THE EFFECT OF COMPS-BASED PROBLEM POSING INTERVENTION ON ENHANCING MATH PERFORMANCE OF STUDENTS WITH LEARNING DISABILITIES

by

Xuan Yang

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THE PURDUE UNIVERSITY GRADUATE SCHOOL
STATEMENT OF COMMITTEE APPROVAL

Dr. Yan Ping Xin, Chair
Department of Educational Studies

Dr. Denise Whitford
Department of Educational Studies

Dr. David Sears
Department of Educational Studies

Dr. Signe Kastberg
Department of Curriculum and Instruction

Approved by:
Dr. Janet M. Alsup

Dedicated to my loving family

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ABSTRACT

In educational research, the cognitive activity of problem posing is recognized as an important component of mathematics teaching and learning. Compared to the prevailing educational paradigm of problem solving, problem posing features less commonly in classroom instruction. During the past 20 years, numerous studies examining the use of problem posing in school mathematics instruction have documented positive outcomes in terms of students' knowledge, problem-solving abilities, creativity, and attitudes and beliefs regarding the study of mathematics. However, despite these promising results, problem posing in mathematics instruction has rarely been studied in the population of students with learning disabilities (LDs). This study describes a problem-posing intervention that draws on existing Conceptual Model-based Problem Solving program (COMPS, Xin, 2012) and conceptual research into the problem posing task. The COMPS-based problem posing intervention is designed to teach word problem posing skills to students with LDs under structured problem posing situations. The study applies a single-subject multiple-baseline design across three participants to investigate the effects on participants' word problem solving and problem posing skills. The results showed that all three students demonstrated increased math performance on both problem solving and problem posing tests when the COMPS-based Problem Posing intervention was used. In addition, both immediate and maintenance effects on student learning were noted.

CHAPTER 1. INTRODUCTION

1.1 Background

According to the Every Student Succeeds Acts of 2015 (ESSA), the National Council of Teacher of Mathematics (NCTM) Standards (2000), and the Common Core State Standards for Mathematics (CCSSM), students with learning disabilities (LDs) must have equal opportunities to access learning resources, take high-stakes state assessments, and meet the same high standards as their peers. These educational laws express high expectations for all students, including those with LDs. In addition, the ESSA (2015) emphasizes the importance of improving mathematical performance for all students (again, including students with LDs), and the Common Core State Standards Initiative (CCSSI, 2012) emphasizes the importance of improving students' conceptual understanding, cognitive thinking, and ability to form connections between mathematical ideas. These high expectations pose significant challenges to general and special educators in terms of overall student learning and achievement.

According to Miller and Hudson (2007), conceptual understanding refers to making connections between the previous knowledge students learned and new knowledge, and building links between different pieces of knowledge. However, to make connections between mathematical ideas, it is important for students to know which methods or tools they should use to apply their knowledge (Kilpatrick et al., 2001). According to Baroody et al (2007), procedural knowledge and conceptual knowledge are closely related. Without procedural knowledge, conceptual understanding is difficult to obtain. The specific subject of mathematics, therefore, tends to be especially difficult and challenging for students to learn (Miller & Hudson, 2007). This poses significant challenges for students with LDs. In addition, an extensive corpus of research demonstrates that many students with LDs experience memory problems, feelings of anxiety, social-emotional problems, and persistent difficulties learning and retaining knowledge when studying mathematics (Maccini & Hughes, 2000; Owen & Fuchs, 2002; Zentall, 2014). For these reasons, the performance of students with LDs in mathematics tends to fall at least two grade levels behind their peers (Wagner & Blackorby, 1996). Moreover, this state of affairs does not appear to be improving. Over the past 10 years, the mathematical performance of students with LDs has not improved (Xin et al., 2017). According to Geary et al. (2012), about 5% to 8%

of school-aged students have been identified as having a learning disability in mathematics. In sum, while advanced mathematical thinking can be difficult for normal-achieving students, students with LDs experience even greater difficulties, as evidenced by consistently low performance in measures of mathematical proficiency (ESSA, 2015).

Problem solving is central to mathematics education (NCTM, 2000; Stanic & Kilpatrick, 1988). Mathematical problem solving requires students to apply their mathematical ideas, knowledge, skills, and strategies to new context and situations (Fuchs et al., 2004). However, because educators and parents tend to place a major focus on problem-solving drills and practice, in a wide variety of educational contexts, students are not given frequent opportunities to pose problems (Ellerton, 2013). Problem solving also plays a dominant role in the research of mathematics education, as many researchers have been (and still are) inspired by Polya's (1957) *How to Solve It.* In the past few decades, a number of studies have investigated the effects of various cognitive, metacognitive, and affective strategies in mathematical problem solving (Lester, 1994; McLeod & Adams, 1989; Schonefeld, 1985, 1992). Yet, by contrast, problem-posing research has stagnated even as there have been major efforts across the world to integrate problem posing into the teaching of mathematics at all levels (Brink, 1987; Cai, 2003; Cai et al., 2015; Hashimoto, 1987). Nevertheless, some studies have explored the cognitive nature of problem posing. These have made possible the classification of various problem-posing tasks, the delineation of several classes of problem-posing interventions, and the development of promising problem-posing models (Stoyanova, 2005; Leung, 1993; Kopparla et al. 2019).

In contrast to problem solving, problem posing refers to the "generation of new problems and questions" from given situations and the "reformulation of given problems" during the process of solving it (Silver, 1994, p.19). It has also been referred to as problem formulating, problem generation, problem finding, problem creating, creative problem-discovering, and problem sensing (Kilpatrick, 1987; Jay & Perkins, 1997; Dillon, 1982). The value of problem posing has been recognized since 1930s, when Einstein and Infeld (1938) argued that "the formulation of a problem is often more essential than its solution, which may be merely a matter of mathematical or experimental skills. To raise new questions, new possibilities, to regard old questions from a new angle, requires creative imagination and marks real advance in science" (p. 92). By engaging in problem-posing activities, students have opportunities to reflect on what they already know, make connections between mathematical ideas, construct new knowledge based on prior knowledge and experiences, and learn from the structure of the instructional content available. These qualities can provide teachers with strong evidence of students' mastery of any knowledge learned (Brown & Walter, 2005; Silver, 1994).

Like problem solving, problem posing has been found to be a significant component of mathematics instruction and mathematical thinking (Brown & Walter, 2005). According to Dickerson (2000), problem-posing instruction tends to lead to better results for students who do not fare as well under traditional problem-solving instruction. This is because for students to create problems, they must master the key concepts and structure of the problem. Many researchers, practitioners, and professional organizations like the NCTM and the CCSSM advocate that students should engage in problem-posing activities in math class. This is because these activities can foster more creative, flexible thinking, enhance students' problem-posing skills, and promote deep understanding of basic concepts while broadening students' perceptions of mathematics (Brown & Walter 1993; English, 1996). Thus, professional organizations like the NCTM have issued several notable calls to incorporate problem posing into school mathematics curricula (e.g., NCTM, 1989, 2000).

Silver and Cai (2005), in addition to making similar arguments for the important role of problem posing in mathematics instruction, also highlighted the benefit of the fact that problem posing allows students to pose their problems in realistic settings. Similarly, Barlow and Cates (2006) further indicated that problem posing is a student-centered activity. Students, who must think independently when posing problems, gain a sense of real-world proficiency and confidence from their work.

According to Cai et al. (2012), there are at least two reasons why problem-posing activities impact student learning. The first is that problem-posing activities require students to think conceptually and metacognitively (Cai & Hwang, 2002). To pose a problem, students must be able to identify relevant background knowledge and reflect on the utility of that knowledge to make connections to the information they have been provided with. According to Doyle (1983), tasks with different cognitive demands are likely to induce different kinds of learning. Problem posing requires a high level of cognitive demand, which, in turn, can promote students' conceptual understanding, mathematical reasoning, and mathematical communication (Cai & Hwang, 2002). The second reason problem-posing activities impact student learning is that the procedure of problem solving often involves the generation and solution of new problems

(Silver, 1994). Therefore, posing problems not only fosters students' understanding of problem situations, but also nurtures the development of more advanced problem-solving strategies (Cai & Hwang, 2002).

Several past research efforts have studied the relationship between problem posing and problem solving. Most studies have shown that problem solving and problem posing are closely related (e.g., Cai, 2002; Kilpatrick, 1987; Silver, 1995; Stoyanova, 2005). A student who is a good problem solver tends to also be a good problem poser (Cai & Hwang, 2002). The reverse is also true: a good problem poser tends to be a good problem solver (English, 1998, 2003; Perrin, 2007). Studies of problem posing activity have demonstrated its promise for enhancing students' problem-solving skills, strengthening their understanding of basic concepts, and providing students with opportunities to think flexibly and nurture their mathematical thinking (Cifarelli & Sevim, 2015; Brown & Walter, 1993; English, 1996).

The National Council of Teachers of Mathematics' *Curriculum and Evaluation Standard for School Mathematics* (1989) was the first major position statement to advocate the integration of problem posing into classroom instruction. However, it only advocated problem posing interventions for students in grades 9–12. Additional research followed this early statement of support, though most studies mainly focused on exploring the nature of problem posing. This included, for instance, classifying various problem-posing tasks (Stoyanova, 1999), defining the stages of problem-posing activities (Sliver, 1995), and describing the processes that occur during problem posing (Christou et al., 2005; Koichu & Kontorovich, 2013). In a discussion of the educational role of problem posing, Sliver (1994) argued that problem posing should be widely adopted in mathematics classrooms, as its instructional qualities made it well-suited to math instruction. Going further, NCTM (1999, 2000) argued the necessity of integrating problem posing into curricula at all grades.

Given the apparent utility of problem posing in school mathematics, and given widespread calls for educational reform, an increasing number of studies have been conducted to document the effectiveness of problem posing in mathematics instruction. These have demonstrated a variety of positive results in terms of students' knowledge, problem solving skills, cognitive activity, creativity, and attitudes and beliefs regarding mathematics (e.g., Barlow & Cates, 2006; Brown & Walter, 2005; Cai & Hwang, 2003; Crespo & Sinclair, 2008; English, 1997; Kilpatrick, 1987; Silver, 1994; Yuan & Sriraman, 2011). However, without specific instructional activities and strategies, it is difficult for practitioners to implement problem posing effectively. Thus, additional research is needed to develop specific approaches and strategies for implementing problem posing activities in teachers' classrooms.

1.2 The significance of the study

Despite several decades of advocacy for integrating problem posing into classroom instruction, problem posing research is still in its infancy (Cai & Hwang, 2002). Over the past 30 years, educational policies (e.g., NCTM, 1989, 1991, 2000), researchers in mathematics education (e.g., Ellerton, 1986; Silver, 1994; Kipatrick, 1987; Schoenfeld, 1985; Cai & Hwang, 2002), and mathematicians (e.g., Polya, 1957) have recognized the significant potential problem posing offers for mathematics instruction. NCTM (1989, 1991, 2000) has called for an increased emphasis on problem posing activities in the mathematics classroom. However, most studies in problem posing have mainly focused on the following four phenomena: (a) the relationship between problem solving and problem posing (e.g., Silver & Cai, 1996), (b) the classification of problem posing (e.g., Stoyanova & Ellerton, 1996, Silver, 1994), (c) problem posing and creativity (e.g., Lin & Leng, 1996), and (d) processes involved in problem posing tasks (Pelczer & Gamboa, 2009). In addition, most research studies have focused on the problem posing skills of pre-service and in-service mathematics teachers (e.g., Cai et al., 2019; Crespo & Sinclair, 2008; Silver et al., 1996; Xie, 2016). Fewer studies (e.g., English, 1997; English, 1998; Dickerson, 2000) have focused on the problem posing skills of students.

Cai et al.(2013) observed that "little research has been done to identify instructional strategies that can effectively promote productive problem posing or even to determine whether engaging students in problem posing activities is an effective pedagogical strategy" (p. 58). They called for future research that might "focus on ways to integrate problem posing into regular classroom activities" (p. 67). Even though there have been efforts to integrate mathematical problem posing into classroom practice, little research has identified problem-posing-based instructional strategies that can enhance students' word problem-posing and problem-solving skills. This is also not to mention the dearth of research investigating effective problem-posing strategies for students with LDs. For this reason, the field has limited knowledge of effective strategies to facilitate productive problem posing, particularly, in educational situations (e.g., in word problems) (Cai et al., 2015). Reflecting recent reform efforts in math education, the

Common Core State Standards for Mathematics (CCSSM) prioritize conceptual understanding in problem solving, mathematical modeling, higher-order thinking and reasoning, and algebra readiness. Problem-posing activities are usually cognitively demanding tasks that require students to demonstrate conceptual understanding, mathematical thinking, and higher-order thinking in intellectual contexts (Common Core State Standards Initiative [CCSSI], 2012; Cai et al., 2015).

As discussed above, the goal of helping students develop advanced mathematical thinking is one that attracts critical attention from researchers and educators. However, little is known about how students with LDs develop mathematical thinking. Given that students with LDs often suffer from anxiety and working memory problems that impede their ability to solve problems correctly, problem posing may offer a more feasible approach for students who struggle with mathematics (Silver, 1994). However, a search of relevant databases reveals that problem-posing research has not yet been widely pursued in the context of special education. In addition, little instructional strategy has been found to engage students in math word problem-posing activities. Thus, this study offers a chance to understand the characteristics of problem posing in word problems and the cognitive processes students with LDs undergo when they pose and solve word problems.

1.3 The purpose of the study

Given the aforementioned calls for incorporating problem posing in mathematics curricula, the significance of problem posing in mathematics learning, and the limited number of studies on instructional strategies for problem posing that currently exist, there is a necessity for new research to develop strategies that promote this skill. In addition, relevant academic databases were found to contain virtually no research exploring problem posing in students with LDs. Considering the benefits of problem posing in general mathematics education, it would be beneficial to examine problem posing in the context of special education in order to assist students with LDs. The purpose of the study, then, is to explore the effects of the COMPS-based problem-posing intervention on the word-problem-posing and problem-solving skills of 7^{th} graders with LDs. Figure 1 presents all of the topics involved in this study.

The study's three research questions are as follows:

(a) What are the effects of COMPS-based problem posing interventions on participating students' problem-posing and problem-solving performance as measured by a researcher-developed problem-posing test and a multiplication/division word problem-solving test (Xin et al, 2008)?

(b) What is the relationship between problem posing and problem solving in terms of students' test scores?

(c) Will students maintain the acquired problem-posing and problem-solving skills following the termination of the intervention?

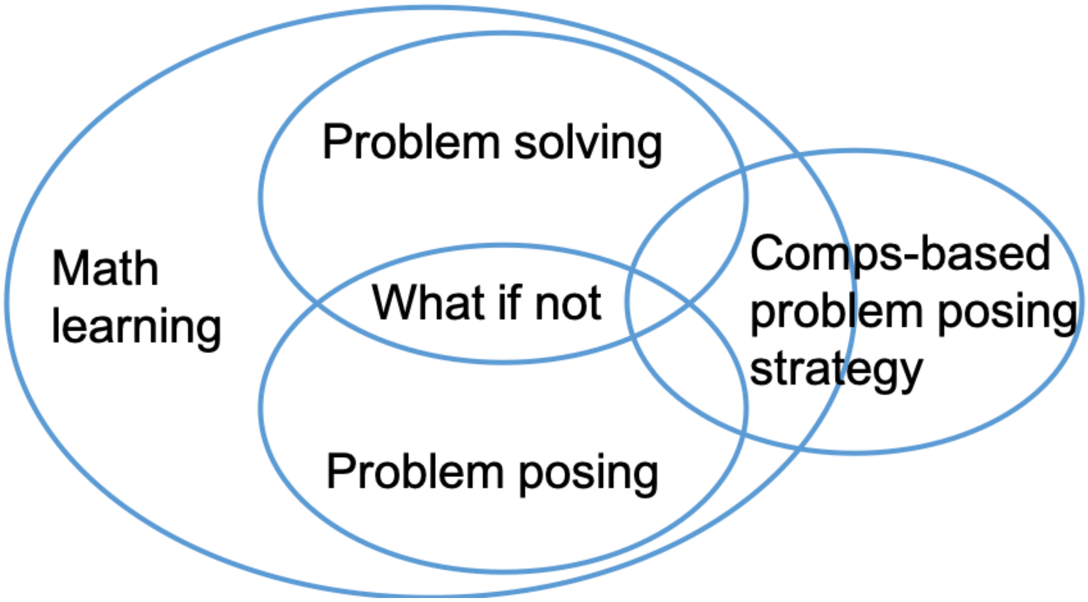


Figure 1. All of the topics involved in the study.

CHAPTER 2. LITERATURE REVIEW

This chapter is organized into three sections. The first section describes the significance of incorporating problem posing in math instruction. The next section reviews and summarizes empirical studies that have demonstrated the potential of problem-posing instruction for improving students' math learning. The last section addresses the performance of students with LDs in math courses and describes the effects of model-based math interventions for students with LDs.

