CAPACITATED NETWORK BASED PARKING MODELS UNDER MIXED TRAFFIC CONDITIONS

by

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ABBREVIATIONS

AV:	Autonomous Vehicles
BPR:	Bureau of Public Roads
CAV:	Autonomous and Connected Vehicles
EV:	Electric Vehicles
FOC:	First Order Condition
LP:	Linear Programming
MUTAPC:	Multi User Traffic Assignment Problem Capacitated
NP:	Class of computational hardness
NV:	Normal Vehicles
P:	Class of computational hardness (Polynomial)
SO:	System Optimum
SOC:	Second Order Condition
TAP:	Traffic Assignment Problem
TAPC:	Traffic Assignment Problem Capacitated
UE:	User Equilibrium

MATHEMATICAL NOTATION

$f = route - flows\ for\ all\ possible\ routes\ for\ all\ OD\ pairs$
$A = link - flows\ incidence\ matrix$
$g_i = convex\ BPR\ function$
$g_i = convex\ BPR\ function$
$B = flow - OD\ pair\ incidence\ matrix\ indicating\ if\ a\ given\ flow\ belongs\ to\ an\ OD\ pair$
$D = demand\ vector\ indicating\ demand\ for\ all\ OD\ pairs$
$C = Cost\ vector\ corresponding\ to\ the\ travel\ cost\ of\ each\ route\ flow\ at\ solution$
$U = Minimum\ cost\ vector\ corresponding\ to\ the\ min\ cost\ route\ for\ each\ OD$
$x_i = link\ flows\ associated\ with\ origin\ i\ going\ to\ all\ destination$
$M = node - link\ incidence\ matrix$
$\bar{X} = link\ flow\ vector$
$OD_i\ \ vector\ of\ OD\ demand\ at\ node\ i$
$f^v = flow\ variables\ associated\ with\ player\ v\ for\ all\ used\ od\ pairs\ (Decision\ variable)$
$x_i^v = flow\ of\ link\ i\ with\ respect\ to\ player\ v$
$X^v = link - flow\ vector\ of\ player\ i$
$A^v = flow - link\ incidence\ matrix\ for\ player\ v$
$B^v = flow - OD\ vector\ matrix\ for\ player\ v$
$D^v = Demand\ vector\ for\ player\ v$
$TF_i^v = Cost\ function\ of\ link\ i\ for\ player\ v$
$PT_v = Parking\ time\ of\ player\ v$
$x_p^v = link\ flows\ of\ parking\ links\ for\ player\ v$
$x_p^v = link\ flows\ of\ parking\ links\ for\ player\ v$
$V =\ set\ of\ players$
$M^v = node - link\ incidence\ matrix\ for\ player\ v$
$C_{AV} = Capacity\ of\ road\ when\ all\ vehicles\ are\ AV$
$C_{NV} = Capacity\ of\ road\ when\ all\ vehicles\ are\ NV$
$OD_i^-\ destination\ nodes\ with\ nonzero\ flow$
$OD_i^+\ origin\ nodes\ with\ nonzero\ flow$
$OD_i^{\emptyset}\ zerp\ nodes\ with\ nonzero\ flow$

ABSTRACT

New technologies such as electric vehicles, Autonomous vehicles and transportation platforms are changing the way humanity move in a dramatic way and cities around the world need to adjust to this rapid change brought by technology. One of the aspects more challenging for urban planners is the parking problem as the new increase or desire for these private technologies may increase traffic congestion and change the parking requirements across the city. For example, Electric vehicles will need parking places for both parking and charging and Autonomous vehicles could increase the congestion by making longer trips in order to search better parking alternatives. Thus, it becomes essential to have clear, precise and practical models for transportation engineers in order to better represent present and future scenarios including normal vehicles, autonomous vehicles and electric vehicles in the context of parking and traffic alike. Classical network model such as traffic assignment have been frequently used for this purpose although they do not take into account essential aspects of parking such as fixed capacities, variety of users and autonomous vehicles. In this work a new methodology for modelling parking for multi class traffic assignment is proposed including autonomous vehicles and hard capacity constraints. The proposed model is presented in the classical Cournot Game formulation based on path flows and in a new link-node formulation which states the traffic assignment problem in terms of link flows instead of path flows. This proposed model allows for the creation of a new algorithm which is more flexible to model requirements such as linear constrains among different players flows and take advantage of fast convergence of Linear programs in the literature and in practice. Also, this link node formulation is used to redefine the network capacity problem as a linear program making it more tractable and easier to calculate. Numerical examples are presented across this work to better exemplify its implications and characteristics. The present work will allow planners to have a clear methodology for modelling parking and traffic in the context of multiusers which can represent diverse characteristics as parking time or type of vehicles. This model will be modified to take into account AV and the necessary assumptions and discussion will be provided.

1. INTRODUCTION

Urban mobility patterns of humanity have drastically changed in the last centuries. During the last century the increase of inflow from rural areas to cities and urban centers have represented one of the main changes in human activity. Today around the world more than 60% of population lives in urban areas [1], which makes the cities to be more important than ever in history and the challenges of supplying all requirements of transportation for such amount of people is an enormous challenge. In fact, around the world there are more than 30 cities with a population higher than 10 million people, implying that the available space for private and public transportation system is smaller. Between the challenges faced by urban areas and prospect cities which are growing exponentially in size are the creation of efficient transit, balance of roads and spaces for private vehicles, electrification of transportation modes, shared mobility and parking of vehicles for freight, commuting and recreational trips.

These problems have been the backbone of transportation engineering during the twenty century [2] and have shifted and shaped the literature in terms of planning aspects. As a result methods such as the classical four stage planning method and more recent methodologies including big data analyses have been created in order to model and plan the movement of people and the need for infrastructure. Historically speaking different places in the world have privileged different transportation modes as a function of local constrains. For example, in the US thanks to the big spaces and available land area the private vehicle gained a leading importance in both rural and urban areas which shaped the planning thinking during the twentieth century. Contrary to this, in most of Europe the transit system gained a leading importance given their space constrains. In Asia, the focus of transportation planner was a combination of both modes as a function of income, with a primary focus in private vehicles as cities densities started to grow and the shift to transit system in the last decades. While in Latin America this shif is just happening with the construction of new transit and metro lines and the modification of bus routes that can compete with the convenience of the motorcycle [3].

One other important aspect of urban planning during the twentieth century was the parking problem, which consists of the planification and policies related with the offering and usage of space for parking [4]. In fact, most of the policies design of cities during the twentieth century in the United States were related or linked with the availability of parking in the central zones of cities in order to increase or decrease their demand of private trips. The research in this aspect was bast and brought policies such as minimal parking requirement for new construction places, introduction of curb parking in central zones and regulation of parking prices in order to shift the mode and destination choice of users [5], [6]. These type of methodologies were supported in economic and choice models which predicted the average usage of parking cells in different places, some of them were based in Nash equilibrium [7] between offer and demand. These types of models brought the ability of analyze the impact of price on total demand in descriptive manner. In this case the main modelling aspect was the usage of Wardrop equilibrium in order to understand how parking search affected the route choice and even the modal split in big cities. Nevertheless, these models were not able to scale well in the size of urban areas such as Chicago or New York which could easily have more than 20000 nodes making its use impractical for the time, which were partially solved with better optimization techniques and an increase in computation power. With the increase in computing power and development of fastest optimization techniques a new modelling approach started to emerge which was the dynamic assignment and agent-based simulation (ABS). These models go into details of specific choices made in a timely manner by users, which are not considered in macro scale models but that are much harder to scale in computation time. These new approaches started to take into account aspects such as visibility of parking availability, complex choice model, lack of information, between others, which made the results more realistic in terms of assumptions. Although these new methodologies for modelling parking became more better understood and accessible for the practitioners, parking planning around the world has not changed much, and simple rules of thumb for number of minimum required parking places still stand to be the norm in terms of policies in cities such as New York City or Chicago [8]. However, new policies such as maximum parking lots have been implemented in countries such as Japan or Singapore showing the parking policy around the world.

One of the main reasons why these simple parking policies are still in use in a lot of areas around the world is because of history and requirements of minimal change in policies that can make stable the decision taking of private enterprises and actors alike. Also, other challenging aspect of the usage of more sophisticated tools for managing parking becomes more important in the third world planning agencies which have the challenge of not making the same mistakes made by other cities in the past while planning for their rapid growth. And, with the introduction of emerging technologies such as electric and autonomous vehicles, the modelling aspect of parking becomes essential. For Electric Vehicles (EV) there is the planning of the locations and number of charging places which would work as parking places. For the Autonomous Vehicles (AV) the automatic search of parking places or the return to home could reduce the parking necessities while increasing the traffic demand of the system. These technologies which are under research in the academic and industry community must have clear modeling tools for planning agencies which can lead to improve decision taking and better transportation performance overall.

