreement point. In the case of dynamic competition, the effects of reservation value and disagreement point can be separated. Intuitively, a seller is able to price her product high when she obtains a significant reservation value when losing a buyer to her opponent. Thus, the equilibrium price is increasing in the seller's reservation value. When the seller has a higher disagreement point, however, she becomes less attractive to the buyer as the negotiated price is higher. In order to compete effectively, the seller needs to lower her posted price and thus to increase the buyer's disagreement point. Therefore, the reservation value and the disagreement point affect the seller's price decision in opposite directions. Only in the last period (or, equivalently, in the static model), the seller's reservation value and disagreement point become the same. In this case, the combined effects of reservation value and disagreement point can lead to an increasing or decreasing posted price, as characterized in Proposition 2.4.1-ii.

2.5.2 The Effect of Inventory on Competition Dynamics

From (2.9) and (2.10), the sellers' reservation values and disagreement points are determined by their stock levels. The existing development on price competition suggests that the seller with a lower inventory level has relatively more chance to deplete her stocks and thus cares less about losing an arriving buyer to the competitor (i.e., has a lower reservation value). The literature on sequential bargaining suggests that the value of an additional item would be higher for a seller with a lower inventory level, suggesting a higher disagreement point. Thus, inventory can influence the reservation values and disagreement points in opposite directions in dynamic competition. The interaction between reservation values and disagreement points leads to new equilibrium behaviors, as we see from the discussion below.

Intuitively, each unit of inventory should provide a nonnegative value to a seller. As her inventory increases, we would expect that the value of an additional unit to a seller decreases. These are consistent with our observations from the dynamic monopolist model analyzed in section 2.4.1 (recall Lemma 2.4.2). We can formally establish these results for a two-period competition model as suggested in the next proposition.

Proposition 2.5.3 *Suppose $T = 2$.*

i) $w_j \geq 0$ and $v_j \geq 0$ are weakly decreasing in n_j and n_i.

ii) s_j^* is weakly increasing in n_j when $n_i = 1$ and $t = 2$, is weakly decreasing in n_j when $n_i \geq 2$ and $t = 2$, and is constant in n_j otherwise; s_j^* is weakly decreasing in n_i.

iii) $V_j(t, n_j, n_i)$ is weakly increasing in n_j when $n_i = 1$ and $t = 2$, is weakly decreasing in n_j when $n_i \geq 2$ and $t = 2$, and is constant in n_j otherwise; $V_j(t, n_j, n_i)$ is weakly decreasing in n_i.

iv) If $n_j \leq n_i$, then $w_j \leq w_i$ and an incoming buyer always weakly prefers seller j to seller i.

In a two-period model, a seller's reservation value and disagreement point are both nonnegative. Also, both are decreasing in the seller's stock level. These observations, consistent with our earlier discussion on the dynamic monopolist model (recall Lemma 2.4.1), have been reported in studies on dynamic duopoly models (see, e.g., Gallego and Hu 2014, Lin and Sibdari 2009). In the face of competition, the seller's profit becomes lower when the opponent holds more inventory. This is because the opponent would tend to compete more aggressively as carrying inventory is less valuable for her (i.e., w_i and v_i are lower with a higher n_i).

The equilibrium price, however, can go either direction as inventory increases. This is because the equilibrium price is increasing in the seller's reservation value but decreasing in her disagreement point (recall Corollary 2.5.1). Consequently, a competing seller's equilibrium profit can be increasing or decreasing in her own inventory level. This observation, due to the nature of competition, is very different from that in the dynamic monopolist model (recall Lemma 2.4.1), where the value of an additional item is always nonnegative to the seller.

In the two-period model, a high stock level held by the opponent induces intense competition, forcing the seller to lower the price and to make less profit. The seller with a lower stock level possesses competitive advantage because she can sell the item at a lower price than her opponent. As a result, the buyer always weakly prefers to trade with a low inventory seller, as suggested by Proposition 2.5.3(iv).

However, all the observations made in Proposition 2.5.3 fail to hold when the selling season extends to more than two periods, where we find much complex equilibrium outcomes. This is characterized in the next proposition.

Proposition 2.5.4 *Suppose $T = 3$, $t = 3$ and $n_j, n_i \in \{1, 2, 3\}$.*

i) $w_j \geq 0$ unless $(n_j, n_i) = (2, 3)$; $v_j \geq 0$ unless $(n_j, n_i) \in \{(2, 2), (2, 3)\}$. w_j and v_j are weakly decreasing in n_j unless $(n_j, n_i) \in \{(2, 2), (2, 3)\}$; w_j and v_j are weakly decreasing in n_i.

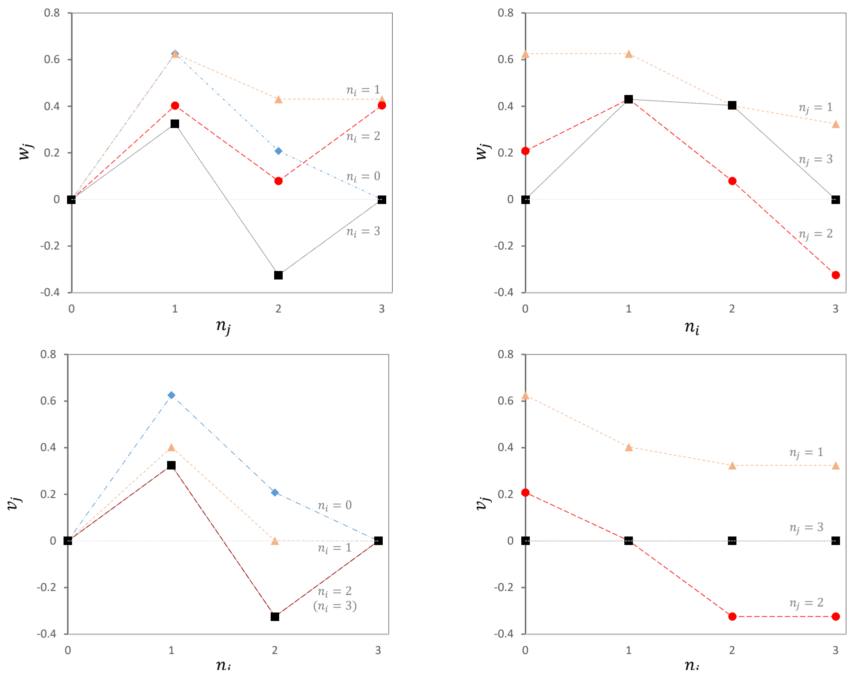
ii) s_j^* is weakly increasing in n_j when $(n_j, n_i) = (2, 2)$, and is weakly decreasing in n_j when $(n_j, n_i) = (2, 3)$; s_j^* is weakly increasing in n_i when $(n_j, n_i) = (2, 2)$, and is weakly decreasing in n_i when $(n_j, n_i) \in \{(2, 1), (3, 2)\}$ or when $n_j = 1$.

iii) $V_j(t, n_j, n_i)$ is weakly increasing in n_j when $(n_j, n_i) = (2, 2)$, and is weakly decreasing in n_j when $(n_j, n_i) = (2, 3)$; $V_j(t, n_j, n_i)$ is weakly decreasing in n_i when $(n_j, n_i) \in \{(1, 2), (2, 1)\}$ or when $n_j = 3$.

iv) If $n_j \leq n_i$, then $w_j < w_i$ and $v_j < v_i$ when $(n_j, n_i) = (2, 3)$, and $w_j \leq w_i$ and $v_j \geq v_i$ otherwise.

An example of a three-period model is presented in Figure 2.3 (also refer to the values for $t = 3, 2, 1$ in Table 2.2). We observe that the sellers' reservation values and disagreement points are no longer monotone in the stock levels when there are three periods to the end of the selling season. Moreover, these values can go negative. For example, when $(n_j, n_i) = (2, 3)$, $w_j < 0$ and $v_j < 0$.

To explain a negative disagreement point $v_j < 0$, we use the numbers computed in Table 2.2 for $t = 3, 2, 1$. Consider seller j's decision in period $t = 3$ when she has 2 units and seller i has 3 units. If seller j makes a sale in period 3, she would be left



Notes. $T = 3$, $t = 3$, $\theta = 0.5$, $\lambda = 1$, $\bar{F}_R(r) = 1 - r^5$, $r \in [0, 1]$.

Figure 2.3. Seller j's reservation value and disagreement point with respect to (n_j, n_i).

with only 1 item in period 2. As a result, seller i's best strategy in period 2 is to set a noncompeting price (i.e., $s_i^* = \bar{r} = 1$ for $(n_j, n_i) = (1, 3)$ and $t = 2$), hoping that seller j would successfully deplete her stock so that seller i can enjoy the monopolist power in the last period. Such a situation corresponds to case (ii-a) in Proposition 2.5.1 and Figure 2.2. If, however, seller j does not sell in period 3, she would be left with 2 items in period 2. It is then impossible for seller i to become a monopolist in the last period, making seller i aggressive in price competition (note that $s_i^* = 0$ for $(n_j, n_i) = (2, 3)$ and $t = 2$). Such a situation corresponds to case (ii) in Lemma 2.5.1 and Figure 2.2. Comparing the two situations, seller j finds her future profit higher when she sells to the buyer than when she does not. As a result, seller j's disagreement point is negative. For a similar reason, seller j would earn less future profit by losing a buyer to seller i in period 3, suggesting a negative reservation value.

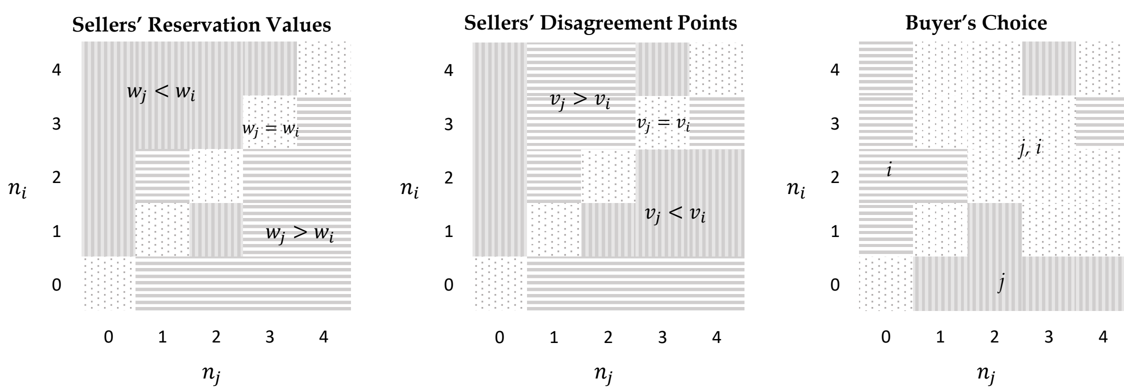
Table 2.2.
The effect of inventories on the dynamic equilibrium.

	$t=4$					$t=3$				$t=2$			$t=1$	
$V_j(t,n_j,n_i)$	$n_i=0$	$n_i=1$	$n_i=2$	$n_i=3$	$n_i=4$	$n_i=0$	$n_i=1$	$n_i=2$	$n_i\geq 3$	$n_i=0$	$n_i=1$	$n_i\geq 2$	$n_i=0$	$n_i\geq 1$
$n_j=1$	0.7969	0.7144	0.6089	0.4309	0.4089	0.7344	0.6089	0.4175	0.3804	0.6254	0.4027	0.3243	0.4167	0
$n_j=2$	1.3573	1.1209	0.7902	0.6756	0.7151	1.1461	0.7995	0.4029	0.5788	0.8333	0.4035	0	0.4167	0
$n_j=3$	1.6147	1.2055	0.7817	0.4035	0.7061	1.25	0.7997	0.4035	0	0.8333	0.4035	0	0.4167	0
$n_j=4$	1.6667	1.2056	0.7791	0.4035	0	1.25	0.7997	0.4035	0	0.8333	0.4035	0	0.4167	0
s_j^*	$n_i=0$	$n_i=1$	$n_i=2$	$n_i=3$	$n_i=4$	$n_i=0$	$n_i=1$	$n_i=2$	$n_i\geq 3$	$n_i=0$	$n_i=1$	$n_i\geq 2$	$n_i=0$	$n_i\geq 1$
$n_j=1$	1	0.8598	1	0.5206	0.514	1	0.8482	0.5369	0.5296	1	0.8333	0.7784	1	0
$n_j=2$	1	1	0.7786	0.5085	0.5818	1	0.8605	0.4826	1	1	1	0	1	0
$n_j=3$	1	0.9008	1	0.2281	1	1	1	1	0	1	1	0	1	0
$n_j=4$	1	1	1	1	0	1	1	1	0	1	1	0	1	0
w_j	$n_i=0$	$n_i=1$	$n_i=2$	$n_i=3$	$n_i=4$	$n_i=0$	$n_i=1$	$n_i=2$	$n_i\geq 3$	$n_i=0$	$n_i=1$	$n_i\geq 2$	$n_i=0$	$n_i\geq 1$
$n_j=1$	0.7344	0.7344	0.6089	0.4175	0.3804	0.6254	0.6254	0.4027	0.3243	0.4167	0.4167	0	0	0
$n_j=2$	0.4117	0.5371	0.3821	0.0226	0.1985	0.2079	0.4306	0.0792	-0.3243	0	0.4167	0	0	0
$n_j=3$	0.1039	0.4505	0.3968	-0.1754	-0.5788	0	0.4299	0.4035	0	0	0.4167	0	0	0
$n_j=4$	0	0.4503	0.3962	0.4035	0	0	0.4299	0.4035	0	0	0.4167	0	0	0
v_j	$n_i=0$	$n_i=1$	$n_i=2$	$n_i=3$	$n_i=4$	$n_i=0$	$n_i=1$	$n_i=2$	$n_i\geq 3$	$n_i=0$	$n_i=1$	$n_i\geq 2$	$n_i=0$	$n_i\geq 1$
$n_j=1$	0.7344	0.6089	0.4175	0.3804	0.3804	0.6254	0.4027	0.3243	0.3243	0.4167	0	0	0	0
$n_j=2$	0.4117	0.1906	-0.0145	0.1985	0.1985	0.2079	0.0008	-0.3243	-0.3243	0	0	0	0	0
$n_j=3$	0.1039	0.0002	0.0005	-0.5788	-0.5788	0	0	0	0	0	0	0	0	0
$n_j=4$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Buyer's choice	$n_i=0$	$n_i=1$	$n_i=2$	$n_i=3$	$n_i=4$	$n_i=0$	$n_i=1$	$n_i=2$	$n_i\geq 3$	$n_i=0$	$n_i=1$	$n_i\geq 2$	$n_i=0$	$n_i\geq 1$
$n_j=1$	j	j,i	i	j,i	j,i	j	j,i	j,i	j,i	j	j,i	j,i	j	j,i
$n_j=2$	j	j	j,i	j,i	j,i	j	j,i	j,i	j	j	j,i	j,i	j	j,i
$n_j=3$	j	j,i	j,i	j,i	j	j	j,i	i	j,i	j	j,i	j,i	j	j,i
$n_j=4$	j	j,i	j,i	i	j,i	j	j,i	i	j,i	j	j,i	j,i	j	j,i

Notes. $T=4$, $\theta=0.5$, $\lambda=1$, $\bar{F}_R(r)=1-r^5$, $r\in[0,1]$.

When the selling season is longer than two periods, a seller's profit is not necessarily decreasing in the opponent's stock level. In particular, a seller's profit can be increasing or decreasing in the opponent's stock level. The increase of the opponent's stock level can induce the opponent to be aggressive in competing for a buyer, potentially reducing the seller's profit. However, if the opponent's stock level exceeds the seller's stock level, the seller obtains a lower reservation value, becoming more attractive to the buyer and potentially earning an increased profit. Depending on which effect dominates, the seller's profit can increase or decrease in the opponent's stock level. Consequently, a seller's margin gain of inventory reduction by her opponent (i.e., $w_j - v_j$) can be negative. For example, when $(n_j, n_i) = (2, 2)$ and $t = 3$, seller j's profit is increasing in n_i. Thus, when $(n_j, n_i) = (2, 3)$ and $t = 4$, we have $w_j < v_j$.

When there are three or fewer periods left before the end of the selling season, the seller with a lower stock level always has a lower reservation value and is more likely to be preferred by an incoming buyer. This observation has been made by Martínez-de-Albéniz and Talluri (2011) in their study of dynamic price competition. Intuitively, the seller with fewer items in stock has relatively more opportunities to sell each item than her opponent who has more items. Thus, the former's expected future profit is less affected when losing a buyer, resulting in a lower reservation value. In our model allowing bargaining, however, this result does not hold when there are more than three periods to the end of the selling season. An example is demonstrated in Figure 2.4. We observe from the example that the seller with a higher inventory level may have a lower reservation value and may make a successful sale.



Note. $T = 4$, $t = 4$, $\theta = 0.5$, $\lambda = 1$, $\bar{F}_R(r) = 1 - r^5$, $r \in [0, 1]$.

Figure 2.4. Sellers' reservation values (left panel), disagreement points (middle panel) and buyer's choice (right panel) as functions of (n_j, n_i).

Such a situation arises when $(n_j, n_i) = (2, 1)$ in the example depicted in Figure 2.4. There is a good chance that all three items possessed by the sellers would be sold in the next four periods. Both sellers choose not to compete and set the highest possible price $s_j^* = s_i^* = 1$ (from Table 2.2), which corresponds to case (a) in Proposition 2.5.2 and Figure 2.2. This makes seller j, with $n_j = 2$, more attractive to an arriving buyer because her disagreement point is lower than that of seller i, who has $n_i = 1$. For seller i, however, losing an arriving buyer to seller j would allow her to be equally attractive to a future buyer compared with seller j, because then each seller would have one item in stock. Consequently, seller i, though holding a lower inventory level, has a higher reservation value than seller j and loses the arriving buyer to seller j.

In summary, when seller firms compete repeatedly, the competition dynamics reveals interesting phenomena that can be very different from those observed in static competition or dynamic monopolist selling. Observations made from this subsection highlight that analysis of a two-period or three-period game is insufficient to fully discover the dynamics of revenue management under competition.

2.5.3 The Effect of Non-homogeneous Buyer's Valuation

In our base model, we have assumed time-invariant buyer's valuation distributions. In this section, we examine the situation where buyers' valuation distributions are non-identical over time. Specifically, we consider the cases where buyers' valuations are stochastically increasing or decreasing over time. For example, airline customers who purchase closer to the departure date may have higher valuations of the tickets and be willing to pay more. Fashion consumers, however, may have lower valuations of the products when it gets closer to the end of the season. To understand how the trend of consumer evaluations affects the firms' competition, we present a four-period example in Table 2.3. The buyers' valuations are stochastically increasing when the parameter $\delta_\alpha > 0$ and stochastically decreasing when $\delta_\alpha < 0$. The absolute value $|\delta_\alpha|$ measures the magnitude of valuation change across periods. When $|\delta_\alpha|$ increases, the buyers' valuation across all periods increases stochastically. Correspondingly, we observe that the firms' expected profits increase (decrease) when facing stochastically increasing (decreasing) valuations.

A seller's expected sales can be increasing or decreasing in the opponent's stock level, which is similar to our observation of the seller's profit in the three-period model in Section 2.5.2. However, the change of a seller's expected sales is not necessarily consistent with that of buyers' valuations. In other words, a seller can sell more or fewer units when facing a stream of buyers with stochastically increasing or decreasing

Table 2.3.
The effect of time-dependent buyer's valuation on the dynamic equilibrium.

	$n_i = 1$ and $t = 4$				$n_i = 3$ and $t = 4$				$n_i = 4$ and $t = 4$			
Stochastically Increasing Buyers' Valuations												
$V_j(t, n_j, n_i)$	$n_j = 1$	$n_j = 2$	$n_j = 3$	$n_j \geq 4$	$n_j = 1$	$n_j = 2$	$n_j = 3$	$n_j \geq 4$	$n_j = 1$	$n_j = 2$	$n_j = 3$	$n_j \geq 4$
$\delta_\alpha = 0$	0.4238	0.5984	0.6417	0.6417	0.2745	0.3351	0.2159	0.2188	0.2513	0.3621	0.3744	0
$\delta_\alpha = 0.2$	0.4763	0.6869	0.7423	0.7423	0.3133	0.3848	0.2703	0.2752	0.2799	0.4225	0.4239	0
$\delta_\alpha = 0.4$	0.5128	0.7514	0.8111	0.8111	0.3407	0.4168	0.3059	0.3125	0.2997	0.4631	0.4482	0
$\delta_\alpha = 0.6$	0.5404	0.7982	0.8619	0.8619	0.3612	0.4399	0.3313	0.3391	0.3145	0.4924	0.4640	0
$\delta_\alpha = 0.8$	0.5622	0.8342	0.9013	0.9014	0.3769	0.4580	0.3505	0.3594	0.3266	0.5147	0.4762	0
$\delta_\alpha = 1$	0.5800	0.8628	0.9331	0.9333	0.3895	0.4726	0.3655	0.3753	0.3370	0.5323	0.4857	0
Expected sales by seller j	$n_j = 1$	$n_j = 2$	$n_j = 3$	$n_j \geq 4$	$n_j = 1$	$n_j = 2$	$n_j = 3$	$n_j \geq 4$	$n_j = 1$	$n_j = 2$	$n_j = 3$	$n_j \geq 4$
$\delta_\alpha = 0$	1	2	3	3.0463	1	1.7554	2	1.1250	0.9537	1.9076	2.8750	2
$\delta_\alpha = 0.2$	1	2	3	3.0543	1	1.7232	2	1.1267	0.9457	1.9004	2.8733	2
$\delta_\alpha = 0.4$	1	2	3	3.0597	1	1.7065	2	1.1274	0.9403	1.8960	2.8726	2
$\delta_\alpha = 0.6$	1	2	2.99997	3.0591	0.999998	1.7043	2	1.1191	0.9409	1.9003	2.8810	2
$\delta_\alpha = 0.8$	1	2	2.99987	3.0555	0.999990	1.7039	2	1.1087	0.9445	1.9073	2.8913	2
$\delta_\alpha = 1$	1	2	2.99971	3.0513	0.999981	1.7044	2	1.1000	0.9487	1.9139	2.9000	2
Stochastically Decreasing Buyers' Valuations												
$V_j(t, n_j, n_i)$	$n_j = 1$	$n_j = 2$	$n_j = 3$	$n_j \geq 4$	$n_j = 1$	$n_j = 2$	$n_j = 3$	$n_j \geq 4$	$n_j = 1$	$n_j = 2$	$n_j = 3$	$n_j \geq 4$
$\delta_\alpha = 0$	0.7144	1.1209	1.2055	1.2056	0.4309	0.6756	0.4035	0.4035	0.4089	0.7151	0.7061	0
$\delta_\alpha = -0.2$	0.7072	1.1083	1.1893	1.1893	0.4262	0.6698	0.3937	0.3937	0.4062	0.7096	0.7028	0
$\delta_\alpha = -0.4$	0.6989	1.0933	1.1698	1.1699	0.4208	0.6633	0.3817	0.3817	0.4031	0.7035	0.6991	0
$\delta_\alpha = -0.6$	0.6889	1.0739	1.1460	1.1460	0.4145	0.6555	0.3668	0.3668	0.3997	0.6961	0.6951	0
$\delta_\alpha = -0.8$	0.6767	1.0480	1.1157	1.1157	0.4071	0.6459	0.3475	0.3475	0.3956	0.6864	0.6904	0
$\delta_\alpha = -1$	0.6612	1.0118	1.0756	1.0756	0.3983	0.6329	0.3210	0.3210	0.3901	0.6713	0.6811	0
Expected sales by Seller j	$n_j = 1$	$n_j = 2$	$n_j = 3$	$n_j \geq 4$	$n_j = 1$	$n_j = 2$	$n_j = 3$	$n_j \geq 4$	$n_j = 1$	$n_j = 2$	$n_j = 3$	$n_j \geq 4$
$\delta_\alpha = 0$	1	2	3	3.0186	1	1.8853	2	1.0714	0.9814	1.9563	2.9286	2
$\delta_\alpha = -0.2$	1	2	3	3.0192	1	1.8848	2	1.0758	0.9808	1.9538	2.9242	2
$\delta_\alpha = -0.4$	1	2	3	3.0194	1	1.8851	2	1.0807	0.9806	1.9524	2.9194	2
$\delta_\alpha = -0.6$	1	2	3	3.0188	1	1.8862	2	1.0862	0.9812	1.9516	2.9138	2
$\delta_\alpha = -0.8$	1	2	3	3.0159	1	1.8914	2	1.0827	0.9841	1.9565	2.9173	2
$\delta_\alpha = -1$	1	2	3	3.0123	1	1.8990	2	1.0741	0.9878	1.9642	2.9259	2

Notes. $T = 4$, $\theta = 0.5$, $\lambda = 1$, $\bar{F}_R(r) = 1 - r^\alpha$, $r \in [0, 1]$, $\alpha = 1 + \delta_\alpha(T - t)$ for $\delta_\alpha > 0$ (upper panel) and $\alpha = 5 + \delta_\alpha(T - t)$ for $\delta_\alpha < 0$ (lower panel).

valuations. To understand such a phenomenon, we need to compare the total number of items against the number of selling opportunities. When the total stock level is lower than the number of selling opportunities (e.g., $(n_j, n_i) = (1, 2)$), there is a good chance that both sellers would fully deplete their stocks by the end of selling season. In this case, a seller's expected sales equals her inventory level. When the total stock level is high (e.g., $(n_j, n_i) = (5, 4)$), it is unlikely that a seller can sell all her stock, making them aggressive in price competition. In this case, sellers would end up equally sharing the selling opportunities and have the same expected sales. The subtle situation arises when only one of the sellers is likely to fully deplete her stock within the selling season. In this case, each seller should carefully evaluate her potential profit by being a future monopolist, competing for the current buyer, or competing for the future buyer. This leads to an inconsistency between the change of a seller's expected sales and that of buyers' valuations (e.g., $(n_j, n_i) = (1, 4)$).

Specifically, in the case that buyers' valuations are stochastically increasing over time, sellers would prefer selling to a future buyer who is likely to have a high valuation. On the one hand, in anticipation of high-valuation buyers in the future, a seller may intentionally give up the selling opportunity in the current period, leading to a decrease in her expected sales. On the other hand, given that the opponent seller may also want to keep the stock for future buyers, the seller may find it less competitive to capture the current buyer. Depending on which effect dominates, a seller's expected sales can be increasing or decreasing as buyers' valuations are stochastically increasing. Generally speaking, sellers' expected sales depend not only on their stock levels but also on the magnitude of how buyers' valuations change.

We further note that sellers' total expected sales can be smaller than the number of selling opportunities even though they have enough inventory. Such a situation arises when $(n_j, n_i) = (3, 1)$ and $\delta_\alpha \geq 0.6$ in Table 2.3. In this case, both sellers have positive disagreement points and thus an arriving buyer with low valuation may walk away without purchase. This makes an interesting contrast to the dynamic monopolist model where the seller's expected sales is always equal to the number of remaining periods or her total stock level, whichever is smaller.

2.6 Concluding Remarks

We analyze the competition between two sellers in sequentially selling a certain amount of inventory or resources to a stream of arriving buyers. Our model, allowing arriving buyers to bargain for a price discount, brings an additional dimension to the competition dynamics. In general, the sellers' competing strategies not only depend on their reservation values of losing an arriving buyer to the competitor but also on their disagreement points of negotiation breakdown with the buyer. Low reservation values induce the sellers to reduce their prices to compete effectively, while low disagreement points force the sellers to increase their prices to ensure a certain profit. Because of the added effect of disagreement points associated with bargaining, the sellers' competitive equilibrium reveals very different behavior from their counterparts in dynamic price competition without bargaining. In particular, we characterize equilibrium regions in which the seller with a higher reservation value, as opposed to the one with a lower reservation value, may post a higher price or may make a successful sale. Moreover, a seller with a higher inventory level, as opposed to the one with a lower inventory level, may have a lower reservation value. Our analysis of different versions of the model also suggests that the insights obtained from a static setting or a dynamic setting in two or three periods may not hold in general.

Our model and analysis can be extended along different directions. For example, we may allow the arrival rate of the buyer stream to be Markov-modulated. The sellers, based on the knowledge of cumulative arrivals and the observation of whether or not an arrival occurs in the current period, can update the probability of an arrival in the next period. In this case, each seller's profit calculation should also include two more state variables, i.e., the number of cumulative arrivals and an indicator for an arrival in the current period. The equilibrium structure does not change with this modification. Our analysis can also be extended to allow the buyers to be heterogeneous in their bargaining powers in negotiation or to allow their valuation distribution to be time-dependent.

3. SUPPLIER-BUYER INTERFACE: NEGOTIATIONS IN COMPETITIVE SUPPLY CHAINS: THE KALAI-SMORODINSKY BARGAINING SOLUTION

3.1 Synopsis

Bargaining is a norm rather than an exception in establishing vertical relations (see, e.g., Draganska et al. 2010, Iyer and Villas-Boas 2003). In most of the business-to-business transactions, deals are reached after rounds of negotiation over contract terms between the trading parties. Applications of bilateral bargaining in operations management have gained increasing attentions in recent years. Almost all of the studies concerning supply chain negotiations adopt the axiomatic Nash bargaining (NB) solution (Nash 1950) and its multi-unit extensions (Davidson 1988, Horn and Wolinsky 1988) to understand buyer-seller interactions.

However, the NB solution assumes *independence of irrelevant alternatives* (IIA), an axiom being widely criticized in the economics literature. In words, this axiom states that the elimination of some unchosen alternatives does not affect the selection of the best option. This assumption has been empirically invalidated, as the available options often impact the way a decision maker evaluates the choices (Arrow 1950). Potentially, a decision model under the IIA assumption can produce unreasonable conclusions.

Specifically in the supply chain context, the NB solution seems to work well in generating useful insights for one-to-one channels (i.e., bilateral monopoly settings), because the choice of feasible profit allocation between the trading parties is determined purely by the set of feasible contracts to be chosen by the trading parties. In competing supply chains, however, the profit allocation set in one trade depends critically on the choice of contract to be executed in other trades. It is thus unclear whether the NB solution always generates appropriate negotiation outcomes. Unfortunately, the answer is no. We demonstrate this through an example in Section 3.3, in which we compare two trading scenarios for a supplier (she) and a retailer (he). The only difference between the two trades is that the retailer in the first scenario is a market monopoly, while the retailer in the second faces potential competition from a rivalry. Given any feasible contract, the supplier always makes a higher profit in the second scenario than in the first, while the NB solution grants the supplier a higher profit in the first scenario than it does in the second. This surprising negotiation outcome is unreasonable and goes against our observations from reality.

The inconsistency between the feasible trade profits and the negotiation outcomes in the aforementioned example is due to the fact that the NB solution ignores the trade prospects of the contracting parties (i.e., the geometric properties of the profit allocation set) under the IIA axiom. While deriving a bargaining solution that accounts for the complete profit allocation set is difficult, it is possible to develop a solution concept that includes more information about the trade than the NB solution does. The most widely applied alternative to the NB solution is the one developed by Kalai and Smorodinsky (1975). Compared with the NB solution, the Kalai-Smorodinsky (KS) bargaining solution takes into account not only the worst trade outcome (i.e., the disagreement point), but also the best trade outcome (i.e., the maximum profit) of each negotiation party. Despite its popularity in studying bargaining problems, the KS solution has not been introduced to the supply chain studies. The purpose of our study is two-fold. We would like to understand, in the context of competing supply chains, (i) when the IIA axiom may lead to unreasonable outcomes under the NB solution, and (ii) how the KS solution works differently from the NB solution.

To answer the first question, we examine how the trade prospects (i.e., the trade profits) change with respect to the firms' competitive positions and compare that against the changes in negotiated profits. This comparison allows us to identify the inconsistency induced by the IIA axiom. To answer the second question, we compute the bargaining power distribution implied by the negotiation equilibrium under the

KS solution. This analysis allows us to understand how the KS solution corrects the symmetric NB solution when accounting for the best trade outcome of each trading party.

Specifically, we consider competing supply chains consisting of either a common upstream supplier trading with two competing retailers (the one-to-two channel) or two competing suppliers trading with a common downstream retailer (the two-to-one channel). The market price of a product is the linear function of the outputs generated from the competing channels. While economics studies on negotiations focus on gain allocation between the trading parties, negotiations in supply chains are commonly over specific contract terms. We focus on the wholesale-price contract, which is the most widely used in practice and is most often studied in the operations literature. Because of the presence of horizontal competition, there are two bargaining units, each consisting of a supplier and a retailer, in the supply chain, and the negotiation outcome of one unit is dependent on that of the other.

When the two contracts are negotiated simultaneously and the bilaterally agreed contracts are executed without additional contingency terms, the equilibrium KS solution corresponds to the NB solution with the downstream retailer's bargaining power being 0.6, not 0.5. This suggests that by considering the trading parties' best prospects, the KS solution captures the retailers' power through their ability to set competition parameters (i.e., the output quantities). When negotiations are conducted sequentially, however, the implied downstream bargaining powers are different in the two trades. The equilibrium KS solution corresponds to the NB solution which grants the retailer in the second trade a bargaining power of 0.6. The retailer in the first trade possesses an implied bargaining power below 0.6 in the one-to-two channel and has a bargaining power above 0.6 in the two-to-one channel. These results suggest that, in sequential negotiations, the KS solution takes into account the advantage enjoyed by the common trading party, who is able to leverage the anticipated gain from the second trade in the first negotiation. Moreover, the implied downstream bargaining power decreases in the one-to-two channel when the retail competition becomes more intense, and increases in the two-to-one channel when the supply competition becomes more aggressive. In all the aforementioned bargaining settings, we find that the inconsistency between the trade prospects and the negotiated profits does not arise in the NB solution. Nevertheless, compared with the NB solution, the KS solution, relaxing the IIA axiom, allows one to identify the negotiation power shift induced by the decision ownership, the negotiation sequence, the vertical relationships, and the competition intensity.

In reality, contingency terms may be imposed for contract execution. This may happen when the upstream supplier would only establish the trading relationship with a retailer provided that the supplier can also penetrate the market through other retailers. Alternatively, the downstream retailer would only purchase from a supplier if the retailer has access to carrying similar products offered by other suppliers. To model such situations, we allow the common trading party, the supplier in the one-to-two channel or the retailer in the two-to-one channel, to impose contingency terms such that a negotiation contract is executed only when the negotiation in the other trade is also successful. We show that with contingency contracts, the KS solution tends to grant the supplier a higher equivalent bargaining power in the one-to-two channel, while leaving the retailer a higher equivalent bargaining power in the two-to-one channel. This implies a disadvantage faced by the competing firms when the restriction on contract execution is imposed through the contingency terms.

We further discuss extensions of our analysis to consider general nonlinear demands and demonstrate that the main insights obtained preserve. We also show that if the wholesale-price contracts are replaced by bilateral coordinating contracts, the KS solution always corresponds to the symmetric NB solution.

The reminder of the paper is organized as follows. In Section 3.2, we review the related literature and articulate our contributions. In Section 3.3, we introduce the Kalai-Smorodinsky bargaining solution. We describe the model in Section 3.4 and derive the bargaining solutions in Section 3.5. We analyze the firms' negotiation sequence preferences and discuss two model variations in Section 3.6. We conclude in Section 3.7. Proofs of all formal results are relegated to the appendix B.

3.2 Literature Review

Supply chain contracting has been extensively researched in economics and operations management. Studies in this literature examine contractual relationships in situations where a monopoly supplier sells to a monopoly retailer (e.g., Cui et al. 2007, Taylor 2002, Tsay 1999), a common supplier selling to competing retailers (e.g., Bernstein and Federgruen 2005, Cachon and Lariviere 2005) and competing suppliers selling to a common retailer (e.g., Cachon and Kök 2010, Shang et al. 2015). Most part of this research is based on the Stackelberg framework, where a bilateral contract is offered by one party as a take-it-or-leave-it offer to the other party. Under this setting, it is generally believed that the wholesale-price contract is nearly always found to be inefficient for the supply chain, while more sophisticated contracts, such as two-part tariffs, revenue-sharing contracts, and quantity-flexibility contracts, can be used to coordinate the system.

In more recent studies (e.g., Chen et al. 2016, Dukes et al. 2006, Feng and Lu 2012, 2013b, Guo and Iyer 2013, Gurnani and Shi 2006, Hsu et al. 2016, Huh and Park 2010, Lovejoy 2010, Nagarajan and Bassok 2008, Van Mieghem 1999, Zhong et al. 2016), the Nash bargaining framework has been applied to replace the Stackelberg framework. Bernstein and Nagarajan (2012) provide a comprehensive summary of these developments. As Feng and Lu (2013a) point out, the outcome of a Stackelberg game does not necessarily coincide with that of a bargaining game when granting the contract offering parties dominating power. When competition is considered, there are multiple bilateral negotiations within the supply chain. If the negotiations take place in parallel (e.g., Chu et al. 2019, Feng and Lu 2013b), the negotiated contracts are the Nash equilibrium of parallel Nash bargaining solutions, which is termed the Nash-Nash solution (Davidson 1988, Horn and Wolinsky 1988). If the negotiations take place sequentially (e.g., Feng and Shanthikumar 2018a), the anticipated future bargaining outcome determines the disagreement points (i.e., the trading parties' profits in the event of negotiation breakdown) in the current negotiation.

The Nash bargaining solution (Nash 1950) is developed based on a set of axioms that is easy to interpret and apply. However, there is significant criticism on this solution concept, the most significant one being on the axiom of *independence of irrelevant alternatives*; see the discussions in §§3.3. To address this issue, Kalai and Smorodinsky (1975) propose an alternative axiom, called *individual monotonicity*. The key difference between these two solution concepts lies in the fact that the Kalai-Smorodinsky solution accounts for more geometric properties of the profit allocation set by specifically involving the trading parties' best prospects (i.e., the maximum possible trade profits). Moulin (1984) shows that the Kalai-Smorodinsky solution corresponds to a subgame perfect equilibrium of a *fractions of dictatorship* auction game. Livne (1989), Rachmilevitch (2014), Thomson (1983) propose different alternative axioms to characterize the Kalai-Smorodinsky solution. The Kalai-Smorodinsky solution has been broadly applied in various fields such as economics (e.g., Alexander 1992, Chun and Thomson 1988, Driesen et al. 2011, Manser and Brown 1980, Monroy et al. 2017), electrical engineering (e.g., Chee et al. 2006, Chen and Swindlehurst 2009, Fattahi and Paganini 2005, Ibing and Boche 2007, Park and van der Schaar 2007, Shrestha et al. 2008, Yang et al. 2010, Zhang and Zhao 2014) and green economy (e.g., Carfì and Schiliro 2012, Carfì and Trunfio 2011).

The studies on supply chain contracting, however, have not adopted the Kalai-Smorodinsky solution. The only exception is Gerchak (2015). He compares the properties of the Nash bargaining solution and the Kalai-Smorodinsky solution in a bilateral monopoly setting, i.e., one-to-one channel. Our study is arguably the first to examine the differences and connections of the two bargaining solution concepts in competing supply chains.

3.3 Preliminaries: The Kalai-Smorodinsky Bargaining Solution

The bargaining solution concept is the key to determine the equilibrium contracts. In the supply chain literature, the Nash bargaining solution (Nash 1950) is widely applied because it often yields an analytically tractable solution that is easy to interpret. There is, however, significant criticism on this solution concept (see, e.g., Osborne and Rubinstein 1990). In an attempt to (partially) address the criticism, several authors have proposed alternative bargaining solutions. Among those, the solution proposed by Kalai and Smorodinsky (1975) has been mostly applied in many areas, but yet not in the supply chain contexts. In this section, we briefly review the concept of the Nash bargaining (NB) solution and introduce the Kalai-Smorodinsky (KS) solution to facilitate our comparison between the two solution concepts in the next section.

Consider the bilateral negotiation between a supplier and a retailer. If a contract C within the feasible set $\mathcal{C}$ is agreed upon, the supplier makes a profit of $\Pi(C)$ and the retailer makes a profit of $\pi(C)$. Define

$$\mathbf{\Pi} = \{(\Pi(C), \pi(C)) \in \mathbb{R}^2_+ : C \in \mathcal{C}\}$$

as the set of feasible profit allocation. If, however, no agreement is reached, the supplier makes a profit of D and the retailer makes a profit of d, which are termed the *disagreement points.* To appropriately define the bargaining problem, one usually assumes that $\mathbf{\Pi}$ is a compact and convex set, and $(D, d) \in \mathbf{\Pi}$.

Let $\mathbf{U}$ denote the set of pairs $(\mathbf{\Pi}, (D, d))$. A solution to the bilateral bargaining problem is a mapping $f : \mathbf{U} \to \mathbf{\Pi}$ that identifies a profit allocation (Π^*, π^*). That is, $\Pi^* = f_1(\mathbf{\Pi}, (D, d))$ and $\pi^* = f_2(\mathbf{\Pi}, (D, d))$.

Definition 1 (Nash 1950) A solution $f : \mathbf{U} \to \mathbf{\Pi}$ satisfies the following axioms is called the **Nash bargaining (NB) solution**:

(i) *Pareto optimality.* For every $(\mathbf{\Pi}, (D, d)) \in \mathbf{U}$, there does not exist any $(\Pi', \pi') \in \mathcal{B}$ such that $(\Pi', \pi') \geq f(\mathbf{\Pi}, (D, d))$ and $(\Pi', \pi') \neq f(\mathbf{\Pi}, (D, d))$.

(ii) *Invariance with respect to affine transformations.* Let $A = (A_1, A_2) : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $A_i(x) = \alpha_i x + \beta_i, i = 1, 2$ with $\alpha_i > 0$. Then $f(A(\mathbf{\Pi}), A(D, d)) = A(f(\mathbf{\Pi}, (D, d)))$.

(iii) *Independence of irrelevant alternatives.* If $\mathbf{\Pi}_1 \subseteq \mathbf{\Pi}_2$ and $f(\mathbf{\Pi}_2, (D, d)) \in \mathbf{\Pi}_1$, then $f(\mathbf{\Pi}_1, (D, d)) = f(\mathbf{\Pi}_2, (D, d))$.

(iv) *Symmetry.* Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T((x_1, x_2)) = (x_2, x_1)$. Then for every $(\mathbf{\Pi}, (D, d)) \in U$, $f(T(\mathbf{\Pi}), T((D, d))) = T(f(\mathbf{\Pi}, (D, d)))$.

Nash shows that a solution satisfying the above axioms solves

$$\max\{\Omega \equiv (\pi - d)(\Pi - D) : (\Pi, \pi) \in \mathbf{\Pi}\}.$$

The Nash product Ω balances off the retailer's trade surplus $(\pi - d)$ and the supplier's trade surplus $(\Pi - D)$. Binmore et al. (1986) relax condition (iv) and introduce a parameter $\theta \in [0, 1]$, which represents the retailer's bargaining power vis-à-vis the supplier. With this relaxation, the generalized Nash bargaining solution solves

$$\max\{\Omega \equiv (\pi - d)^{\theta}(\Pi - D)^{1-\theta} : (\Pi, \pi) \in \mathbf{\Pi}\}. \tag{3.1}$$

The original Nash bargaining solution, which is often termed the symmetric Nash bargaining solution, corresponds to the situation of $\theta = 0.5$.

Despite its elegance and wide application, the NB solution has been criticized for the axiom of independence of irrelevant alternatives (IIA) (see, e.g., Roth 1977a,b). The issue associated with the IIA axiom in the collective decision making is first discussed in the seminal work by Arrow (1950). To understand the implication of the IIA axiom in the application of supply chain contracting, we take the following example.

Example 1 Consider a supplier negotiating with her primary retailer. The market price for the product is $p(q) = 1 - q$, where q is the total output of the supplier's product. The production cost is $c = 0$ per unit. The supplier has two options:

A. Negotiate exclusively with the primary retailer.

B. Negotiate first with the primary retailer. If a deal is agreed upon, negotiate with a second retailer; otherwise, do not trade with any.

Intuitively, we would expect that the supplier makes a higher profit and the primary retailer makes a lower profit under option B than under option A. This is simply because option B introduces retail competition and allows the supplier to expand her market presence. With some derivation (by applying Lemma 3.4.1 in the appendix B), we can show that for a given wholesale price $w \in [0, 1]$ between the supplier and the primary retailer,

$$\begin{aligned} \Pi_B(w) &= \frac{(1-w)w}{2} + \frac{2}{75} = \Pi_A(w) + \frac{2}{75} > \Pi_A(w), \text{ and} \\ \pi_B(w) &= \frac{\left((1-w) - \frac{4}{15}\right)^2}{4} < \frac{(1-w)^2}{4} = \pi_A(w), \end{aligned}$$

where $\Pi_i(w)$ and $\pi_i(w)$ are respectively the supplier's and the primary retailer's profits under option $i = A, B$. It is clear that for *any* wholesale price w, the supplier is better off under option B while the retailer is better off under option A.

When applying the NB solution with $\theta = 0.6$, we obtain a negotiated wholesale price of $w_A^{NB} = 0.2$ and negotiated profits $(\Pi_A(w_A^{NB}), \pi_A(w_A^{NB})) = (0.08, 0.16)$ under option A, and $w_B^{NB} \approx 0.118$ and $(\Pi_B(w_B^{NB}), \pi_B(w_B^{NB})) \approx (0.079, 0.095)$ under option B. The supplier is worse off when the primary retailer faces potential competition. This is a surprising outcome!

Example 1 suggests that NB solution ignores some geometric properties of the feasible profit allocation set $\{(\Pi(w), \pi(w)) : w \in [0, 1]\}$. Specifically, while the negotiation parties' worst case scenarios (i.e., their disagreement points) remain unchanged, the supplier has a better prospect and the retailer has a worse prospect under option B than under option A. The NB solution, however, is inconsistent with the trade prospects. The issue is due to the restriction of Axiom (iii), independence of irrelevant alternatives. This axiom, though technically appealing, may generate unreasonable solutions. To (partially) address this issue, Kalai and Smorodinsky (1975) propose an alternative axiom, called *individual monotonicity*.

Definition 2 (Kalai and Smorodinsky 1975) A solution $f : \mathbf{U} \to \mathbf{\Pi}$ satisfying axioms (i), (ii), (iv) in Definition 1 and the following axiom is called the **Kalai-Smorodinsky (KS) bargaining solution**:

(iii') *Individual monotonicity.* Let $\overline{\Pi} = \max\{\Pi : (\Pi, \pi) \in \mathbf{\Pi}\}$ and $\overline{\pi} = \max\{\pi : (\Pi, \pi) \in \mathbf{\Pi}\}$. For $(\mathbf{\Pi}_1, (D, d))$ and $(\mathbf{\Pi}_2, (D, d))$, if $\mathbf{\Pi}_1 \subseteq \mathbf{\Pi}_2$, $\overline{\Pi}_1 = \overline{\Pi}_2$ and $\overline{\pi}_1 \le \overline{\pi}_2$ (or $\overline{\pi}_1 = \overline{\pi}_2$ and $\overline{\Pi}_1 \le \overline{\Pi}_2$), then $f(\mathbf{\Pi}_1, (D, d)) \le f(\mathbf{\Pi}_2, (D, d))$.

Kalai and Smorodinsky (1975) show that the KS solution can be derived from the following equation:

$$\frac{\pi - d}{\Pi - D} = \frac{\overline{\pi} - d}{\overline{\Pi} - D}. \tag{3.2}$$

Compared with the NB solution, the KS solution allocates negotiation parties' trade surpluses proportionately according to their maximum trade surpluses (i.e., $\overline{\pi} - d$ and $\overline{\Pi} - D$). Applying the KS solution concept to Example 1, we find that under scenario A, the KS solution coincides with the NB solution, i.e., $(\Pi_A^{KS}, \pi_A^{KS}) = (0.08, 0.16)$. Under option B, the KS solution yields a profit allocation of $(\Pi_B^{KS}, \pi_B^{KS}) \approx (0.087, 0.088)$, suggesting that the supplier benefits and the primary retailer loses from involving a second retailer. This outcome, compared with the NB solution, takes into account the best prospects for the negotiation parties and appears more practical.

We shall note that the KS solution, while incorporating more information of trade prospects than the NB solution, may not completely resolve the issue associated with the NB solution. Ideally, a bargaining solution should be a function of all possible trade prospects (i.e., the geometry of the entire set $\mathbf{\Pi}$) to fully avoid the issue associated with the IIA axiom. However, such a solution, even can be developed, would be difficult to compute and implement.

3.4 The Problem of Multi-Unit Bilateral Bargaining

In supply chains, the negotiation parties' trading prospects depend critically on market competition. Thus, we focus our analysis of KS bargaining solutions on competing channels. Specifically, we consider two-tier supply chains consisting of one or two suppliers (she) selling products to one or two retailers (he), as shown in Figure 3.1. Two market structures are commonly analyzed in the literature. In a *one-to-two* channel, a common supplier sells a product to two competing retailers. In a *two-to-one* channel, two suppliers sell substitutable products to a common retailer. Though one may also consider a general two-to-two channel that involves both supply and retail competition, the main insights obtained from the one-to-two and two-to-one settings are sufficient to explain the general equilibrium behaviors, as suggested by Feng and Lu (2013b). Thus, to isolate the effects of upstream and downstream competitions and to simplify the exposition, we focus only on these two settings.

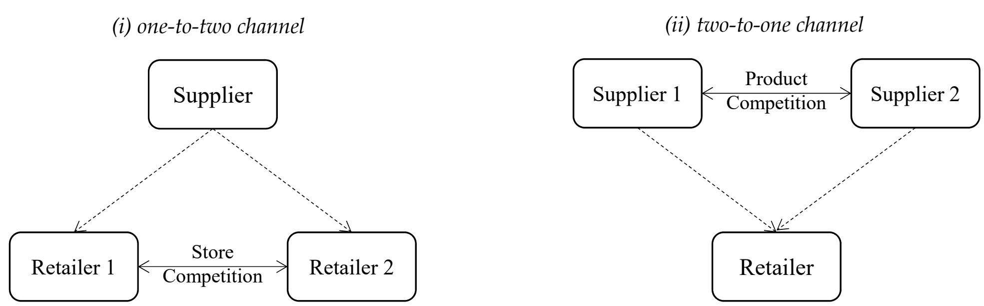


Figure 3.1. The two-tier supply chain models: one-to-two (left panel) and two-to-one (right panel).

The trade between a supplier and a retailer takes place through a bilateral negotiation over a wholesale-price contract, denoted by $w_i, i \in \{1, 2\}$. We focus on the wholesale-price contract because it is widely used in practice and because it yields interesting comparison under different negotiation frameworks. In §3.6.3, we also discuss the implications of alternative contracts. In our models, there are two bargaining pairs or bargaining units, each consisting of a supplier and a retailer. We allow for the possibility that the two negotiations are *parallel* (see, e.g., Feng and

Lu 2013a) or *sequential* (see, e.g., Feng and Shanthikumar 2018a). The setting of parallel bargaining describes the situation in which the party involved in both units simultaneously interacts with two trading partners to determine the contracts. The setting of sequential bargaining is suitable when one contract is determined before the negotiation starts for the other contract. As we lay out the model in the next section, it becomes clear that negotiation sequence has a major impact on the trading parties' walk-away values, which in turn determine the trade outcome. We also distinguish the situations with contingency from those without. Specifically, a trading party may impose contingency terms on a contract that the negotiated contract will be executed only if the other negotiation is successful (see, e.g., Chu et al. 2019). For example, a supplier, who intends to enter a local market, may simultaneously negotiate with two retailers with the condition that she either fully serves the market (i.e., trading with both) or does not serve at all (i.e., trading with none). Likewise, a retailer may only carry a product line when he has successful deals with both suppliers.

Once the contract terms are agreed upon, each retailer determines the quantity that he orders from each supplier according to the contract terms. To isolate the effects of channel structure and negotiation sequence, we assume that the competing parties are symmetric. A supplier produces the product at a marginal cost of c, and a retailer can sell the product at a price that is decreasing in his own output as well as the opponent retailer's output, if any. For the most of our discussion, we assume that the price p_i of product in channel i is a linear function of the quantity q_i in channel i and the quantity q_j in the other channel, i.e.,

$$p_i(q_i, q_j) = a - bq_i - \eta b q_j, \quad i = 1, 2, \ j \neq i,$$

where $a > c$, $b > 0$, and $\eta \in [0, 1]$. The parameter η captures the level of store substitutability in the one-to-two channel and that of product substitutability in the two-to-one channel. This demand function is the most commonly used in studies of competing supply chains. (In §§3.6.2, we discuss implications of nonlinear price-demand relationship.) Following the standard analysis (see, e.g., Davidson 1988, Feng and Lu 2013a), we have the following result.

Lemma 3.4.1 (Retail Quantities and Prices) *Given the negotiated contracts* $\mathbf{w} = (w_1, w_2)$,

i) **One-to-two channel:** *the competing retailers set the following equilibrium quantities and prices: (a) if* $\frac{a-w_i}{a-w_j} \leq \frac{\eta}{2}$, $p_i^*(\mathbf{w}) = \frac{2a-\eta(a-w_j)}{2}$ *and* $q_i^*(\mathbf{w}) = 0$; *(b) if* $\frac{\eta}{2} < \frac{a-w_i}{a-w_j} < \frac{2}{\eta}$, $p_i^*(\mathbf{w}) = w_i + \frac{2(a-w_i)-\eta(a-w_j)}{4-\eta^2}$ *and* $q_i^*(\mathbf{w}) = \frac{2(a-w_i)-\eta(a-w_j)}{b(4-\eta^2)}$; *(c) if* $\frac{a-w_i}{a-w_j} \geq \frac{2}{\eta}$, $p_i^*(\mathbf{w}) = \frac{a+w_i}{2}$ *and* $q_i^*(\mathbf{w}) = \frac{a-w_i}{2b}$, $i, j \in \{1, 2\}$ *and* $i \neq j$.

ii) **two-to-one channel:** *the common retailer sets the following quantities and prices: (a) if* $\frac{a-w_i}{a-w_j} \leq \eta$, $p_i^*(\mathbf{w}) = \frac{2a-\eta(a-w_j)}{2}$ *and* $q_i^*(\mathbf{w}) = 0$; *(b) if* $\eta < \frac{a-w_i}{a-w_j} < \frac{1}{\eta}$, $p_i^*(\mathbf{w}) = \frac{a+w_i}{2}$ *and* $q_i^*(\mathbf{w}) = \frac{(a-w_i)-\eta(a-w_j)}{2b(1-\eta^2)}$; *(c) if* $\frac{a-w_i}{a-w_j} \geq \frac{1}{\eta}$, $p_i^*(\mathbf{w}) = \frac{a+w_i}{2}$ *and* $q_i^*(\mathbf{w}) = \frac{a-w_i}{2b}$, $i, j \in \{1, 2\}$ *and* $i \neq j$.

With the result in Lemma 3.4.1, we can compute the supplier's and retailer's profits from trade i as

$$R_i(w_i, w_j) = q_i^*(\mathbf{w})(w_i - c) \text{ and } r_i(w_i, w_j) = q_i^*(\mathbf{w})(p_i^*(\mathbf{w}) - w_i), \tag{3.3}$$

respectively. These expressions allow us to compute the trading parties' profit functions for our analysis of the negotiation outcomes in §3.5. For ease of exposition, we use $\Pi_i(w_i, w_j)$ to denote the supplier's profit and $\pi_i(w_i, w_j)$ to denote the retailer's profit in trade $i \in \{1, 2\}$, with the understanding that $\Pi(w_1, w_2) = \Pi_i(w_i, w_j) = \Pi_j(w_j, w_i)$ for the common supplier in the one-to-two channel and $\pi(w_1, w_2) = \pi_i(w_i, w_j) = \pi_j(w_j, w_i)$ for the common retailer in the two-to-one channel. Specifically, in the one-to-two channel, we derive

$$\Pi_i(w_i, w_j) = \Pi_j(w_j, w_i) = R_i(w_i, w_j) + R_j(w_j, w_i) \quad \text{and} \quad \pi_i(w_i, w_j) = r_i(w_i, w_j). \tag{3.4}$$

In the two-to-one channel, we have

$$\Pi_i(w_i, w_j) = R_i(w_i, w_j) \quad \text{and} \quad \pi_i(w_i, w_j) = \pi_j(w_j, w_i) = r_i(w_i, w_j) + r_j(w_j, w_i). \tag{3.5}$$

Because we are interested in the competition equilibrium, it is natural to restrict the feasible contract parameter set to be

$$\hat{\mathcal{C}} = \left\{(w_i, w_j) : \frac{\eta}{2} \leq \frac{a - w_i}{a - w_j} \leq \frac{2}{\eta}\right\} \quad \text{and} \quad \check{\mathcal{C}} = \left\{(w_i, w_j) : \eta \leq \frac{a - w_i}{a - w_j} \leq \frac{1}{\eta}\right\} \tag{3.6}$$

for the one-to-two channel and for the two-to-one channel, respectively.

We shall remark that the set of profit allocation $\mathbf{\Pi}$ under the wholesale-price contract is *not* convex. In general, one needs to convexify the feasible region by extending the negotiation to randomized contracts (i.e., specifying a distribution over the feasible range of wholesale prices). When the contract execution does not require contingency terms (see, e.g., Feng and Lu 2013a), the feasible profit allocation region contains the upper boundary (i.e., the Pareto set) of the convexified region and thus the negotiated contract is nonrandomized. As a result, convexification is not necessary in such settings. When negotiating over contingency contracts, however, the feasible region may not fully contain the Pareto set of the convexified region. Such a situation only arises when the competition is intense (i.e., when η is close to 1). In this case, we choose to extend the Pareto-dominated region, instead of convexifying the entire feasible region, to ensure the continuity of the bargaining solution. The detailed discussion on this can be found in the appendix E. For all of the formal results presented below, we focus only on the situations where the equilibrium contracts are non-randomized.

3.4.1 A Benchmark Model: The One-to-One Channel

In this subsection, we consider the channel with one supplier and one retailer. The retailer faces an inverse demand of

$$p(q) = a - bq,$$

where $a > c$, $b > 0$, and $q \geq 0$ is the output level of the retailer. Based on case (c) of Lemma 3.4.1, the retailer's optimal output level is $q^*(w) = \frac{a-w}{2b}$, resulting a price of $p^*(w) = \frac{a+w}{2}$. We can then compute the supplier's and the retailer's profits, respectively, as

$$\Pi(w) = q^*(w)(w - c) = \frac{(a - w)(w - c)}{2b} \quad \text{and} \quad \pi(w) = q^*(w)(p^*(w) - w) = \frac{(a - w)^2}{4b}.$$

We shall note that regardless of the bargaining framework applied, the feasible set of profit allocation is defined by

$$\mathbf{\Pi} = \{(\Pi(w), \pi(w)) : c \leq w \leq a\}.$$

The trading parties' disagreement points are $D = d = 0$ and their maximum profits are

$$\begin{aligned}
\overline{\Pi} &= \max\{\Pi(w) : \pi(w) \geq 0\} = \Pi(\frac{a+c}{2}) = \frac{(a-c)^2}{8b}, \\
\overline{\pi} &= \max\{\pi(w) : \Pi(w) \geq 0\} = \pi(c) = \frac{(a-c)^2}{4b}.
\end{aligned}$$

Proposition 3.4.1 (Benchmark: The One-to-One Channel) *In the one-to-one channel the KS solution yields a wholesale price of*

$$w^{KS} = c + \frac{a-c}{5},$$

which corresponds to the NB solution with the retailer's bargaining power being $\theta^{KS} =$ 0.6.

In the NB solution, the bargaining power determines the relative profit allocation between the trading parties. The larger the bargaining power is, the more portion of profit the firm obtains through negotiation. Proposition 3.4.1 suggests that in the two-tier supply chain the KS solution does not coincide with the symmetric Nash bargaining solution (i.e., that with $\theta = 0.5$). In other words, the trading parties' best prospects can have a major impact on supply chain negotiation. In particular, the retailer, who is able to influence the channel profit through his output decision, has a higher maximum profit than the supplier does. From this perspective, the retailer is granted more 'control' over the negotiation despite the symmetric structure of the bargaining problem.

3.5 Negotiations in Competing Supply Chains

In this section, we analyze in detail the equilibrium negotiation outcomes in competing channels. Section 3.5.1 focuses on the case when no contingency terms are imposed on contract execution and Section 3.5.2 discusses the case with contingency terms.

3.5.1 Negotiations without Contingencies

In this subsection, we consider the situation in which no contingency terms are imposed on executing the negotiated contracts, which is the most studied situation in the literature. When no contingency of contract execution is imposed, if a bargaining unit successfully reaches an agreement, the agreed contract is implemented regardless of the negotiation outcome in the other bargaining unit.

3.5.1.1 The Interdependence between the Trades and the NB Solution

Because there are two trades, the common trading party's profit depends on the outcome of both negotiations. Also, the common trading party may still make profit from the other trade in the event of one negotiation breakdown, imposing a potential threat to the negotiation partner. As a result, the trade prospects for the negotiation parties involved in one bargaining unit certainly depend on the negotiation in the other. To make the idea explicit, we consider the situation where contract w_j is given and focus on understanding the negotiation within bargaining unit i. Such a situation arises in the simultaneous game when unit i is determining its best response to unit j, or in the sequential negotiation when unit i is determining the negotiation outcome after unit j has reached an agreement.

In the one-to-two channel, the retailers have zero disagreement points (i.e., $d_i(w_j) = 0$), while the common supplier has a nonzero disagreement point. Given the contract w_j, the supplier's disagreement point in bargaining unit i is the profit she makes if only trading with retailer j with contract w_j. Such a situation corresponds to Lemma 3.4.1(i-a), which is equivalent to setting a wholesale price w_i such that $(a - w_i)/(a - w_j) \leq \eta/2$. Thus, the supplier's disagreement point is

$$D_i(w_j) = R_j(w_j, a - \eta(a - w_j)/2). \tag{3.7}$$

Similarly, in the two-to-one channel, the suppliers have zero disagreement points (i.e., $D_i(w_j) = 0$). The common retailer's disagreement point in bargaining unit i is the profit that he makes if only selling supplier j's product. This corresponds to Lemma 3.4.1(ii-a) that w_i satisfies $(a - w_i)/(a - w_j) \leq \eta$. Thus, the retailer's disagreement point is

$$d_i(w_j) = r_j(w_j, a - \eta(a - w_j)). \tag{3.8}$$

With the trading parties' profits in (3.4)-(3.5) and disagreement points in (3.7)-(3.8), we can apply (3.1) to derive the negotiated NB contract $w_i^{NB}(w_j)$ for bargaining unit i as the best response to a w_j chosen by bargaining unit j.