2.1 Incorporating problem posing in math instruction

Problem posing refers to the generation of new problems and questions from a given situation as well as to the reformulation of a problem during the process of solving it (Silver, 1994). An example of a problem-posing activity is as follows: students are given the information "there are 6 towers with 3 cubes in each," and are asked to pose multiplicative word problems. One could pose the problem, for instance, "How many cubes are there in all?" This is an example of the generation of a new problem from given situation. After solving the problem by finding the answer of 18, a student may subsequently ask, "What if the number of towers is not known?" or "What if the number of cubes in each tower is not known?" In these cases, the subsequent problems posed might take forms like "Jose uses 18 cubes to make towers. Each tower is built from 3 cubes. How many towers can Jose make?" or "Jose used 18 cubes to make towers. He made 6 towers. How many cubes were each of towers?" Posing these problems is an example of reformulating a previously solved problem.

Problem posing is not only a phenomenon that occurs in schools. It is also a skill that is regularly employed in everyday actions and behaviors. Even young children pose problems when they ask, for instance, why 1 and 1 equals 2. Research suggests that the process of posing problems encourages us to think, explore, make connections between new and old knowledge, and understand the world better (Xie, 2016). However, research also reveals that, because teachers focus more on students' problem-solving practices (as well as measures of achievement like standardized test scores), students have few opportunities to pose problems at school (Ellerton, 2013).

In addition, both teachers and students may place such great faith in course textbooks—including the utility of the questions and exercises they provide—that they seldom pursue opportunities to actively pose problems. For example, when I was an elementary school student (and even later, when I was a high school student), I was not taught to pose problems in class. I assumed that all my textbooks provided valid, worthwhile problems and exercises. Thus, I never considered whether the questions from my books were as helpful, comprehensive, and educationally effective as possible. Later, I became a college teacher, and I was baffled to find that my students frequently asked me why some statements in the book were not written in a different way. Experiences like this point to the fact that, in addition to textbooks, students and teachers can also be the source of good problems.

Studies have shown that problem posing and problem solving are linked. The connection if fundamental: for a problem to be solved, the problem must be posed first (Silver, 1994). However, compared to problem solving, problem posing was a virtually unstudied topic in mathematics education before 1960 (Silver, 1994). Despite the publishing of some problem posing studies between 1960 and 1970, problem posing only started receiving scholarly attention in earnest after the release of the *Curriculum and Evaluation Standard for School Mathematics* (NCTM, 1989) and the *Professional Standards for Teaching Mathematics* (NCTM, 1991). These important, discipline-shaping standards explicitly recognized the importance of offering student's problem-posing opportunities at school. Later, the NCTM (1999, 2001) highlighted problem posing as an important target for ongoing efforts to reform mathematics education and emphasized that problem posing could be used in the classroom to promote mathematics as a worthy intellectual activity. During the past two decades, problem posing research has received increased attention in mathematics education (Cai, et al., 2015). However, nowadays, students still do not have enough problem-posing opportunities at school, as their textbooks tend to offer relatively few problem-posing activities (Cai & Jiang, 2017).

Stoyanova (1999) stated that problem posing is a teaching activity as well as a learning activity. For example, in mathematics classes, teachers frequently pose problems for students to solve. This is an example of problem posing as a teaching activity. On the other hand, students can also pose problems for teachers or their peers to solve. This is an example of a learning activity. The latter type of problem posing is a cognitively demanding activity that requires students to either generate new problems based on the given information or reformulate existing problems by changing the context, changing the information provided in the problem, and so on (Silver, 1994). Researchers and practitioners have advocated for incorporating problem posing into classroom instruction, as experience problem posing can promote students’ engagement in authentic mathematical activity by making them explore problems and solutions in depth. It can also increase students’ inclination to look for new problems, alternative methods, and novel solutions (Silver & Cai, 2005). Providing students with opportunities to create problems not only engages students in advanced thinking, but also promotes students’ cognitive growth (English, 1997; Lowrie, 2002; NCTM, 2000).

2.2 Frameworks of problem posing

Empirical studies have produced frameworks that classify problem-posing tasks based on their structure, context, problem solving stages involved, and problem-posing processes involved.

2.2.1 The structure of problem posing

Stoyanova and Ellerton (1996) classified problem-posing tasks via three categories: free problem posing, semi-structured problem posing, and structured problem posing. In free situations, the problem is not provided. Students are asked to pose problems related to a natural situation without any restrictions. Directions may take the form of prompts like “John received 20 gifts last Christmas. Use the information to make up as many problems as you can.” In semi-structured problem-posing situations, students are provided with open situations that require them to pose problems based on knowledge that they have previously learned. To do this, they must draw on mathematics experience and make connections between their previously-learned knowledge and any new knowledge provided to them. For example, students may be required to develop word problem based on a given equation (e.g., $2 + a = 8$) or to pose problems based on provided diagrams and pictures. Structured problem-posing situations refer to situations where students pose problems by reformulating already-solved problems or by varying the qualities of given problems. For example, students may be asked to pose new problems after being presented with the following word problem: “David received 6 boxes of candies from his friend. Each box has 10 candies. How many candies did David receive from his friends? Explain how you found your answer." Previous research on mathematical problem posing suggests that individuals are more successful posing mathematical problems under semi-structured/structured posing situations than under free posing situations (Silber & Cai, 2017). For this reason, and also due to the deficits in working memory, attention, and information processing that are often observed in students with LDs (Zental, 2014), this study employed structured problem-posing situations.

2.2.2 Problem-posing intervention types

Based on Stoyanova's three problem-posing structures, Kopparla et al. (2019) posited three problem-posing intervention types: problem posing using informal context, problem posing using visual representation, and problem posing using symbolic representation. In informal context problem-posing activities, students are expected to make connections between their own experiences and an informal context they experience or encounter. For example, after students visit a zoo, they may be asked to generate mathematical problems about animals they saw. Since informal context problem-posing tasks are open-ended, they are not typically good choices for students with LDs, who are likely to have attention and memory deficits. In problem-posing activities involving visual representation, students are asked to generate problems based on given visual representations (e.g., pictures or graphs). For example, if a picture displaying a variety of animals on a farm is shown to students, students may generate problems like "How many pigs are on the farm?" While a variety of valid problems may be associated with a single picture, any problems students pose must be relevant to the information in the picture. In problem-solving activities involving symbolic representations, students are provided with a symbolic representation of some relationship or phenomenon, like the equation $5a = 20$, and asked to create questions based on it. The symbolic representation problem-posing intervention is an effective instructional strategy to evaluate students' conceptual understanding of an equation and develop students' mathematical thinking (Kopparla et al. 2019), therefore, the study employed the symbolic representation problem-posing intervention.

2.2.3 Problem posing classification in the process of problem solving

Silver (1994) classified problem posing in terms of the stages of the problem solving process when problem posing could occur. In Silver's system, problem posing can occur before, during, and after problem solving. These problem posing phenomena are referred to as pre-solution, within-solution, and post-solution, respectively. Prior to solving a given problem, new problems can be generated from the stimuli presented (e.g., a story, a representation, or a diagram). Similarly, during the process of solving problems, students may intentionally change the goals and conditions of the problems presented to them. This leads to a process of reformulation as the given mathematical problem (or information associated with it) is reinterpreted into a new problem during the process of solving it. Finally, after solving a problem, students may apply the experience and information gained from the solved problem to a new situation, generating a new problem. The new problem may be a simple extension or modification, or it may be totally different from the original. Silver's problem-posing framework implies that students can make connections between what they learn from solving various problems and can reformulate new problems during the processes of solving existent problems and posing new ones.

2.2.4 Classification of posed problems

Leung (2013) classified students' posed problems into five categories: (1) responses that are not problems, (2) non-math problems, (3) impossible problems, (4) insufficient problems, and (5) sufficient or extraneous problems. A response that is not a problem would occur if, for example, a student who is asked to pose a word problem responds with "I played with Jane yesterday." This, of course, is not a word problem. A non-math problem occurs when the student is able to pose a problem, but the problem does not involve mathematics. For example, if a student is asked to pose a one-step multiplicative word problem, she will have posed a non-math problem if she responds with "How did Jose get the candies from her uncle?" An impossible problem is a problem that is in mathematical form but not solvable. An example would be "Tommy has 10 books. Emily has 12 books. How many more books does Tommy have than Emily?" An insufficient problem is a mathematical problem that cannot be solved because insufficient information has been provided with it. An example would be if a student posed a word problem, "David delivered pizzas last weekend. How many pizzas did he deliver on Saturday?" In this case, the given information is not sufficient to find the solution. Finally, a sufficient or extraneous problem is a solvable mathematical problem. Though the categories of the posed problem are well-defined, in practice, teachers can have difficulties distinguishing responses that are not a problem, non-math problems, impossible problems, and insufficient problems (Xie, 2016).

In addition, Winograd (1992) examined students' posed problem types by having them write down life experiences and create questions based on these everyday experiences. Winograd classified students' posed problems into four categories: (1) problems containing new mathematical concepts, (2) problems that required knowledge of a mathematical procedure (for example, a student writes a multiplicative math word problem, but has not yet obtained the knowledge that can help him solve the problem successfully), (3) problems that require problem-solving knowledge the student does not yet possess, and (4) problems the students understand but make minor errors in solving. The study showed that students in fifth grade were able to successfully pose mathematics problems, but had difficulty solving or understanding the problems they posed.

2.2.5 Criteria of quality posed problems

Silver and Cai (2005) analyzed students' mathematical problem posing based on three criteria: quantity, originality, and complexity. Quantity refers to the number of correct responses generated from a problem-posing task by students. According to Silver and Cai, "a core feature of problem-posing tasks is that they allow for the generation of multiple correct responses" (p. 131). Given this opportunity, some students may simply rephrase or change the wording of the posed problem. Originality is a response feature that may be used as a criterion to measure students' creativity. For example, if a teacher asks students to name the uses of a blanket, and students are encouraged to pose as many answers as they can, students' responses can be compared with typical responses to see if the students are able to generate atypical (i.e., original) responses. Typical responses may be "keep warm," "sleep on it," and so on, while the atypical responses may be ones like "decoration" or "fire extinguisher." The atypical responses suggest a degree of originality that is not present in the first set of responses.

Similarly, the complexity of student responses can be analyzed from different perspectives and classified into different levels. Silver and Cai (2005) classified complexity into four categories: (1) sophistication of the mathematical relationships embedded in problems, (2) linguistic complexity, (3) problem difficulty, and (4) mathematical complexity. Sophistication refers to the complexity of the steps needed to obtain the answer or the complexity of the mathematical relationships. A word problem involving multiple steps to obtain the solution, for instance, is more sophisticated than a one-step word problem. Also, a word problem involving multiplicative relationships is more sophisticated that an additive word problem. Linguistic complexity refers to the degree to which the student uses advanced or highly proficient language constructions to pose the problem. For example, the problem “Lucy has 5 towers of 6 cubes each. Jay has a total of 36 cubes. How many more cubes does Jay have than Lucy?” is more difficult than the problem “Jay has 5 towers of 6 cubes each. How many cubes does he have in all?” for students to solve. It is also more linguistically complex. According to Mayer et al. (1992), problem difficulty is related to linguistic or syntactic structures in the posed problems and the presence of assignment, relational, and conditional propositions in posed problems is the measuring criteria of problem difficulty. For example, an assignment proposition refers to a question “How many pizzas did Tom deliver during the weekend?” A relational proposition refers to a question “How many more pizzas did Tom deliver than Jim?” A conditional proposition refers to a question “If Tom delivered 16 pizzas more than Jim, how many pizzas did Tom deliver.” The presence of relational and conditional propositions is the indication of problem difficulty as the problems with relational and conditional propositions are more difficult than the problems with only assignment propositions (Mayer et al., 1992).

Finally, Silver and Cai (2005) define the mathematical complexity of posed problems in terms of the semantic structural relations developed by Marshall (1995). These semantic structural relations fall into five categories: (1) change, (2) group, (3) compare, (4) restate, and (5) vary. According to Silver and Cai (2005), the degree of mathematical complexity is determined by the number of semantic structural relations presented in the posed problems. In other words, the more semantic structural relations presented in the problem, the more mathematically complex it is. For example, the problem “Did Jay have more cubes than Lucy?” contains no structural relations. By contrast, the problem “How many more cubes does Jay have than Lucy” has multiple relations (the compare and restate relations).

The National Assessment of Educational progress (NAPE) also reported a mathematics framework that describes three different levels of mathematical complexity: low, moderate and high (2005). In reference to the NAPE framework, Lin and Leng (2008) pointed out that “[e]ach level of complexity includes the aspects of knowing and doing mathematics, such as reasoning, performing procedures, understanding concepts, or solving problems” (p. 5). A low level of mathematical complexity may require a student to recall or recognize mathematical concepts they have learned previously: for example, students may need to recognize multiplication as a required skill to solve a one-step multiplicative word problem. A moderate level of mathematical complexity may require a student to think flexibly and make connections between two quantities: for example, a more difficult problem may require the student to represent a situation mathematically in more than one way. A high level of complexity may require a student to think creatively and engage in abstract reasoning.

2.3 Problem-posing strategies

What-If-Not Strategy. Brown and Walter's *The Art of Problem Posing* (1993) introduces a general strategy for posing new problems by manipulating each attribute of a given problem. The strategy, called "What-If-Not," is used to design a method for posing new questions. The problem posing process is classified into two phases: accepting the given and using "what if not" to challenge the given. This strategy requires students to list all attributes of a problem and then challenge the given by asking "what if not this attribute?"

"Acting-Out" Strategy. Students are encouraged to manipulate tangible, concrete objects to help them pose problems (Walter, 1992). For example, if given a piece of paper, students may fold the paper into triangles and pose questions like "how many triangles are made after folding 3 times?" In addition, the acting-out strategy can be used help students generate problems during role play (Dickerson, 1999). For example, a student might be asked to act out an imaginary shopping trip during which he bought a box of chocolate for 8 dollars and a teddy bear for 10 dollars. At the end of the act, the student is asked to report how much he spent. In this scenario, the student creates problems based on what he experienced during the role play.

Chaining Strategy. According to Silver et al. (1996), chaining requires students to solve one problem in order to generate a new problem dependent on the logic of the first. For example, a student might be presented with the word problem "Carl has 5 boxes of chocolates with 12 chocolates in each. How many chocolates does he have?" After solving the problem by indicating that Carl has 60 chocolates, the student can expand the problem like so: "If Carl gives his chocolates to 3 friends equally, how many chocolates does each of his friends get?". Thus, the new problem is linked to the previous problem.

Open-ended Problem-posing Strategy. This approach is based on Stoyanova's free problem-posing structure. It involves presenting students with a story problem starter prompt that does not contain sufficient information to solve the problem. This forces students to brainstorm new questions based on the given information. For example, students may be given a starter prompt like this: "Carol wants to buy pizzas for her friends, and she has a total of 30 dollars she can spend." Starting from the information in this statement, students add details and pose mathematical questions to generate new problems.

Semi-structured Problem-posing Strategy. This approach is based on Stoyanova's semi-structured problem-posing situations. Students are provided with an open-ended situation and are required to pose problems by drawing on previously-obtained knowledge, new knowledge, and the mathematical skills in which they have competency. For example, students may be provided with a graph displaying a line and the corresponding linear equation and asked to develop new problems or exercises from this starting information.

Structured Problem-posing Strategy. This approach is based on Stoyanova's structured problem-posing situations. This approach allows the teacher the greatest control over the situation the students must pose problems from. For example, students may be presented with a story problem that has already been solved and asked to develop new problems by modifying or reinterpreting the details of the story, the question being posed in the prompt, and so on.

2.4 Factors that may affect students' abilities to pose problems

As discussed above, problem-posing skills are mainly affected by two factors. The first is the student's level of experience. Stoyanova (2005) pointed out that problem-posing skills are like other skills insofar as they can be developed and nurtured over time. Given sufficient exposure to problem-posing activities, students can gain the ability to confidently pose their own mathematical problems (Chen et al., 2010). Students learn how to analyze task structures and how to make connections between previously learned knowledge and new knowledge from immersion in problem-posing activities. Therefore, additional exposure to problem-posing activities at school is essential for students to reap the benefits to cognitive thinking and conceptual understanding associated with problem posing.

The second factor affecting students' problem-posing skill is problem-solving ability. Studies have reported possible relationships between mathematical problem-solving and problem-posing skills. According to the *NCTM Principles and Standards,* “Good problem solvers tend naturally to pose problems based on the situations they see” (NCTM, 2000, p. 53). Thus, the connections teachers make between problem solving and problem posing in classroom may play an important role in students’ problem-posing skill development.

In this study, both problem-solving and problem-posing skills are taught via exercises that draw on the same basic knowledge and that use the same word problem elements. Per the stages of problems posing (Silver, 1994), one way to teach problem posing is to ask students to pose problems based on problems they have just solved. The other way is to teach the problem posing before or during the process of solving problems. In these cases, students may be asked to change some information or aspects of the problem structure to generate new problems.

2.5 Review of exiting research studies that investigate problem posing

Most studies of problem posing have focused on pre-service and in-service mathematics teachers (e.g., Chen et al., 2010; Xie, 2016), though a few have focused on students (e.g., English 1997; English, 1998). Existing studies have provided evidence that problem posing has a positive impact on both teachers and students’ content knowledge (English, 1997a; Lavy & Bershadsky, 2003), mathematical thinking (Brown & Walter, 2005; Cunningham, 2004), problem-solving performance (Traylor, 2005), problem-posing abilities (English, 1997a, 1997b, 1998; Winogard, 1992), creative thinking (Yuan & Sriranman, 2001), attitudes and beliefs (Balow & Cates; 2006), and disposition toward mathematics (English, 1997).