Therefore, it becomes essential for planning agencies to better understand difference, pros and cons of different modeling aspects going from macro simulation to microsimulations, how to interpret the results and easy to use tools and scalable algorithms that can allow these models to be used in practice. Specifically, one of the most used models in practice is the static assignment which is based on the ideas of User Equilibrium and in a generalized manner can be analyzed under the general framework of network games from a theoretical perspective. The static traffic assignment generally speaking has the advantage of providing good algorithm scaling, and good theorical analysis. Specifically, in parking the static traffic assignment model has been worked in the perspective of using one mode and type of user describing capacity of parking lots as their available cells. Nevertheless, in this model the differentiation among users is classically not taken into account, which has major importance as difference user's choice different parking spaces based on the type of vehicle or parking duration that they may have. In this sense it becomes essential to have static model which consider a multiuser modeling scenario taking into account parking duration, capacity constraints of parking and traffic links into a unique model. This type of work has received special attention in last years although not clear relationship of parking duration and parking capacity has been stablished. Also, one big missing factor in the literature is the analysis of network capacity, which in a sense is an extension of the capacity of individual links but in the case of networks where the overall capacity comprises

1.1 Description of the Parking Problem

Parking is one of the main ways to control decisions and modal choice in city planning, and has been traditionally used as a main way to force and shape the landscape of cities in terms of land usage, mode decision and traffic congestion. Nonetheless, initially in terms of modelling the aspects taken into account were simple compared to the development of traffic assignment. In fact, although for traffic assignment the software and mathematical models are bast, for parking most of the rules and analysis done before the twenty-one century were based on general models which stablish the relationship in the macro scale of offer and demand for parking. Parking by itself consist on the allocation of physicals space to stop while the user does some other activity. For different cities around the world the increase in parking space followed a differentiated behavior, for example for UK there was a big increase in parking places during the 1960-1980 time period, where cities such has Arlington, Berkeley and Cambridge experienced an increase on its number of parking places of more than 50 percent while in the following twenty years it growth by 11 percent [9] . This tendency was repeated in multiple cities around the world that consolidated their urban growth at the end of the twentieth centuries, although is still on the rise in Asian cities. This differentiated behavior has become a fertile opportunity for studying the implications of different parking policies, but the results are inconclusive. The parking analysis have covered topics such as parking choice, effect of parking on urban and economic growth and its impact on congestion [4]. In terms of parking choice some authors specify that the most important choice decision in parking choice are cost and walking distance to final destination, but results may vary greatly by city and type of trip [10]. Urban and economic impact of parking demand studies demonstrated that parking availability may not have a big effect on the decision of users to go to certain place in central areas of cities, although may have an impact in not centric ones. In terms of congestion the analysis of parking has been scarce since most of the analysis center on the relationship of offer and demand of parking, nevertheless since the 1980 parking has taken a central role in analysis of congestion.

Urban and mobility planning in general focus on policies and infrastructure planning of traffic and transit. Parking infrastructure, historically was generally provided by private institutions making planner to take it for granted and sometimes to not take it into account in city planning. Because of this during most part of the twenty century and even today, a lot of city plans reference minimum parking requirements based on constructed area and functionality. This resulted in an increase of traffic attraction to city centers, and fast increase in congestion. This due to the increase of urban density but a not proportional increase in transit offer. For a long time, the mentality of city planners consisted in the increase of road supply as a response to the increase of demand in order to reduce congestion. Nevertheless, the experience showed that it was impossible to keep up with the rapid increase in motorization which made planners to move away from this idea. Some European cities experienced with this thematic in the last decades incentivizing substantial increase in the usage of public transit and reduction of the usage of private modes. This change in the last decades brought benefits for the society in term of health and money savings. Nevertheless, as populations gets older and new technologies like self-driving and electric vehicles start to enter the market, planners are starting to face new struggles in terms of parking design. The parking problem is highly connected with the development of technology such as electric vehicles for charging or autonomous parking for the decrease in number of required parking cells.

1.2 Description of Network Capacity

Network capacity is a known term in communications networks and in other areas where it plays a central role in network and protocol design [11]. Nevertheless, in transportation science the capacity analysis has been mostly focused on individual link analysis, where the capacity of different systems such as transit lines, metros and traffic lanes are analyzed. Thus, explicit functions for the capacity of a multitude of transportation lanes or links exist, nevertheless when the whole network is considered the problem received less attention. The definition of network capacity can be stated as what is the maximum throughput of a given transportation network in a given period of time and under some behavior and flow patterns. In the case of static assignment the Capacity of networks is considered when hard capacity constrains are taken into account [12]. In this formulation the capacity is defined as the maximum OD matrix that can be putted on it until the problem remains feasible given the link flow constrains. When the sum of such a matrix is considered an unique value describes the maximum throughput of the system in terms of Maximum number of vehicles per study period which for static models tends to be between 1 to 3 hours [13]. When analyzed over parking networks classic literature defines the capacity of parking lots as the number of parking spaces in a given place. However, the problem with this definition is that it does not take into account the parking duration factor of users, which would be the equivalent of calculating the capacity of traffic links without taking into account the safe space which each user considers when driver under certain velocity. Therefore, it becomes clear that in order to make a simile of infrastructure capacity of traffic links and parking links the demand characteristics should be taken into account. This implies that the parking duration of users define the capacity of a parking link. Thus, if in studied during a certain period of time the capacity of a parking link should be equal to the total amount of vehicles that can be attended in a given period of time. In average the capacity of a parking link is equal to the number of cells multiplied by the study period and divided by the average parking duration. The importance of this definition is that when considered as a whole each link in a network can represent a traffic segmented or a parking location. The capacity of each link must be clearly defined and be consistent among them. With this definition the network capacity is measure of the maximum throughput that as a system the network could serve under a given OD matrix and in a period of time. The usage of this concept can be enormous as for example it could be used to measure the quality of service of a network, identify inefficient links and even improve the design of the network with capacity increase. Also with the introduction of new technologies such as AV the network capacity concept can become as a metric which helps to understand the effect of automatization as this technology enters the market. In this work a precise definition and review of the literature of network capacity will be included and algorithms for calculating it will be proposed based on the classic traffic assignment with hard capacity constraints and the multi user traffic assignment model proposed.

1.3 Motivation and Objectives

Parking has been and keep being an ongoing problem of city planning which is being related with a lot of new technologies such as electric vehicles, autonomous vehicles and other tendencies of technology of the twenty-one century. As explained before the problem has had a lot of different perspectives and treatments from traffic modelers and planning agencies alike which have not come in unified methods or manuals to understand differences, advantages and cons of the different modelling methodologies. These methodologies range from the more classical economic modelling to static traffic assignment to the newer ones being the dynamic traffic assignment and data analysis which have become popular as computation power and accessibility has increased drastically. Therefore, this work focuses on proposing a clear methodology form modelling parking based on network models starting with the proposed model for static parking modelling taking into account different users such as Autonomous Vehicles. Thus, comparison of these models can be used by traffic modelers in order to take better decision when planning parking plans for a city. Allowing them to understand the pros, cons and requirements of each model. Also, in this work we propose a faster and more tractable logic to find the capacity of the network in the static case and give a definition and procedure on how to calculate it. The network capacity has an important meaning from the planning aspect since it allows to check the efficiency of the system to attend a given demand pattern and could be used in network design problems.

In the case of the objectives this work has 4 main objectives which one for each modelling aspect which correspond to static assignment of parking, static assignment of parking including autonomous vehicles, Network capacity definition of static model with normal vehicles and Network capacity definition including Autonomous Vehicles. These are described in detail below.

- Propose a clear methodology and mathematical formulation for the Multi User Traffic Assignment with Hard capacity constraints of parking and traffic combined, including efficient algorithm for calculating solutions to the problem and parameters required to modify inputs such as Travel cost functions.
- Propose a clear methodology and mathematical formulation for the Multi User Traffic Assignment with Hard capacity constraints of parking and traffic combined with autonomous vehicles including efficient algorithms for calculating solutions to the problem and parameters required to modify inputs such as Travel cost functions.
- Definition of network capacity in the scenario of Normal vehicles in a multi class network including traffic and parking links and propose efficient algorithm for solving it.
- Definition of network capacity in the scenario including Autonomous and Normal vehicles in a multi class network including traffic and parking links, and propose efficient algorithm for solving it.

Overall, this work is organized 7 chapters with the first chapter being the introduction giving an overall overview of the problem, motivation and objective. The second chapter is an overall literature review of the parking research area including the different type of approaches given in the literature with respect to economic analysis, policy analysis, network models, dynamic models and data driven works which are related to parking including at the end the contribution of this work on the existing literature and its importance. The third chapter includes a review of network models for parking analysis including differences between User Equilibrium and System optimum will be important in autonomous driving. The chapter includes mathematical formulation, solution algorithms, network capacity definition and small numerical cases for the different models. The fourth chapter presents the proposed formulation to the multi user hard capacitated network model including parking, solution algorithm, numerical examples and the presentation of the study case of Bello city in Colombia. The fifth chapter presents the proposed formulation for the autonomous vehicle modelling in the perspective of network models, mathematical formulation, differences that autonomous vehicles bring over travel delay functions and users dynamic, small numerical examples and the application on the same study case in Bello city. The sixth chapter gives a comparison of the results of the described models, advantages and disadvantages, recommendation for modelling in real scenarios and future work.