Lemma 3.5.1 (Trade Prospects and the NB Solution) *Consider the trade in bargaining unit i with the contract w_j in unit j given. Suppose (w_i, w_j) satisfies condition (3.6) and the profit allocation within unit i belongs to the Pareto set.*

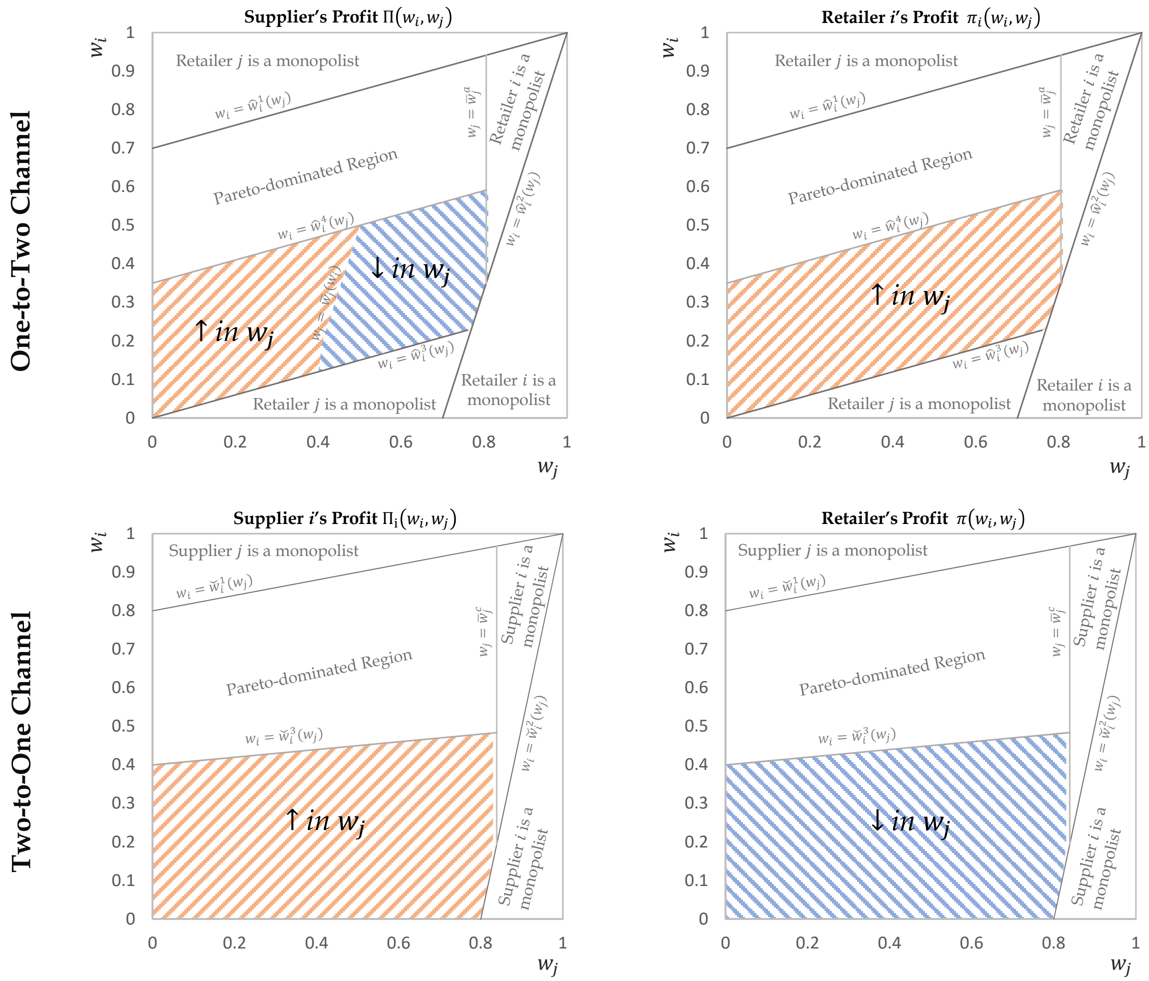
i) In the one-to-two channel, the supplier's profit $\Pi(w_i, w_j)$ is increasing [decreasing] in w_j for $w_j \leq [>] \bar{w}_j(w_i)$, where $\bar{w}_j(\cdot)$ is an increasing function, and retailer i's profit $\pi_i(w_i, w_j)$ is increasing in w_j. Under the NB solution, the supplier's negotiated profit $\Pi(w_i^{NB}(w_j), w_j)$ is increasing [decreasing] in w_j for $w_j \leq [>] \bar{w}_j(w_i^{NB}((a + c)/2)) = (a + c)/2$, and retailer i's negotiated profit $\pi_i(w_i^{NB}(w_j), w_j)$ is constant in w_j.

ii) In the two-to-one channel, supplier i's profit $\Pi_i(w_i, w_j)$ is increasing in w_j and the retailer's profit $\pi(w_i, w_j)$ is decreasing in w_j. Under the NB solution, supplier i's negotiated profit $\Pi_i(w_i^{NB}(w_j), w_j)$ is increasing in w_j and the retailer's negotiated profit $\pi(w_i^{NB}(w_j), w_j)$ is decreasing in w_j.

Because the interdependence between the two trades disappears when one of the products monopolizes the market, we focus only on the region in which both products have positive outputs in Lemma 3.5.1. We also restrict our attention to wholesale prices that lead to a Pareto profit allocation, because a Pareto-dominated profit allocation cannot arise in the negotiation outcome as suggested by Axiom (i). Figure 3.2 demonstrates the properties of the trade prospects (i.e., feasible trade profits). The corresponding negotiated trade profits under the NB solution are depicted in Figure 3.3.

In the one-to-two channel, an increased w_j suggests a weakened competition from bargaining unit j, it does not necessarily lead to improved trade profits obtained by trading parties in unit i. In particular, retailer i's profit is always increasing in w_j due to the reduced competitive pressure, but the supplier's profit is not necessarily monotone in w_j. On the one hand, an increased w_j implies enhanced competitiveness of the product carried by retailer i, leading to an increased trade profit of product i to be shared by the supplier. On the other hand, an increased w_j leads to a reduced profit that the supplier can claim from the product sold in the other bargaining unit, leading to a reduced trade profit contributed by the supplier. The first effect dominates the second for a small w_j, while the second effect is much enhanced for a large w_j. Consequently, the supplier's profit is increasing in w_j when w_j is small and is decreasing when w_j is large. In the two-to-one channel, because the competition between the products is internalized by a common retailer, an increased w_j naturally benefits the supplier i but hurts the retailer.

We observe that whenever a trading party's profit is increasing or decreasing in w_j (for all possible choice of w_i), so does the corresponding negotiated profit under the NB solution. In other words, the potential inconsistency between the trade prospects and the negotiation outcome due to the IIA axiom does not arise in dependent trades when no contingencies are imposed.



Notes. $a = 1$, $b = 1$, $c = 0$, $\theta = 0.6$, $\eta = 0.6$ in the one-to-two channel and $\eta = 0.2$ in the two-to-one channel.

Figure 3.2. The trade profits in bargaining unit i for a given w_j, as characterized in Lemma 3.5.1.

3.5.1.2 The Connection between the KS and NB Solutions

Under simultaneous negotiations, each bargaining unit reaches an agreement as a best response to the contract of the other unit. The game outcome is the Nash equilibrium of the two bargaining problems. Because the NB solution for this problem has been derived by Feng and Lu (2013a), we focus on analyzing the KS solution and establish its connection to the NB solution.

To derive the KS solution, we need to compute the trading parties' maximum profits for any given w_j:

$$\overline{\Pi}_i(w_j) = \max\{\Pi_i(w_i, w_j) : \pi_i(w_i, w_j) \geq d_i(w_j)\}, \tag{3.9}$$

$$\overline{\pi}_i(w_j) = \max\{\pi_i(w_i, w_j) : \Pi_i(w_i, w_j) \geq D_i(w_j)\}. \tag{3.10}$$

Substituting the trade profits in (3.4)-(3.5), the disagreement points in (3.7)-(3.8), and the maximum profits (3.9)-(3.10) into (3.2), we can obtain the KS solution $w_i^{KS}(w_j)$ of bargaining unit i as a best response to a given w_j. The equilibrium of the best responses $w_i^{KS}(w_j)$ and $w_j^{KS}(w_i)$ is the simultaneously negotiated KS contracts.

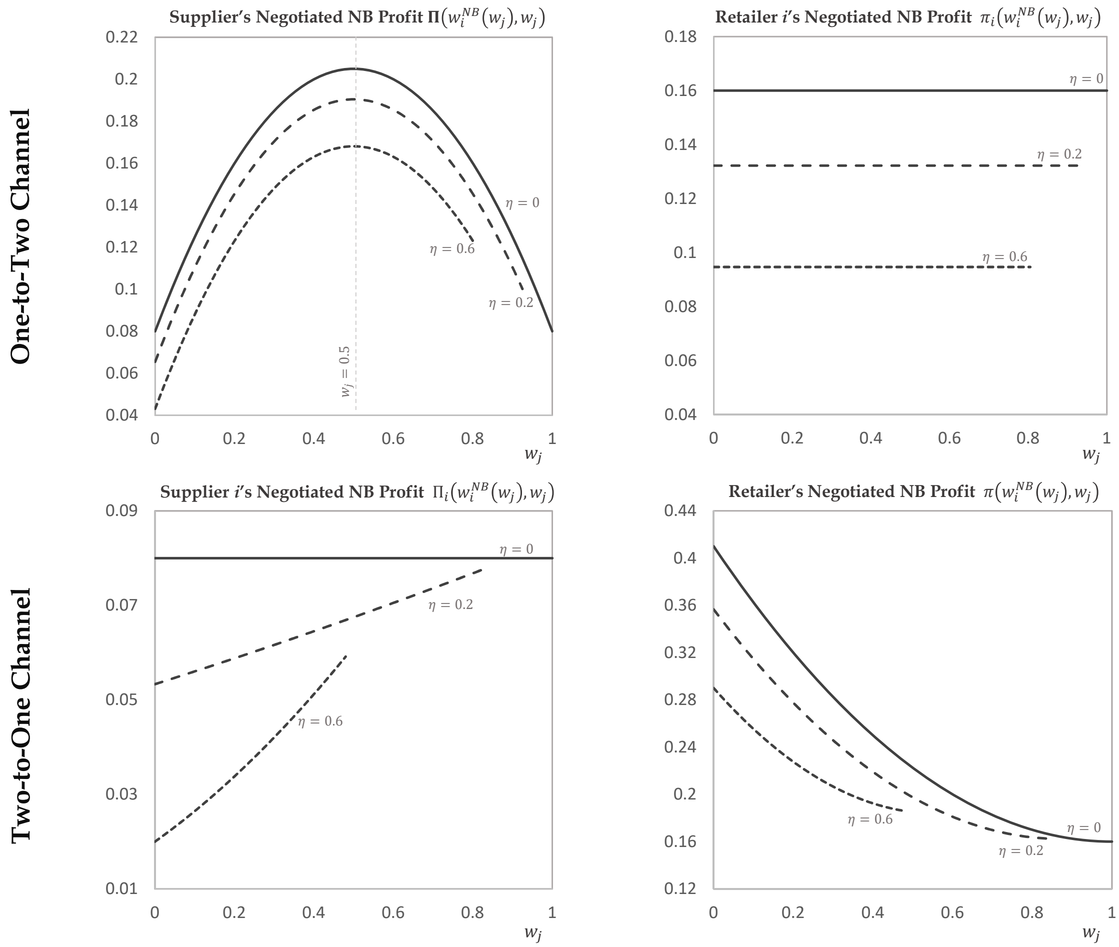
Proposition 3.5.1 (Simultaneous Bargaining without Contingency) *Applying the KS solution, the negotiated wholesale prices in the one-to-two and two-to-one channel are, respectively,*

$$\hat{w}_{sim}^{KS} = c + \frac{a-c}{5} \text{ and } \check{w}_{sim}^{KS} = c + \frac{(1-\eta)(a-c)}{5-\eta},$$

which equal to the corresponding NB solutions with the retailers' bargaining power being $\hat{\theta}_{sim}^{KS} = \check{\theta}_{sim}^{KS} = 0.6$.

The negotiated wholesale price in the one-to-two channel coincides with that of the one-to-one channel, while that in the two-to-one channel is generally smaller and is decreasing in the level of supply competition represented by η. In all these settings, the KS solution corresponds to the NB solution with the retailer's bargaining power being 0.6.

Now we consider the situation of sequential negotiations. Given that bargaining unit j reaches an agreement of w_j, the solution for the second negotiation in bargaining unit i is simply $w_i^{KS}(w_j)$, which is the same as the best response of bargaining unit i to a given w_j in the simultaneous bargaining. Thus, we only need to focus on deriving the negotiation outcome for the first negotiation in bargaining unit j. If the negotiation breaks down in unit j, then unit i reduces to a one-to-one channel, suggesting $w_i = c + (a-c)/5$ by Proposition 3.4.1. Thus, in anticipation of the



Notes. $a = 1$, $b = 1$, $c = 0$, and $\theta = 0.6$. We only compute the profits for w_j values for which unit j sells a positive quantity.

Figure 3.3. The negotiated NB profits in bargaining unit i as characterized in Lemma 3.5.1.

latter negotiation within unit i, the disagreement points of bargaining unit j satisfy $D_j = D_j(c + (a - c)/5)$ and $d_j = 0$ in the one-to-two channel, and $D_j = 0$ and

$d_j = d_j(c + (a - c)/5)$ in the two-to-one channel. Also, the trading parties' maximum profits in bargaining unit j are

$$\begin{aligned}\overline{\Pi}_j &= \max\{\Pi_j(w_j, w_i^{KS}(w_j)) : \pi_j(w_j, w_i^{KS}(w_j)) \geq d_j\},\\ \overline{\pi}_j &= \max\{\pi_j(w_j, w_i^{KS}(w_j)) : \Pi_j(w_j, w_i^{KS}(w_j)) \geq D_j\}.\end{aligned}$$

Proposition 3.5.2 (Sequential Bargaining without Contingency) *Applying the KS solution, the equilibrium satisfies the following.*

i) In the one-to-two channel, the negotiated wholesale prices in the first and second trades are both increasing in η. This equilibrium outcome corresponds to that under the NB solution with the retailer's bargaining power in the first and second trades being, respectively,

$$\hat{\theta}_{seq1}^{KS}(\eta) \in (0.5, 0.6] \quad \textit{and} \quad \hat{\theta}_{seq2}^{KS} = 0.6,$$

where $\hat{\theta}_{seq1}^{KS}(\cdot)$ is decreasing.

ii) In the two-to-one channel, the negotiated wholesale prices in the first and second trades are both decreasing in η. This equilibrium outcome corresponds to that under the NB solution with the retailer's bargaining power in the first and second trades being

$$\check{\theta}_{seq1}^{KS}(\eta) \in [0.6, 1] \quad \textit{and} \quad \check{\theta}_{seq2}^{KS} = 0.6,$$

where $\check{\theta}_{seq1}^{KS}(\cdot)$ is increasing.

According to Proposition 3.5.2, an intensified retail competition leads to increased trade prices in both negotiations, while an intensified supply competition leads to reduced trading prices. In the one-to-two channel, a competing retailer gains a reduced portion of the trade surplus if he, as opposed to his opponent, negotiates first with the common supplier. This is because the supplier can leverage her anticipated negotiation with the second retailer in the first negotiation, leading to an increase in her equivalent bargaining power (to be above 0.4). In the two-to-one channel, similarly, the common retailer's equivalent bargaining power increases (to be above 0.6) in his first negotiation. These results suggest that the KS solution, compared with the NB solution, appropriately captures the disadvantage faced by the competing firms when reaching early deals with common vertical partners.

3.5.2 Negotiations with Contingencies

Contingency contract negotiations are not uncommon in practice (Chu et al. 2019). A supplier, who intends to enter a new territory, may only serve that market if she can have significant presence. Without a large enough market potential, the investments involved in setting up the logistics lanes, in catering for local preference, and in dealing with local trade environment may not be justified. Thus, before entering the negotiations, the supplier may impose a contingency term that she would also not execute the contract with one retailer if a deal with the other retailer fails. Such a contingency term makes the supplier's disagreement point in either trade zero, no matter she negotiates with the retailers in parallel or in sequence. Likewise, a retailer, who plans to add a new product family to his assortment, would only do so if he can carry two products. This may be due to the significant effort needed to market and manage a new product family, which may not be justified by the sales generated from a single product. In this case, the retailer may specify a term that an agreed-upon contract with one supplier is only executed if the retailer also strikes a deal with the other supplier. The contingency-negotiation problems can be formulated similarly as their counterparts in §§3.5.1.1 with $D_i(w_j) = d_i(w_j) = 0$ and those in §§3.5.1.2 with $D_j = d_j = 0$.

Intuitively, the contingency term imposes an execution constraint to both trading parties. On the one hand, the competing firms can credibly threat each other with the possibility of negotiation breakdown. On the other hand, the common trading party now faces a disadvantage in that it no longer enjoys positive disagreement points to leverage the two trades, while still having to share its profit from both products in each trade.

3.5.2.1 The Interdependence between the Trades and the NB Solution

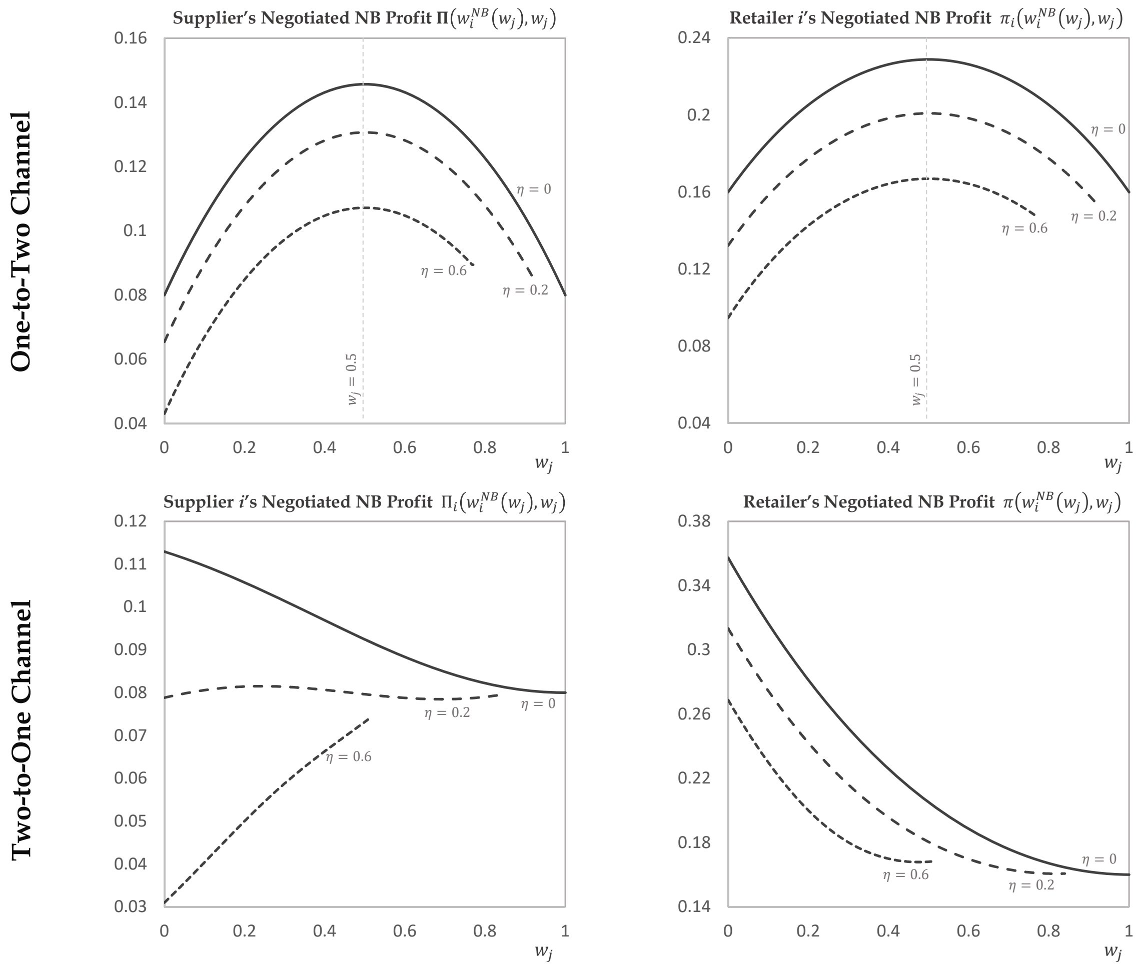
When contingencies are imposed, the issue of the NB solution due to the IIA axiom indeed arises. Both the feasible set of trade profits and the negotiated profits in one trade depend on the contract of the other trade. However, the effects on the feasible set and the negotiated profits can go in opposite directions as suggested by Lemma 3.5.2. This is also evident from Figure 3.4, where we plot the negotiated profits.

Lemma 3.5.2 (Trade Prospects and the NB solution) *Consider the trade in bargaining unit i with the contract w_j in unit j given. Suppose (w_i, w_j) satisfies (3.6) and the profit allocation within unit i belongs to the Pareto set. The trading parties' profits as functions of (w_i, w_j) exhibit the same property as in Lemma 3.5.1. Moreover, the NB solution leads to the following negotiation outcomes.*

i) In the one-to-two channel, the supplier's negotiated profit $\Pi(w_i^{NB}(w_j), w_j)$ and the retailer i's negotiated profit $\pi_i(w_i^{NB}(w_j), w_j)$ are increasing [decreasing] in w_j for $w_j \leq [>](a+c)/2$.

ii) In the two-to-one channel, supplier i's negotiated profit $\Pi_i(w_i^{NB}(w_j), w_j)$ and the retailer's negotiated profit $\pi(w_i^{NB}(w_j), w_j)$ can be increasing or decreasing in w_j.

We relate Lemma 3.5.2 to Lemma 3.5.1 to understand the effect of contingency terms. In the one-to-two channel, a higher w_j leads to a larger profit obtained by retailer i for any feasible w_i (recall Lemma 3.5.1-i). However, the retailer's negotiated profit is not necessarily increasing with w_j, as suggested by Lemma 3.5.2-i (see also the demonstration in the upper right panel of Figure 3.4). In other words, despite the improved trade prospect for the retailer, the retailer's negotiated profit may reduce.



Notes. $a = 1$, $b = 1$, $c = 0$, and $\theta = 0.6$. We only compute the profits for w_j values for which unit j sells a positive quantity.

Figure 3.4. The negotiated NB profits in bargaining unit i for a given w_j, as characterized in Lemma 3.5.2.

Similar situations arise in the two-to-one channel. As w_j increases, the supplier's trade profit always increases while retailer i's trade profit always decreases (recall Lemma 3.5.1-ii). However, both parties may end up with increased or decreased negotiated profits.

These findings, in direct contrast to their counterparts in contracting without contingencies, suggest that the NB solution may not be appropriate for studies involving contract contingency and channel competition.

3.5.2.2 The Connection between the KS and NB Solutions

The next proposition characterizes the equilibrium contracts under the KS solution and their relationships to the NB solution for simultaneous negotiations over contingency contracts.

Proposition 3.5.3 (Simultaneous Bargaining with Contingency) *Applying the KS solution, the equilibrium contracts satisfy the following.*

i) In the one-to-two channel, the negotiated wholesale price is decreasing in η. This equilibrium outcome corresponds to that under the NB solution with the retailers' bargaining power being $\hat{\theta}^{KS}_{sim}(\eta) \in (0.57, 0.59)$, where $\hat{\theta}^{KS}_{sim}(\cdot)$ is decreasing.

ii) In the two-to-one channel, the negotiated wholesale price is decreasing in η. This equilibrium outcome corresponds to that under the NB solution with the retailers' bargaining power being $\check{\theta}^{KS}_{sim}(\eta) \in (0.68, 1]$, where $\check{\theta}^{KS}_{sim}(\cdot)$ is increasing.

Compared with Proposition 3.5.1, Proposition 3.5.3 suggests that the imposed contingencies on contract execution effectively enhance the equivalent bargaining power of the common trading party. Specifically, the supplier in the one-to-two channel possesses an equivalent bargaining power above 0.4 and the retailer in the two-to-one channel obtains an equivalent bargaining power above 0.6.

When negotiations are conducted in sequence, the analysis becomes complex and we resort to numerical computation. Note from our earlier discussions that the implied retailer bargaining power under the KS solution, as shown in Figure 3.5, depends only on the competition parameter η but not on any other model inputs. We observe that the implied retailer's bargaining power in the second trade under the sequential bargaining is close to that under the simultaneous bargaining. In the one-to-two channel, the KS solution grants a lower bargaining power to the retailer in the first trade than to the one in the second, which is consistent with the observation from Proposition 3.5.2 for the case without contingency terms. In the two-to-one channel, however, the KS solution leads to a higher retailer's bargaining power in the first trade than that in the second only when the supply competition is not intense. This observation makes a direct contrast to that from Proposition 3.5.2.

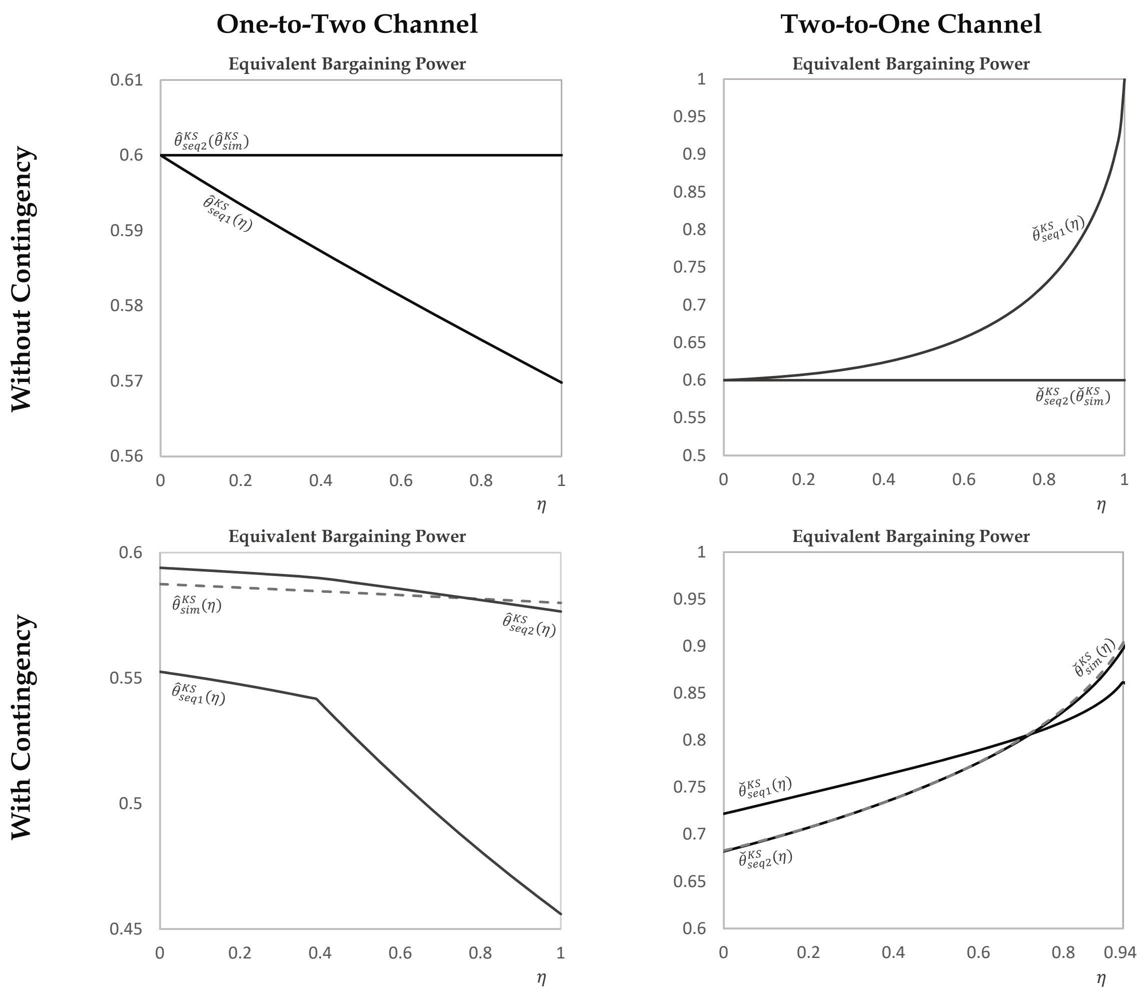


Figure 3.5. The retailer's equivalent bargaining power implied by the KS solution.

Comparing the upper and the lower panels of Figure 3.5, we observe that contingency terms grant the supplier a higher equivalent bargaining power in the one-to-two channel, while leaving the retailer a higher equivalent bargaining power in the two-to-one channel (unless the competition is highly intense). This suggests that the KS solution captures the disadvantage that the competing firms face when the contract contingency is imposed in vertical relationships.

3.6 Discussions

In this section, we discuss several aspects of our models. Section 3.6.1 analyzes the effect of negotiation sequence, Section 3.6.2 addresses the implication of nonlinear demand functions, and Section 3.6.3 considers negotiation outcomes under alternative contracts.

3.6.1 The Effect of Negotiation Sequence

In competing supply chains, the sequence with which firms negotiate can have a major implication on the negotiating firms' power distribution, as suggested in our analysis in the previous section. An alternative dimension to examine the effect of negotiation sequence is the profit allocation characterized below. Following our earlier discussions, we use the subscripts 'sim', 'seq1', 'seq2' to index the firms in the simultaneous negotiation, in the first of the sequential negotiation, and in the second of the sequential negotiation, respectively.

Proposition 3.6.1 (Bargaining Sequence: without Contingency) *Regardless of the channel structure and the bargaining solution applied, a supplier's negotiated profit is higher under sequential bargaining, while a retailer's negotiated profit is higher under simultaneous bargaining. Moreover, the following holds.*

i) In the one-to-two channel, the negotiated retailers' profits satisfy

$$\hat{\pi}^i_{seq1} \leq \hat{\pi}^i_{seq2} = \hat{\pi}^i_{sim},\ i \in \{NB, KS\}.$$

ii) In the two-to-one channel, there exists an $\bar{\eta}^i$ such that the negotiated suppliers' profits satisfy

$$\check{\Pi}^i_{sim} \leq \check{\Pi}^i_{seq2} \leq \check{\Pi}^i_{seq1} \quad [\check{\Pi}^i_{sim} \leq \check{\Pi}^i_{seq1} \leq \check{\Pi}^i_{seq2}]$$

for $\eta \leq [\geq] \bar{\eta}^i$ and $i \in \{NB, KS\}$. Under the NB solution, $\bar{\eta}^{NB}$ is increasing in θ.

Moreover, equalities hold in the above comparisons when $\eta = 0$ in (i) and $\eta \in \{0, 1\}$ in (ii).

When no contingency term is imposed on contract execution, a supplier always prefers sequential negotiation, while a retailer always prefers simultaneous negotiation, regardless of the channel structure or bargaining solution applied. The sequential negotiation, as opposed to the simultaneous negotiation, softens the competition among the competing firm, and grants the common negotiating party the opportunity to leverage the second trade as a credible threat against the first trade. In the one-to-two channel, the common supplier can enjoy the benefit of both softened retail competition and high negotiated wholesale prices with sequential trading. In the two-to-one channel, the common retailer partially internalizes the competition between the two products, because the retailer also determines the competing parameters, i.e., quantities. This reduces the retailer's incentive of leveraging between the trades to push down the wholesale prices. As a result, the effect of softened supply competition dominates, leading to increased suppliers' profits under sequential negotiations.

When negotiations are conducted sequentially, the competing retailers in the one-to-two channel would like to participate in the second negotiation instead of the first, because the common supplier can demand an aggressive wholesale price by leveraging the second trade. This observation is in line with our earlier discussion of Proposition 3.5.2 that the equivalent bargaining power of the retailer in the first trade is lower than that in the second. For the competing suppliers in the two-to-one channel, however, the preference of negotiation sequence depends also on the competition intensity. Despite that the implied retailer's bargaining power is higher (recall Proposition 3.5.2), the negotiated wholesale price is higher in the first trade than that in the second. This is because if the first negotiation breaks down, the supplier becomes a monopoly in the second trade, suggesting a lower disagreement point possessed by the retailer in the first trade than that in the second. As a result, the retailer has to agree on a higher wholesale price with the first supplier than that with the second. When the suppliers' products are highly differentiated (i.e., when η is low), the first supplier enjoys a large profit due to the high wholesale price. When the products are very similar (i.e., when η is high), the first supplier suffers a low order quantity from the retailer, as the latter would tend to shift the allocation toward the second, low-price supplier. These observations highlight that under either NB or KS solution, the bargaining power alone cannot determine the negotiated profits. The interdependence between the trades plays a crucial role in competitive environment.

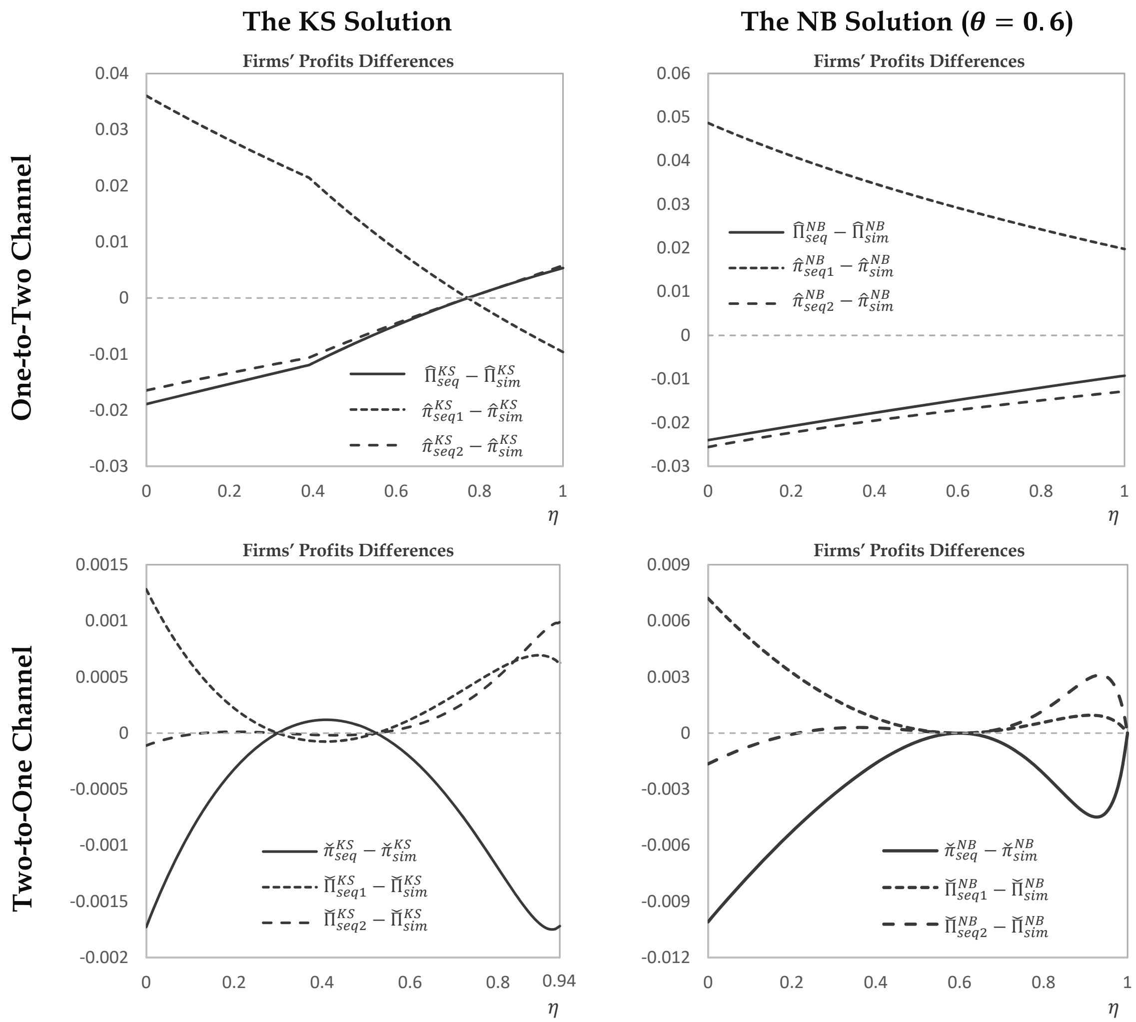
Now we turn to the case with contingency terms on contract execution. In this case, both the upstream and downstream firms may prefer either simultaneous or sequential negotiations depending on the level of competition.

Proposition 3.6.2 (Bargaining Sequence: with Contingency) *In the one-to-two channel, there exists an $\bar{\eta}^i$ such that the negotiated retailers' profits satisfy*

$$\hat{\pi}^i_{seq1} \geq \hat{\pi}^i_{sim} \geq \hat{\pi}^i_{seq2} \ [\hat{\pi}^i_{seq1} \leq \hat{\pi}^i_{sim} \leq \hat{\pi}^i_{seq2}]$$

and the negotiated supplier's profits satisfy $\hat{\Pi}^i_{sim} \geq [\leq] \hat{\Pi}^i_{seq}$ for $\eta \leq [>] \bar{\eta}^i$ and $i \in \{NB, KS\}$. Under the NB solution, $\bar{\eta}^{NB}$ is increasing in θ.

When the contract execution is contingent on the success of the other trade, the common trading party is no longer able to leverage one trade against the other as the disagreement point becomes zero. Therefore, the retailer in the first trade may not necessarily earn a lower profit than that in the second trade, as suggested by Proposition 3.6.2. When the retail competition is intense (i.e., when η is close to 1), the retailer's profit in the first trade is indeed lower than that in the second. When



Notes. $a = 1$, $b = 1$, $c = 0$, and in the right panel $\theta = 0.6$.

Figure 3.6. The profit comparison in the case with contingency.

retail competition is weak (i.e., when η is close to 0), however, the retailer in the first trade enjoys a higher profit than that in the second.

To understand this effect, we consider the extreme case without retail competition (i.e., $\eta = 0$). If there were no contingency terms in the contract, the two trades would become completely independent and the retailers would make the same profits in the simultaneous game and in the sequential game (recall Proposition 3.6.1).

With contingency contracts, however, the common supplier's trade profit comes from successfully selling to both retailers. Thus the two trades are interdependent because part of the profit of one product is shared in the trade between the common supplier and the other retailer. This grants the retailer in the first trade a credible threat to the one in the first trade through the common supplier. Consequently, the first retailer makes a higher profit than the second.

In the two-to-one channel, the comparison between simultaneous and sequential negotiations reveals complex structure as illustrated in Figure 3.6. In general, depending on the level of competition, there can be disjoint regions in which a firm makes a higher or lower profit in one setting than that in the other.

3.6.2 Alternative Demand Functions

In our base model analyzed in §3.5, we have assumed a linear price-demand relationship. In this section, we examine general nonlinear demand functions. There are two major difficulties in dealing with general demand functions. First, unlike in the case of linear demands where the effects of competing quantities are separable in affecting the market prices, nonlinear demands often involve interactions among competing quantities and prices. Second, it is often impossible to derive an explicit form of the competing equilibrium under nonlinear demands. In view of these difficulties, we present the formal results for the one-to-one channel, while confirming their implication to competing channels through a numerical analysis.

Consider the situation when only one product is sold in the market. The retailer's revenue is a concave function $R(q) = p(q)q$, where $p(q)$ is the market price when the output quantity is q. In general, a concave function $R_a(\cdot)$ is said to be more (equally, less) concave than another concave function $R_b(\cdot)$ if $R_a''(x) < (=, >) R_b''(x)$ for $x \in \mathbb{R}$, where $R''(\cdot)$ is the second-order derivative of $R(\cdot)$. The next corollary gives explicit results for the commonly used polynomial, log-linear and isoelastic demand functions. The parameter k in each example below measures the degree of concavity of the associated revenue function.

Corollary 3.6.1 (Example Demand Functions: One-to-One Channel)

i) **Polynomial demand function:** *Suppose* $p(q) = a - bq^k$*, where* $a > c$*,* $b > 0$ *and* $k > 0$*. The KS solution yields a wholesale price of*

$$w = c + \frac{k}{(k+1)^{(k+1)/k} + k}(a - c),$$

which corresponds to the NB solution with the retailer's bargaining power being $\theta^{KS}(k) = 1 - \frac{k+1}{(k+1)^{(k+1)/k}+k} \in (1/2, 1 - 1/e)$*, where* $\theta^{KS}(\cdot)$ *is decreasing.*

ii) **Log-linear demand function:** *Suppose* $p(q) = a - k \log q$*, where* $a > c$ *and* $k > 0$*. The KS solution yields a wholesale price of*

$$w = c + \frac{k}{e}.$$

which corresponds to the NB solution with the retailer's bargaining power being $\theta^{KS} = 1 - 1/e$*.*

iii) **Isoelastic demand function:** *Suppose* $p(q) = aq^{-k}$*, where* $a > c$ *and* $0 < k < 1$*. The KS solution yields a wholesale price of*

$$w = \frac{1-k}{1 - k - k(1-k)^{1/k}}c,$$

which corresponds to the NB solution with the retailer's bargaining power being $\theta^{KS}(k) = \frac{1-k-(1-k)^{1/k}}{1-k-k(1-k)^{1/k}} \in (1 - 1/e, 1)$*, where* $\theta^{KS}(\cdot)$ *is increasing.*

The polynomial demand $p(q) = a - bq^k$ can capture a wide range of functional shapes, as any smooth function can be approximated using polynomial functions. The associated revenue function $R(q) = p(q)q$ becomes more concave in $q \geq 1$ as k increases, leading to a smaller implied bargaining power of the retailer under the KS solution. There can be two potential extensions of the polynomial demand function for two products:

$$p_i(q_i, q_j) = a - b_i q_i^k - \eta b_j q_j^k, \quad \text{or} \tag{3.11}$$

$$p_i(q_i, q_j) = a - b_i(q_i + \eta q_j)^k. \tag{3.12}$$

When the log-linear demand $p(q) = a - k \log q$ is applied, the associated revenue function $R(q)$ becomes more concave in $q \geq 0$ for a larger value of k. However, the retailer's bargaining power implied by the KS solution does not change with k. The two-product extensions of this demand function can take the following forms:

$$p_i(q_i, q_j) = a - k \log q_i - k\eta \log q_j, \quad \text{or} \tag{3.13}$$

$$p_i(q_i, q_j) = a - k \log(q_i + \eta q_j). \tag{3.14}$$

The iso-elastic demand $p(q) = aq^{-k}$ induces a more concave revenue function $R(q)$ when $q \leq \exp(\frac{1-2k}{(1-k)k})$ as k increases, resulting in a smaller equivalent bargaining power of the retailer implied by the KS solution. The two-product extensions of this demand function can take the follow forms:

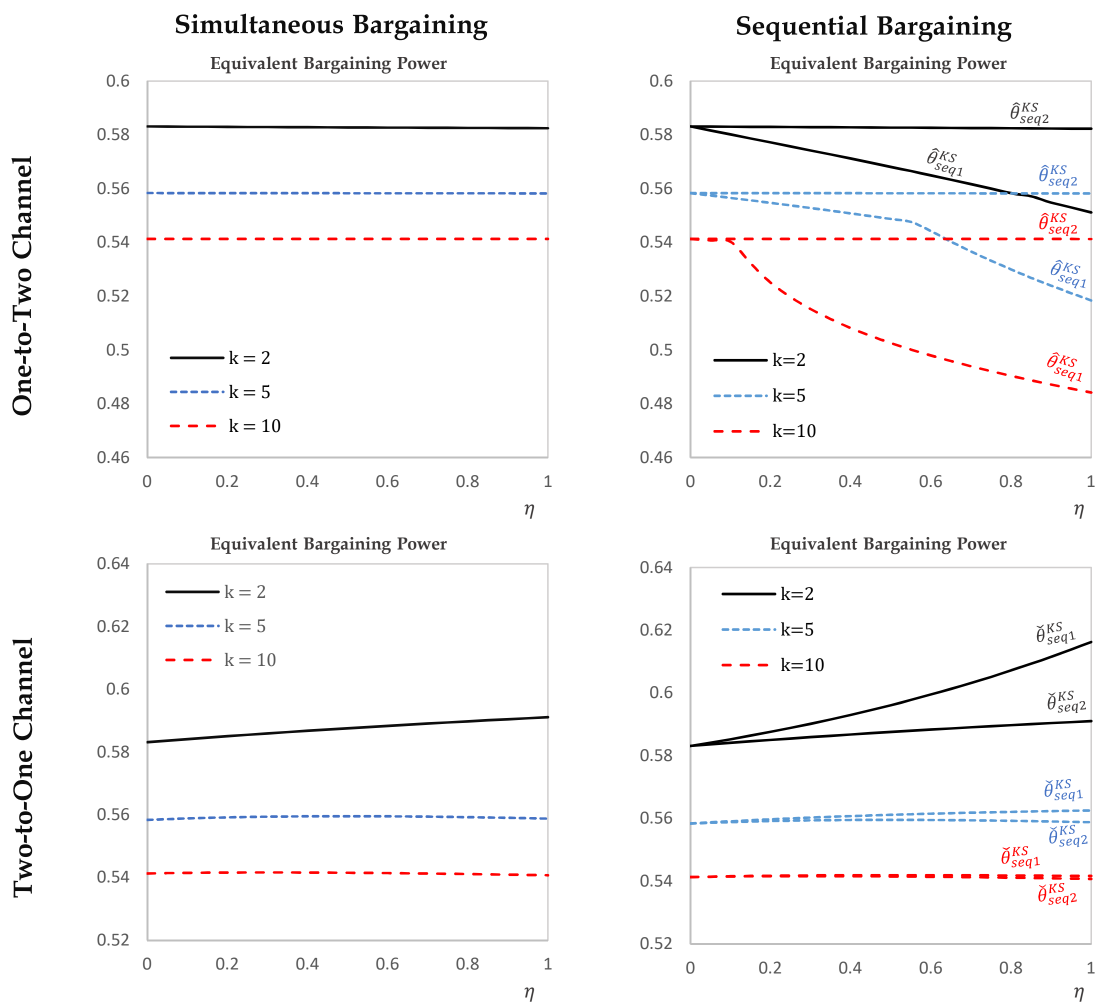
$$p_i(q_i, q_j) = a(q_i + \eta q_j)^{-k}, \quad \text{or} \tag{3.15}$$

$$p_i(q_i, q_j) = a(q_i q_j^\eta)^{-k}. \tag{3.16}$$

We demonstrate the results for competing channels in Figure 3.7 using the polynomial demand in (3.11). The observations from other demands are similar. We find that the equilibrium negotiation outcome resembles a similar pattern as is in the case of one-to-one channel. Specifically, when the revenue function becomes more concave (i.e., when k becomes larger), the retailers' equivalent bargaining power implied by the KS solution becomes lower.

3.6.3 Bilaterally Coordinating Contracts

In this subsection, we analyze the negotiations over alternative contracts. We consider the two-part tariff and revenue-sharing contracts, both extensively studied



Note. $(a-c)^2/b = 1$.

Figure 3.7. The retailer's equivalent bargaining power in the case without contingency.

in the supply chain contracting literature (e.g., Cachon and Kök 2010, Cachon and Lariviere 2005, Cai et al. 2012). It is well known that a bilaterally coordinating contract leads to the maximized joint profit of the two negotiation parties. However, the entire supply chain profit may not be maximized when retail competition presents (Feng and Lu 2013a,b).

A two-part tariff $(v_i, F_i), i = 1, 2$, consists of a unit payment v_i and a fixed payment F_i from a retailer to a supplier. With this contract, the supplier's and the retailer's profits from trade i in (3.3) become

$$R_i(v_i, v_j, F_i) = q_i^*(\mathbf{w})(w_i - c) + F_i \quad \text{and} \quad r_i(v_i, v_j, F_i) = q_i^*(\mathbf{w})(p_i^*(\mathbf{w}) - w_i) - F_i.$$

Replacing $R_i(w_i, w_j)$ by $R_i(v_i, v_j, F_i)$ and $r_i(w_i, w_j)$ by $r_i(v_i, v_j, F_i)$ in the profit computation, we can derive the negotiated two-part tariffs in equilibrium.

Proposition 3.6.3 (The Two-Part Tariff Contracts) *Regardless of the channel structure, negotiation sequence and contract contingency, under two-part tariffs, the equilibrium outcome under the KS solution corresponds to that under the NB solution with the retailer's bargaining power being* $\theta^{KS} = 1/2$.

With the fixed payment in the two-part tariff contract, each trading party's maximum trade surplus equals the joint trade surplus. As a result, the KS solution leads to equal allocation of the joint trade surplus between the supplier and the retailer engaged in the negotiation. Thus, the KS solution is equivalent to the symmetric NB solution.

It is well known that a revenue sharing contract can also achieve bilateral negotiation. Specifically, a revenue-sharing contract $(u_i, \ell_i), i = 1, 2$, consists of a unit payment u_i and a revenue sharing portion ℓ_i. Under this contract, the supplier's and the retailer's profits in (3.3) are modified to

$$\begin{aligned} R_i(u_i, u_j, \ell_i, \ell_j) &= q_i^*\Big(\frac{u_i}{1-\ell_i}, \frac{u_j}{1-\ell_j}\Big)\Big(\ell_i p_i^*\Big(\frac{u_i}{1-\ell_i}, \frac{u_j}{1-\ell_j}\Big) + u_i - c\Big), \\ r_i(u_i, u_j, \ell_i, \ell_j) &= q_i^*\Big(\frac{u_i}{1-\ell_i}, \frac{u_j}{1-\ell_j}\Big)\Big((1-\ell_i) p_i^*\Big(\frac{u_i}{1-\ell_i}, \frac{u_j}{1-\ell_j}\Big) - u_i\Big). \end{aligned}$$

In the above expressions, we note that because a retailer retains only $(1-\ell_i)$ portion of the revenue, his optimal order quantity coincides that when he retains all the revenue while paying a unit cost of $u_i/(1-\ell_i)$, i.e., $\arg\max_{q_i \geq 0}\{(1-\ell_i)(a-bq_i-\eta bq_j)q_i - q_i u_i\} = \arg\max_{q_i \geq 0}\{(a-bq_i-\eta bq_j)q_i - q_i u_i/(1-\ell_i)\}$. Let $\tilde{v}_i = z_i/(1-l_i)$ and $\tilde{F}_i = \ell_i r_i(\tilde{v}_i, \tilde{v}_j)$. Then, we can derive

$$R_i(u_i, u_j, \ell_i, \ell_j) = R_i(\tilde{v}_i, \tilde{v}_j, \tilde{F}_i) \quad \text{and} \quad r_i(u_i, u_j, \ell_i, \ell_j) = r_i(\tilde{v}_i, \tilde{v}_j, \tilde{F}_i).$$

In other words, given any revenue sharing contract, there exists a two-part tariff that generates the same trade profits. Thus, with the revenue sharing contract, the KS solution corresponds to the NB solution when the trading parties are equally powerful.

Relating the above observations to those obtained with the wholesale-price contracts in §3.5, we find that, under the KS solution, wholesale-price contracts generally grant the downstream retailers more bargaining power than bilaterally coordinating contracts do. The only exception occurs in highly competitive one-to-two channels with sequentially contingent trades–The retailer in the first trade possesses an equivalent bargaining power below 0.5 (recall Figure 3.5).

3.7 Concluding Remarks

This study introduces the Kalai-Smorodinsky (KS) bargaining solution, as an alternative to the Nash bargaining (NB) solution, to study contracting in competing supply chains. Compared with the NB solution, the KS solution takes into account not only the worst trade scenarios, but also the best trade scenarios for the trading parties, partially fixing the issue associated with the axiom of independent of irrelevant alternatives assumed in deriving the NB solution.

Our analysis suggests that when contracting without additional contingency terms in competing supply chains, the firms' negotiated profits under the NB solution change consistently with the direction of how the firms' trade prospects change (i.e., the set of firms' trade profits under all feasible contracts). When contract contingency is imposed, however, the NB solution may grant a firm a lower negotiated profit, when the firm's trade profit becomes higher for all feasible contracts. This suggests that one needs to be cautious when applying the NB solution to study vertical relationships in supply chains when contingency terms are imposed on contract executions.

Using the equivalent bargaining power implied by the KS solution, we demonstrate that the KS solution appropriately captures the negotiation power shift induced by the decision ownership, the negotiation sequence, the vertical relationship, the competition intensity, the contract contingency and the contract type.

There are wide applications of Nash bargaining framework in supply chains problems including outsourcing (Feng and Lu 2013b, Wang et al. 2013), assortment planning (Aydin and Heese 2014), product line extension (Chen et al. 2016), procurement (Chu et al. 2019, Wang et al. 2017), quality assurance (Leng et al. 2016), supplier encroachment (Yang et al. 2018) and environmental and social responsibility (Feng et al. 2019). Our study calls for a careful scrutiny of interpreting firms behaviors based on Nash bargaining solutions.

4. FIRM-GOVERNMENT INTERFACE: SUBSIDY FOR PRODUCERS IN A FRAGMENTED MARKET

4.1 Synopsis

Subsidies to agriculture industries are common in many countries. Governments provide aid to the producers to expand production, reduce poverty, increase local consumption, or encourage export. The US federal government spends over $20 billion every year on subsidizing farming.[1] The Chinese government, aiming to offset the rising production cost, initiates the general agricultural input subsidy since 2006 (Gale 2013) and the total amount has reached about 109 million dollars by 2017.[2]

Typically, the subsidy programs for agricultural products are aimed toward increasing the output. Sufficient agricultural supply can not only help alleviate poverty but also ensure domestic food supply security. In 2009, the Haiti government, with the guidance of Food and Agriculture Organization of the United Nations, initiated a $10.2 million scheme to distribute and multiply quality seeds, raising the outputs and profits of small producers.[3] In 2014, Chinese "Number One Document" emphasized that China's primary food sources should be domestic supplies — The government should boost domestic production capacity to ensure self-sufficiency policy and reduce reliance on food import (Gale et al. 2015). Alternatively, the subsidy programs can be adopted to promote agricultural export. For example, the dairy farmers in India struggled with oversupply in the domestic market. To alleviate this, two state governments offer $700 per ton for dairy exports while the national government ap-

[1] See https://www.downsizinggovernment.org/agriculture/subsidies (last accessed May 26, 2020)

[2] See http://english.agri.gov.cn/overview/201703/t20170301_247343.htm (last accessed May 26, 2020)

[3] See https://web.archive.org/web/20090827021358/http://www.afriquejet.com/news/africa-news/haiti's-seed-multiplication-programme-yields-fruits-2009082133773.html (last accessed May 26, 2020)

proved an extra 10 percent subsidy.[4] In the long term, the government intends to improve efficiency in agricultural production. For example, the Chinese government encourages the substitution of machinery for human labor by offering subsidies on farming machinery purchase since 2004 (Gale 2013).

Among tremendous subsidy programs, two kinds of aids are commonly offered. Plantation subsidies allow the producers to obtain needed inputs, including fertilizers, seeds, kerosene and machinery, at below market prices. For example, the governments in sub-Saharan Africa offer vouchers for fertilizers and various crop seeds including rice, maize, potatoes and oilseeds (Hemming et al. 2018, Jayne et al. 2018). The Indian government spent over \$22 million to provide farmers with support for irrigation, fertilizers and electricity in 2017.[5] Alternatively, harvesting subsidies are provided to aid the output collection, storage, and distribution process. For example, the Indian government approved a transport subsidy program to promote agriculture exports in 2019,[6] and the Thailand government offers 50 dollars per metric ton to rice farmers.[7]

Despite the popularity of planting and harvesting subsidies, the implication of subsidies on farmers' output decisions as well as their welfare distribution, however, are not well understood. Recently, Tang et al. (2019) demonstrated the differential effects of two subsidies by showing that the harvesting subsidy always widens the income gap between two farmers while the planting subsidy narrows the income gap. Building on the modeling framework of Tang et al. (2019), we attempt to further understand the impact of two subsidies on farmers' welfare distribution by concerning two important factors: (i) the farmers' market is fragmented; (ii) the government attempts to increase market output.

[4]See https://www.farmpolicyfacts.org/2018/10/subsidy-spotlight-india-2/ (last accessed May 26, 2020)

[5]See https://economictimes.indiatimes.com/news/economy/agriculture/india-refuses-to-cap-farm-input-subsidies /articleshow/74485419.cms?from=mdr (last accessed May 26, 2020)

[6]See https://dipp.gov.in/programmes-and-schemes/himalayan-north-eastern/transport-subsidy-scheme (last accessed May 26, 2020)

[7]See https://gain.fas.usda.gov (last accessed May 26, 2020)

Specifically, we consider a stylized model in which farmers differ in their productivity levels, reflected by their input-to-output ratios in farming. This reflects the fact that farmers usually face different soil, water and weather conditions, making discrepancy among their productivity levels. The market price of the agricultural product is the linear function of the outputs generated from all farmers. While the aim of a subsidy program can be multi-dimensional, we focus on the the case in which the government attempts to increase market output, as it is most commonly observed in practice. The government would set the target for overall output and announce the formats of subsidy program at the season beginning.

When the government offers a *combined subsidy* (i.e., the farmers can receive payments for both plantation and harvesting), an increased harvesting subsidy incentivizes all farmers to plant more, harvest more and earn more, while an increased planting subsidy grants farmers with lower productivity greater competitive advantage, reducing the outputs from farmers with higher productivity. Consequently, a higher harvesting subsidy widens the gaps among the farmers in both their outputs and profits, while a higher planting subsidy leads to a more balanced output distribution when the plantation is not overly subsidized. When the government offers a *selective subsidy* (i.e., the farmers are forced to choose either of both payments), intuitively, an increased harvesting subsidy benefits the high-yield farmers choosing harvesting subsidy and hurts those low-yield farmers choosing planting subsidy. Interestingly, the planting subsidy only benefits the least productive farmers. Similar to the observation under the combined subsidy, a higher harvesting subsidy widens the output and profit discrepancy among the farmers while a higher planting subsidy reduces the output gap, given that the planting subsidy is below planting cost.

To achieve the target output with minimum budget, the government always prefers to implement a combined subsidy regardless of the target output level. Moreover, under the combined subsidy, the overall output is the most evenly distributed among the farmers when the target output level is either relatively low or sufficiently large. The combined subsidy, however, may require excessive needed input or induce undesirable social welfare. This suggests that although combined subsidy is appealing to the government due to the low budget cost, alternative subsidy program can be offered if multiple aspects besides budget are concerned. In practice, the government may choose to offer only one payment to all farmers because offering both payments or choice of preferred payment to farmers may incur additional administrative cost. In this case, the government prefers planting subsidy over harvesting subsidy due to lower budget cost and more evenly distributed farmers' outputs only when the target output level is not too far from the overall output without subsidy. Moreover, the harvesting subsidy always induces a more favorable overall input and net social welfare than planting subsidy does.

The reminder of the paper is organized as follows. In Section 4.2, we review the related literature and articulate our contributions. We describe the model in Section 4.3 and derive the farmers' equilibrium outcomes under various subsidy programs in Section 4.4. In Section 4.5, we analyze the government's problem. We conclude in Section 4.6. Proofs of all formal results are relegated to appendix C.

4.2 Literature Review

Many authors have contributed to the literature on agricultural operations (see the survey by Lowe and Preckel 2004, Sodhi and Tang 2014). The existing studies have taken various angles, including plantation planning (Boyabatlı et al. 2019, Maatman et al. 2002, Zhang and Swaminathan 2020), harvesting scheduling (Allen and Schuster 2004, Lejeune and Kettunen 2017), irrigation allocation (Huh and Lall 2013), capacity and production planning (Boyabatlı et al. 2017, Kazaz 2004, Kazaz and Webster 2011), information disclosure (Chen and Tang 2015, Chen et al. 2013a,b), strategic behavior (Hu et al. 2019), adulteration (Levi et al. 2020), gleaning operations (Ata et al. 2019), agricultural cooperatives (An et al. 2015), contract farming (Federgruen et al. 2019) and data-driven operations (Devalkar et al. 2018).

None of the aforementioned studies consider the effect of subsidy on agricultural operations. Recently, there is a small but growing stream of studies concerning the role of government subsidies or interventions in agricultural operations literature. For example, Tang et al. (2015) model two symmetric farmers, who each decide their own outputs, choose to utilize the market information to reduce the demand uncertainty or adopt agricultural advice to improve operations efficiency. They show that the agricultural advice leads to increased farmers' profits only when the upfront investment is relatively low, and highlight that the government should consider offering subsidies to reduce the investment cost. Kazaz et al. (2016) analyze different interventions for the artemisinin-based malaria medicine supply chain and conclude that the supported price scheme can efficiently improve the supply and reduce price fluctuations of the artemisinin-based malaria medicine. Chintapalli and Tang (2018) consider the government offering the supported price schemes for two different crops to the strategic farmers with different productivity levels. They find that there exist supported price schemes for both crops, leading to Pareto improvements for all farmers. Guda et al. (2019) examine the case in which the government with a budget constraint, adopting the supported price scheme, procures the crops from farmers to support the poor population. Gupta et al. (2017) incorporate the fact that the government's procurement capacity is limited and thus farmers are forced to sell their crops on the open market due to the holding cost of their products. They predict the farmers' welfare loss by developing a stochastic dynamic programming model and validate it by using the real-world data. Alizamir et al. (2019) analyze two commonly-used subsidy schemes (Price Loss Coverage (PLC) and Agriculture Risk Coverage (ARC) programs) in the U.S. and find that PLC always incentivizes farmers to plant more compared to the case without subsidy, while ARC may lead to a decrease in farmers' plantation quantity. None of these studies consider the role of planting and harvesting subsidies.