2.5.1 Studies that investigated student’s problem posing skills

English (1997a, 1997b, 1998) investigated students’ skills to generate problems in separate studies involving third, fifth and seventh graders (respectively). The goal of the first study (English, 1998) was to investigate the problem-posing skills of third graders who displayed different achievement profiles in terms of number sense and novel problem solving. The 54 students who participated in the study were evaluated in the categories of problem solving and number sense. Although the students had experience with a range of addition and subtraction problems from their regular class activities, they had not been exposed to the types of problem-posing tasks included in the study. Number sense and problem-solving tests were designed by

English. According to English (1998), the number sense test was designed to align with students' school curricula and to provide insight into children's number sense. The problem-solving test was designed to represent novel and meaningful situations that required students to use reasoning processes which are important to students' mathematical development. English classified students into four categories: students with strong number sense and strong problem-solving skills (SN/SP), students with strong number sense and weak problem-solving skills (SN/WP), students with weak number sense and strong problem-solving skills (WN/ SP), and students with weak number sense and weak problem-solving skills (WN/WP). The study assessed the student problem-posing ability via a pre- and posttest. Students were asked to pose problems in both formal and informal contexts. The pretest results showed students had difficulty generating problems in both formal and informal contexts. Students then participated in a problem-posing program consisting of six 45-minute instructional sessions. The posttest results showed students were able to create several novel problems, largely by changing the contexts of the problems. However, the program did not increase the diversity of the problem types students generated. Notably, the students in the SN/WP group were the least creative, while the WN/SP group showed the greatest diversity in terms of problems created.

Later, English (1997a) conducted a similar one-year study that involved designing and implementing a problem-posing program for students in the 5th grade. The purpose of the study was to investigate the extent to which students' number sense and novel problem-solving skills correlated to the problem-posing skills used in routine and non-routine situations. Routine situations involve procedures, while non-routine situations involve various reasoning processes. According to English (1997), "a framework developed for the study encompassed three main components: (1) children's recognition and utilization of problem structures, (2) their perceptions of, and preferences for, different problem types, and (3) their development of diverse mathematical thinking" (p. 188). Like the students in the previous study, students were grouped into four categories: SN/SP, SN/WP, WN/SP, and WN/WP. In contrast to the previous study, however, the fifth-grade students increased their understanding of problem structures, including their ability to identify problem structures that correspond to a given situation or problem. They also improved their ability to create new problems similar to problems they had been provided with, generate new story contexts, and pose more complex and diverse problems. Students in the WN/WP group struggled most at generating diverse problems.

Further, English (1997b) conducted a third study during the final year of the project to investigate the problem-posing skills of students in grade seven. Students were assessed across a range of mathematical situations in order to identify the relationship between students' problem-solving and problem-posing activities, monitor students' perceptions and attitudes toward problem solving and problem posing, and monitor students' metacognitive activity. A 3-month problem-posing program was developed to promote an inquiry-oriented classroom. 23 students were selected to participate in the study based on their responses to tests of number sense and novel problem-solving ability. Another six students were selected to serve as a small control group. During the pretest assessment, students posed many non-solvable math problems, and some students were unable to pose any problems. During the problem-posing program, students were instructed to pose problems from a provided set of information. Students were also shown sample math problems and asked to pose problems related to the samples. During the posttest assessment, every student was able to pose problems adequately. The number of non-solvable problems decreased, and complex problems were posed more frequently. 68% of the students reported that they became better at solving and posing mathematical problems.

Silver and Cai (1996) examined middle school students' problem-posing skills. 509 students participated in the study. Given a set of "story-problem" descriptions, students were asked to pose problems. Each student had to pose three problems based on a given situation and eight given problem-solving tasks. The problems the students posed were analyzed by solvability, linguistic, and mathematical complexity. Relationships within the sets of posed problems were also noted based on the data coding scheme the researchers developed. Problems were first divided into three categories: non-mathematical questions, mathematical questions, and statements (i.e., non-questions). The mathematical questions were then divided into solvable and non-solvable questions and analyzed for complexity. Students in the study provided 1,465 responses. Among the responses, more than 70% of the responses were classified as mathematical questions, 20% of the responses were statements, and 10% of the responses were non-mathematical questions. Among the mathematical questions, more than 90% of the mathematical problems posed were mathematically solvable. Silver and Cai also grouped students into "HI" and "LO" groups based on their problem-solving skills. The HI group students generated a greater portion of the mathematical questions than the LO group. The LO group generated significantly more statements and nonmathematical questions. HI group students also posed more complex mathematical questions, and they posed a greater number of multi-relational problems as well. To summarize, the study's results indicated that middle school students were able to pose a large number of solvable mathematical problems when presented with a story prompt similar to the prompts often provided in textbooks.

Lin and Leng (1996) described how teachers implemented problem-posing tasks in the classroom via an analysis of the problems posed by 120 high-ability students from a secondary school. The study measured students' performances by analyzing the complexity of their proposed problems. The results showed that students demonstrated what they had mastered in class through the problem-posing tasks. As part of the study, teachers were invited to observe patterns in students' problem posing, mathematical learning, and thinking. In a later study, Cai and Silver (2005) suggested expanding the role of problem posing to have it serve as an assessment of students' mathematical understanding. The researchers argued that problem-posing tasks and activities can be used as assessment tools to inform teachers about students' learning and inform researchers about the effectiveness of proposed interventions.

2.5.2 Studies that investigated the effect of problem posing

Some studies have reported that problem posing has effects on students' beliefs and attitudes about mathematics and mathematics instruction. Balow and Cates (2006), for instance, investigated elementary teachers' beliefs about incorporating problem posing into elementary classrooms. 61 teachers from three elementary schools participated in a year-long staff development project aimed at promoting the use of problem posing in classrooms. Pre- and post-surveys were used to examine participants' beliefs about mathematics and mathematics instruction. The results showed that when teachers incorporated problem-posing interventions into their curricula, students showed greater levels of active involvement while creating and solving their posed problems. Cunningham (2004) examined the benefits resulting from a classroom activity in which students practiced problem posing. Students in the study were highly engaged in the problem-posing activity and felt a sense of ownership for the problems they proposed. The study also reported that when students posed their own problems, their senses of responsibility increased, as they felt empowered by directing their own understanding.

Moreover, Brown & Walter (1993) and Silver (1994) noted that problem posing may alleviate the anxiety some students experience while studying mathematics. When students create problems, they don't have to worry about the correctness of their answers, which can lead them (potentially) to feel that the math classroom is a less-threatening environment. Since students with LDs tend to experience math-related anxiety and a fear of making mistakes, having them engage in problem-posing activities has potential benefits.

To summarize, studies on problem posing have showed promising impact in terms of students' knowledge, problem-solving skills, problem-posing skills, creativity, and disposition toward mathematics in general education (e.g., Cai, 1998, 2003; Cai & Hwang, 2002; English, 1997, 1998; Lavy & Bershadsky, 2003; Silver et al., 1996; Stoyanova, 1999; Yuan & Sriraman, 2011). However, literature reveals limited knowledge about these aspects of problem posing for students with disabilities even though educational laws require all students receive the same quality of learning resources. So far, no studies have investigated multiplicative word problem-posing skills and the relationship between problem-solving and problem-posing skills for students with LDs.

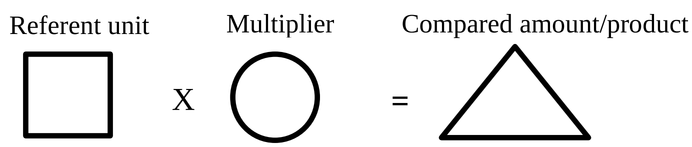
2.6 COMPS model-based instruction

Mathematical modeling is the process of using different kinds of mathematical structures (including graphs, equations, diagrams, and bars) to represent real-world situations (Annenberg Lerner, 2017). According to Lesh and Lehrer (2003), mathematical models can be used to facilitate students' conceptual understanding of word problems. The NCTM suggests "Modeling involves identifying and selecting relevant features of a real-world situation, representing those features symbolically, analyzing and reasoning about the model and the characteristics of the situation, and considering the accuracy and limitations of the model" (*Principles and Standards for School Mathematics*, p. 302). Studies have shown that mathematical modeling can promote students' problem-solving and mathematical thinking skills (Lesh et al., 2003). By studying mathematical models, students not only see the surface features of the model, but also gain a deep understanding of problem structures (Lesh & Lehrer, 2003). In a separate study, learning via mathematical models was also found to help students achieve a deep understanding of each element represented in the model and apply the model to mathematical learning activities (Hamson, 2003), including both problem solving and problem posing.

Xin and colleagues (Xin, 2012; Xin et al., 2008; Xin et al., 2011) have developed Conceptual Model-based Problem Solving (COMPS) approach, which focuses on conceptual understanding and representing word problems in mathematical model equations. Figure 2 presents the “Multiplicative Compare (MC) concept model including MC word problem story gramma to promote self-generated questions in guiding the representation of the information onto the MC diagram equation” (Xin, 2012, p. 123).

Multiplicative Compare (MC)

A MC problem describes one quantity as a multiple or part of the other quantity



Which sentences (or question) describes one quantity as a multiple or part of the other? Detect the two things (people) being compared and who (the compared) to whom (the referent unit). Name “whom” and “who” in the diagram. Fill in the relation (e.g. “2 times” or “1/2”) in the circle.

What is the referent unit? Write that quantity in the referent unit box?

what is the compared quantity or product? Write that quantity in the triangle on one side of the equation by itself.

Figure 2. MC WP story grammar poster (adapted from Xin, 2012, p. 123)

This approach, which emphasizes representing word problems in COMPS diagram equations before solving the problem, prevents students from relying on “keyword” or “cue word” strategies or other mechanical rules for operation sign for solving the problems. The COMPS diagrams help students develop a solution plan based on the model equations, provide students with opportunities to represent a variety of word problems, and promote students’ understanding of the underlying problem structure and the meanings of the key elements of the problems (Xin, 2012). Students are taught to learn the mathematical relations in the diagram and use the model to solve for unknown quantities in the math equation. As shown in Figure 2, the MC COMPS model provides three components that correspond to the three key elements in MC word problems. Students are instructed to identify the three elements in the word problem, map the correct elements into the COMPS diagram equation, and finally solve the problem. By practicing in a range of problem contexts and solving problems expressed as algebraic equations with unknown quantities occupying various positions, students achieve abstract understanding of mathematical concepts via the COMPS approach. They also learn how to transfer the skills they use to problems with similar constructions in other contexts (Xin et al., 2008). Moreover, studies (e.g., Xin & Zhang, 2009; Xin et al., 2011) have shown that the COMPS approach is particularly helpful for students with LDs. Since students with LDs tend to have poor working memory, the COMPS approach can alleviate students' memorization burden by removing irrelevant information that would be provided in the problem.

Studies conducted during the past 10 years have shown that the COMPS approach is effective in promoting the conceptual understanding of mathematics topics and word problem-solving skills for students with learning disabilities. For example, Xin et al (2008) investigated the effect of teaching "word problem story grammar" on the ability of students to solve word problems. A single-subject design was used to explore the functional relationship between students' word problem-solving performance and the educational intervention. Five students in grades 4 and 5 who had, or were at risk for, mathematics disabilities participated in the study. According to Xin et al. (2008), the concept of "story grammar" was borrowed from reading comprehension research, in which "grammar" refers to the "elements" of a reading passage (p. 5). The study developed a series of grammar-promoting questions designed to help students identify the three elements in the problem quickly. For example, the guiding questions included "Which sentence tells about the referent unit?" (p.127) "Which sentence tells about the compared amount" (p.127), and "Which sentence tells about relation?" (p.127). In addition to helping students map the grammatical elements into the corresponding places in the model, the COMPS model presents a mathematical equation that represents the structure and relationships of the three elements. After representing three key elements in the diagram equation correctly, students just need to follow a simple procedure to find the answer of the unknown quantity (e.g., if one of the two factors on one side of the equation is unknown, division may be able to find the unknown quantity). The results of this study indicated that conceptual model-based representation promoted by word problem story grammar improved students' performance on arithmetic word problem solving and improved prealgebra concept and skill acquisition.

Xin and Zhang (2009) investigated the effects of COMPS instruction on improving students' prealgebra concepts and skills in different problem structure and situations. Participants were three 4^{th} and 5^{th} grade students with or at risk for mathematics disabilities. The study used adapted multi-probe design (Horner & Baer, 1978) to evaluate the functional relationship between the COMPS instruction and students' word problem solving performance on criterion word problem-solving tests, KeyMath Revised Normative Update (Connolly, 1998) and prealgebra test. Participants in this study received two equal group (EG) problem solving sessions, two mixed EG problems. three session on solving MC problems and two sessions on solving problems with all types. Students were taught to identify the key elements in EG problems (i.e., unit rate, number of units, and product) and MC problems (i.e., compared amount, referent, or multiplicative relations) in different situations. The results showed all students improved their performance on the criterion test and prealgebra test as well as the KeyMath Revised Normative Update subtest.

Xin et al. (2011) compared COMPS approach with general heuristic instructional (GHI) approach regarding teaching multiplication-division word problem to students with learning problems. The study used a pretest-posttest, comparison group design with random assignment of participants to COMPS group and GHI group to examine the effect of the two approach. Participants who were 29 elementary students with learning problems were assigned into the two groups by using a stratified random-sampling procedure based on students' grade, gender and pretest score. Students in COMPS group received explicit strategy and modeling instruction on the concept of equal groups, using the COMPS approach to solve EG and MC problems as well as mixed EG and MC problems. While, students in GHI group received guided instruction on following a problem-solving checklist SOVE (Search-Organize-Look-Visualize-Evaluate) which was a part of school's curriculum and teaching practice to solve problems. Results showed that students in COMPS group have much better word problem solving performance than the students in GHI group.

2.6.1 COMPS-based problem-posing intervention

This study used a COMPS-based problem-posing intervention to teach problem-posing skills for students with LDs. The COMPS-based problem-posing intervention was developed based on the key elements of the COMPS model-based problem-solving diagram (Xin, 2012) and the “What-If-Not” strategy (Brown and Walter, 1993). Figure 3 presents the conceptual framework of the COMPS-based problem-posing intervention for posing MC word problems in structured situations. First, a MC story is presented. For example, “Edward has 192 oranges. Brandon has 16 times as many oranges as Edward. Brandon has 3,072 oranges.” Then, students are prompted to map these information onto the diagram equation and then list the three components of the story based on the COMPS model (the referent unit is “Edward has 192 oranges,” the multiplier is “Brandon has 16 times as many oranges as Edward,” and the product is “Brandon has 3072 oranges”). Next, students write down the math equation (192 x 16 =3072) based on the COMPS model. After this, students select one component to challenge. If a student decides to challenge the referent unit, for instance, the students applies the “What-if-Not” strategy and asks, “What if the referent unit is not known? What would the posed problem look like then?” Next, students use the variable “a” to represent the unknown quantity in the equation, that is, a x 16=3072. Finally, students pose their new questions based on the equation. In this example, the new question might read as follows: “Brandon has 3072 oranges. Brandon has 16 times as many oranges as Edward. How many oranges does Edward have?” A rubric has been developed to analyze students’ posed problems.