2. LITERATURE REVIEW

Parking analysis has been a broad interest topic for transportation engineers, general research in the topic could be divided in five main categories: economic and policy analysis of parking, Mode choice selection of parking, static assignment models of traffic and parking assignment, dynamic and simulation models for parking and traffic simulation and optimal parking network design. The main focus of this work is the static assignment of traffic and parking, so a more profound review will be given of parking policy, Traffic and parking assignment. However, a brief overall review of the literature review will be described.

2.1 General description of Parking Research

The research on the topic dates back to 1922, where N.P Lewis and HML Lewis [14] pointed out to the parking necessities of the expansion of cities in the USA. They recommended and recorded some of the first rules of minimum parking provision per constructed area. This started an analysis of the best policies and methodologies for parking assessment. Later, on the decade of the forties, with the implementation of out of street regulations and parking meters, research started to focus on the analysis of these measurements. Problems such as congestion where studied by authors such as Brinkman, et al [15] Pointing out to some statistics and basic analysis of these measurements. Modern authors started to use econometric, traffic assignment and simulations to analyze the parking problems. Young W, et al [16] made a review of the types of models used in parking. They explained how multiple authors in the 80's used econometric and choice models to describe the choice of parking options and the important characteristics associated with them. In fact, Hunt et al [17], created a logit for choice model finding that distance, price and type or parking as the most relevant variables in parking choice. On the other hand, traffic assignment has been used to analyze the situation. R Thompson and A.J Richardson [7] developed a traffic assignment model with integrated parking facilities choosing as part of the equilibrium formulation. Recently simulation has been one of the main choices for parking modelling, where multiple researchers have created frameworks taking into account parking choice, distance, time variation of parking demand among other factors into account. Horni et al. [18] created an Agent Based Simulation (ABS) for parking cruising including traffic assignment, they applied the model to the study case of Zurich. One of the main drawbacks of simulation, is the high complexity of the algorithms which make it slow to run in bigger instances, but generally a big part of its internal calculations can be made in parallel.

Although there is a rich literature for the modelling of parking, one of the main lacks is the definition of network capacity. This concept is generally assumed to be the sum of the capacity of all the cells in the parking network, nevertheless the problem with this assumption is that it lacks the ability to capture the reality of rotation and different parking behavior around the city. On the other hand on traffic assignment H Yang, MGH Bell, Q Meng [19] , proposed a Network capacity definition as the maximum amount of demand that can be assigned in a given set of OD-pairs, such that throughput of the network is maximized. The importance of this definitions is that it takes into account capacity constraints on links and produces a maximum throughput estimate of the network under a given demand characterization. Nevertheless this definition has a high mathematical complexity as the problem is defined as a bilevel optimization program, which in general is hard to solve [20]. Also, a discussion of the properties of such a problem have not been discussed and its implication in practical use are still low.

2.2 Economic and Policy Analysis of Parking

Economic and policy analysis of parking problems plays a central role in overall urban planning and policy making. Specially since the beginning of this research area, economic analysis was one of the main drivers for policy making and planners alike [21]. Initially the authors started to model the problem of parking in terms of possible financial gains or cost for the public institutions, so the parking supply side was made a requirement for the private land developers as a function of the project total area. Because of this, a great concern was created around city planner and research in order to find ways to fund the creation of parking spaces where supply was not enough [22]. This marked the start of the usage of policies such as curb parking, public parking facilities and discount for private companies investing in parking supply [23]. In modelling terms the economic analysis of parking focused on a multi scenario analysis of offer and demand and correspondent estimation of benefits and cost [24]. For this, simple rules of demand and relationship between behavior of users was taken into account which were required given the low computational power of the moment so simplicity was chosen for any modelling mechanism applied in real cases. However, with the increase of the complications of parking over cities and a rapidly increasing computation power offered by multiple companies more realistic and complex models were taken into account by the research academy. In these models the prices of parking are taking into account and final destinations estimated on relationships between costs and behavior of demand until certain defined equilibrium is reached [25]. Other type of models are based in optimal design of parking fares for curb parking, were optimization scenarios are created in order to optimally select the fares of parking facilities [26]. Generally speaking, the resulting models are NLP (Non Linear Programs) which are solved by successive linearization algorithms by solving an LP (Linear Program) per iteration, making them feasible for big instances of problems. In terms of policy analysis, the literature focuses on the effect of different policies on parking and traffic combined. One of the best reviews in the topic was given in [27] were the authors reviewed different parking policies and implications around the USA and Europe, concluding that parking diminution is correlated with private vehicle usage. Lastly these type of analysis have become an essential part of any real implementation of policies and in general new literature in the topic uses economic analysis to study benefits and cons of policies such as Autonomous vehicles or maximum parking laws [28].

2.3 Static Traffic Assignment with Parking.

Static models have been the preferred choose for practitioner and academia alike for traffic simulation in the last century. When parking is considered the literature on the topic becomes large although is relatively new with respect to other areas of traffic modeling with a more active research since the decade of the eighties. In this regard, one of the most important papers on the topic was given by [29], where parking was combined with traffic classical traffic assignment by the usage of adequate cost performance functions. This concept was further extended by [30] with the usage of multilayer network representation including the parking, traffic and pedestrian network which allowed for the usage of classic traffic assignment algorithm with convex BPR functions. However, hard capacity constraints of links where still out of radar in the literature, although formulation and algorithm for the Traffic Assignment Problem Constrained (TAPC) was already developed [31]. This problem in specific was studied in multiple cases, and one of the best performing algorithms was given by [32] with the introduction of the Augmented Lagrangian algorithm

Dynamic model for parking including Agent Based simulation Dual Simplicial Decomposition, whose idea is the usage of the augmented Lagrangian method to put the capacity constrains in the objective function and the solution of the resultant Assignment problem by using the DSD Algorithm. Combination of capacity constrains for parking and traffic constrains was worked out by [33] by an extended formulation of the problem including hard capacity constraints, stochastic assignment, parking links and transit routes. Nevertheless, for the solution of the problem the author just enumerated some paths and solve the equivalent formulation using GAMS, which makes the solution not equivalent to the real UE as not guarantee exist that the preselected paths where the optimal ones. The advancement in modelling techniques and the usage of Variational Inequalities [34] for problem formulation attracted new research in the parking modelling topic. Multi User traffic assignment for example was analyzed in the context of parking by [35] with the introduction of multiple modes in a multinetwork representation and multiple users formulation including parking prices. However, this formulation does not bring advantages in terms of computation time since the all simple path enumeration is NP hard problem which implies that the number of paths would be incredibly large and therefore the problem size would become intractable. Recent works in this topic [36] have included the similar problem formulation although the usage of the fixed point algorithm is considered, detailed algorithm characteristics are not specified which makes harder to verify or implement.

2.4 Big data and Parking Analysis

The last decade has seen an increase in the data available for analyzing different aspect of human behavior and mobility patterns. In the literature the usage of data related with vehicle locations and in some cases parking occupations has allowed authors to understand the dynamic of parking. The work in this area has boomed since 2015. The work in this area focuses on the usage of the data to give better recommendations for parking availability to users, optimize parking lots location and diagnosis of demand requirements. [37] Presented a first glance of the usage of Internet of Things and big data analysis to provide parking services information for users, which has a big implication in real systems as cruising for parking is estimated to take around 30% of total trip travel time [38]. Nevertheless his description was just theoretical and demonstrative. [39] Described a system were such data collection and processing were putted into a real case study with data in the city of Aarhus in Denmark. In the study the authors give a clear description of the usage of cluster and parallel collection techniques based on the Hadoop technology. The results presented are of interest for the planning authorities as real time and historic analysis of parking occupation is given which could be used for parking planning or dynamic fare estimation.[40] further works into these type of analysis although with the usage of Deep learning is able to give an estimate of available cells in parking locations around the Federal University of Parana, which allows users to better plan their trips and reduce the cruising time. Other type of application of big data is in the planning of parking locations for normal and future Autonomous vehicles. In this aspect [41] used trip data from taxis in the city of Beijing in order to optimize the selection of parking and charging location for Electric Vehicles. For this study the authors used big data of taxi trips in order to stablish the potential demand of different parking location for charging. On the same line [42] proposed a more robust framework contrasting existing and potential locations and using data from cellphone users which allowed them to have more data and better analysis. In general, all these studies focused mainly on the collection and processing of the data while giving recommendations and results analysis over study cases. Nevertheless, it is expected that in the future and with the implementation of systems based on IoT more analysis will be perform comparing predicted outcomes with real measurements. This is of great importance as one of the biggest problems in parking and transportation modelling in general is the lack of experimental data to verify the mathematical models used to predict the behavior of users. Therefore, the usage of data analysis and machine learning in parking planning analysis has a big potential of improving the usage of existing parking layouts and give better recommendations to the creation of new ones. Also, as AV and EV market share increase this data will be vastly available and efficient algorithm for its exploitation and usage will become necessary.