In a recent work, Tang et al. (2019) study the effects of planting and harvesting subsidies on a local market involved two farmers. They find that a higher planting subsidy leads to a reduced farmers' income gap, while a higher harvesting subsidy exaggerates the discrepancy between the farmers' incomes. They also show that it is never optimal for the government to provide a combined subsidy to farmers. Our study differs from Tang et al. (2019) along three important dimensions. First, they assume that only two farmers compete in a local market, which is a common assumption in the literature. In reality, the local farming market is usually fragmented. In light of this, we analyze the competition among multiple farmers with different productivity levels. This modeling change no longer allows us to compare the fairness issue based on the difference in farmers' profits. To address this issue, we adopt the concept of majorization (see the discussion in Section 4.3.2). Second, we assume that the government can overly subsidize or tax the farmers, which may lead to contrasting equilibrium outcomes. Specifically, when the plantation subsidy is above the planting cost, farmers with lower productivity levels produce more and a further increased planting subsidy would lead to more dispersed output and profit distributions among the farmers. More interestingly, when the target output level is small, subsidizing based on plantation and taxing based on harvesting lead to more evenly distributed outputs and profits than planting only or harvesting only subsidy. Third, we focus on understanding the incentive via subsidies when the government aims toward improving the market output. Instead, Tang et al. (2019) study the case in which the government attempts to improve the farmers' welfare.

There is a growing literature on studying the government subsidy in other operations contexts such as improving the availability of vaccines (Adida et al. 2013, Arifoğlu et al. 2012, Mamani et al. 2012), green technology adoption (Alizamir et al. 2016, Cohen et al. 2016, Lobel and Perakis 2011) and coordination between the parties involved (Raz and Ovchinnikov 2015, Yu et al. 2020). Several papers are more closely related to ours in which the government implements input-based and/or output-based subsidies. Taylor and Xiao (2014) consider a model in which both purchase and sales subsidies are offered to the for-profit firm for distributing malaria drugs. They find that the government should only subsidize input for products with long life cycle.

Berenguer et al. (2017) study the effect of purchase and sales subsidies in a newsvendor setting. They show that the insights from Taylor and Xiao (2014) only hold when the for-profit firm is a price taker or the government has a budget constraint. They also find that the government should always offer the purchase subsidy to the non-for-profit firm. Cohen et al. (2016) consider the case in which the government provides a sales subsidy to incentivize the for-profit firm to achieve a target consumption level. They show that the government can significantly miss the target level when ignoring the demand uncertainty. Yu et al. (2018) consider the case in which the government attempts to improve consumer welfare. They find that the government can increase consumer welfare by implementing subsidy programs that involve competing firms with different market potentials.

4.3 The Problem

Consider the farming industry for a certain agriculture product, say a crop. The farmers' market is fragmented, and the farmers vary from one another in their productivity levels, reflected by their input-to-output ratios in farming. The government, aiming toward increasing the overall market output to alleviate the crop shortage in the consumer market, initiates a farmer subsidy program.

The Farmers' Market. There are n farmers in the market, index by $j \in N = \{1, 2, \ldots, n\}$, who may grow the crop. The farmers plant at the beginning of the season and harvest at the end of the season. The cost for plantation (covering, e.g., labor, tools, seeds and fertilizers) is c_P per unit of input and the cost for harvesting (covering, e.g., labor, tools, packaging, and storage) is c_H per unit of output. The farmers vary from one another in the soil condition, capability and productivity, and technology used, which results in different yield rates at harvesting. Specifically, for farmer j to harvest q_j units, an input of $x_j = z_j q_j$ units needs to be planted. In other words, the input-to-output ratio is z_j, or the yield rate is $1/z_j$. Without loss of generality, we assume

$$z_1 \leq z_2 \leq \cdots \leq z_n.$$

In other words, farmer 1 is most productive, while farmer n is least productive.

We also denote

$$\bar{z} = \frac{\sum_{i \in N} z_i}{n} \text{ and } v_z = \frac{\sum_{i \in N} z_i^2}{n} - \bar{z}^2 \tag{4.1}$$

as the average and the variability of the productivity, respectively.

Upon harvesting, the farmers bring their outputs into the market for sale. The market price of the crop is determined by the overall output. Specifically, the market clearing price for the output vector $\mathbf{q} = (q_1, q_2, \ldots, q_n) \geq 0$ is

$$p = \alpha - \beta \sum_{i \in N} q_i,$$

for $\alpha > 0$ and $\beta > 0$.

The Government's Subsidy Program. Typically, subsidy programs for agriculture products aim toward increasing the output. The government may set a target for the overall market output level and attempt to efficiently implement the program with minimum budget. Alternatively, the government may try to maximize the overall output while refining the subsidy spending within an allocated budget. Mathematically, the two scenarios are dual to each other.

There are two types of subsidies widely used. One is given based on the plantation effort, i.e., a payment of s_P is offered to a farmer for each unit planted. The other is granted based on the output, i.e., an amount of s_H is paid to a farmer for each harvested unit. The government may choose to offer a single subsidy in the form of either the *planting subsidy* or the *harvesting subsidy*. The government may also offer a *combined subsidy* of (s_P, s_H), with which farmers receive payments for both plantation and harvesting. Alternatively, a *selective subsidy* of (s_P, s_H) can be implemented, with which farmers choose to receive payments based on either the plantation quantity or the harvesting quantity, but not both. We use a superscript P, H, C, S to denote, respectively, planting, harvesting, combined and selective subsidy programs.

We should also note that, while increasing the output level is the primary goal of most subsidy programs, the resulting market evolution and wealth distribution can be an important concern. Because the farmers' market is often highly fragmented, a subsidy program that induces uneven wealth distribution can be highly undesired. Under such a program, farmers with less capabilities or less assets could be eventually driven out of the market, leading to increased poverty or unemployment.

The Sequence of Events. The subsidy program is executed with the following sequence of events.

1. The government sets the market output target and locates the budget for the subsidy program. The format of the program is announced.

2. Given the subsidy program, farmer j determines a plantation quantity x_j at the beginning of the season, which leads to a harvest quantity $q_j = x_j/z_j$ at the end of the season.

3. Farmers bring their output to the market for sale, and the market clearing price is determined. Subsidy payments are made to the farmers based on the format of the program.

4.3.1 A Benchmark Model: Without Subsidy

In this subsection, we briefly study the case when no subsidy is offered to the farmers. We will build on these results to analyze the effect of subsidies in the next sections. Without any subsidy, farmer j's profit is

$$\pi_j(\mathbf{q}) = \left(\alpha - z_j c_P - c_H - \beta \sum_{i \in N} q_i\right) q_j, \quad j \in N. \tag{4.2}$$

The next lemma characterizes conditions for farmers to produce.

Lemma 4.3.1 (Condition for Production) *Farmer $j \in N$ produces a positive amount if and only if*

$$g(j) \leq \frac{\alpha - c_H}{c_P},$$

where $g(j) = (j+1)z_j - \sum_{i=1}^{j} z_i$ is increasing in j.

The ratio $(\alpha - c_P)/c_H$ is an index for the profitability of the crop, as this ratio increases with the market potential α and decreases with the farmers' costs. The function $g(j)$ defines farmer j's efficiency level relative to those who are more productive than farmer j in the crop supply market. Because both $g(j)$ and z_j are increasing in j, the higher the farmer's input-to-output ratio, the less efficient is the farmer. Lemma 4.3.1 suggests that when the farmer's efficiency level exceeds the index of crop profitability, the farmer produces a positive amount. With this result, we can identify a threshold $j \in N$ so that farmers in the set $\{1, 2, \ldots, j\}$ produce, while the remainders do not. With this observation, we can now characterize the market equilibrium in the next proposition.

Proposition 4.3.1 (Production Equilibrium: Without Subsidy) *Suppose $g(n) \leq \frac{\alpha - c_H}{c_P}$. In equilibrium, farmer j's output quantity is*

$$q_j^* = \frac{1}{\beta}\left(\frac{\alpha - c_H - \bar{z}c_P}{n+1} - (z_j - \bar{z})c_P\right), \quad j \in N,$$

and farmer j's profit is

$$\pi_j^* = \beta(q_j^*)^2, \quad j \in N.$$

The overall input quantity is

$$X = \frac{n}{\beta(n+1)}\big(\bar{z}(\alpha - c_H - \bar{z}c_P) - (n+1)v_z c_p\big),$$

the overall output quantity is

$$Q = \frac{n}{\beta(n+1)}\big(\alpha - c_H - \bar{z}c_P\big),$$

and the overall farmer profit is

$$\Pi = \frac{n}{\beta(n+1)^2}\big((\alpha - c_H - \bar{z}c_P)^2 + (n+1)^2 v_z c_P^2\big).$$

Moreover, the following results hold.

i) q_j^* *is decreasing in* j *and is increasing in* $\bar{z}$.

ii) When the average productivity $\bar{z}$ *increases while the variability in productivity* v_z *is kept constant, the overall input quantity increases [decreases] in the average productivity* $\bar{z}$ *when* $\bar{z} < [>](\alpha - c_H)/(2c_P)$*, the overall output quantity decreases, and the overall farmer profit decreases.*

iii) When the variability in productivity v_z *increases while the average productivity* $\bar{z}$ *is kept constant, the overall input quantity decreases, the overall output quantity does not change, and the overall farmer profit increases.*

We observe from Proposition 4.3.1 that a farmer's output level and profit depend on the productivity distribution among all farmers only through the average $\bar{z}$. A lower average productivity (i.e., a higher $\bar{z}$) leads to a higher output from farmer j when we keep z_j unchanged. The total market output level also depends on the market productivity distribution only through the average. Thus, neither the individual output nor the market output is affected when the variability among the farmers' productivity levels changes.

The overall input level X, however, exhibits a very different response to the productivity distribution. A decrease in the average market productivity leads to an increased market input when the average productivity level is high (i.e., $\bar{z}$ is small), making the production at the average productivity level profitable (i.e., the marginal profit $\alpha - 2\bar{z}c_P - c_H \geq 0$). In this case, the market responds to a reduced average productivity through an increased plantation. When the average market productivity level is low (i.e., $\bar{z}$ is large), producing at the average productivity level is not profitable (i.e., the marginal profit $\alpha - 2\bar{z}c_P - c_H < 0$). In this case, the reduced average productivity further reduces the profitability of plantation, leading to a reduced market input. An increased variability among farmers' productivity, increasing the differences between the efficient farmers and the inefficient ones, makes the inefficient farmers less competitive in the market. As a result, the inefficient farmers reduce their input quantities in view of the competition pressure, while the efficient farmers also reduce their input quantities in view of the increased yields.

The equilibrium derived in Proposition 4.3.1 characterizes the overall market outcome of farmers' competition. From the policy maker's standpoint, the distribution of individual farmers' inputs, outputs, and wealth are also important concerns. To further understand the implication of the competition equilibrium, we discuss the concept of majorization in the next subsection.

4.3.2 Preliminaries: The Majorization Order

The notion of fairness or evenness in distribution has always been an important aspects in evaluating government policies. Specifically in the government subsidy programs, the incentive provided to the farmers and the resulting wealth allocation among the farmers need to be carefully examined to understand the strategic social implications of the program. In studies involving only two producers (Tang et al. 2019), one can evaluate the fairness of wealth distribution by taking the difference of the two producers' profits. When a large number of producers are involved, however, such an approach does not work. For our problem with a fragmented farmers' market, we apply the concept of majorization. This concept has been used to study resource allocation problems (e.g., Feng and Shanthikumar 2018b, Tong 1997, Yao 1987).

Definition 3 (Majorization) A vector $\mathbf{u}$ of size n *majorizes* another vector $\mathbf{v}$ of the same size, written as $\mathbf{u} \geq^m \mathbf{v}$ if

i) if $\sum_{i=1}^{k} u_{[i]} \geq \sum_{i=1}^{k} v_{[i]}, k = 1, \ldots, n$, and $\sum_{i=1}^{n} u_{[i]} = \sum_{i=1}^{n} v_{[i]}$, or, equivalently,

ii) if $\sum_{i=1}^{k} u_{(i)} \leq \sum_{i=1}^{k} v_{(i)}, k = 1, \ldots, n$, and $\sum_{i=1}^{n} u_{(i)} = \sum_{i=1}^{n} v_{(i)}$, where $u_{(i)}$ and $u_{[i]}$ [$v_{(i)}$ and $v_{[i]}$] are, respectively, the i-th smallest and largest elements in $\mathbf{u}$ [$\mathbf{v}$].

When the condition of equal total sums is removed in (i), we say $\mathbf{u}$ *weakly sub-majorizes* $\mathbf{v}$, written as $\mathbf{u} \geq_{wm} \mathbf{v}$. When the condition of equal total sums is removed in (ii), we say $\mathbf{u}$ *weakly sup-majorizes* $\mathbf{v}$, written as $\mathbf{u} \geq^{wm} \mathbf{v}$.

To understand the concept of majorization, consider two otherwise identical markets with the farmers' productivity vectors $\mathbf{z}_A$ and $\mathbf{z}_B$. Suppose $\mathbf{z}_A = (1, 0, \ldots, 0)$ and $\mathbf{z}_B = (\frac{1}{n}, \ldots, \frac{1}{n})$. It is clear that the average productivity levels in the two markets are the same, i.e., $\bar{z}_A = \bar{z}_B$, and $\sum_{i=1}^{k} z_{A[i]} = 1 \leq k/n = \sum_{i=1}^{k} z_{B[i]}$ for $k = 1, 2, \ldots, n$. Thus, $\mathbf{z}_A \geq^m \mathbf{z}_B$ suggesting that the farmers in market A are more dispersed in their productivity levels than those in market B.

Proposition 4.3.2 (Distribution among Farmers: Without Subsidy) *Consider two otherwise identical markets indexed by A and B. Suppose all farmers produce positive quantities and* $\mathbf{z}_A \geq^m \mathbf{z}_B$. *The following results hold.*

i) $\mathbf{q}_A^* \geq^m \mathbf{q}_B^*$.

ii) $\boldsymbol{\pi}_A^* \geq_{wm} \boldsymbol{\pi}_B^*$.

By the definition, $\mathbf{z}_A \geq^m \mathbf{z}_B$ implies that the average productivity levels are the same in the two markets. Then, the market output levels are the same in the two markets according to Proposition 4.3.1. The distribution of the farmers' output, however, becomes more balanced when their input-to-output ratios are more even, as suggested by Proposition 4.3.2(i). Moreover, the farmers' profits become more evenly distributed, as suggested by Proposition 4.3.2(ii). In the next section, we examine whether the subsidy programs may increase or decrease the disparity among the farmers.

4.4 The Effect of Subsidies on the Farmers' Incentives

In this section, we analyze the farmers' market responses to the subsidy programs. Section 4.4.1 focuses on the case when a combined subsidy is offered and section 4.4.2 discusses the case when a selective subsidy is implemented. The planting or harvesting subsidy can be regarded as a special case of a combined subsidy by setting $s_H = 0$ or $s_P = 0$ accordingly.

4.4.1 Combined Subsidy

Under a combined subsidy, the government announces (s_P, s_H) with s_P paid for each unit of planting input and s_P for each unit of harvesting output. Effectively, the subsidies change a farmer's production cost to $c_P - s_P$ and harvesting cost to $c_H - s_H$. Then, farmer j's profit under an output vector $\mathbf{q}$ becomes

$$\pi_j(\mathbf{q}, s_P, s_H) = \left(a_j(s_P, s_H) - \beta \sum_{i \in N} q_i\right) q_j, \quad j \in N, \tag{4.3}$$

where

$$a_j(s_P, s_H) = \alpha - z_j(c_P - s_P) - (c_H - s_H) \tag{4.4}$$

is the market potential (i.e., the highest possible margin) of farmer j.

The next lemma characterizes conditions for the farmers to produce under the combined subsidy program.

Lemma 4.4.1 (Condition for Production: Combined Subsidy) *Farmer $j \in N$ produces a positive amount under a combined subsidy (s_P, s_H) if and only if*

$$\mathbb{I}_{\{s_P \le c_P\}} g(j) + \mathbb{I}_{\{s_P > c_P\}} \tilde{g}(j) \le \frac{\alpha - c_H + s_H}{|c_P - s_P|},$$

where $g(j)$ is defined in Lemma 4.3.1 and $\tilde{g}(j) = \sum_{i=j}^{n} z_i - (n-j+2)z_j$ is decreasing in j.

Compared with the no-subsidy case in Lemma 4.3.1, the implementation of the subsidy program has a direct impact on the farmers' production incentive. Intuitively, the subsidies reduce the farmers' costs and induces more farmers to produce. This is only true when the plantation subsidy is below the plantation cost (i.e., $s_P \leq c_P$). In this case, the profitability index under the subsidy (i.e., $(\alpha - c_H + s_H)/(c_P - s_P)$) is higher than that without subsidy (i.e., $(\alpha - c_H)/c_P$), increasing the number of farmers meeting the condition for production. Interestingly, when the farmers can make money by just planting (i.e., $s_P > c_P$), the farmers with lower productivity levels would benefit more from the plantation subsidy because they need to invest more on input than the farmers with higher productivity levels do. In this case, a further increased planting subsidy can induce aggressive competition from the low-yield farmers, driving the high-yield farmers out of the market. This observation suggests that overly subsidizing the plantation can hurt the overall market productivity.

Replacing a farmer's planting cost to $c_P - s_P$ and harvesting cost to $c_H - s_H$ in Proposition 4.3.1, we can directly derive the equilibrium outcome when the combined subsidy is offered.

Lemma 4.4.2 (Production Equilibrium: Combined Subsidy) *Suppose that the government implements a combined subsidy (s_P, s_H) and all farmers produce positive quantities. In equilibrium, farmer j's output quantity is*

$$q_j^C(s_P, s_H) \quad = \quad \frac{1}{\beta}\left(\frac{\bar{a}(s_P, s_H)}{n+1} - (z_j - \bar{z})(c_P - s_P)\right), \tag{4.5}$$

and farmer j's profit is

$$\pi_j^C(s_P, s_H) \quad = \quad \beta\big(q_j^C(s_P, s_H)\big)^2, \quad j \in N. \tag{4.6}$$

The overall input quantity is

$$X^C(s_P, s_H) \quad = \quad \frac{n}{\beta(n+1)}\big(\bar{z}\bar{a}(s_P, s_H) - (n+1)v_z(c_p - s_P)\big), \tag{4.7}$$

the overall output quantity is

$$Q^C(s_P, s_H) \quad = \quad \frac{n}{\beta(n+1)}\bar{a}(s_P, s_H), \tag{4.8}$$

and the overall profit is

$$\Pi^C(s_P, s_H) \quad = \quad \frac{n}{\beta(n+1)^2}\big(\bar{a}^2(s_P, s_H) + (n+1)^2 v_z (c_P - s_P)^2\big), \tag{4.9}$$

where $\bar{a}(s_P, s_H) = \alpha - \bar{z}(c_P - s_P) - (c_H - s_H)$.

The next proposition characterizes the properties of these quantities derived in equilibrium.

Proposition 4.4.1 *Suppose the government implements a combined subsidy* (s_P, s_H) *and all farmers produce positive quantities. The following results hold.*

i) $q_j^C(s_P, s_H)$ *is decreasing [increasing] in* j *for* $s_P \le [>] c_P$.

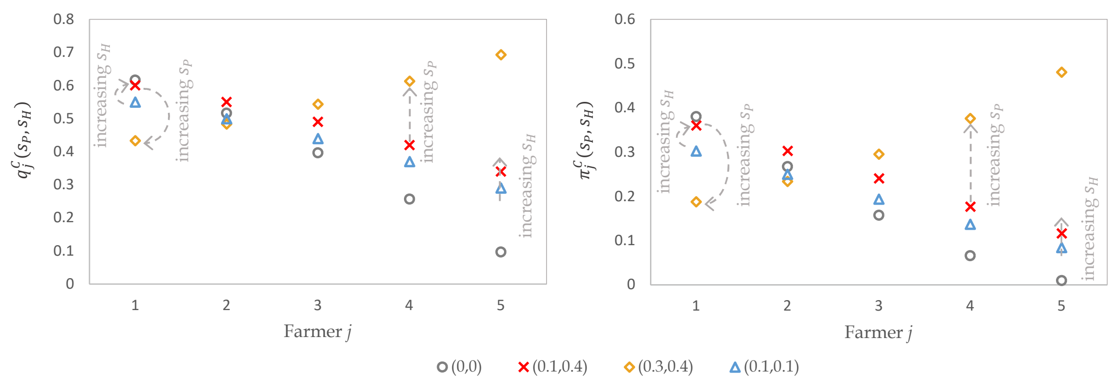
ii) Each farmer's input quantity, output quantity and profit is increasing [decreasing] in s_P *when* $z_j \ge [\le] \frac{n\bar{z}}{n+1}$. *Each farmer's input quantity, output quantity and profit are increasing in* s_H.

iii) When the average productivity $\bar{z}$ *increases while the variability in productivity* v_z *remains constant, the total input quantity decreases [increases] for* $s_P \le c_P$ *and* $\bar{z} \ge (\alpha - c_H + s_H)/(2(c_P - s_P))$ *[otherwise], the total output quantity decreases [increases] for* $s_P \le [>] c_P$ *and total profit decreases [increases]* $s_P \le [>] c_P$.

iv) When the variability in productivity v_z *increases while the average productivity* $\bar{z}$ *is kept constant, the total input quantity decreases [increases] for* $s_P \le [>] c_P$, *the total output quantity does not change, and total profit increases.*

We demonstrate the individual farmer's output level and profit in Figure 4.1. When the plantation subsidy is below the planting cost, a more productive farmer produces more and makes more money. The market output level increases with the farmers' average productivity level and is not affected by the variability among the farmers' productivity levels. These observations are consistent with their counterparts in the case without subsidy.

While a larger harvesting subsidy incentivizes all farmers to plant more, harvest more and profit more, a larger plantation subsidy can discourage high-yield farmers (with a smaller z_j) to produce, as suggested by Proposition 4.4.1(ii). In this case, a farmer's output level and profit both decrease in his productivity level. Moreover, when the planting subsidy exceeds the planting cost, the farmer with a lower productivity level plants more and harvest more than one with a higher productivity level. In this case, the market input, the market output, and the total farmer profit always increase with the average farmer productivity level. This echoes the message from Lemma 4.4.1.



Notes. $\alpha = 3$, $b = 1$, $c_P = 0.2$, $c_H = 0.3$ and $\mathbf{z} = (1, 1.5, 2.1, 2.8, 3.6)$.

Figure 4.1. The equilibrium output levels (left panel) and profits (right panel) under the combined subsidy (s_P, s_H).

Proposition 4.4.1 characterizes the first-order effect of the subsidy program (i.e., how the farmers' outputs and profits change with the subsidies). To understand the second-order effect (i.e., how the subsidies impact the distributions among the farmers), we evaluate the responses of the individual farmer's output level to the increase of planting subsidy and harvesting subsidy, i.e.,

$$\Delta_{s_P} q_j^C \equiv q_j^C(s_P + \delta, s_H) - q_j^C(s_P, s_H) \quad \text{and} \quad \Delta_{s_H} q_j^C \equiv q_j^C(s_P, s_H + \delta) - q_j^C(s_P, s_H),$$

and the responses of the individual farmer's profit, i.e.,

$$\Delta_{s_P}\pi_j^C \equiv \pi_j^C(s_P+\delta, s_H) - \pi_j^C(s_P, s_H) \text{ and } \Delta_{s_H}\pi_j^C \equiv \pi_j^C(s_P, s_H+\delta) - \pi_j^C(s_P, s_H).$$

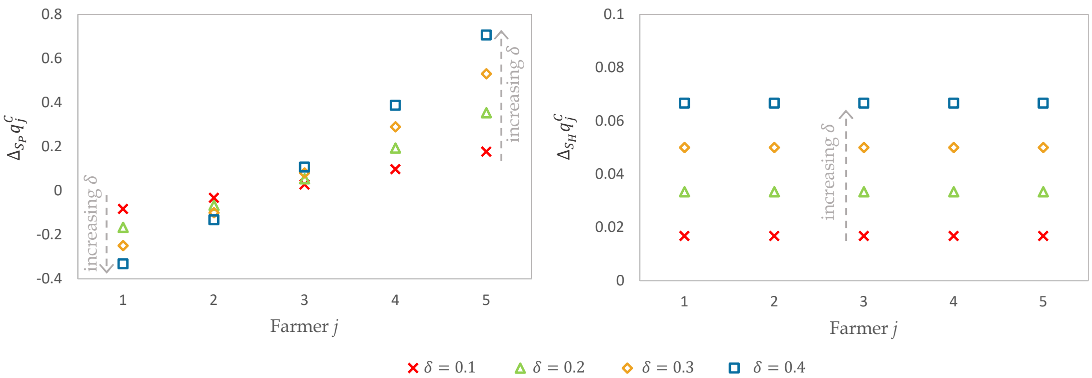
Corollary 4.4.1 (Individual Farmer's Response to the Combined Subsidies)

i) **Planting subsidy:** $\Delta_{s_P}q_j^C$ *is increasing in* j*. When* $c_P \leq s_{P1} < s_{P2}$*,* $\mathbf{q}^C(s_{P2}, s_H) \geq_{wm} \mathbf{q}^C(s_{P1}, s_H)$ *and* $\boldsymbol{\pi}^C(s_{P2}, s_H) \geq_{wm} \boldsymbol{\pi}^C(s_{P1}, s_H)$*, and when* $s_{P1} < s_{P2} \leq c_P$*,* $\mathbf{q}^C(s_{P1}, s_H) \geq^{wm} \mathbf{q}^C(s_{P2}, s_H)$*.*

ii) **Harvesting subsidy:** $\Delta_{s_H}q_j^C$ *is constant in* j *and* $\Delta_{s_H}\pi_j^C$ *is decreasing [increasing] in* j *for* $s_P \leq [>]c_P$*.* $\mathbf{q}^C(s_P, s_{H2}) \geq_{wm} \mathbf{q}^C(s_P, s_{H1})$ *and* $\boldsymbol{\pi}^C(s_P, s_{H2}) \geq_{wm} \boldsymbol{\pi}^C(s_P, s_{H1})$ *for* $s_{H1} < s_{H2}$*.*

We explain Corollary 4.4.1 with the reference to Figure 4.2. The farmers with lower productivity increase their output more when the planting subsidy is larger. This suggests that, when $s_P < c_P$ (i.e., the farmer's output level is increasing in their productivity), the increased planting subsidy leads to a more balanced output distribution among the farmers because an increased planting subsidy leads to increased outputs from the low-yield farmers and decreased outputs from the high-yield farmers. When $s_P > c_P$, the planting subsidy can overly remix the distribution, making the high-yield farmers produce less than low-yield farmers. In this case, the increased planting subsidy leads to a more dispersed output distribution. It is worth noting that, when $s_P = c_P$, the farmers' outputs are not affected by their productivity levels and thus all farmers produce the same amounts. The distribution of the farmers' profits exhibit similar responses to the planting subsidy as that of the farmers' outputs.

An increase in the harvesting subsidy leads to the same amount of output increase for all farmers. However, the high-yield farmers gain a larger profit increase than the low-yield farmer only if the plantation is not overly subsidized (i.e., $s_P \leq c_P$).



Notes. $\alpha = 4$, $b = 1$, $c_P = c_H = 0.3$, $s_P = 0.1$, $s_H = 0.2$, and $\mathbf{z} = (1, 1.5, 2.1, 2.8, 3.6)$.

Figure 4.2. The increased output distribution with respect to the increased planting (left) and harvesting (right) subsidies.

Overall, an increased harvesting subsidy widens the gaps among the farmers in both their outputs and profits.

The next result, examining the impact of productivity distribution, extends our earlier observation in Lemma 4.3.2.

Proposition 4.4.2 (Distribution among Farmers: Combined subsidy) *Consider two otherwise identical systems indexed by A and B. Suppose all farmers produce positive quantities for given combined subsidies and $\mathbf{z}_A \geq^m \mathbf{z}_B$. The following results hold.*

i) $\mathbf{q}_A^C(s_P, s_H) \geq^m \mathbf{q}_B^C(s_P, s_H)$, $\boldsymbol{\Delta}_{s_P}\mathbf{q}_A^C \geq^m \boldsymbol{\Delta}_{s_P}\mathbf{q}_B^C$, *and* $\boldsymbol{\Delta}_{s_H}\mathbf{q}_A^C =^m \boldsymbol{\Delta}_{s_H}\mathbf{q}_B^C$.

ii) $\boldsymbol{\pi}_A^C(s_P, s_H) \geq_{wm} \boldsymbol{\pi}_B^C(s_P, s_H)$, *and* $\boldsymbol{\Delta}_{s_H}\boldsymbol{\pi}_A^C \geq^m \boldsymbol{\Delta}_{s_H}\boldsymbol{\pi}_B^C$.

Proposition 4.4.2 suggests that when the farmers exhibit more evenly distributed productivity, their output levels and profits are more evenly distributed under the subsidy programs. The subtle difference is that the majorization order can be established for the output distribution, while only the weak majorization order is derived for the profit distribution. Moreover, the increased outputs and profits induced by the increase in the subsidies are more evenly distributed among the farmers. The difference between the responses to the planting subsidies and the harvesting subsidies lies in the fact that the effect of former on the individual farmer's output depends on the farmer's productivity, but the effect of the latter does not.

4.4.2 Selective Subsidy

Under a selective subsidy, the farmers are offered (s_P, s_H) and each choose to get paid based on plantation or harvesting. The farmer's choice depends on the comparison between the per unit plantation payment $z_j s_P$ and the per unit harvesting payment s_H. Let $m = \max\{j \in N : z_j s_P \leq s_H\}$ (when $z_1 > s_H/s_P$, we set $m = 0$). It is easy to see that farmer $j \in N$ chooses harvesting [planting] subsidy when $j \leq [>]m$. The next lemma characterizes the farmers' production incentive.

Lemma 4.4.3 (Condition for Production: Selective Subsidy) *Farmer $j \in N$ produces a positive amount under a selective subsidy (s_P, s_H) if and only if*

i) when $s_P \leq c_P$,

$$g(j) \leq \begin{cases} \dfrac{\alpha - c_H + s_H}{c_P} & \text{for } j \leq m, \\ \dfrac{\alpha - c_H - m s_H + \sum_{i=1}^{m} z_i s_P}{c_P - s_P} & \text{for } j > m, \end{cases}$$

where $g(j)$ is defined in Lemma 4.3.1.

ii) when $s_P > c_P$,

$$\begin{cases} g^a(j) \leq \dfrac{\alpha - (c_H - s_H) + (n - i^a(j))s_H - \sum_{i=i^a(j)+1}^{n} z_i s_P}{c_P} & \text{for } j \leq m, \\ g^b(j) \leq \dfrac{\alpha - c_H - (i^b(j) - 1)s_H + \sum_{i=1}^{i^b(j)-1} z_i s_P}{s_P - c_P} & \text{for } j > m, \end{cases}$$

where $g^a(j) = (n - i^a(j) + j)z_j + \sum_{i=j}^{i^a(j)} z_i - n\bar{z}$, $g^b(j) = n\bar{z} - \sum_{i=i^b(j)}^{j} z_i - (n - j + i^b(j))z_j$, $i^a(j) = \max\{i \in \{m+1, \ldots, n\} : z_i(s_P - c_P) \leq s_H - z_j c_P\}$ (when $z_{m+1}(s_P - c_P) > s_H - z_j c_P$, we set $i^a(j) = m$), and $i^b(j) = \min\{i \in$

$\{1,2,\ldots,m\} : z_i c_P \geq s_H - z_j(s_P - c_P)\}$ (when $z_m c_P < s_H - z_j(s_P - c_P)$, we set $i^b(j) = m+1$). Moreover, $i^a(j)$ and $i^b(j)$ are decreasing in j.

When the planting subsidy is below the planting cost (i.e., $s_P \leq c_P$), a less productive farmer would not produce unless those with higher productivity produce (see the left panel of Figure 4.3). Because the most productive farmers would choose the harvesting subsidy s_H, the offered harvesting subsidy s_P is irrelevant to their production incentive, as suggested by Lemma 4.4.3(i). For farmers with lower productivity, however, their participation depends on s_H, even though they would choose the planting subsidy. This is due to the concern of competition from the more productive farmers.

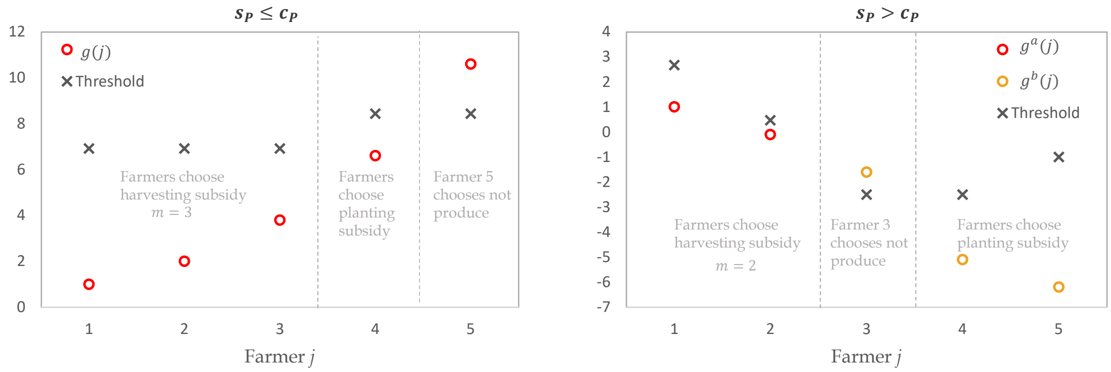
When the planting subsidy is above the plantation cost (i.e., $s_P > c_P$), Lemma 4.4.3(ii) suggests a very different situation (see the right panel of Figure 4.3). With $s_P > c_P$, the least productive farmers would plant a large amount and compete aggressively in the market. Among these farmers, the lower their productivity, the larger their plantation amount. Thus, a high-yield farmer ($j \leq m$) would need to take into account the low-yield farmers who show competitive advantage (i.e., farmer i's with $i > j^a(j)$). Similarly, when a low-yield farmer ($j > m$) determines whether or not to participate, he needs to take into account those highly productive farmers who compete efficiently with him (i.e., farmer i's with $i < j^b(j)$). Because of the different incentives from the two groups of the farmers, the farmers with the highest and lowest productivity levels have the strongest incentive to produce, while the farmers with intermediate productivity levels (i.e., those around m) may drop out of the market.

Lemma 4.4.4 (Production Equilibrium: Selective Subsidy) *Suppose that the government implements a selective subsidy (s_P, s_H) and all farmers produce positive quantities. In equilibrium, farmer j's output quantity is*

$$q_j^S(s_P, s_H) = q_j^C(s_P, s_H) - \begin{cases} \dfrac{1}{\beta}\Big(z_j s_P - \dfrac{\sum_{i=1}^m z_i s_P + (n-m)s_H}{n+1}\Big) & \text{for } j \leq m, \\ \dfrac{1}{\beta}\Big(s_H - \dfrac{\sum_{i=1}^m z_i s_P + (n-m)s_H}{n+1}\Big) & \text{for } j > m, \end{cases} \tag{4.10}$$

and farmer j's profit is

$$\pi_j^S(s_P, s_H) = \beta\big(q_j^S(s_P, s_H)\big)^2. \tag{4.11}$$



Notes. $b = 1$, $c_P = c_H = 0.3$ and $\mathbf{z} = (1, 1.5, 2.1, 2.8, 3.6)$. In the left panel, $\alpha = 2$, $s_P = 0.15$, $s_H = 0.375$. In the right panel, $\alpha = 0.5$, $s_P = 0.35$, $s_H = 0.6$.

Figure 4.3. Condition for production: selective subsidy.

The total input quantity is

$$X^S(s_P, s_H) = X^C(s_P, s_H) - \frac{1}{\beta}\Bigg(\sum_{i=1}^{m} z_i^2 s_P + \sum_{i=m+1}^{n} z_i s_H - \frac{n\bar{z}}{n+1}\Big(\sum_{i=1}^{m} z_i s_P + (n-m)s_H\Big)\Bigg) \quad (4.12)$$

and the total output quantity as

$$Q^S(s_P, s_H) = Q^C(s_P, s_H) - \frac{1}{\beta(n+1)}\Bigg(\sum_{i=1}^{m} z_i s_P + (n-m)s_H\Bigg). \quad (4.13)$$

Given the farmers' choices, the selective subsidy, unlike the combined subsidy, segments the farmer market. The resulting equilibrium quantities depend on the individual farmers' productivity levels, and thus their choices of subsidies, as shown in Lemma 4.4.4 . Thus, the overall market statistics, $\bar{z}$ and v_z, are no longer sufficient to characterize the farmers' behaviors.

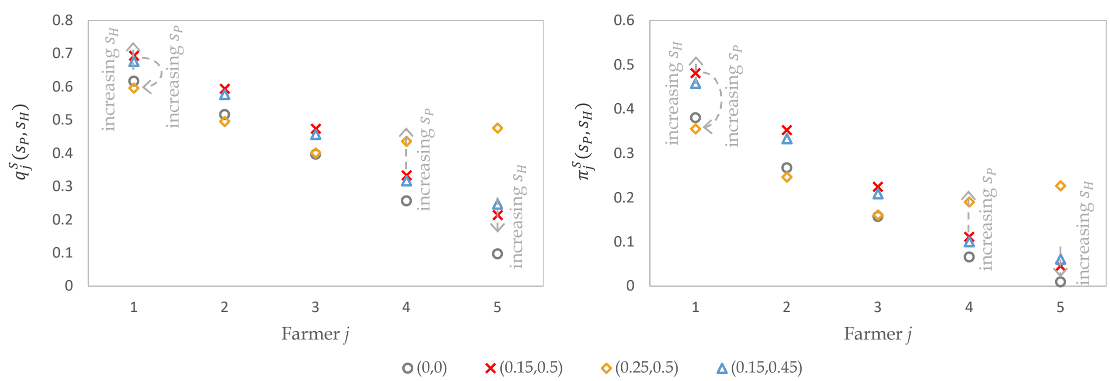
Proposition 4.4.3 *Suppose that the government implements a selective subsidy* (s_P, s_H) *and all farmers produce positive quantities. The following results hold.*

i) When $s_P \leq c_P$, $q_j^S(s_P, s_H)$ *is decreasing in* j. *When* $s_P > c_P$, $q_j^S(s_P, s_H)$ *is decreasing [increasing] in* j *for* $j < [>] m$ *and* $q_m^S(s_P, s_H) \leq [>] q_{m+1}^S(s_P, s_H)$ *for* $(z_{m+1}s_P - s_H) + (z_m - z_{m+1})c_P \geq [<] 0$.

ii) Each farmer's input quantity, output quantity and profit is increasing [decreasing] in s_P when $z_j \geq \frac{\sum_{i=m+1}^{n} z_i}{n+1}$ and $j > m$ [otherwise]. Each farmer's input quantity, output quantity and profit are increasing [decreasing] in s_H when $j \leq m$ [$j > m$].

The result in Proposition 4.4.3 is demonstrated in Figure 4.4. When the planting subsidy is below the plantation cost, the farmers' output levels are increasing in their productivity levels. When planting is overly subsidized, however, this is only true for highly productive farmers, who choose the harvesting subsidy. For those who choose the plantation subsidy, however, the farmers' output levels are decreasing in their productivity levels.

It is intuitive that an increased harvesting subsidy s_H benefits the farmers who choose the harvesting subsidy, while hurts those who choose the planting subsidy. An increased plantation subsidy s_P certainly hurts the farmers who choose harvesting subsidy, as the increase makes the other farmers more efficient in competition. Interestingly, not all farmers choosing the planting subsidy benefit from an increased s_P. Rather, only the least productive ones do, as suggested by Proposition 4.4.3(ii).



Notes. $\alpha = 3$, $b = 1$, $c_P = 0.2$, $c_H = 0.3$ and $\mathbf{z} = (1, 1.5, 2.1, 2.8, 3.6)$.

Figure 4.4. Equilibrium outcome under the selective subsidy (s_P, s_H): output level (left panel) and profit (right panel).

Corollary 4.4.2 (Individual Farmer's Response to the Selective Subsidies) *Suppose* $s_P \leq c_P$.

i) **Planting Subsidy:** $\Delta_{s_P} q_j^S$ *is increasing in* j. *For* $s_{P1} < s_{P2}$, $\mathbf{q}^S(s_{P1}, s_H) \geq^{wm} \mathbf{q}^S(s_{P2}, s_H)$.

ii) **Harvesting Subsidy:** $\Delta_{s_H} q_j^S$ *is decreasing in* j. *For* $s_{H1} < s_{H2}$, $\mathbf{q}^S(s_P, s_{H2}) \geq_{wm} \mathbf{q}^S(s_P, s_{H1})$ *and,* $\boldsymbol{\pi}^S(s_P, s_{H2}) \geq_{wm} \boldsymbol{\pi}^S(s_P, s_{H1})$.

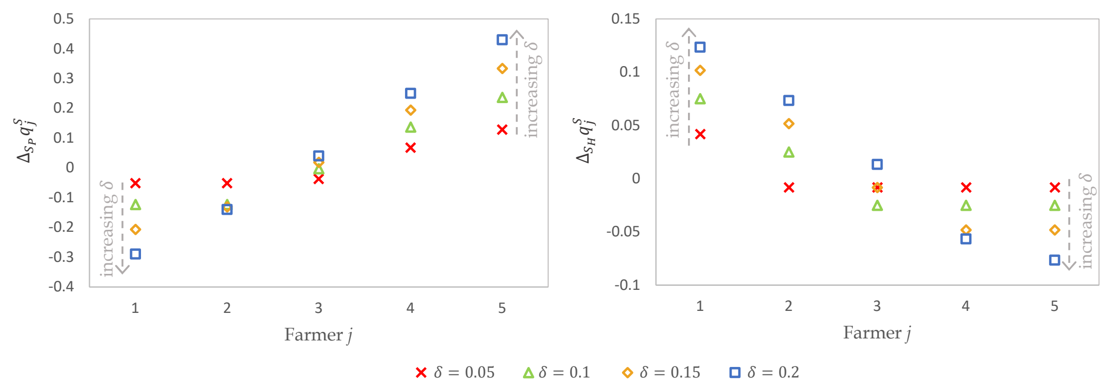
We explain Corollary 4.4.2 with the reference to Figure 4.5. An increased planting subsidy leads to more evenly distributed outputs, while an increased harvesting subsidy induces a larger dispersion in the outputs, provided that the plantation is not overly subsidized. As the planting subsidy increases, a low-yield farmer raises his output more than a high-yield one does, leading to a reduced gap among the farmers' outputs. This is consistent with our discussion on the planting subsidy in Corollary 4.4.1. There is, however, a worth noting difference here. Under a selective subsidy, only the low-yield farmers receive planting subsidy, and their output levels increases as the planting subsidy increases. This, in turn, makes the high-yield farmers less competitive in the market.

For the same reason, when the harvesting subsidy increases, the increased amount of output is contributed more by a high-yield farmer than by a low-yield one. As a result, the dispersion of the output distribution increases.

Now we examine the effect of productivity distribution when the selective subsidy is offered.

Proposition 4.4.4 (Distribution among Farmers: Selective subsidy) *Consider two otherwise identical markets indexed by* A *and* B. *Suppose all farmers produce positive quantities for given selective subsidies and* $\mathbf{z}_A \geq^m \mathbf{z}_B$. *Let* $m_k = \max\{j \in N : z_{kj} s_P \leq s_H\}$, $k = A, B$ *(when* $z_{k1} > s_H/s_P$, *we set* $m_k = 0$*). We assume*

a) $m_A \leq m_B$,



Notes. $\alpha = 4$, $b = 1$, $c_P = c_H = 0.3$, $s_P = 0.1$, and $\mathbf{z} = (1, 1.5, 2.1, 2.8, 3.6)$. In the left panel, $s_H = 0.3$. In the right panel, $s_H = 0.1$.

Figure 4.5. The increased output distribution with respect to the increased planting (left) and harvesting (right) subsidies.

b) $\frac{1}{l}\sum_{j=1}^{l}(z_{Bj} - z_{Aj})c_P \geq \frac{1}{n+1}\left(\sum_{i=m_A+1}^{m_B}(z_{Ai}s_P - s_H) + \sum_{i=m_B+1}^{n}(z_{Ai} - z_{Bi})s_P\right)$ *for* $1 \leq l \leq m_A$.

Then the following results hold.

i) $\mathbf{q}_A^S(s_P, s_H) \geq_{wm} \mathbf{q}_B^S(s_P, s_H)$ *for* $s_P \leq c_P$.

ii) $\boldsymbol{\pi}_A^S(s_P, s_H) \geq_{wm} \boldsymbol{\pi}_B^S(s_P, s_H)$ *for* $s_P \leq c_P$.

Proposition 4.4.4 echoes the message obtained under the combined subsidy in Proposition 4.4.2 that more evenly distributed farmers' productivity lead to more evenly distributed farmers' outputs and profits. The difference is that this conclusion is true for selective subsidy with additional conditions. In particular, more farmers choose harvesting subsidy over the planting subsidy (condition a) when the productivity becomes more evenly distributed. At the same time, the highly productive farmers, who would choose harvesting subsidy, obtain significantly increased plantation cost (condition b).

4.4.3 Comparisons

In the previous section, we have derived the farmers' equilibrium outcomes under different subsidy programs. This allows us to compare various scenarios to understand the incentives offered via subsidies on farmers' output decisions.

Proposition 4.4.5 *Suppose that all farmers produce positive quantities for given combined and selective subsidy* (s_P, s_H)*. The following results hold.*

i) *There exists a* $j^o \leq m$ *such that* $q_j^C(s_P, s_H) \leq [>] q_j^S(s_P, s_H)$ *and* $\pi_j^C(s_P, s_H) \leq [>] \pi_j^S(s_P, s_H)$ *for* $j \leq [>] j^o$.

ii) $\mathbf{q}^S(s_P, s_H) \geq^{wm} \mathbf{q}^C(s_P, s_H)$ *for* $s_P \leq c_P$.

iii) *There exists a* j_P *such that* $\Delta_{s_P} q_j^C \geq [<] \Delta_{s_P} q_j^S$ *for* $j \leq [>] j_P$*, and* $\boldsymbol{\Delta}_{s_P}\mathbf{q}^S \geq^{wm} \boldsymbol{\Delta}_{s_P}\mathbf{q}^C$ *if* $(n+1)z_1 \geq \sum_{i=1}^m z_i$.

iv) *There exists a* j_H *such that* $\Delta_{s_H} q_j^C \leq [>] \Delta_{s_H} q_j^S$ *for* $j \leq [>] j_H$*, and* $\boldsymbol{\Delta}_{s_H}\mathbf{q}^S \geq^{wm} \boldsymbol{\Delta}_{s_H}\mathbf{q}^C$.

According to Proposition 4.4.5, the highly productive farmers have a stronger incentive to increase their outputs when offering the choice over subsidies than they do when the combination of the same subsidies is offered to them. While these farmers get only one payment, as opposed to two payments, their increase in plantation is resulted from the market competition—The selective subsidy induces lower overall outputs from farmers with low productivity than the combined subsidy does.

Under the combined subsidy, not only the overall output is more evenly distributed among the farmers, but also the increased amount of outputs induced by either planting subsidy or harvesting subsidy are more evenly distributed among the farmer, than under the selective subsidy.

4.5 The Government's Objectives and Subsidy Design

After understanding the farmers' equilibrium behaviors, we are now ready to analyze the government's subsidy design. As mentioned in Section 4.1, increasing market output to meet the consumption need is a common objective in agriculture subsidy programs. The output-oriented subsidies have been discussed in the single-firm setting by Berenguer et al. (2017), Cohen et al. (2016), Taylor and Xiao (2014). In our problem, in contrast, the farmers' market is fragmented. Thus, the government must take into account the fact that offering of a subsidy induces different incentives for farmers with different productivity levels.

Given the output-oriented objective, we can formulate the government's subsidy design problem as

$$\min_{s_P, s_H} \quad \left\{ b^C(s_P, s_H) \equiv s_P X^C(s_P, s_H) + s_H Q^C(s_P, s_H) : Q^C(s_P, s_H) \geq \bar{Q} \right\}.$$

When the government plans to offer planting only subsidy or harvesting only subsidy, a constraint of $s_P = 0$ or $s_H = 0$, respectively, is added to the above optimization problem.

If a selective subsidy is offered, the government's problem becomes

$$\min_{s_P, s_H} \quad \left\{ b^S(s_P, s_H) \equiv s_H \sum_{i=1}^{m} q_i^S(s_P, s_H) + s_P \sum_{i=m+1}^{n} z_i q_i^S(s_P, s_H) : Q^S(s_P, s_H) \geq \bar{Q} \right\}.$$

We use s_P^P, s_H^H, (s_P^C, s_H^C) and (s_P^S, s_H^S) to denote, respectively, the government's optimal planting only, harvesting only, combined, and selective subsidy programs. We should assume that $\bar{Q} \geq Q^C(0,0) = Q^S(0,0)$ so that the government's target output level cannot be achieved if no subsidy is offered. Otherwise, the problem reduces to the one analyzed in Section 4.3.1.

4.5.1 The Optimal Subsidy Design

The next two lemmas characterize the optimal subsidy schemes for a given overall output level $\bar{Q}$.

Lemma 4.5.1 (Subsidy Design: Combined Subsidy) *The government's optimal subsidy scheme is characterized as follows: (Recall that* $\bar{a}(0,0) = \alpha - \bar{z}c_P - c_H$*.)*

i) When the planting only subsidy is offered, $s_P^P = (\frac{\beta(n+1)}{n}\bar{Q} - \bar{a}(0,0))/\bar{z}$*.*

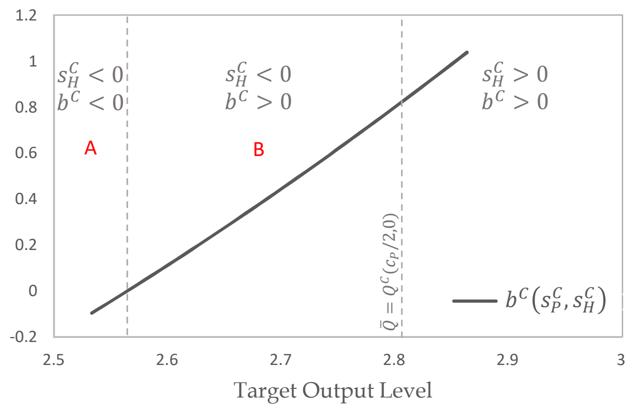
ii) When the harvesting only subsidy is offered, $s_H^H = \frac{\beta(n+1)}{n}\bar{Q} - \bar{a}(0,0)$*.*

iii) When the combined subsidy is offered, $s_P^C = c_P/2$ *and* $s_H^C = \frac{\beta(n+1)}{n}\bar{Q} - \bar{a}(0,0) - \frac{c_P\bar{z}}{2}$*.*

Several interesting observations can be made from Lemma 4.5.1. It is known in the context of subsidizing a single-firm (e.g., Berenguer et al. 2017, Taylor and Xiao 2014) that if the ratio of planting subsidy to harvesting subsidy equals to the firm's input-to-output ratio, then implementing either subsidy leads to the same output level. Parts i) and ii) suggests that the same result holds in our model (as $s_H^H = s_P^P\bar{z}$) with multiple subsidized farmers. Therefore, using the average input-to-output $\bar{z}$, we can convert the planting subsidy to its output-equivalent harvesting subsidy as $s_P^P\bar{z}$. We further note that market competition has a major impact on the government's subsidy design. When the market price is more sensitive to the overall output level (i.e., when β is large), the government needs to subsidize more per unit on planting or harvesting. When the number of producers increases, however, the government would reduce the per unit subsidy (as $(n+1)/n$ is decreasing in n).

When the government subsidizes both planting and harvesting, the strategy is to cover half of the farmers' plantation cost (i.e., $s_P^C = c_P/2$), while keeping the equivalent subsidy per unit output remain unchanged (i.e., $s_P^C\bar{z} + s_H^C = s_H^H = s_P^P\bar{z}$).

Interestingly, under the combined subsidy, the government may tax the farmers if the target output level is not too far from the overall output without subsidy, i.e., $\bar{Q} \in [Q^C(0,0), Q^C(c_P/2, 0)]$ (also labeled as regions A and B in Figure 4.6). In this case, $s_C^H \leq 0$, suggesting a larger tax payment from the farmers with higher productivity levels. Moreover, if the target output level is sufficiently close to the overall output without subsidy (i.e., region A in Figure 4.6), the government even earns money as the tax payment from harvesting exceeds the subsidy payment to plantation.



Notes. $\alpha = 4$, $b = 1$, $c_P = c_H = 0.3$, and $\mathbf{z} = (1, 1.5, 2.1, 2.8, 3.6)$.

Figure 4.6. The budget with respect to the overall output level under the combined subsidy.

Lemma 4.5.2 (Subsidy Design: Selective Subsidy) *Let* $m^S = \max\{j \in N : z_j s_P^S \leq s_H^S\}$ *(when* $z_1 > s_H^S / s_P^S$*, we set* $m^S = 0$*). When the selective subsidy is offered,* (s_P^S, s_H^S) *should satisfy the following conditions:*

i) When $\bar{Q} < Q^S(\frac{c_P}{2}, 0)$, $s_P^S = (\frac{\beta(n+1)}{n}\bar{Q} - \bar{a}(0,0))/\bar{z}$ *and* s_H^S *can be any value within* $[0, z_1 s_P^S]$ *(i.e.,* $m^S = 0$*).*

ii) When $\bar{Q} \geq Q^S(\frac{c_P}{2}, 0)$, $(s_P^S,\ s_H^S)$ *belongs to the set of critical points, i.e.,*

$$\left\{ (s_P, s_H) \;\middle|\; \begin{array}{l} s_P = c(m)\big(\frac{\beta(n+1)}{n}\bar{Q} - \bar{a}(0,0)\big)/\bar{z} + (1 - c(m))(c_P/2), \\ s_H = \big(\beta(n+1)\bar{Q} - n\bar{a}(0,0) - \sum_{i=m+1}^n z_i s_P\big)/m, \end{array} \quad \text{for } m \in M \right\},$$

where $c(m) = n\bar{z}\sum_{i=m+1}^n z_i / \big(\sum_{i=m+1}^n z_i^2 m + \big(\sum_{i=m+1}^n z_i\big)^2\big)$, $M = \{m \in N \backslash \{1\} : \phi^l(m) \leq 2(\beta(n+1)\bar{Q} - n\bar{a}(0,0))/c_P < \phi^u(m)\}$ *and*

$$\phi^l(m) = \frac{m z_m + \sum_{i=m+1}^n z_i}{m \sum_{i=m+1}^n z_i (z_i - z_m)} \left(\sum_{i=m+1}^n z_i^2 m - \Big(\sum_{i=1}^m z_i\Big)\Big(\sum_{i=m+1}^n z_i\Big) \right),$$

$$\phi^u(m) = \frac{m z_{m+1} + \sum_{i=m+1}^n z_i}{m \sum_{i=m+1}^n z_i (z_i - z_{m+1})} \left(\sum_{i=m+1}^n z_i^2 m - \Big(\sum_{i=1}^m z_i\Big)\Big(\sum_{i=m+1}^n z_i\Big) \right).$$

Moreover, $c(m)$ is decreasing in m, and m^S is increasing in $\bar{Q}$.

According to Lemma 4.5.2, the choice of optimal selective subsidy varies a lot, depending on the target output level (also refer to Figure 4.7). When the target output level is not too high (i.e., $\bar{Q} \in [Q^S(0,0), Q^S(c_P/2,0)]$), the selective subsidy coincides with the planting only subsidy, and payment for harvesting (i.e., s_H^S) becomes vacuous and can be any value within the range $[0, z_1 s_P^S]$. That is, the government would incentivize all farmers to choose planting subsidy over harvesting subsidy (i.e., $m^S = 0$). When the target output level is relatively high (i.e., $\bar{Q} > Q^S(c_P/2,0)$), the strategy is to set the payment for plantation as a combination of that under the combined subsidy and that under the planting only subsidy given the same output level, while keeping the equivalent subsidy per unit output remain unchanged (i.e., $s_H^H = s_P^P \bar{z} = s_H^C + s_P^C \bar{z} = (m^S s_H^S + \sum_{i=m^S+1}^{n} s_P^S \bar{z})/n$). This observation highlights that each subsidy program leads to the same equivalent subsidy per unit output.

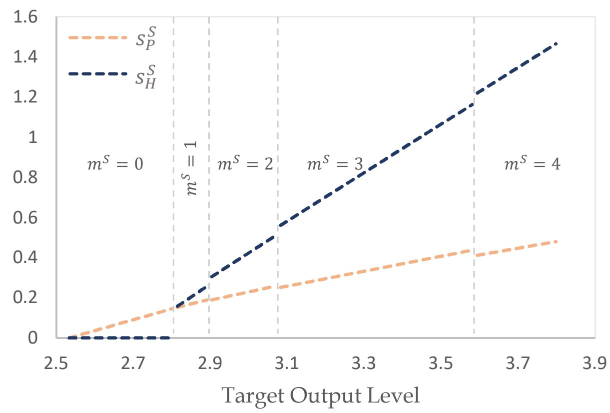
Moreover, the higher the target output level, the more the farmers are motivated to choose to receive payment based on harvesting rather than plantation (as m^S is increasing in $\bar{Q}$). Interestingly, not all farmers are incentivized to choose harvesting subsidy even though the target output level is sufficiently high. Rather, the least productive farmer (i.e., farmer n) is always induced to choose planting subsidy, leveraging to those farmers with higher productivity.

4.5.2 Comparisons Among Subsidy Programs

Having derived the optimal subsidy schemes in the previous section, we can now compare different subsidy programs to understand their effects on budget, total input and distribution among farmers.

Proposition 4.5.1 (Budget Comparison) *Suppose that the government has a target output $\bar{Q}$ to achieve and all farmers choose to produce. The following results hold.*

i) $b^C(s_P^C, s_H^C) \le b^S(s_P^S, s_H^S) \le b^C(s_P^P, 0) \le b^C(0, s_H^H)$ *for* $\bar{Q} \le Q^C(c_P, 0)$.



Notes. $\alpha = 4$, $b = 1$, $c_P = c_H = 0.3$, and $\mathbf{z} = (1, 1.5, 2.1, 2.8, 3.6)$.

Figure 4.7. The optimal selective subsidy with respect to the target output level.

ii) $b^C(s_P^C, s_H^C) \leq b^S(s_P^S, s_H^S) \leq b^C(0, s_H^H) \leq b^C(s_P^P, 0)$ *for* $\bar{Q} > Q^C(c_P, 0)$.

Moreover, $b^C(s_P^P, 0) = b^C(0, s_H^H) = b^C(s_P^C, s_H^C) = b^S(s_P^S, s_H^S)$ *for* $v_z = 0$.

To achieve the same overall output, the government always prefers to implement the subsidy program with minimum cost. In the special case in which the farmers have the same input-to-output ratios (including the case involved only a producer), the government is indifferent between the subsidy programs and farmers each obtain the same subsidy payment per unit output. This observation has been reported on the study of newsvendor model (Berenguer et al. 2017). They find that when the demand becomes deterministic, the planting and harvesting subsidies become equivalent to the subsidized firm. In our competition model considering farmers with different productivity levels, the combined subsidy leads to the minimum budget regardless of the target output level. This observation highlights the importance of modeling the heterogeneity of farmers' input-to-output ratios.

Intuitively, a lower budget is needed under the selective subsidy than under either planting subsidy or harvesting subsidy. Thus, the selective subsidy induces the second lowest budget. Using the combined subsidy as the benchmark, we further note that the increased budget induced by the selective subsidy is not significant unless the target output level is close to the overall output without subsidy (also refer to Figure 4.8).

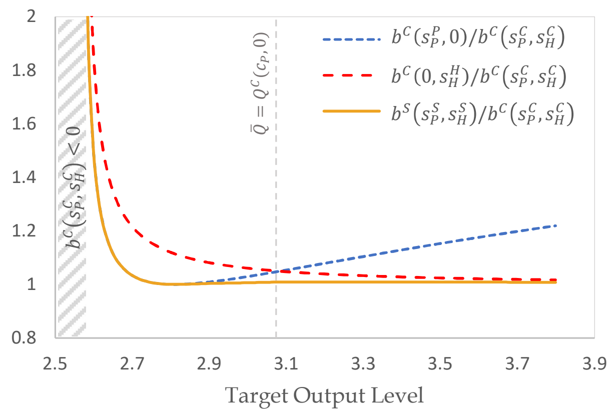
In practice, the government, considering the administrative costs to implement the subsidy program, may be in favor of uniformly offering one subsidy payment to all farmers. In light of this, planting subsidy or harvesting subsidy can be attractive to the policymakers. The comparison between two subsidy programs critically depends on the target output level. When the target output level is relatively low, i.e., $\bar{Q} \in [Q^C(0,0), Q^C(c_P,0)]$, the planting subsidy ($s_P^P \leq c_P$) leads to a lower budget than the harvesting subsidy does. As suggested by Corollary 4.4.1, an increased planting subsidy reduces the output gap among the farmers when planting subsidy is below planting cost. Therefore, under the planting subsidy, not only the lower budget is needed by the government, but also the overall output is more evenly distributed among the farmers than under the harvesting subsidy. When the target output level further increases (i.e., $\bar{Q} > Q^C(c_P,0)$), the harvesting subsidy induces a lower budget as the planting subsidy excessively benefits the farmers with lower productivity (i.e., $s_P^P > c_P$). In the case, the budget discrepancy under selective subsidy and harvesting subsidy also diminishes, as more farmers are motivated to receiver payment based on output quantity rather than input quantity under the selective subsidy (recall Lemma 4.5.2).

Proposition 4.5.2 (Overall Input Comparison) *Suppose that the government has a target output $\bar{Q}$ to achieve and all farmers choose to produce. The following results hold.*

i) $X^C(0, s_H^H) \leq X^S(s_P^S, s_H^S) = X^C(s_P^P, 0) \leq X^C(s_P^C, s_H^C)$ *for* $\bar{Q} \leq Q^C(\frac{c_P}{2}, 0)$.

ii) $X^C(0, s_H^H) < X^C(s_P^C, s_H^C) < X^C(s_P^P, 0)$ *and* $X^C(0, s_H^H) < X^S(s_P^S, s_H^S) < X^C(s_P^P, 0)$ *for* $\bar{Q} > Q^C(\frac{c_P}{2}, 0)$.

iii) $X^C(0, s_H^H)/\bar{Q} \leq X^C(s_P^C, s_H^C)/\bar{Q} \leq \bar{z}$, *and* $X^C(s_P^P, 0)/\bar{Q} \leq [>]\bar{z}$ *for* $\bar{Q} \leq [>]Q^C(c_P, 0)$.