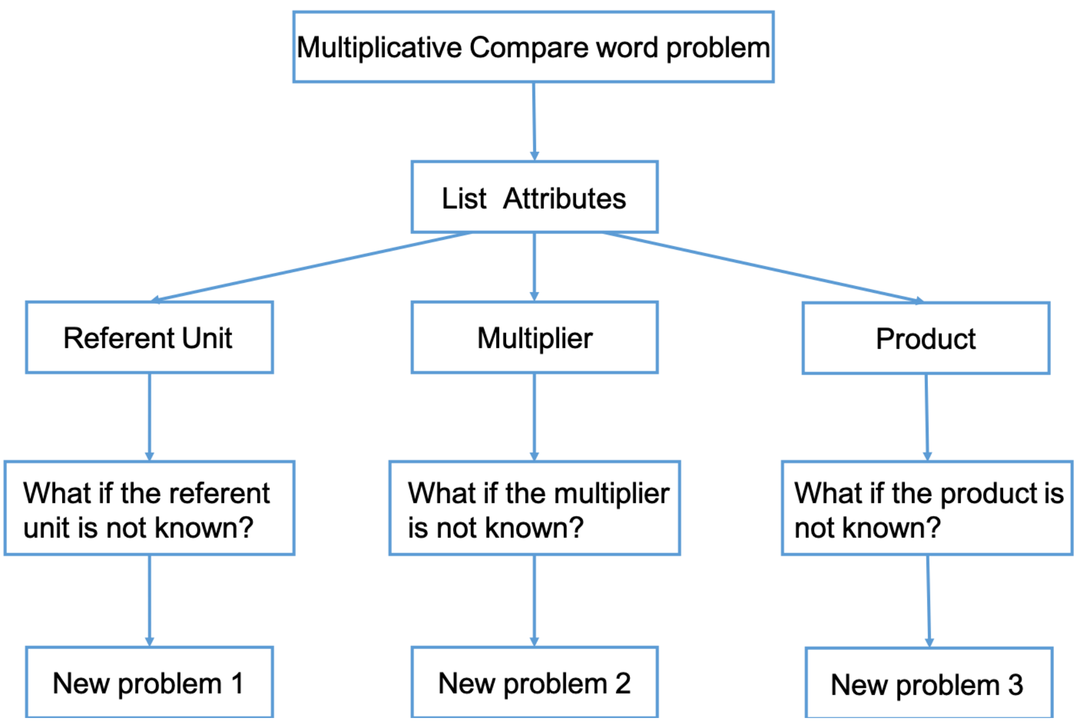


Figure 3. COMPS-based problem posing intervention flowchart

2.7 Students with learning disabilities

According to the Individuals with Disabilities Education Improvement Act (IDEA, 2004), specific learning disabilities (SLDs) are defined as "disorder[s] in one or more of the basic psychological processes involved in understanding or using languages, spoken or written, which may manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or do mathematical calculations" (U.S. Office of Education, 1977, p. 65083). In the past, the discrepancy model was commonly used as a major diagnostic criterion for SLDs. For example, a child who has normal IQ for his or her age but who fails to achieve at a level commensurate with that IQ would embody the discrepancy referred to in the name of the model (Raymond, 2000; Vaughn & Fuchs, 2003). However, this model is a "wait until fail" model. In other words, students with SLDs may not receive accommodations and interventions until they have already failed to achieve. However, nowadays, the identification process for learning disabilities has shifted away from the traditional IQ-achievement discrepancy model and towards the "Response to Intervention" (RtI) model. RtI is a multi-tiered approach for identifying students with learning disabilities who have not responded to evidence-based interventions (Fuchs & Fuchs, 2006; Lembke et al., 2012). The RtI model typically encompasses three tiers. In the first Tier, all students in a general classroom receive universal screening and high-quality instruction. Those who do not respond to the instruction well proceed to Tier 2. In Tier 2, the potential at-risk students receive a targeted intervention, typically in the form of small group instruction. Those who does not make enough progress in Tier 2 proceed to Tier 3. In Tier 3, students receive substantial interventions. Typically, they receive one-on-one instruction to address their learning needs. Those who do not respond to Tier 3 instruction well are referred for eligibility. The RtI model is a preventative approach that identifies potential at-risk students via early screening before they fail to achieve. The impact it has on the educational trajectories of students with learning disabilities is profound, but it has been found to particularly impact students' mathematics performance (Geary, 2003; Geary et al., 2012).

2.7.1 Characteristics of students with learning disabilities

To assess educational interventions that aim to help students with LDs, an understanding of the characteristics that students with LDs tend to share is necessary. According to the

Diagnostic and Statistical Manual of Mental Disorders (DSM-IV-TR, 2000), students are diagnosed as having LDs when they are "substantially below that [which is] expected given the person's chronological age, measured intelligence, and age-appropriate education" (p. 53). The characteristics of students with LDs provide an explanation for why these students have difficulties in learning in field. First, students with LDs tend to have working memory issues that lead to poor performance in terms of acquiring math knowledge and using appropriate strategies to solve mathematics problems (Kroesbergen & Van Luit, 2003). Memory problems may affect students' math performance in several ways. For example, memory problems may interfere with students' ability to retrieve basic arithmetic facts quickly (Geary, 2003). In upper grades, memory problems may affect students' ability to recall the steps needed to solve more difficult word problems. Students may exhibit inconsistent performance, leading teachers to question why, for instance, a student who appeared to know the facts yesterday can't seem to remember them today. Due to their poor memory, students with LDs have difficulty connecting knowledge they have previously learned to new knowledge to generate mathematical ideas (Kroesbergen & Van Luit, 2003).

Second, students with LDs tend to have relatively short attention spans (Zentall, 2014). Students with LDs who have short attention spans can easily get distracted if they see something, hear something, smell something, or feel something unrelated to the task at hand, preventing them from focusing on the task (Lerner, 2000).

Furthermore, students with LDs tend to have delays in cognitive development, which hinders their learning and information processing (Zentall, 2014). This can lead to problems understanding relationships between numbers and operations (e.g., fractions and decimals, addition and subtraction, multiplication and division), solving word problems, and using effective counting strategies. Faced with these disadvantages, students with LDs can have difficulties getting fully involved in mathematics problem solving, which has been proposed in reform-based classes (Miller & Hudson, 2007).

Literature shows that students with LDs easily get lost in reform-based instruction and "seemed to disappear during whole-class discussions" (Baxter et al., 2001, p. 545). A factor that contributes to the academic struggles of students with LDs is working memory (Schuchardt et al., 2010). Working memory not only stores the set of information relevant to the task at hand, but also processes any additional sets of information encountered, finally combining both sets of information in a cohesive mental whole (Allen et al., 2006; Baddeley, 2003). Conceptual understanding does not require that students spend much time remembering facts and knowledge. Therefore, a COMPS-based problem-posing intervention may be effective for students with LDs to solve and pose problems.

CHAPTER 3. RESEARCH METHODOLOGY

3.1 Pilot Study

The purpose of the pilot study was to explore the problems students pose in a mathematics classroom environment in order to develop criteria for students' posed problems. Students' problems were analyzed based on the framework provided by Leung (2013). This framework classified posed problems into five categories: (1) responses that are not problems, (2) non-math problems, (3) impossible problems, (4) insufficient problems, and (5) sufficient or extraneous problems.

3.1.2 Participants

Participants were four 7th grade students with LDs receiving learning support in their mathematics class, who met every morning between 9:12am and 10:05am. The participants in the pilot study would not be enrolled in the intervention study. Two students spent 62.5% of their time in the general classroom and were in special education for four years. The other two students spent 75% of their time in general education. Of the latter two students, one was in special education for five years and the other student was in special education for three years. These students were chosen because the special education teacher said they had not previously been exposed to problem posing, which meant that the problems they would create would be relatively untainted by prior exposure to problem-posing activities. The students were given alternative MC problem-posing tests (see Table 3) which required them to create a solvable problem based on the given scenario and equation with one factor unknown. Testing took place in the students' math classroom. Each test had six questions, and each testing session lasted about 53 minutes. Each student participated in eight sessions, thus completing 48 problem-posing questions in total. Therefore, four students completed a total of 192 problems. An example demonstrating how to write word problems and a sample scenario were provided to students upon request.

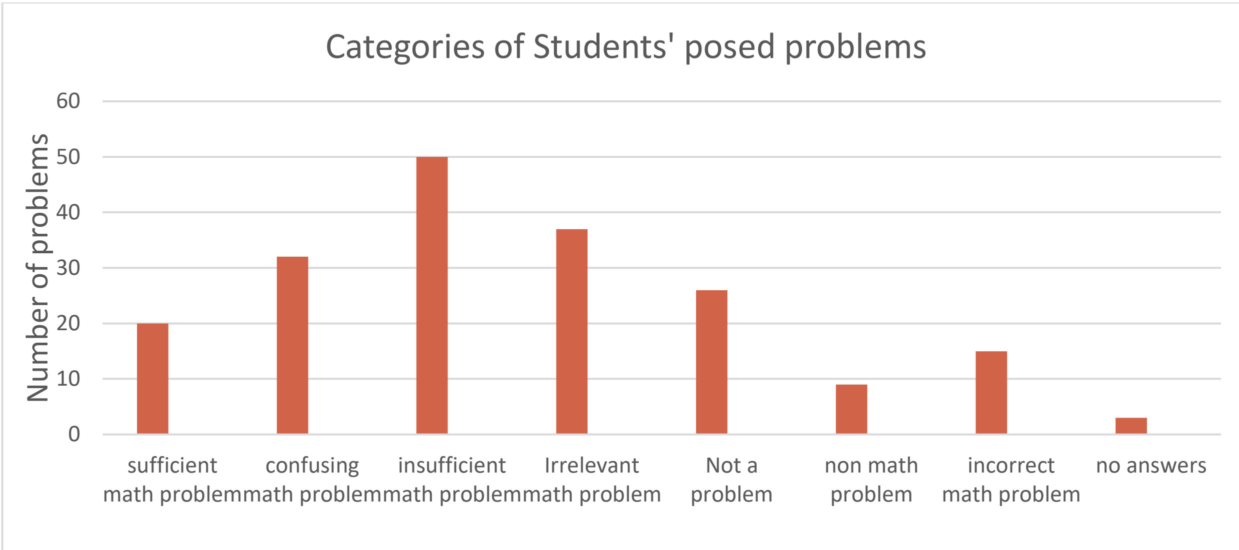


Figure 4. Categories of students' posed problem

3.1.3 Results

Adapted from Leung's problem posing classification framework, an analysis of students' tests revealed that their posed problems fell into eight categories: (1) sufficient math problems, (2) confusing math problems, (3) insufficient math problems, (4) irrelevant math problems, (5) not problems, (6) non-math problems, (7) incorrect math problems and (8) no answers. This classification provided more detailed information to analyze the characteristics of students' posed problems. On three questions, students did not pose any problems, instead leaving the answer field blank. Among the completed answers, 48 problems were labeled "insufficient math problems," as students did not provide sufficient information to make the problems solvable. 37 problems were labeled "irrelevant math problems." Irrelevant math problems type is an added type as some students' posed problems were irrelevant to the given information. For example, the posed problems have no relationships with the numbers showing in the equation and scenario. Thirty-two problems were labeled "confusing math problems" which is also an added problem type. In these problems, the student was able to express the existence of an unknown quantity. However, the student was unable to write a comprehensible problem directing the reader to find that unknown quantity, given other vital pieces of information. An example of a confusing problem is "Richard has 54 marbles. Chris has 3. How many marbles does Chris have?" 15 problems were labeled "incorrect math problems," which is also an added problem type, as the posed problems do not correctly reflect the mathematical relationships of the given equation. Figure 4 presents the problems types students created by category. The problem posing test rubric (see Table 5) was developed based on the findings of the pilot study.

3.1.4 Implication of the Pilot Study

The pilot study found that most of students were able to create questions. However, due to a poor conceptual understanding of how mathematical concepts are relayed in MC word problems and a lack of experience with problem-posing strategies, they often struggled to pose questions correctly. In addition, in cases when students could not immediately identify a strategy to create a problem, they tended to quickly tire and give up. Some students could not even distinguish between writing that conveyed a problem and writing that did not, as some of their posed problems did not qualify as problems at all. These findings suggest a need to explore COMPS-based problem-posing interventions that may improve both students' problem-posing and problem-solving performance.

The following sections of the chapter describe the methodology of this study designed to examine the effectiveness of a COMPS-based problem-posing intervention on the word-problem-posing and problem-solving skills of 7th graders with LDs.

3.2 Participants and Setting

The study was conducted in an urban public middle school in the southern United States. To be included in this study, students needed to satisfy the following criteria: (a) having been identified with a learning disability, (b) having been identified by their teachers as students struggling with mathematic problem solving, (c) having a pre-intervention performance on the problem-posing and problem-solving sections of the criterion test of less than 70% (as, according to Montague and Bos (1986), 70% correct corresponds to an average grade), and (d) having no prior experience with COMPS diagram and the "what if not" problem posing strategy. The qualifying participants were three seventh grade students with LDs. Table 1 reports demographic information about these students.

Table 1. Demographics

Variable	**Breana**	**Taylor**	**Amy**
Gender	Female	Male	Female
Ethnicity	Caucasian	African-American	African-American
Age	13	13	13
Grade	7	7	7
Classification	LD	LD	LD
Reduced/Free Lunch	Y	Y	Y
Years in Special Education	4	4	3
Learning Support Classroom	Applied Math/ELA/ Study Skills	Applied Math/ Reading/ ELA	Applied Math/ Study Skills
Percentage of time in general education class	62.5	62.5	75
IQ test	WISC-IV	WISC-IV	WISC-IV
Full Scale	81	81	110
Verbal	93	84	126
Performance (non-verbal)	102	105	125
State Assessment	STAAR 2018	STAAR 2018	STAAR 2018
Math	1468 (did not pass)	Not present	1592 Pass
English/Language Arts	1431 (did not pass)	Not Present	1651 pass

Note. LD=Learning Disability WISC-IV = Wechsler Intelligence Scale for Children – Fourth Edition (Wechsler, 2003);STAAR = The State of Texas Assessment of Academic Readiness

The participants were led to a library study room between 9:07am and 10:05 am in the morning and between 13:50-14:40 in the afternoon from Monday to Friday to participate in the study. The library room contained tables and chairs, printers, and bookshelves with hundreds of books. Pencils, scratch paper, all necessary test booklets, and calculators were provided to the students during each session. Each student was seated at a table directly across from the researcher. This study consisted of 15 total sessions of data collection for each student. Each session took approximately 45 minutes. The students' class progress and coursework schedules were not significantly affected by the study.

3.3. Dependent Measures

Dependent measures included students' performance on COMPS criterion word problem-solving tests as well as researcher-developed problem-posing tests and transfer tests. In addition, to examine the social validity of the problem posing instruction program, a questionnaire was developed to assess students' perception of their experience and evaluation to the instruction was given to students following the study.

3.3.1 Problem Solving Criterion Test

The criterion test and its alternate forms were adapted from Xin (2012), which were designed to assess the students' multiplicative reasoning and problem-solving acquisition and maintenance. The criterion tests consisted of six one-step MC word problems. According to Xin (2012), these criterion word problem-solving tests were aligned with the curriculum and the National Council of Teachers of Mathematics standards (NCTM, 2000) which emphasizes a conceptual and structural understanding of word problems. In addition, these criterion tests were informed by academic literature from the fields of mathematics and special education, as well as input from both mathematics education researchers and educators. According to Xin et al. (2011), the test-retest reliability of the criterion test was .86, and the parallel-form reliability of the alternate forms of the criterion test was .85. The MC problems that comprised the test used the conceptual model "referent unit × multiplier = compared amount/product" (Xin, 2012). Each test worksheet was composed of four "referent unit" unknown problems, one "multiplier" unknown problems, and one "product/compared amount" unknown problems. The order of the four types of problems in each worksheet was randomized. For each problem, the participants were required to provide their reasoning (i.e., to explain the process they used to solve the problem) and to provide their final answer in the form of a number. Table 2 presents the three components of the MC problem structure. Sample of actual test sheet can be found in Appendix A.

Table 2. Three Components of MC Problem Structure

MC Problem Components	Sample Problem Situation
The “referent unit” is unknown.	Earl has 374 pennies in a jar. Earl has 17 times as many pennies as his sister Stacy. How many pennies does Stacy have?
The “multiplier” is unknown.	Paul has 378 points. Carey has 18 points. Paul has how many times as many points as Carey?
The “Product” is unknown.	Frank has been on the basketball team for 28 days. His friend Mike has been on the basketball team 14 times as long as Frank. How long has Mike been on the basketball team?

3.3.2 Problem-Posing Test

The researcher developed the problem-posing test which was used during the pre-test, the intervention, the post-test, and the maintenance phases of the study. To determine the content validity of the test, the researcher consulted with educators and professors in the fields of special education and math education. Students were required to pose problems according to a predetermined structure. Each of the six MC problems on the test provided a scenario and an equation containing an unknown factor. Students were required to create problems based on the equation. Table 3 presents a sample item from the problem-posing test. Actual test sheet can be found in Appendix B.

Table 3. The Components of Problem Posing Test

Given scenario: Richard and Chris would like to compare the number of marbles they have.	
54 x 3= a	Posed problem 1
54 x a =162	Posed problem 2
a x 3 = 162	Posed problem 3

3.3.2 Problem-Solving Transfer Test

The format of problem-solving transfer test is identical to the criterion test but had six two-step word problems taken from assessments recorded in the State of Texas Assessments of Academic Readiness (STAAR) database over the past six years. The transfer test consisted of six two-steps word problems with the operation of multiplication division, multiplication multiplication, division division and division multiplication. The problem-posing transfer test was conducted toward the end of the pre-test, posttest, and maintenance test phases, respectively. One data point was collected in each phase (i.e., each student took the transfer test three times). Appendix C presents a sample problem-solving transfer test.

3.4 Data Scoring

All of the tests taken by the participants were analyzed for two dependent variables: accuracy of problem solving and quality of problem posing.

3.4.1 Accuracy of Problem Solving

To determine the accuracy of a student's problem solving, the total points earned by the student were divided by the total possible points to give the percentage of problems solved correctly in each test. For the criterion test and its alternate forms, each problem was worth two points, so the total possible score for each test was 12 points. A rubric (see Table 4) was developed to ensure accurate scoring. If a participant only wrote the correct final answer or provided the correct answer with problem solving process, two points were awarded. If a participant provided a problem-solving process that reflected the correct reasoning but did not obtain the correct answer or if the participant reached the correct answer but provided an incorrect reasoning process, one point was awarded. If the participant only provided an incorrect answer with no accompanying explanation or if the participant provided both incorrect reasoning and an incorrect, zero points were awarded.