2.5 Dynamic Modelling of traffic and parking

Dynamic models in traffic engineering refer to network model that take time into account the time dimension and specific behavior characteristics of individual users in order to obtain aggerated and individual results of travel time, cost, etc. These model started being developed in the seventies and one of the best references was given by [43] presenting an non convex optimization framework to calculate system optimum dynamic traffic assignment (DTA). Although this model did not take into account the UE, while also using simplistic assumptions as fixed demand, single-destination, single-commodity. Furthermore [44] reformulates the problem as a well-behaved convex nonlinear, gaining some theoretical and algorithm gains performance wise. Nonetheless these formulations had the problem of the violation of First in First Out (FIFO) component which is related with dynamically how users are assigned into the network. These types of model belong to the class of analytical formulation of DTA, where a mathematical formulation of the problem is available which can have insights on algorithm, behavior or parties such a uniqueness and existence. Nevertheless, these model even today lack the ability to be sued for generalized networks and do not include a lot of realistic properties such as advanced behavior models control strategies, thus the most popular way to apply DTA consist of simulation-based modelling combining with equilibrium conditions. This type of models have been worked out by multiple authors, with [45] being one of the more referenced. In that worked the selected model which consisted of demand and supply generator which interact based on a UE search principle, were the assignment is finalized when a UE dynamic equilibrium criterion is met. The model is mesoscopic which implies that some way of cost delay function is still taken into account, avoiding the extra complexity brought by Agent Based simulation. The main difficulty with DTA formulation is the concept of equilibrium which implies that user's selection of routes must follow an overall pattern across time and space. On the other hand, Agent Based simulation (ABS) which as discussed can be used as an input for DTA is based on a set of logical rules which determine the agents and rules among them which can dynamically vary. Thus, ABS is an easy way to implement a lot of realistic scenarios although the computational cost can be expensive. In this regard one of the most prominent works was given by [46] using a general ABS model called MATSIM, including specific following behavior showing good computational power and comparing to real data from a real case scenario of Zurich. When it comes to parking ABS has been the primary focus of research as it can allow for complex behavior in parking search and decision making. In this regard [47] developed a Abs model taking into account lack of information of available parking and occupation realization based on line of sight. When taking into account autonomous vehicles [48] developed a simulation framework for shared AV which would serve around 2% of population and would avoid parking in a shared mobility scheme. The study case for that study was the city is run on a hypothetical study case, although some precaution was taken in order to make it as realistic as possible. Results show a fast reduction of parking requirement with fleet increase, as the system is more efficient in the usage of available vehicles. Overall, the simulation methodology has the advantage of model advanced characteristics such as reaction time or connection among vehicles, shared mobility among other. Overall, the literature presents a rich environment of modeling scenarios and characteristic, although results are only theoretical as real implementation cases are still not available. More analysis needs to be done in order to compare static and dynamic results and to understand how good can they predict reality.

3. NETWORK MODELS FOR TRAFFIC ASSIGNMENT

This chapter presents a description of classical network models for traffic assignment in the perspective of static assignment which is based on the concept of equilibrium, analysis of classical methods and proposed algorithms for solving the problems are presented. Classical Traffic assignment and Hard capacity constrained traffic assignment are studied as they are the backbone of the proposed multiuser framework for both normal users and autonomous vehicles which will be detailed in chapter 4 and 5 respectively. An overview of the literature and the mathematical formulation is given and some numerical examples studied and compared for both cases.

3.1 Transportation Assignment Problem (TAP)

Static models refer to the classical form of traffic assignment where the route choice of users in the network follows a given behavior stablished as User Equilibrium (UE) which was introduced for the first time in the 1952 by John Glen Wardrop. In this equilibrium the premise is that for any user Origin-Destination (OD) pair all used routes will have an equal and minimal travel cost while for the unused ones the travel time can be any value and flow will be zero. Later this principle of equilibrium was stablished to be a Nash Equilibrium where a simultaneous game is played between different agents and each agent just wants to minimize its own travel time [49]. These types of models are based on finding a certain structure on the optimality conditions which show that the given equilibrium is followed and on its simple version it can be reduced to a single convex optimization format. While on its extended version Variational Inequalities are required in order to stablish the optimality conditions and formulate the problem in the right manner. The classical formulation of the traffic assignment assumes that there is just one type of user to be assigned which has complete knowledge of the network. This implies that all users know the exact cost performance functions of all links in the networks and all paths that can be followed between a pair and destination. The usage of these methodologies is still on high demand and although its assumptions are generally not true in reality, metanalysis have concluded that it has reasonable performance in real implementations. On its basic form this model assumes perfect knowledge from users of the overall state of the network, convex cost performance functions and a strict guidance of user behavior under the UE principle. In this chapter some of these assumptions will be relaxed and discusses as capacity constraints of links should be considered. Also, algorithm and difference between System Optimum and User equilibrium will be discussed as those will play a central role in parking and Autonomous vehicles modelling.

3.1.1 Mathematical Formulation

Classical mathematical modelling of this problem included algorithm and behavior description of users which became important in the context of solution algorithm. Nevertheless, the problem was later found to have an equivalent optimization framework formulation, which could be proven to reduce to the UE conditions. For this an integral transformation of the cost functions is required and which work in the case where the cost performance function is self-dependent on their own link flow. In terms of behavior all users follow the classical UE, in this context this formulation is reduced to the following optimization problem.

$$\min_{f} \sum_{i=0}^{n} \int_{0}^{x_i} g_i(s)ds \ (1)$$
$$Af = x \ (2)$$
$$Bf = D \ (3)$$
$$f \geq 0$$

Equation 1. TAP mathematical formulation

$$f = route - flows\ for\ all\ possible\ routes\ for\ all\ OD\ pairs$$
$$A = link - flows\ incidence\ matrix$$
$$g_i = convex\ BPR\ function$$
$$B = flow - OD\ pair\ incidence\ matrix\ indicating\ if\ a\ given\ flow\ belongs\ to\ an\ OD\ pair$$
$$D = demand\ vector\ indicating\ demand\ for\ all\ OD\ pairs$$

This problem has multiple properties, as can be checked the constrains are linear, therefore as long as feasible they will describe a polytope, also since the BPR or travel cost function are convex so will be the overall objective function which is the sum of the integrals of them. It is not evident that an answer to this problem would results in UE, and in order to check it, the optimality conditions of the answer most be studied. For this the KKT conditions of the problem must be obtained which in this case correspond to:

$$f(C-U)=0\ (1)$$
$$C-U\geq 0\ (2)$$
$$Bf=D\ (3)$$
$$f\geq 0\ (4)$$

Equation 2. First order optimality conditions for TAP

$$C = Cost\ vector\ corresponding\ to\ the\ travel\ cost\ of\ each\ route\ flow\ at\ solution$$
$$U = Minimum\ cost\ vector\ corresponding\ to\ the\ min\ cost\ route\ for\ each\ OD$$

From the first condition it can be seen that there are two possible outcomes for routes cost and flows, either the flow is zero and the associated cost can be greater than the minimum one, or the flow can be greater than zero but it's cost must be equal to the minimum one of all routes between that OD pair. This condition corresponds to the definition of UE and the solution to this optimization problem corresponds a solution of traffic equilibrium. The second important thing to consider in this problem is checking whether the problem is or not convex, since it can foresee properties such as uniqueness of the solution. To check this the second derivative of the Lagrangian projected into the feasible set must be checked, nevertheless since in this case the constrains form a polytope which is compact and convex, it is only needed to check the Hessian of the objective function. The aforementioned mathematical program has the path-flows as decision variables which imply that every link flow is a function of a set of path flows. Although for every OD a given path flow will just appear once in every link flow. Since the link flow cost function are dependent only on their own link flow, the hessian matrix with respect to link flows would be

$$\frac{\partial F(X)}{\partial X} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & 0 \\ \vdots & \ddots & 0 \\ 0 & 0 & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

Equation 3. Hessian Matrix of TAP objective function

Which is a diagonal matrix with positive entries since, all cost function are assumed to be convex and strictly positive, therefore this matrix is positive definite and the optimization program would be convex [50]. Nevertheless this result just holds with respect to link flows, but in terms of path flows the problem is not convex so multiple solution can be found that follow a unique link flow pattern, which has been further analyzed in the literature [51].