Notes. $\alpha = 4$, $b = 1$, $c_P = c_H = 0.3$, and $\mathbf{z} = (1, 1.5, 2.1, 2.8, 3.6)$.

Figure 4.8. Budget Comparison.

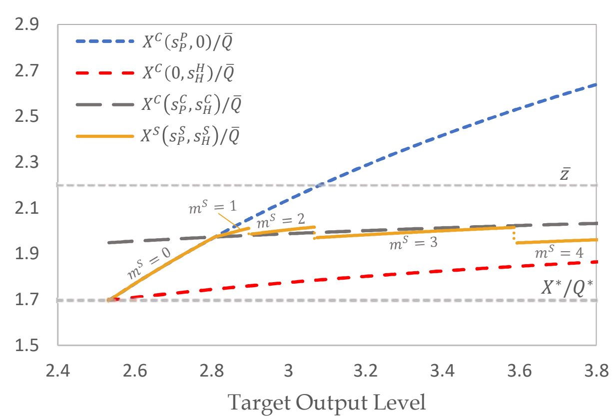
iv) Regardless of the subsidy program implemented, the overall input-to-output ratio under subsidy is higher than that without subsidy. That is, $X^C(0, s_H^H)/\bar{Q}$, $X^C(s_P^P, 0)/\bar{Q}$, $X^C(s_P^C, s_H^C)/\bar{Q}$ *and* $X^S(s_P^S, s_H^S)$ *are higher than* X^*/Q^*.

The farmers' overall input can be an important concern from the governments perspective. It is known that subsidies may cause overproduction, which draws the low-quality farmland into active production. Subsidies may also encourage farmers to overly use fertilizers and pesticides, which results in water contamination problem. Thus, the government should take into account overall input when evaluating a subsidy program.

To achieve the same overall output level, the harvesting subsidy calls for the lowest overall input among the subsidy programs. This is because the highly productive farmers have the strongest incentive to plant more and harvest more under harvesting subsidy, reducing the input from farmers with lower productivity.

Generally speaking, the overall input comparison can boil down to comparing the unit payment to plantation among the subsidy programs. When the target output level is relatively small (i.e., $\bar{Q} \in [Q^C(0, 0), Q^C(\frac{c_P}{2}, 0)]$), the combined subsidy leads to the highest overall input (as $s_P^C > s_P^P = s_P^S$). When the target level is relatively large (i.e., $\bar{Q} > Q^C(\frac{c_P}{2}, 0)$), the planting subsidy grants the low-yield farmers the strongest incentive to plant more (as $s_P^P > s_P^S > s_P^C$) and thus induces the highest overall input. Interestingly, the selective subsidy does not necessarily require a higher overall input than the combined subsidy does. This is true only when the target output level is not sufficiently high. When the target output level is sufficiently high, fewer farmers would choose planting subsidy over harvesting one, making the overall input increase less rapidly.

Alternatively, we can compare the overall input-to-output ratios under various subsidy programs to understand how subsidies affect the market productivity on average (also refer to Figure 4.9). Compared to the overall input-to-output ratio without subsidy (i.e., X^*/Q^*), a subsidy program always leads to a decreased overall input-to-output ratio because the low-yield farmers are better off and produce more under the subsidy program. Due to the nature of competition, the high-yield farmers produce more than those low-yield farmers, making the overall input-to-output ratio being below $\bar{z}$ (recall Proposition 4.3.1). This tuition remains when the subsidy program is introduced. The only exception is when the government, offering the planting only program, attempts to achieve a high target output level (i.e., $\bar{Q} > Q^C(c_P, 0)$). In this case, overly subsidizing the plantation induces aggressive competition from the low-yield farmers, hurting the overall market productivity.



Notes. $\alpha = 4$, $b = 1$, $c_P = c_H = 0.3$, and $\mathbf{z} = (1, 1.5, 2.1, 2.8, 3.6)$.

Figure 4.9. Overall Input Comparison.

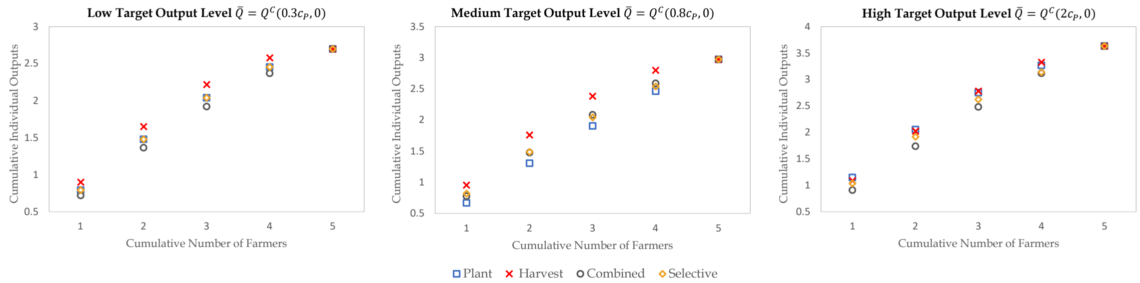
Proposition 4.5.3 (Distribution among Farmers) *Suppose that the government has a target output $\bar{Q}$ to achieve and all farmers choose to produce. The following results hold.*

i) $\mathbf{q}^C(0, s_H^H) \geq^m \mathbf{q}^C(s_P^P, 0) =^m \mathbf{q}^S(s_P^S, s_H^S) \geq^m \mathbf{q}^C(s_P^C, s_H^C)$ *for* $\bar{Q} \leq Q^C(\frac{c_P}{2}, 0)$; $\mathbf{q}^C(0, s_H^H) \geq^m \mathbf{q}^C(s_P^C, s_H^C) \geq^m \mathbf{q}^C(s_P^P, 0)$ *and* $\mathbf{q}^C(0, s_H^H) \geq^m \mathbf{q}^S(s_P^S, s_H^S) \geq^m \mathbf{q}^C(s_P^P, 0)$ *for* $Q^C(\frac{c_P}{2}, 0) < \bar{Q} \leq Q^C(c_P, 0)$; $\mathbf{q}^C(0, s_H^H) \geq^m \mathbf{q}^C(s_P^C, s_H^C)$ *for* $\bar{Q} > Q^C(c_P, 0)$.

ii) $\boldsymbol{\pi}^C(0, s_H^H) \geq_{wm} \boldsymbol{\pi}^C(s_P^P, 0) =^m \boldsymbol{\pi}^S(s_P^S, s_H^S) \geq_{wm} \boldsymbol{\pi}^C(s_P^C, s_H^C)$ *for* $\bar{Q} \leq Q^C(\frac{c_P}{2}, 0)$; $\boldsymbol{\pi}^C(0, s_H^H) \geq_{wm} \boldsymbol{\pi}^C(s_P^C, s_H^C) \geq_{wm} \boldsymbol{\pi}^C(s_P^P, 0)$ *and* $\boldsymbol{\pi}^C(0, s_H^H) \geq_{wm} \boldsymbol{\pi}^S(s_P^S, s_H^S) \geq_{wm} \boldsymbol{\pi}^C(s_P^P, 0)$ *for* $Q^C(\frac{c_P}{2}, 0) < \bar{Q} \leq Q^C(c_P, 0)$; $\boldsymbol{\pi}^C(0, s_H^H) \geq^m \boldsymbol{\pi}^C(s_P^C, s_H^C)$ *for* $\bar{Q} > Q^C(c_P, 0)$.

We explain Proposition 4.5.3 with the reference to Figure 4.10. One immediate observation is that overall output is more evenly distributed among the farmers under the combined subsidy than under the harvesting subsidy as latter one grants the low-yield farmers more incentive to produce than the former one. We further note that the combined subsidy leads to most balanced output distribution when the target output level is close to the overall output without subsidy (i.e., $\bar{Q} \in [Q^C(0,0), Q^C(\frac{c_P}{2}, 0)]$). This makes an interesting contrast to that obtained by Tang et al. (2019). They find that it is never optimal to provide the combined subsidy to the farmers when the government aims towards improving the farmer's welfare (i.e., reducing the farmers' profit gap). In our model allowing for taxation (recall that $s_H^C < 0$ for $\bar{Q} < Q^C(\frac{c_P}{2}, 0)$ from Lemma 4.5.1), it is possible that farmers' welfare is most evenly distributed under the combined subsidy.

Similar comparisons can be carried out among profit distributions induced by subsidy programs. The subtle difference is that the majorization order can be established for the output distribution, while only the weak majorization order is derived for the profit distribution.



Notes. $\alpha = 4$, $b = 1$, $c_P = c_H = 0.3$, $\mathbf{z} = (1, 1.5, 2.1, 2.8, 3.6)$. The farmers' outputs are added in descending order.

Figure 4.10. The output distribution with respect to the target output level.

The next corollary characterizes how the target output level affects the distribution among the farmers.

Corollary 4.5.1 (Response to The Increased Target Output Level)

i) $\Delta^P q_j$ *is increasing in* j*, and* $\Delta^H q_j$ *and* $\Delta^C q_j$ *are constant in* j*.* $\boldsymbol{\Delta}^P \mathbf{q} \geq^m \boldsymbol{\Delta}^H \mathbf{q} =^m \boldsymbol{\Delta}^C \mathbf{q}$.

ii) $\Delta^H \pi_j$ *and* $\Delta^C \pi_j$ *are decreasing in* j*.* $\boldsymbol{\Delta}^H \boldsymbol{\pi} \geq^m \boldsymbol{\Delta}^C \boldsymbol{\pi}$.

Corollary 4.5.1 echoes the message from Corollary 4.4.1. Under the planting subsidy, the farmers with lower productivity increases their outputs more when the target output level is higher. Under the harvesting subsidy and combined subsidy, an increase in target output level leads to the same amount of output increase for all farmers. As a result, the increased outputs induced by the increase in target output level are more evenly distributed under the planting subsidy than under harvesting subsidy and combined subsidy. Interestingly, the distribution of the farmers' increased profits is more balanced under the combined subsidy than under the harvesting subsidy.

4.5.3 Implications on Social Welfare

As introduced in Section 4.1, subsidy programs for agricultural products are primarily aimed toward increasing the overall output. In reality, the social welfare is an aspect that the government cannot afford to ignore. In operations literature, some studies have considered the effect of welfare-oriented subsidies (e.g., Cohen et al. 2016, Yu et al. 2018). In this section, rather than reformulating the problem to maximize social welfare, we provide some insights on how social welfare is affected by output-oriented subsidy programs. We focus on the case when the combined subsidy is offered.

For ease of exposition, we use $CS^C(s_P, s_H)$ to denote the consumer welfare under the combined subsidy (s_P, s_H). We can drive

$$CS^C(s_P, s_H) = \frac{1}{2}\sum_{i\in N}(\alpha_i(s_P, s_H) - p)q_i^C(s_P, s_H) = \frac{\beta}{2}(Q^C(s_P, s_H))^2.$$

We then define the social welfare and net social welfare, respectively, as

$$W^C(s_P, s_H) = \Pi^C(s_P, s_H) + CS^C(s_P, s_H), \tag{4.14}$$

$$NW^C(s_P, s_H) = \Pi^C(s_P, s_H) + CS^C(s_P, s_H) - b^C(s_P, s_H). \tag{4.15}$$

We observe that that consumer welfare depends on the subsidy program only through the overall output induced by subsidy. Thus, consumer welfare is not affected by the specific format of the output-oriented subsidy program. In other words, the effect of subsidy programs on social welfare is equivalent to that on farmer overall profit.

The next proposition characterizes the effect of output-oriented subsidy programs on social welfare.

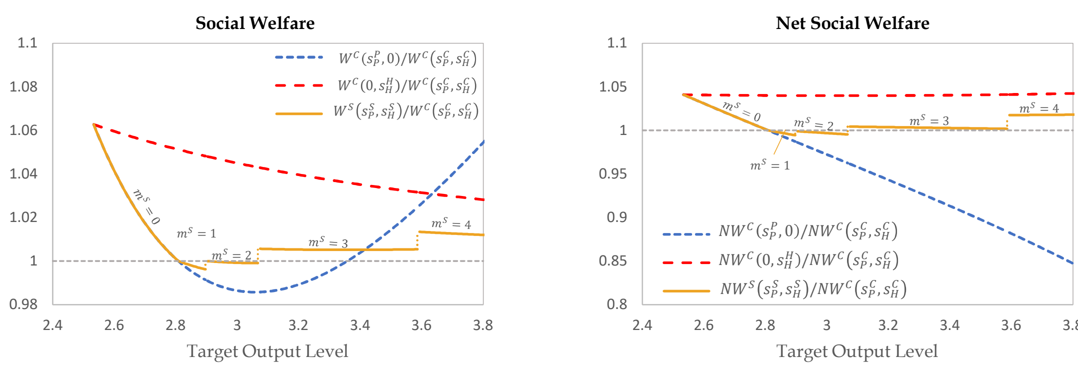
Proposition 4.5.4 (Social Welfare) *Suppose that the government has a target output $\bar{Q}$ to achieve and all farmers choose to produce. Then the following results hold.*

i) When $\bar{Q} \le Q^C(\frac{c_P}{2}, 0)$ or $Q^C(\frac{3c_P}{2}, 0) \le \bar{Q} \le Q^C(2c_P, 0)$, $W^C(s_P^C, s_H^C) \le W^C(s_P^P, 0) \le W^C(0, s_H^H)$. When $Q^C(\frac{c_P}{2}, 0) \le \bar{Q} \le Q^C(\frac{3c_P}{2}, 0)$, $W^C(s_P^P, 0) \le W^C(s_P^C, s_H^C) \le W^C(0, s_H^H)$. When $\bar{Q} \ge Q^C(2c_P, 0)$, $W^C(s_P^P, 0) \ge W^C(0, s_H^H) \ge W^C(s_P^C, s_H^C)$.

ii) When $\bar{Q} \le Q^C(\frac{c_P}{2}, 0)$, $NW^C(s_P^C, s_H^C) \le NW^C(s_P^P, 0) \le NW^C(0, s_H^H)$. When $\bar{Q} \ge Q^C(\frac{c_P}{2}, 0)$, $NW^C(s_P^P, 0) \le NW^C(s_P^C, s_H^C) \le NW^C(0, s_H^H)$.

We explain Proposition 4.5.4 with the reference to Figure 4.11). The harvesting subsidy leads to the most favorable social welfare when the target output level is not too high (i.e., $\bar{Q} \leq Q^C(2c_P, 0)$), while the highest social welfare is achieved under the planting subsidy when the target output level is sufficiently high (i.e., $\bar{Q} > Q^C(2c_P, 0)$). The combined subsidy, however, induces the least favorable social welfare when the target output level is either relatively low or sufficiently high (i.e., $\bar{Q} \leq Q^C(\frac{c_P}{2}, 0)$ or $\bar{Q} \geq Q^C(\frac{3c_P}{2}, 0)$). Together with the observation from previous propositions, we observe that although the combined subsidy is appealing in low cost and fair profit allocation, it requires additional needed input and induce low social welfare. This suggests that alternative subsidy programs can be attractive if the government has adequate budget and concerns other aspects.

When the government gives consideration to the net social welfare, the government' preference over subsidy programs coincides with that in which overall input is taken into account — The harvesting subsidy leads to the highest net social welfare, regardless of the target output level.



Notes. $\alpha = 4$, $b = 1$, $c_P = c_H = 0.3$, $\mathbf{z} = (1, 1.5, 2.1, 2.8, 3.6)$.

Figure 4.11. The social welfare comparison.

4.6 Conclusion

In this study, we offer a stylized model to understand the effect of output-oriented subsidy programs on farmers' output decisions as well as their welfare distribution. We consider two types of subsidies, planting subsidy and harvesting subsidy. We allow for the possibility that a farmer receives payments for both plantation and harvesting or chooses to receive either of them.

We observe that planting and harvesting subsidies exhibit different effects on farmers' outputs. Under the combined subsidy, a larger harvesting subsidy incentivizes all farmers to produce more, while a larger planting subsidy can discourage high-yield farmers to produce. Under the selective subsidy, an increased harvesting subsidy benefits the farmers who choose the harvesting subsidy, while hurting those who choose the planting subsidy. The planting subsidy, however, only benefits the least productive farmers and may hurt the farmers choosing the planting subsidy. Generally speaking, an increased harvesting subsidy widens the gaps among the farmers in both their outputs and profits, while an increased planting subsidy can lead to a more balanced farmers' output distribution when the plantation is not overly subsidized. Moreover, the farmers' overall output is more evenly distributed under the combined subsidy than under the selective subsidy.

When the government chooses the subsidy program that minimizes the budget, the combined subsidy leads to the minimum budget cost among subsidy programs. However, the combined subsidy requires additional input and induces undesirable social welfare when the target output level is relatively low. In contrast, although the harvesting only subsidy induces the highest budget cost, it gives the most favorable overall input as well as net social welfare.

5. CONCLUSION AND DIRECTION FOR FUTURE RESEARCH

This chapter concludes the findings of this research, and discusses the scope and directions of future research. This study considered the vertical relationship in the supply chain at different levels, namely, firm-consumer interface, supplier-buyer interface, and firm-government interface. Three specific problems are examined. The first problem considers the firms' dynamic pricing strategies with the possibility of bargaining. The second problem introduces the Kalai-Smorodinsky (KS) bargaining solution to study contracting in competing supply chains. The third problem focuses on the design of subsidy programs for producers in a fragmented market.

For the first problem, we use a dynamic programming framework to study firms' pricing strategies. Our model, allowing arriving buyers to bargain for a price discount, brings an additional dimension to the competition dynamics. In general, the sellers' competing strategies not only depend on their reservation values of losing an arriving buyer to the competitor but also on their disagreement points of negotiation breakdown with the buyer. We also show that it is not necessarily the case that the seller with a lower stock level can deplete her inventory first, as it is in the pure price competition. Because of the possibility of bargaining, the buyer may end up purchasing from a seller who has a higher inventory level. Interestingly, such a phenomenon only appears when the length of selling season is long enough. In view of the common occurrence of negotiation in buyer-seller interactions, our study, as a first step to analyze dynamic competition with bargaining, calls for additional research in this area. In our model, we have assumed that the buyers, in the event of no purchase, would leave the market (or seek other alternatives). Consideration of returned buyers naturally requires modeling of the strategic behavior of the buyers, an aspect extensively researched in the revenue management literature (e.g., Dasu and Tong 2010, Levin et al. 2009, 2010, Liu and Zhang 2013, Shen and Su 2007, Zhang and Cooper 2008). Understanding the effect of bargaining in competing for strategic buyers is important, yet challenging. Another important aspect in competitive sequential selling is the possession of private information by different parties. It is known that bargaining under asymmetric information is a difficult problem (see, e.g., Bhandari and Secomandi 2011, Feng et al. 2014). More research is needed to understand the role of information in the competition dynamics.

For the second problem, we formulate a two-tier supply chain consisting of one or two suppliers selling products to one or two retailers. Because of the axiom of *independence of irrelevant alternatives*, we uncover an important observation that the Nash bargaining solution may lead to unreasonable negotiation outcomes in competing supply chains with contingency terms. Instead, we apply the Kalai-Smorodinsky solution to study contract negotiations in competing supply chains, and analyze its connection to and difference from the Nash bargaining solution. We find that the KS solution appropriately captures the negotiation power shift induced by the decision ownership, the negotiation sequence, the vertical relationship, the competition intensity, the contract contingency and the contract type.

For the third problem, we analyze the design of government subsidy programs to induce socially improved farmers' decisions. We find that a higher harvesting subsidy widens the gaps among the farmers in both their outputs and profits, while a higher planting subsidy narrows the gap in farmers' outputs when the plantation is not overly subsidized. Further, when the government attempts to achieve the target output level with minimum budget, the combined subsidy is always preferred regardless of target output level. Our model and analysis can be extended along different directions. In our model, we have assumed that farmers' input-to-output ratios are deterministic. Consideration of yield certainty naturally requires modeling the order of farmers' productivity distributions, which requires the theory of stochastic orders (see Shaked and Shanthikumar (2007) for commonly used orders and closure properties). Understanding the effect of subsidy programs in a fragmented market with yield uncertainty is important, yet challenging. Another potential aspect is to consider alternative formats of subsidy programs such as supported price (e.g., Chintapalli and Tang 2018, Guda et al. 2019, Gupta et al. 2017) as well as PLC and ARC programs (e.g., Alizamir et al. 2019).

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A. Proofs of Formal Results in Chapter 2

Proof of Lemma 2.4.1. By definition, $s_B(r,s,v)$ is increasing in s and hence the result follows. $\square$

Proof of Proposition 2.4.1. We first note that it is without loss of generality to restrict the posted prices within $[v_j, \bar{r}]$. If $s_j > \bar{r}$, the resulting profit to seller j is the same as $s_j = \bar{r}$ for any equilibrium outcome. If $s_j < v_j$, seller j would be better off not selling the item than selling it at s_j.

To see part (i), we compute seller j's expected profit provided that her opponent sets a price s_i. By (2.5), we have

$$\Psi_j^S(v,s_j,s_i) = \begin{cases} \Psi_{j,1}^S(v,s_j,s_i) & \text{for } s_j < s_i, \\ \Psi_{j,2}^S(v,s_j,s_i) & \text{for } s_j > s_i, \\ \Psi_{j,3}^S(v,s_j,s_i) & \text{for } s_j = s_i, \end{cases}$$

where

$$\begin{aligned} \Psi_{j,1}^S(v,s_j,s_i) &\equiv \mathbb{E}[\mathbb{I}_{\{R<v\}}v] + \mathbb{E}[\mathbb{I}_{\{s_j<R\}}s_{Bj}(R,s_j,v)] + \mathbb{E}\left[\mathbb{I}_{\{v\le R\le s_j\}}\frac{s_{Bj}(R,s_j,v)+v}{2}\right], \\ \Psi_{j,2}^S(v,s_j,s_i) &\equiv \mathbb{E}[\mathbb{I}_{\{R<v\}\cup\{s_i<R\}}v] + \mathbb{E}\left[\mathbb{I}_{\{v\le R\le s_i\}}\frac{s_{Bj}(R,s_j,v)+v}{2}\right], \\ \Psi_{j,3}^S(v,s_j,s_i) &\equiv \mathbb{E}[\mathbb{I}_{\{R<v\}}v] + \mathbb{E}\left[\mathbb{I}_{\{v\le R\}}\frac{s_{Bj}(R,s_j,v)+v}{2}\right]. \end{aligned}$$

We have two cases to consider to derive the best response $\hat{s}_j(s_i)$ for seller j.

Case(a): $s_i = v$. We note that seller j would not post a price below v and thus $s_j \ge v = s_i$. For any $s_j \ge v$, $\Psi_{j,2}^S(v,s_j,s_i)$ is constant in s_j and $\Psi_{j,3}^S(v,v,v) - \Psi_{j,2}^S(v,s_j,v) = 0$. As a result, seller j is indifferent among any prices above v. In other words, seller j's best response is $\hat{s}_j(s_i) \in [v,\bar{r}]$ for $s_i = v$.

Case(b): $s_i > v$. We have

$$\Psi_{j,3}^S(v,s_i,s_i) - \Psi_{j,2}^S(v,s_j,s_i) = \mathbb{E}\left[\mathbb{I}_{\{s_i<R\}}\frac{s_{Bj}(R,s_i,v)-v}{2}\right] \ge 0.$$

The equality holds only if $s_i = \bar{r}$. This implies that the best response $\hat{s}_j(s_i)$ must be within $[v, s_i]$. Also note that

$$\frac{\partial}{\partial s_j}\Psi^S_{j,1}(v, s_j, s_i) = \frac{1-\theta}{2}\bar{F}_R(s_j)(2 - (s_j - v)h_R(s_j)).$$

Because $(s_j - v)h_R(s_j)$ is increasing in s_j for $s_j \geq v$, the maximizer of $\Psi^S_{j,1}(v, s_j, s_i)$ over $s_j \in [v, s_i)$ is unique and is denoted by $\bar{s}_j(v) = \max\{s \in [v, \bar{r}] : (s-v)h_R(s) \leq 2\}$. Also note that

$$\begin{aligned}
&\Psi^S_{j,1}(v, s_i - \epsilon, s_i) - \Psi^S_{j,3}(v, s_i, s_i) \\
&= \mathbb{E}\left[\mathbb{I}_{\{s_i - \epsilon < R\}}\left(s_{Bj}(R, s_i - \epsilon, v) - \frac{s_{Bj}(R, s_i, v) + v}{2}\right)\right] > 0.
\end{aligned}$$

Thus, seller j's best response to an s_i above v is $\hat{s}_j(s_i) = (s_i - \epsilon) \wedge \bar{s}_j(v)$ for a sufficiently small $\epsilon > 0$. Hence seller j's best response to an $s_i \geq v$ is

$$\hat{s}_j(s_i) = \begin{cases} [v, \bar{r}], & s_i = v, \\ s_i - \epsilon, & v < s_i \leq \bar{s}_j(v), \\ \bar{s}_j(v), & \bar{s}_j(v) < s_i \leq \bar{r}. \end{cases}$$

Because sellers' disagreement points are the same, we can derive seller i's best response $\hat{s}_i(s_j)$ symmetrically. As a result, the sellers engage in symmetric Bertrand price competition and the equilibrium prices are $(s^*_j, s^*_i) = (v, v)$.

To see part (ii) and (iii), we compute seller $j(i)$'s expected profit provided that her opponent sets a price $s_i(s_j)$. By (2.7) and (2.8), we have

$$\Psi^{A:s}_j(v_j, v_i, s_j, s_i) = \begin{cases} \Psi^{A:s}_{j,1}(v_j, v_i, s_j, s_i) & \text{for } s_j < s_i + \frac{\theta(v_i - v_j)}{1-\theta}, \\ \Psi^{A:s}_{j,2}(v_j, v_i, s_j, s_i) & \text{for } s_j > s_i + \frac{\theta(v_i - v_j)}{1-\theta}, \\ \Psi^{A:s}_{j,3}(v_j, v_i, s_j, s_i) & \text{for } s_j = s_i + \frac{\theta(v_i - v_j)}{1-\theta}, \end{cases}$$

$$\Psi^{A:b}_i(v_i, v_j, s_i, s_j) = \begin{cases} \Psi^{A:b}_{i,1}(v_i, v_j, s_i, s_j) & \text{for } s_j < s_i + \frac{\theta(v_i - v_j)}{1-\theta}, \\ \Psi^{A:b}_{i,2}(v_i, v_j, s_i, s_j) & \text{for } s_j > s_i + \frac{\theta(v_i - v_j)}{1-\theta}, \\ \Psi^{A:b}_{i,3}(v_i, v_j, s_i, s_j) & \text{for } s_j = s_i + \frac{\theta(v_i - v_j)}{1-\theta}, \end{cases}$$

where

$$\begin{aligned}
\Psi^{A:s}_{j,1}(v_j, v_i, s_j, s_i) &\equiv \mathbb{E}[\mathbb{I}_{\{R<v_j\}} v_j] + \mathbb{E}[\mathbb{I}_{\{v_j \le R\}} s_{Bj}(R, s_j, v_j)], \\
\Psi^{A:s}_{j,2}(v_j, v_i, s_j, s_i) &\equiv \mathbb{E}[\mathbb{I}_{\{R<v_j\}\cup\{R\ge s_i+\frac{\theta(v_i-v_j)}{1-\theta}\}} v_j] + \mathbb{E}\big[\mathbb{I}_{\{v_j \le R < s_i+\frac{\theta(v_i-v_j)}{1-\theta}\}} s_{Bj}(R, s_j, v_j)\big], \\
\Psi^{A:s}_{j,3}(v_j, v_i, s_j, s_i) &\equiv \mathbb{E}[\mathbb{I}_{\{R<v_j\}} v_j] + \mathbb{E}\big[\mathbb{I}_{\{v_j \le R < s_i+\frac{\theta(v_i-v_j)}{1-\theta}\}} s_{Bj}(R, s_j, v_j)\big] \\
&\quad + \mathbb{E}\left[\mathbb{I}_{\{R\ge s_i+\frac{\theta(v_i-v_j)}{1-\theta}\}} \frac{s_{Bj}(R, s_j, v_j) + v_j}{2}\right], \\
\Psi^{A:b}_{i,1}(v_i, v_j, s_i, s_j) &\equiv v_i, \\
\Psi^{A:b}_{i,2}(v_i, v_j, s_i, s_j) &\equiv \mathbb{E}[\mathbb{I}_{\{R<s_i+\frac{\theta(v_i-v_j)}{1-\theta}\}} v_i] + \mathbb{E}\big[\mathbb{I}_{\{R\ge s_i+\frac{\theta(v_i-v_j)}{1-\theta}\}} s_{Bi}(R, s_i, v_i)\big], \\
\Psi^{A:b}_{i,3}(v_i, v_j, s_i, s_j) &\equiv \mathbb{E}[\mathbb{I}_{\{R<s_i+\frac{\theta(v_i-v_j)}{1-\theta}\}} v_i] + \mathbb{E}\left[\mathbb{I}_{\{R\ge s_i+\frac{\theta(v_i-v_j)}{1-\theta}\}} \frac{s_{Bi}(R, s_i, v_i) + v_i}{2}\right].
\end{aligned}$$

To derive seller j's best response $\hat{s}_j(s_i)$, we have two cases to consider.

Case(a): $v_i \le s_i < \bar{r} - \frac{\theta(v_i-v_j)}{1-\theta}$. A necessary condition for this case to be valid is that $v_i < (1-\theta)\bar{r} + \theta v_j$. For any $s_j \ge v_j$, $\Psi^{A:s}_{j,2}(v_j, v_i, s_j, s_i)$ is constant in s_j and

$$\begin{aligned}
&\Psi^{A:s}_{j,3}(v_j, v_i, s_i + \frac{\theta(v_i-v_j)}{1-\theta}, s_i) - \Psi^{A:s}_{j,2}(v_j, v_i, s_j, s_i) \\
&= \mathbb{E}\left[\mathbb{I}_{\{R\ge s_i+\frac{\theta(v_i-v_j)}{1-\theta}\}} \frac{s_{Bj}(R, s_i + \frac{\theta(v_i-v_j)}{1-\theta}, v_j) - v_j}{2}\right] > 0,
\end{aligned}$$

implying that best response $\hat{s}_j(s_i)$ must be within $[v_j, s_i + \frac{\theta(v_i-v_j)}{1-\theta}]$. For any $s_j \ge v_j$, $\Psi^{A:s}_{j,1}(v_j, v_i, s_j, s_i)$ is increasing in s_j and

$$\begin{aligned}
&\Psi^{A:s}_{j,1}(v_j, v_i, s_i + \frac{\theta(v_i-v_j)}{1-\theta} - \epsilon, s_i) - \Psi^{A:s}_{j,3}(v_j, v_i, s_i + \frac{\theta(v_i-v_j)}{1-\theta}, s_i) \\
= \;& \mathbb{E}\left[\mathbb{I}_{\{R\ge s_i+\frac{\theta(v_i-v_j)}{1-\theta}\}}\left(s_{Bj}(R, s_i + \frac{\theta(v_i-v_j)}{1-\theta} - \epsilon, v_j) - \frac{s_{Bj}(R, s_i + \frac{\theta(v_i-v_j)}{1-\theta}, v_j) + v_j}{2}\right)\right] \\
& + \mathbb{E}\left[\mathbb{I}_{\{s_i+\frac{\theta(v_i-v_j)}{1-\theta} - \epsilon \le R < s_i+\frac{\theta(v_i-v_j)}{1-\theta}\}}\left(s_{Bj}(R, s_i + \frac{\theta(v_i-v_j)}{1-\theta} - \epsilon, v_j) - s_{Bj}(R, s_i + \frac{\theta(v_i-v_j)}{1-\theta}, v_j)\right)\right] \\
> \;& 0.
\end{aligned}$$

Thus, seller j's best response in this case is $\hat{s}_j(s_i) = s_i + \frac{\theta(v_i-v_j)}{1-\theta} - \epsilon$ for a sufficiently small $\epsilon > 0$.

Case(b): $s_i \ge \bar{r} - \frac{\theta(v_i-v_j)}{1-\theta}$. We note that seller j would not post a price above $\bar{r}$ and thus $s_j \le \bar{r} \le s_i + \frac{\theta(v_i-v_j)}{1-\theta}$. Because $\Psi^{A:s}_{j,1}(v_j, v_i, s_j, s_i)$ is increasing in s_j, seller j's best response in this case is $\hat{s}_j(s_i) = \bar{r}$.

Combining cases (a) and (b), we obtain seller j's best response for different values of (v_j, v_i):

Case I: If $v_i < (1-\theta)\bar{r} + \theta v_j$,

$$\hat{s}_j(s_i) = \begin{cases} s_i + \frac{\theta(v_i - v_j)}{1-\theta} - \epsilon, & v_i \leq s_i < \bar{r} - \frac{\theta(v_i - v_j)}{1-\theta}, \\ \bar{r}, & \bar{r} - \frac{\theta(v_i - v_j)}{1-\theta} \leq s_i \leq \bar{r}. \end{cases} \tag{A.1}$$

Case II: If $v_i \geq (1-\theta)\bar{r} + \theta v_j$,

$$\hat{s}_j(s_i) = \bar{r}, \quad v_i \leq s_i \leq \bar{r}. \tag{A.2}$$

To derive seller i's best response $\hat{s}_i(s_j)$, we have two cases to consider.

Case(a): $s_j \leq v_i + \frac{\theta(v_i - v_j)}{1-\theta}$. We note that seller i would not post a price below v_i and thus $s_i \geq v_i \geq s_j - \frac{\theta(v_i - v_j)}{1-\theta}$. For any $s_i \geq v_i$, $\Psi_{i,1}^{A:b}(v_i, v_j, s_i, s_j)$ is constant in s_i and

$$\Psi_{i,1}^{A:b}(v_i, v_j, s_i, s_j) - \Psi_{i,3}^{A:b}(v_i, v_j, v_i, v_i + \frac{\theta(v_i - v_j)}{1-\theta}) = 0.$$

As a result, seller i is indifferent among any prices above v_i. In other words, seller i's best response in this case is $\hat{s}_i(s_j) \in [v_i, \bar{r}]$.

Case(b): $v_i + \frac{\theta(v_i - v_j)}{1-\theta} < s_j \leq \bar{r}$. A necessary condition for this case to be valid is that $v_i < (1-\theta)\bar{r} + \theta v_j$. We have

$$\begin{aligned} &\Psi_{i,3}^{A:b}(v_i, v_j, s_j - \frac{\theta(v_i - v_j)}{1-\theta}, s_j) - \Psi_{i,1}^{A:b}(v_i, v_j, s_i, s_j) \\ &= \mathbb{E}\Big[\mathbb{I}_{\{R \geq s_j\}} \frac{s_{Bi}(R, s_j - \frac{\theta(v_i - v_j)}{1-\theta}, v_i) - v_i}{2}\Big] \geq 0. \end{aligned}$$

The equality holds only if $s_j = \bar{r}$. This implies that the best response $\hat{s}_i(s_j)$ must be within $[v_i, s_j - \frac{\theta(v_i - v_j)}{1-\theta}]$. Also note that

$$\frac{\partial}{\partial s_i}\Psi_{i,2}^{A:b}(v_i, v_j, s_i, s_j) = (1-\theta)\bar{F}_R\big(s_i + \frac{\theta(v_i - v_j)}{1-\theta}\big)\big(1 - (s_i - v_i)h_R\big(s_i + \frac{\theta(v_i - v_j)}{1-\theta}\big)\big).$$

Because $(s_i - v_i)h_R\big(s_i + \frac{\theta(v_i - v_j)}{1-\theta}\big)$ is increasing in s_i for $s_i \geq v_i$, the maximizer of $\Psi_{i,2}^{A:b}(v_i, v_j, s_i, s_j)$ over $s_i \in [v_i, s_j - \frac{\theta(v_i - v_j)}{1-\theta})$ is unique and we denote the maximizer as

$$\bar{s}_i(v_i, v_j) = \max\big\{s \in \big[v_i, \bar{r} - \frac{\theta(v_i - v_j)}{1-\theta}\big] : (s - v_i)h_R\big(s + \frac{\theta(v_i - v_j)}{1-\theta}\big) \leq 1\big\}.$$

Also note that

$$
\begin{aligned}
&\Psi_{i,2}^{A:b}(v_i, v_j, s_j - \frac{\theta(v_i - v_j)}{1-\theta} - \epsilon, s_j) - \Psi_{i,3}^{A:b}(v_i, v_j, s_j - \frac{\theta(v_i - v_j)}{1-\theta}, s_j) \\
&= \mathbb{E}\Bigg[\mathbb{I}_{\{R \geq s_j\}}\Bigg(s_{Bi}(R, s_j - \frac{\theta(v_i - v_j)}{1-\theta} - \epsilon, v_i) - \frac{s_{Bi}(R, s_j - \frac{\theta(v_i-v_j)}{1-\theta}, v_i) + v_i}{2}\Bigg)\Bigg] \\
&\quad + \mathbb{E}\Bigg[\mathbb{I}_{\{s_j - \epsilon \leq R < s_j\}}\Bigg(s_{Bi}(R, s_j - \frac{\theta(v_i - v_j)}{1-\theta} - \epsilon, v_i) - v_i\Bigg)\Bigg] > 0.
\end{aligned}
$$

Thus, seller i's best response in this case is $\hat{s}_i(s_j) = (s_j - \frac{\theta(v_i-v_j)}{1-\theta} - \epsilon) \wedge \bar{s}_i(v_i, v_j)$ for a sufficiently small $\epsilon > 0$.

Combining cases (a) and (b), we obtain seller i's best response for different values of (v_i, v_j):

Case I: If $v_i < (1-\theta)\bar{r} + \theta v_j$,

$$
\hat{s}_i(s_j) = \begin{cases} [v_i, \bar{r}], & v_j \leq s_j \leq v_i + \frac{\theta(v_i-v_j)}{1-\theta}, \\ s_j - \frac{\theta(v_i-v_j)}{1-\theta} - \epsilon, & v_i + \frac{\theta(v_i-v_j)}{1-\theta} < s_j \leq \bar{s}_i(v_i, v_j) + \frac{\theta(v_i-v_j)}{1-\theta}, \\ \bar{s}_i(v_i, v_j), & \bar{s}_i(v_i, v_j) + \frac{\theta(v_i-v_j)}{1-\theta} < s_j \leq \bar{r}. \end{cases} \tag{A.3}
$$

Case II: If $v_i \geq (1-\theta)\bar{r} + \theta v_j$,

$$
\hat{s}_i(s_j) = [v_i, \bar{r}], \quad v_j \leq s_j \leq \bar{r}. \tag{A.4}
$$

Now we are ready to derive the equilibrium using the expressions of the best responses $\hat{s}_j(s_i)$ and $\hat{s}_i(s_j)$. In the case that a pure strategy Nash equilibrium does not exist, we follow the notion of ϵ-equilibrium (Tijs 1981), which states that each player can benefit ϵ by deviating from her strategy.

Case I: $v_i < (1-\theta)\bar{r} + \theta v_j$. We claim that the equilibrium prices are $(s_j^*, s_i^*) = (\frac{v_i - \theta v_j}{1-\theta} - \epsilon, v_i)$. To verify this equilibrium, we note from (A.1), seller j's best response to $s_i = v_i$ is $\hat{s}_j(v_i) = \frac{v_i-\theta v_j}{1-\theta} - \epsilon$. Also by (A.3), seller i's best response to $s_j = \frac{v_i - \theta v_j}{1-\theta} - \epsilon$ is $\hat{s}_i(\frac{v_i-\theta v_j}{1-\theta} - \epsilon) = v_i \in [v_i, \bar{r}]$. Thus, $(\frac{v_i-\theta v_j}{1-\theta} - \epsilon, v_i)$ is an equilibrium.

Next we show no other equilibrium exists in this case. Suppose in equilibrium $s_i^o \in (v_i, \bar{r} - \frac{\theta(v_i-v_j)}{1-\theta})$. By (A.1), equilibrium price set by seller j is $s_j^o = \hat{s}_j(s_i^o) = s_i^o + \frac{\theta(v_i-v_j)}{1-\theta} - \epsilon \in (\frac{v_i-\theta v_j}{1-\theta} - \epsilon, \bar{r} - \epsilon)$. Given seller j's price s_j^o, by (A.3), seller i's best response is $\hat{s}_i(s_j^o) = (s_i^o - \epsilon) \wedge \bar{s}_i(v_i, v_j) \neq s_i^o$. Thus, we cannot have an equilibrium with $s_i = s_i^o \in (v_i, \bar{r} - \frac{\theta(v_i - v_j)}{1-\theta})$. Similarly, we can prove that no equilibrium exists with $s_i = s_i^o \in [\bar{r} - \frac{\theta(v_i - v_j)}{1-\theta}, \bar{r}]$.

Case II: $v_i \geq (1-\theta)\bar{r} + \theta v_j$. It is immediate from (A.2) and (A.4) to see that the equilibrium prices are $(s_j^*, s_i^*) = (\bar{r}, [v_i, \bar{r}])$. □

Proof of Lemma 2.5.1. We compute seller j's expected profit provided that her opponent sets a price s_i. By (2.12), we have

$$\Psi_j^S(v, w_j, s_j, s_i) = \begin{cases} \Psi_{j,1}^S(v, w_j, s_j, s_i) & \text{for } v \leq s_j < s_i, \\ \Psi_{j,2}^S(v, w_j, s_j, s_i) & \text{for } s_j < s_i \wedge v, \\ \Psi_{j,3}^S(v, w_j, s_j, s_i) & \text{for } v \leq s_i < s_j, \\ \Psi_{j,4}^S(v, w_j, s_j, s_i) & \text{for } s_i < s_j \wedge v, \\ \Psi_{j,5}^S(v, w_j, s_j, s_i) & \text{for } v \leq s_j = s_i, \\ \Psi_{j,6}^S(v, w_j, s_j, s_i) & \text{for } s_j = s_i < v, \end{cases}$$

where

$$\begin{aligned} \Psi_{j,1}^S(v, w_j, s_j, s_i) &\equiv \mathbb{E}[\mathbb{I}_{\{R<v\}} v] + \mathbb{E}[\mathbb{I}_{\{s_j<R\}} s_{Bj}(R, s_j, v)] + \mathbb{E}\left[\mathbb{I}_{\{v \leq R \leq s_j\}} \frac{s_{Bj}(R, s_j, v) + w_j}{2}\right], \\ \Psi_{j,2}^S(v, w_j, s_j, s_i) &\equiv \mathbb{E}[\mathbb{I}_{\{R<s_j\}} v] + \mathbb{E}[\mathbb{I}_{\{s_j \leq R\}} s_j], \\ \Psi_{j,3}^S(v, w_j, s_j, s_i) &\equiv \mathbb{E}[\mathbb{I}_{\{R<v\}} v] + \mathbb{E}[\mathbb{I}_{\{s_i<R\}} w_j] + \mathbb{E}\left[\mathbb{I}_{\{v \leq R \leq s_i\}} \frac{s_{Bj}(R, s_j, v) + w_j}{2}\right], \\ \Psi_{j,4}^S(v, w_j, s_j, s_i) &\equiv \mathbb{E}[\mathbb{I}_{\{R<s_i\}} v] + \mathbb{E}[\mathbb{I}_{\{s_i \leq R\}} w_j], \\ \Psi_{j,5}^S(v, w_j, s_j, s_i) &\equiv \mathbb{E}[\mathbb{I}_{\{R<v\}} v] + \mathbb{E}\left[\mathbb{I}_{\{v \leq R\}} \frac{s_{Bj}(R, s_j, v) + w_j}{2}\right], \\ \Psi_{j,6}^S(v, w_j, s_j, s_i) &\equiv \mathbb{E}[\mathbb{I}_{\{R<s_j\}} v] + \mathbb{E}\left[\mathbb{I}_{\{s_j \leq R\}} \frac{s_j + w_j}{2}\right]. \end{aligned}$$

We have two cases to consider to derive the best response $\hat{s}_j(s_i)$ for seller j.

Case(a): $s_i \leq v$. We note that

$$\frac{\partial}{\partial s_j} \Psi_{j,2}^S(v, w_j, s_j, s_i) = \bar{F}_R(s_j)(1 - (s_j - v) h_R(s_j)).$$

Because $(s_j - v) h_R(s_j)$ is increasing in s_j for $s_j \geq v$, the maximizer of $\Psi_{j,2}^S(v, w_j, s_j, s_i)$ over $s_j < s_i$ is unique and is denoted by $\bar{s}_j(v) = \max\{s \in [v, \bar{r}] : (s - v) h_R(s) \leq 1\}$.

We further note that $\bar{s}_j(v) \geq v$ and thus seller j's best response provided that $s_j < s_i$ is $\hat{s}_j(s_i) = s_i - \epsilon$. For any $s_j > s_i$, $\Psi^S_{j,4}(v, w_j, s_j, s_i)$ is constant in s_j and

$$\begin{aligned}
&\Psi^S_{j,2}(v, w_j, s_i - \epsilon, s_i) - \Psi^S_{j,4}(v, w_j, s_j, s_i) \\
&= \mathbb{E}[\mathbb{I}_{\{s_i \leq R\}}(s_i - \epsilon - w_j)] + \mathbb{E}[\mathbb{I}_{\{s_i - \epsilon \leq R < s_i\}}(s_i - \epsilon - v)], \\
&\Psi^S_{j,2}(v, w_j, s_i - \epsilon, s_i) - \Psi^S_{j,6}(v, w_j, s_i, s_i) \\
&= \mathbb{E}\left[\mathbb{I}_{\{s_i \leq R\}}\left(\frac{s_i - w_j}{2} - \epsilon\right)\right] + \mathbb{E}[\mathbb{I}_{\{s_i - \epsilon \leq R < s_i\}}(s_i - \epsilon - v)].
\end{aligned}$$

For $s_i = v$, $\Psi^S_{j,6}(v, w_j, v, v) = \Psi^S_{j,5}(v, w_j, v, v)$ and $\Psi^S_{j,4}(v, w_j, s_j, v) = \Psi^S_{j,3}(v, w_j, s_j, v)$. Note that the right-hand sides of both equations are strictly positive only if $s_i \in (w_j, \bar{r})$. As a result, seller j's best response to an s_i above w_j is $\hat{s}_j(s_i) = s_i - \epsilon$ for a sufficiently small $\epsilon > 0$. Also note that

$$\Psi^S_{j,4}(v, w_j, s_j, s_i) - \Psi^S_{j,6}(v, w_j, s_i, s_i) = \mathbb{E}\left[\mathbb{I}_{\{s_i \leq R\}}\frac{w_j - s_i}{2}\right].$$

The right-hand side of the equation is above (equal to) zero only if $s_i < (=) w_j$. As a result, seller j's best response is $\hat{s}_j(s_i) \in [s_i + \epsilon, \bar{r}]$ for an $s_i < w_j$, and is $\hat{s}_j(s_i) \in [w_j, \bar{r}]$ for $s_i = w_j$. Hence seller j's best response in this case is

$$\hat{s}_j(s_i) = \begin{cases} [s_i + \epsilon, \bar{r}], & s_i < w_j, \\ [w_j, \bar{r}], & s_i = w_j, \\ s_i - \epsilon, & w_j < s_i < \bar{r}. \end{cases}$$

Case(b): $s_i > v$. We have $\Psi^S_{j,1}(v, w_j, v, s_i) = v > \Psi^S_{j,2}(v, w_j, v - \epsilon, s_i)$, implying that the best response $\hat{s}_j(s_i)$ provided that $s_j < s_i$ must be within $[v, s_i)$. Also note that

$$\frac{\partial}{\partial s_j}\Psi^S_{j,1}(v, w_j, s_j, s_i) = \frac{1-\theta}{2}\bar{F}_R(s_j)\big(2 - \big(s_j - \frac{w_j - \theta v}{1-\theta}\big)h_R(s_j)\big).$$

Because $(s_j - \frac{w_j - \theta v}{1-\theta})h_R(s_j)$ is increasing in s_j for $s_j \geq \frac{w_j - \theta v}{1-\theta}$, the maximizer of $\Psi^S_{j,1}(v, w_j, s_j, s_i)$ over $s_j \in [v, s_i)$ is unique. We denote the maximizer as

$$\bar{s}_j(v, w_j) = \max\big\{v, \max\big\{s \in \big[\frac{w_j - \theta v}{1-\theta} \wedge \bar{r}, \bar{r}\big] :\ \big(s - \frac{w_j - \theta v}{1-\theta}\big)h_R(s) \leq 2\big\}\big\}.$$

Thus, seller j's best response provided that $s_j < s_i$ is $\hat{s}_j(s_i) = (s_i - \epsilon) \wedge \bar{s}_j(v, w_j)$ for a sufficiently small $\epsilon > 0$. For any $s_j \geq v$, $\Psi^S_{j,3}(v, w_j, s_j, s_i)$ is constant in s_j and

$$\begin{aligned}
& \Psi^S_{j,1}(v, w_j, s_i - \epsilon, s_i) - \Psi^S_{j,3}(v, w_j, s_j, s_i) \\
= \ & \mathbb{E}[\mathbb{I}_{\{s_i < R\}}(s_{Bj}(R, s_i - \epsilon, v) - w_j)] \\
& + \mathbb{E}\left[\mathbb{I}_{\{s_i - \epsilon < R \leq s_i\}}\left(s_{Bj}(R, s_i - \epsilon, v) - \frac{s_{Bj}(R, s_i, v) + w_j}{2}\right)\right], \\
& \Psi^S_{j,1}(v, w_j, s_i - \epsilon, s_i) - \Psi^S_{j,5}(v, w_j, s_i, s_i) \\
= \ & \mathbb{E}\left[\mathbb{I}_{\{s_i - \epsilon < R\}}\left(s_{Bj}(R, s_i - \epsilon, v) - \frac{s_{Bj}(R, s_i, v) + w_j}{2}\right)\right].
\end{aligned}$$

For $s_i = \bar{r}$, the first equation is not valid since seller j would not set a price above $\bar{r}$. The right-hand sides of both equations are strictly positive only if $s_i \in (\frac{w_j - \theta v}{1-\theta}, \bar{r}]$. As a result, seller j's best response to an $s_i \in (\frac{w_j - \theta v}{1-\theta}, \bar{r}]$ is $\hat{s}_j(s_i) = (s_i - \epsilon) \wedge \bar{s}_j(v, w_j)$ for a sufficiently small $\epsilon > 0$. Also note that

$$\Psi^S_{j,3}(v, w_j, s_j, s_i) - \Psi^S_{j,5}(v, w_j, s_i, s_i) = \mathbb{E}\left[\mathbb{I}_{\{s_i < R\}} \frac{w_j - s_{Bj}(R, s_i, v)}{2}\right].$$

The right-hand side of the equation is above zero only if $s_i < \frac{w_j - \theta v}{1-\theta} \wedge \bar{r}$, and equals zero only if $s_i = \frac{w_j - \theta v}{1-\theta} < \bar{r}$. As a result, seller j's best response is $\hat{s}_j(s_i) \in [s_i + \epsilon, \bar{r}]$ for an $s_i < \frac{w_j - \theta v}{1-\theta} \wedge \bar{r}$, and is $\hat{s}_j(s_i) \in [\frac{w_j - \theta v}{1-\theta} \wedge \bar{r}, \bar{r}]$ for $s_i = \frac{w_j - \theta v}{1-\theta} \wedge \bar{r}$. Hence seller j's best response in this case is

$$\hat{s}_j(s_i) = \begin{cases} [s_i + \epsilon, \bar{r}], & s_i < \frac{w_j - \theta v}{1-\theta} \wedge \bar{r}, \\ [\frac{w_j - \theta v}{1-\theta} \wedge \bar{r}, \bar{r}], & s_i = \frac{w_j - \theta v}{1-\theta} \wedge \bar{r}, \\ s_i - \epsilon, & \frac{w_j - \theta v}{1-\theta} < s_i \leq \bar{s}_j(v, w_j), \\ \bar{s}_j(v, w_j), & \bar{s}_j(v, w_j) < s_i \leq \bar{r}. \end{cases}$$

We note that $\frac{w_j - \theta v}{1-\theta} \geq \bar{s}_j(v, w_j) = \bar{r}$ if and only if $w_j \geq (1-\theta)\bar{r} + \theta v$. Combining cases (a) and (b), we obtain seller j's best response for different values of (v, w_j):

Case I: If $w_j \geq (1-\theta)\bar{s}_j(v, w_j) + \theta v (\geq v)$,

$$\hat{s}_j(s_i) = \begin{cases} [s_i + \epsilon, \bar{r}], & s_i < \bar{r}, \\ \bar{r}, & s_i = \bar{r}. \end{cases}$$

Case II: If $v \le w_j < (1-\theta)\bar{s}_j(v,w_j) + \theta v$,

$$\hat{s}_j(s_i) = \begin{cases} [s_i + \epsilon, \bar{r}], & s_i < \frac{w_j - \theta v}{1-\theta}, \\ [\frac{w_j - \theta v}{1-\theta}, \bar{r}], & s_i = \frac{w_j - \theta v}{1-\theta}, \\ s_i - \epsilon, & \frac{w_j - \theta v}{1-\theta} < s_i \le \bar{s}_j(v,w_j), \\ \bar{s}_j(v,w_j), & \bar{s}_j(v,w_j) < s_i \le \bar{r}. \end{cases}$$

Case III: If $w_j \le v$,

$$\hat{s}_j(s_i) = \begin{cases} [s_i + \epsilon, \bar{r}], & s_i < w_j, \\ [w_j, \bar{r}], & s_i = w_j, \\ s_i - \epsilon, & w_j < s_i \le \bar{s}_j(v,w_j), \\ \bar{s}_j(v,w_j), & \bar{s}_j(v,w_j) < s_i \le \bar{r}. \end{cases}$$

Because sellers' disagreement points are the same, we can derive seller i's best response $\hat{s}_i(s_j)$ symmetrically. Now we are ready to derive the equilibrium using the expressions of the best responses $\hat{s}_j(s_i)$ and $\hat{s}_i(s_j)$. We note that because sellers are symmetric in their reservation values and disagreement points, the equilibrium prices (if any) must be the same. Following a similar analysis of that used in Proposition 2.4.1, we have three cases to consider.

Case I: $w \ge (1-\theta)\bar{r} + \theta v$. The equilibrium prices are $(s_j^*, s_i^*) = (\bar{r}, \bar{r})$.

Case II: $v \le w < (1-\theta)\bar{r} + \theta v$. The equilibrium prices are $(s_j^*, s_i^*) = (\frac{w-\theta v}{1-\theta}, \frac{w-\theta v}{1-\theta})$.

Case III: $w < v$. The equilibrium prices are $(s_j^*, s_i^*) = (w, w)$. □

Proof of Proposition 2.5.1. By definition, $\bar{s}_j(v,w_j)$ is increasing in w_j and hence $\bar{s}_j(v,w_j) \le \bar{s}_i(v,w_i)$ for $w_j < w_i$. If (i) $w_i > w_j \ge (1-\theta)\bar{s}_j(v,w_j) + \theta v$, or (ii) $w_j < w_i < v$ and $\bar{s}_i(v,w_i) = v$, then the equality is achieved (i.e., $\bar{s}_j(v,w_j) = \bar{s}_i(v,w_i)$). Given the expressions of the best responses of $\hat{s}_j(s_i)$ and $\hat{s}_i(s_j)$ derived in Lemma 2.5.1, we are ready to derive the equilibrium prices.

Case I: $w_i > w_j \ge (1-\theta)\bar{s}_j(v,w_j) + \theta v$. The equilibrium prices are $(s_j^*, s_i^*) = (\bar{r}, \bar{r})$.

Case II: $v < w_j < (1-\theta)\bar{s}_j(v,w_j) + \theta v$ and $w_i \ge (1-\theta)\bar{s}_i(v,w_i) + \theta v$. The equilibrium prices are $(s_j^*, s_i^*) \in \{(s_a - \epsilon, s_a) \text{ or } (\bar{s}_j(v,w_j), (\bar{s}_j(v,w_j), \bar{r}]) : s_a \in (\frac{w_j - \theta v}{1-\theta}, \bar{s}_j(v,w_j)]\}$.

Case III: $v < w_j < (1-\theta)\bar{s}_j(v, w_j) + \theta v$ and $v < w_i < (1-\theta)\bar{s}_i(v, w_i) + \theta v$. We have two subcases to consider.

(III-a) If $w_i \geq (1-\theta)\bar{s}_j(v, w_j) + \theta v$, the equilibrium prices are the same as in case II.

(III-b) If $w_i < (1-\theta)\bar{s}_j(v, w_j) + \theta v$, the equilibrium prices are $(s_j^*, s_i^*) = (s_b - \epsilon, s_b)$ for $s_b \in (\frac{w_j - \theta v}{1-\theta}, \frac{w_i - \theta v}{1-\theta}]$.

Case IV: $w_j \leq v$ and $w_i \geq (1-\theta)\bar{s}_i(v, w_i) + \theta v$. The equilibrium prices are $(s_j^*, s_i^*) \in \{(s_c - \epsilon, s_c) \text{ or } (\bar{s}_j(v, w_j), (\bar{s}_j(v, w_j), \bar{r}]) : s_c \in (w_j, \bar{s}_j(v, w_j)]\}$.

Case V: $w_j \leq v$ and $v < w_i < (1-\theta)\bar{s}_i(v, w_i) + \theta v$. We have two subcases to consider.

(V-a) If $w_i \geq (1-\theta)\bar{s}_j(v, w_j) + \theta v$, the equilibrium prices are the same as in case IV.

(V-b) If $w_i < (1-\theta)\bar{s}_j(v, w_j) + \theta v$, the equilibrium prices are $(s_j^*, s_i^*) = (s_d - \epsilon, s_d)$ for $s_d \in (w_j, \frac{w_i - \theta v}{1-\theta}]$.

Case VI: $w_j \leq v$ and $w_i \leq v$. The equilibrium prices are $(s_j^*, s_i^*) = (s_e - \epsilon, s_e)$ for $s_e \in (w_j, w_i]$.