Table 4. Scoring Rubric for MC Problem Solving Test (adapted from Liu, 2017, p. 136)

Earl has 374 pennies in a jar. Earl has 17 times as many pennies as his sister Stacy. How many pennies does Stacy have?	
2 points	(1) The participant only wrote the correct number as the result. e.g., “22”. (2) The participant wrote the correct answer and provided correct problem solving process (no matter what strategy was used), and it led to the correct number as the result. such as: (a). ″374÷17= 22″ (b). “374÷22 =17”
1 point	(1) The participant provided a problem solving process that reflected the correct understanding and reasoning, but ended up without a number as the result or with an incorrect number as the result. e.g., ″374÷17=27″. (2) The participant had the correct number as the result, but the problem solving process was incorrect (reflecting incorrect understanding or reasoning of the problem context). e.g., ″374 – 17-17-17…”, so 27″.
0 points	(1) Both the problem solving process and the answer were incorrect. e.g., ″374 x 17 =6358″. (2) Only an incorrect number (e.g., ″6358″) or nothing was written as the answer

3.4.2 Quality of Problem Posing

The participants’ problem-posing tests were scored to measure the students’ problem-posing proficiency. Each problem-posing test contains two scenarios and each scenario has three equations with one factor unknown, therefore, there is a total of six problem posing tasks in each test. For the problem-posing test and its alternative forms, each item was assigned two points, so the total possible score for each test was 12 points. Table 5 illustrates the scoring rubric and provides sample problems corresponding to 2-point, 1-point, and 0-point scores. The scoring rubric was developed from results of the pilot study and Leung’s classification of posed problems (2013). Students’ posed problems were classified into eight categories, and each category was awarded a corresponding number of points. Students were required to create questions based on equations with either the referent unit, multiplier, or product unknown. They were allowed to write their questions according to provided scenarios if they wished. However, students were also allowed to use scenarios they created.

Table 5. Sample Scoring Rubric for Structured Problem−Posing Test

Richard and Chris would like to compare the number of marbles they have. 54 × 3=a	
Please pose your questions for the unknown "a" based on the equation and the scenario.	
2 points	A sufficient problem. The problem should involve the three factors: referent unit, multiplier, and product. The posed problem expresses clearly and correctly the mathematical relationship and asks a question for the unknown "a", that is the product; it shows a correct conceptual understanding of the problem. An example: Richard has 54 marbles. Chris has 3 times as many marbles as Richard. How many marbles does Chris have?
1 point	(a). A vague and confusing problem. The student was able to express the existence of an unknown quantity. However, the student was unable to write a comprehensible problem directing the reader to find that unknown quantity, given other vital pieces of information. Example: Richard has 54 marbles. Chris has 3. How many marbles does Chris have? (b). An insufficient problem. A mathematical problem with insufficient information and cannot be solved but the posed question reflects the student knows the product is not known. Example: Richard has 54 marbles. How many marbles does Chris have?
0 points	(a). An irrelevant problem. The posed problems have no relationships with the numbers showing in the equation and scenario. Example: I ate some fruits this morning. How many apples did I eat? (b). A non-math problem. The posed problem is irrelevant with mathematics. Example: How did Chris get her marbles? (c). Not a problem. Example: Chris played marbles yesterday. (d) an incorrect math problem. The posed problems do not correctly reflect the mathematical relationships of the given equation/condition. Example: Students posed EG problem instead of MC problem. (e) no answers. Students did not provide any answers and leave the filed blank.

3.4.3 Accuracy of Problem-Solving Transfer Test

There were a total of six items in the transfer test. Each problem had two steps and each step was worth two points, so each problem was worth four points, and the transfer test was worth 24 points in total. The grading rubric for this test was identical to the criterion test rubric. Actual sample transfer test can be found in Appendix C.

3.5 Social Validity

To determine the social validity of the intervention, a survey was developed to measure the participants' general perceptions of the experience of solving and posing MC word problems. The survey was administered to all participating students. These questions focused on the benefits of the intervention in terms of helping the students improve their MC math word problem-solving and -posing performance. A five-point Likert scale was used to measure the participants' agreements with statements about the intervention, with "1" corresponding to "strongly disagree", "2" to "disagree", "3" to "neutral", "4" to "agree", and "5" to "strongly agree". Items in this survey included statements such as "I like posing problems to other people" and "The teacher's explanation really helped me to clarify my mathematical thinking." See Appendix D for additional information about the survey.

3.6 Design

A multiple baseline design across participants (Horner et al., 2005) was used to evaluate the functional relation between the problem posing intervention and participants' problem-posing and word problem-solving performance. A single-subject research design was chosen because this research method is "particularly appropriate for use in special education research" (p. 174) to examine the functional relationship between dependent and independent variables and allow detailed analysis of individuals (Gast & Spriggs, 2010). Intervention effects in a multiple baseline design can be demonstrated by introducing the intervention to different subjects after stable responses patterns are established in each subject's baseline. If each baseline changes only when the intervention is introduced, a functional relationship is demonstrated and the effects can be attributed to the intervention (Kazdin, 1982; Barlow, Nock, & Hersen, 2009; Kennedy, 2005).

3.7 Procedure

Students participated in the study five times per week. The researcher worked with participants individually on site at their school. Each session took approximately 50 minutes.

3.7.1 Baseline

At the beginning of the experiment, the researcher told participants that they were going to solve some problems. The researcher directed the students to write down their answers and provide an explanation for how they arrived at their answer under each item. Students were allowed use the calculators and scratch paper provided to them, and the researcher offered to read the problems to the students if they needed help. However, the researcher notified the participants that they could not receive any feedback on their answers during the test. All three participants completed one criterion test in the morning and one problem-posing test in the afternoon of the same day. They were told to use as much time as possible to finish the test, but notified that the total time for each session was 50 minutes. Participants were required to read through each problem and to write down their solution process, including math equations, for each problem on the criterion test. For the problem-posing test, students were required to create problems based on the provided equations and scenario. However, students were allowed to create their own scenario when posing problems. A problem-posing example was given to students before each problem-posing test. Appendix E presents the problem posing example.

The three student participants were Taylor, Breanna, and Amy (pseudonyms). Testing proceeded at a staggered pace. A stable baseline was first established for Taylor. Following this, the intervention was employed on Taylor. During the intervention phase, while Taylor's scores on the problem-solving test showed an upward trend, Breanna's baseline was established, and the intervention was subsequently employed on her. After Breanna's performance on the problem-solving test improved and a stable baseline for Amy was established, the intervention was introduced to Amy. All students took one transfer test following the establishment of their baseline.

3.7.2 Intervention

After the baseline sessions, the students received a COMPS-based problem-posing intervention that consisted of the following two components: (1) representing and mapping the three components of MC problems in the COMPS diagram equation (Xin, 2012, see Figure 5) and solving for the unknown factor and (2) "what if not" problem posing strategies. Before instruction, the researcher distributed a teaching sheet with a MC problem story along with the

MC diagram equation (Xin, 2012,), a set of “what if not” questions, and several types of problems based on the MC story (see Figure 5). Then, the researcher taught the student how to represent an MC problem by using the MC diagram equation, which helped the student understand the multiplicative relationships in the MC problem structure. For example, in one instance, the researcher told the student,

> “The story is about the number of crayons Ray and Crystal have. It compares the number of crayons Crystal has to the number of crayons Ray has, and it involves a multiple relation (five times a number). In a MC problem, the relationship statement ‘Crystal has five times as many crayons as Ray’ provides the comparison and determines who is compared to whom (i.e., it determines that Crystal is compared to Ray). Therefore, the number of crayons Ray has is the referent unit, the number of crayons Crystal has is the compared amount (product), and five times is the multiple relation when the two quantities are compared.” (adapted from Xin, 2012).

The student was taught to write down three sentences, one corresponding to each of the components (see Figure 5 upper panel), and map the three components into the diagram. After this mapping, it is expected that the student would understand how a MC problem can be represented in the COMPS diagram equation.

The “what if not” strategy was subsequently introduced to help the student create questions based on the MC problem. Brown and Walter (1993) separated the problem-posing process into two stages: accepting the given problems and challenging the given problems. New questions can be raised in the latter stage. To challenge the given problems (i.e., the MC stories), the researcher asked, “what if the referent unit is not known in this story?” Since the referent unit in the sample story is the number of crayons Ray has, this question refers to a situation in which the number of Ray’s crayons is not known. The student was instructed to rewrite the MC equation and use the letter “a” to represent the unknown quantity of crayons Ray has. Then, the student was taught to create a new question based on the equation “a x 5 =625”. During the intervention phase, the student was taught to create problems in which either the unit referent, multiplier, or the compared amount was the unknown.

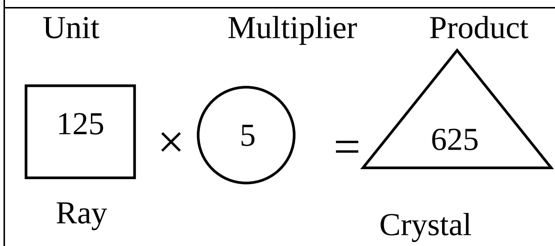
The student was also instructed to follow the “Detect-Transform-Create” checklist to create questions. First, detect the missing component in an MC equation—in other words, the quantity that the unknown “a” represents. Second, transform the equation into the diagram.

Third, create questions based on the unknown quantity. The intervention phase was carried out until the participant's problem-solving and -posing scores showed a stable trend of no less than 80% correct. However, to meet the convention of single subject design, in the cases that students reached 80% correct at the second or the third session in the intervention, the intervention continued until at least five data points were collected for the monitoring of the steadiness of the performance (Kratochwill et al., 2010).

A Multiplicative Comparison Problem

Ray has 125 crayons. Crystal has 5 times as many crayons as Ray. Crystal has 625 crayons.

Unit: Ray has 125 crayons
Multiplier: Crystal has 5 times as many crayons as Ray
Product: Crystal has 625 crayons



(Upper panel adapted from Xin, 2012, Unit 9)

Problem posing:
What if the unit is not known?
$a \times 5 = 625$
Crystal has 625 crayons. Crystal has 5 times as many crayons as Ray. How many crayons does Ray have?
What if the multiplier is not known?
$125 \times a = 625$
Crystal has 625 crayons. Ray has 125 crayons. Crystal has how many times as many crayons as Ray?
What if the product is not known?
$125 \times 5 = a$
Ray has 125 crayons. Crystal has 5 times as many crayons as Ray. How many crayons does Crystal have?

Figure 5. COMPS-based problem posing Instruction sheet

Post-test: At the end of the intervention, students took three criterion tests, three problem-posing tests, and a transfer test. This phase was identical to the baseline procedure.

Maintenance: One month after the intervention, students took the alternative forms of the problem-posing test and problem-solving tests.

3.8 Treatment Fidelity

Based on the work of Horner et al. (2005), a checklist (see Appendix F) that contains the treatment components and each component's corresponding application context was developed to assess the researcher's adherence to the intervention. One third of the sessions were observed by a schoolteacher to monitor the delivery of each component listed on the fidelity checklist. The researcher explained each step on the fidelity checklist to the observer before starting the study. The adherence of the researcher's implementation was judged according to the presence or absence of the features listed on the fidelity checklist. Overall treatment fidelity, calculated as the percentage of correctly implemented treatment components, was 98%.

3.9 Interrater Reliability

After collecting all of the completed participant test sheets, the researcher made photocopies of these tests. The researcher scored the copied version of each test using an answer key and the aforementioned rubrics. A research assistant (RA) who was blind to the purpose of the study independently rescored 30% of the test items. The interrater reliability was calculated by dividing the number of agreements by the sum of agreements and disagreements and multiplied by 100 %. The interrater reliability was 100% for the problem-solving test scores and 98% for the problem-posing test scores.

3.10 Data Analysis

Each participant's performance on the criterion test, problem-posing quality tests (including their alternative forms), and the three transfer tests (corresponding to the pre-test, the post-test, and the maintenance test phases) was plotted in a graphic display. Visual inspection was used for data analysis to determine the effectiveness of the intervention (Kennedy, 2005). Specifically, the visual inspection focused on six features to determine within- and between-phase data patterns, including level, trend, variability, immediacy of the effect, overlap, and consistency of data patterns across similar phases (Fisher, Kelley, & Lomas, 2003; Kazdin, 1982; Kennedy, 2005).

The percentage of non-overlapping data (PND) was used to calculate the effect sizes of the intervention. PND is most commonly used effect size index in single-subject studies (Gast & Spriggs, 2010). It was obtained by dividing the number of intervention data points exceeding the highest baseline data point in an expected direction by the total number of data points in that intervention phase (Scruggs et al., 1987).

CHAPTER 4. RESULTS

Figure 6 presents the three participants' performance on the criterion test, problem-posing tests, and transfer tests during the baseline, intervention, post-test, and maintenance assessment phases. Performance is interpreted as the percentages of correct responses on the criterion tests, problem posing tests, and transfer tests. Data points for each of the targeted skills are indicated with different shapes. For example, participants' progress in terms of problem-solving skills is marked with a blue rectangle.

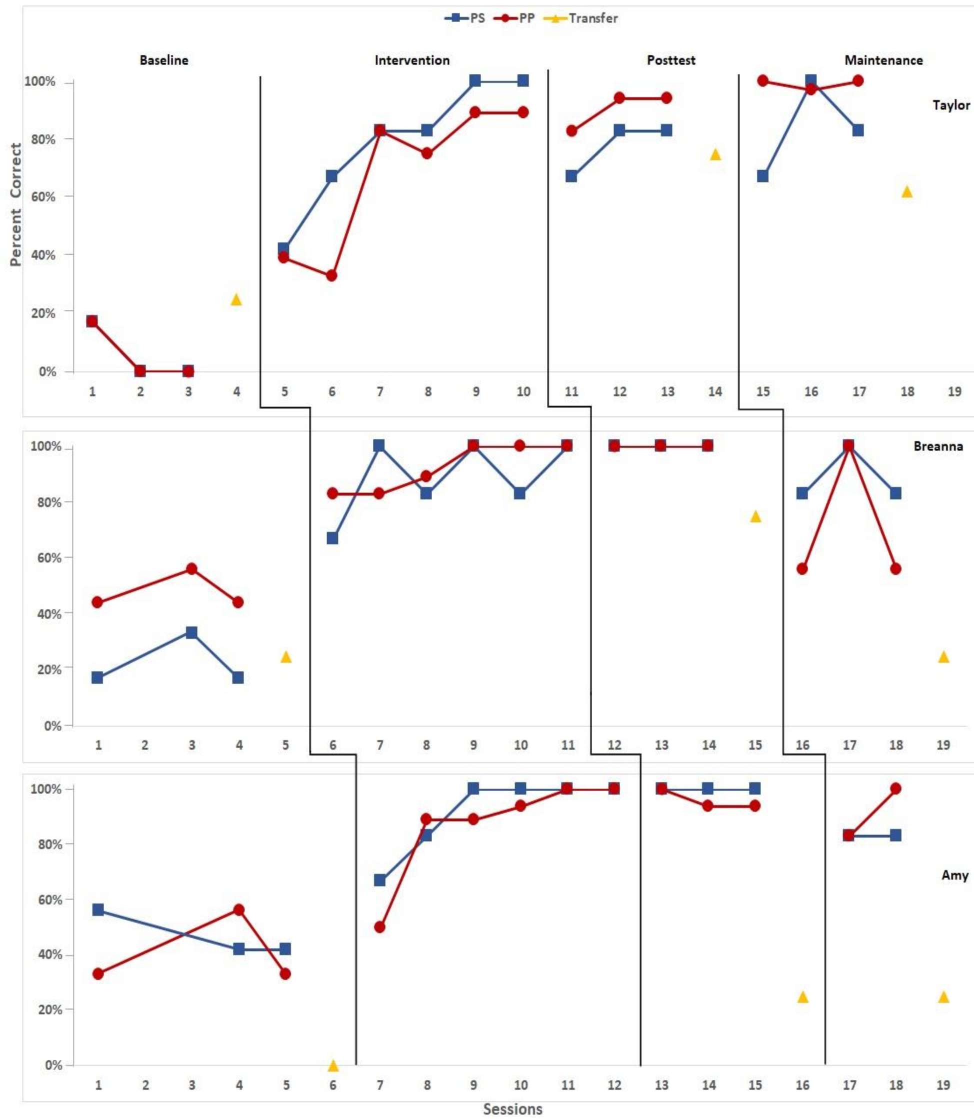


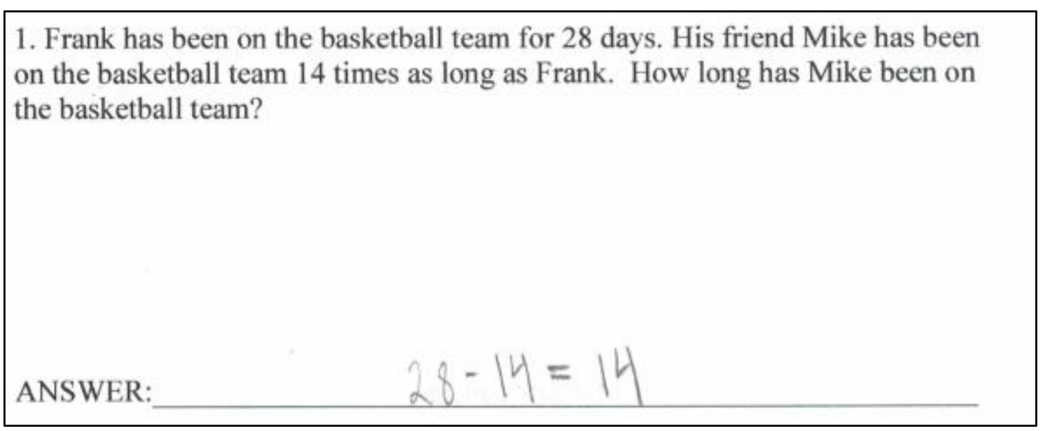
Figure 6. Percentage Correct for MC Word Problem Solving (PS) and Problem Posing (PP) for the Criterion Test and the Transfer Test During the Baseline, Intervention, and Posttest Phases for the Three Participants

4.1 Baseline Analysis

Taylor. (1) Problem solving: Taylor scored 17% correct on the first criterion test and scored 0% correct on the second and third criterion tests with a median score of 0% (mean = 5.7%) (see Table 6). According to his math teacher, Taylor, who has a history of reading difficulties, struggled to interpret the problems. Most of time, he used his fingers to point to every word while reading each problem aloud. At times, he appeared to use a "keyword" strategy in which he circled certain key words in the word problems. For example, on one occasion, when he encountered the word "times," he immediately circled the word and opted to use multiplication to solve the problem. It is possible that one reason that he scored so few questions correctly was that he approached each problem as a gamble, identifying key words and guessing how they figured into the task set before him. This would seem to be supported by the fact that, on some occasions, he simply added or subtracted the two numbers provided in the problem (see Figure 7).