3.1.2 Solution Algorithms for the TAP

This formulation has been widely used by practitioners and researchers alike and stablished an important correlation between optimization and traffic assignment which does not hold in general. In terms of the algorithms the literature has proposed multiple procedures to solve the problem, and these algorithms divide in two categories. The first category is link flow based which gives solution in terms of link flows while not conserving information of path flows. The second category is path flow based which gives solution for both the link path and flow paths variables although the path flows can be not unique. The classical approach for solving the classical traffic assignment for UE consists of the Modified Frank Wolf method which takes a linear approximation of the objective functions and proceeds to solve the resulting linear program (LP), then the new estimate for each iteration is found by performing a line search between the sequential solutions to the LP. This new solution is assured to be inside the feasible region since the region is a convex set thus an affine combination inside the feasible set will belong to it. This algorithm improved greatly with respect to past iteration algorithm such as capacity constrain iterations. One further improvement of this algorithm was proposed by [52] by introducing the concept of conjugate direction, which makes the Frank wolf of each iterations to be conjugate between with respect to the hessian of the objective function $d_i H d_i^T = 0$. Also since most of the execution time is spent in the All or Nothing Assignment and the shortest path identification, further progress by [53] was made in this regard by formulating the subproblems in terms of link incidence matrix and using a tree structure representation to speed up its performance in factors up to 2x. This algorithm was shown to have a better practical and theoretical convergence in the original paper and has been implemented in commercial algorithm such as TransCAD, a detailed description of this algorithm is given in Figure 1.

Algorithm 1 Conjugate Frank Wolf

Input:
$G = (V, N,),$, V=Vertexes,N=Nodes
X_0, D =Demand vector
Output: X^*
Step 0: k=0 $x_0 = 0$
Step 1: $y_k^{FW} = AON(G, D, T)$ set $d_k^{FW} = y_k^{FW} - X_k$
Step 2:
if $k = 0$ **then**
    $d_k^{CFW} = d_k^{FW}$
else
    Calculate α_k
    Calculate s_k^{CFW}
    Set $d_k^{CFW} = s_k^{CFW} - x_k$
end if
Step 3:find τ_k performing line search in the direction $[x_k, x_k + d_k^{CFW}]$
Step 4: set $x_{k+1} = x_k + \tau_k * d_k^{CFW}$ and $k = k + 1$
Step 5: Check convergence if not met go back to step 1

Figure 1 Conjugate Franw Wolf Algorithm

$$\propto_k = \begin{cases} \dfrac{N_k}{D_k} \, if \ D_k \neq 0 \ and \ \dfrac{N_k}{D_k} \in [0,1-\delta] \\ 1-\delta \ if \ D_k \neq 0 \ and \ \dfrac{N_k}{D_k} > 1-\delta \\ 0 \ otherwise \end{cases} ; N_k = \bar{d}_{k-1}^T H_k d_k^{FW} \, ; D_k = \bar{d}_{k-1}^T H_k (d_k^{FW} - \bar{d}_{k-1})$$

$$\bar{d}_{k-1} = s_k^{CFW} - x_k \, ; s_k^{CFW} = \propto_k s_{k-1}^{CFW} + (1 - \propto_k) y_k^{FW}$$

To have a further understanding and proofs of the convergence of the algorithm the reader may refer to [52]. The experiments show that the method has an overall faster convergence than classical FW method and the author go further showing a Bi conjugate method, which makes the new direction at each iteration to be conjugate with respect to the last two search direction further improving its convergence but for high levels of precision. On the other hand, the same problem can be solved by enumerating a set of feasible paths before formulation of the problem though some heuristic and then simply using some known and fast algorithm or commercial software to solve the equivalent problem in term of path flows using the optimization formulation. Nevertheless, this method suffers from the path computation problem. This is the case since computing such paths is as hard as solving the optimization problem, and resembles the problem of obtaining a feasible solution for LPS, where the problem of finding a good starting point is as hard as solving the original LP [54], [55]. Thus, must of the literature in the topic and current applications use variant of the Frank Wolf method when link-flow variables are required and the best algorithms known generally use a combination of conjugate direction method with fast solution algorithm for the all or Nothing Assignment.

3.1.3 Difference between UE and SO in TAP

It is important to notice that this formulation can also be used to analyze the system optimal behavior of the system, for this the objective function of the problem changes to be the overall travel time of the whole system, the equivalent formulation is shown in Equation 4. Equivalently the problem can be stated as UE, but with a modification of the cost function of each link, by adding the marginal cost of each function. This implies that the SO system is equivalent to solving for UE with BPR function $\bar{f}(x) = f_i(x) + x_i \frac{df_i(x_i)}{dx_i}$, which has an great importance since it implies that the same algorithm used for UE can be used for SO by just changing the BPR functions.

$$\min_{f} \sum_{i=0}^{n} x_i \bar{f}_i(x_i)$$
$$Af = x \ (2)$$
$$Bf = D \ (3)$$
$$f \geq 0$$

Equation 4 System Optimal (SO formulation)

One important question to address once the system optimal is stated is how different are the solution of one problem with respect to the other both in terms of link flows and in terms of total travel time for the system. The literature in the topic is broad, although the existence of theorical results imply the imposition of constrains on the functional form of the cost functions. One of the most generalized results in the topic was given by [56] where the author claimed that the worst case scenarios (although not generally) occur in simple parallel networks and the difference between UE and SO can be unbounded as a function of the functional form of the cost function. When this cost function takes the form of a polynomial of degree p then the author shows that for parallel networks the relationship between UE and SO time is of the form $O(p/\log(p))$, this relationship is generally denominated as price of Anarchy in traffic assignment and the topic is also of high interest in economic fields. Nevertheless, this result holds only for classical traffic assignment without hard capacity constrains which will be introduced later in this chapter and that force the maximum travel time of a network to be bounded, thus giving a maximum amount of travel time for the system and making the value of the price of Anarchy to be bounded as well.

3.1.4 Discussion of TAP and its Problems

Generally speaking, the classical Traffic Assignment Problem (TAP) has brought big advantages for traffic modelers and has been widely use in real cases all around the world. Nonetheless since its beginning this formulation has suffered from multiple criticisms of its simplistic assumptions and inability to model dynamic behaviors.

In terms of parking modelling this classical formulation has received some major or small changes in order to take parking into account which include the addition of cost function of parking links which stablish the approximate cruising time if a vehicle where to park in this segment. However, there are a source of inconsistencies in the assumption or modelling aspects of this classical traffic assignment. On one hand all users are assumed to behave the same, have perfect information and perception of travel times, which has been modified in other formulations such as stochastic traffic assignment [57]. Other assumption is that the link traffic flow can take an infinity value, which is not true as clearly specified by the capacity of a given link which has been extensively researched [19], [58]. For these reasons there have been simple modifications for the classic formulation which include the addition of hard constrains for the maximum link volume. This change drastically modifies the problem, as a change in the constraint set of the optimization problem makes the problem much harder to solve. Also, it makes the problem to have an additional complexity for which the problem may not be feasible for any OD matrix since it may the network capacity of some od pairs. For this reason, this work will introduce the addition of the hard capacity constraints in classical traffic assignment.

3.2 Traffic Assignment with Hard capacity Constrains

The traffic Assignment problem with Hard capacity constrains correspond to the same problem formulation of the TAP, but with the addition of extra constrains on the link flows which allow for a better representation of real conditions as real traffic links flows are bounded by maximum amounts. As discussed before, TAP although highly used in real modelling scenarios suffers from a lot of assumptions which are important to address in order to have realistic results. For this reason during the 1980 the research community started to work with the modelling of the problem when adding hard constrains that represent the upper bound of link-flow [31], [59]. This formulation plays a central role in the modelling of parking as this phenomenon is highly constrained by physical capacity described by parking time and parking cells. In this chapter a detailed description of this formulation will be given including formulation, solution algorithm, difference between UE and SO and network capacity concept.

3.2.1 Mathematical Formulation

This approach is simple in terms on formulation and its mathematical equivalence is identical to TAP with the addition of a linear inequality term representing relationship of path flows and link flows with their Upper Bound value or Capacity as seen in Equation 5

$$\begin{gathered}\min_{f} \sum_{i=0}^{n} \int_{0}^{x_i} f_i(s)ds \ (1) \\ Af = x \ (2) \\ Bf = D \ (3) \\ Af \leq HC \ (4) \\ f \geq 0\end{gathered}$$

Equation 5 TAP-C formulation

The TAP-C (Traffic Assignment problem Constrained) formulation is equivalent to the TAP but with the addition of the linear constrains of capacity. Therefore the feasible region is still a convex polytope which as long as feasible is convex, closed and compact [60]. In terms of the formulation the changes are minor, but the solution algorithm, interpretation of the model and possible results has a high impact. To verify that the actual problem formulation is a UE, the FOC (First Order Conditions) must be analyzed which are shown in Equation 6

$$f(C - U + \alpha) = 0 \ (1)$$
$$\alpha(HC - Af) = 0$$
$$(C - U + \alpha) \geq 0$$
$$Af = x \ (2)$$
$$Bf = D \ (3)$$
$$Af \leq HC$$
$$f \geq 0 \ (4)$$

Equation 6 FOC TAP-C

Where ∝ is the associated Lagrange multiplier of the capacity constrains, therefore if we define a generalized cost of each route to be the sum of the travel time plus the LaGrange multiplier of all links associated to each path condition 1 would imply that at equilibrium the generalized cost of the used routes is either minimal and have a positive flow or greater than that but with a flow of zero. Nevertheless, the problem with this definition is that the there is no longer guaranteed that the actual travel time of all used routes is equal which greatly differs with the TAP formulation. This implies that in a congested network the pure UE in cost that can be measured directly in the network does not hold. This is essential since it will change not just the solution difference with respect to the TAP problem but also change the performance of system travel time with respect to the TAP-C when considering the SO solution. The formulation in that case would be the same as with the TAP problem but changing the objective function, also the same replacement could be done to solve the SO TAP-C as UE problem, by using the replaced cost function plus link flow by its derivative.

$$\min_f \sum_{i=0}^{n} x f_i(x) \ (1)$$
$$Af = x \ (2)$$
$$Bf = D \ (3)$$
$$Af \leq HC \ (4)$$
$$f \geq 0$$

Equation 7 Link flow formulation of TAPC subproblems

This formulation is also convex as the multiplication of a linear term with a strictly convex function does not alter the convexity of the resultant product. Thus, the problem also has a unique solution and as with the case of TAP can be solved by directly solving with the formulation or by transforming the problem into an equivalent UEC formulation. Thereby the transformed cost function is still equal to the original function with the marginal cost of the link added.