Proposition 2.5.1-i corresponds to case I; Proposition 2.5.1-ii(a) corresponds to cases II, III-a, IV and V-a by picking $(s_j^*, s_i^*) = (\bar{s}_j(v, w_j), \bar{r})$; Proposition 2.5.1-ii(b) corresponds to cases III-b and V-b by picking $(s_j^*, s_i^*) = (\frac{w_i - \theta v}{1-\theta} - \epsilon, \frac{w_i - \theta v}{1-\theta})$ and Proposition 2.5.1-iii corresponds to case VI by picking $(s_j^*, s_i^*) = (w_i - \epsilon, w_i)$. $\square$

Proof of Proposition 2.5.2. We compute seller $j(i)$'s expected profit provided that her opponent sets a price $s_i(s_j)$. By (2.13) and (2.14), we have

$$\Psi_j^{A:s}(v_j, v_i, w_j, s_j, s_i) = \begin{cases} \Psi_{j,1}^{A:s}(v_j, v_i, w_j, s_j, s_i) & \text{for } v_j \leq s_j < s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta}, \\ \Psi_{j,2}^{A:s}(v_j, v_i, w_j, s_j, s_i) & \text{for } s_j > s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta} \text{ and } s_i \geq v_j, \\ \Psi_{j,3}^{A:s}(v_j, v_i, w_j, s_j, s_i) & \text{for } s_j < s_i \wedge v_j, \\ \Psi_{j,4}^{A:s}(v_j, v_i, w_j, s_j, s_i) & \text{for } s_i < s_j \wedge v_j, \\ \Psi_{j,5}^{A:s}(v_j, v_i, w_j, s_j, s_i) & \text{for } s_j = s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta} \text{ and } s_i \geq v_j, \\ \Psi_{j,6}^{A:s}(v_j, v_i, w_j, s_j, s_i) & \text{for } s_j = s_i < v_j, \end{cases}$$

$$\Psi_i^{A:b}(v_i,v_j,w_i,s_i,s_j) = \begin{cases} \Psi_{i,1}^{A:b}(v_i,v_j,w_i,s_i,s_j) & \text{for } v_j \le s_j < s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta}, \\ \Psi_{i,2}^{A:b}(v_i,v_j,w_i,s_i,s_j) & \text{for } s_j > s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta} \text{ and } s_i \ge v_j, \\ \Psi_{i,3}^{A:b}(v_i,v_j,w_i,s_i,s_j) & \text{for } s_j < s_i \wedge v_j, \\ \Psi_{i,4}^{A:b}(v_i,v_j,w_i,s_i,s_j) & \text{for } s_i < s_j \wedge v_j, \\ \Psi_{i,5}^{A:b}(v_i,v_j,w_i,s_i,s_j) & \text{for } s_j = s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta} \text{ and } s_i \ge v_j, \\ \Psi_{i,6}^{A:b}(v_i,v_j,w_i,s_i,s_j) & \text{for } s_j = s_i < v_j, \end{cases}$$

where

$$\begin{aligned}
\Psi_{j,1}^{A:s}(v_j,v_i,w_j,s_j,s_i) &\equiv \mathbb{E}[\mathbb{I}_{\{R<v_j\}}v_j] + \mathbb{E}[\mathbb{I}_{\{v_j \le R\}} s_{Bj}(R,s_j,v_j)], \\
\Psi_{j,2}^{A:s}(v_j,v_i,w_j,s_j,s_i) &\equiv \mathbb{E}[\mathbb{I}_{\{R<v_j\}}v_j] + \mathbb{E}[\mathbb{I}_{\{v_j \le R < s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta}\}} s_{Bj}(R,s_j,v_j)] \\
&\quad + \mathbb{E}[\mathbb{I}_{\{R \ge s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta}\}} w_j], \\
\Psi_{j,3}^{A:s}(v_j,v_i,w_j,s_j,s_i) &\equiv \mathbb{E}[\mathbb{I}_{\{R<s_j\}}v_j] + \mathbb{E}[\mathbb{I}_{\{R \ge s_j\}} s_j], \\
\Psi_{j,4}^{A:s}(v_j,v_i,w_j,s_j,s_i) &\equiv \mathbb{E}[\mathbb{I}_{\{R<s_i\}}v_j] + \mathbb{E}[\mathbb{I}_{\{R \ge s_i\}} w_j], \\
\Psi_{j,5}^{A:s}(v_j,v_i,w_j,s_j,s_i) &\equiv \mathbb{E}[\mathbb{I}_{\{R<v_j\}}v_j] + \mathbb{E}[\mathbb{I}_{\{v_j \le R < s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta}\}} s_{Bj}(R,s_j,v_j)] \\
&\quad + \mathbb{E}\left[\mathbb{I}_{\{R \ge s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta}\}} \frac{s_{Bj}(R,s_j,v_j) + w_j}{2}\right], \\
\Psi_{j,6}^{A:s}(v_j,v_i,w_j,s_j,s_i) &\equiv \mathbb{E}[\mathbb{I}_{\{R<s_i\}}v_j] + \mathbb{E}\left[\mathbb{I}_{\{R \ge s_i\}} \frac{s_j + w_j}{2}\right], \\
\Psi_{i,1}^{A:b}(v_i,v_j,w_i,s_i,s_j) &\equiv \mathbb{E}[\mathbb{I}_{\{R<v_j\}}v_i] + \mathbb{E}[\mathbb{I}_{\{v_j \le R\}} w_i], \\
\Psi_{i,2}^{A:b}(v_i,v_j,w_i,s_i,s_j) &\equiv \mathbb{E}[\mathbb{I}_{\{R<v_j\}}v_i] + \mathbb{E}[\mathbb{I}_{\{v_j \le R < s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta}\}} w_i] \\
&\quad + \mathbb{E}[\mathbb{I}_{\{R \ge s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta}\}} (\mathbb{I}_{\{s_i<v_i\}} s_i + \mathbb{I}_{\{s_i \ge v_i\}} s_{Bi}(R,s_i,v_i))], \\
\Psi_{i,3}^{A:b}(v_i,v_j,w_i,s_i,s_j) &\equiv \mathbb{E}[\mathbb{I}_{\{R<s_j\}}v_i] + \mathbb{E}[\mathbb{I}_{\{R \ge s_j\}} w_i], \\
\Psi_{i,4}^{A:b}(v_i,v_j,w_i,s_i,s_j) &\equiv \mathbb{E}[\mathbb{I}_{\{R<s_i\}}v_i] + \mathbb{E}[\mathbb{I}_{\{R \ge s_i\}} s_i], \\
\Psi_{i,5}^{A:b}(v_i,v_j,w_i,s_i,s_j) &\equiv \mathbb{E}[\mathbb{I}_{\{R<v_j\}}v_i] + \mathbb{E}[\mathbb{I}_{\{v_j \le R < s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta}\}} w_i] \\
&\quad + \mathbb{E}\left[\mathbb{I}_{\{R \ge s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta}\}} \frac{\mathbb{I}_{\{s_i<v_i\}} s_i + \mathbb{I}_{\{s_i \ge v_i\}} s_{Bi}(R,s_i,v_i) + w_i}{2}\right], \\
\Psi_{i,6}^{A:b}(v_i,v_j,w_i,s_i,s_j) &\equiv \mathbb{E}[\mathbb{I}_{\{R<s_j\}}v_i] + \mathbb{E}\left[\mathbb{I}_{\{R \ge s_j\}} \frac{s_i + w_i}{2}\right].
\end{aligned}$$

For ease of exposition, define $\bar{s}_j^{A:s}(v_j, v_i, s_i) = \mathbb{I}_{\{s_i<v_j\}}s_i + \mathbb{I}_{\{v_j\le s_i<v_i\}}\frac{s_i-\theta v_j}{1-\theta} + \mathbb{I}_{\{v_i\le s_i\}}(s_i + \frac{\theta(v_i-v_j)}{1-\theta})$ and $\bar{s}_i^{A:b}(v_i, v_j, s_j) = \mathbb{I}_{\{s_j<v_j\}}s_j + \mathbb{I}_{\{v_j\le s_j<\frac{v_i-\theta v_j}{1-\theta}\}}((1-\theta)s_j+\theta v_j) + \mathbb{I}_{\{\frac{v_i-\theta v_j}{1-\theta}\le s_j\le\bar{r}\}}(s_j - \frac{\theta(v_i-v_j)}{1-\theta})$.

To derive seller j's best response $\hat{s}_j(s_i)$, we have two cases to consider.

Case(a): $s_i \le v_j$ (i.e., $\bar{s}_j^{A:s}(v_j, v_i, s_i) = s_i \le v_j$). The best response is the same as the case (a) in the proof of Lemma 2.5.1. Specifically, seller j's best response in this case is

$$\hat{s}_j(s_i) = \begin{cases} [\bar{s}_j^{A:s}(v_j, v_i, s_i) + \epsilon, \bar{r}], & s_i < w_j, \\ [w_j, \bar{r}], & s_i = w_j, \\ \bar{s}_j^{A:s}(v_j, v_i, s_i) - \epsilon, & w_j < s_i < \bar{r}. \end{cases}$$

Case(b): $s_i > v_j$ (i.e., $\bar{s}_j^{A:s}(v_j, v_i, s_i) > v_j$). We have $\Psi_{j,1}^{A:s}(v_j, v_i, w_j, v_j, s_i) = v_j > \Psi_{j,3}^{A:s}(v_j, v_i, w_j,$
$v_j-\epsilon, s_i)$, implying that the best response $\hat{s}_j(s_i)$ provided that $s_j < \bar{s}_j^{A:s}(v_j, v_i, s_i)$ must be within $[v_j, \bar{s}_j^{A:s}(v_j, v_i, s_i))$. For any $s_j \ge v_j$, $\Psi_{j,1}^{A:s}(v_j, v_i, w_j, s_j, s_i)$ is increasing in s_j and thus the best response provided that $s_j < \bar{s}_j^{A:s}(v_j, v_i, s_i)$ is $\hat{s}_j(s_i) = \bar{s}_j^{A:s}(v_j, v_i, s_i) - \epsilon$ for a sufficiently small $\epsilon > 0$. Also note that

$$\begin{aligned} & \Psi_{j,1}^{A:s}(v_j, v_i, w_j, \bar{s}_j^{A:s}(v_j, v_i, s_i) - \epsilon, s_i) - \Psi_{j,2}^{A:s}(v_j, v_i, w_j, s_j, s_i) \\ = \; & \mathbb{E}[\mathbb{I}_{\{R\ge\bar{s}_j^{A:s}(v_j,v_i,s_i)\}}(s_{Bj}(R, \bar{s}_j^{A:s}(v_j, v_i, s_i) - \epsilon, v_j) - w_j)] \\ & +\mathbb{E}[\mathbb{I}_{\{\bar{s}_j^{A:s}(v_j,v_i,s_i)-\epsilon\le R<\bar{s}_j^{A:s}(v_j,v_i,s_i)\}}(s_{Bj}(R, \bar{s}_j^{A:s}(v_j, v_i, s_i) - \epsilon, v_j) - s_{Bj}(R, \bar{s}_j^{A:s}(v_j, v_i, s_i), v_j))], \\ & \Psi_{j,1}^{A:s}(v_j, v_i, w_j, \bar{s}_j^{A:s}(v_j, v_i, s_i) - \epsilon, s_i) - \Psi_{j,5}^{A:s}(v_j, v_i, w_j, \bar{s}_j^{A:s}(v_j, v_i, s_i), s_i) \\ = \; & \mathbb{E}\left[\mathbb{I}_{\{R\ge\bar{s}_j^{A:s}(v_j,v_i,s_i)\}}\left(s_{Bj}(R, \bar{s}_j^{A:s}(v_j, v_i, s_i) - \epsilon, v_j) - \frac{s_{Bj}(R, \bar{s}_j^{A:s}(v_j, v_i, s_i), v_j) + w_j}{2}\right)\right] \\ & +\mathbb{E}[\mathbb{I}_{\{\bar{s}_j^{A:s}(v_j,v_i,s_i)-\epsilon\le R<\bar{s}_j^{A:s}(v_j,v_i,s_i)\}}(s_{Bj}(R, \bar{s}_j^{A:s}(v_j, v_i, s_i) - \epsilon, v_j) - s_{Bj}(R, \bar{s}_j^{A:s}(v_j, v_i, s_i), v_j))]. \end{aligned}$$

The right-hand sides of both equations are strictly positive only if $\bar{s}_j^{A:s}(v_j, v_i, s_i) \in (\frac{w_j-\theta v_j}{1-\theta}, \bar{r})$, i.e., $s_i \in (\frac{w_j-\theta(w_j\wedge v_i)}{1-\theta}, \tilde{s})$, where $\tilde{s} = \mathbb{I}_{\{v_i\ge(1-\theta)\bar{r}+\theta v_j\}}((1-\theta)\bar{r} + \theta v_j) + \mathbb{I}_{\{v_i<(1-\theta)\bar{r}+\theta v_j\}}(\bar{r} - \frac{\theta(v_i-v_j)}{1-\theta}) = \frac{((1-\theta)\bar{r}+\theta v_j)-\theta(((1-\theta)\bar{r}+\theta v_j)\wedge v_i)}{1-\theta}$. As a result, seller j's best response to an $s_i \in (\frac{w_j-\theta(w_j\wedge v_i)}{1-\theta}, \tilde{s})$ is $\hat{s}_j(s_i) = \bar{s}_j^{A:s}(v_j, v_i, s_i) - \epsilon$.

Also note that

$$
\begin{aligned}
&\Psi_{j,2}^{A:s}(v_j, v_i, w_j, s_j, s_i) - \Psi_{j,5}^{A:s}(v_j, v_i, w_j, \bar{s}_j^{A:s}(v_j, v_i, s_i), s_i) \\
&= \mathbb{E}\left[\mathbb{I}_{\{R \geq \bar{s}_j^{A:s}(v_j, v_i, s_i)\}} \frac{w_j - s_{Bj}(R, \bar{s}_j^{A:s}(v_j, v_i, s_i), v_j)}{2}\right].
\end{aligned}
$$

The right-hand side of the equation is above zero only if $\bar{s}_j^{A:s}(v_j, v_i, s_i) < \frac{w_j - \theta v_j}{1-\theta}$ and $\bar{s}_j^{A:s}(v_j, v_i, s_i) < \bar{r}$ (i.e., $s_i < (\frac{w_j - \theta(w_j \wedge v_i)}{1-\theta}) \wedge \tilde{s}$); equals zero if $\bar{s}_j^{A:s}(v_j, v_i, s_i) = \frac{w_j - \theta v_j}{1-\theta}$ or $\bar{s}_j^{A:s}(v_j, v_i, s_i) \geq \bar{r}$ (i.e., $s_i = (\frac{w_j - \theta(w_j \wedge v_i)}{1-\theta})$ or $s_i \geq \tilde{s}$). As a result, seller j's best response is $\hat{s}_j(s_i) \in [\bar{s}_j^{A:s}(v_j, v_i, s_i) + \epsilon, \bar{r}]$ for an $s_i < \min\{\frac{w_j - \theta(w_j \wedge v_i)}{1-\theta}, \tilde{s}\}$ and is $\hat{s}_j(s_i) \in [\frac{w_j - \theta v_j}{1-\theta}, \bar{r}]$ for $s_i = \frac{w_j - \theta(w_j \wedge v_i)}{1-\theta} < \tilde{s}$. Finally, note that $\hat{s}_j(s_i) = \bar{r}$ for an $s_i \in [\tilde{s}, \bar{r}]$. Hence seller j's best response in this case is

$$
\hat{s}_j(s_i) = \begin{cases}
[\bar{s}_j^{A:s}(v_j, v_i, s_i) + \epsilon, \bar{r}], & s_i < \min\{\frac{w_j - \theta(w_j \wedge v_i)}{1-\theta}, \tilde{s}\}, \\
[\frac{w_j - \theta v_j}{1-\theta}, \bar{r}], & s_i = \frac{w_j - \theta(w_j \wedge v_i)}{1-\theta} < \tilde{s}, \\
\bar{s}_j^{A:s}(v_j, v_i, s_i) - \epsilon, & \frac{w_j - \theta(w_j \wedge v_i)}{1-\theta} < s_i < \tilde{s}, \\
\bar{r}, & \tilde{s} \leq s_i \leq \bar{r}.
\end{cases}
$$

Combining cases (a) and (b), we obtain seller j's best response for different values of (v_j, v_i, w_j):

Case I-j: If $w_j \geq (1-\theta)\bar{r} + \theta v_j$,

$$
\hat{s}_j(s_i) = \begin{cases}
[\bar{s}_j^{A:s}(v_j, v_i, s_i) + \epsilon, \bar{r}], & s_i < \tilde{s}, \\
\bar{r}, & \tilde{s} \leq s_i \leq \bar{r}.
\end{cases}
$$

Case II-j: If $w_j < (1-\theta)\bar{r} + \theta v_j$,

$$
\hat{s}_j(s_i) = \begin{cases}
[\bar{s}_j^{A:s}(v_j, v_i, s_i) + \epsilon, \bar{r}], & s_i < \frac{w_j - \theta(w_j \wedge v_i)}{1-\theta}, \\
[\frac{w_j - \theta(w_j \wedge v_j)}{1-\theta}, \bar{r}], & s_i = \frac{w_j - \theta(w_j \wedge v_i)}{1-\theta}, \\
\bar{s}_j^{A:s}(v_j, v_i, s_i) - \epsilon, & \frac{w_j - \theta(w_j \wedge v_i)}{1-\theta} < s_i < \tilde{s}, \\
\bar{r}, & \tilde{s} \leq s_i \leq \bar{r}.
\end{cases}
$$

To derive seller i's best response $\hat{s}_i(s_j)$, we have two cases to consider.

Case(a): $s_j \le v_j$ (i.e., $\bar{s}_i^{A:b}(v_i, v_j, s_j) = s_j \le v_j$). The best response is the same as case (a) in the proof of Lemma 2.5.1. Specifically, seller i's best response in this case is

$$s_i(s_j) = \begin{cases} [\bar{s}_i^{A:b}(v_i, v_j, s_j) + \epsilon, \bar{r}], & s_j < w_i, \\ [w_i, \bar{r}], & s_j = w_i, \\ \bar{s}_i^{A:b}(v_i, v_j, s_j) - \epsilon, & w_i < s_j < \bar{r}. \end{cases}$$

Case(b): $s_j > v_j$ (i.e., $\bar{s}_i^{A:b}(v_i, v_j, s_j) > v_j$). We have $\Psi_{i,2}^{A:b}(v_i, v_j, w_i, v_j, s_j) > \Psi_{i,4}^{A:b}(v_i, v_j, w_i, v_j - \epsilon, s_j)$, implying that the best response $\hat{s}_i(s_j)$ over $s_i < \bar{s}_i^{A:b}(v_i, v_j, s_j)$ must be within $[v_j, \bar{s}_i^{A:b}(v_i, v_j, s_j))$. Also note that first-order condition of $\Psi_{i,2}^{A:b}(v_i, v_j, w_i, s_i, s_j)$ gives

$$\begin{cases} \bar{F}_R\big(\frac{s_i - \theta v_j}{1-\theta}\big)\big(1 - (\frac{s_i - w_i}{1-\theta})h_R(\frac{s_i - \theta v_j}{1-\theta})\big), & v_j \le s_i < v_i, \\ (1-\theta)\bar{F}_R\big(s_i + \frac{\theta(v_i - v_j)}{1-\theta}\big)\big(1 - (s_i - \frac{w_i - \theta v_i}{1-\theta})h_R(s_i + \frac{\theta(v_i - v_j)}{1-\theta})\big), & v_i \le s_i < \bar{r} - \frac{\theta(v_i - v_j)}{1-\theta}. \end{cases}$$

Because $(s_i - w_i)h_R(\frac{s_i - \theta v_j}{1-\theta})$ is increasing in s_i for $s_i \ge w_i$ and $(s_i - \frac{w_i - \theta v_i}{1-\theta})h_R(s_i + \frac{\theta(v_i - v_j)}{1-\theta})$ is increasing in s_i for $s_i \ge \frac{w_i - \theta v_i}{1-\theta}$, the maximizer of $\Psi_{i,2}^{A:b}(v_i, v_j, w_i, s_i, s_j)$ over $s_i \in [v_j, \bar{s}_i^{A:b}(v_j, v_i, s_j))$ is unique. We denote the maximizer as $\frac{\tilde{s}_i(v_j, w_i) - \theta(\tilde{s}_i(v_j, w_i) \wedge v_i)}{1-\theta}$, where

$$\begin{aligned} \tilde{s}_i(v_j, w_i) \quad = \quad & \max\big\{v_j, \max\big\{s \in [w_i \wedge ((1-\theta)\bar{r} + \theta v_j), (1-\theta)\bar{r} + \theta v_j] : \\ & \qquad (\frac{s - w_i}{1-\theta})h_R(\frac{s - \theta v_j}{1-\theta}) \le 1\big\}\big\}. \end{aligned}$$

Thus the best response provided that $s_i < \bar{s}_i^{A:b}(v_i, v_j, s_j)$ is

$$\hat{s}_i(s_j) = (\bar{s}_i^{A:b}(v_i, v_j, s_j) - \epsilon) \wedge \frac{\tilde{s}_i(v_j, w_i) - \theta(\tilde{s}_i(v_j, w_i) \wedge v_i)}{1-\theta}$$

for a sufficiently small $\epsilon > 0$. For any $s_i \ge v_j$, $\Psi_{i,1}^{A:b}(v_i, v_j, w_i, s_i, s_j)$ is constant in s_i and

$$\begin{aligned} & \Psi_{i,2}^{A:b}\big(v_i, v_j, w_i, \bar{s}_i^{A:b}(v_i, v_j, s_j) - \epsilon, s_j\big) - \Psi_{i,1}^{A:b}(v_i, v_j, w_i, s_i, s_j) \\ = \quad & \mathbb{E}[\mathbb{I}_{\{R \ge s_j - \epsilon\}}((1-\theta)s_j + \theta v_j - w_i - \epsilon)], \\ & \Psi_{i,2}^{A:b}\big(v_i, v_j, w_i, \bar{s}_i^{A:b}(v_i, v_j, s_j) - \epsilon, s_j\big) - \Psi_{i,5}^{A:b}\big(v_i, v_j, w_i, \bar{s}_i^{A:b}(v_i, v_j, s_j), s_j\big) \\ = \quad & \mathbb{E}\bigg[\mathbb{I}_{\{R \ge s_j\}}\bigg(\frac{(1-\theta)s_j + \theta v_j - w_i}{2} - \epsilon\bigg)\bigg] + \mathbb{E}[\mathbb{I}_{\{s_j - \epsilon \le R < s_j\}}((1-\theta)s_j + \theta v_j - w_i - \epsilon)]. \end{aligned}$$

The right-hand sides of both equations are strictly positive only if $s_j \in (\frac{w_i-\theta v_j}{1-\theta}, \bar{r}]$. As a result, seller i's best response to an $s_j \in (\frac{w_i-\theta v_j}{1-\theta}, \bar{r}]$ is $\hat{s}_i(s_j) = (\bar{s}_i^{A:b}(v_i, v_j, s_j) - \epsilon) \wedge \frac{\tilde{s}_i(v_j,w_i)-\theta(\tilde{s}_i(v_j,w_i)\wedge v_i)}{1-\theta}$. Also note that

$$\Psi_{i,1}^{A:b}(v_i, v_j, w_i, s_i, s_j) - \Psi_{i,5}^{A:b}(v_i, v_j, w_i, \bar{s}_i^{A:b}(v_i, v_j, s_j), s_j) = \mathbb{E}\left[\mathbb{I}_{\{R\geq s_j\}}\frac{w_i - ((1-\theta)s_j + \theta v_j)}{2}\right].$$

The right-hand side of the equation is above (equal to) zero only if $s_j < (=) \frac{w_i-\theta v_j}{1-\theta} \wedge \bar{r}$. As a result, seller i's best response is $\hat{s}_i(s_j) \in [\bar{s}_i^{A:b}(v_i, v_j, s_j) + \epsilon, \bar{r}]$ for an $s_j < \frac{w_i-\theta v_j}{1-\theta} \wedge \bar{r}$, and is $\hat{s}_i(s_j) \in [\frac{w_i-\theta(w_i\wedge v_i)}{1-\theta} \wedge \bar{r}, \bar{r}]$ for $s_j = \frac{w_i-\theta v_j}{1-\theta} \wedge \bar{r}$. Thus, seller i's best response in this case is

$$\hat{s}_i(s_j) = \begin{cases} [\bar{s}_i^{A:b}(v_i, v_j, s_j) + \epsilon, \bar{r}] & s_j < \frac{w_i-\theta v_j}{1-\theta} \wedge \bar{r}, \\ [\frac{w_i-\theta(w_i\wedge v_i)}{1-\theta} \wedge \bar{r}, \bar{r}], & s_j = \frac{w_i-\theta v_j}{1-\theta} \wedge \bar{r}, \\ \bar{s}_i^{A:b}(v_i, v_j, s_j) - \epsilon, & \frac{w_i-\theta v_j}{1-\theta} < s_j \leq \frac{\tilde{s}_i(v_j,w_i)-\theta v_j}{1-\theta}, \\ \frac{\tilde{s}_i(v_j,w_i)-\theta(\tilde{s}_i(v_j,w_i)\wedge v_i)}{1-\theta}, & \frac{\tilde{s}_i(v_j,w_i)-\theta v_j}{1-\theta} < s_j \leq \bar{r}. \end{cases}$$

Combining cases (a) and (b), we obtain seller i's best response for different values of (v_i, v_j, w_i):

Case I-i: If $w_i \geq (1-\theta)\bar{r} + \theta v_j$,

$$\hat{s}_i(s_j) = \begin{cases} [\bar{s}_i^{A:b}(v_i, v_j, s_j) + \epsilon, \bar{r}], & s_j < \bar{r}, \\ [\tilde{s}, \bar{r}], & s_j = \bar{r}. \end{cases}$$

Case II-i: If $w_i < (1-\theta)\bar{r} + \theta v_j$,

$$\hat{s}_i(s_j) = \begin{cases} [\bar{s}_i^{A:b}(v_i, v_j, s_j) + \epsilon, \bar{r}], & s_j \leq \frac{w_i-\theta(w_i\wedge v_j)}{1-\theta}, \\ \frac{w_i-\theta(w_i\wedge v_i)}{1-\theta}, & s_j = \frac{w_i-\theta(w_i\wedge v_j)}{1-\theta}, \\ \bar{s}_i^{A:b}(v_i, v_j, s_j) - \epsilon, & \frac{w_i-\theta(w_i\wedge v_j)}{1-\theta} < s_j \leq \frac{\tilde{s}_i(v_j,w_i)-\theta v_j}{1-\theta}, \\ \frac{\tilde{s}_i(v_j,w_i)-\theta(\tilde{s}_i(v_j,w_i)\wedge v_i)}{1-\theta}, & \frac{\tilde{s}_i(v_j,w_i)-\theta v_j}{1-\theta} < s_j \leq \bar{r}. \end{cases}$$

Now we are ready to drive the equilibrium using the expressions of the best responses $\hat{s}_j(s_i)$ and $\hat{s}_i(s_j)$.

Case I: $\min\{w_j, w_i\} \geq (1-\theta)\bar{r} + \theta v_j$. The equilibrium prices are $(s_j^*, s_i^*) = (\bar{r}, [\tilde{s}, \bar{r}])$.

Case II: $w_j < (1-\theta)\bar{r} + \theta v_j$ and $w_i \geq (1-\theta)\bar{r} + \theta v_j$. The equilibrium prices are $(s_j^*, s_i^*) \in \{(\bar{s}_j^{A:s}(v_j, v_i, s_a) - \epsilon, s_a) \text{ or } (\bar{r}, [\tilde{s}, \bar{r}]) : s_a \in (\frac{w_j - \theta(w_j \wedge v_i)}{1-\theta}, \tilde{s})\}$.

Case III: $w_j \geq (1-\theta)\bar{r} + \theta v_j$ and $w_i < (1-\theta)\bar{r} + \theta v_j$. The equilibrium prices are $(s_j^*, s_i^*) \in \{(s_b, \bar{s}_i^{A:b}(v_i, v_j, s_b) - \epsilon) \text{ or } ((\frac{\tilde{s}_i(v_j, w_i) - \theta v_j}{1-\theta}, \bar{r}], \frac{\tilde{s}_i(v_j, w_i) - \theta(\tilde{s}_i(v_j, w_i) \wedge v_i)}{1-\theta}) : s_b \in (\frac{w_i - \theta(w_i \wedge v_j)}{1-\theta}, \frac{\tilde{s}_i(v_j, w_i) - \theta v_j}{1-\theta}]\}$.

Case IV: $\max\{w_j, w_i\} < (1-\theta)\bar{r} + \theta v_j$. We have four subcases to consider.

(IV-a) If $w_j < w_i$, the equilibrium prices are $(s_j^*, s_i^*) = (\bar{s}_j^{A:s}(v_j, v_i, s_c) - \epsilon, s_c)$ for $s_c \in (\frac{w_j - \theta(w_j \wedge v_i)}{1-\theta}, \frac{w_i - \theta(w_i \wedge v_i)}{1-\theta}]$.

(IV-b) If $w_j = w_i$, the equilibrium prices are $(s_j^*, s_i^*) = (\frac{w_j - \theta(w_j \wedge v_j)}{1-\theta}, \frac{w_i - \theta(w_i \wedge v_i)}{1-\theta})$.

(IV-c) If $w_i < w_j < \tilde{s}_i(v_j, w_i)$, the equilibrium prices are $(s_j^*, s_i^*) = (s_d, \bar{s}_i^{A:b}(v_i, v_j, s_d) - \epsilon)$ for $s_d \in (\frac{w_i - \theta(w_i \wedge v_j)}{1-\theta}, \frac{w_j - \theta(w_j \wedge v_j)}{1-\theta}]$.

(IV-d) If $w_j \geq \tilde{s}_i(v_j, w_i)$, the equilibrium prices are the same as in case III.

Proposition 2.5.2-a corresponds to case I and II by picking $(s_j^*, s_i^*) = (\bar{r}, \bar{r})$; Proposition 2.5.2-b corresponds to case IV-a by picking $(s_j^*, s_i^*) = (\frac{w_i - \theta(w_i \wedge v_j)}{1-\theta} - \epsilon, \frac{w_i - \theta(w_i \wedge v_i)}{1-\theta})$; Proposition 2.5.2-c corresponds to case IV-b by picking $(s_j^*, s_i^*) = (\frac{w_j - \theta(w_j \wedge v_j)}{1-\theta}, \frac{w_i - \theta(w_i \wedge v_i)}{1-\theta})$; Proposition 2.5.2-d corresponds to case IV-c by picking $(s_j^*, s_i^*) = (\frac{w_j - \theta(w_j \wedge v_j)}{1-\theta}, \frac{w_j - \theta(w_j \wedge v_i)}{1-\theta} - \epsilon)$ and Proposition 2.5.2-e corresponds to cases III and IV-d by $(s_j^*, s_i^*) = (\bar{r}, \frac{\tilde{s}_i(v_j, w_i) - \theta(\tilde{s}_i(v_j, w_i) \wedge v_i)}{1-\theta})$. □

Proof of Corollary 2.5.1. The results follow immediately from Proposition 2.5.2. □

Proof of Proposition 2.5.3. In period 1, we have $w_j = v_j = 0$ for any states (n_j, n_i). For $n_j, n_i \geq 1$, by Lemma 2.5.1, $s_j^* = V_j(1, n_j, n_i) = 0$. For $n_j \geq 1$ and $n_i = 0$, by Lemma 2.1, $s_j^* = \bar{r}$ and $V_j(1, n_j, 0) = \lambda\Psi(0) = \lambda(1-\theta)\mu$, where $\mu = \mathbb{E}[R]$.

In period 2, it is immediate from (2.9) and (2.10) to see that w_j and v_j are constant in $n_j(n_i) \geq 2$ for a fixed $n_i(n_j)$. As a result, s_j^* and $V_j(2, n_j, n_i)$ are constant in $n_j(n_i) \geq 2$ for a fixed $n_i(n_j)$. Therefore, we can restrict (n_j, n_i) within $n_j, n_i \in \{1, 2\}$. These values are summarized in Table A.1. Because most of the derivation is algebraic, we only sketch the derivation of s_j^* and $V_j(2, n_j, n_i)$. We have two cases to consider.

Table A.1.
The effect of inventories on the dynamic equilibrium in the two-period game.

	$t=2$			$t=1$	
$V_j(t,n_j,n_i)$	$n_i=0$	$n_i=1$	$n_i\geq 2$	$n_i=0$	$n_i\geq 1$
$n_j=1$	$\lambda(1-\theta)\big(\int_{\lambda(1-\theta)\mu}^{\bar{r}}\bar{F}_R(r)\mathrm{d}r+\mu\big)$	$\frac{\lambda(1-\theta)}{2}\big(\int_0^{\lambda\mu}\bar{F}_R(r)\mathrm{d}r+\lambda\mu\big)$	$\frac{\lambda(1-\theta)}{2}\big(\int_0^{s^*}\bar{F}_R(r)\mathrm{d}r+s^*\bar{F}_R(s^*)\big)$	$\lambda(1-\theta)\mu$	0
$n_j\geq 2$	$2\lambda(1-\theta)\mu$	$\frac{\lambda(1-\theta)}{2}\big(\int_0^{s^*}\bar{F}_R(r)\mathrm{d}r+(\lambda\mu-s^*)\bar{F}_R(s^*)+\lambda\mu\big)$	0	$\lambda(1-\theta)\mu$	0
s_j^*	$n_i=0$	$n_i=1$	$n_i\geq 2$	$n_i=0$	$n_i\geq 1$
$n_j=1$	$\bar{r}$	$\lambda\mu$	s^*	$\bar{r}$	0
$n_j\geq 2$	$\bar{r}$	$\mathbb{I}_{\{\lambda\mu\geq s_j(0,0)\}}\bar{r}+\mathbb{I}_{\{\lambda\mu<s_j(0,0)\}}\lambda\mu$	0	$\bar{r}$	0
w_j	$n_i=0$	$n_i=1$	$n_i\geq 2$	$n_i=0$	$n_i\geq 1$
$n_j=1$	$\lambda(1-\theta)\mu$	$\lambda(1-\theta)\mu$	0	0	0
$n_j\geq 2$	0	$\lambda(1-\theta)\mu$	0	0	0
v_j	$n_i=0$	$n_i=1$	$n_i\geq 2$	$n_i=0$	$n_i\geq 1$
$n_j=1$	$\lambda(1-\theta)\mu$	0	0	0	0
$n_j\geq 2$	0	0	0	0	0

Notes. $\bar{s}_j(0,0)=\max\{s\in[0,\bar{r}]:\ sh_R(s)\leq 2\}$, $s^*=\mathbb{I}_{\{\lambda\mu\geq\bar{s}_j(0,0)\}}\bar{s}_j(0,0)+\mathbb{I}_{\{\lambda\mu<\bar{s}_j(0,0)\}}(\lambda\mu-\epsilon)$.

Case (a): $(n_j,n_i)=(1,1)$. We note that $v_j<w_j<(1-\theta)\bar{r}+\theta v_j$. By Lemma 2.5.1-ii, $s_j^*=(w_j-\theta v_j)/(1-\theta)=\lambda\mu$ and $V_j(2,1,1)=\frac{\lambda(1-\theta)}{2}(\int_0^{\lambda\mu}\bar{F}_R(r)\mathrm{d}r+\lambda\mu)$.

Case (b): $(n_j,n_i)=(1,2)$. We note that $v_j=v_i=0$ and $v_j<w_j<w_i<(1-\theta)\bar{r}+\theta v_j$. By Proposition 2.5.1-ii, sellers' equilibrium prices are

$$
\begin{aligned}
s_j^* &= \mathbb{I}_{\{\lambda\mu\geq\bar{s}_j(0,0)\}}\bar{s}_j(0,0)+\mathbb{I}_{\{\lambda\mu<\bar{s}_j(0,0)\}}(\lambda\mu-\epsilon),\\
s_i^* &= \mathbb{I}_{\{\lambda\mu\geq\bar{s}_j(0,0)\}}\bar{r}+\mathbb{I}_{\{\lambda\mu<\bar{s}_j(0,0)\}}\lambda\mu,
\end{aligned}
$$

where $\bar{s}_j(0,0)=\max\{s\in[0,\bar{r}]:\ sh_R(s)\leq 2\}$. We define $s^*\equiv s_j^*$. Sellers' equilibrium profits are

$$
\begin{aligned}
V_j(2,1,2) &= \frac{\lambda(1-\theta)}{2}\bigg(\int_0^{s^*}\bar{F}_R(r)\mathrm{d}r+s^*\bar{F}_R(s^*)\bigg),\\
V_j(2,2,1) &= \frac{\lambda(1-\theta)}{2}\bigg(\int_0^{s^*}\bar{F}_R(r)\mathrm{d}r+(\lambda\mu-s^*)\bar{F}_R(s^*)+\lambda\mu\bigg).
\end{aligned}
$$

Part (i), (ii) and (iv) follows immediately. To see part (iii), we note that $\lambda\mu\geq s^*$ and thus

$$
\int_0^{\lambda\mu}\bar{F}_R(r)\mathrm{d}r=\int_0^{s^*}\bar{F}_R(r)\mathrm{d}r+\int_{s^*}^{\lambda\mu}\bar{F}_R(r)\mathrm{d}r\leq\int_0^{s^*}\bar{F}_R(r)\mathrm{d}r+(\lambda\mu-s^*)\bar{F}_R(s^*) \tag{A.5}
$$

From the above inequality, we have $V_j(2,2,1)\geq V_j(2,1,1)\geq V_j(2,1,2)$. $\square$

Proof of Proposition 2.5.4. To see part (i), we summarize seller j's reservation values and disagreement points in period 3 in Table A.2. The non-positivity of w_j and v_j follows immediately.

Table A.2.
The effect of inventories on the dynamic equilibrium for $t = 3$.

w_j	$n_i = 1$	$n_i = 2$	$n_i = 3$
$n_j = 1$	$\lambda(1-\theta)\big(2\mu - \int_0^{\lambda(1-\theta)\mu} \bar{F}_R(r)\mathrm{d}r\big)$	$\frac{\lambda(1-\theta)}{2}\big(\int_0^{\lambda\mu} \bar{F}_R(r)\mathrm{d}r + \lambda\mu\big)$	$\frac{\lambda(1-\theta)}{2}\big(\int_0^{s^*} \bar{F}_R(r)\mathrm{d}r + s^*\bar{F}_R(s^*)\big)$
$n_j = 2$	$2\lambda(1-\theta)\mu - \frac{\lambda(1-\theta)}{2}\big(\int_0^{\lambda\mu} \bar{F}_R(r)\mathrm{d}r + \lambda\mu\big)$	$\frac{\lambda(1-\theta)}{2}\big((\lambda\mu - 2s^*)\bar{F}_R(s^*) + \lambda\mu\big)$	$-\frac{\lambda(1-\theta)}{2}\big(\int_0^{s^*} \bar{F}_R(r)\mathrm{d}r + s^*\bar{F}_R(s^*)\big)$
$n_j = 3$	$2\lambda(1-\theta)\mu - \frac{\lambda(1-\theta)}{2}\big(\int_0^{s^*} \bar{F}_R(r)\mathrm{d}r + (\lambda\mu - s^*)\bar{F}_R(s^*) + \lambda\mu\big)$	$\frac{\lambda(1-\theta)}{2}\big(\int_0^{s^*} \bar{F}_R(r)\mathrm{d}r + (\lambda\mu - s^*)\bar{F}_R(s^*) + \lambda\mu\big)$	0
v_j	$n_i = 1$	$n_i = 2$	$n_i = 3$
$n_j = 1$	$\frac{\lambda(1-\theta)}{2}\big(\int_0^{\lambda\mu} \bar{F}_R(r)\mathrm{d}r + \lambda\mu\big)$	$\frac{\lambda(1-\theta)}{2}\big(\int_0^{s^*} \bar{F}_R(r)\mathrm{d}r + s^*\bar{F}_R(s^*)\big)$	$\frac{\lambda(1-\theta)}{2}\big(\int_0^{s^*} \bar{F}_R(r)\mathrm{d}r + s^*\bar{F}_R(s^*)\big)$
$n_j = 2$	$\frac{\lambda(1-\theta)}{2}\big((\lambda\mu - s^*)\bar{F}_R(s^*) - \int_{s^*}^{\lambda\mu} \bar{F}_R(r)\mathrm{d}r\big)$	$-\frac{\lambda(1-\theta)}{2}\big(\int_0^{s^*} \bar{F}_R(r)\mathrm{d}r + s^*\bar{F}_R(s^*)\big)$	$-\frac{\lambda(1-\theta)}{2}\big(\int_0^{s^*} \bar{F}_R(r)\mathrm{d}r + s^*\bar{F}_R(s^*)\big)$
$n_j = 3$	0	0	0

Notes. $\bar{s}_j(0,0) = \max\{s \in [0, \bar{r}] : \ sh_R(s) \leq 2\}$, $s^* = \mathbb{I}_{\{\lambda\mu \geq \bar{s}_j(0,0)\}}\bar{s}_j(0,0) + \mathbb{I}_{\{\lambda\mu < \bar{s}_j(0,0)\}}(\lambda\mu - \epsilon)$.

For ease of exposition, we define $w_j(n_j, n_i)$, $v_j(n_j, n_i)$ and $s_j^*(n_j, n_i)$ as seller j's reservation value, disagreement point and equilibrium posted price with states (n_j, n_i) for $t = 3$, respectively. To see the non-monotonicity of w_j with respect to n_j, we have three cases to consider.

Case (a): Fix $n_i = 1$. We have $\int_0^{\lambda(1-\theta)\mu} \bar{F}_R(r)\mathrm{d}r \leq \int_0^{\lambda\mu} \bar{F}_R(r)\mathrm{d}r \leq \lambda\mu$ and thus $w_j(1,1) \geq w_j(2,1)$. $w_j(2,1) \geq w_j(3,1)$ follows from (A.5).

Case (b): For $n_i = 2$. We have

$$\int_0^{\lambda\mu} \bar{F}_R(r)\mathrm{d}r \geq \lambda\mu - \int_{s^*}^{\bar{r}} \bar{F}_R(r)\mathrm{d}r \geq (\lambda\mu - 2s^*)\bar{F}_R(s^*). \tag{A.6}$$

The first inequality follows from $\lambda\mu \geq s^*$ and the second inequality follows from $\bar{s}_j(0,0)\bar{F}_R(\bar{s}_j(0,0)) - \int_{\bar{s}_j(0,0)}^{\bar{r}} \bar{F}_R(r)\mathrm{d}r \geq 0$. This implies that $w_j(1,2) \geq w_j(2,2)$. From (A.5), we have $w_j(3,2) \geq w_j(1,2)$.

Case (c): Fix $n_i = 3$. It is immediate to see that $w_j(1,3) > w_j(3,3)(= 0) > w_j(2,3)$.

To see the monotonicity of w_j with respect to n_i, we have three cases to consider.

Case (a): Fix $n_j = 1$. We note that $w_j(1,1) > \lambda(1-\theta)\mu > w_j(1,2) > w_j(1,3)$. The third inequality comes from (A.5).

Case (b): Fix $n_j = 2$. We have $w_j(2,1) > \lambda(1-\theta)\mu > w_j(2,2) > 0 > w_j(2,3)$.

Case (c): Fix $n_j = 3$. We have $w_j(3,1) > \lambda(1-\theta)\mu > w_j(3,2) > w_j(3,3)(=0)$.

The proof of v_j with respect to n_j and n_i follows a similar way and hence omitted. This concludes the proof of part (i).

To see part (iv), It is immediate to see that $w_j = w_i$ and $v_j = v_i$ for $n_j = n_i$. For $n_j < n_i$, we have three cases to consider.

Case (a): $(n_j, n_i) = (1,2)$. We have $w_j(1,2) < \lambda(1-\theta)\mu < w_j(2,1)$ and $v_j(1,2) > v_j(2,1)$ by (A.6).

Case (b): $(n_j, n_i) = (1,3)$. We have $w_j(1,3) < \lambda(1-\theta)\mu < w_j(3,1)$ and $v_j(1,3) > v_j(3,1)(=0)$.

Case (c): $(n_j, n_i) = (2,3)$. We have $w_j(2,3) < 0 < w_j(3,2)$ and $v_j(2,3) < v_j(3,2)(=0)$. This concludes the proof of part (iv).

To see part (ii), we have $s_j^*(2,2) = (w_j(2,2) - \theta v_j(2,2))/(1-\theta) < \bar{r}$ (which corresponds to case (ii) in Lemma 2.5.1). Also note that $s_j^*(3,2) \geq (w_j(3,2) - \theta v_j(3,2))/(1-\theta)$, and $s_j^*(2,3) = ((w_j(3,2) - \theta v_j(2,3))/(1-\theta)) \wedge \bar{r}$ (which corresponds to cases (a) and (b) in Proposition 2.5.2). Thus, $s_j^*(2,2) \leq (w_j(2,2) - v_j(2,2))/(1-\theta) = w_j(3,2)/(1-\theta) \leq s_j^*(3,2)$ and $s_j^*(2,3) \geq s_j^*(3,3)(=0)$. This concludes the proof of non-monotonicity of s_j^* with respect to n_j.

To see the non-monotonicity of s_j^* with respect to n_i, we have three cases to consider.

Case (a): Fix $n_j = 1$. We note that $s_j^*(1,1) = ((w_j(1,1) - \theta v_j(1,1))/(1-\theta)) \wedge \bar{r}$ (which corresponds to cases (i) and (ii) in Lemma 2.5.1), and $s_j^*(1,2) \leq ((w_j(2,1) - \theta v_j(1,2))/(1-\theta)) \wedge \bar{r}$ (which corresponds to cases (d) and (e) in Proposition 2.5.2). Because $(w_j(1,1) - \theta v_j(1,1))/(1-\theta) \geq (w_j(2,1) - \theta v_j(1,2))/(1-\theta)$, $s_j^*(1,1) \geq s_j^*(1,2)$. $s_j^*(1,2) \geq s_j^*(1,3)$ follows from Corollary 2.5.1.

Case (b): Fix $n_j = 2$. We have $s_j^*(2,1) \geq (w_j(2,1) - \theta v_j(2,1))/(1-\theta) \geq \lambda\mu$ (which corresponds to cases (d) and (e) in Proposition 2.5.2) and $s_j(2,2) = (w_j(2,2) - \theta v_j(2,2))/(1-\theta) \leq \lambda\mu$, and thus $s_j^*(2,1) \geq s_j^*(2,2)$. Because $w_j(2,2) \leq w_j(3,2)$ and $v_j(2,2) = v_j(2,3)$, $s_j^*(2,2) \leq s_j^*(2,3)$.

Case (c): Fix $n_j = 3$, it is immediate to see that $s_j^*(3,2) \geq s_j^*(3,3)(=0)$. We conclude part (ii).

To see part (iii), we have $V_j(3,2,2) \leq \lambda w_j(3,2)$ and $V_j(3,3,2) = \lambda w_j(3,2)$ and thus $V_j(3,2,2) \leq V_j(3,3,2)$. Also note that $V_j(3,2,3) > V_j(3,3,3)(=0)$. This concludes the proof of non-monotonicity of $V_j(3,n_j,n_i)$ with respect to n_j.

Finally, we prove the non-monotonicity of $V_j(3,n_j,n_i)$ with respect to n_i. We have three cases to consider.

Case (a): Fix $n_j = 1$. We first note that when (v_j, v_i, w_j, w_i) satisfy equilibrium conditions of cases (d) and (e) in Proposition 2.5.2 and $v_i < w_i$, seller i's equilibrium profit is

$$\Psi_i^{A:b}(v_i, v_j, w_i, s_j^*) = \bar{F}_R\Big(\frac{\tilde{s}_i(v_j, w_i) \wedge w_j - \theta v_j}{1-\theta}\Big)(\tilde{s}_i(v_j, w_i) \wedge w_j - w_i) + \bar{F}_R(v_j)(w_i - v_i) + v_i.$$

The right-hand side of the above equation is increasing in w_j and w_i. We also note that

$$\Psi_i^{A:b}(v_i, v_j, w_i, s_j^*) \geq \bar{F}_R\Big(\frac{\tilde{s}_i(v_j, w_i) \wedge w_j - \theta v_j}{1-\theta}\Big)(\tilde{s}_i(v_j, w_i) \wedge w_j - v_i) + v_i.$$

The right-hand side of the above inequality is increasing in v_j and the equality holds if $w_i = v_i$. Combining the above equation and inequality, we observe that

$$\begin{aligned} \Psi_i^{A:b}(v_j(1,2), v_j(2,1), w_j(1,2), s_{j,1}^*) &\geq \Psi_i^{A:b}(v_j(1,2), v_j(2,1), w_j(1,3), s_{j,2}^*) \\ &\geq \Psi_i^{A:b}(v_j(1,3), v_j(3,1), w_j(1,3), s_{j,3}^*), \end{aligned}$$

where

$$\begin{aligned} s_{j,1}^* &= \frac{\tilde{s}_i(v_j(2,1), w_j(1,2)) \wedge w_j(2,1) - \theta v_j(1,2)}{1-\theta}, \\ s_{j,2}^* &= \frac{\tilde{s}_i(v_j(2,1), w_j(1,3)) \wedge w_j(3,1) - \theta v_j(1,2)}{1-\theta}, \\ s_{j,3}^* &= \frac{\tilde{s}_i(v_j(3,1), w_j(1,3)) \wedge w_j(3,1) - \theta v_j(1,3)}{1-\theta}. \end{aligned}$$

The first inequality follows from $w_j(1,2) > w_j(1,3)$ and $w_j(2,1) > w_j(3,1)$. The second inequality follows from $v_j(2,1) \geq v_j(3,1)$ and $w_j(1,3) = v_j(1,3)$. Because $V_j(2,0,2) = V_j(2,0,3) = 0$ and $V_j(2,1,2) = V_j(2,1,3)$, we have $V_j(3,1,2) \geq V_j(3,1,3)$.

Case (b): Fix $n_j = 2$. We have $V_j(3,2,2) \leq \lambda(w_j(2,2) - v_j(2,2)) = \lambda(V_j(2,2,1) - V_j(2,2,2)) = \lambda V_j(2,2,1)$ and $V_j(3,2,1) \geq V_j(2,2,1)$, and thus $V_j(3,2,1) \geq V_j(3,2,2)$.

Case (c): Fix $n_j = 3$. We have $V_j(3,3,1) \geq V_j(2,3,1) \geq \lambda w_j(3,2)$, and thus $V_j(3,3,1) \geq V_j(3,3,2) \geq V_j(3,3,3) = 0$. $\square$

B. Proofs of Formal Results in Chapter 3

Proof of Proposition 3.4.1. Substituting the expressions of $\Pi(w)$, $\pi(w)$, $\overline{\Pi}$ and $\overline{\pi}$ into (3.2) yields

$$\frac{(1-w)^2/4}{w(1-w)/2} = \frac{1/4}{1/8}.$$

Solving for w leads to $w^{KS} = 1/5$.

To compare with the NB solution, we differentiate the Nash product in (3.1) with respect to w to obtain

$$\frac{\partial \Omega}{\partial w} = \frac{(1-w)^{\theta}}{2^{\theta+1} w^{\theta}} \big((1-\theta)(1-w) - (1+\theta)w\big) = 0.$$

Solving for w leads to $w^{NB} = (1-\theta)/2$. Setting $w^{KS} = w^{NB}$ gives $\theta = 0.6$. □

Proof of Lemma 3.5.1. To see part (i), we first list the expressions of boundaries of the feasible region of (w_i, w_j) (see the top panels of Figure 3.2). The derivation has been provided in the proof of Lemmas F.1.1 and F.1.2. We have $\hat{w}_i^1(w_j) = 1-(\eta/2)(1-w_j)$, $\hat{w}_i^2(w_j) = 1-(2/\eta)(1-w_j)$, $\hat{w}_i^3(w_j) = (\eta/2)w_j$ and $\hat{w}_i^4(w_j) = (2\eta w_j + 2 - \eta)/4$. It is easy to check that the second piece of $\Pi_i(w_i, w_j)$ in (F.1) is increasing (decreasing) in w_j for $w_j < (>)(2\eta w_i + 2 - \eta)/4 \equiv \bar{w}_j(w_i)$ and the second piece of $\pi_i(w_i, w_j)$ in (F.2) is increasing in w_j. We then substitute the expression of (F.6) into (F.1) and (F.2) to derive the supplier's and retailer j's negotiated profits as

$$\Pi_i(w_i^{NB}(w_j), w_j) = \frac{w_j(1-w_j)}{2} + \frac{(2-\eta^2)(1-\theta^2)}{8(2+\eta)} \text{ and } \pi_i(w_i^{NB}(w_j), w_j) = \frac{(1+\theta)^2}{4(2+\eta)^2}$$

for $w_j < \bar{w}_j^a$ (i.e., both products have positive outputs). It is easy to see that $\Pi_i(w_i^{NB}(w_j), w_j)$ is increasing (decreasing) in w_j for $w_j < (>)1/2$ and $\pi_i(w_i^{NB}(w_j), w_j)$ is constant in w_j. This concludes part (i).

To see part (ii), we first list the expressions of boundaries of the feasible region of (w_i, w_j) (see the bottom panels of Figure 3.2). The derivation has been provided in the proof of Lemmas F.2.1 and F.2.2. We have $\breve{w}_i^1(w_j) = 1 - \eta(1 - w_j)$, $\breve{w}_i^2(w_j) = 1 - (1/\eta)(1 - w_j)$ and $\breve{w}_i^3(w_j) = (1 - \eta + \eta w_j)/2$. It is easy to see that the second piece of $\Pi_i(w_i, w_j)$ in (F.19) is increasing in w_j and the second piece of $\pi_i(w_i, w_j)$ in (F.20) is decreasing in w_j. We now substitute the expression of (F.24) into (F.19) and (F.20) to derive the supplier j's and retailer's negotiated profits as

$$\Pi_i(w_i^{NB}(w_j), w_j) = \tfrac{(1-\theta^2)(1-\eta+\eta w_j)^2}{8(1-\eta^2)} \text{ and } \pi_i(w_i^{NB}(w_j), w_j) = \tfrac{(1+\theta)^2(1-\eta+\eta w_j)^2}{16(1-\eta^2)} + \tfrac{(1-w_j)^2}{4}$$

for $w_j < 1 - \eta(1 + \theta)/(2 - \eta^2(1 - \theta))$. It is easy to see that $\Pi_i(w_i^{NB}(w_j), w_j)$ is increasing in w_j and $\pi_i(w_i^{NB}(w_j), w_j)$ is decreasing in w_j. We conclude part (ii). $\square$

Proof of Lemma 3.5.2. To see part (i), we substitute the expression of (B.8) into (B.4) and (B.5) to derive the supplier's and retailer j's negotiated profits as

$$\begin{aligned}
\Pi_i(w_i^{NB}(w_j), w_j) &= (1-\theta)\left[\tfrac{(1-w_j)w_j}{2} + \tfrac{(1-\theta)(2-\eta)+\sqrt{(2-\eta)^2(1+\theta)^2+16\theta(4-\eta^2)(1-w_j)w_j}}{16(2+\eta)}\right], \\
\pi_i(w_i^{NB}(w_j), w_j) &= \frac{\left((1+\theta)(2-\eta) + \sqrt{(2-\eta)^2(1+\theta)^2 + 16\theta(4-\eta^2)(1-w_j)w_j}\right)^2}{16(4-\eta^2)^2},
\end{aligned}$$

for $w_j < (2-\eta)(4+\eta(1-\theta))/(8-2\eta^2(1-\theta))$. In the case that $w_j > (2-\eta)(4+\eta(1-\theta))/(8-2\eta^2(1-\theta))$, retailer j only earns ϵ profit and equilibrium prices never arise under this range. Thus, we omit this range. It is easy to see that $\Pi_i(w_i^{NB}(w_j), w_j)$ and $\pi_i(w_i^{NB}(w_j), w_j)$ are increasing (decreasing) in w_j for $w_j < (>)1/2$. This concludes part (i). $\square$

Proof of Proposition 3.5.1. First, we consider the one-to-two channel. By (F.6), (F.7) and symmetry, we can derive negotiated prices under the NB and KS solutions as $\hat{w}_{sim}^{NB} = (1-\theta)/2$ and $\hat{w}_{sim}^{KS} = 1/5$, respectively. Setting $\hat{w}_{sim}^{NB} = \hat{w}_{sim}^{KS}$ gives $\hat{\theta}_{sim}^{KS} = 0.6$.

Now we consider the two-to-one channel. By (F.24), (F.25) and symmetry, we can derive negotiated prices under the NB and KS solutions as $\breve{w}_{sim}^{NB} = (1-\theta)(1-\eta)/(2-(1-\theta)\eta)$ and $\breve{w}_{sim}^{KS} = (1-\eta)/(5-\eta)$, respectively. Setting $\breve{w}_{sim}^{NB} = \breve{w}_{sim}^{KS}$ gives $\breve{\theta}_{sim}^{KS} = 0.6$. $\square$

Proof of Proposition 3.5.2. To see part (i), we apply (F.8), (F.9), (F.10) and $d_j = 0$ to (3.1) and obtain the Nash product for unit j as

$$\Omega_j(w_j) = \begin{cases} \left(\frac{w_j(1-w_j)}{2} + \frac{(2-\eta)(1-\theta_i^2)}{8(2+\eta)} - \frac{1-\theta_i^2}{8}\right)^{1-\theta_j}\left(\frac{(4+\eta(1-\theta_i)-2(2+\eta)w_j)^2}{16(2+\eta)^2}\right)^{\theta_j} & w_j < \bar{w}_j^a, \\ 0 & w_j > \bar{w}_j^a. \end{cases}$$

Setting $\partial \ln \Omega_j(w_j)/\partial w_j = 0$ in the first case gives

$$(1-\theta_j)\frac{1-2w_j}{2(2+\eta)w_j(1-w_j) - \eta(1-\theta_i^2)} + 2\theta_j \frac{-1}{4+\eta(1-\theta_i) - 2(2+\eta)w_j} = 0.$$

This gives

$$\hat{w}_j^{NB} = \frac{6 - 2\theta_j + \eta(2-\theta_i+\theta_i\theta_j) - \sqrt{(2-\eta\theta_i)^2(1+\theta_j)^2 + 4\eta\theta_j((4+\eta)\theta_i^2 + 4\theta_i - \eta)}}{4(\eta+2)} \tag{B.1}$$

Note that the above expression has two roots within $[0, 1]$ and the maximizer of Ω_j should be the smaller root.

We then substitute the expressions of (F.12), (F.13), (F.14), (F.15), (F.16) and $d_j = 0$ into (3.2) and obtain

$$\frac{(10+\eta-5(2+\eta)w_j)^2/(100(2+\eta)^2)}{w_j(1-w_j)/2 + 2(2-\eta)/(25(2+\eta)) - 2/25} = \frac{(10-3\eta+\sqrt{(50-7\eta)(2+\eta)})^2/(400(2+\eta)^2)}{(82+9\eta)/(200(2+\eta)) - 2/25}$$

for $w_j < 2/(2+\eta)$. We omit the expressions for w_j above the maxima that attain $\overline{\Pi}_j$ (i.e., $w_j > 1/2$) as the equilibrium prices should lead to a Pareto profit allocation. This gives

$$\hat{w}_j^{KS} = \frac{2(10-\eta)(2+\eta)(50+\eta) + (5(2+\eta)(10-3\eta) - \sqrt{2\Delta_3})\sqrt{(50-7\eta)(2+\eta)}}{5(2+\eta)(300+12\eta-5\eta^2+(20-6\eta)\sqrt{(50-7\eta)(2+\eta)})}, \tag{B.2}$$

where $\Delta_3 = (2+\eta)(50-7\eta)(100-36\eta+5\eta^2) - (10-3\eta)(100+12\eta-3\eta^2)\sqrt{(50-7\eta)(2+\eta)}$. Note that the above expression has two roots within $[0, 1]$ and the price should be the smaller root. It is easy to check that $\partial \hat{w}_j^{KS}/\partial\eta > 0$ and thus $\hat{w}_j^{KS} \in [0.2, 0.27)$. It follows that $\partial \hat{w}_i^{KS}/\partial\eta = (\eta/2)\partial \hat{w}_j^{KS}/\partial\eta + (5\hat{w}_j^{KS} - 1)/10 > 0$.

Now we consider the equivalent bargaining power. Recall that the supplier negotiates with retailer j first. To see $\hat{\theta}_i^{KS}$, it is easy to check that $\hat{\theta}_i^{KS}$ should lead to the same expression of the first pieces in (F.6) and (F.7), which implies that $\hat{\theta}_i^{KS} = 0.6$. To see $\hat{\theta}_j^{KS}$, setting $\hat{w}_j^{NB}|_{\theta_i=0.6} = \hat{w}_j^{KS}$ yields

$$\hat{\theta}_j^{KS}(\eta) = \frac{5(1-2\hat{w}_j^{KS})(10+\eta-5(2+\eta)\hat{w}_j^{KS})}{50(1-\hat{w}_j^{KS})-\eta(11-15\hat{w}_j^{KS})}.$$

It is easy to check that $\partial\hat{\theta}_j^{KS}(\eta)/\partial\eta < 0$ for $\eta \in (-2, 6)$ and thus $\hat{\theta}_j^{KS}(\eta) \in (0.5, 0.6]$. We conclude part (i).

To see part (ii), we apply (F.27), (F.28), (F.29) and $D_j = 0$ to (3.1) and obtain the Nash product for unit j as

$$\Omega_j(w_j) = \begin{cases} \left(\frac{w_j((2-\eta^2(1-\theta_i))(1-w_j)-\eta(1+\theta_i))}{4(1-\eta^2)}\right)^{1-\theta_j}\left(\frac{(1-w_j)^2}{4}+\frac{(1+\theta_i)^2(1-\eta+\eta w_j)^2}{16(1-\eta^2)}-\frac{(1+\theta_i)^2}{16}\right)^{\theta_j} & w_j < w^a, \\ 0 & w_j > w^a, \end{cases}$$

where $w^a = 1-\eta(1+\theta_i)/(2-\eta^2(1-\theta_i))$. Setting $\partial \ln \Omega_j(w_j)/\partial w_j = 0$ in the first case gives

$$\frac{(1-\theta_j)((2-\eta^2(1-\theta_i))(1-2w_j)-\eta(1+\theta_i))}{w_j((2-\eta^2(1-\theta_i))(1-w_j)-\eta(1+\theta_i))} + \frac{\theta_j(-8(1-\eta^2)w_j+2\eta(1+\theta_i)^2(1-\eta+\eta w_j))}{(1+\theta_i)^2(1-\eta+\eta w_j)^2+(1-\eta^2)(4(1-w_j)^2-(1+\theta_i)^2)} = 0.$$

Rearranging the terms gives

$$\Phi(w_j) = \Gamma_0 + \Gamma_1 w_j + \Gamma_2 w_j^2 + \Gamma_3 w_j^3, \tag{B.3}$$

where $\Gamma_0 = 2(1-\eta)^2(1-\theta_j)(2+\eta(1-\theta_i))(2+\eta(1-2\theta_i-\theta_i^2))$, $\Gamma_1 = 2(1-\eta)\big((1-\theta_i)(5-6\theta_i-3\theta_i^2-2(1-2\theta_i-\theta_i^2)\theta_j)\eta^3+(1-\theta_i)(11+4\theta_i+\theta_i^2-4\theta_j)\eta^2-2(3-8\theta_i-3\theta_i^2-2(1-2\theta_i-\theta_i^2)\theta_j)\eta-8(2-\theta_j)\big)$, $\Gamma_2 = (1-\eta)\big(8(5-\theta_j)+4(1-\theta_i)(7+2\theta_i-(\theta_i+2)\theta_j)\eta-2(1-\theta_i)(11+\theta_i-(1-\theta_i)\theta_j)\eta^2-(1-\theta_i)^2(3+\theta_i)(5-\theta_j)\eta^3\big)$ and $\Gamma_3 = -16+4(1-\theta_i)(5+\theta_i)\eta^2-2(1-\theta_i)^2(3+\theta_i)\eta^4$.

We note that $\Phi(w_j)$ is a cubic function with the coefficient of w_j^3 being negative (i.e., $\Gamma_3 < 0$). Also, note that $\Phi(w_j) = \Gamma_0 \geq 0$ and

$$\Phi(w_j^o) = -\frac{(1-\eta)^3(2+\eta-\theta_i\eta)^2\Big(8+4(2+\theta_i)(1-\theta_i)\eta-2(1-\theta_i)^2\eta^2-(1-\theta_i)^2(3+\theta_i)\eta^3\Big)\theta_j}{4(2-(1-\theta_i)\eta^2)^2} < 0$$

for $\eta < 1$ and $\theta_j < 1$, where $w_j^o = (1/2)w^a$. In the case that $\eta = 1$ or $\theta_j = 1$, it is easy to check that $\breve{w}_j^{NB} = 0$. This implies that there exists at least one root within the range $[0, w_j^o]$. Note that $\partial\Phi(w_j)/\partial w_j$ is a quadratic function with the coefficient of w_j^2 being negative (i.e., $3\Gamma_3 < 0$) and is maximized at $w_j^q = -(1/3)\Gamma_2/\Gamma_3 > 0$. Also, note that $\partial\Phi(w_j)/\partial w_j|_{w_j=0} = \Gamma_1 < 0$ and $w_j^q > w_j^o$, which implies that the sign of $\partial\Phi(w_j)/\partial w_j$ changes at most once within the range $[0, w_j^o]$ and the change is from negative to positive. This suggests that there exists a unique root that is the maximizer of $\Phi(w_j)$ within the range $[0, w_j^o]$. Let $\breve{w}_j^{NB}$ denote this root.