Table 6. Percentage Correct for Taylor's Problem Solving and Problem Posing Performances during the Baseline Condition

	Session 1	Session 2	Session 3	Median
PS	17%	0%	0%	0%
PP	17%	0%	0%	0%

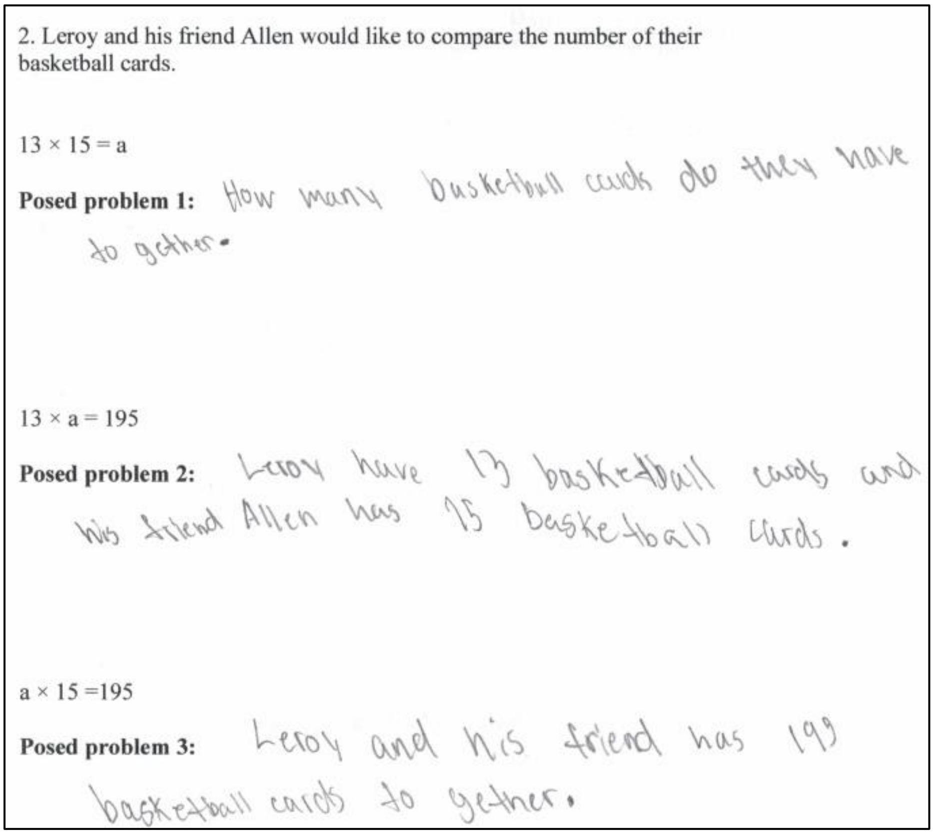

1. Frank has been on the basketball team for 28 days. His friend Mike has been on the basketball team 14 times as long as Frank. How long has Mike been on the basketball team?

ANSWER: 28 - 14 = 14

Figure 7. Sample of Taylor's Problem Solving Worksheet in Baseline

(2) Problem posing: Taylor scored 17% correct on the first test and scored 0% correct on both the second and third criterion tests with a median score of 0% (mean = 5.7%). On the first test, he created several insufficient questions, incorrect questions and non-problem questions (see

Figure 8), and, on the last two tests, he tended to simply use single sentences to create irrelevant problems or non-problem questions.


2. Leroy and his friend Allen would like to compare the number of their basketball cards.

13 × 15 = a

Posed problem 1: How many basketball cards do they have to gether.

13 × a = 195

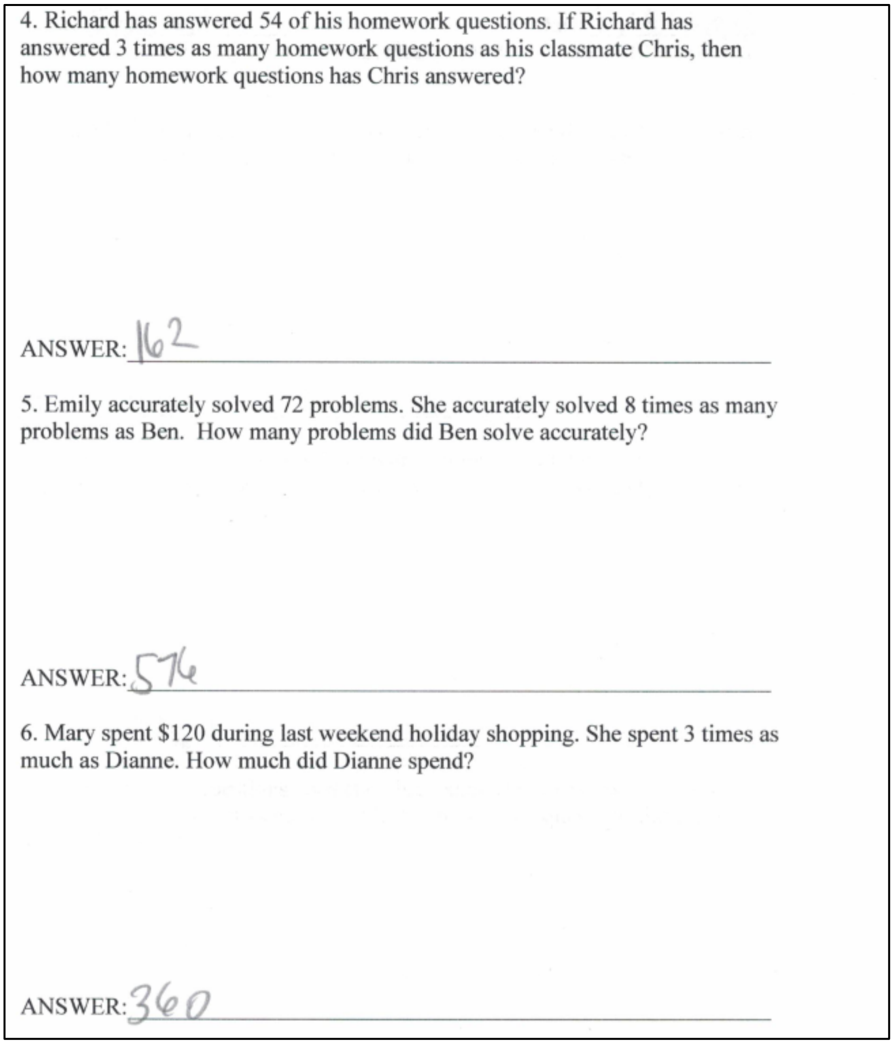
Posed problem 2: Leroy have 13 basketball cards and his friend Allen has 15 basketball cards.

a × 15 =195

Posed problem 3: Leroy and his friend has 199 basketball cards to gether.

Figure 8. Sample of Taylor's Problem Posing Worksheet in Baseline

Breanna. (1) Problem solving: Breanna scored 17% correct on the first criterion test, 33% correct on the second, and 17% correct on the third with a median of 17% correct (mean = 22.3%). She used multiplication to solve most questions on the criterion tests (see Figure 9), though she occasionally used division. Unfortunately, Breanna was not forthcoming in terms of explaining her thought process. On multiple occasions, when I asked her to explain how she had arrived at a certain conclusion or why she had used multiplication or division, she remained silent and smiled. According to her teacher, Breanna did not talk much at school.


4. Richard has answered 54 of his homework questions. If Richard has answered 3 times as many homework questions as his classmate Chris, then how many homework questions has Chris answered?

ANSWER: 162

5. Emily accurately solved 72 problems. She accurately solved 8 times as many problems as Ben. How many problems did Ben solve accurately?

ANSWER: 576

6. Mary spent $120 during last weekend holiday shopping. She spent 3 times as much as Dianne. How much did Dianne spend?

ANSWER: 360

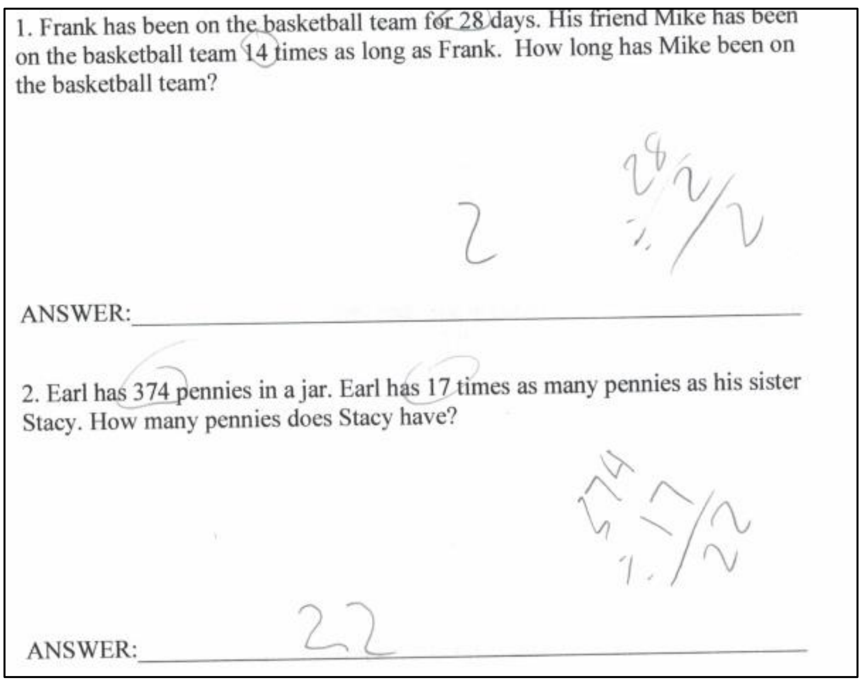
Figure 9. Sample of Breanna’s Problem Solving Worksheet in Baseline

(2) Problem posing: Breanna scored 44%, 56%, and 44% correct on the three criterion tests respectively. Breanna’s data showed a low degree of variability, with a median of 44% correct (mean = 48% correct). While most of her posed questions were indeed math problems, many were written in a confusing way or did not make logical sense. She did not appear to have a fluent understanding of the mathematical relationships expressed in most of the problems, as she often simply copied the wording from the problem-posing model that was provided to her.

Table 7. Percentage Correct for Breanna's Problem Solving and Problem Posing Performances during the Baseline Condition

	Session 1	Session 2	Session 3	Median
PS	17%	33%	17%	17%
PP	44%	56%	44%	44%

Amy. (1) Problem solving: Amy scored 56% on the first criterion test and 42% on both of the subsequent tests (see table 8). Amy's scores, which had median of 42% correct (mean = 46.7% correct), demonstrated a downward trend. Amy's strategy was to circle any numbers in the word problem, then use either multiplication or division to solve the problems, seemingly choosing one of the operations at random (see Figure 10). In cases when she arrived at a large number using multiplication, she was observed to erase her answers and rework the problem using division. The calculator played an important role in Amy's problem solving process, as Amy relied on it to complete virtually every problem. However, unlike the other two participants, Amy could consistently describe her process for finding a solution to each question.


1. Frank has been on the basketball team for 28 days. His friend Mike has been on the basketball team 14 times as long as Frank. How long has Mike been on the basketball team?

ANSWER:

2. Earl has 374 pennies in a jar. Earl has 17 times as many pennies as his sister Stacy. How many pennies does Stacy have?

ANSWER: 22

Figure 10. Sample of Amy's Problem Solving in Baseline

(2) Problem posing: Amy scored 33%, 56%, and 33% (respectively) on the first three problem-posing tests, with a median of 33% correct (mean= 40.67% correct). One notable aspect

of Amy's performance was that she did not use the provided scenarios to create questions. Instead, she chose to create her own. However, rather than posing MC problems, Amy mostly posed simple additive problems, tending to use the phrase "more than" in her posed problems to direct readers to perform addition (e.g., "How many more marbles does Chris have than Christian?").

Table 8. Percentage Correct for Amy's Problem Solving and Problem Posing Performances during the Baseline Condition

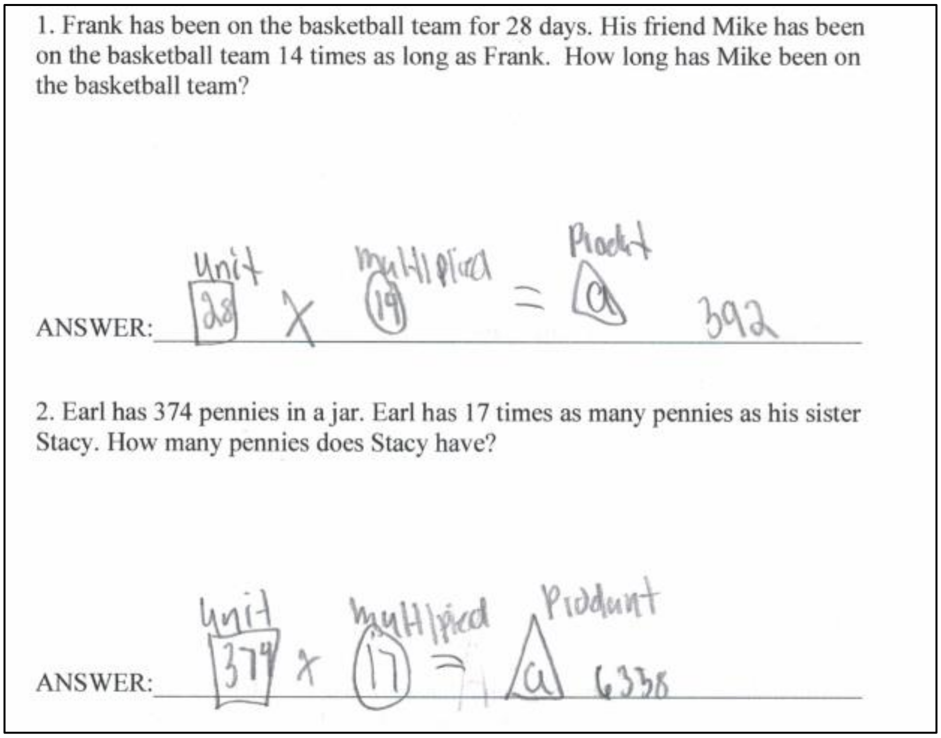
	Session 1	Session 2	Session 3	Median
PS	56%	42%	42%	42%
PP	33%	56%	33%	33%

4.2 Intervention Analysis

Taylor. (1) Problem solving: Taylor's problem-solving performance increased from 0% to 42% correct during the initial testing session of the intervention phase. Over the following sessions, his problem solving performance increased gradually. Taylor finally achieved 100% correct in the fifth session. Taylor's mean word problem solving performance increased from 5.7% correct in the baseline tests to 79.1% correct during the intervention. Taylor's median problem score during the intervention was 83% correct. The PND of Taylor's improved problem solving scores was 100%. He wrote down the COMPS diagram on the testing sheet and mapped all of the information from story problem into the diagram (see figure 11).

Table 9. Percentage Correct for Taylor's Problem Solving and Problem Posing Performances during the Intervention Condition

	Session 1	Session 2	Session 3	Session 4	Session 5	Session 6	Median
PS	42%	67%	83%	83%	100%	100%	83%
PP	39%	33%	83%	75%	89%	89%	79%



1. Frank has been on the basketball team for 28 days. His friend Mike has been on the basketball team 14 times as long as Frank. How long has Mike been on the basketball team?

ANSWER: Unit 28 x multiplied 19 = Product a 392

2. Earl has 374 pennies in a jar. Earl has 17 times as many pennies as his sister Stacy. How many pennies does Stacy have?

ANSWER: Unit 374 x multiplied 17 = Product a 6358

Figure 11. Sample of Taylor's Problem Solving Worksheet in Intervention

(2) Problem posing: Taylor's problem posing performance increased from 0% correct to 39% correct in the first testing session conducted under intervention conditions. In the following sessions, his performance increased gradually and finally reached 89% correct. Taylor's mean problem-posing performance increased from 5.7% correct during the baseline phase to 68% correct during the intervention phase, with a median score of 79% correct in the latter phase. The PND of Taylor's problem-posing performance was 100%. For the first two sessions of the intervention, most of his posed problems qualified as insufficient or incorrect problems. For example, for a question based on an unknown quantity "a" in the equation "54 x a = 162," Taylor posed the problem "How many do Richard and Chris have in total?," a response that was labeled as incorrect answers and earned zero points. However, starting from the third session, he started to circle "a" in the equation and labeled the quantity in the equation (see Figure 12). Taylor began to create some sufficient problems. As a result, he finally reached a score of 89% at the end of the intervention.