3.2.2 Solution Algorithms

In terms of solution algorithm, the addition of these constrains completely change the problem, since for example if using a first order Taylor approximation of the objective function the resultant Linear programs do not equate to the AON assignment, as the feasible set is changed. This implies that these subproblems cannot be solved using the independent shortest path for all used od pairs and assigning the traffic accordingly, but the subproblems would equate to the multi commodity flow problem with capacity constrains [61]. Because of this the solution algorithm for this problem proposed in the literature often rely on the replacement of the capacity constrains by some penalization function in the objective function. This would be equivalent to estimating at each iteration the equivalent Lagrange multiplier of each constrain and adding it as a generalized cost. One of the simplest algorithms for solving this problem takes the form of penalty functions, which change the objective function in such a way that the overall problem is solved as a sequence of simplified problems, where the desired constrains are incorporated in the objective function and their value increased as the constrain violation increases. This method was studied thoughtfully by multiple authors [59], [62] where the results pointed out to a good performance given the simplicity of the constrains. On the other hand, augmented Lagrangian methods which use a similar technique but add an equivalent term to a Lagrange multiplier which is a function of the constrain penalty value. This method was also studied in the traffic applications by some authors [32], [59], the authors find that these method are more reliable and still tractable in terms of extra computation time, since the additional calculations increase linearly with the number of links. Based on this, the method of solution adopted in this work is a combination of the conjugate Frank wolf method with the augmented Lagrangian penalty function. The algorithm is described below.

Algorithm 2 Conjugate Capacitated Frank Wolf CCFW

Input:

$G = (V,N), \mu_0, C_0, X_0, tol1, tol2$ V=Vertexes,N=Nodes

Output: X^*

Step 0: k=0 modify the original functions

$f_k = F + P_a$

$P_a = \frac{(\mu_L - C_k * X)^2 - \mu_k}{2 * C_k}$ if $X - HC >= 0$

Step 1: Solve a unrestricted Tap with f_k using CFW to get new link flow (just 1 iteration)

if $X - HC <= tol2$ and $||X_k - X_{k-1}|| <= tol1$ **then**

  Finish the solution is optimal and feasible

else

  Calculate New $C_{k+1} = k * C_k$ if $X - HC >= 0$

  Calculate $\mu_{k+1} = \mu + cX$

  Set $d_k^{CFW} = s_k^{CFW} - x_k$

  go back to Step 0

end if

Figure 2 Algorithm for TAP-C (CCFW)

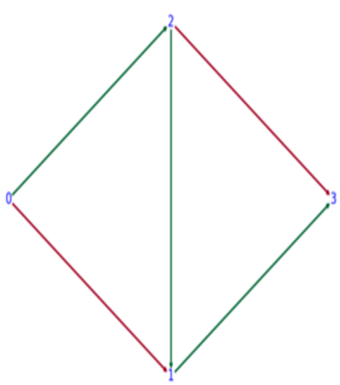
This algorithm has the property of updating the link flow functions at each iteration and making the last search direction conjugate with respect to the varying Hessian matrix. The convergence of this algorithm is assured when the function P_a tends to infinity as the constrain violation becomes larger and an extensive proof of this is presented in [32]. Nevertheless, the problem with this formulation is that it does not address the initial feasible solution which stablish whether or not the OD matrix can be assigned in the given network given the imposed hard capacity constraints. Thus, it becomes necessary to make transform the problem into a two-phase optimization problem with the first level checking feasibility and the second one improving upon the feasible solution. The reason why this is important is given by the fact that the algorithm will not end if not feasible solution can be found. In order to solve this, the first phase requires that the constrain set to be further analyzed, which as discussed is a polytope and therefore can be threaded with linear programing. The main problem with this approach is that it would require an enumeration of all paths which makes it infeasible for big size networks as the simple paths problems is a known NP-Hard problem [63]. Therefore, a link incidence formulation is proposed, for this all real links are assigned a new variable for each Origin with non-zero destination in the network. The link flow becomes the sum of all the proportions of link flow of each Origin. This formulation is presented in Equation 8

$$\min_{x_i} \sum_{i=0}^{n} \bar{X} f_i$$
$$M x_i = OD_i \ \forall \ i$$
$$\bar{X} = \sum_{i=0}^{r} x_i$$
$$\bar{X} \leq HC$$
$$x_i \geq 0$$

Equation 8 Link flow formulation of Multicommodity flow problem

$$x_i = link\ flows\ associated\ with\ origin\ i\ going\ to\ all\ destination$$
$$M = node - link\ incidence\ matrix$$
$$\bar{X} = link\ flow\ vector$$
$$OD_i\ \ vector\ of\ OD\ demand\ at\ node\ i$$

This problem is similar to the ones exposed in [53] where for the acceleration of the subproblem a link incidence formulation was used in combination with fast tree structure method for the construction of the path tree. In order to check the equivalence of the problem the following example is presented and the flow path and link flow formulations given.



$$f(x) = 5, OD_0 = [40,10,0,30]; HC = [20,20,30,10,20];$$

$$M = \begin{matrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 \end{matrix}; x_0 = \begin{matrix} x_{0-1} \\ x_{0-2} \\ x_{2-1} \\ x_{2-3} \\ x_{1-3} \end{matrix}; A = \begin{matrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{matrix} f = \begin{matrix} Route\ [0,1,3] \\ Route\ [0,2,1,3] \\ Route\ [0,2,3] \\ Route\ [0,1] \\ Route\ [0,2,1] \end{matrix}; F = \begin{matrix} f_{0-3}^{0} \\ f_{0-3}^{1} \\ f_{0-3}^{2} \\ f_{0-1}^{0} \\ f_{0-1}^{1} \end{matrix}$$

Figure 3 Multi commodity example

$$\min_{x_0}[5,5,5,5,5]x_o$$

$$\begin{matrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 & 0 \end{matrix} * \begin{matrix} x_{0-1} \\ x_{0-2} \\ x_{2-1} \\ x_{2-3} \\ x_{1-3} \end{matrix} = \begin{matrix} 40 \\ -10 \\ 0 \\ -30 \end{matrix}$$

$$0 \leq x_0 \leq HC$$

$$solution\ x_0 = [20\ 20\ 10\ 10\ 20]$$

Equation 9 Node-link formulation Linearization example

$$\min_{f}[5,5,5,5,5] * A * f$$

$$\begin{matrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix} * F = \begin{matrix} 30 \\ 10 \end{matrix}$$

$$A * F \leq HC$$

$$F \geq 0$$

$$solution\ F = [20\ 0\ 10\ 0\ 10]; X = [20\ 20\ 10\ 10\ 20]$$

Equation 10 path-flow formulation Linearization example

Clearly both problems have similar solutions although the solution properties and advantages are different, in the case of the flow formulation more information is incorporated but the disadvantage becomes the intractability of the computation of all simple paths. On the other hand, the link formulation creates new variables equivalent to non-zero origins, then the number of variables in the worst-case scenario (All OD pairs used) increases in a factor of $O(VN)$ where V is the number of vertexes and N the number of nodes in the networks. However realistically traffic assignment problems usually have very sparse OD matrix so this factor will be around 5% of the Nodes in the networks create flow since most of the models assign OD demand matrix that are related with zones covering multiple nodes. The subproblems which result from the linearization of the Frank Wolf problem can be formulated despite size of the networks and solved using efficient interior point methods. These subproblems have the capacity problem incorporated so if the implemented methods are efficient there might be an efficiency gain with respect to the CCFW algorithm. Therefore, the search direction would be generated by solving these subproblems and then the new direction could be made to be H orthogonal to the n past directions.

With this formulation the finalized TAPC algorithm becomes a two-stage algorithm, in the first stage a node link linearization of the problem is taken assuming the travel time of the links as $t_a(0)$. If the above LP is feasible this solution is used to solve the CCFW algorithm, if not then it means that the capacity of the network is not able to accommodate such an OD matrix and some OD flows must be reduced.