We then substitute the expressions of (F.31), (F.32), (F.33), (F.34), (F.35) and $D_j = 0$ into (3.2) and obtain

$$\frac{(1-w_j)^2/4+4(1-\eta+\eta w_j)^2/(25(1-\eta^2))-4/25}{w_j((5-\eta^2)(1-w_j)-4\eta)/(10(1-\eta^2))} = \frac{(41+9\eta)/(100(1+\eta))-4/25}{(1-\eta)(5+\eta)^2/(40(1+\eta)(5-\eta^2))}$$

for $w_j < 1-\eta$. We omit the expressions for w_j above the maxima that attain $\overline{\Pi}_j$ (i.e., $w_j > (1-\eta)(5-\eta)/(2(5-\eta^2)))$ as the equilibrium prices should lead to a Pareto profit allocation. This gives

$$\breve{w}_j^{KS} = \frac{(1-\eta)(5+\eta)\big(375 - 125\eta - 111\eta^2 + 5\eta^3 - 2\sqrt{\Delta_4}\big)}{3125 - 1075\eta - 1450\eta^2 + 390\eta^3 + 181\eta^4 - 19\eta^5},$$

where $\Delta_4 = (1-\eta)(5+\eta)(3125 + 250\eta - 1120\eta^2 - 266\eta^3 + 27\eta^4)$. Note that the above expression has two roots within $[0, 1]$ and the price should be the smaller root. It is easy to check that $\partial\breve{w}_j^{KS}/\partial\eta < 0$ and thus $\breve{w}_j^{KS} \in [0, 0.2]$. It follows that $\partial\breve{w}_i^{KS}/\partial\eta = (\eta/5)\partial\breve{w}_j^{KS}/\partial\eta - (1-\breve{w}_j^{KS})/5 < 0$.

Now we consider the equivalent bargaining power. Recall that the supplier j negotiates with retailer first. To see $\breve{\theta}_i^{KS}$, it is easy to check that $\breve{\theta}_i^{KS}$ should lead to the same expression of the first pieces in (F.24) and (F.25), which implies that $\breve{\theta}_i^{KS} = 0.6$. To see $\breve{\theta}_j^{KS}$, we substitute $\theta_i = 0.6$ into (B.3) and obtain

$$\begin{aligned}
&\frac{125}{8}\Phi(w_j)\Big|_{\theta_i=0.6}\\
&= (1-\eta)^2(5+\eta)(25-7\eta)(1-\theta_j)\\
&\quad +2(1-\eta)\big((2+7\theta_j)\eta^3 + (86-25\theta_j)\eta^2 + (90-35\theta_j)\eta - 125(2-\theta_j)\big)w_j\\
&\quad -(1-\eta)\big(9(5-\theta_j)\eta^3 + 5(29-\theta_j)\eta^2 - (205-65\theta_j)\eta - 125(5-\theta_j)\big)w_j^2\\
&\quad -2(125-70\eta^2+9\eta^4)w_j^3 = 0.
\end{aligned}$$

Substituting $\check{w}_j^{KS}$ into the above expression yields

$$\check{\theta}_j^{KS}(\eta)=\frac{\big((1-\eta)(5+\eta)-2(5-\eta^2)\check{w}_j^{KS}\big)\big((1-\eta)(25-7\eta)-2(1-\eta)(25+9\eta)\check{w}_j^{KS}+(25-9\eta^2)(\check{w}_j^{KS})^2\big)}{(1-\eta)\big((1-\eta)(5+\eta)(25-7\eta)-2(25-7\eta)(5-\eta^2)\check{w}_j^{KS}+(125+65\eta-5\eta^2-9\eta^3)(\check{w}_j^{KS})^2\big)}.$$

It is easy to check that $\partial\check{\theta}_j^{KS}(\eta)/\partial\eta > 0$ for $\eta\in(-0.26,1)$ and thus $\check{\theta}_j^{KS}(\eta)\in[0.6,1]$. We conclude part (ii). Finally, we note that parameters with subscript j (i) correspond to those with subscript $seq1$ ($seq2$). □

Proof of Proposition 3.5.3. To see part (i), we can modify (F.1) and (F.2) to derive the trade profits for the supplier and the retailers, respectively, as

$$\Pi_i(w_i,w_j) = \begin{cases} \sum_{i=1}^2 w_i\frac{2(1-w_i)-\eta(1-w_j)}{4-\eta^2} & \frac{\eta}{2}(1-w_j)<1-w_i<\frac{2}{\eta}(1-w_j),\\ 0 & 1-w_i\le\frac{\eta}{2}(1-w_j) \text{ or } 1-w_i\ge\frac{2}{\eta}(1-w_j).\end{cases} \tag{B.4}$$

$$\pi_i(w_i,w_j) = \begin{cases} \frac{(2(1-w_i)-\eta(1-w_j))^2}{(4-\eta^2)^2} & \frac{\eta}{2}(1-w_j)<1-w_i<\frac{2}{\eta}(1-w_j),\\ 0 & 1-w_i\le\frac{\eta}{2}(1-w_j) \text{ or } 1-w_i\ge\frac{2}{\eta}(1-w_j).\end{cases} \tag{B.5}$$

The supplier's disagreement point under simultaneous negotiation with contingency is 0. Their maximum profits are

$$\overline{\Pi}_i(w_j) = \begin{cases} \Pi_i(\frac{2-\eta+2\eta w_j}{4},w_j)=\frac{2-\eta}{8(2+\eta)}+\frac{1}{2}w_j(1-w_j) & w_j<1-\frac{\eta}{2(2+\eta)},\\ \Pi_i(\frac{2w_j-2+\eta}{\eta}+\epsilon,w_j)=\frac{(1-w_j)(2w_j-2+\eta)}{\eta^2}-\epsilon & w_j>1-\frac{\eta}{2(2+\eta)},\end{cases} \tag{B.6}$$

$$\overline{\pi}_i(w_j) = \begin{cases} \pi_i(w^a,w_j)=\frac{(2-\eta+\sqrt{(2-\eta)^2+4(4-\eta^2)(1-w_j)w_j})^2}{4(4-\eta^2)^2} & w_j<\frac{2-\eta}{2},\\ \pi_i(\frac{2w_j-2+\eta}{\eta}+\epsilon,w_j)=\frac{(1-w_j)^2}{\eta^2}-\epsilon & w_j>\frac{2-\eta}{2},\end{cases} \tag{B.7}$$

where $w^a=(1/4)((2-\eta+2\eta w_j)-\sqrt{(2-\eta)^2+4(4-\eta^2)(1-w_j)w_j})$. We first note that any feasible (w_i,w_j) should lead to nonnegative trade surpluses $\Pi_i(w_i,w_j)$ and $\pi_i(w_i,w_j)$ and thus we have $w_i\ge w^a$ and $1/2-\sqrt{1/(2+\eta)}\le w_j\le 1$ in the first pieces in (B.4) and (B.5), respectively.

To derive (B.6), we note that the first piece of $\Pi_i(w_i,w_j)$ in (B.4) is maximized at $w_i^m=(2-\eta+2\eta w_j)/4$ and leads to a maximum value of $\Pi^m=(2-\eta)/(8(2+\eta))+(1/2)w_j(1-w_j)$. For w_i^m be the maxima, we must have $1-w_i^m<(2/\eta)(1-w_j)$ (or $w_j<w^b\equiv 1-\eta/(2(2+\eta)))$. In the case that $w_j>w^b$, it is easy to check that the maxima is $w_i^r=1-(2/\eta)(1-w_j)+\epsilon$ and leads to a maximum value of $\Pi^r=(1-w_j)(2w_j-2+\eta)/\eta^2-\epsilon$, where ϵ is a sufficiently small positive number.

To derive (B.7), we note that the first piece of $\pi_i(w_i, w_j)$ in (B.5) is maximized at $w_i^q = w^a$ and leads to a maximum value of $\pi^q = (2-\eta+\sqrt{(2-\eta)^2+4(4-\eta^2)(1-w_j)w_j})^2/(4(4-\eta^2)^2)$. For w^a be the maxima, we must have $1 - w^a < (2/\eta)(1-w_j)$ (or $w_j < (2-\eta)/2$. In the case that $w_j > (2-\eta)/2$, it is easy to check that the maxima is $w_i^r = 1 - (2/\eta)(1-w_j) + \epsilon$. This leads to the expression of (B.7).

Now we derive the equilibrium prices under the NB and KS solutions. Applying (B.4), (B.5) and $D_i(w_j) = d_i(w_j) = 0$, the Nash product for trade i is

$$\Omega_i(w_i, w_j) = \begin{cases} (\sum_{i=1}^2 w_i \frac{2(1-w_i)-\eta(1-w_j)}{4-\eta^2})^{1-\theta} (\frac{(2(1-w_i)-\eta(1-w_j))^2}{(4-\eta^2)^2})^{\theta} & \frac{\eta}{2} < \frac{1-w_i}{1-w_j} < \frac{2}{\eta}, \\ 0 & \frac{1-w_i}{1-w_j} \le \frac{\eta}{2} \text{ or } \frac{1-w_i}{1-w_j} \ge \frac{2}{\eta}. \end{cases}$$

Setting $\partial \ln \Omega_i(w_i, w_j)/\partial w_i = 0$ in the first case gives

$$(1-\theta)\frac{2(1-2w_i)-\eta(1-2w_j)}{\sum_{i=1}^2 w_i(2(1-w_i)-\eta(1-w_j))} + 2\theta \frac{-2}{2(1-w_i)-\eta(1-w_j)} = 0.$$

This gives $w_i^s = (1/8)\big((3-\theta)(2-\eta)+4\eta w_j - \sqrt{(2-\eta)^2(1+\theta)^2+16\theta(4-\eta^2)(1-w_j)w_j}\big)$. For w_i^s be the best response, we must have $1 - w_i^s < (2/\eta)(1-w_j)$ (or $w_j < w^c \equiv (2-\eta)(4+\eta(1-\theta))/(8-2\eta^2(1-\theta))$). In the case that $w_j > w^c$, it is easy to see that the best response should be $1-(2/\eta)(1-w_j)+\epsilon$. This gives

$$w_i^{NB}(w_j) = \begin{cases} \frac{(3-\theta)(2-\eta)+4\eta w_j - \sqrt{(2-\eta)^2(1+\theta)^2+16\theta(4-\eta^2)(1-w_j)w_j}}{8} & w_j < \frac{(2-\eta)(4+\eta(1-\theta))}{8-2\eta^2(1-\theta)}, \\ \frac{2w_j-2+\eta}{\eta} + \epsilon & w_j > \frac{(2-\eta)(4+\eta(1-\theta))}{8-2\eta^2(1-\theta)}. \end{cases} \quad \text{(B.8)}$$

By symmetry, we have

$$\hat{w}_{sim}^{NB} = \frac{(2-\eta)(1-\theta)}{2(2-\eta+2\theta+\eta\theta)}. \quad \text{(B.9)}$$

We then substitute (B.4), (B.5), (B.6), (B.7) and $D_i(w_j) = d_i(w_j) = 0$ into (3.2) and obtain

$$\frac{(2(1-w_i)-\eta(1-w_j))^2/(4-\eta^2)^2}{\sum_{i=1}^2 w_i(2(1-w_i)-\eta(1-w_j))/(4-\eta^2)} = \begin{cases} \frac{(2-\eta+\sqrt{(2-\eta)^2+4(4-\eta^2)(1-w_j)w_j})^2/4(4-\eta^2)^2}{(2-\eta)/(8(2+\eta))+(1/2)w_j(1-w_j)} & w_j < \frac{2-\eta}{2}, \\ \frac{(1-w_j)^2/\eta^2-\epsilon}{(2-\eta)/(8(2+\eta))+(1/2)w_j(1-w_j)} & \frac{2-\eta}{2} < w_j < \frac{4+\eta}{2(2+\eta)}, \\ \frac{(1-w_j)^2/\eta^2-\epsilon}{(1-w_j)(2w_j-2+\eta)/\eta^2-\epsilon} & w_j > \frac{4+\eta}{2(2+\eta)}. \end{cases}$$

The first piece gives $w_i^m = \big(2(2-\eta)^2+3(2-\eta)(4+3\eta)w_j-2(2+\eta)(6-7\eta)w_j^2-8\eta(2+\eta)w_j^3+(2-\eta+2\eta w_j-\sqrt{2\Delta_5})\sqrt{\Delta_6}\big)/(2(6-3\eta+8(\eta+2)(1-w_j)w_j+2\sqrt{\Delta_6}))$, where $\Delta_5 =$

$(2-\eta+(2+\eta)(1-w_j)w_j)(2-\eta+4(2+\eta)(1-w_j)w_j)+(2-\eta+3(2+\eta)(1-w_j)w_j)\sqrt{\Delta_6}$ and $\Delta_6 = (2-\eta)^2 + 4(4-\eta^2)(1-w_j)w_j$. Note that the first piece has two roots and the best response should be the smaller root.

The second piece gives $w_i^q = \eta w_j/2 + \big((2-\eta)^2(8+8\eta+3\eta^2) - 8(4-\eta^2)(2-\eta^2)w_j + 2(4-3\eta^2)(4-\eta^2)w_j^2 - \sqrt{\Delta_7}\big)/(2(2-\eta)(16+16\eta+5\eta^2) - 8(2+\eta)(8-3\eta^2)w_j + 16(2+\eta)(2-\eta^2)w_j^2) + \epsilon$, where $\Delta_7 = (1-w_j)^3(2-\eta)^2(2+\eta)^3(2-\eta+4(2+\eta)(1-w_j)w_j)(2+(2-\eta^2)(1-2w_j))$. Note that the second piece has two roots and the best response should be the smaller root.

The third piece gives $w_i^p = (2w_j - 2 + \eta)/\eta + \epsilon$.

Combining the above cases leads to

$$
\begin{aligned}
& w_i^{KS}(w_j) \\
&= \begin{cases} \frac{2(2-\eta)^2+3(2-\eta)(4+3\eta)w_j-2(2+\eta)(6-7\eta)w_j^2-8\eta(2+\eta)w_j^3+(2-\eta+2\eta w_j-\sqrt{2\Delta_5})\sqrt{\Delta_6}}{2(6-3\eta+8(\eta+2)(1-w_j)w_j+2\sqrt{\Delta_6})} & w_j < \frac{2-\eta}{2}, \\ \frac{\eta w_j}{2} + \frac{(2-\eta)^2(8+8\eta+3\eta^2)-8(4-\eta^2)(2-\eta^2)w_j+2(4-3\eta^2)(4-\eta^2)w_j^2-\sqrt{\Delta_7}}{2(2-\eta)(16+16\eta+5\eta^2)-8(2+\eta)(8-3\eta^2)w_j+16(2+\eta)(2-\eta^2)w_j^2} + \epsilon & \frac{2-\eta}{2} < w_j < \frac{\eta+4}{2(\eta+2)}, \\ \frac{2w_j-2+\eta}{\eta} + \epsilon & w_j > \frac{\eta+4}{2(\eta+2)}. \end{cases}
\end{aligned}
\tag{B.10}
$$

The first piece and symmetry (i.e., setting $w_j = w_i = w$) give

$$
\begin{aligned}
\Phi(w) &= 16(6-\eta)^2(2+\eta)^2 w^5 - 16(6-\eta)(10-3\eta)(2+\eta)^2 w^4 \\
&\quad +8(4-\eta^2)(28+32\eta-7\eta^2)w^3 + 32(3-\eta)(2-\eta)^2(2+\eta)w^2 \\
&\quad +(2-\eta)^3(2-9\eta)w - (2-\eta)^4.
\end{aligned}
\tag{B.11}
$$

Note that $\Phi(0) = -(2-\eta)^4 < 0$ and $\Phi(1/5) = (4/3125)(2+\eta)(7688 - 6732\eta + 1350\eta^2 - 81\eta^3) > 0$, which implies that there exists at least one root within the range $[0, 1/5]$. Also, note that

$$
\begin{aligned}
\frac{1}{16(2+\eta)} \frac{\partial^2 \Phi(w)}{\partial w^2} &= 20(2+\eta)(6-\eta)^2 w^3 - 12(2+\eta)(6-\eta)(10-3\eta)w^2 \\
&\quad +3(2-\eta)(28+32\eta-7\eta^2)w + 4(3-\eta)(2-\eta)^2 > 0
\end{aligned}
$$

for $w \in [0, 1/5]$. Thus, there exists a unique root within the range $[0, 1/5]$. Let $\hat{w}_{sim}^{KS}$ denote this root. We have $\hat{w}_{sim}^{KS}|_{\eta=0} \approx 0.1299 < 0.13$, $\hat{w}_{sim}^{KS}|_{\eta=1/2} \approx 0.1055$, $\hat{w}_{sim}^{KS}|_{\eta=1} \approx 0.0767 > 0.07$.

Let $\Psi(\eta) \equiv \partial\Phi(w)/\partial\eta$. We have

$$\begin{aligned}\Psi(\eta) &= 32(1-3w-20w^2+32w^3-64w^4+48w^5)\\ &\quad -16(1-2w)^2(3-3w-16w^2+4w^3)\eta\\ &\quad +24(1-2w)^3(1-w+2w^2)\eta^2-4(1-2w)^4(1-w)\eta^3.\end{aligned}$$

Note that $\Psi(\eta)$ is a cubic function with the coefficient of η^3 being negative. It is easy to check that $\Psi(0) > 0$, $\Psi(1/2) > 0$ and $\partial\Psi(\eta)/\partial\eta < 0$. Also, $\Psi(1) > (< 0)$ for $w < (>)0.107$. We deduce that $\Phi(w)$ is increasing in $\eta \in [0, 1/2]$ for $w \in [0.07, 0.13]$ and is increasing in $\eta \in [1/2, 1]$ for $w \in [0.07, 0.107]$. This implies that $\hat{w}^{KS}_{sim}$ is decreasing in η.

The second piece and symmetry give $w = 1/2 \pm 2\sqrt{16+32\eta+18\eta^2+\eta^3-\eta^4}/(16+16\eta+2\eta^2-\eta^3)$, which are not within the range $[(2-\eta)/2, 1-\eta/(2(2+\eta))]$. It is easy to check that the third piece and symmetry give no feasible solution.

Now we consider the equivalent bargaining power. Substituting (B.9) into (B.11) yields

$$\Phi\Big(\frac{(2-\eta)(1-\theta)}{2(2-\eta+2\theta+\eta\theta)}\Big) = \frac{128(2-\eta)^4}{(2(1+\theta)-\eta(1-\theta))^5}\Xi(\theta) = 0,$$

where $\Xi(\theta) = (1-\eta)(2-\eta) + (6+3\eta-5\eta^2)\theta - 2(2+\eta)(2-5\eta)\theta^2 - 2(2+\eta)(6+5\eta)\theta^3 + 5(2+\eta)^2\theta^4 - (2+\eta)^2\theta^5$. Note that $\Xi(1/2) = (1/32)(6-\eta)^2 > 0$ and $\Xi(3/5) = -(2/3125)(286+111\eta-16\eta^2) < 0$, which implies that there exists at least one root within the range $[1/2, 3/5]$. Because $\hat{w}^{NB}_{sim}$ is (strictly) decreasing in θ, there must exist a unique root within the range. Let $\hat{\theta}^{KS}_{sim}(\eta)$ denote this root. We note that $\partial\Xi(\theta)/\partial\eta = 2\eta(1-\theta)^5 - 3 + 3\theta + 16\theta^2 - 32\theta^3 + 20\theta^4 - 4\theta^5 < 0$ for $\theta \in [0, 3/5]$. This implies that $\Xi(\theta)$ is decreasing in η, which is equivalent to $\hat{\theta}^{KS}_{sim}(\eta)$ being decreasing in η. Finally, note that $\hat{\theta}^{KS}_{sim}(0) < 0.59$ and $\hat{\theta}^{KS}_{sim}(1) > 0.57$. This concludes part (i).

To see part (ii), we can modify (F.19) and (F.20) to derive the trade profits for the suppliers and the retailer, respectively as

$$\Pi_i(w_i, w_j) = \begin{cases} w_i \frac{(1-w_i)-\eta(1-w_j)}{2(1-\eta^2)} & \eta(1-w_j) < 1-w_i < \frac{1}{\eta}(1-w_j), \\ 0 & 1-w_i \leq \eta(1-w_j) \text{ or } 1-w_i \geq \frac{1}{\eta}(1-w_j). \end{cases} \quad (B.12)$$

$$\pi_i(w_i, w_j) = \begin{cases} \sum_{i=1}^{2} \frac{(1-w_i)((1-w_i)-\eta(1-w_j))}{4(1-\eta^2)} & \eta(1-w_j) < 1-w_i < \frac{1}{\eta}(1-w_j), \\ 0 & 1-w_i \leq \eta(1-w_j) \text{ or } 1-w_i \geq \frac{1}{\eta}(1-w_j). \end{cases} \quad (B.13)$$

The retailer's disagreement point under simultaneous negotiation with contingency is 0. We can modify (F.22) and (F.23) to derive the suppliers and retailer's maximum profits as

$$\overline{\Pi}_i(w_j) = \begin{cases} \Pi_i(\frac{1-\eta+\eta w_j}{2}, w_j) = \frac{(1-\eta(1-w_j))^2}{8(1-\eta^2)} & w_j < \frac{(2+\eta)(1-\eta)}{2-\eta^2}, \\ \Pi_i(\frac{w_j-1+\eta}{\eta}+\epsilon, w_j) = \frac{(1-w_j)(w_j-1+\eta)}{2\eta^2} - \epsilon & w_j > \frac{(2+\eta)(1-\eta)}{2-\eta^2}, \end{cases} \quad (B.14)$$

$$\overline{\pi}_i(w_j) = \begin{cases} \pi_i(0, w_j) = \frac{(1-w_j)^2 - 2\eta(1-w_j)+1}{4(1-\eta^2)} & w_j < 1-\eta, \\ \pi_i(\frac{w_j-1+\eta}{\eta}+\epsilon, w_j) = \frac{(1-w_j)^2}{4\eta^2} - \epsilon & w_j > 1-\eta. \end{cases} \quad (B.15)$$

Now we are ready to the equilibrium prices under the NB and KS solutions. Applying (B.12), (B.13) and $D_i(w_j) = d_i(w_j) = 0$, the Nash product for trade i is

$$\Omega_i(w_i, w_j) = \begin{cases} (w_i \frac{(1-w_i)-\eta(1-w_j)}{2(1-\eta^2)})^{1-\theta} (\sum_{i=1}^{2} \frac{(1-w_i)((1-w_i)-\eta(1-w_j))}{4(1-\eta^2)})^{\theta} & \eta < \frac{1-w_i}{1-w_j} < \frac{1}{\eta}, \\ 0 & \frac{1-w_i}{1-w_j} \leq \eta \text{ or } \frac{1-w_i}{1-w_j} \geq \frac{1}{\eta}. \end{cases}$$

Setting $\partial \ln \Omega_i(w_i, w_j)/\partial w_i = 0$ in the first case gives

$$(1-\theta)\frac{1-2w_i-\eta(1-w_j)}{w_i(1-w_i-\eta(1-w_j))} + \theta \frac{-2(1-w_i-\eta(1-w_j))}{\sum_{i=1}^{2}(1-w_i)((1-w_i)-\eta(1-w_j))} = 0.$$

By symmetry, we have

$$\check{w}_{sim}^{NB} = \frac{(1-\eta)(1-\theta)}{2-\eta-\theta}. \quad (B.16)$$

We then substitute (B.12), (B.13), (B.14), (B.15) and $D_i(w_j) = d_i(w_j) = 0$ into (3.2) and obtain

$$\frac{\sum_{i=1}^{2}(1-w_i)((1-w_i)-\eta(1-w_j))/(4(1-\eta^2))}{w_i((1-w_i)-\eta(1-w_j))/(2(1-\eta^2))} = \begin{cases} \frac{((1-w_j)^2-2\eta(1-w_j)+1)/(4(1-\eta^2))}{(1-\eta(1-w_j))^2/(8(1-\eta^2))} & w_j < 1-\eta, \\ \frac{(1-w_j)^2/(4\eta^2)-\epsilon}{(1-\eta(1-w_j))^2/(8(1-\eta^2))} & 1-\eta < w_j < \frac{(2+\eta)(1-\eta)}{2-\eta^2}, \\ \frac{(1-w_j)^2/(4\eta^2)-\epsilon}{(1-w_j)(w_j-1+\eta)/(2\eta^2)-\epsilon} & w_j > \frac{(2+\eta)(1-\eta)}{2-\eta^2}. \end{cases}$$

The first piece and symmetry (i.e., setting $w = w_j = w_i$) give

$$\Gamma(w) = (2+\eta^2)w^3 - (4-6\eta+3\eta^2)w^2 + (1-\eta)(5-3\eta)w - (1-\eta)^2. \quad \text{(B.17)}$$

Note that $\Gamma(w)$ is a cubic function with the coefficient of w^3 being positive. For $\eta = 1$, it is easy to check that $\check{w}_j^{KS} = 0$. For $\eta < 1$, we have $\Gamma(w) = -(1-\eta)^2 < 0$, $\Gamma((2\eta - 3 + \sqrt{9-8\eta})/(2\eta)) = (1/\eta^3)(-27 + 27\eta + 5\eta^2 - 7\eta^3 + (9 - 5\eta - 3\eta^2 + \eta^3)\sqrt{9-8\eta}) > 0$ and $\partial\Gamma(w)/\partial w > 0$. This implies that there exists a unique root within the range $[0, (2\eta - 3 + \sqrt{9-8\eta})/(2\eta)]$. Let $\check{w}_{sim}^{KS}$ denote this root. We note that $\partial\Gamma(w)/\partial\eta = 2(1-w)(1-\eta-(3-2\eta)w - \eta w^2) > 0$ for $w < (2\eta - 3 + \sqrt{9-8\eta})/(2\eta)$, which is equivalent to $\check{w}_{sim}^{KS}$ being decreasing in η.

The second piece and symmetry give $w = (1-\eta^2-\eta^3+\eta^4 \pm \sqrt{1-4\eta^2+2\eta^3+3\eta^4-2\eta^5})$ $/(2-2\eta^2+\eta^4)$. The smaller root is not within the range $[1-\eta, (2+\eta)(1-\eta)/(2-\eta^2)]$ and the bigger root leads to a Pareto-dominated profit allocation. It is easy to check that the third piece and symmetry give no feasible solution.

Now we consider the equivalent bargaining power. Substituting (B.16) into (B.17) yields

$$\Gamma\Big(\frac{(1-\eta)(1-\theta)}{2-\eta-\theta}\Big) = \frac{(1-\theta)^3}{(2-\eta-\theta)^3}\Big[6 - 2(7+2\eta)\theta + 9(1+\eta)\theta^2 - (2+\eta)(1+\eta)\theta^3\Big] = 0.$$

Setting the second term being zero gives $\eta(\theta) = (-4+9\theta-3\theta^2-(1-\theta)\sqrt{16-16\theta+\theta^2})/(2\theta^2)$. Note that $\partial\eta(\theta)/\partial\theta > 0$ for $\theta \in (0,1)$ and $\eta(\theta) \in (-\infty, 1]$, which is equivalent to $\check{\theta}_{sim}^{KS}(\eta)$ being increasing in η. Finally, note that $\check{\theta}_{sim}^{KS}(0) > 0.68$ and $\check{\theta}_{sim}^{KS}(1) = 1$. We conclude part (ii). $\square$

Proof of Proposition 3.6.1. To see part (i), we first consider the case when the KS solution is applied. By (F.1), (F.2), Lemma F.1.3 and Proposition 3.5.2, we have

$$\hat{\Pi}_{sim}^{KS} = \frac{8}{25(2+\eta)} \le \frac{(1-\hat{w}_{seq1}^{KS})\hat{w}_{seq1}^{KS}}{2} + \frac{2(2-\eta)}{25(2+\eta)} = \hat{\Pi}_{seq}^{KS},$$
$$\hat{\pi}_{sim}^{KS} = \hat{\pi}_{seq2}^{KS} = \frac{16}{25(2+\eta)^2} \ge \frac{(10+\eta-5(2+\eta)\hat{w}_{seq1}^{KS})^2}{100(2+\eta)^2} = \hat{\pi}_{seq1}^{KS}.$$

The inequalities follow because $\hat{w}_{seq1}^{KS} \in [0.2, 0.27)$ from the proof of Proposition 3.5.2.

Now we consider the case when the NB solution is applied. Similarly, we have

$$\hat{\Pi}^{NB}_{sim} = \frac{(1-\theta^2)}{2(2+\eta)} \leq \frac{(1-\hat{w}^{NB}_{seq1})\hat{w}^{NB}_{seq1}}{2} + \frac{(2-\eta)(1-\theta^2)}{8(2+\eta)} = \hat{\Pi}^{NB}_{seq},$$

$$\hat{\pi}^{NB}_{sim} = \hat{\pi}^{NB}_{seq2} = \frac{(1+\theta)^2}{4(2+\eta)^2} \geq \frac{(4+\eta(1-\theta)-2(2+\eta)\hat{w}^{NB}_{seq1})^2}{16(2+\eta)^2} = \hat{\pi}^{NB}_{seq1}.$$

To see the inequalities, we can set $\theta_i = \theta_j = \theta$ into (B.1) and obtain

$$\hat{w}^{NB}_{seq1} = \frac{6-2\theta+\eta(2-\theta+\theta^2) - \sqrt{(2-\eta\theta)^2(1+\theta)^2 + 4\eta\theta((4+\eta)\theta^2+4\theta-\eta)}}{4(\eta+2)}.$$

It is easy to check that $\partial\hat{w}^{NB}_{seq1}/\partial\theta \leq 0$ and $\partial\hat{w}^{NB}_{seq1}/\partial\eta \geq 0$ and thus $\hat{w}^{NB}_{seq1} \geq \hat{w}^{NB}_{seq1}|_{\eta=0} = (1-\theta)/2$. The inequalities then follow. We conclude part (i).

To see part (ii), we first consider the case when the KS solution is applied. By (F.19), (F.20), Lemma F.2.3 and Proposition 3.5.2, we can compute the suppliers' and retailer's profits, respectively, as

$$\begin{aligned}
\check{\Pi}^{KS}_{sim} &= \frac{2(1-\eta)}{(1+\eta)(5-\eta)^2}, \\
\check{\pi}^{KS}_{sim} &= \frac{8}{(1+\eta)(5-\eta)^2}, \\
\check{\Pi}^{KS}_{seq1} &= \frac{\check{w}^{KS}_{seq1}((5-\eta^2)(1-\check{w}^{KS}_{seq1})-4\eta)}{10(1-\eta^2)}, \\
\check{\Pi}^{KS}_{seq2} &= \frac{2(1-\eta+\eta\check{w}^{KS}_{seq1})^2}{25(1-\eta^2)}, \\
\check{\pi}^{KS}_{seq} &= \frac{(25-9\eta^2)(1-\check{w}^{KS}_{seq1})^2 - 32\eta(1-\check{w}^{KS}_{seq1})+16}{100(1-\eta^2)}.
\end{aligned}$$

Comparing $\check{\Pi}^{KS}_{sim}$ and $\check{\Pi}^{KS}_{seq1}$, we obtain

$$\check{\Pi}^{KS}_{sim} - \check{\Pi}^{KS}_{seq1} = \frac{1}{10(1-\eta^2)(5-\eta)^2}\Xi_1(\eta),$$

where $\Xi_1(\eta) = 20(1-\eta)^2 - (5-\eta)^2\check{w}^{KS}_{seq1}((5-\eta^2)(1-\check{w}^{KS}_{seq1})-4\eta)$. Note that $\Xi_1(\eta)$ is a quadratic function with the coefficient of ${\check{w}^{KS}_{seq1}}^2$ being positive. $\Xi_1(\eta)=0$ has two roots, $w^{(1)} = (1-\eta)/(5-\eta)$ and $w^{(2)} = 20(1-\eta)/((5-\eta)(5-\eta^2))$. We have $w^{(1)} \leq \check{w}^{KS}_{seq1} \leq (1-\eta)(5+\eta)/(2(5-\eta^2)) \leq w^{(2)}$ and thus $\Xi_1(\eta) \leq 0$.

Comparing $\check{\Pi}^{KS}_{sim}$ and $\check{\Pi}^{KS}_{seq2}$, we obtain

$$\check{\Pi}^{KS}_{sim} - \check{\Pi}^{KS}_{seq2} = \frac{2\eta}{25(1-\eta^2)(5-\eta)^2}\Xi_2(\eta),$$

where $\Xi_2(\eta) = (10-\eta)(1-\eta)^2 - (5-\eta)^2 \check{w}_{seq1}^{KS}(2(1-\eta)+\eta\check{w}_{seq1}^{KS})$. Note that $\Xi_2(\eta)$ is a quadratic function with the coefficient of $\check{w}_{seq1}^{KS\,2}$ being negative. $\Xi_2(\eta) = 0$ has two roots, $w^{(1)} = (1-\eta)/(5-\eta)$ and $w^{(3)} = -(1-\eta)(10-\eta)/(\eta(5-\eta))$. We have $w^{(3)} \leq 0 \leq w^{(1)} \leq \check{w}_{seq1}^{KS}$ and thus $\Xi_2(\eta) \leq 0$.

Comparing $\check{\Pi}_{seq1}^{KS}$ and $\check{\Pi}_{seq2}^{KS}$, we obtain

$$\check{\Pi}_{seq1}^{KS} - \check{\Pi}_{seq2}^{KS} = \frac{1}{50(1-\eta^2)}\Xi_3(\eta),$$

where $\Xi_3(\eta) = -4(1-\eta)^2 + (25-\eta)(3-\eta)\check{w}_{seq1}^{KS} - (25-\eta^2)\check{w}_{seq1}^{KS\,2}$. Note that $\Xi_3(\eta)$ is a quadratic function with the coefficient of $\check{w}_{seq1}^{KS\,2}$ being negative. $\Xi_3(\eta) = 0$ has two roots, $w^{(1)} = (1-\eta)/(5-\eta)$ and $w^{(4)} = 4(1-\eta)/(5+\eta)$. We have $w^{(1)} \leq \check{w}_{seq1}^{KS}$ and $\check{w}_{seq1}^{KS} < (>) w^{(4)}$ for $\eta < (>) \bar{\eta}^{KS} \approx 0.994576$ and thus $\Xi_3(\eta) \geq (\leq) 0$ for $\eta \leq (\geq) \bar{\eta}^{KS}$.

Comparing $\check{\pi}_{sim}^{KS}$ and $\check{\pi}_{seq}^{KS}$, we obtain

$$\check{\pi}_{sim}^{KS} - \check{\pi}_{seq}^{KS} = \frac{1}{100(1-\eta^2)(5-\eta)^2}\Xi_4(\eta),$$

where $\Xi_4(\eta) = -(1-\eta)^2(225+40\eta-9\eta^2) + 2(5-\eta)^2(25-16\eta-9\eta^2)\check{w}_{seq1}^{KS} - (5-\eta)^2(25-9\eta^2)\check{w}_{seq1}^{KS\,2}$. Note that $\Xi_4(\eta)$ is a quadratic function with the coefficient of $\check{w}_{seq1}^{KS\,2}$ being negative. $\Xi_4(\eta) = 0$ has two roots, $w^{(1)} = (1-\eta)/(5-\eta)$ and $w^{(5)} = (225-185\eta-49\eta^2+9\eta^3)/(125-25\eta-45\eta^2+9\eta^3)$. We have $w^{(1)} \leq \check{w}_{seq1}^{KS} \leq w^{(5)}$ and thus $\Xi_4(\eta) \geq 0$.

We now consider the case when the NB solution is applied. Similarly, we can compute the suppliers' and retailer's profits, respectively, as

$$\begin{aligned}
\check{\Pi}_{sim}^{NB} &= \frac{(1-\eta)(1-\theta^2)}{2(1+\eta)(2-\eta(1-\theta))^2}, \\
\check{\pi}_{sim}^{NB} &= \frac{(1+\theta)^2}{2(1+\eta)(2-\eta(1-\theta))^2}, \\
\check{\Pi}_{seq1}^{NB} &= \frac{\check{w}_{seq1}^{NB}((2-\eta^2(1-\theta))(1-\check{w}_{seq1}^{NB}) - \eta(1+\theta))}{4(1-\eta^2)}, \\
\check{\Pi}_{seq2}^{NB} &= \frac{(1-\theta^2)(1-\eta+\eta\check{w}_{seq1}^{NB})^2}{8(1-\eta^2)}, \\
\check{\pi}_{seq}^{NB} &= \frac{(1-\check{w}_{seq1}^{NB})^2}{4} + \frac{(1+\theta)^2(1-\eta+\eta\check{w}_{seq1}^{NB})^2}{16(1-\eta^2)}.
\end{aligned}$$

We first note that $w^l \equiv (1-\theta)(1-\eta)/(2-\eta(1-\theta)) \leq \check{w}_{seq1}^{NB} \leq (1-\eta)(2+\eta(1-\theta))/(2(2-\eta^2(1-\theta))) \equiv w^u$. To see the inequality, we can set $\theta_i = \theta_j = \theta$ in (B.3) and obtain $\Psi(w^l) \geq 0$ and $\Psi(w^u) \leq 0$.

Comparing $\check{\Pi}_{sim}^{NB}$ and $\check{\Pi}_{seq1}^{NB}$, we obtain

$$\check{\Pi}_{sim}^{NB} - \check{\Pi}_{seq1}^{NB} = \frac{1}{4(1-\eta^2)(2-\eta(1-\theta))^2}\Xi_1(\eta,\theta),$$

where $\Xi_1(\eta,\theta) = 2(1-\eta)^2(1-\theta^2) - (2-\eta(1-\theta))^2\check{w}_{seq1}^{NB}((2-\eta^2(1-\theta))(1-\check{w}_{seq1}^{NB}) - \eta(1+\theta))$. Note that $\Xi_1(\eta,\theta)$ is a quadratic function with the coefficient of ${\check{w}_{seq1}^{NB}}^2$ being positive. $\Xi_1(\eta,\theta) = 0$ has two roots, $w^{(a)} = (1-\theta)(1-\eta)/(2-\eta(1-\theta))$ and $w^{(b)} = 2(1-\eta)(1+\theta)/(4-2\eta(1+\eta)(1-\theta)+\eta^3(1-\theta)^2)$. We have $w^{(a)} = w^l \leq \check{w}_{seq1}^{NB} \leq w^u \leq w^{(b)}$ and thus $\Xi_1(\eta,\theta) \leq 0$.

Comparing $\check{\Pi}_{sim}^{NB}$ and $\check{\Pi}_{seq2}^{NB}$, we obtain

$$\check{\Pi}_{sim}^{NB} - \check{\Pi}_{seq2}^{NB} = \frac{(1-\theta^2)}{8(1-\eta^2)(2-\eta(1-\theta))^2}\Xi_2(\eta,\theta),$$

where $\Xi_2(\eta,\theta) = 4(1-\eta)^2 - (2-\eta(1-\theta))^2(1-\eta+\eta\check{w}_{seq1}^{NB})^2$. Note that $\Xi_2(\eta,\theta)$ is a quadratic function with the coefficient of ${\check{w}_{seq1}^{NB}}^2$ being negative. $\Xi_2(\eta,\theta) = 0$ has two roots, $w^{(a)} = (1-\theta)(1-\eta)/(2-\eta(1-\theta))$ and $w^{(c)} = -(1-\eta)(4-\eta(1-\theta))/(\eta(2-\eta(1-\theta)))$. We have $w^{(c)} \leq 0 \leq w^{(a)} \leq \check{w}_{seq1}^{NB}$ and thus $\Xi_2(\eta,\theta) \leq 0$.

Comparing $\check{\Pi}_{seq1}^{NB}$ and $\check{\Pi}_{seq2}^{NB}$, we obtain

$$\check{\Pi}_{seq1}^{NB} - \check{\Pi}_{seq2}^{NB} = \frac{1}{8(1-\eta^2)}\Xi_3(\eta,\theta),$$

where $\Xi_3(\eta,\theta) = 2\check{w}_{seq1}^{NB}((2-\eta^2(1-\theta))(1-\check{w}_{seq1}^{NB}) - \eta(1+\theta)) - (1-\theta^2)(1-\eta+\eta\check{w}_{seq1}^{NB})^2$. Note that $\Xi_3(\eta,\theta)$ is a quadratic function with the coefficient of ${\check{w}_{seq1}^{NB}}^2$ being negative. $\Xi_3(\eta,\theta) = 0$ has two roots, $w^{(a)} = (1-\theta)(1-\eta)/(2-\eta(1-\theta))$ and $w^{(d)} = (1+\theta)(1-\eta)/(2+\eta(1-\theta))$. It is easy to check that $w^{(a)} \geq (\leq) w^{(d)}$ for $\eta \geq (\leq) 2\theta/(1-\theta)$. We have two cases depending on the value of η.

Case 1: $\eta \geq 2\theta/(1-\theta)$. We have $w^{(d)} \leq w^{(a)} \leq \check{w}_{seq1}^{NB}$ and thus $\Xi_3(\eta,\theta) \leq 0$.

Case 2: $\eta < 2\theta/(1-\theta)$. By (B.3), we have

$$\frac{(2+\eta(1-\theta))^3}{(1-\eta)^2(1-\theta)^2}\Psi(w^{(d)}) = \Gamma_0 + \Gamma_1\eta + \Gamma_2\eta^2 + \Gamma_3\eta^3 + \Gamma_4\eta^4 + \Gamma_5\eta^5 \equiv G(\eta,\theta),$$

where $\Gamma_0 = -32\theta$, $\Gamma_1 = 8(2 - 19\theta + \theta^3)$, $\Gamma_2 = 4(19 - 68\theta - 2\theta^2 + 4\theta^3 - \theta^4)$, $\Gamma_3 = 4(1-\theta)(35 - 13\theta - 7\theta^2 + \theta^3)$, $\Gamma_4 = 113 - 29\theta + 50\theta^2 + 30\theta^3 - 3\theta^4 - \theta^5$ and $\Gamma_5 = (3+\theta)(11 + 6\theta^2 - \theta^4)$. We note that $G(0,\theta) = \Gamma_0 \leq 0$, $\partial G(\eta,\theta)/\partial\eta|_{\eta=0} = \Gamma_1 \geq (\leq)0$ for $\theta \geq (\leq)0.105$, $\partial^2 G(\eta,\theta)/\partial\eta^2|_{\eta=0} = 2\Gamma_2 \geq (\leq)0.278$ and $\partial^3 G(\eta,\theta)/\partial\eta^3 = 6\Gamma_3 + 24\Gamma_4\eta + 60\Gamma_5\eta^2 > 0$. This implies that $\partial G(\eta,\theta)/\partial\eta$ is convex in $\eta \in [0,1]$. Moreover, the sign of $\partial G(\eta,\theta)/\partial\eta$ (i) is always positive or (ii) changes at most once and the change is from negative to positive. This further implies that $G(\eta,\theta) = 0$ has at most one root within the range $\eta \in [0,1]$. Let $\bar{\eta}^{NB}$ denote this root. In the case that no root exists within the range, we set $\bar{\eta}^{NB} = 1$. Similarly, we can show that $\partial G(\eta,\theta)/\partial\theta < 0$ and thus $\bar{\eta}^{NB}$ is increasing in θ. Thus, $\Xi_3(\eta,\theta) \geq (\leq)0$ for $\eta \leq (\geq)\bar{\eta}^{NB}$.

Comparing $\breve{\pi}_{sim}^{NB}$ and $\breve{\pi}_{seq}^{NB}$, we obtain

$$\breve{\pi}_{sim}^{NB} - \breve{\pi}_{seq}^{NB} = \frac{1}{16(1-\eta^2)(2-\eta(1-\theta))^2}\Xi_4(\eta,\theta),$$

where $\Xi_4(\eta,\theta) = 8(1+\theta)^2(1-\eta) - (2-\eta(1-\theta))^2(4(1-\eta)^2(1-\breve{w}_{seq1}^{NB})^2 + (1+\theta)^2(1-\eta+\eta\breve{w}_{seq1}^{NB})^2)$. Note that $\Xi_4(\eta,\theta)$ is a quadratic function with the coefficient of $\breve{w}_{seq1}^{NB\,2}$ being negative. $\Xi_4(\eta,\theta) = 0$ has two roots, $w^{(a)} = (1-\theta)(1-\eta)/(2-\eta(1-\theta))$ and $w^{(e)} = (1-\eta)((4-\eta^2(1-\theta)^2)(3+\theta) + 4\eta(1-\theta^2))/(8 - 4\eta(1-\theta) - 2\eta^2(3+\theta)(1-\theta) + \eta^3(1-\theta)^2(3+\theta))$. We have $w^{(a)} \leq \breve{w}_{seq1}^{NB} \leq w^u \leq w^{(e)}$ and thus $\Xi_4(\eta,\theta) \geq 0$. This concludes part (ii). □

Proof of Proposition 3.6.2. We first consider the case when the NB solution is applied. From the proof of Lemma 3.5.2, it is easy to check that $\hat{\Pi}_{sim}^{NB} \geq (\leq)\hat{\Pi}_{seq}^{NB}$ and $\hat{\pi}_{seq1}^{NB} \geq \hat{\pi}_{sim}^{NB} \geq \hat{\pi}_{seq2}^{NB}$ ($\hat{\pi}_{seq1}^{NB} \leq \hat{\pi}_{sim}^{NB} \leq \hat{\pi}_{seq2}^{NB}$) for $\hat{w}_{sim}^{NB} \geq (\leq)\hat{w}_{seq1}^{NB}$. Thus, it suffices to compare the difference between $\hat{w}_{sim}^{NB}$ and $\hat{w}_{seq1}^{NB}$. Note that the Nash product for the first trade in sequential negotiation is

$$\Omega_1(w_1) = \Pi_1(w_1, \hat{w}_2^{NB}(w_1))^{1-\theta}\pi_1(w_1, \hat{w}_2^{NB}(w_1))^{\theta},$$

where $\Pi_1(\cdot,\cdot)$ and $\pi_1(\cdot,\cdot)$ are respectively first pieces in (B.4) and (B.5), and $\hat{w}_2^{NB}(\cdot)$ is given by (B.8).

Setting $\partial \ln \Omega_1(w_1)/\partial w_1 = 0$ gives

$$\begin{aligned} 0 &= \frac{\partial}{\partial w_1}\Big[(1-\theta)\ln \Pi_1(w_1,w_2) + \theta \ln \pi_1(w_1,w_2)\Big] \\ &\quad + \frac{\partial}{\partial w_2}\Big[(1-\theta)\ln \Pi_1(w_1,w_2) + \theta \ln \pi_1(w_1,w_2)\Big] \frac{\partial \hat{w}_2^{NB}(w_1)}{\partial w_1}\bigg|_{w_2=\hat{w}_2^{NB}(w_1)} \equiv \Gamma_1 + \Gamma_2. \end{aligned}$$

We first note that $\Gamma_1 \geq (\leq) 0$ for $w_1 \leq (\geq) \hat{w}_{sim}^{NB}$. Also note that the first term in Γ_2 is above zero because $\Pi_1(w_1,w_2)$ and $\pi_1(w_1,w_2)$ are increasing in w_2 for $w_2 \leq 1/2$ by Lemma 3.5.2. The second term in Γ_2 gives

$$\frac{\partial \hat{w}_2^{NB}(w_1)}{\partial w_1}\bigg|_{w_1=\hat{w}_{sim}^{NB}} = \frac{\eta}{2} + \frac{4(2+\eta)\theta^2}{\eta(1-\theta)^2 + 2(\theta^2 - 4\theta - 1)} \geq 0$$

for $\eta > (1+5\theta - \sqrt{(1+\theta)(1+9\theta)})/(1-\theta)$ and $0 < \theta < (5+2\sqrt{13})/27$. In the case that $\theta > (5+2\sqrt{13})/27$, we have $(1+5\theta-\sqrt{(1+\theta)(1+9\theta)})/(1-\theta) > 1$. Thus, we have $\bar{\eta}^{NB} = ((1+5\theta - \sqrt{(1+\theta)(1+9\theta)})/(1-\theta)) \wedge 1$ and $\bar{\eta}^{NB}$ is increasing in θ. This implies that $\hat{w}_{sim}^{NB} \leq (\geq) \hat{w}_{seq1}^{NB}$ for $\eta \geq (\leq) \bar{\eta}^{NB}$.

Now we consider the case when the KS solution is applied. We focus on the case when no extension of Pareto profit allocation set is needed (see the detailed discussion in Appendix E). Thus, we restrict the range of η within $[0, 0.39]$. Similar to that under the NB solution, the profit comparison boils down to compare the difference between $\hat{w}_{sim}^{KS}$ and $\hat{w}_{seq1}^{KS}$.

Let $\Pi_1(w_1) \equiv \Pi_1(w_1, \hat{w}_2^{KS}(w_1))$ and $\pi_1(w_1) \equiv \pi_1(w_1, \hat{w}_2^{KS}(w_1))$, where $\hat{w}_2^{KS}(\cdot)$ is given by (B.10). By Proposition 3.5.3, we have the supplier and retailer 1's maximum profits as $\overline{\Pi}_1 = \Pi_1(1/2)$ and $\overline{\pi}_1 = \pi_1(1/2 - \sqrt{1/(2+\eta)})$, respectively. By (3.2), we note that $\hat{w}_{seq1}^{KS}$ and $\hat{w}_{sim}^{KS}$ should satisfy

$$\frac{\pi_1(w_1)}{\Pi_1(w_1)} = \frac{\overline{\pi}_1}{\overline{\Pi}_1} \quad \text{and} \quad \frac{\pi_1(w_1)}{\Pi_1(w_1)} = \frac{\overline{\pi}_1(\hat{w}_2^{KS}(w_1))}{\overline{\Pi}_1(\hat{w}_2^{KS}(w_1))},$$

respectively. Recall that $\overline{\Pi}_1(\cdot)$ and $\overline{\pi}_1(\cdot)$ are respectively first pieces in (B.6) and (B.7). We first note that $\pi_1(w_1)/\Pi_1(w_1)$ is decreasing in w_1. We also note that $\overline{\pi}_1(\hat{w}_2^{KS}(w_1))/\overline{\Pi}_1(\hat{w}_2^{KS}(w_1))$ is increasing in w_1 for $w_1 \leq 0.13$ because (i) $\overline{\pi}_1(w_2)/\overline{\Pi}_1(w_2)$ is decreasing in w_2 for $w_2 \leq 1/2$; (ii) $\hat{w}_2^{KS}(w_1)$ is decreasing in w_1 and $\hat{w}_2^{KS}(w_1) \leq 1/2$

for $w_1 \leq 0.13$. Finally, note that $\overline{\pi}_1/\overline{\Pi}_1 > \overline{\pi}_1(\hat{w}_2^{KS}(w_1))/\overline{\Pi}_1(\hat{w}_2^{KS}(w_1))|_{w_1=0.13}$ and thus we have $\hat{w}_{seq1}^{KS} < \hat{w}_{sim}^{KS} < \hat{w}_{seq2}^{KS}$. $\square$

Proof of Corollary 3.6.1. To see part (i), we can compute the supplier's and retailer's profits as

$$\Pi(w) = wq^*(w) = w\big(\frac{1-w}{k+1}\big)^{1/k} \quad \text{and} \quad \pi(w) = q^*(w)(p^*(w)-w) = k\big(\frac{1-w}{k+1}\big)^{1/k+1},$$

and their maximum profits are

$$\overline{\Pi} = \Pi\big(\frac{k}{k+1}\big) = k\big(\frac{1}{k+1}\big)^{2/k+1} \quad \text{and} \quad \overline{\pi} = \pi(0) = k\big(\frac{1}{k+1}\big)^{1/k+1}.$$

Substituting the above expressions into (3.1) and (3.2) gives $w^{NB} = k(1-\theta)/(k+1)$ and $w^{KS} = k/((k+1)^{1/k+1}+k)$, respectively. Setting $w^{NB} = w^{KS}$ gives $\theta^{KS}(k) = 1-(k+1)/((k+1)^{1/k+1}+k)$. This concludes part (i).

To see part (ii), we can compute the supplier's and retailer's profits as

$$\Pi(w) = wq^*(w) = w\exp\big(\frac{1-k-w}{k}\big) \quad \text{and} \quad \pi(w) = q^*(w)(p^*(w)-w) = k\exp\big(\frac{1-k-w}{k}\big),$$

and their maximum profits are

$$\overline{\Pi} = \Pi(k) = k\exp\big(\frac{1-2k}{k}\big) \quad \text{and} \quad \overline{\pi} = \pi(0) = k\exp\big(\frac{1-k}{k}\big).$$

Substituting the above expressions into (3.1) and (3.2) gives $w^{NB} = k(1-\theta)$ and $w^{KS} = k/e$, respectively. Setting $w^{NB} = w^{KS}$ gives $\theta^{KS}(k) = 1-1/e$. This concludes part (ii).

To see part (iii), we can compute the supplier's and retailer's profits as

$$\Pi(w) = (w-c)q^*(w) = (w-c)\big(\frac{1-k}{w}\big)^{1/k}, \quad \text{and} \quad \pi(w) = q^*(w)(p^*(w)-w) = k\big(\frac{1-k}{w}\big)^{1/k-1},$$

and their maximum profits are

$$\overline{\Pi} = \Pi\big(\frac{c}{1-k}\big) = \frac{kc}{1-k}\big(\frac{(1-k)^2}{c}\big)^{1/k} \quad \text{and} \quad \overline{\pi} = \pi(c) = k\big(\frac{1-k}{c}\big)^{1/k-1}.$$

Substituting the above expressions into (3.1) and (3.2) gives $w^{NB} = c(1-k\theta)/(1-k)$ and $w^{KS} = c/(1-k(1-k)^{1/k-1})$. Setting $w^{NB} = w^{KS}$ gives $\theta^{KS}(k) = (1-(1-k)^{1/k-1})/(1-k(1-k)^{1/k-1})$. This concludes part (iii). $\square$

Proof of Proposition 3.6.3. There are eight cases to analyze, depending on the industry structures, contingency terms and negotiation sequence. We present the detailed analysis for one case and omit the others as they follow in the similar way. Specifically, we focus on the simultaneous bargaining without contingency in the one-to-two channel. We consider the negotiation in unit i for a given (v_j, F_j) from unit j. The supplier's and retailer i's profits are

$$\Pi_i(v_i, v_j, F_i, F_j) = R_i(v_i, v_j, F_i) + R_j(v_j, v_i, F_j) \text{ and } \pi_i(v_i, v_j, F_i) = r_i(v_i, v_j, F_i).$$

The retailer i has zero disagreement point (i.e., $d_i(v_j, F_j) = 0$) and the supplier's disagreement point is $D_i(v_j, F_j) = R_j(v_j, a - \eta(a - v_j)/2, F_j)$. Their maximum profits are

$$\begin{aligned}
\overline{\Pi}(v_j, F_j) &= \max\{\Pi_i(v_i, v_j, F_i, F_j) : \pi_i(v_i, v_j, F_i) \geq d_i(v_j, F_j)\},\\
\overline{\pi}(v_j, F_j) &= \max\{\pi_i(v_i, v_j, F_i) : \Pi_i(v_i, v_j, F_i, F_j) \geq D_i(v_j, F_j)\}.
\end{aligned}$$

We note that though trade parties' profits and disagreement points may depend on F_j, their total trade surplus $\Pi_i(v_i, v_j, F_i, F_j) + \pi_i(v_i, v_j, F_i) - D_i(v_j, F_j) - d_i(v_j, F_j)$ is independent of F_j. This implies that total trade surplus can be allocated between the supplier and the retailer i by varying F_i. Thus, we have $\overline{\Pi}(v_j, F_j) - D_i(v_j, F_j) = \overline{\pi}(v_j, F_j) - d_i(v_j, F_j)$. Consequently, F_i should split the total surplus evenly between the trade parties and unit payment v_i should maximize the total trade surplus. The KS solution, in turn, coincides with the symmetric NB solution. $\square$

C. Proofs of Formal Results in Chapter 4

Proof of Lemma 4.3.1. According to Proposition 4.3.1, q_j^* decreases in j. For farmer $j \in N$ to participate production, his output quantity should be positive even when farmers $j+1, j+2, ..., n$ quit the market. By Proposition 4.3.1, farmer j's output quantity should be

$$q_j^* = \frac{1}{\beta}\left(\frac{\alpha - c_H - (\frac{1}{j}\sum_{i=1}^{j} z_i)c_P}{j+1} - (z_j - \frac{1}{j}\sum_{i=1}^{j} z_i)c_P\right).$$

$q_j^* \geq 0$ gives

$$g(j) \leq \frac{\alpha - c_H}{c_P},$$

where

$$g(j) = (j+1)z_j - \sum_{i=1}^{j} z_i.$$

Let j be any positive integer where $j < n$,

$$g(j+1) - g(j) = (j+1)(z_{j+1} - z_j) \geq 0.$$

Thus, $g(j)$ is increasing in j. $\square$

Proof of Proposition 4.3.1. Note that farmer j's profit in (4.3) is concave in q_j. Thus, farmer j's optimal output quantity can be derived from the first-order condition of (4.3) as

$$q_j = \frac{1}{2\beta}\left(\alpha - c_H - z_j c_P - \beta \sum_{i \in N\setminus\{j\}} q_i\right) = \frac{1}{2\beta}\left(\alpha - c_H - z_j c_P + \beta q_j - \beta \sum_{i \in N} q_i\right), \quad j \in N.$$

The equilibrium output quantities is the unique solution of the above system of linear equations:

$$q_j^* = \frac{1}{\beta(n+1)}\big(\alpha - c_H - \bar{z}c_P - (n+1)(z_j - \bar{z})c_P\big), \quad j \in N.$$

In the right-hand side of the above expression, the coefficient of z_j is $-nc_p/(\beta(n+1))$. Thus, q_j^* is decreasing in z_j, which gives rise to part (i).

The equilibrium total input quantity is

$$X = \sum_{i \in N} z_i q_i^* = \frac{n}{\beta(n+1)}\big(\bar{z}(\alpha - c_H - \bar{z}c_P) - (n+1)v_z c_p\big). \tag{C.1}$$

We deduce that

$$\frac{\partial X}{\partial \bar{z}} = \frac{n}{\beta(n+1)}\big(\alpha - c_H - 2c_P\bar{z}\big).$$

It follows that X is increasing [decreasing] in $\bar{z}$ when $\bar{z} < [>](\alpha - c_H)/(2c_P)$.

The equilibrium total output quantity is

$$Q = \sum_{i \in N} q_i^* = \frac{n}{\beta(n+1)}\big(\alpha - c_H - \bar{z}c_P\big), \tag{C.2}$$

which is clearly decreasing in $\bar{z}$. Thus, we obtain part (ii).

The total farmer profit in equilibrium is

$$\Pi = \sum_{i \in N} \pi_i^* = \frac{n}{\beta(n+1)^2}\big((\alpha - c_H - \bar{z}c_P)^2 + (n+1)^2 v_z c_P^2\big), \tag{C.3}$$

which is clearly decreasing in $\bar{z}$.

Part (iii) follows immediately by inspecting (C.1), (C.2) and (C.3). □

Proof of Proposition 4.3.2. Note that q_j^* is affine in z_j for all j. Thus, by Theorem A.1.f. of Marshall et al. (1979), $\mathbf{q}_A^C(s_P, s_H) \geq^m \mathbf{q}_B^C(s_P, s_H)$, as majorization order is preserved under affine transformation. From Theorem A.1. of Marshall et al. (1979), $\boldsymbol{\pi}_A^C(s_P, s_H) \geq_{wm} \boldsymbol{\pi}_B^C(s_P, s_H)$. This concludes the proof. □

Proof of Lemma 4.4.1. By Proposition 4.4.1, q_j^* decreases in j for $s_P \leq c_P$. Following the lines of the proof of Lemma 4.3.1, we have $g(k) \leq (\alpha - c_H + s_H)/(c_P - s_P)$. For $s_P > c_P$, q_j^* increases in j. For farmer $j \in N$ to participate production, his output quantity should be positive even when farmers $1, 2, \ldots, j-1$ quit the market. By Proposition 4.4.2, farmer j's output quantity should be

$$q_j^*(s_P, s_H) = \tfrac{1}{\beta}\left(\tfrac{\alpha - (c_H - s_H) - \left(\frac{1}{n-j+1}\sum_{i=j}^n z_i\right)(c_P - s_P)}{n-j+2} - \left(z_j - \tfrac{1}{n-j+1}\sum_{i=j}^n z_i\right)(c_P - s_P)\right).$$

$q_j^*(s_P, s_H) \geq 0$ gives

$$\tilde{g}(j) \leq \frac{\alpha - c_H + s_H}{s_P - c_P},$$

where

$$\tilde{g}(j) = \sum_{i=j}^{n} z_i - (n-k+2)z_j.$$

Let j be any positive integer where $j < n$,

$$\tilde{g}(j+1) - \tilde{g}(j) = (n-j+2)(z_j - z_{j+1}) \leq 0.$$

Thus, $\tilde{g}(j)$ is decreasing in j. □

Proof of Lemma 4.4.2. With subsidies (s_P, s_H), farmer j's profit is

$$\pi_j(\mathbf{q}; s_P, s_H) = \left(a_j(s_P, s_H) - \beta \sum_{i \in N} q_i(s_P, s_H)\right) q_j, \quad j \in N.$$

It is clear that π_j is concave in q_j and the first-order condition of q_j gives

$$\begin{aligned} q_j(s_P, s_H) &= \frac{1}{2\beta}\left(a_j(s_P, s_H) - \beta \sum_{i \in N \setminus \{j\}} q_i(s_P, s_H)\right) \\ &= \frac{1}{2\beta}\left(a_j(s_P, s_H) + \beta q_j(s_P, s_H) - \beta \sum_{i \in N} q_i(s_P, s_H)\right), \quad j \in N. \end{aligned}$$

The equilibrium is thus obtained by solving the above system of linear equations:

$$\begin{aligned} q_j^C(s_P, s_H) &= \frac{1}{\beta}\left(a_j(s_P, s_H) - \frac{1}{n+1}\sum_{i \in N} a_i(s_P, s_H)\right) \\ &= \frac{1}{\beta}\left(\alpha - z_j(c_P - s_P) - (c_H - s_H) - \frac{1}{n+1}\sum_{i \in N}\big(\alpha - z_i(c_P - s_P) - (c_H - s_H)\big)\right) \\ &= \frac{1}{\beta}\left(\frac{\bar{a}(s_P, s_H)}{n+1} - (z_j - \bar{z})(c_P - s_P)\right). \end{aligned}$$

It follows that the farmer's equilibrium profit can be computed as

$$\begin{aligned} \pi_j^C(s_P, s_H) &= \pi_j(\mathbf{q}^C(s_P, s_H); s_P, s_H) \\ &= \frac{1}{\beta}\left(a_j(s_P, s_H) - \frac{1}{n+1}\sum_{i \in N} a_i(s_P, s_H)\right)^2 \\ &= \frac{1}{\beta}\left(\frac{\bar{a}(s_P, s_H)}{n+1} - (z_j - \bar{z})(c_P - s_P)\right)^2, \quad j \in N. \end{aligned}$$

The total input quantity is

$$\begin{aligned} X^C(s_P,s_H) &= \sum_{i\in N} z_i q_i^C(s_P,s_H) = \frac{1}{\beta}\Bigg(\sum_{i\in N} z_i a_i(s_P,s_H) - \frac{1}{n+1}\sum_{i\in N} z_i \sum_{i\in N} a_i(s_P,s_H)\Bigg) \\ &= \frac{n}{\beta(n+1)}\big(\bar{z}\bar{a}(s_P,s_H) - (n+1)v_z(c_p - s_P)\big), \end{aligned}$$

and the total output quantity is

$$Q^C(s_P,s_H) = \sum_{i\in N} q_i^C(s_P,s_H) = \frac{1}{\beta(n+1)}\sum_{i\in N} a_i(s_P,s_H) = \frac{n}{\beta(n+1)}\bar{a}(s_P,s_H),$$

and the total profit is

$$\Pi^C(s_P,s_H) = \sum_{i\in N} \pi_i^C(s_P,s_H) = \frac{n}{\beta(n+1)^2}\big(\bar{a}^2(s_P,s_H) + (n+1)^2 v_z(c_P - s_P)^2\big).$$

Hence, we conclude the proof. $\square$

Proof of Proposition 4.4.1. From (4.5), we have

$$q_{j+1}^C(s_P,s_H) - q_j^C(s_P,s_H) = \frac{1}{\beta}(c_P - s_P)(z_j - z_{j+1})\begin{cases} \leq 0 & \text{for } s_P \leq c_P, \\ \geq 0 & \text{for } s_P > c_P. \end{cases}$$

Thus, we conclude (i).