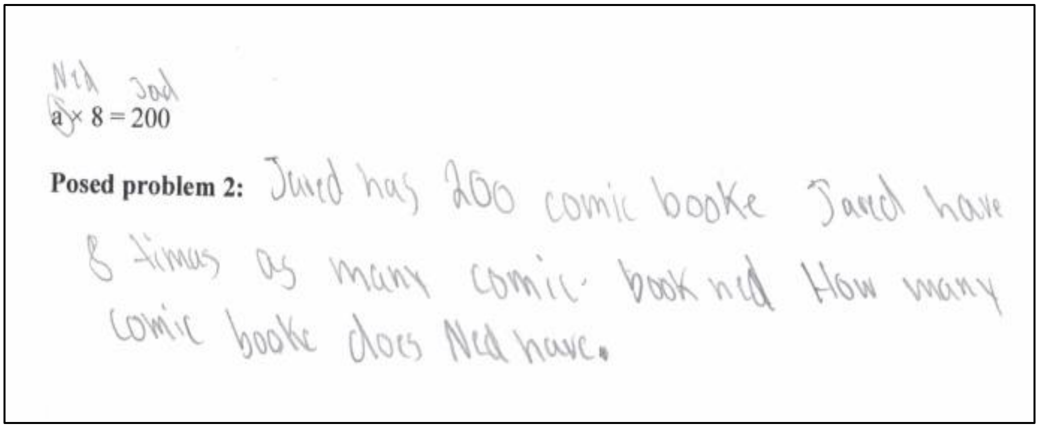


Figure 12. Sample of Taylor's Problem Posing Worksheet in Intervention

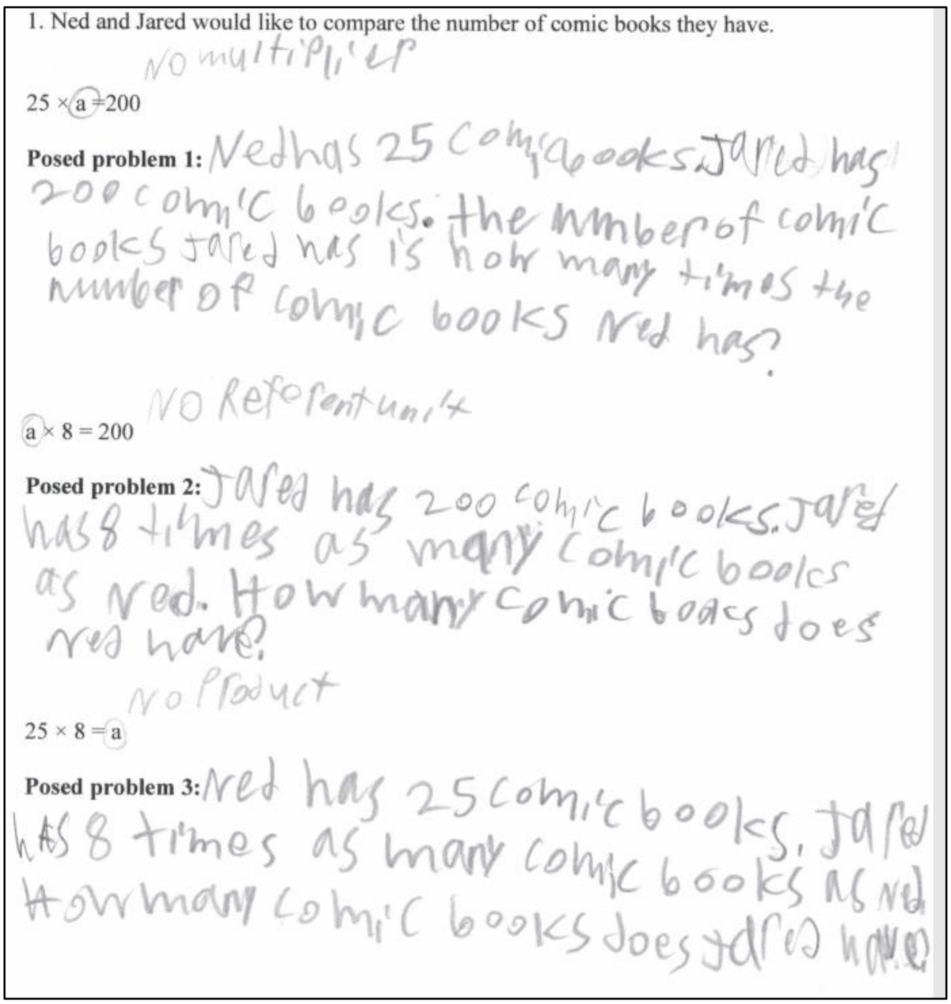
Breanna. (1) Problem solving: Breanna's problem-solving performance increased from 17% correct to 67% correct in the first session conducted under intervention conditions. In the subsequent sessions, her problem solving performance increased rapidly. She scored 100% correct in the second session and maintained a score of over 83% correct in the following four sessions. Breanna's mean word problem solving score increased from 22.3% in the baseline tests to 88.8% correct while receiving the intervention (see Table 10). Her median problem solving score during the intervention was 91.5% correct, and her PND was 100%. During the first session of the intervention phase, Breanna drew the MC diagram on her test sheet, and then mapped the information from the question into the diagram, using "a" to represent the unknown quantity. This helped her solve problems quickly and accurately. During the following sessions, she did not use the MC diagram, but did demonstrate that she understood MC problem structure in her explanations of her problem-solving process.

Table 10. Percentage Correct for Breanna's Problem Solving and Problem Posing Performances during the Intervention Condition

	Session 1	Session 2	Session 3	Session 4	Session 5	Session 6	Median
PS	67%	100%	83%	100%	83%	100%	91.5%
PP	83%	83%	89%	100%	100%	100%	94.5%

(2) Problem posing: Breanna's problem-posing performance showed an immediate increase from 44% to 83% correct in the first session conducted during the intervention. In subsequent sessions, her performance increased gradually, hitting 100% correct in the fourth

session and maintaining a score of 100% in the next two sessions. Breanna's mean problem-posing score increased from 22.3% correct during the baseline phase to 92.5% correct during the intervention phase, with a median score of 94.5% correct. The PND of Breanna's problem-posing score improvement was 100%. Starting with the first session of intervention, Breanna circled "a" in equations provided for all the questions. In session 3, she circled "a" in the given equation and also wrote either "no multiplier," "no referent unit," or "no product" on top of the "a" (see Figure 13). Breanna's posed problems in the last three sessions all qualified as sufficient. However, she did create some confusing problems and incorrect problems during the first three sessions.


1. Ned and Jared would like to compare the number of comic books they have.

no multiplier

$25 \times a = 200$

Posed problem 1: Ned has 25 comic books. Jared has 200 comic books. the number of comic books Jared has is how many times the number of comic books Ned has?

No Referent unit

$a \times 8 = 200$

Posed problem 2: Jared has 200 comic books. Jared has 8 times as many comic books as Ned. How many comic books does Ned have?

no Product

$25 \times 8 = a$

Posed problem 3: Ned has 25 comic books. Jared has 8 times as many comic books as Ned. How many comic books does Jared have?

Figure 13. Sample of Breanna's Problem Posing worksheet in Intervention

Amy. (1) Problem solving: Amy's problem-solving performance increased from 42% correct to 67% and 83% correct in the first and second sessions respectively. She proceeded to reach 100% correct in the third session, and she maintained that score for all of the remaining sessions. The average for Amy's word problem performance increased from 46.7% correct during the baseline phase to 91.7% correct during the intervention. Amy achieved a median problem solving score of 100% correct during the intervention, and the PND of Amy's problem solving scores was 100%. Amy was able to articulate her solving process on all of the criterion test problems, and she circled all of the numbers in the question during most sessions in the intervention phase.

Table 11. Percentage Correct for Amy's Problem Solving and Problem Posing Performances during the Intervention Condition

	Session 1	Session 2	Session 3	Session 4	Session 5	Session 6	Median
PS	67%	83%	100%	100%	100%	100%	100%
PP	50%	89%	89%	94%	100%	100%	91.5%

(2) Problem posing: Amy's problem-posing performance increased to 50% correct in the first session of intervention condition (from 33% correct during the baseline phase). Over the next several sessions, her performance slowly increased, and she achieved 100% correct in the fifth and sixth sessions. Amy's average problem-posing score increased to 87% correct under intervention conditions (from her previous level of 29%). Amy's problem-posing scores during the intervention had a median value of 91.5% correct. The PND of Amy's improvement in problem posing was 83%.

To a greater extent than the other participants, Amy liked to invent her own scenarios in the questions she posed. This tendency did not appear to affect her performance, however, which improved without an accompanying change in whether or not she used her own scenarios. In the first session, half of the problems she created were "not a problem" questions. For example, given the scenario "Devin and Sandra would like to compare the number of piano songs they can play" and the accompanying equation "35 x a =165," when asked to create a question for the unknown quantity "a," Amy wrote, "Sandra has 165 songs. Devin has 35 songs." For this response, she earned zero points, as she did not create a question that could be solved for the unknown "a." In the second and third sessions, she still created some "not a problem" questions despite demonstrating an understanding of the mathematical relationships involved in some of the problems (i.e., her problems did not ask for the unknown "a"). In these situations, she still did not earn points for her responses (see Figure 14). From the fourth session onward, however, she no longer posed a “not a problem” question. Instead, she mostly posed “sufficient” problems.

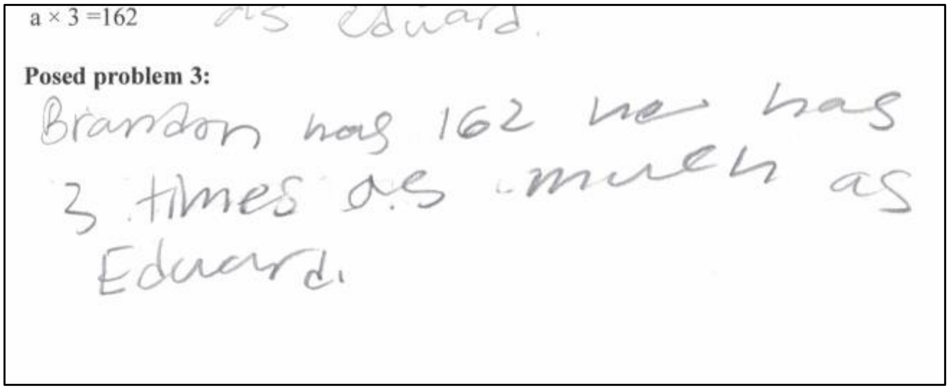


Figure 14. Sample of Amy’s Problem Posing Worksheet in Intervention

4.3 Post-test

Post-tests of problem solving and problem posing were administered to students immediately after they completed the intervention. For the problem-solving sessions, Breanna and Amy got 100% correct for all sessions, while Taylor got 67%, 83%, and 83% correct. However, Taylor’s median problem solving score for the post-test (83% correct) was equal to his corresponding median during the intervention phase. Breanna and Amy appeared very confident in the post-test, with both finishing each test in less than 20 minutes and neither erasing or rewriting any of their answers. Breanna scored 100% across all problem-posing sessions, while Amy scored 100% correct on the first post-test and then scored 94% correct on the last two sessions. Thus, Amy’s median score was 94%, which was still above her median during the intervention (91.5%). Taylor scored 83%, 94%, and 94% correct consecutively for the post-tests, and his median was 94%, which was far above the median (79%) for the intervention phase. In sum, all students maintained their high performance in the post-test phase.

Table 12. Percentage Correct for all Three Students’ Problem Solving and Problem Posing Performances during the Post-test Condition

	Session 1		Session 2		Session 3		Median	
	WP solving	Problem posing	WP solving	Problem posing	WP solving	Problem posing	WP solving	Problem posing
Taylor	67%	83%	83%	94%	83%	94%	83%	94%
Breanna	100%	100%	100%	100%	100%	100%	100%	100%
Amy	100%	100%	100%	94%	100%	94%	100%	94%

4.4 Maintenance Test

One month after the completion of post-test, all three students were given the maintenance test. It is important to note, however, that Amy was absent from school for several days after finishing the first two sessions. Because the end of the semester was approaching, she was unable to finish the final session. Thus, only two data points were collected for Amy's problem-solving and -posing tests. Amy scored 83% correct on both problem-solving tests, and the mean of her problem-posing scores was 91.5%. Taylor's median problem-solving performance on the maintenance test (83%) was the same as his median on the intervention test. However, Taylor's problem-posing performance increased from a median of 79% during the intervention to 94% during the post-test and 100% on the maintenance test. This may indicate that Taylor was still learning problem-posing strategies after the intervention. Breanna maintained her performance on problem-solving items, with a median score of 83%. However, her performance for problem-posing decreased to a median of 56%, which may indicate that she did not use the strategies for creating problems she learned during the intervention afterward.

Table 13. Percentage Correct for all Three Students' Problem Solving and Problem Posing Performances during the Maintenance Condition

	Session 1		Session 2		Session 3		Median	
	WP solving	Problem posing	WP solving	Problem posing	WP solving	Problem posing	WP solving	Problem posing
Taylor	67%	100%	100%	97%	83%	100%	83%	100%
Breanna	83%	56%	100%	100%	83%	56%	83%	56%
Amy	83%	83%	83%	100%			83%	91.%%

4.5 Transfer Test

Students completed one transfer test after the baseline, post-test and maintenance test phases (i.e., each student completed three transfer tests in total). Taylor's performance on the transfer tests was the best. His score increased from 25% correct to 75% correct following the intervention, and decreased a relatively slight amount to 62% after the maintenance test. Breanna's performance increased from 25% correct to 75% correct after the intervention, but it decreased to 25% after the maintenance test. By contrast, Amy scored 0% on the baseline, increasing her score to 25% after the intervention and maintaining her performance during the maintenance test.

Table 14. Percentage Correct for all Three Students' Problem-Solving Performance in the Transfer Test

	Baseline	Posttest	Maintenance
Taylor	25%	75%	62%
Breanna	25%	75%	25%
Amy	0%	25%	25%

4.6 Social Validity

All three students took the Student Perception and Satisfaction Survey. The survey was worth a total of 30 points. Taylor scored 27 points, while Breana scored 26 points. Both indicated that they found the problem-posing strategy they learned to be beneficial for solving and creating problems. For example, both chose "agree" when presented with the statement "The problem posing strategy the teacher taught me helped me solve and pose new word problems." All participants indicated that they liked solving and posing problems and enjoyed the teaching process overall. Amy scored slightly lower on the survey with a total of 24 points. Though her answer for "I like solving and posing word problems" was "neutral," she indicated that the strategy she learned was helpful for solving and posing new word problems. Overall, the survey's results indicated that all three students enjoyed the teaching process and thought the strategy was helpful.

CHAPTER 5. DISCUSSION

5.1 Effectiveness of COMPS-based Problem Posing Intervention

This study was to explore the effects of a COMPS-based problem-posing intervention on the word-problem-posing and problem-solving skill of students with LDs. On baseline tests, all participating students performed poorly in terms of both problem solving and problem posing. However, once the students received the intervention, their performance in both skill areas increased immediately. The results of the study suggest that all three participants used skills imparted by the problem posing intervention to solve and pose MC problems more effectively. Two participants, Taylor and Breanna, had baseline and intervention performances that did not overlap at all. In other words, the PND for their problem-solving and -posing performance was 100%. The third participant, Amy, had only a small amount of overlap: her problem-posing performance had a PND of 83% and her problem-solving performance had a PND of 100%.

In the intervention phase, Taylor took three sessions to reach above 80% correct for problem solving and maintained this performance for the rest of the intervention sessions. A few additional factors related to Taylor's solving performance bear mentioning. Since Taylor experienced reading difficulties, he was provided with sufficient time to read each problem, and, upon his request, problems were read to him aloud. It took Taylor two sessions to understand the multiplicative relationships in the MC problem structure and quickly identify which quantity was unknown in the given equation. This became clear as he thought through problems out loud in the earliest sessions, and, in doing so, betrayed a lack of understanding of the task before him. For example, during the first session, he mentioned that "125 times 5 equals 625, so there must be a total of 625 crayons for Crystal." He further explained that "because Crystal has five times as many crayons as Ray−here, you can see the word 'times'—I must need to use multiplication." His explanation illustrated why he used multiplication to solve most problems on the baseline criterion tests. After he was introduced to the three-component COMPS diagram and had the mathematical relationship of the story explained to him, I gave him two MC problem examples which showed that the word "times" does not always mean the reader must use multiplication. Starting from the third session, Taylor's problem posing had a sudden surge as he began to use strategies that demonstrated a clearer understanding of the mathematical relationships at play.

Before posing any problems, Taylor used the MC diagram to label each quantity in the equation. For example, he labeled the number "54" in the equation "Richard" and labeled "3" as "times," then labeled the unknown quantity "a" "Chris." This labeling strategy helped him quickly identify the mathematical relationships involved in the story. Then, he wrote, "what if Chris is not known" on right side of the test sheet and used the "what if not" strategy to ask for the value of the unknown quantity "a" as he wrote his own questions. Following the third session, he began to use this strategy consistently, and he learned to refer to the provided problem-posing example to find language that he could use in his own problems. He continued this strategy for the rest of the intervention. Taylor's problem-posing performance increased from a median of 79% during the intervention to 100% during the maintenance phase. By contrast, his problem-solving performance remained at 83%. Taylor was very hardworking and well-behaved despite being labeled as having LD. He was very interested in problem posing and asked many questions during the intervention. His maintenance test performance indicated that he would likely use the problem-posing strategy he had been taught after the intervention.