3.2.3 Differences between UEC and SOC

There are multiple differences analysis to be done over the TAPC one of the most important ones being the difference of UEC (User Equilibrium Capacitated) and UEC (User equilibrium Capacitated) with respect to the classical TAP. This analysis becomes highly important since as expressed before for generalized function and network forms in the TAP problems there is not bound on the price of anarchy. On the other hand, on the TAPC problem there is a clear upper bound for the system travel time. This plays a huge role, since it implies that as demand increases the travel time of the system will tend to an upper bound regardless of the distribution pattern of users as long as all links have a hard capacity constrain which is the case of real modelling scenarios. In order to compare the differences a small example network was used, on which one OD pair is used and the demand increases until it reaches its maximum. For this a random planar graph was used, which is created by using a simple generation procedure which is described in Figure 4. This algorithm consisting on random sampling from an initial Strongly connected graph and then proceeding to recover its Strong Connectivity property, this process is similar to the creation of free scale networks proposed in the literature [64]. Although in this case the random planar are clearly biased towards rectangular grids, but since it is the case of most transportation networks the algorithm can be used for this case.

Algorithm 3 Random planar graph generation

Input:
$H(n,m)$=rectangular grid of points,
$P(H(n,m))$=Position vector of every point in the grid
Output: GR
Step 0: Generate a graph GC with all links which connect consecutive nodes
(Including Horizontal,vertical,diagonal to right,diagonal to left)
Step 1: Select a random sample of links from GC creating the graph GR
Step 2: find all Strongly connected components(SCC) of GR
Step 3: Add all links belonging to shortest path beetwen all (SCC) to GC
return GR

Figure 4. Algorithm for random planar graph generation

The generated graph was created using a square grid of 10 by 10 points, the resulting connected graph has 144 links and 58 nodes, the capacity of all links is set to 100 with the exception of 4 links related with the paths that connect node 40 and 43. In this case most of the links are considered bi directional a plot of the network is shown in Figure 5. For this experiment the OD used will be between node 40 and 89 and a simple analysis of the network allows us to determine that the maximum flow achievable between these two od pairs is 300, so the experiment will calculate the resultant flows between these two nodes solving for UE,SO, UEC and SOC and changing the demand from 0 to 300.

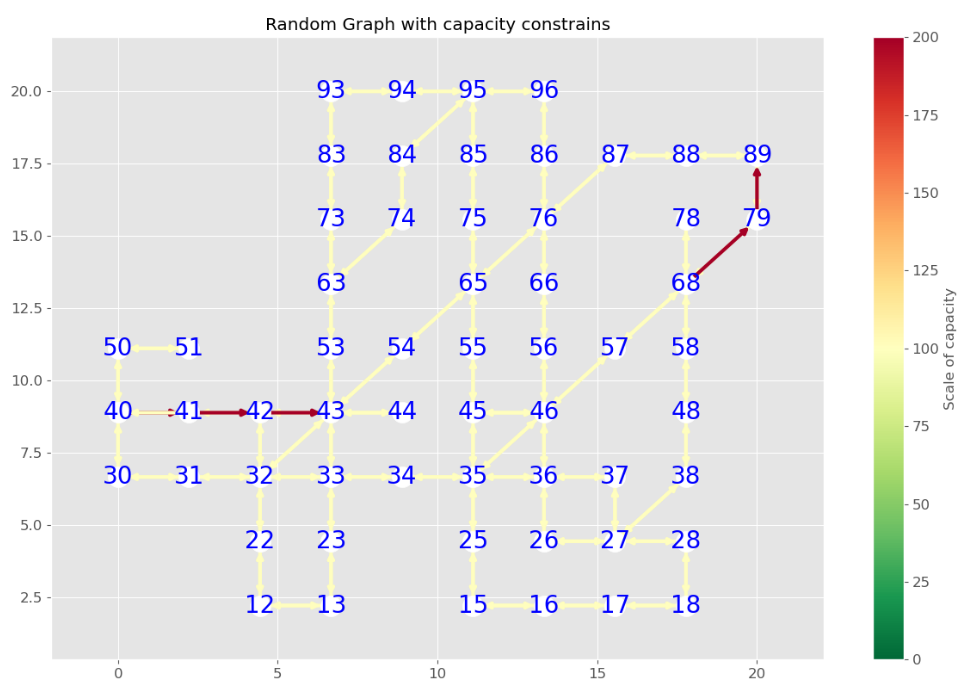


Figure 5. Example network capacity plot

The results indicate that the four types of assignments although different in link flows are similar in overall system travel time until some of the links capacity start to be reached. At this point the UEC starts to be closer to the SOC, which implies that the hard constrains force users to act in a more efficient way, as their real times cannot longer be exactly equal. Therefore, the difference between the SO and UE in the constrain setting is much lesser than in the unconstrained setting, implying that congested networks have a lower chance of improvement on travel time savings by applying route choice control of users. On the other hand, the route choice behavior is also similar as the congestion in the networks starts to appear, since routes possibilities get reduced by the congested links.

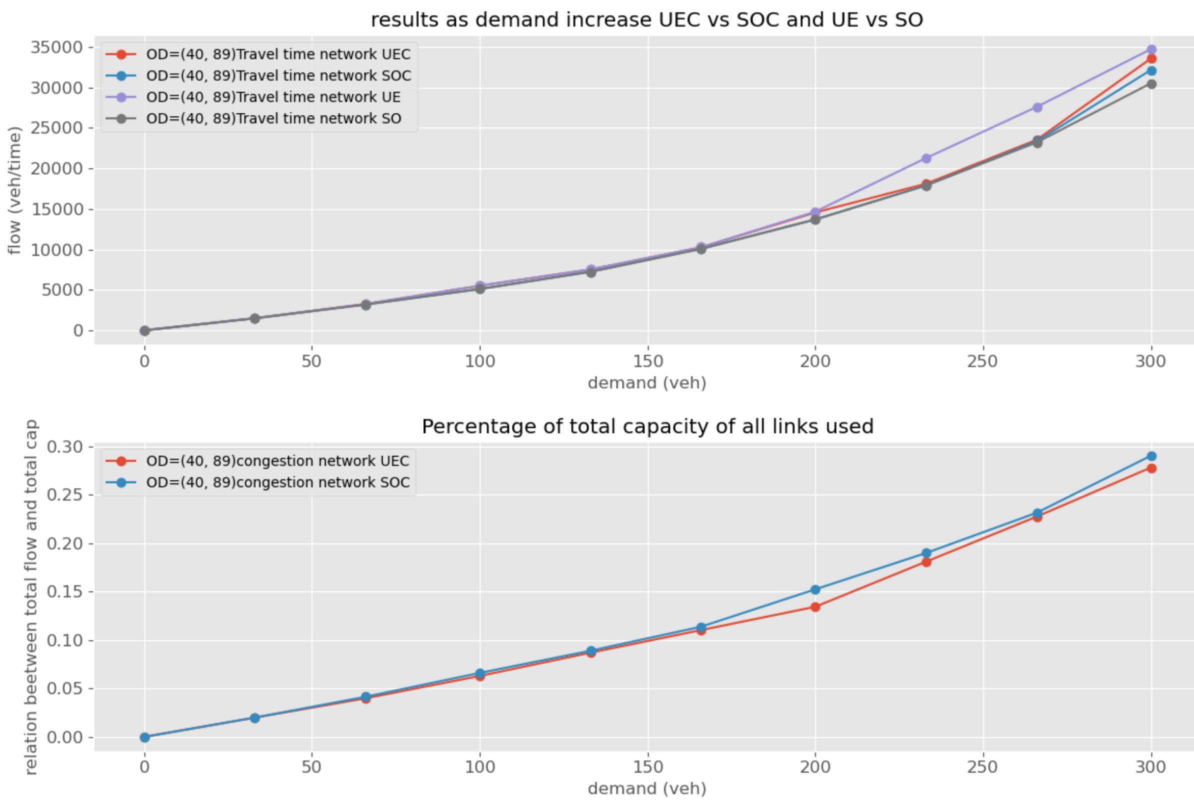


Figure 6 Comparison of UE and SO for TAPC example

Figure 7 and Figure 8 show the difference in the links flows as demand approaches capacity and as seen in the previous results the link flows are similar until total demand goes over 200, as links start to get to congested levels and traffic is forced to used other routes. This route patterns at congestion of both UEC and SOC start to get closer to the routes of the SO, which means that users start to use distribute themselves in more diverse routes although their real travel time is no longer equal to similar users in other routes.