To show part (ii), we differentiate (4.5) with respect to s_P and s_H, respectively, to obtain

$$\frac{\partial q_j^C(s_P,s_H)}{\partial s_P} = \frac{(n+1)z_j - n\bar{z}}{\beta(n+1)} \quad \text{and} \quad \frac{\partial q_j^C(s_P,s_H)}{\partial s_H} = \frac{1}{\beta(n+1)}.$$

It is clear that farmer j's output quantity is increasing [decreasing] in s_P for $z_j \geq [\leq]\, n\bar{z}/(n+1)$ and is increasing in s_H. Since the input quantity and the output quantity vary by a scale of z_j, farmer j's input quantity exhibits the same monotone property with respect to s_P and s_H. Moreover, from (4.5) and (4.6), we have $\pi_j^C(s_P,s_H) = \beta(q_j^C(s_P,s_H))^2$. Thus, farmer j's profit π_j^C exhibits the same monotone property as q_j^C with respect to s_P and s_H. Hence we obtain part (ii).

To see part (iii), we substitute $\bar{a}(s_P,s_H) = \alpha - \bar{z}(c_P - s_P) - (c_H - s_H)$ into (4.7) to obtain

$$X^C(s_P,s_H) = \frac{n}{\beta(n+1)}\big((s_P - c_P)\bar{z}^2 + (\alpha - c_H + s_H)\bar{z} + (n+1)(s_P - c_P)v_z\big).$$

Differentiating with respect to $\bar{z}$ yields

$$\frac{\partial X^C(s_P, s_H)}{\partial \bar{z}} = \frac{n}{\beta(n+1)}\bigg(2(s_P - c_P)\bar{z} + \alpha - c_H + s_H\bigg).$$

Thus, $X^C(s_P, s_H)$ is increasing in $\bar{z}$ when $s_P > c_P$. For $s_P \leq c_P$, $X^C(s_P, s_H)$ is increasing [decreasing] in $\bar{z}$ when $\bar{z} \leq [\geq](\alpha - c_H + s_H)/(2(c_P - s_P))$.

Now substituting $\bar{a}(s_P, s_H) = \alpha - \bar{z}(c_P - s_P) - (c_H - s_H)$ into (4.8), we obtain

$$Q^C(s_P, s_H) = \frac{n}{\beta(n+1)}\bigg(\alpha + \bar{z}(s_P - c_P) - c_H + s_H\bigg).$$

It is easy to see that $Q^C(s_P, s_H)$ decreases [increases] in $\bar{z}$ for $s_P \leq [>]c_P$. We conclude part (iii).

To see part (iv), we note from (4.8) that Q^C is independent of v_z. Differentiating (4.7) with respect to v_z yields

$$\frac{\partial X^C(s_P, s_H)}{\partial v_z} = \frac{n}{\beta}(s_P - c_P).$$

Thus, $X^C(s_P, s_H)$ decreases [increases] in v_z for $s_P \leq [>]c_P$. We conclude part (iv). □

Proof of Corollary 4.4.1. To see part (i), we have

$$\Delta_{s_P} q_j^C = \frac{\delta}{\beta}\bigg(z_j - \frac{n}{n+1}\bar{z}\bigg).$$

Thus, $\Delta_{s_P} q_j^C$ increases in j. Next, we note that

$$\sum_{j=l}^{n} \Delta_{s_P} q_j^C = \frac{\delta}{\beta}\bigg(\sum_{j=l}^{n} z_j - \frac{(n-l+1)n}{n+1}\bar{z}\bigg) > 0.$$

This suggests that $\mathbf{q}^C(s_{P1}, s_H)$ weakly sup-majorizes $\mathbf{q}^C(s_{P2}, s_H)$ for $s_{P1} < s_{P2} \leq c_P$, as $q_j^C(s_P, s_H)$ decreases in j for $s_P \leq c_P$. Because $q_j^C(s_P, s_H)$ increases in j for $s_P > c_P$, we have $\mathbf{q}^C(s_{P2}, s_H)$ weakly sub-majorizes $\mathbf{q}^C(s_{P1}, s_H)$ for $c_P \leq s_{P1} < s_{P2}$. Because $\pi_j^C(s_P, s_H) = \beta(q_j^C(s_P, s_H))^2$, from Theorem A.1. of Marshall et al. (1979), $\boldsymbol{\pi}^C(s_{P2}, s_H)$ weakly sub-majorizes $\boldsymbol{\pi}^C(s_{P1}, s_H)$.

To see part (ii), we have

$$\Delta_{s_H} q_j^C = \frac{\delta}{\beta(n+1)} \quad \text{and} \quad \Delta_{s_H} \pi_j^C = 2(\Delta_{s_H} q_j^C) q_j^C(s_P, s_H + \delta/2).$$

Thus, $\Delta_{s_H} q_j^C$ is constant in j, and $\Delta_{s_H} \pi_j^C$ exhibits the same monotone property as $q_j^C(s_P, s_H + \delta/2)$ with respect to j. Because $\Delta_{s_H} q_j^C > 0$ and $\Delta_{s_H} \pi_j^C > 0$ for all j, we have $\mathbf{q}^C(s_P, s_{H2}) \geq_{wm} \mathbf{q}^C(s_P, s_{H1})$ and $\boldsymbol{\pi}^C(s_P, s_{H2}) \geq_{wm} \boldsymbol{\pi}^C(s_P, s_{H1})$ for $s_{H1} < s_{H2}$. This concludes the proof. $\square$

Proof of Proposition 4.4.2. To see part (i), we note that $q_j^C(s_P, s_H)$ is affine in z_j for all j. Thus, by Theorem A.1.f. of Marshall et al. (1979), $\mathbf{q}_A^C(s_P, s_H) \geq^m \mathbf{q}_B^C(s_P, s_H)$, as majorization order is preserved under affine transformation. Because $\Delta_{s_P} q_j^C$ is affine in z_j (see the proof of Corollary 4.4.1), $\boldsymbol{\Delta}_{s_P} \mathbf{q}_A^C \geq^m \boldsymbol{\Delta}_{s_P} \mathbf{q}_B^C$. Because $\Delta_{s_H} q_j^C$ is independent of z_j, we have $\boldsymbol{\Delta}_{s_H} \mathbf{q}_A^C =^m \boldsymbol{\Delta}_{s_H} \mathbf{q}_B^C$. This concludes part (i).

To see part (ii), from Theorem A.1. of Marshall et al. (1979), $\boldsymbol{\pi}_A^C(s_P, s_H) \geq_{wm} \boldsymbol{\pi}_B^C(s_P, s_H)$. Because $\Delta_{s_H} \pi_j^C$ is affine in z_j for all j, we have $\boldsymbol{\Delta}_{s_H} \boldsymbol{\pi}_A^C \geq^m \boldsymbol{\Delta}_{s_H} \boldsymbol{\pi}_B^C$. This concludes part (ii). $\square$

Proof of Lemma 4.4.3. By Proposition 4.4.3, $q_j^S(s_p, s_H)$ decreases in j for $s_P \leq c_P$. For farmer $j \in N$ to participate production, his output quantity should be positive even when farmers $j+1, j+2, ..., n$ quit the market. If $j \leq m$, all farmers choose harvesting subsidy and farmer j's output becomes

$$q_j^S(s_P, s_H) = q_j^C(0, s_H) = \frac{1}{\beta}\left(\frac{\alpha - (\frac{1}{j}\sum_{i=1}^j z_i)c_P - (c_H - s_H)}{j+1} - (z_j - \frac{1}{j}\sum_{i=1}^j z_i)c_P\right).$$

$q_j^S(s_P, s_H) \geq 0$ gives

$$g(j) \leq \frac{\alpha - c_H + s_H}{c_P}.$$

If $j > m$, farmer j should choose planting subsidy. By Lemma 4.4.4, farmer j's output should be

$$\begin{aligned} q_j^S(s_P, s_H) \;&=\; \frac{1}{\beta}\left(\frac{\alpha - (\frac{1}{j}\sum_{i=1}^j z_i)(c_P - s_P) - (c_H - s_H)}{j+1} - \left(z_j - \frac{1}{j}\sum_{i=1}^j z_i\right)(c_P - s_P)\right) \\ &\quad -\frac{1}{\beta}\left(s_H - \frac{\sum_{i=1}^m z_i s_P + (j-m)s_H}{j+1}\right). \end{aligned}$$

$q_j^S(s_P, s_H) \geq 0$ gives

$$g(j) \leq \frac{\alpha - c_H - m s_H + \sum_{i=1}^m z_i s_P}{c_P - s_P}.$$

For $s_P > c_P$, $q_j^S(s_P, s_H)$ decreases [increases] in j for $j < [>]m$. If $j \leq m$, farmer k's output should be positive even when farmers $j+1, \ldots, i^a$ quit the market, where $i^a = \max\{i \in \{m+1, \ldots, n\} : q_i^S(s_P, s_H) \leq q_j^S(s_P, s_H)\} = \max\{i \in \{m+1, \ldots, n\} : z_i(s_P - c_P) \leq s_H - z_j c_P\}$ (when $z_{m+1}(s_P - c_P) > s_H - z_j c_P$, we set $i^a = m$). The farmer j's output becomes

$$\begin{aligned} q_j^S(s_P, s_H) &= \frac{1}{\beta}\Bigg(\frac{\alpha - \frac{1}{n-i^a+j}(n\bar{z} - \sum_{i=j+1}^{i^a} z_i)(c_P - s_P) - (c_H - s_H)}{n - i^a + j + 1} \\ &\quad - \Big(z_j - \frac{n\bar{z} - \sum_{i=j+1}^{i^a} z_i}{n - i^a + j}\Big)(c_P - s_P)\Bigg) - \frac{1}{\beta}\Bigg(z_j s_P - \frac{\sum_{i=1}^{j} z_i s_P + (n - i^a) s_H}{n - i^a + j + 1}\Bigg). \end{aligned}$$

$q_j^S(s_P, s_H) \geq 0$ gives

$$(n - i^a + j) z_j - n\bar{z} + \sum_{i=j}^{i^a} z_i \leq \frac{\alpha - c_H + (n - i^a + 1) s_H - \sum_{i=i^a+1}^{n} z_i s_P}{c_P}.$$

If $j > m$, farmer j's output quantity should be positive even when farmers $i^b, i^b + 1, \ldots, j-1$ quit the market, where $i^b = \min\{i \in \{1, 2, \ldots, m\} : q_i^S(s_P, s_H) \leq q_j^S(s_P, s_H)\} = \min\{i \in \{1, 2, \ldots, m\} : z_i c_P \geq s_H - z_j(s_P - c_P)\}$ (when $z_m c_P < s_H - z_j(s_P - c_P)$, we set $i^b = m + 1$). The farmer j's output becomes

$$\begin{aligned} q_j^S(s_P, s_H) &= \frac{1}{\beta}\Bigg(\frac{\alpha - \frac{1}{n-j+i^b}(n\bar{z} - \sum_{i=i^b}^{j-1} z_i)(c_P - s_P) - (c_H - s_H)}{n - j + i^b + 1} \\ &\quad - \Big(z_j - \frac{n\bar{z} - \sum_{i=i^b}^{j-1} z_i}{n - j + i^b}\Big)(c_P - s_P)\Bigg) - \frac{1}{\beta}\Bigg(s_H - \frac{\sum_{i=1}^{i^b-1} z_i s_P + (n - j + 1) s_H}{n - j + i^b + 1}\Bigg). \end{aligned}$$

$q_j^S(s_P, s_H) \geq 0$ gives

$$n\bar{z} - \sum_{i=i^b}^{j} z_i - (n - j + i^b) z_j \leq \frac{\alpha - c_H - (i^b - 1) s_H + \sum_{i=1}^{i^b-1} z_i s_P}{s_P - c_P}.$$

This concludes the proof. $\square$

Proof of Lemma 4.4.4. From (4.3) and (4.4), it is straightforward to see that a farmer with $z_j s_P < [>, =] s_H$ would choose harvesting subsidy [choose planting subsidy, be indifferent]. Thus, the farmer's profit is

$$\begin{aligned}\pi_j(\mathbf{q}) &= \Big(a_j(s_P, s_H) - \beta \sum_{i\in N} q_i\Big) q_j, j \in N \\ &= \begin{cases} \big(a_j(0, s_H) - \beta \sum_{i\in N} q_i\big) q_j, & \text{for } j \leq m, \\ \big(a_j(s_P, 0) - \beta \sum_{i\in N} q_i\big) q_j, & \text{for } j > m. \end{cases}\end{aligned}$$

Define $\hat{a}_j(s_P, s_H) = a_j(0, s_H)\mathbb{I}_{\{j\leq m\}} + a_j(s_P, 0)\mathbb{I}_{\{j>m\}}$. Then the analysis directly follows that for Lemma 4.4.2 with $a_j(s_P, s_H)$ replaced by $\hat{a}_j(s_P, s_H)$. We also note that

$$a_j(0, s_H) = a_j(s_P, s_H) - z_j s_P \text{ and } a_j(s_P, 0) = a_j(s_P, s_H) - s_H.$$

We deduce, from the proof of Lemma 4.4.2 and (4.5),

$$\begin{aligned} q_j^S(s_P, s_H) &= \begin{cases} \frac{1}{\beta}\Big(a_j(0, s_H) - \frac{1}{n+1}\big(\sum_{i=1}^{m} a_i(0, s_H) + \sum_{i=m+1}^{n} a_i(s_P, 0)\big)\Big) & \text{for } j \leq m, \\ \frac{1}{\beta}\Big(a_j(s_P, 0) - \frac{1}{n+1}\big(\sum_{i=1}^{m} a_i(0, s_H) + \sum_{i=m+1}^{n} a_i(s_P, 0)\big)\Big) & \text{for } j > m, \end{cases} \\ &= q_j^C(s_P, s_H) - \begin{cases} \frac{1}{\beta}\Big(z_j s_P - \frac{\sum_{i=1}^m z_i s_P + (n-m)s_H}{n+1}\Big) & \text{for } j \leq m, \\ \frac{1}{\beta}\Big(s_H - \frac{\sum_{i=1}^m z_i s_P + (n-m)s_H}{n+1}\Big) & \text{for } j > m. \end{cases} \end{aligned}$$

We can then compute

$$\begin{aligned} X^S(s_P, s_H) &= \sum_{i\in N} z_i q_i^S(s_P, s_H) \\ &= X^C(s_P, s_H) - \frac{1}{\beta}\Bigg(\sum_{i=1}^{m} z_i^2 s_P + \sum_{i=m+1}^{n} z_i s_H - \frac{n\bar{z}}{n+1}\big(\sum_{i=1}^{m} z_i s_P + (n-m)s_H\big)\Bigg) \end{aligned}$$

and

$$Q^S(s_P, s_H) = \sum_{i\in N} q_i^S(s_P, s_H) = Q^C(s_P, s_H) - \frac{1}{\beta(n+1)}\Bigg(\sum_{i=1}^{m} z_i s_P + (n-m)s_H\Bigg).$$

Relating to the results in Lemma 4.4.2, we conclude the proof. □

Proof of Proposition 4.4.3. From (4.10), we have

$$q_{j+1}^S(s_P,s_H)-q_j^S(s_P,s_H)=\frac{1}{\beta}\times\begin{cases} c_P(z_j-z_{j+1}) & \text{for } j<m,\\ ((c_P-s_P)(z_m-z_{m+1})+(z_ms_P-s_H)) & \text{for } j=m,\\ (c_P-s_P)(z_j-z_{j+1}) & \text{for } j>m.\end{cases}$$

Note that $z_ms_P\le s_H$ and $z_j\le z_{j+1}$, we have $q_{j+1}^S(s_P,s_H)-q_j^S(s_P,s_H)\le 0$ when $s_P\le c_P$ and $q_{j+1}^S(s_P,s_H)-q_j^S(s_P,s_H)\le[\ge]0$ for $j<[>]m$ when $s_P>c_P$. We conclude part (i).

To see part (ii), we differentiate (4.10) with respect to s_P and s_H, respectively, to obtain

$$\frac{\partial q_j^S(s_P,s_H)}{\partial s_P}=\begin{cases} -\frac{\sum_{i=m+1}^n z_i}{\beta(n+1)} & \text{for } j\le m,\\ \frac{(n+1)z_j-\sum_{i=m+1}^n z_i}{\beta(n+1)} & \text{for } j>m,\end{cases} \quad\text{and}\quad \frac{\partial q_j^S(s_P,s_H)}{\partial s_H}=\begin{cases} \frac{n-m+1}{\beta(n+1)} & \text{for } j\le m,\\ -\frac{m}{\beta(n+1)} & \text{for } j>m.\end{cases}$$

It is clear that farmer j's output quantity is increasing [decreasing] in s_P for $z_j\ge\frac{\sum_{i=m+1}^n z_i}{n+1}$ and $j>m$ [*otherwise*], and is increasing [decreasing] in s_H when $j\le m$ [$j>m$]. Since the input quantity and the output quantity vary by a scale of z_j, farmer j's input quantity exhibits the same monotone property with respect to s_P and s_H. Moreover, from (4.11), we have $\pi_j^S(s_P,s_H)=\beta(q_j^S(s_P,s_H))^2$. Thus, farmer j's profit π_j^S exhibits the same monotone property as q_j^S with respect to s_P and s_H. Hence we obtain part (ii). $\square$

Proof of Corollary 4.4.2. To see part (i), we define $m_\delta=\max\{j\in N: z_j(s_P+\delta)\le s_H\}$ (when $z_1>s_H/(s_P+\delta)$, we set $m_\delta=0$). Note that $m\ge m_\delta$. We also define

$$\Delta_P \equiv \sum_{i=1}^{m_\delta} z_i\delta+\sum_{i=m_\delta+1}^{m}(s_H-z_is_P).$$

It is clear that $0<\Delta_P<\sum_{i=1}^m z_i\delta$. Note that

$$\Delta_{s_P}q_j^S=\begin{cases} \frac{\Delta_P-\delta n\bar z}{\beta(n+1)} & \text{for } j\le m_\delta,\\ \frac{\Delta_P-\delta n\bar z}{\beta(n+1)}+\frac{(s_P+\delta)z_j-s_H}{\beta} & \text{for } m_\delta<j\le m,\\ \frac{\Delta_P-\delta n\bar z}{\beta(n+1)}+\frac{\delta z_j}{\beta} & \text{for } m<j.\end{cases}$$

The first piece is constant in j, and the second and third pieces increase in j. Because $\delta z_{m+1}-((s_P+\delta)z_m-s_H)>\delta(z_{m+1}-z_m)>0$, $\Delta_{s_P}q_j^S$ increases in j. Because

$\Delta_{s_P} q_n^S > 0$ and $\sum_{i\in N} \Delta_{s_P} q_i^S > 0$, $\sum_{j=l}^n \Delta_{s_P} q_j^S > 0$ for any $1 \le l \le n$. This suggests that $\mathbf{q}^S(s_{P1}, s_H)$ weakly sup-majorizes $\mathbf{q}^S(s_{P2}, s_H)$ for $s_{P1} < s_{P2} \le c_P$. We conclude part (i).

To see part (ii), $m_\delta = \max\{j \in N : z_j s_P \le (s_H + \delta)\}$ (when $z_1 > (s_H+\delta)/s_P$, we set $m_\delta = 0$). Note that $m \le m_\delta$. We also define

$$\Delta_H \equiv (n - m_\delta)\delta + \sum_{i=m+1}^{m_\delta} (z_i s_P - s_H).$$

It is clear that $0 < \Delta_H < (n-m)\delta$. Note that

$$\Delta_{s_H} q_j^S = \begin{cases} \frac{\Delta_H+\delta}{\beta(n+1)} & \text{for } j \le m, \\ \frac{\Delta_H+\delta}{\beta(n+1)} - \frac{z_j s_P - s_H}{\beta} & \text{for } m < j \le m_\delta, \\ \frac{\Delta_H+\delta}{\beta(n+1)} - \frac{\delta}{\beta} & \text{for } m_\delta < j. \end{cases}$$

The first and third pieces are constant in j, and the second piece decreases in j. Because $(z_{m_\delta} s_P - s_H) - \delta < 0$, $\Delta_{s_H} q_j^S$ decreases in j. Because $\Delta_{s_H} q_1^S > 0$ and $\sum_{i\in N} \Delta_{s_H} q_i^S > 0$, we have $\sum_{j=1}^l \Delta_{s_H} q_j^S > 0$ for any $1 \le l \le n$. This suggests that $\mathbf{q}^S(s_P, s_{H2})$ sub-majorizes $\mathbf{q}^S(s_P, s_{H1})$. By Theorem A.2.(i) of Marshall et al. (1979), $\boldsymbol{\pi}^S(s_P, s_{H2})$ sub-majorizes $\boldsymbol{\pi}^S(s_P, s_{H1})$, as $\pi_j^S(s_P, s_H) = \beta\big(q_j^S(s_P, s_H)\big)^2$. We conclude part (ii). □

Proof of Proposition 4.4.4. We define

$$\begin{aligned} \Delta &\equiv \left(m_A s_H + \sum_{i=m_A+1}^n z_{Ai} s_P\right) - \left(m_B s_H + \sum_{i=m_B+1}^n z_{Bi} s_P\right) \\ &= \sum_{i=m_B+1}^n (z_{Ai} - z_{Bi}) s_P + \sum_{i=m_A+1}^{m_B} (z_{Ai} s_P - s_H) \ge 0. \end{aligned}$$

The inequality follows from $\mathbf{z}_A \ge^m \mathbf{z}_B$ and $z_{Ai} s_P > s_H$ for $i > m_A$. We note that

$$\begin{aligned} &\sum_{j=1}^l \big(q_{Aj}^S(s_P, s_H) - q_{Bj}^S(s_P, s_H)\big) \\ &= \begin{cases} \frac{c_P}{\beta} \sum_{j=1}^l (z_{Bj} - z_{Aj}) - \frac{l\Delta}{\beta(n+1)} & \text{for } l \le m_A, \\ \frac{(c_P - s_P)}{\beta} \sum_{j=1}^l (z_{Bj} - z_{Aj}) + \frac{(n-l+1)\Delta}{\beta(n+1)} + \frac{1}{\beta} \sum_{j=l+1}^{m_B} (s_H - z_{Bj} s_P) & \text{for } m_A < l < m_B, \\ \frac{(c_P - s_P)}{\beta} \sum_{j=1}^l (z_{Bj} - z_{Aj}) + \frac{(n+1-l)\Delta}{\beta(n+1)} & \text{for } l \ge m_B. \end{cases} \end{aligned}$$

It is clear that $\sum_{j=1}^{l}(q_{Aj}^{S}(s_P,s_H)-q_{Bj}^{S}(s_P,s_H))\geq 0$ for any $1\leq l\leq n$. This suggests that $\mathbf{q}_A^S(s_P,s_H)$ weakly sub-majorizes $\mathbf{q}_B^S(s_P,s_H)$. By Theorem A.2.(i) of Marshall et al. (1979), $\boldsymbol{\pi}_A^S(s_P,s_H)$ weakly sub-majorizes $\boldsymbol{\pi}_B^S(s_P,s_H)$. This concludes the proof.

□

Proof of Proposition 4.4.5. To see part (i), we have, from (4.5) and (4.10),

$$q_j^C(s_P,s_H)-q_j^S(s_P,s_H)=\frac{1}{\beta(n+1)}\times\begin{cases}(n+1)z_js_P-\left(\sum_{i=1}^{m}z_is_P+(n-m)s_H\right) & \text{for } j\leq m,\\ (m+1)s_H-\sum_{i=1}^{m}z_is_P & \text{for } j>m.\end{cases}$$

Because $((m+1)s_H-\sum_{i=1}^{m}z_is_P)-((n+1)z_ms_P-(\sum_{i=1}^{m}z_is_P+(n-m)s_H))=(n+1)(s_H-z_ms_P)\geq 0$, $q_j^C(s_P,s_H)-q_j^S(s_P,s_H)$ increases in j. Because the second piece is above zero, $j^o\leq m$. We conclude part (i).

To see part (ii), we note that $\sum_{i\in N}(q_j^C(s_P,s_H)-q_j^S(s_P,s_H))>0$ and $q_n^C(s_P,s_H)>q_n^S(s_P,s_H)$. This suggests that $\sum_{j=l}^{n}(q_j^C(s_P,s_H)-q_j^S(s_P,s_H))>0$ for any $1\leq l\leq n$. Thus, $\mathbf{q}^S(s_P,s_H)$ weakly sub-majorizes $\mathbf{q}^C(s_P,s_H)$ for $s_P\leq c_P$. We conclude part (ii).

To see part (iii), we note that

$$\phi_j\equiv\Delta_{s_P}q_j^C-\Delta_{s_P}q_j^S=\begin{cases}\frac{\delta z_j}{\beta}-\frac{\Delta_P}{\beta(n+1)} & \text{for } j\leq m_\delta,\\ \frac{s_H-s_Pz_j}{\beta}-\frac{\Delta_P}{\beta(n+1)} & \text{for } m_\delta<j\leq m,\\ -\frac{\Delta_P}{\beta(n+1)} & \text{for } m<j.\end{cases}$$

The first piece increases in j, the second piece decreases in j and the third piece is constant in j. Because $s_H\geq z_ms_P$, ϕ_j decreases in j for $j>m_\delta$. Because $(n+1)z_1\delta\geq\sum_{i=1}^{m}z_i\delta>\Delta_P$, we have $\phi_1>0$. This suggests that there exists a j_P such that $\phi_j\geq[<]0$ for $j\leq[>]j_P$. Note that $\sum_{i\in N}\phi_i>0$. We conclude that $\sum_{j=1}^{l}\phi_j>0$ for any $1\leq l\leq n$ and thus $\boldsymbol{\Delta}_{s_P}\mathbf{q}^S$ weakly sup-majorizes $\boldsymbol{\Delta}_{s_P}\mathbf{q}^C$.

To see part (iv), we note that

$$\psi_j\equiv\Delta_{s_H}q_j^C-\Delta_{s_H}q_j^S=\begin{cases}-\frac{\Delta_H}{\beta(n+1)} & \text{for } j\leq m,\\ \frac{z_js_P-s_H}{\beta}-\frac{\Delta_H}{\beta(n+1)} & \text{for } m<j\leq m_\delta,\\ \frac{\delta}{\beta}-\frac{\Delta_H}{\beta(n+1)} & \text{for } m_\delta<j.\end{cases}$$

Because $\Delta_{s_H} q_j^C$ is constant in j and $\Delta_{s_H} q_j^S$ decreases in j, ψ_j increases in j. Because $\psi_1 < 0$ and $\psi_n > 0$, there exists a j_H such that $\psi_j \leq [>] 0$ for $j \leq [>] j_H$. Note that $\sum_{i \in N} \psi_i > 0$. We conclude that $\sum_{j=l}^{n} \psi_j > 0$ for any $1 \leq l \leq n$ and thus $\mathbf{\Delta}_{s_H} \mathbf{q}^S$ weakly sup-majorizes $\mathbf{\Delta}_{s_H} \mathbf{q}^C$. $\square$

Proof of Lemma 4.5.1. Because $Q^C(s_P, s_H) = \bar{Q}$, we have

$$\bar{z} s_P + s_H = \frac{\beta(n+1)}{n} \bar{Q} - \bar{a}(0,0). \tag{C.4}$$

i) When only a planting subsidy is offered (i.e., $s_H = 0$), we must have

$$s_P^P = \frac{1}{\bar{z}} \left(\frac{\beta(n+1)}{n} \bar{Q} - \bar{a}(0,0) \right) = \left(\frac{\beta(n+1)\bar{Q}}{n} - \bar{a}(0,0) \right) / \bar{z},$$

and the optimal budget is

$$b(s_P^P, 0) = X^C(s_P^P, 0) s_P^P = \bar{Q} \left(\frac{\beta(n+1)}{n} \bar{Q} - \bar{a}(0,0) \right) - \frac{n v_z}{\beta} (c_P - s_P^P) s_P^P. \tag{C.5}$$

ii) When only a harvesting subsidy is offered (i.e., $s_P = 0$), we must have

$$s_H^H = \frac{\beta(n+1)}{n} \bar{Q} - \bar{a}(0,0),$$

and the optimal budget is

$$b(0, s_H^H) = Q^C(0, s_H^H) s_H^H = \bar{Q} \left(\frac{\beta(n+1)}{n} \bar{Q} - \bar{a}(0,0) \right). \tag{C.6}$$

iii) When both subsidies are given, (C.4) imply that $\mathrm{d}s_H / \mathrm{d}s_P = -\bar{z}$. By Lemma 4.4.2, we have

$$\frac{\partial X^C(s_P, s_H)}{\partial s_P} = \frac{n}{\beta(n+1)} \big((n+1) v_z + \bar{z}^2 \big) \quad \text{and} \quad \frac{\partial Q^C(s_P, s_H)}{\partial s_P} = \frac{n}{\beta(n+1)} \bar{z},$$
$$\frac{\partial X^C(s_P, s_H)}{\partial s_H} = \frac{n}{\beta(n+1)} \bar{z} \quad \text{and} \quad \frac{\partial Q^C(s_P, s_H)}{\partial s_H} = \frac{n}{\beta(n+1)}.$$

Now differentiating $b^C(s_P, s_H)$ with respect to s_P, we have

$$\begin{aligned} \frac{\mathrm{d}b^C(s_P, s_H)}{\mathrm{d}s_P} &= \frac{\partial b^C(s_P, s_H)}{\partial s_P} + \frac{\partial b^C(s_P, s_H)}{\partial s_H} \frac{\mathrm{d}s_H}{\mathrm{d}s_P} \\ &= \frac{n}{\beta(n+1)} \big(-(n+1) v_z c_P + 2(n+1) v_z s_P \big) \\ &= \frac{n v_z}{\beta} (-c_P + 2 s_P). \end{aligned}$$

Because the right-hand side is increasing in s_P, b is convex in s_P and is minimized at $s_P^C = c_P/2$. Substituting this into (C.4), we obtain $s_H^C = \frac{\beta(n+1)}{n}\bar{Q} - \bar{a}(0,0) - \frac{c_P\bar{z}}{2}$ and the optimal budget is

$$b(s_P^C, s_H^C) = \bar{Q}\left(\frac{\beta(n+1)}{n}\bar{Q} - \bar{a}(0,0)\right) - \frac{nv_z c_P^2}{4\beta}. \tag{C.7}$$

Hence, we conclude the proof. □

Proof of Lemma 4.5.2. Because $\hat{Q}(s_P, s_H) = \bar{Q}$, we have

$$\left(ms_H + s_P\sum_{i=m+1}^{n} z_i\right)/n = \frac{\beta(n+1)}{n}\bar{Q} - \bar{a}(0,0) \equiv C. \tag{C.8}$$

This implies that $\mathrm{d}s_H/\mathrm{d}s_P = -\sum_{i=m+1}^{n} z_i/m$. By the definition of m, we have

$$\begin{cases} \bar{z}s_P = C \text{ and } s_H < (z_1/\bar{z})C & \text{for } m = 0, \\ \left(mz_m + \sum_{i=m+1}^{n} z_i\right)s_P \le nC < \left(mz_{m+1} + \sum_{i=m+1}^{n} z_i\right)s_P & \text{for } 1 \le m < n, \\ z_n s_P \le C \text{ and } s_H = C & \text{for } m = n. \end{cases} \tag{C.9}$$

Now differentiating $b^S(s_P, s_H)$ with respect to s_P, we have

$$\begin{aligned} \frac{\mathrm{d}b^S(s_P,s_H)}{\mathrm{d}s_P} &= \frac{\partial b^S(s_P,s_H)}{\partial s_P} + \frac{\partial b^S(s_P,s_H)}{\partial s_H}\frac{\mathrm{d}s_H}{\mathrm{d}s_P} \\ &= \frac{2}{\beta}\left(\sum_{i=m+1}^{n} z_i^2 s_P - \sum_{i=m+1}^{n} z_i s_H\right) + \frac{c_P}{m\beta}\left(\left(\sum_{i=1}^{m} z_i\right)\left(\sum_{i=m+1}^{n} z_i\right) - \sum_{i=m+1}^{n} z_i^2 m\right) \\ &= \frac{1}{m\beta}\left(\left(\sum_{i=m+1}^{n} z_i^2 m + \left(\sum_{i=m+1}^{n} z_i\right)^2\right)(2s_P - c_P) + \sum_{i=m+1}^{n} z_i n(\bar{z}c_P - 2C)\right). \end{aligned}$$

Because the right-hand side is increasing in s_P for each m, $b^S(s_P, s_H)$ is convex in s_P and is locally minimized at $s_{P,m} = c(m)C/\bar{z} + (1 - c(m))(c_P/2)$ for $1 \le m < n$. Note that $s_{P,m}$ should satisfy the second piece of (C.9). Substituting $s_{P,m}$ into the second piece of (C.9), we derive $\phi^l(m) \le 2nC/c_P < \phi^u(m)$. We note that $\phi^l(1) \le \phi^l(m)$, as $\phi^l(m) - \phi^l(1) = (mz_m - \sum_{i=1}^{m} z_i)\big(\sum_{i=m+1}^{n} z_i^2 m + \big(\sum_{i=m+1}^{n} z_i\big)^2\big)/(m\sum_{i=m+1}^{n} z_i(z_i - z_m)) \ge 0$, and $\phi^u(n-1) = \infty$. Also, note that

$$\begin{aligned} \phi^l(m+1) &= \frac{(m+1)z_{m+1} + \sum_{i=m+2}^{n} z_i}{(m+1)\sum_{i=m+2}^{n} z_i(z_i - z_{m+1})}\left(\sum_{i=m+2}^{n} z_i^2(m+1) - \left(\sum_{i=1}^{m+1} z_i\right)\left(\sum_{i=m+2}^{n} z_i\right)\right) \\ &= \phi^u(m) - \frac{(mz_{m+1} + \sum_{i=m+1}^{n} z_i)^2(mz_{m+1} - \sum_{i=1}^{m} z_i)}{m(m+1)\sum_{i=m+1}^{n} z_i(z_i - z_{m+1})}. \end{aligned}$$

Since the second term is positive, we have $\phi^l(m+1) \leq \phi^u(m)$. This suggests that the collection of intervals $\{[\phi^l(m), \phi^u(m)) : 1 \leq m < n\}$ covers the range above $\phi^l(1)$. Thus, we conclude that for any $2nC/c_P \geq \phi^l(1) = n\bar{z}$, $s_{P,m}$ satisfies the second piece of (C.9) for some $1 \leq m < n$. For $2nC/c_P < n\bar{z}$ (or $C < \bar{z}c_P/2$), it is easy to see that the optimal subsidy scheme is $s_P^S = C/\bar{z}$ and $s_H^S < (z_1/\bar{z})C$ (i.e., $m^S = 0$).

Next we deduce that $b^S(s_P, s_H)$ is not minimized at $s_P \leq C/z_n$ (i.e., $m^S = n$) or any kink points of $b^S(s_P, s_H)$. This follows from the observation that for any kink point $s_P^o = nC/(mz_{m+1} + \sum_{i=m+1}^n z_i)$, $1 \leq m < n$,

$$
\begin{aligned}
&\frac{\mathrm{d}b^S(s_P, s_H)}{\mathrm{d}s_P}\Big|_{s_P \to s_P^{o}{}^-} \\
&= \frac{2}{\beta}\left(\sum_{i=m+2}^{n} z_i^2 s_P^o - \sum_{i=m+2}^{n} z_i z_{m+1} s_P^o\right) + \frac{c_P}{(m+1)\beta}\left(\left(\sum_{i=1}^{m+1} z_i\right)\left(\sum_{i=m+2}^{n} z_i\right) - \sum_{i=m+2}^{n} z_i^2 (m+1)\right) \\
&= \frac{\mathrm{d}b^S(s_P, s_H)}{\mathrm{d}s_P}\Big|_{s_P \to s_P^{o}{}^+} + \frac{c_P}{m(m+1)\beta}\left(mz_{m+1} - \sum_{i=1}^{m} z_i\right)\left(mz_{m+1} + \sum_{i=m+1}^{n} z_i\right).
\end{aligned}
$$

Since the second term is positive, the above inequality suggests that the left derivative of $b^S(s_P, s_H)$ is greater than the right derivative of $b^S(s_P, s_H)$ at s_P^o. For $s_P < C/z_n$, $b^S(s_P, s_H)$ is constant in s_P. Thus, we conclude that $b^S(s_P, s_H)$ is not minimized at $s_P < C/z_n$ or any kink points.

Substituting this into (C.8), we obtain $s_{H,m} = \big(nC - \sum_{i=m+1}^n z_i s_{P,m}\big)/m$ and the budget is

$$
\begin{aligned}
&b(s_{P,m}, s_{H,m}) \\
&= \bar{Q}C + \frac{1}{\beta}\left(ms_{H,m}^2 + \sum_{i=m+1}^{n} z_i^2 s_{P,m}^2 - \left(\sum_{i=1}^{m}(z_i - \bar{z})s_{H,m} + \sum_{i=m+1}^{n}(z_i - \bar{z})z_i s_{P,m}\right)c_P - nC^2\right) \\
&= \bar{Q}C + \frac{n}{4\beta}\left(c(m)\frac{\sum_{i=m+1}^n z_i^2}{\bar{z}\sum_{i=m+1}^n z_i} - 1\right)(2C - \bar{z}c_P)^2 + \left(\sum_{i=1}^{m} z_i^2 - \frac{1}{m}\left(\sum_{i=1}^{m} z_i\right)^2 - nv_z\right)\frac{c_P^2}{4\beta}.
\end{aligned}
$$

Now we focus on establishing the monotonicity of $c(m)$. This follows from the observation that

$$\begin{aligned}
&\frac{n\bar{z}}{c(m+1)} - \frac{n\bar{z}}{c(m)} \\
&= \frac{(m+1)\sum_{i=m+2}^{n} z_i^2}{\sum_{i=m+2}^{n} z_i} - \frac{m\sum_{i=m+1}^{n} z_i^2}{\sum_{i=m+1}^{n} z_i} - z_{m+1} \\
&= \frac{\sum_{i=m+1}^{n} z_i^2 \sum_{i=m+1}^{n} z_i - z_{m+1}(\sum_{i=m+1}^{n} z_i)^2 + m z_{m+1}\sum_{i=m+1}^{n} z_i(z_i - z_{m+1})}{(\sum_{i=m+1}^{n} z_i)(\sum_{i=m+2}^{n} z_i)} \\
&\geq z_{m+1}\frac{\sum_{i=m+1}^{n} z_i^2(n-m) - (\sum_{i=m+1}^{n} z_i)^2 + m\sum_{i=m+1}^{n} z_i(z_i - z_{m+1})}{(\sum_{i=m+1}^{n} z_i)(\sum_{i=m+2}^{n} z_i)} \geq 0.
\end{aligned}$$

The first inequality follows because z_j decreases in j. The second inequality follows from the fact that $\sum_{i=m+1}^{n} z_i^2(n-m) \geq (\sum_{i=m+1}^{n} z_i)^2$, by the Cauchy-Schwartz inequality. Thus, we conclude that $c(m)$ decreases in m.

Now we focus on establishing the monotonicity of m^S with respect to $\bar{Q}$ (or C). We note that the third term of $b(s_{P,m}, s_{H,m})$ increases in m as

$$\left(\sum_{i=1}^{m+1} z_i^2 - \frac{1}{m+1}\Big(\sum_{i=1}^{m+1} z_i\Big)^2\right) - \left(\sum_{i=1}^{m} z_i^2 - \frac{1}{m}\Big(\sum_{i=1}^{m} z_i\Big)^2\right) = \frac{(m z_{m+1} - \sum_{i=1}^{m} z_i)^2}{m(m+1)} \geq 0.$$

Thus, for $m_1 < m_2$, the sign of $b(s_{P,m_1}, s_{H,m_1}) - b(s_{P,m_2}, s_{H,m_2})$ changes at most once and the change is from negative to positive. Consider a output level $\hat{C}$ such that the number of farmers who choose the harvesting subsidy is $\hat{m}$. We have $2n\hat{C}/c_P \in [\phi^l(\hat{m}), \phi^u(\hat{m}))$. We deduce that for any $C_o \in [\hat{C}, \phi^u(\hat{m})c_P/(2n))$ with the number of farmers who choose the harvesting subsidy being m_o, $m_o \geq \hat{m}$. If not, suppose for some $C_o \in [\hat{C}, \phi^u(\hat{m})c_P/(2n))$, $m_o < \hat{m}$. Then, $2nC_o/c_P \in [\phi^l(m_o), \phi^u(m_o))$. We have two cases.

Case (a): $2n\hat{C}/c_P \in [\phi^l(m_o), \phi^u(m_o))$. This contradicts the fact that the sign of $b(s_{P,m_o}, s_{H,m_o}) - b(s_{P,\hat{m}}, s_{H,\hat{m}})$ changes at most once and change is from negative to positive.

Case (b): $2n\hat{C}/c_P < \phi^l(m_o)$. Let $\tilde{C}$ denote the single-crossing point of $b(s_{P,m_o}, s_{H,m_o})$ and $b(s_{P,\hat{m}}, s_{H,\hat{m}})$. We must have $\phi^l(m_o) < 2n\tilde{C}/c_P$. Thus, m_o is optimal for $C = \phi^l(m_o)c_P/(2n)$. However, for $C = \phi^l(m_o)c_P/(2n)$, $b^S(s_P, s_H)$ is not minimized at $s_P = s_{P,m_o}$, as s_{P,m_o} is the kink point of $b^S(s_P, s_H)$. This leads to a contradiction.

This suggests that for $C_a \in [\hat{C}, \phi^u(\hat{m})c_P/(2n))$, the number of farmers who choose the harvesting subsidy would be $m_a > m_o$. Then we can repeat the above argument for any $C \in [C_a, \phi^u(m_a)c_P/(2n))$. Thus, we conclude that m^S increases in $\bar{Q}$. This concludes the proof. $\square$

Proof of Proposition 4.5.1. It is straightforward to see that either combined subsidy or selective subsidy leads to a lower budget than the planting/harvesting only subsidy. Also, note that

$$b(0, s_H^H) - b(s_P^P, 0) = (nv_z/\beta)(c_P - s_P^P)s_P^P.$$

It is clear that $b(0, s_H^H) - b(s_P^P, 0) \geq [<]0$ for $s_P^P \leq [>]c_P$. Finally, note that

$$\begin{aligned}
&b(s_P^S, s_H^S) - b(s_P^C, s_H^C) \\
&= \frac{n}{4\beta}\left(c(m^S)\frac{\sum_{i=m^S+1}^n z_i^2}{\bar{z}\sum_{i=m^S+1}^n z_i} - 1\right)(2C - \bar{z}c_P)^2 + \left(\sum_{i=1}^{m^S} z_i^2 - \frac{1}{m^S}\Big(\sum_{i=1}^{m^S} z_i\Big)^2\right)\frac{c_P^2}{4\beta}.
\end{aligned}$$

It is clear that the both terms are both nonnegative as $(n - m^S)\sum_{j=m^S+1}^n z_j^2 \geq (\sum_{j=m^S+1}^n z_j)^2$ and $\sum_{j=1}^{m^S} z_j^2 \geq (\sum_{j=1}^{m^S} z_j)^2/m^S$, by the Cauchy-Schwartz inequality. Thus, $b(s_P^S, s_H^S) - b(s_P^C, s_H^C) \geq 0$. This concludes the proof. $\square$

Proof of Proposition 4.5.2. Substituting $(s_P^P, 0)$, $(0, s_H^H)$ and (s_P^C, s_H^C) into (4.7) and (s_P^S, s_H^S) into (4.12), respectively, we obtain

$$\begin{aligned}
&X^C(s_P^P, 0) = \bar{z}\bar{Q} - \frac{nv_z}{\beta}(c_P - s_P^P), \quad X^C(0, s_H^H) = \bar{z}\bar{Q} - \frac{nv_z}{\beta}c_P, \\
&X^C(s_P^C, s_H^C) = \bar{z}\bar{Q} - \frac{nv_z}{\beta}(c_P/2), \\
&X^S(s_P^S, s_H^S) = \bar{z}\bar{Q} - \frac{nv_z}{\beta}(c_P - s_P^S) + \frac{1}{\beta}\left(\sum_{i=1}^{m^S} z_i(\bar{z} - z_i)s_P^S + \sum_{i=m^S+1}^{n}(\bar{z} - z_i)s_H^S\right).
\end{aligned}$$

It is clear that $X^C(s_P^P, 0) - X^C(0, s_H^H) = nv_z s_P^P/\beta > 0$, $X^C(s_P^C, s_H^C) - X^C(0, s_H^H) = nv_z c_P/(2\beta) > 0$, and $X^C(s_P^C, s_H^C) - X^C(s_P^P, 0) = nv_z(c_P/2 - s_P^P)/\beta \geq [<]0$ for $s_P^P \leq [>]c_P/2$. For $\bar{Q} > Q^C(\frac{c_P}{2}, 0)$,

$$\frac{\mathrm{d}X^S(s_P^S, s_H^S)}{\mathrm{d}s_P^S} = \frac{\partial X^S}{\partial s_P^S} + \frac{\partial X^S}{\partial s_H^S}\frac{\mathrm{d}s_H^S}{\mathrm{d}s_P^S} = \frac{1}{m^S\beta}\left(\sum_{i=m^S+1}^{n} z_i^2 m - \Big(\sum_{i=1}^{m^S} z_i\Big)\Big(\sum_{i=m^S+1}^{n} z_i\Big)\right) \geq 0.$$

This suggests that $X^C(0, s_H^H) < X^S(s_P^S, s_H^S) < X^C(s_P^P, 0)$. This conclude parts i) and ii).

To see part iii), we note that

$$\frac{X^C(s_P^P,0)}{\bar{Q}} = \bar{z} - \frac{nv_z}{\beta\bar{Q}}(c_P - s_P^P), \quad \frac{X^C(0,s_H^H)}{\bar{Q}} = \bar{z} - \frac{nv_z c_P}{\beta\bar{Q}}, \quad \frac{X^C(s_P^C,s_H^C)}{\bar{Q}} = \bar{z} - \frac{nv_z c_P}{2\beta\bar{Q}}.$$

It is clear that $X^C(s_P^P, 0)/\bar{Q} \le [>]\bar{z}$ for $s_P^P \le [>]c_P$, $X^C(0, s_H^H)/\bar{Q} \le \bar{z}$ and $X^C(s_P^C, s_H^C)/\bar{Q} \le \bar{z}$. This concludes part iii).

To see part iv), it is clear that $X^C(0, s_H^H)/\bar{Q}$ is increasing in $\bar{Q}$ and thus we conclude that $X^C(0, s_H^H)/\bar{Q} \ge X^*/Q^*$. Because of parts i) and ii), we conclude that $X^C(s_P^P, 0)/\bar{Q} \ge X^*/Q^*$, $X^C(s_P^C, s_H^C)/\bar{Q} \ge X^*/Q^*$ and $X^S(s_P^S, s_H^S) \ge X^*/Q^*$. This concludes part iv). $\square$

Proof of Proposition 4.5.3. From Lemmas 4.5.1 and 4.5.2, we have

$$\begin{aligned}
\sum_{j=1}^{l} q_j^C(s_P^P,0) &= \frac{l\bar{Q}}{n} + \frac{c_P}{\beta}\Big(l\bar{z} - \sum_{j=1}^{l} z_j\Big) - \frac{C}{\beta\bar{z}}\Big(l\bar{z} - \sum_{j=1}^{l} z_j\Big), \\
\sum_{j=1}^{l} q_j^C(0,s_H^H) &= \frac{l\bar{Q}}{n} + \frac{c_P}{\beta}\Big(l\bar{z} - \sum_{j=1}^{l} z_j\Big), \\
\sum_{j=1}^{l} q_j^C(s_P^C,s_H^C) &= \frac{l\bar{Q}}{n} + \frac{c_P}{2\beta}\Big(l\bar{z} - \sum_{j=1}^{l} z_j\Big), \\
\sum_{j=1}^{l} q_j^S(s_P^S,s_H^S) &= \frac{l\bar{Q}}{n} + \frac{c_P}{\beta}\Big(l\bar{z} - \sum_{j=1}^{l} z_j\Big) + \frac{1}{\beta}\Big((m^S \wedge l)s_H^S + \mathbb{I}_{\{l>m^S\}}\sum_{j=m^S+1}^{l} z_j s_P^S - lC\Big).
\end{aligned}$$

Because $\sum_{j=1}^{l}(q_j^C(0, s_H^H) - q_j^C(s_P^P, 0)) = (l\bar{z} - \sum_{j=1}^{l} z_j)C/(\beta\bar{z}) \ge 0$, we have $\mathbf{q}^C(0, s_H^H)$ majorizes $\mathbf{q}^C(s_P^P, 0)$. Because $\sum_{j=1}^{l}(q_j^C(s_P^P, 0) - q_j^C(s_P^C, s_H^C)) = (\bar{z}c_P/2 - C)(l\bar{z} - \sum_{j=1}^{l} z_j)/(\beta\bar{z}) \ge [<]0$ for $C \le [>]\bar{z}c_P/2$, $\mathbf{q}^C(s_P^P, 0)$ majorizes $\mathbf{q}^C(s_P^C, s_H^C)$ for $C \le \bar{z}c_P/2$ (or $\bar{Q} \ge Q^C(c_P/2, 0)$) and $\mathbf{q}^C(s_P^C, s_H^C)$ majorizes $\mathbf{q}^C(s_P^P, 0)$ for $\bar{z}c_P/2 < C \le \bar{z}c_P$ (or $Q^C(c_P/2, 0) < \bar{Q} \le Q^C(c_P, 0)$).

Note that $s_H^S < C$ and thus $\sum_{j=1}^{l}\left(q_j^C(0, s_H^H) - q_j^S(s_P^S, s_H^S)\right) \ge 0$. We conclude that $\mathbf{q}^C(0, s_H^H)$ majorizes $\mathbf{q}^S(s_P^S, s_H^S)$ for $C \le \bar{z}c_P$. Also note that $s_H^S > z_1 C/\bar{z}$ and $s_P^S < C/\bar{z}$ and thus the sign of $\mathbb{I}_{\{j\le m\}}s_H^S + \mathbb{I}_{\{j>m\}}z_j s_P^S - z_j C/\bar{z}$ changes once and the change is from positive to negative. Thus, $\sum_{j=1}^{l}\left(q_j^S(s_P^S, s_H^S) - q_j^C(s_P^P, 0)\right) \geq 0$ and we conclude that $\mathbf{q}^S(s_P^S, s_H^S)$ majorizes $\mathbf{q}^C(s_P^P, 0)$ for $C \leq \bar{z}c_P$. This concludes part (i).

Part (ii) follows directly from Theorem A.2.(i) of Marshall et al. (1979), as $\pi_j^C = \beta(q_j^C)^2$ and $\pi_j^S = \beta(q_j^S)^2$. This concludes part (ii). $\square$

Proof of Corollary 4.5.1. To see part i), we have, from Lemmas 4.4.2 and 4.5.1,

$$\Delta^P q_j = \left(\frac{(n+1)z_j}{n\bar{z}} - 1\right)\delta \quad \text{and} \quad \Delta^H q_j = \Delta^C q_j = \frac{\delta}{n}.$$

It is clear that $\Delta^P q_j$ is increasing in j, and $\Delta^H q_j$ and $\Delta^C q_j$ are constant in j. Note that

$$\sum_{j=1}^{l}(\Delta^H q_j - \Delta^P q_j) = \frac{(n+1)\delta}{n\bar{z}}\left(l\bar{z} - \sum_{j=1}^{l} z_j\right) \geq 0.$$

The inequality follows from the fact that z_j is increasing in j. Because $\sum_{i\in N}(\Delta^H q_i - \Delta^P q_i) = 0$ and we conclude that $\mathbf{\Delta}^P\mathbf{q}$ majorizes $\mathbf{\Delta}^H\mathbf{q}$ and $\mathbf{\Delta}^C\mathbf{q}$.

To see part ii), we have

$$\Delta^H \pi_j = \frac{\delta}{n}\left(\frac{\beta(2\bar{Q}+\delta)}{n} - 2(z_j - \bar{z})c_P\right) \quad \text{and} \quad \Delta^C \pi_j = \frac{\delta}{n}\left(\frac{\beta(2\bar{Q}+\delta)}{n} - (z_j - \bar{z})c_P\right).$$

It is clear that both $\Delta^H \pi_j$ and $\Delta^C \pi_j$ are decreasing in j. Note that

$$\sum_{j=1}^{l}(\Delta^H \pi_j - \Delta^C \pi_j) = \frac{\delta c_P}{n}\left(l\bar{z} - \sum_{j=1}^{l} z_j\right) \geq 0.$$

This suggests that $\mathbf{\Delta}_H\boldsymbol{\pi}$ majorizes $\mathbf{\Delta}_C\boldsymbol{\pi}$. We conclude the proof. $\square$

Proof of Proposition 4.5.4. We substitute (4.8) into (4.14) to obtain

$$W^C(s_P, s_H) = \frac{1}{\beta}\left(\frac{\beta^2(n^2+2n)}{2n^2}\bar{Q}^2 + nv_z(c_P - s_P)^2\right).$$

It is clear that $W^C(0, s_H^H) - W^C(s_P^C, s_H^C) = 3nv_z c_P^2/(4\beta) > 0$. Note that

$$\begin{aligned} W^C(s_P^P, 0) - W^C(0, s_H^H) &= \frac{nv_z s_P^P}{\beta}(s_P^P - 2c_P) \\ W^C(s_P^P, 0) - W^C(s_P^C, s_H^C) &= \frac{nv_z}{4\beta}(c_P - 2s_P^P)(3c_P - 2s_P^P). \end{aligned}$$

Thus, when $c_P/2 \le s_P^P \le 3c_P/2$, $W^C(s_P^C, s_H^C) \ge W^C(s_P^P, 0)$. When $s_P^P \le c_P/2$ or $3c_P/2 \le s_P^P \le 2c_P$, $W^C(s_P^C, s_H^C) \le W^C(s_P^P, 0) \le W^C(0, s_H^H)$. When $s_P^P \ge 2c_P$, $W^C(s_P^P, 0) \ge W^C(0, s_H^H)$.

Substituting (4.8) into (4.15), we obtain

$$NW^C(s_P, s_H) = \frac{1}{\beta}\left(\frac{\beta^2(n^2+2n)}{2n^2}\bar{Q}^2 - \bar{Q}\left(\frac{\beta(n+1)}{n}\bar{Q} - \bar{a}(0,0)\right) + nv_z c_P(c_P - s_P)\right).$$

It is clear that $NW^C(0, s_H^H) - NW^C(s_P^C, s_H^C) = nv_z c_P^2/(2\beta) > 0$ and $NW^C(0, s_H^H) - NW^C(s_P^P, 0) = nv_z c_P s_P^P/\beta \ge 0$. Also note that $NW^C(s_P^P, 0) - NW^C(s_P^C, s_H^C) = nv_z c_P(c_P/2 - s_P^P)/\beta$. Thus, $NW^C(s_P^P, 0) \ge [<] NW^C(s_P^C, s_H^C)$ for $s_P^P \le [>] c_P/2$. $\square$

D. Derivation of the Sellers' Profit Functions in Section 2.5.1

In this section, we provide the derivation of sellers' profit functions in the dynamic competition in Section §2.5. To do so, we need to understand the buyer's choice between the sellers, i.e., which seller the buyer would purchase from. Following a similar argument as that for static competition in Section §§2.4.2, the buyer should evaluate the sign of

$$\Delta^D = \left(\mathbb{I}_{\{s_j<v_j\}}s_j + \mathbb{I}_{\{s_j\geq v_j\}}s_{Bj}(r, s_j, v_j)\right) - \left(\mathbb{I}_{\{s_i<v_i\}}s_i + \mathbb{I}_{\{s_i\geq v_i\}}s_{Bi}(r, s_i, v_i)\right).$$

The buyer chooses seller $j(i)$ if Δ^D is negative (positive), and is indifferent between the sellers if $\Delta^D = 0$. We have two cases to consider, depending on whether the sellers have the same disagreement points.

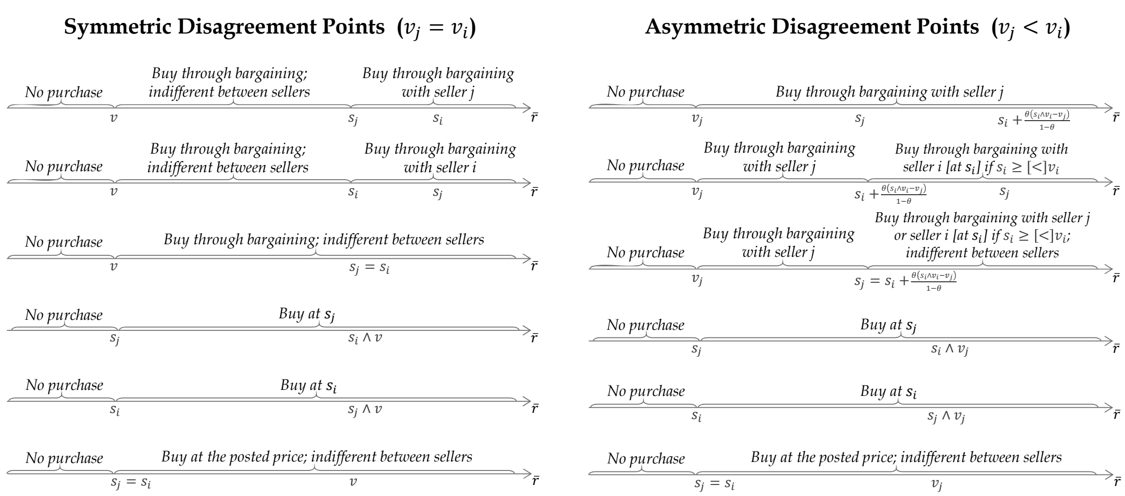


Figure D.1. Summary of buyer's choice in the dynamic competition, as characterized in (2.12), (2.13) and (2.14).

If the sellers' disagreement points happen to be the same (i.e., $v_j = v_i = v$), the buyer would face six potential scenarios, which are demonstrated in the left panel of Figure D.1.

Case(a): $v \le s_j < s_i$. If $r < v$, the buyer quits the market. If $v \le r \le s_j$, the buyer is indifferent between two sellers because $s_{Bj}(r, s_j, v) = (1-\theta)r + \theta v = s_{Bi}(r, s_i, v)$. If $s_j < r$, the buyer purchases from seller j at the negotiated price because $s_{Bj}(r, s_j, v) = (1-\theta)s_j + \theta v < (1-\theta)(r \wedge s_i) + \theta v = s_{Bi}(r, s_i, v)$.

Case(b): $v \le s_i < s_j$. Similar to that in Case (a), if $r < v$, the buyer quits the market. If $v \le r \le s_i$, the buyer purchases at the negotiated price and is indifferent between two sellers. If $s_i < r$, the buyer purchases from seller i at the negotiated price.

Case(c): $v \le s_i = s_j = s$. If $r < v$, the buyer quits the market. If $v \le r$, the buyer purchases at the negotiated price and is indifferent between two sellers because $s_{Bj}(r, s_j, v) = (1-\theta)(r \wedge s) + \theta v = s_{Bi}(r, s_i, v)$.

Case(d): $s_j < v \wedge s_i$. If $r < s_j$, the buyer quits the market. If $s_j \le r$, the buyer purchases from seller j at the posted price because $s_j < \mathbb{I}_{\{s_i < v\}} s_i + \mathbb{I}_{\{s_i \ge v\}} s_{Bi}(r, s_i, v)$.

Case(e): $s_i < v \wedge s_j$. Similar to that in Case (e), if $r < s_i$, the buyer quits the market. If $s_i \le r$, the buyer purchases from seller i at the posted price.

Case(f): $s_j = s_i = s < v$. If $r < s$, the buyer quits the market. If $s \le r$, the buyer purchase at the posted price and is indifferent between the sellers.

In the above cases, each seller obtains a value of v if the buyer walks away. If seller j is chosen, seller j obtains a value of the trading price s_{Bj} or s_j and seller i obtains her reservation value w_i. Combining the above cases, we obtain the expressions of S_i, $i \in \{1, 2, \ldots, 6\}$ in (2.12).