Breanna was unique among the participants in that she took only two sessions to achieve a score of 100% on her problem-solving tests. It is possible that she benefitted from mapping her thoughts on paper using the COMPS problem-posing strategy she learned. When Breanna seemed confused while attempting problem-solving items, I often prompted her with questions like "if the reference unit/multiplier/product is not known, what does your equation look like?" After I did this, she instantly knew what to do. In terms of problem posing, Breanna only took one session to reach a score of 83%, and she maintained that score for the rest of the intervention sessions. In one instance when she was stuck on a problem-posing question, I prompted her with the question, "what is unknown in this equation?" Following this, she immediately circled the unknown "a" and wrote down the corresponding quantity above "a". After doing this, I pointed to the "a" and asked her, "what if the quantity is not known? What does your question look like then?" She then quickly created her own question.

As noted previously, Breanna did not talk too much at school. During my instruction, she usually opted not to speak, instead simply nodding her head for "yes" and shaking her head for "no." When a more complex answer was needed, I often wrote down several options and asked her to point to one to signal her response. For example, when teaching the mathematical relationship about the MC story problem, I wrote down the options "compared amount" “multiplier” and “referent unit”, she had to point to correct options based on my questions. Other times, to clarify certain important points, I asked her to write her thoughts on scratch paper.

Though she scored 100% correct on all problem-posing post-test sessions, her scores on maintenance tests fluctuated. She scored 56% correct for the first and third sessions, and she scored 100% correct for the second session. However, on the day of the third session, Breanna happened to be late for the class session, and she took the test very quickly even though I told her that she had more than enough time. This may be the reason for her low score in the final session.

Amy took only two sessions to reach a score of 80% correct for both tests, and she maintained this performance throughout the whole intervention. Through my communication with her teachers, I learned that Amy took an active role in classroom instruction and enjoyed conversation with teachers. She demonstrated this penchant for conversation during the study. When I introduced the MC diagram, she repeatedly asked me, “What is the referent unit (or product, or multiplier) in this problem?” while pointing at a specific problem. When I introduced the “what if not” strategy, she even asked me, “What if the referent unit is not known? How would you create your question?” In addition to asking for help, Amy seemed to develop effective organizational strategies that helped her solve and pose problems on her own. When solving problems, she quickly mapped her thoughts on the paper using the COMPS strategy and labeled the corresponding components in the story problem. When posing problems, she copied the given equation to the MC diagram and made a note reading “what if the number of marbles Chris has is not known?” Then, she used the problem-posing example to organize her language to create problems. For all sections of the maintenance tests, Amy’s performance remained above 83% correct.

The intervention’s effect on the transfer test was clear for Taylor and Amy. Taylor’s problem-solving performance increased from 25% correct in the baseline test to 75% correct in the post-test and 62% correct in the maintenance test. Amy’s problem solving performance increased from 0% correct in the baseline test to 25% in the post-test and 25% correct in the maintenance test. However, in the case of Breanna, even though her problem-solving performance increased from 25% correct in the baseline test to 75% correct in the post-test, her maintenance test score was the same as her baseline score. It is important to note, that the transfer test involves the phrase like “each week” For example, “the small copier makes 437 copies each day. The large copier makes 4 times as many copies each day. How many copies does the large copier make each week?" It is possible that students' transfer test performance decreased in the maintenance test is related to their reading difficulties rather than their mathematical thinking. Overall, the intervention seemed to help all students effectively transfer their skills from one-step MC word problems to two-step multiplication/division word problems.

5.1.1 The development of students' problem posing skills

The results of this study indicated that middle school students were able to pose a variety of solvable mathematical problems when presented with a story prompt, which was consistent with existing research (e.g. English, 1997b; Silver & Cai, 1996). In the baseline phase, all three students posed mainly insufficient, incorrect, and non-math problems. Taylor did not pose any sufficient problems at all. Most of the problems he posed were insufficient questions, non-problem questions, and irrelevant problems. By contrast, Breanna and Amy posed some sufficient questions in the baseline although most of the problems they posed were insufficient problems or incorrect problems. For these two students, the "no referent unit" equation was the most difficult item for them to pose problems for. They typically did not usually earn points for these items, as they did not seem to fully understand the mathematical relationships implied in the problems and thus could not identify constructive ways to create new problems.

More generally, there appeared to be multiple reasons for why students failed to pose sufficient problems. The first was unclear wording. For example, when the multiplier was unknown in the equation, students often wrote responses like "how many more times does Chris have than Christian?" The second reason was a lack of sufficient information. For example, students sometimes asked for the unknown quantity using only one sentence without giving any other information that the reader would need to solve the problem. The third reason was a failure to write a mathematical problem. Some students seemingly did not have a firm understanding of the form of a mathematical problem. This became clear, for instance, when students' posed problems contained no reference to a computational procedure (e.g., multiplication). The fourth reason was a failure to ask a question. This was evident when students created problems that did not ask the reader to provide any unknown quantity or information.

Overall, these results suggest that students may not have previously been exposed to problem-posing tasks in the classroom and that they thus might have had little or no experience posing problems. They might also have also had difficulty interpreting mathematical language and understanding mathematical reasoning. However, after the intervention phase began, students immediately began to identify the mathematical relationships provided in the questions and use the “what if not” strategy to pose problems involving the unknown quantity. Over time, they posed more and more sufficient problems. Therefore, the intervention appeared to help the students develop a conceptual understanding of the mathematical relationships in story problems and use the “what if not” strategy to create their own problems.

5.1.2 The development of students’ MC problem solving skills

In the baseline phase, most students struggled to find an effective strategy for MC word problem solving. Taylor used a “keyword” strategy, simply multiplying the numbers provided in the problem when he saw the word “times.” Breanna mainly solved problems using her calculator. She used multiplication as a first resort for most problems. If this strategy resulted in a large number, she used division instead. Finally, Amy chose an operation at random to solve most of the problems. For these reasons, all three students struggled especially with “no referent unit” problems. This is consistent with findings from existing research (Xin, 2007, Xin et al., 2012). Because the students lacked the conceptual understanding required to identify the appropriate operation, they could not initially find the correct answers for these problems. However, after entering the intervention phase, the students rapidly learned to understand the structure of the problems. For instance, they learned that in MC problems containing the phrase “who is compared to whom,” in which “whom” is the “referent unit” and “Who” is the product (Xin al., 2008). After they identified the relevant attributes of a MC problem, the “What-if-Not” strategy required them to make one attributes unknown, and then they need to solve for the unknown. The COMPS intervention helped them understand the problems’ structures which promoted productive problem solving.

5.2 The relationship between problem solving and problem posing

This study’s results indicated that students’ problem-solving and problem-posing performances were closely related, which was consistent with findings from existing studies (e.g. Cai, 2002; Silver, 1995; Stoyanova, 2005). Throughout all sessions, Taylor’s problem-solving and problem-posing performances were roughly 73% consistent. Similarly, Breanna's problem-solving and problem-posing performances showed around 87% consistency, while Amy's performances showed around 71% consistency. These statistics indicate that, as students gained proficiency in the problem-posing strategy, this also supported their problem solving. One likely explanation for this is that posing problems improved students' conceptual understanding of general problem structure, thus enhancing students' problem-solving performance. Another possible explanation is that the processes students used to perform problem posing informed their thinking while solving problem. For example, after students listed the three attributes of a problem, they selected one component on which to apply the "What-if-Not" strategy to create a problem. As this process became a habit, it may have helped cement a conceptual understanding of the problems they were presented with and thus eventually helped them solve problems. Conversely, the training in understanding the COMPS model, that is, three key elements that make up the MC mathematical model as well as the problem-solving process perhaps contributed significantly to students' problem posing. Students were initially more familiar with problem solving than problem posing. The problem-solving process may have provided them with the concepts and linguistic knowledge needed to start creating problems. In particular, the "word problem story grammar" prompting questions (Xin et al, 2008) helped students to understand the key elements in the MC problem structure. For example, in this study, students were given problem-solving tests before being given problem-posing tests. They may have learned the structure and language commonly used in word problems from the problem-solving tasks, which then helped them pose better problems. It is possible that the strategy of teaching problem structure through the COMPS Program (Xin, 2012) before problem posing may have contributed to the positive findings of this study. Therefore, placing problem solving instruction before the problem posing instruction may be particularly beneficial to students with LDs. Existing research studies have shown strong evidence of the COMPS intervention programs in facilitating students' skill acquisition and generalization (Xin, et al., 2011; Xin et al, 2017). This study has added to the existing evidence of the COMPS program and more importantly, extended the existing research in facilitating students' conceptual understanding of word problem solving through engaging students in problem posing.

5.3 Implications

This study is the first known study to show problem-posing to be an effective strategy for the math education of students with LDs. The study investigated how students with LDs improved their MC problem-solving and problem-posing skills by using a COMPS-based problem posing intervention strategy. Students with LDs often face great challenges in math courses, as they can suffer from anxiety in addition to deficits in their working memory, cognitive development, and language processing ability. In this study, problem posing, as an extension of problem solving, helped students with LDs to overcome these kinds of hurdles as they worked toward stronger conceptual understandings of math problems.

This study’s results appear to support current reform efforts in mathematics education that advocate for an increased focus on problem-posing activities in the mathematics classroom (NCTM, 2000). Special education teachers can easily implement these reforms in their math classes. For example, they can provide students with problems, then ask them to create questions for their peers. If the results of this study are an indication, when students are made to develop problem-posing skills, their problem-solving skills will increase as well.

Adding to the reform efforts of professional organizations like the NCTM, researchers have recently called for the integration of problem posing into math classrooms and teacher preparation programs (Crespo & Sinclair, 2008; Ellerton, 2013). Therefore, policy makers should consider making problem posing as required component in mathematics curriculum as well as address it in the context of special education policy. In addition, these researchers argue that it is important for pre-service teachers to have more experience posing problems and working with students who initially struggle to pose problems. Based on the results of this study, frequently having teachers compare well-written mathematical problems to poorly-written problems would help them develop their own internal quality rubrics for judging students’ posed problems.

In another instance of agreement with other studies, this study also found that problem solving and problem posing were closely related skills. This study provided evidence in support of a strong positive relationship between problem posing and problem solving. This suggests that special education teachers may use problem-posing class activities to improve students’ problem-solving skills. As Cai (2012) has suggested, problem posing can also be used as a measure of the effect of curricula on students’ learning. Thus, special education teachers should develop problem-posing rubrics to evaluate their students’ performance.

However, the fact that students in this study were given problem-solving tests first and problem-posing tests second may have played some role in the relationship between problem solving and problem posing. The mathematical relationships and linguistic structures emphasized in the COMPS program (Xin, 2012), which is part of the problem-solving tasks, may have impacted students' ensuing problem-posing performance. Special education teachers should carefully consider the particular needs of their students when deciding whether to teach problem solving or problem posing first. Students' individual needs and characteristics should always be the top concern when teachers design problem-posing and problem-solving tasks.

In the present study, students began to pose high-quality problems after the intervention phase began, but they also continued to pose some insufficient, confusing, and non-math problems. As mentioned above, the results of the study suggest several factors that may have contributed to the persistence of some poor problem posing performance. For example, students may have lacked a strong conceptual understanding of story problems because of the lack of instruction that explicitly illustrated what a story problem was. Future researchers may want to develop a framework to help students understand the differences between well-structured story problems and poorly-structured ones.

Previous research on mathematical problem posing suggests that individuals pose mathematical problems more successfully under semi-structured and structured posing situations than under free posing situations (Silber & Cai, 2017).While students in the present study successfully solved and posed MC problems by using the problem-posing strategy they were taught under structured situation on the basis of the COMPS program (Xin, 2012), it is unclear whether students with LDs would be able to successfully pose MC problems under free or semi-structured conditions. Special education researchers may consider developing problem-posing strategies to help students with LDs become better problem solvers and problem posers in semi-structured situations. Additionally, as mentioned above, because students' performance may be affected by the sequence of teaching problem solving or problem posing first, special education researchers should explore the order of presenting students with problem-solving and problem-posing tasks to determine which way is more beneficial.

5.4 Limitations

This study had several limitations. First, a sample size of three participants is relatively small. Future studies should include more participants (ideally at least six) or employ a group design to enhance the external validity of the study. This would make it possible to draw firmer conclusions about the intervention's effect and students' performance. Second, since the study used a single-subject design, each student received personalized one-on-one instruction and extensive attention. However, if students had not received this sort of personalized instruction, their performance may not have improved so quickly, and it may not have reached the same peaks. Third, the study did not allow students to distinguish their addition/subtraction and multiplication/division skill and did not include EG problem types, as the problems were all MC problems. In the baseline phase, students occasionally used addition and subtraction. After the intervention began, they gradually realized that the tests were all about multiplication and division. It is difficult to speculate how their performance may have changed if the study had also allowed them to demonstrate their addition and subtraction skill. Fourth, students were given problem-solving tests first and problem-posing tests second. The results may have been different if the participants had engaged in problem posing first and then problem solving, as the experience of taking problem-solving tests may have helped them analyze problems' structures when presented with posing-problem tasks. Future research should compare the sequence of administering problem-solving and problem-posing interventions to see which sequence helps students achieve better learning outcomes. Also, future study may explore alternative problem posing interventions without structured COMPS instruction.

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APPENDIX A. SAMPLE MC WORD PROBLEM SOLVING ASSESSMENT

Please solve the following word problems. Please write down EVERYTHING you did while solving the problems.

1. Frank has been on the basketball team for 28 days. His friend Mike has been on the basketball team 14 times as long as Frank. How long has Mike been on the basketball team?

ANSWER:____

2. Earl has 374 pennies in a jar. Earl has 17 times as many pennies as his sister Stacy. How many pennies does Stacy have?

ANSWER:____

3. Paul has 378 points. Carey has 18 points. Paul has how many times as many points as Carey?

ANSWER:____

APPENDIX B. PROBLEM POSING ASSESSMENT

Name: ____________________ **Date: ____________________**

Please pose your questions for the unknown "a" based on the equation and the scenario.

1. Richard and Chris would like to compare the number of marbles they have**.**

$54 \times 3 = a$

Posed problem 1:

$54 \times a = 162$

Posed problem 2:

$a \times 3 = 162$

Posed problem 3:

2. Leroy and his friend Allen would like to compare the number of their basketball cards.

$13 \times 15 = a$

Posed problem 1:

$13 \times a = 195$

Posed problem 2:

$a \times 15 = 195$

Posed problem 3:

APPENDIX C. SAMPLE TRANSFER ASSESSMENT

Please solve the following word problems. Please write down EVERYTHING you did while solving the problems.

1. The small copier makes 437 copies each day. The large copier makes 4 times as many copies each day. How many copies does the large copier make each week?

ANSWER:____

APPENDIX D. STUDENT PERCEPTION AND SATISFACTION SURVEY

Name: _____________ Date: __________________

Please circle the choice that best describes your opinions

I like doing math word problems.

1=strongly disagree, 2=disagree, 3=neutral, 4=agree, 5=strongly agree

I like posing problems to other people.

1=strongly disagree, 2=disagree, 3=neutral, 4=agree, 5=strongly agree

The problem posing strategy the teacher taught me helped me solve new word problems

1=strongly disagree, 2=disagree, 3=neutral, 4=agree, 5=strongly agree

The problem posing strategy the teacher taught me helped me pose new word problems

1=strongly disagree, 2=disagree, 3=neutral, 4=agree, 5=strongly agree

The teacher's explanation really helped me to clarify my mathematical thinking

1=strongly disagree, 2=disagree, 3=neutral, 4=agree, 5=strongly agree

I enjoy the entire section with the teacher

1=strongly disagree, 2=disagree, 3=neutral, 4=agree, 5=strongly agree

APPENDIX E. PROBLEM POSING EXAMPLE

Instruction: Please pose your questions for the unknown "a" based on equation and the scenario. (use multiplicative compare)

Example:

Edward and Brandon would like to compare the quantity of their oranges.

$192 \times 16 = a$

Posed problem 1: Edward has 192 oranges. Brandon has 16 times as many oranges as Edward. How many oranges does Brandon have?

$a \times 16 = 3072$

Posed problem 2: Brandon has 3072 oranges. Brandon has 16 times as many oranges as Edward. How many oranges does Edward have?

$192 \times a = 3072$

Posed problem 3: Brandon has 3072 oranges. Edward has 192 oranges. The number of oranges Brandon has is how many times as many as the number of oranges Edward has?

APPENDIX F. FIDELITY CHECKLIST

Components	Yes	No	Comments
The investigator provided direction. For example, “what if the “product” is not known? Can you pose a problem with the factor “product” unknown based on the given equation? (pointing to the “product”)			
The investigator provided general request in the form of “Do you want to try again to make it better?” when the participant provided an initial posed problem which was insufficient, irrelevant, confusing problems and non-question problems.			
The investigator requested for specification/clarification after the general direction when participant provided a new problem which need further information. “Could you tell me more about the new problem you generated?”			
The investigator provided modeling of a problem-posing task when the general request did not work for students.			