If the sellers' disagreement points are not the same (i.e., $v_j < v_i$), the buyer would face six potential scenarios, which are demonstrated in the right panel of Figure D.1.

Case(a′): $v_j \le s_j < s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta}$. If $r < v_j$, the buyer quits the market. If $v_j \le r$, the buyer purchases from seller j at the negotiated price because $s_{Bj}(r, s_j, v_j) =$

$(1-\theta)(s_j \wedge r) + \theta v_j < ((1-\theta)s_i + \theta(s_i \wedge v_i)) \wedge ((1-\theta)r + \theta v_i) = \mathbb{I}_{\{s_i < v_i\}}(s_i \wedge ((1-\theta)r + \theta v_i)) + \mathbb{I}_{\{s_i \geq v_i\}} s_{Bi}(r, s_i, v_i)$.

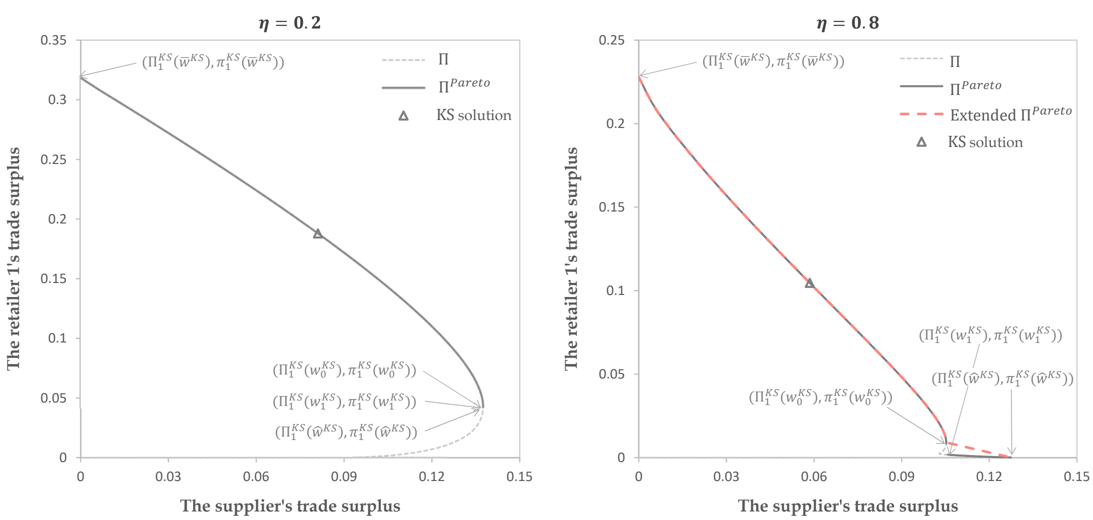
Case(b′): $v_j \leq s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta} < s_j$. Similar to that in Case (a′), if $r < v_j$, the buyer quits the market. If $v_j \leq r < s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta}$, the buyer purchases from seller j at the negotiated price. If $s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta} < r$, the buyer purchases from seller i at the negotiated [posted] price for $s_i \geq [<] v_i$.

Case(c′): $v_j \leq s_j = s_i + \frac{\theta(s_i \wedge v_i - v_j)}{1-\theta}$. If $r < v_j$, the buyer quits the market. If $v_j \leq r < s_j$, the buyer purchases from seller j at the negotiated price. If $s_j \leq r$, the buyer is indifferent between two sellers and purchase from seller j at the negotiated price or from seller i at the negotiated [posted] price for $s_i \geq [<] v_i$.

Cases (d′), (e′) and (f′) are equivalent to that of Cases (d), (e) and (f). Combining the above cases, we obtain the expressions of A_i, $i \in \{1, 2, \ldots, 8\}$ in (2.13) and (2.14).

E. The Feasible Profit Allocation Set

In general, the feasible set of profit allocation is a curve, which is not a convex set under the wholesale-price contract. To facilitate our analysis with the nonconvex feasible set, the common approach is to allow randomized contracts. That is, the decision becomes a choice of probability distribution over all possible wholesale prices. This randomization convexifies the profit allocation set. When no contingency terms are imposed on contracting (see Feng and Lu 2013a), the convexified region is determined by the original profit allocation curve and the two axis. The Pareto set of the convexified set, where the negotiation outcome lies, is a subset of the original profit allocation curve. Thus, it is sufficient to consider only non-randomized contracts in this case.



Note. $a = 1$, $b = 1$, $c = 0$.

When contingency terms are imposed in contract execution, however, the profit allocation set can reveal complex structure. To see that, consider the first trade in the sequential negotiation over contingent contracts. The profit allocation set is $\mathbf{\Pi} =$

$\{(\Pi_1^i(w_1), \pi_1^i(w_1)) : \bar{w}^i \leq w_1 \leq a\}$, where $\bar{w}^i = \min\{w_1 : \Pi_1^i(w_1) \geq 0\}$, $i \in \{NB, KS\}$, and the Pareto set is $\mathbf{\Pi}^{Pareto} = \{(\Pi_1^{KS}(w_1), \pi_1^{KS}(w_1)) : \bar{w}^i \leq w_1 \leq w_0^i$ and $w_1^i \leq w_1 \leq \hat{w}^i\}$, where $\hat{w}^i = \arg\max\{\Pi_1^i(w_1)\}$, $w_0^i = \min\{w_1 : \mathrm{d}\Pi_1^i(w_1)/\mathrm{d}w_1 \leq 0\}$ and $w_1^i = \mathbb{I}_{\{\hat{w}^i = w_0^i\}} w_0^i + \mathbb{I}_{\{\hat{w}^i > w_0^i\}} \min\{w_1 : w_1 > w_0^i$ and $\Pi_1^i(w_1) \geq \Pi_1^i(w_0^i)\}$. When the level of competition is low (i.e., when η is small), indeed $\mathbf{\Pi}^{Pareto} \subset \mathbf{\Pi}$ (or $w_0^i = w_1^i = \hat{w}^i$). For a large η, however, this is not true. In this case, the convexified region for the entire set of $\{\mathbf{\Pi}\}$ would make part of the original Pareto set non-Pareto, inducing discontinuity of the bargaining solution. As we would like to focus on non-randomized strategies, we take an alternative approach. Specifically, we extend the Pareto set $\mathbf{\Pi}^{Pareto}$ by including the segment connecting $(\Pi_1^i(\hat{w}^i), \pi_1^i(\hat{w}^i))$ and $(\Pi_1^i(w_0^i), \pi_1^i(w_0^i))$.

F. Derivation of the Bargaining Solutions

In this section, we provide the expressions of the firms' profit functions and the derivation of the bargaining solutions. We shall note that under the linear demands, the firms' profits can be expressed as a product of $(a-c)^2/b$ and some function of η. Thus, it is without loss of generality to take $a=1$, $b=1$ and $c=0$ in our analysis. For general values of a, b and c, the corresponding firms' profits would be $(a-c)^2/b\Pi$ and $(a-c)^2/b\pi$, and the corresponding wholesale price would be $c+(a-c)w$, where Π, π and w are computed for $a=1$, $b=1$ and $c=0$.

F.1 One-to-Two Channel

By Lemma 3.4.1, we can derive the trade profits for the supplier and the retailers, respectively, as

$$\begin{aligned}
\Pi_i(w_i,w_j) &= \sum_{i=1}^{2} w_i q_i^*(\mathbf{w}) \\
&= \begin{cases} w_j \frac{1-w_j}{2} & 1-w_i \le \frac{\eta}{2}(1-w_j), \\ \sum_{i=1}^{2} w_i \frac{2(1-w_i)-\eta(1-w_j)}{4-\eta^2} & \frac{\eta}{2}(1-w_j) < 1-w_i < \frac{2}{\eta}(1-w_j), \\ w_i \frac{1-w_i}{2} & 1-w_i \ge \frac{2}{\eta}(1-w_j). \end{cases} \qquad \text{(F.1)} \\
\pi_i(w_i,w_j) &= (p_i^*(\mathbf{w})-w_i)q_i^*(\mathbf{w}) \\
&= \begin{cases} 0 & 1-w_i \le \frac{\eta}{2}(1-w_j), \\ \frac{(2(1-w_i)-\eta(1-w_j))^2}{(4-\eta^2)^2} & \frac{\eta}{2}(1-w_j) < 1-w_i < \frac{2}{\eta}(1-w_j), \\ \frac{(1-w_i)^2}{4} & 1-w_i \ge \frac{2}{\eta}(1-w_j). \end{cases} \qquad \text{(F.2)}
\end{aligned}$$

The supplier's disagreement point under simultaneous negotiation without contingency is

$$D_i(w_j) = w_j \frac{1-w_j}{2}. \qquad \text{(F.3)}$$

Lemma F.1.1 *In the one-to-two channel without contingency, the maximum profits of the supplier and the retailers are, respectively,*

$$\overline{\Pi}_i(w_j) = \begin{cases} \Pi_i(\frac{2-\eta+2\eta w_j}{4}, w_j) = \frac{2-\eta}{8(2+\eta)} + \frac{1}{2}w_j(1-w_j) & w_j < \frac{1}{2} + \frac{1}{2}\sqrt{\frac{2-\eta}{2+\eta}}, \\ \Pi_i(\frac{1}{2}, w_j) = \frac{1}{8} & w_j > \frac{1}{2} + \frac{1}{2}\sqrt{\frac{2-\eta}{2+\eta}}, \end{cases} \tag{F.4}$$

$$\overline{\pi}_i(w_j) = \begin{cases} \pi_i(\frac{\eta w_j}{2}, w_j) = \frac{1}{(2+\eta)^2} & w_j < \frac{2}{2+\eta}, \\ \pi_i(1-w_j, w_j) = \frac{w_j^2}{4} & w_j > \frac{2}{2+\eta}. \end{cases} \tag{F.5}$$

Moreover, the profit allocation $(\Pi_i(w_i, w_j), \pi_i(w_i, w_j))$ *is Pareto-dominated for* w_i *above the maxima that attain* $\overline{\Pi}_i(w_j)$.

Proof. We first note that any feasible (w_i, w_j) should lead to nonnegative trade surpluses $\Pi_i(w_i, w_j) - D_i(w_j)$ and $\pi_i(w_i, w_j) - d_i(w_j)$, and thus we have $w_i \geq \eta w_j/2$ in the second pieces and $w_i \geq 1 - w_j$ in the third pieces in (F.1) and (F.2), respectively.

To derive (F.4), we note that $\Pi_i(w_i, w_j)$ is constant in w_i in the first case in (F.1). The second piece is maximized at $w_i^m = (2 - \eta + 2\eta w_j)/4$ and leads to a maximum value of $\Pi^m = (2-\eta)/(8(2+\eta)) + (1/2)w_j(1-w_j)$. The third piece is maximized at $w_i^r = 1/2$ and leads to a maximum value of $\Pi^r = 1/8$. It is easy to check that $(2-\eta)/4 < w_i^m < (2+\eta)/4$ and $(2-\eta)/4 < (1 - w_i^m) < (2+\eta)/4$.

For w_i^r be the maxima, we must have two cases: *Case (i):* $1 - w_i^r > (2/\eta)(1-w_j)$ (or $w_j > w^a \equiv 1 - \eta/4$), $1 - w_i^m < (2/\eta)(1-w_j)$ (or $w_j < w^b \equiv 1 - \eta/(2(2+\eta))$) and $\Pi^m < \Pi^r$ (or $w_j < w^c \equiv (1/2) - \sqrt{(2-\eta)/(2+\eta)}/2$ or $w_j > w^d \equiv (1/2) + \sqrt{(2-\eta)/(2+\eta)}/2$). It is easy to show that $w^a < w^d < w^b$ and the above conditions lead to $w^d < w_j < w^b$. *Case (ii):* $1 - w_i^r > (2/\eta)(1-w_j)$ (or $w_j > w^a$) and $1 - w_i^m > (2/\eta)(1-w_j)$ (or $w_j > w^b$), which leads to $w_j > w^b$.

For w_i^m to be the maxima, we can have two cases: *Case (i):* $1 - w_i^r > (2/\eta)(1-w_j)$ (or $w_j > w^a$), $1 - w_i^m < (2/\eta)(1-w_j)$ (or $w_j < w^b$) and $\Pi^m > \Pi^r$ (or $w^c < w_j < w^d$), which leads to $w^a < w_j < w^d$. *Case (ii):* $1 - w_i^r < (2/\eta)(1-w_j)$ (or $w_j < w^a$) and $1 - w_i^m < (2/\eta)(1-w_j)$ (or $w_j < w^b$). These give the relation $w_j < w^a$.

Combining the above cases, we obtain the expression of (F.4).

Now we note that the second piece of $\pi_i(w_i, w_j)$ in (F.2) is decreasing in w_i and is maximized at $w_i = \eta w_j/2 \equiv w_i^p$. The third piece is decreasing in w_i and is maximized at $w_i = 1 - w_j \equiv w_i^q$. For w_i^p to be maxima, we must have $1 - w_i^p < (2/\eta)(1 - w_j)$ (or $w_j < 2/(2+\eta) \equiv w^e$). For w_i^q to be maxima, we must have $1 - w_i^q > (2/\eta)(1 - w_j)$ (or $w_j > w^e$). This leads to the expression of (F.5). □

Lemma F.1.2 *In the one-to-two channel without contingency, for a given w_j in bargaining unit j, the negotiated wholesale prices in unit i under the NB and KS solutions are*

$$w_i^{NB}(w_j) = \begin{cases} \frac{(2-\eta)(1-\theta)+2\eta w_j}{4} & w_j < \bar{w}_j^a, \\ \frac{3-\theta-\sqrt{(1+\theta)^2-16\theta w_j(1-w_j)}}{4} & w_j > \bar{w}_j^a, \end{cases} \tag{F.6}$$

$$w_i^{KS}(w_j) = \begin{cases} \frac{\eta w_j}{2} + \frac{2-\eta}{10} & w_j < \frac{2}{2+\eta}, \\ \frac{\eta w_j}{2} + \frac{2-\eta}{2+2(2+\eta)^2 w_j^2} & \frac{2}{2+\eta} < w_j < \bar{w}_j^b, \\ \frac{1}{2} - \frac{-2+\eta+4\sqrt{w_j^3(2+\eta)(4w_j-4(2+\eta)w_j^2(1-w_j)-2+\eta)}}{2(2-\eta+4(2+\eta)w_j^2)} & w_j^b < w_j < \frac{1}{2} + \frac{1}{2}\sqrt{\frac{2-\eta}{2+\eta}}, \\ \frac{1}{2} - \frac{-(2w_j-1)^2+4\sqrt{w_j^3(2w_j-1)^3}}{2(1-4w_j+8w_j^2)} & w_j > \frac{1}{2} + \frac{1}{2}\sqrt{\frac{2-\eta}{2+\eta}}, \end{cases} \tag{F.7}$$

where $\bar{w}_j^a$ is some value within $[(4-\eta(1+\theta))/(4-\eta^2\theta), (4+\eta(1-\theta))/(2(2+\eta))]$ and $\bar{w}_j^b$ is some value within $[2/(2+\eta), (4-\eta)/4]$.

Proof. Applying (F.1), (F.2), (F.3) and $d_i(w_j) = 0$, the Nash product for trade i is

$$\Omega_i(w_i, w_j) = \begin{cases} 0 & 1 - w_i \le \frac{\eta(1-w_j)}{2}, \\ \left(\frac{(2w_i-\eta w_j)(2(1-w_i)-\eta(1-w_j))}{2(4-\eta^2)}\right)^{1-\theta}\left(\frac{(2(1-w_i)-\eta(1-w_j))^2}{(4-\eta^2)^2}\right)^{\theta} & \frac{\eta(1-w_j)}{2} < 1 - w_i < \frac{2(1-w_j)}{\eta}, \\ \left(\frac{w_i(1-w_i)-w_j(1-w_j)}{2}\right)^{1-\theta}\left(\frac{(1-w_i)^2}{4}\right)^{\theta} & 1 - w_i \ge \frac{2(1-w_j)}{\eta}. \end{cases}$$

Setting $\partial \ln(\Omega)/\partial w_i = 0$ in the second case gives

$$(1-\theta)\frac{2}{2w_i - \eta w_j} + (1+\theta)\frac{-2}{2(1-w_i) - \eta(1-w_j)} = 0.$$

This gives $w_i^m = ((1-\theta)(2-\eta) + 2\eta w_j)/4$. We shall note that $w_i^m \ge \eta w_j/2$ so that the supplier's surplus is nonnegative under this solution. Setting $\partial \ln(\Omega)/\partial w_i = 0$ in the third case gives

$$(1-\theta)\frac{1 - 2w_i}{w_i - w_i^2 - w_j + w_j^2} + 2\theta\frac{-1}{1 - w_i} = 0.$$

This gives $w_i^r = (3-\theta-\sqrt{(1+\theta)^2-16\theta w_j(1-w_j)})/4 \le 1/2$. Note that the above expression has two roots within $[0,1]$ and the maximizer of Ω_i should be the smaller root.

For w_i^m be the best response, we can have two cases: *Case (i):* $1-w_i^m < (2/\eta)(1-w_j)$ (or $w_j < w^a \equiv (4+\eta(1-\theta))/(2(2+\eta)))$ and $1-w_i^r < (2/\eta)(1-w_j)$ (or $w_j < w^b \equiv (4-\eta(1+\theta))/(4-\eta^2\theta))$. It is easy to show that $w^a > w^b$ and the above conditions lead to $w_j < w^b$. *Case (ii):* $1-w_i^m < (2/\eta)(1-w_j)$ (or $w_j < w^a$), $1-w_i^r > (2/\eta)(1-w_j)$ (or $w_j > w^b$) and $\Omega_i(w_i^m, w_j) > \Omega_i(w_i^r, w_j)$. Note that $\Omega_i(w_i^m, w_j) = ((2-\eta)(1-\theta^2))^{1-\theta}(1+\theta)^{2\theta}/(2^{3-\theta}(2+\eta)^{1+\theta})$ and is constant in w_j. Also, note that $\Omega_i(w_i^r, w_j) = ((1-\theta)(1+\theta-8w_j(1-w_j)+\sqrt{(1+\theta)^2-16\theta w_j(1-w_j)}))^{1-\theta}(1+\theta+\sqrt{(1+\theta)^2-16\theta w_j(1-w_j)})^{2\theta}/2^{4+2\theta}$ and is increasing in w_j for $w_j > w^a (> 1/2)$. Finally, note that $\Omega_i(w_i^r, w^b) < \Omega_i(w_i^m, w_j) < \Omega_i(w_i^r, w^a)$. We deduce that there exists a $\bar{w}_j^a$ such that $\Omega_i(w_i^m, w_j) > (<) \Omega_i(w_i^r, w_j)$ for $w_j < (>) \bar{w}_j^a$. These give the relation $w^b < w_j < \bar{w}_j^a$.

For w_i^r be the best response, we must have two cases: *Case (i):* $1-w_i^m > (2/\eta)(1-w_j)$ (or $w_j > w^a$) and $1-w_i^r > (2/\eta)(1-w_j)$ (or $w_j > w^b$), which leads to $w_j > w^a$. *Case (ii):* $1-w_i^m < (2/\eta)(1-w_j)$ (or $w_j < w^a$), $1-w_i^r > (2/\eta)(1-w_j)$ (or $w_j > w^b$) and $\Omega_i(w_i^m, w_j) < \Omega_i(w_i^r, w_j)$ (or $w_j > \bar{w}_j^a$), which leads to $\bar{w}_j^a < w_j < w^a$.

Combining the above cases, we obtain the expression of (F.6).

To derive (F.7), we substitute the expressions of (F.1), (F.2), (F.4), (F.5), (F.3) and $d_i(w_j) = 0$ into (3.2) and obtain

$$\begin{cases} \frac{(2(1-w_i)-\eta(1-w_j))^2/(4-\eta^2)^2}{(2w_i-\eta w_j)(2(1-w_i)-\eta(1-w_j))/(2(4-\eta^2))} = \frac{1/(2+\eta)^2}{(2-\eta)/(8(2+\eta))} & w_j < \frac{2}{2+\eta}, \\ \frac{(2(1-w_i)-\eta(1-w_j))^2/(4-\eta^2)^2}{(2w_i-\eta w_j)(2(1-w_i)-\eta(1-w_j))/(2(4-\eta^2))} = \frac{w_j^2/4}{(2-\eta)/(8(2+\eta))} & \frac{2}{2+\eta} < w_j < \frac{1}{2}+\frac{1}{2}\sqrt{\frac{2-\eta}{2+\eta}} \text{ and } \frac{\eta}{2} < \frac{1-w_i}{1-w_j} < \frac{2}{\eta}, \\ \frac{(1-w_i)^2/4}{(w_i(1-w_i)-w_j(1-w_j))/2} = \frac{w_j^2/4}{(2-\eta)/(8(2+\eta))} & \frac{2}{2+\eta} < w_j < \frac{1}{2}+\frac{1}{2}\sqrt{\frac{2-\eta}{2+\eta}} \text{ and } \frac{1-w_i}{1-w_j} \ge \frac{2}{\eta}, \\ \frac{(1-w_i)^2/4}{(w_i(1-w_i)-w_j(1-w_j))/2} = \frac{w_j^2/4}{1/8-w_j(1-w_j)/2} & w_j > \frac{1}{2}+\frac{1}{2}\sqrt{\frac{2-\eta}{2+\eta}}. \end{cases}$$

We omit the expressions for w_i above the maxima that attain $\overline{\Pi}_i(w_j)$ as the best response should lead to a Pareto profit allocation. Let $w^c \equiv 2/(2+\eta)$ and $w^d \equiv (1/2)+\sqrt{(2-\eta)/(2+\eta)}/2$.

The first piece gives $w_i^p = (2-\eta+5\eta w_j)/10$.

The second piece gives $w_i^q = \eta w_j/2 + (2-\eta)/(2+2(2+\eta)^2 w_j^2)$. For w_i^q be the best response, we must have $1 - w_i^q < (2/\eta)(1-w_j)$, which yields

$$(4\eta - \eta^3)(1 + (2+\eta)^2 w_j^2)(1 - w_j + 2(2+\eta)w_j^2 - (2+\eta)^2 w_j^3) > 0.$$

Note that the first and second terms are positive and the third term is a cubic function in w_j with the coefficient of w_j^3 being negative. Also, the third term equals $\eta/(2+\eta) > 0$ at $w_j = w^c$, equals $-\eta(48 - 32\eta - 12\eta^2 + 8\eta^3 - \eta^4)/64 < 0$ at $w_j = (4-\eta)/4 \equiv w^e$, and its first-order condition gives $-1 + 4(2+\eta)w_j - 3(2+\eta)^2 w_j^2$, which is negative for w_j within $[w^c, w^e]$. Hence, there exists only one root within $[w^c, w^e]$. Let $\bar{w}_j^b$ denote this root. These give the relation $w^c < w_j < \bar{w}_j^b$.

The third piece gives $w_i^s = 1/2 - \big(1/(2(2-\eta+4(2+\eta)w_j^2))\big)\big(-2+\eta+4\sqrt{w_j^3(2+\eta)(4w_j - 4(2+\eta)w_j^2(1-w_j) - 2+\eta)}\big)$. Note that the third piece has two roots within $[0,1]$ and the best response should be the smaller root. For w_i^s be the best response, we must have $1 - w_i^s > (2/\eta)(1-w_j)$, which implies

$$(1-w_j)(2-\eta)(2-\eta+4(2+\eta)w_j^2)(1 - w_j + 2(2+\eta)w_j^2 - (2+\eta)^2 w_j^3) < 0,$$

and thus $w_j > \bar{w}_j^b$. These yield the relation $\bar{w}_j^b < w_j < w^d$. We shall note that for a w_j within $[w^e, w^d]$, the profit allocation $(\Pi_i(w_i, w_j), \pi_i(w_i, w_j))$ is not Pareto for $w_i \leq (2-\eta+2\eta w_j)/4$ (i.e., the maxima that attain $\overline{\Pi}_i(w_j)$) because $(\Pi_i(w_i, w_j), \pi_i(w_i, w_j))$ is Pareto-dominated for $w_i \in (1/2, 1-(2/\eta)(1-w_j))$. Since $w_i^s < 1/2$ for w_j within $[w^e, w^d]$, w_i^s always leads to a Pareto profit allocation.

The fourth piece gives $w_i^t = 1/2 - (-(2w_j-1)^2 + 4\sqrt{w_j^3(2w_j-1)^3})/(2(1-4w_j+8w_j^2)) < 1/2$. Note that the fourth piece has two roots within $[0,1]$ and the best response should be the smaller root. Combining the above cases, we obtain the expression of (F.7). $\square$

Lemma F.1.3 *In the one-to-two channel under sequential negotiation, suppose the supplier first negotiates with retailer j.*

i) Without contingency and the negotiated price in unit i is given by (F.6), the supplier and retailer j's profits are

$$\begin{aligned}
\Pi_j(w_j) &\equiv \Pi_j(w_j, w_i^{NB}(w_j)) \\
&= \begin{cases} \frac{w_j(1-w_j)}{2} + \frac{(2-\eta)(1-\theta^2)}{8(2+\eta)} & w_j < \bar{w}_j^a, \\ \frac{8\theta w_j(1-w_j)+(1-\theta)(1+\theta+\sqrt{(1+\theta)^2-16\theta w_j(1-w_j)})}{16} & w_j > \bar{w}_j^a, \end{cases} \qquad \text{(F.8)} \\
\pi_j(w_j) &\equiv \pi_j(w_j, w_i^{NB}(w_j)) \\
&= \begin{cases} \frac{(4+\eta(1-\theta)-2(2+\eta)w_j)^2}{16(2+\eta)^2} & w_j < \bar{w}_j^a, \\ 0 & w_j > \bar{w}_j^a, \end{cases} \qquad \text{(F.9)}
\end{aligned}$$

and the supplier's disagreement point is

$$D_j = \frac{1-\theta^2}{8}. \qquad \text{(F.10)}$$

ii) With contingency and the negotiated price in unit i is given by (F.6), the supplier's profit is

$$\Pi_j(w_j) = \begin{cases} \frac{w_j(1-w_j)}{2} + \frac{(2-\eta)(1-\theta^2)}{8(2+\eta)} & w_j < \bar{w}_j^a, \\ 0 & w_j > \bar{w}_j^a, \end{cases} \qquad \text{(F.11)}$$

and retailer j's profit is same as that without contingency, and the supplier's disagreement point is 0.

iii) Without contingency and the negotiated price in unit i is given by (F.7), the supplier and retailer j's profits are

$$\begin{aligned}
\Pi_j(w_j) &\equiv \Pi_j(w_j, w_i^{KS}(w_j)) \\
&= \begin{cases} \frac{w_j(1-w_j)}{2} + \frac{2(2-\eta)}{25(2+\eta)} & w_j < \frac{2}{2+\eta}, \\ \frac{w_j(1+(3-\eta^2)w_j+(2+\eta)^2 w_j^2(1-w_j)(2+(2+\eta)^2 w_j^2))}{2(1+(2+\eta)^2 w_j^2)^2} & \frac{2}{2+\eta} < w_j < \bar{w}_j^b, \\ \frac{1}{8} - \frac{(-2+\eta+4\sqrt{w_j^3(2+\eta)(4w_j-4(2+\eta)w_j^2(1-w_j)-2+\eta)})^2}{8(2-\eta+4(2+\eta)w_j^2)^2} & \bar{w}_j^b < w_j < \frac{1}{2}+\frac{1}{2}\sqrt{\frac{2-\eta}{2+\eta}}, \\ \frac{1}{8} - \frac{(-(2w_j-1)^2+4\sqrt{w_j^3(2w_j-1)^3})^2}{8(1-4w_j+8w_j^2)^2} & w_j > \frac{1}{2}+\frac{1}{2}\sqrt{\frac{2-\eta}{2+\eta}}, \end{cases} \qquad \text{(F.12)} \\
\pi_j(w_j) &\equiv \pi_j(w_j, w_i^{KS}(w_j)) \\
&= \begin{cases} \frac{(10+\eta-5(2+\eta)w_j)^2}{100(2+\eta)^2} & w_j < \frac{2}{2+\eta}, \\ \frac{(1-w_j+2(2+\eta)w_j^2-(2+\eta)^2 w_j^3)^2}{4(1+(2+\eta)^2 w_j^2)^2} & \frac{2}{2+\eta} < w_j < \bar{w}_j^b, \\ 0 & w_j > \bar{w}_j^b, \end{cases} \qquad \text{(F.13)}
\end{aligned}$$

their maximum profits are

$$\overline{\Pi}_j = \Pi_j\big(\frac{1}{2}\big) = \frac{82+9\eta}{200(2+\eta)}, \tag{F.14}$$

$$\overline{\pi}_j = \pi_j\big(\frac{1}{2} - \frac{1}{10}\sqrt{\frac{50-7\eta}{2+\eta}}\big) = \frac{\big(10-3\eta+\sqrt{(50-7\eta)(2+\eta)}\big)^2}{400(2+\eta)^2}, \tag{F.15}$$

and the supplier's disagreement point is

$$D_j = \frac{2}{25}. \tag{F.16}$$

iv) *With contingency and the negotiated price in unit i is given by (F.7), the supplier's profit is*

$$\Pi_j(w_j) = \begin{cases} \frac{w_j(1-w_j)}{2} + \frac{2(2-\eta)}{25(2+\eta)} & w_j < \frac{2}{2+\eta}, \\ \frac{w_j(1+(3-\eta^2)w_j+(2+\eta)^2w_j^2(1-w_j)(2+(2+\eta)^2w_j^2))}{2(1+(2+\eta)^2w_j^2)^2} & \frac{2}{2+\eta} < w_j < \bar{w}_j^b, \\ 0 & w_j > \bar{w}_j^b, \end{cases} \tag{F.17}$$

and retailer j's profit is same as that of without contingency. Moreover, the supplier's maximum profit is same as that of without contingency and retailer j's maximum profit is

$$\overline{\pi}_j = \pi_j\big(\frac{1}{2} - \frac{1}{10}\sqrt{\frac{82+9\eta}{2+\eta}}\big) = \frac{(10-3\eta+\sqrt{(82+9\eta)(2+\eta)})^2}{400(2+\eta)^2}, \tag{F.18}$$

and the supplier's disagreement point is 0.

Moreover, the profit allocation $(\Pi_j(w_j), \pi_j(w_j))$ is Pareto-dominated for w_j above the maxima that attain $\overline{\Pi}_j$.

Proof. To see part (i), we substitute the expression of (F.6) into (F.1) and (F.2) to derive (F.8) and (F.9), respectively. We substitute the negotiated price $w^{NB} = (1-\theta)/2$ (see Proposition 3.4.1) in the one-to-one channel into (F.3) to derive (F.10). This concludes part (i).

To see part (ii), we note that when the supplier negotiates with retailer j with contingency, her profit becomes zero if retailer j does not order positive quantity (i.e.,

$w_j > \bar{w}_j^a$). Modifying the second piece in (F.8) leads to that of (F.11). We conclude part (ii).

To see part (iii), we substitute the expression of (F.7) into (F.1) and (F.2) to derive (F.12) and (F.13), respectively. We substitute the negotiated price $w^{KS} = 1/5$ (see Proposition 3.4.1) in the one-to-one channel into (F.3) to derive (F.16).

We now consider the supplier and retailer j's maximum profits. We shall note that any feasible w_j should lead to nonnegative trade surpluses $\Pi_j(w_j) - D_j$ and $\pi_j(w_j) - d_j$ and thus we have $w_j \geq 1/2 - (1/10)\sqrt{(50 - 7\eta)/(2 + \eta)} \equiv w^a$ in the first pieces in (F.12) and (F.13), respectively.

To derive (F.14), we note that the first piece of $\Pi_j(w_j)$ in (F.12) is maximized at $w_j^m = 1/2$ and leads to a maximum value of $\Pi^m = 1/8 + (2/25)(2 - \eta)/(2 + \eta)$. The second piece is decreasing in w_j and is maximized at $w_j^r = 2/(2 + \eta)$, which leads to a maximum value of $\Pi^r = \eta/(2 + \eta)^2 + (2/25)(2 - \eta)/(2 + \eta)$. It is easy to check that the maximum values of the third and fourth pieces are both smaller than 1/8. Note that $\Pi^m > \max\{1/8, \Pi^r\}$, which leads to the expression of (F.14).

To derive (F.15), we note that the first piece of $\pi_j(w_j)$ in (F.13) is decreasing in w_j and is maximized at $w_j^p = w^a$, which leads to a maximum value of $\pi^p = (10 - 3\eta + \sqrt{(50 - 7\eta)(2 + \eta)})^2/(400(2 + \eta)^2)$. The second piece is convex in w_j and thus the maximum value $\pi^q < \max\{\pi_j(2/(2+\eta)), \pi_j(\bar{w}_j^b)\} < \eta^2(2+\eta)^2/(4(5+4\eta+\eta^2)^2) \equiv \overline{\pi}^q$. The second inequality follows from setting $w_j = 1$ for the second piece. It is easy to check that $\pi^p > \overline{\pi}^q$. We conclude part (iii).

To see part (iv), we note that when the supplier negotiates with retailer j with contingency, her profit becomes zero if retailer j does not order positive quantity (i.e., $w_j > \bar{w}_j^b$), which leads to the expression of (F.17). We note that any feasible w_j should lead to nonnegative trade surpluses and thus we have $w_j \geq 1/2 - (1/10)\sqrt{(82 + 9\eta)/(2 + \eta)} \equiv w^c$. It is then easy to see that (F.13) is maximized at w^c, which gives the expression of (F.18). We conclude part (iv). $\square$

F.2 Two-to-One Channel

By Lemma 3.4.1, we can derive the trade profits for the suppliers and the retailer, respectively, as

$$\begin{aligned}\Pi_i(w_i, w_j) &= w_i q_i^*(\mathbf{w}) \\ &= \begin{cases} 0 & 1-w_i \leq \eta(1-w_j), \\ w_i \frac{(1-w_i)-\eta(1-w_j)}{2(1-\eta^2)} & \eta(1-w_j) < 1-w_i < \frac{1}{\eta}(1-w_j), \\ w_i \frac{1-w_i}{2} & 1-w_i \geq \frac{1}{\eta}(1-w_j). \end{cases} \qquad \text{(F.19)}\end{aligned}$$

$$\begin{aligned}\pi_i(w_i, w_j) &= \sum_{i=1}^{2}(p_i^*(\mathbf{w}) - w_i) q_i^*(\mathbf{w}) \\ &= \begin{cases} \frac{(1-w_j)^2}{4} & 1-w_i \leq \eta(1-w_j), \\ \frac{\sum_{i=1}^{2}(1-w_i)((1-w_i)-\eta(1-w_j))}{4(1-\eta^2)} & \eta(1-w_j) < 1-w_i < \frac{1}{\eta}(1-w_j), \\ \frac{(1-w_i)^2}{4} & 1-w_i \geq \frac{1}{\eta}(1-w_j). \end{cases} \qquad \text{(F.20)}\end{aligned}$$

The retailer's disagreement point under simultaneous negotiation without contingency is

$$d_i(w_j) = \frac{(1-w_j)^2}{4}. \qquad \text{(F.21)}$$

Lemma F.2.1 *In the two-to-one channel without contingency, the maximum profits of the supplier and the retailers are, respectively,*

$$\overline{\Pi}_i(w_j) = \begin{cases} \Pi_i(\frac{1-\eta+\eta w_j}{2}, w_j) = \frac{(1-\eta(1-w_j))^2}{8(1-\eta^2)} & w_j < \frac{(2+\eta)(1-\eta)}{2-\eta^2}, \\ \Pi_i(\frac{w_j-1+\eta}{\eta}, w_j) = \frac{(1-w_j)(w_j-1+\eta)}{2\eta^2} & \frac{(2+\eta)(1-\eta)}{2-\eta^2} < w_j < \frac{2-\eta}{2}, \\ \Pi_i(\frac{1}{2}, w_j) = \frac{1}{8} & w_j > \frac{2-\eta}{2}, \end{cases} \qquad \text{(F.22)}$$

$$\overline{\pi}_i(w_j) = \begin{cases} \pi_i(0, w_j) = \frac{(1-w_j)^2 - 2\eta(1-w_j)+1}{4(1-\eta^2)} & w_j < 1-\eta, \\ \pi_i(0, w_j) = \frac{1}{4} & w_j > 1-\eta. \end{cases} \qquad \text{(F.23)}$$

Moreover, the profit allocation $(\Pi_i(w_i, w_j), \pi_i(w_i, w_j))$ is Pareto-dominated for w_i above the maxima that attain $\overline{\Pi}_i(w_j)$.

Proof. We first note that any feasible (w_i, w_j) should lead to nonnegative trade surpluses $\Pi_i(w_i, w_j) - D_i(w_j)$ and $\pi_i(w_i, w_j) - d_i(w_j)$, and thus we have $w_i \geq 0$ in the second and third pieces in (F.19) and (F.20), respectively.

To derive (F.22), we note that $\Pi_i(w_i, w_j)$ is constant in w_i in the first case in (F.19). The second piece is maximized at $w_i^m = (1 - \eta + \eta w_j)/2$ and leads to a maximum value of $\Pi^m = (1 - \eta(1 - w_j))^2/(8(1 - \eta^2))$. The third piece is maximized at $w_i^r = 1/2$ and leads to a maximum value of $\Pi^r = 1/8$.

For w_i^r be the maxima, we must have two cases: *Case (i):* $1 - w_i^r > (1/\eta)(1 - w_j)$ (or $w_j > w^a \equiv 1 - \eta/2$), $1 - w_i^m < (1/\eta)(1 - w_j)$ (or $w_j < w^b \equiv (2+\eta)(1-\eta)/(2-\eta^2)$) and $\Pi^m < \Pi^r$ (or $w_j > w^c \equiv 1 - (1/\eta)(1 + \sqrt{1 - \eta^2})$ and $w_j < w^d \equiv 1 - (1/\eta)(1 - \sqrt{1 - \eta^2})$). It is easy to show that $w^b < w^d < w^a$ and $w^c < 0$ and no feasible w_i satisfies the above conditions. *Case (ii):* $1 - w_i^r > (1/\eta)(1 - w_j)$ (or $w_j > w^a$) and $1 - w_i^m > (1/\eta)(1 - w_j)$ (or $w_j > w^b$), which leads to $w_j > w^a$.

For w_i^m to be the maxima, we can have two cases: *Case (i):* $1 - w_i^r > (1/\eta)(1 - w_j)$ (or $w_j > w^a$), $1 - w_i^m < (1/\eta)(1 - w_j)$ (or $w_j < w^b$) and $\Pi^m > \Pi^r$ (or $w_j < w^c$ or $w_j > w^d$). We note that no feasible w_i satisfies the above conditions. *Case (ii):* $1 - w_i^r < (1/\eta)(1 - w_j)$ (or $w_j < w^a$) and $1 - w_i^m < (1/\eta)(1 - w_j)$ (or $w_j < w^b$). These give the relation $w_j < w^b$.

For $w^a < w_j < w^b$, neither w_i^r nor w_i^m is attainable and thus the maxima should be $w_i^o = 1 - (1/\eta)(1 - w_j)$, which leads to a maximum value of $\Pi^o = (1 - w_j)(w_j - 1 + \eta)/(2\eta^2)$.

Combining the above cases, we obtain the expression of (F.22).

Now we note that the second and third pieces of $\pi_i(w_i, w_j)$ in (F.20) are both decreasing in w_i, and thus $\pi_i(w_i, w_j)$ is maximized at $w_i = 0$. This leads to the expression of (F.23). $\square$

Lemma F.2.2 *In the two-to-one channel without contingency, for a given w_j in bargaining unit j, the negotiated wholesale prices in unit i under the NB and KS solutions are*

$$w_i^{NB}(w_j) = \begin{cases} \frac{(1-\theta)(1-\eta+\eta w_j)}{2} & w_j < 1 - \frac{\eta(1+\theta)}{2-\eta^2(1-\theta)}, \\ \frac{\eta-1+w_j}{\eta} & 1 - \frac{\eta(1+\theta)}{2-\eta^2(1-\theta)} < w_j < 1 - \frac{\eta(1+\theta)-\eta^3(1-\theta)}{2(1-\eta^2(1-\theta))}, \\ w_i^{\alpha} & w_j > 1 - \frac{\eta(1+\theta)-\eta^3(1-\theta)}{2(1-\eta^2(1-\theta))}, \end{cases} \quad \text{(F.24)}$$

$$w_i^{KS}(w_j) = \begin{cases} \frac{1-\eta+\eta w_j}{5} & w_j < 1-\eta, \\ \frac{(1-\eta+\eta w_j)^3}{(1-\eta)^2+2(4+\eta-5\eta^2)w_j-(4-5\eta^2)w_j^2} & 1-\eta < w_j < \bar{w}_j^c, \\ \frac{(1-\eta)^2+2(2+\eta-3\eta^2)w_j-(2-3\eta^2)w_j^2-\sqrt{\Delta_1}}{(1-\eta)^2+2(4+\eta-5\eta^2)w_j-(4-5\eta^2)w_j^2} & \bar{w}_j^c < w_j < \frac{(2+\eta)(1-\eta)}{2-\eta^2}, \\ 1 + \frac{\eta^2(2w_j-w_j^2)+\sqrt{\Delta_2}}{2(1-\eta-(2-\eta+2\eta^2)w_j+(1+\eta^2)w_j^2)} & \frac{(2+\eta)(1-\eta)}{2-\eta^2} < w_j < \frac{2-\eta}{2}, \\ \frac{1+4w_j-2w_j^2-\sqrt{1+6w_j-3w_j^2}}{1+8w_j-4w_j^2} & w_j > \frac{2-\eta}{2}, \end{cases} \quad \text{(F.25)}$$

where w_i^{α} is the unique root of (F.26) within the range $[0,1/2]$, $\Delta_1 = (1-\eta)^4 + 2(1-\eta)^3(3+7\eta)w_j - (1-\eta)^2(3-38\eta-67\eta^2)w_j^2 - 4\eta(17-14\eta-28\eta^2+25\eta^3)w_j^3 + \eta(40-54\eta-60\eta^2+75\eta^3)w_j^4 - 2\eta(4-12\eta-6\eta^2+15\eta^3)w_j^5 - \eta^2(4-5\eta^2)w_j^6$, $\Delta_2 = 4(1-\eta)^2 - 8(3-5\eta+3\eta^2-\eta^3)w_j + 4(15-20\eta+15\eta^2-7\eta^3+\eta^4)w_j^2 - 4(20-20\eta+20\eta^2-9\eta^3+\eta^4)w_j^3 + (60-40\eta+60\eta^2-20\eta^3+\eta^4)w_j^4 - 4(6-2\eta+6\eta^2-\eta^3)w_j^5 + 4(1+\eta^2)w_j^6$ and $\bar{w}_j^c$ is some value within $[1-\eta, (2+\eta)(1-\eta)/(2-\eta^2)]$.

Proof. Applying (F.19), (F.20), (F.21) and $D_i(w_j) = 0$, the Nash product for trade i is

$$\Omega_i(w_i, w_j) = \begin{cases} 0 & 1-w_i \le \eta(1-w_j), \\ \left(\frac{w_i(1-w_i-\eta(1-w_j))}{2(1-\eta^2)}\right)^{1-\theta}\left(\frac{(1-w_i-\eta(1-w_j))^2}{4(1-\eta^2)}\right)^{\theta} & \eta(1-w_j) < 1-w_i < \frac{1-w_j}{\eta}, \\ \left(\frac{w_i(1-w_i)}{2}\right)^{1-\theta}\left(\frac{(1-w_i)^2-(1-w_j)^2}{4}\right)^{\theta} & 1-w_i \ge \frac{1-w_j}{\eta}. \end{cases}$$

Setting $\partial \ln(\Omega)/\partial w_i = 0$ in the second case gives

$$(1-\theta)\frac{1}{w_i} + (1+\theta)\frac{-1}{(1-w_i)-\eta(1-w_j)} = 0.$$

This gives $w_i^m = (1-\theta)(1-\eta+\eta w_j)/2$. Setting $\partial \ln(\Omega)/\partial w_i = 0$ in the third case gives

$$(1-\theta)\frac{1-2w_i}{w_i(1-w_i)} + \theta\frac{-2(1-w_i)}{(1-w_i)^2-(1-w_j)^2} = 0.$$

Rearranging the terms, we obtain

$$\Phi(w_i) \equiv -2w_i^3 + (5-\theta)w_i^2 - 2\big(1 + (1-\theta)(2w_j - w_j^2)\big)w_i + (1-\theta)(2w_j - w_j^2) = 0 \quad (F.26)$$

We note that $\Phi(w_i)$ is a cubic function in w_i with the coefficient of w_i^3 being negative. Since $\Phi(0) = (1-\theta)(2w_j - w_j^2) > 0$ and $\Phi(1/2) = -\theta/4 < 0$, there exists at least one root in $[0, 1/2]$. Moreover,

$$\frac{\partial}{\partial w_i}\Phi(w_i) = -6\Big(w_i - \frac{5-\theta}{6}\Big)^2 + 2(1-\theta)(1-w_j)^2 + \frac{(1+\theta)^2}{6}.$$

The above function is symmetric with respect to $w_i = (5-\theta)/6 > 1/2$ and equals $2(1-\theta)(1-w_j)^2 - 2(2-\theta) < 0$ at $w_i = 0$. Thus, the sign of $\partial\Phi(w_i)/\partial w_i$ changes at most once and the change is from negative to positive. This suggests that $\Phi(w_i)$ has a unique root within the range $[0, 1/2]$ that is the maximizer of Ω_i. Let w_i^α denote this root.

For w_i^m be the best response, we can have two cases: *Case (i):* $1 - w_i^m < (1/\eta)(1 - w_j)$ (or $w_j < w^a \equiv 1 - \eta(1+\theta)/(2 - \eta^2(1-\theta)))$ and $1 - w_i^\alpha < (1/\eta)(1-w_j)$ (or $\Phi(1 - (1/\eta)(1-w_j)) > 0$). It is easy to show that $\Phi(1-(1/\eta)(1-w_j)) > 0$ gives $w_j < w^b \equiv 1 - (\eta(1+\theta) - \eta^3(1-\theta))/(2(1-\eta^2(1-\theta)))$ and $w^a < w^b$. These give the relation $w_j < w^a$. *Case (ii):* $1 - w_i^m < (1/\eta)(1-w_j)$ (or $w_j < w^a$), $1 - w_i^\alpha > (1/\eta)(1-w_j)$ (or $w_j > w^b$) and $\Omega_i(w_i^m, w_j) > \Omega_i(w_i^\alpha, w_j)$. It is easy to see that no feasible w_i satisfies the first two conditions.

For w_i^α be the best response, we must have two cases: *Case (i):* $1 - w_i^m > (1/\eta)(1-w_j)$ (or $w_j > w^a$) and $1 - w_i^\alpha > (1/\eta)(1-w_j)$ (or $w_j > w^b$), which leads to $w_j > w^b$. *Case (ii):* $1 - w_i^m < (1/\eta)(1-w_j)$ (or $w_j < w^a$), $1 - w_i^\alpha > (1/\eta)(1-w_j)$ (or $w_j > w^b$) and $\Omega_i(w_i^m, w_j) < \Omega_i(w_i^\alpha, w_j)$, which suggests that no feasible w_i satisfies the conditions.

For $w^a < w_j < w^b$, neither w_i^α nor w_i^m is attainable and thus the maxima should be $w_i^o = 1 - (1/\eta)(1-w_j)$.

Combining the above cases, we obtain the expression of (F.24).

To derive (F.25), we substitute the expressions of (F.19), (F.20), (F.22), (F.23), (F.21) and $D_i(w_j) = 0$ into (3.2) and obtain

$$\begin{cases} \frac{(1-w_i-\eta(1-w_j))^2/(4(1-\eta^2))}{w_i(1-w_i-\eta(1-w_j))/(2(1-\eta^2))} = \frac{(1-\eta(1-w_j))^2/(4(1-\eta^2))}{(1-\eta(1-w_j))^2/(8(1-\eta^2))} & w_j < 1-\eta, \\ \frac{(1-w_i-\eta(1-w_j))^2/(4(1-\eta^2))}{w_i(1-w_i-\eta(1-w_j))/(2(1-\eta^2))} = \frac{1/4-(1-w_j)^2/4}{(1-\eta(1-w_j))^2/(8(1-\eta^2))} & 1-\eta < w_j < \frac{(2+\eta)(1-\eta)}{2-\eta^2} \text{ and } \eta < \frac{1-w_i}{1-w_j} < \frac{1}{\eta}, \\ \frac{(1-w_i)^2/4-(1-w_j)^2/4}{w_i(1-w_i)/2} = \frac{1/4-(1-w_j)^2/4}{(1-\eta(1-w_j))^2/(8(1-\eta^2))} & 1-\eta < w_j < \frac{(2+\eta)(1-\eta)}{2-\eta^2} \text{ and } \frac{1-w_i}{1-w_j} > \frac{1}{\eta}, \\ \frac{(1-w_i)^2/4-(1-w_j)^2/4}{w_i(1-w_i)/2} = \frac{1/4-(1-w_j)^2/4}{(1-w_j)(w_j-1+\eta)/(2\eta^2)} & \frac{(2+\eta)(1-\eta)}{2-\eta^2} < w_j < \frac{2-\eta}{2}, \\ \frac{(1-w_i)^2/4-(1-w_j)^2/4}{w_i(1-w_i)/2} = \frac{1/4-(1-w_j)^2/4}{1/8} & w_j > \frac{2-\eta}{2}. \end{cases}$$

We omit the expressions for w_i above the maxima that attain $\overline{\Pi}_i(w_j)$ as the best response should lead to a Pareto profit allocation. Let $w^c \equiv 1-\eta$ and $w^d \equiv (2+\eta)(1-\eta)/(2-\eta^2)$.

The first piece gives $w_i^p = (1-\eta+\eta w_j)/5$.

The second piece gives $w_i^q = (1-\eta+\eta w_j)^3/\big((1-\eta)^2+2(4+\eta-5\eta^2)w_j-(4-5\eta^2)w_j^2\big)$. For w_i^q be the best response, we must have $1-w_i^q < (1/\eta)(1-w_j)$, which yields

$$\Phi(w_j) = (1-\eta)^2 - (3\eta^2+4\eta-7)w_j + (3\eta^2+2\eta-12)w_j^2 + (4-\eta^2)w_j^3 > 0.$$

We note that $\Phi(w_j)$ is a cubic function in w_j with the coefficient of w_j^3 being positive. Also, $\Phi(w^c) = \eta(1-\eta^2)^2 > 0$, $\Phi(w^d) = -4\eta(3-\eta^2)(1-\eta^2)^2/(2-\eta^2)^3 < 0$ and $\partial\Phi(w_j)/\partial w_j = 3(4-\eta^2)(1-w_j)^2 - 4\eta(1-w_j) - 5 < 0$ for w_j within $[w^c, w^d]$. Hence, there exists only one root within $[w^c, w^d]$. Let $\bar{w}_j^c$ denote this root. These give the relation $w^c < w_j < \bar{w}_j^c$.

The third piece gives $w_i^s = \big((1-\eta)^2+2(2+\eta-3\eta^2)w_j-(2-3\eta^2)w_j^2-\sqrt{\Delta_1}\big)/\big((1-\eta)^2+2(4+\eta-5\eta^2)w_j-(4-5\eta^2)w_j^2\big)$, where $\Delta_1 = (1-\eta)^4 + 2(1-\eta)^3(3+7\eta)w_j - (1-\eta)^2(3-38\eta-67\eta^2)w_j^2 - 4\eta(17-14\eta-28\eta^2+25\eta^3)w_j^3 + \eta(40-54\eta-60\eta^2+75\eta^3)w_j^4 - 2\eta(4-12\eta-6\eta^2+15\eta^3)w_j^5 - \eta^2(4-5\eta^2)w_j^6$. Note that the third piece has two roots and the best response should be the smaller root. For w_i^s be the best response, we must have $1-w_i^s > (1/\eta)(1-w_j)$, which implies $\Phi(w_j) < 0$ and thus $w_j > \bar{w}_j^c$. These yield the relation $\bar{w}_j^c < w_j < w^d$.

The fourth piece gives $w_i^t = 1+\big(\eta^2(2w_j-w_j^2)+\sqrt{\Delta_2}\big)/\big(2(1-\eta-(2-\eta+2\eta^2)w_j+(1+\eta^2)w_j^2)\big)$, where $\Delta_2 = 4(1-\eta)^2 - 8(3-5\eta+3\eta^2-\eta^3)w_j + 4(15-20\eta+15\eta^2-$

$7\eta^3+\eta^4)w_j^2-4(20-20\eta+20\eta^2-9\eta^3+\eta^4)w_j^3+(60-40\eta+60\eta^2-20\eta^3+\eta^4)w_j^4-4(6-2\eta+6\eta^2-\eta^3)w_j^5+4(1+\eta^2)w_j^6$. Note that the fourth piece has two roots and the best response should be the smaller root.

The fifth piece gives $w_i^u=(1+4w_j-2w_j^2-\sqrt{1+6w_j-3w_j^2})/(1+8w_j-4w_j^2)$. Note that the fifth piece has two roots and the best response should be the smaller root.

Combining the above cases, we obtain the expression of (F.25). □

Lemma F.2.3 *In the two-to-one channel under sequential negotiation, suppose the retailer first negotiates with supplier j.*

i) Without contingency and the negotiated price in unit i is given by (F.24), the supplier j and retailer's profits are

$$\begin{aligned}\Pi_j(w_j) &\equiv \Pi_j(w_j,w_i^{NB}(w_j))\\ &= \begin{cases}\frac{w_j((2-\eta^2(1-\theta))(1-w_j)-\eta(1+\theta))}{4(1-\eta^2)} & w_j<1-\frac{\eta(1+\theta)}{2-\eta^2(1-\theta)},\\ 0 & w_j>1-\frac{\eta(1+\theta)}{2-\eta^2(1-\theta)},\end{cases}\end{aligned}\tag{F.27}$$

$$\begin{aligned}\pi_j(w_j) &\equiv \pi_j(w_j,w_i^{NB}(w_j))\\ &= \begin{cases}\frac{(1-w_j)^2}{4}+\frac{(1+\theta)^2(1-\eta+\eta w_j)^2}{16(1-\eta^2)} & w_j<1-\frac{\eta(1+\theta)}{2-\eta^2(1-\theta)},\\ \frac{(1-w_j)^2}{4\eta^2} & 1-\frac{\eta(1+\theta)}{2-\eta^2(1-\theta)}<w_j<1-\frac{\eta(1+\theta)-\eta^3(1-\theta)}{2(1-\eta^2(1-\theta))},\\ \frac{(1-w_i^\alpha)^2}{4} & w_j>1-\frac{\eta(1+\theta)-\eta^3(1-\theta)}{2(1-\eta^2(1-\theta))},\end{cases}\end{aligned}\tag{F.28}$$

and the retailer's disagreement point is

$$d_j=\frac{(1+\theta)^2}{16}.\tag{F.29}$$

ii) With contingency and the negotiated price in unit i is given by (F.24), the supplier j's profit is same as that without contingency and retailer's profit is

$$\pi_j(w_j)=\begin{cases}\frac{(1-w_j)^2}{4}+\frac{(1+\theta)^2(1-\eta+\eta w_j)^2}{16(1-\eta^2)} & w_j<1-\frac{\eta(1+\theta)}{2-\eta^2(1-\theta)},\\ 0 & w_j>1-\frac{\eta(1+\theta)}{2-\eta^2(1-\theta)},\end{cases}\tag{F.30}$$

and the retailer's disagreement point is 0.

iii) Without contingency and the negotiated price in unit i is given by (F.25), the supplier j and retailer's profits are

$$\begin{aligned}
\Pi_j(w_j) &\equiv \Pi_j(w_j, w_i^{KS}(w_j)) \\
&= \begin{cases} \frac{w_j((5-\eta^2)(1-w_j)-4\eta)}{10(1-\eta^2)} & w_j < 1-\eta, \\ \frac{w_j(1-\eta-w_j)}{2(1-\eta^2)} + \frac{\eta w_j(1-\eta+\eta w_j)^3}{2(1-\eta^2)\left[(1-\eta)^2+2(4+\eta-5\eta^2)w_j-(4-5\eta^2)w_j^2\right]} & 1-\eta < w_j < \bar{w}_j^c, \\ 0 & w_j > \bar{w}_j^c, \end{cases} \qquad \text{(F.31)} \\
\pi_j(w_j) &\equiv \pi_j(w_j, w_i^{KS}(w_j)) \\
&= \begin{cases} \frac{(1-w_j)^2}{4} + \frac{4(1-\eta+\eta w_j)^2}{25(1-\eta^2)} & w_j < 1-\eta, \\ \frac{(1-w_j)^2}{4} + \frac{4(1-\eta^2)w_j^2(2-w_j)^2(1-\eta+\eta w_j)^2}{((1-\eta)^2+2(4+\eta-5\eta^2)w_j-(4-5\eta^2)w_j^2)^2} & 1-\eta < w_j < \bar{w}_j^c, \\ \frac{(2(1-\eta^2)(2w_j-w_j^2)+\sqrt{\Delta_1})^2}{4((1-\eta)^2+2(4+\eta-5\eta^2)w_j-(4-5\eta^2)w_j^2)^2} & \bar{w}_j^c < w_j < \frac{(2+\eta)(1-\eta)}{2-\eta^2}, \\ \frac{(\eta^2(2w_j-w_j^2)+\sqrt{\Delta_2})^2}{16(1-\eta-(2-\eta+2\eta^2)w_j+(1+\eta^2)w_j^2)^2} & \frac{(2+\eta)(1-\eta)}{2-\eta^2} < w_j < \frac{2-\eta}{2}, \\ \frac{(2(2w_j-w_j^2)+\sqrt{1+6w_j-3w_j^2})^2}{4(1+8w_j-4w_j^2)^2} & w_j > \frac{2-\eta}{2}, \end{cases} \qquad \text{(F.32)}
\end{aligned}$$

their maximum profits are

$$\overline{\Pi}_j = \Pi_j\left(\frac{(1-\eta)(5-\eta)}{2(5-\eta^2)}\right) = \frac{(1-\eta)(5+\eta)^2}{40(1+\eta)(5-\eta^2)}, \qquad \text{(F.33)}$$

$$\overline{\pi}_j = \pi_j(0) = \frac{41+9\eta}{100(1+\eta)}, \qquad \text{(F.34)}$$

and the retailer's disagreement point is

$$d_j = \frac{4}{25}. \qquad \text{(F.35)}$$

iv) With contingency and the negotiated price in unit i is given by (F.25), the supplier j's profit is same as that without contingency and retailer's profit is

$$\pi_j(w_j) = \begin{cases} \frac{(1-w_j)^2}{4} + \frac{4(1-\eta+\eta w_j)^2}{25(1-\eta^2)} & w_j < 1-\eta, \\ \frac{(1-w_j)^2}{4} + \frac{4(1-\eta^2)w_j^2(2-w_j)^2(1-\eta+\eta w_j)^2}{((1-\eta)^2+2(4+\eta-5\eta^2)w_j-(4-5\eta^2)w_j^2)^2} & 1-\eta < w_j < \bar{w}_j^c, \\ 0 & w_j > \bar{w}_j^c, \end{cases} \qquad \text{(F.36)}$$

their maximum profits are respectively same as those without contingency and the retailer's disagreement point is 0.

Moreover, the profit allocation $(\Pi_j(w_j), \pi_j(w_j))$ *is Pareto-dominated for* w_j *above the maxima that attain* $\overline{\Pi}_j$*.*

Proof. To see part (i), we substitute the expression of (F.24) into (F.19) and (F.20) to derive (F.27) and (F.28), respectively. We substitute the negotiated price $w^{NB} = (1-\theta)/2$ (see Proposition 3.4.1) in the one-to-one channel into (F.21) to derive (F.29). This concludes part (i).

To see part (ii), we note that when the retailer negotiates with supplier j with contingency, his profit becomes zero if he does not order positive quantity (i.e., $w_j > 1-\eta(1+\theta)/(2-\eta^2(1-\theta))$), which leads to the expression of (F.30). We conclude part (ii).

To see part (iii), we substitute the expression of (F.25) into (F.19) and (F.20) to derive (F.31) and (F.32), respectively. We substitute the negotiated price $w^{KS} = 1/5$ (see Proposition 3.4.1) in the one-to-one channel into (F.21) to derive (F.35).

Now we consider the supplier j and retailer's maximum profits. We shall note that any feasible w_j should lead to nonnegative trade surpluses $\Pi_j(w_j) - D_j$ and $\pi_j(w_j) - d_j$ and thus we have $w_j \geq 0$ in the first pieces in (F.31) and (F.32), respectively.

To derive (F.33), we note that the first piece in (F.31) is maximized at $w_j^m = (1-\eta)(5-\eta)/(2(5-\eta^2))$ and leads to a maximum value of $\Pi^m = (1-\eta)(5+\eta)^2/(40(1+\eta)(5-\eta^2))$. The second piece is decreasing in w_j and thus is maximized at $w_j^r = 1-\eta$, which leads to a maximum value of $\Pi^r = \eta(1-\eta)/10$. It is easy to check that $\Pi^m > \Pi^r$, which leads to the expression of (F.33).

To derive (F.34), we note that the first piece in (F.32) is decreasing in w_j and is maximized at $w_j^p = 0$, which leads to a maximum value of $\pi^p = (41+9\eta)/(100(1+\eta)) \geq 1/4$. It is easy to check that the second piece is convex in w_j for $w_j \in (1-\eta, (2+\eta)(1-\eta)/(2-\eta^2))$ and is maximized at $w_j^q = 1-\eta$. This leads to a maximum value of $\pi^q = \pi_j(1-\eta) = (9\eta^2+16)/100 < \pi^p$. Note that the third, forth and fifth pieces are degenerated to the one-to-one channel and thus the maximum value in turn must be smaller than 1/4. This concludes part (iii).

To see part (iv), we note that when the retailer negotiates with supplier j with contingency, his profit becomes zero if he does not order positive quantity (i.e., $w_j > \bar{w}_j^c$), which leads to the expression of (F.36). Also note that any feasible w_j should lead to nonnegative trade surpluses and thus we have $w_j \geq 0$, which implies that the maximum profits are same as those without contingency. We conclude part (iv).

$\square$

VITA

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