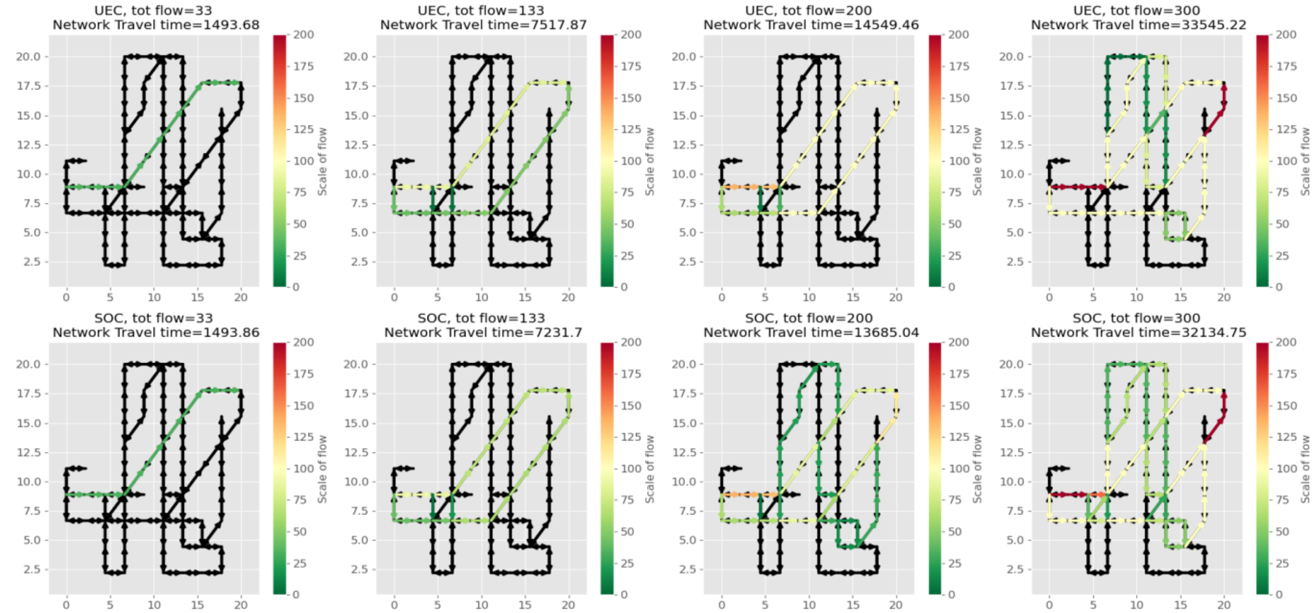


Figure 7 TAPC Results

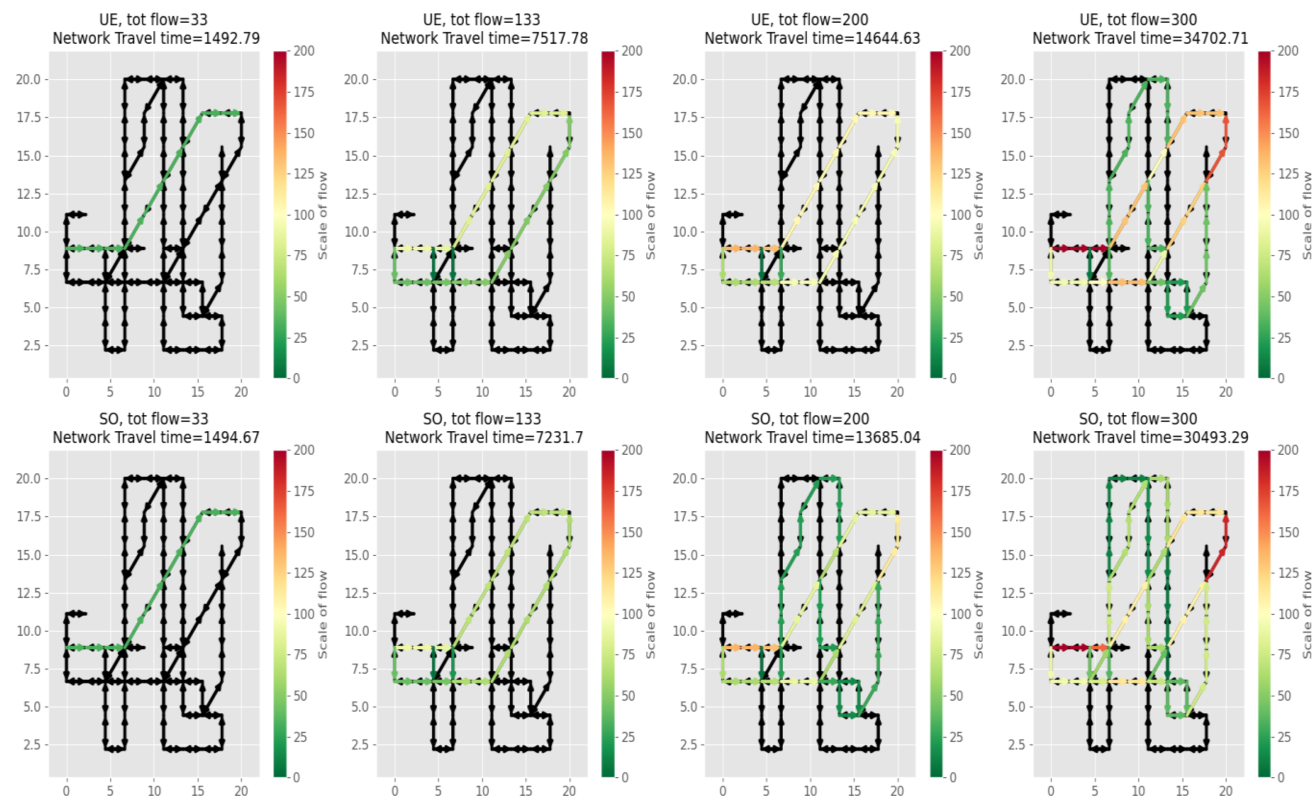


Figure 8 TAP Results

Another critical issue analyzed in classical traffic assignment which is related to the difference of UE and SO is the Braess Paradox [65] which consists on a simple question of network design : Does the addition of a new link increase the performance of the system travel time?. The question is crucial in the context of planning since is a long-standing practice to create more roads in order to reduce congestion. On its simplest version using TAP, past results shows that given the non-cooperative behavior of users under UE, the addition of new links may make the overall system travel time to become larger under some conditions. In the case of TAPC the problem has been less studied although a simple analysis was given by [66] where the author stablished an example and studied the existence of the paradox in the classic Braess network. In order to better illustrate this effect an example was created and the TAP and TAPC where considered, the results are shown in Figure 9.

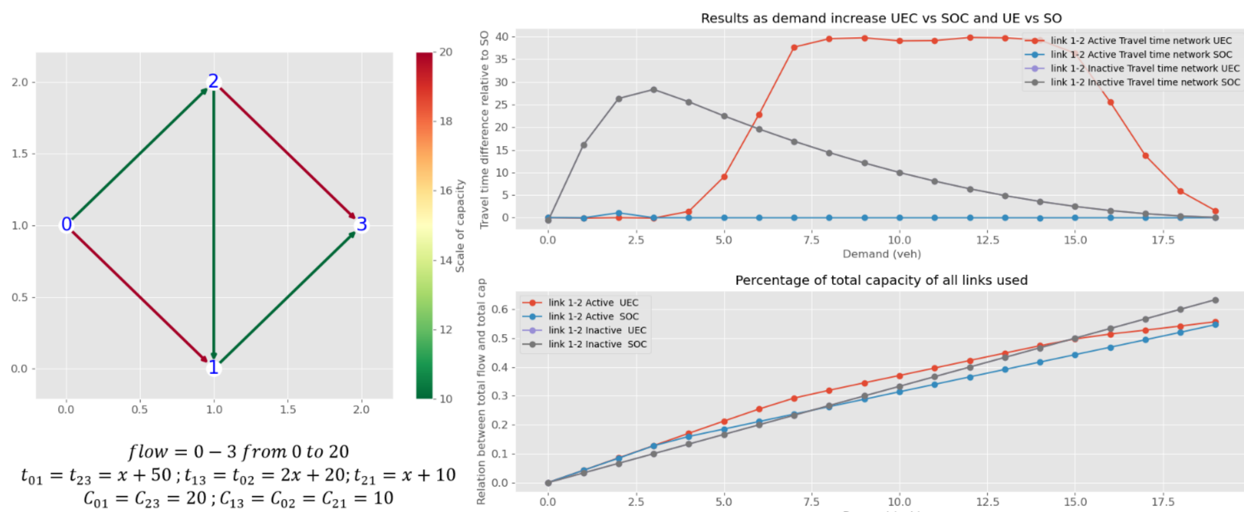


Figure 9. Example of Braess Paradox in TAPC

Notice that the added link does not increase the capacity of the network before which in this case corresponds to 10. As mention before the UEC travel time with respect to the SOC has a maximum difference achieve between some value of $0 \leq x \leq c$ where c is the capacity of the network for that OD flow. In this specific network the UEC and SOC travel time are the same when link 1-2 is not active, thus the Braess paradox occurs for OD flows ≥ 6, although this starts to become smaller as the demand become close to the capacity of the network. All the results of travel time are plotted relative to the SO travel time which is the guaranteed to be smaller than the

SOC, since no hard capacity constrains are present. Nevertheless, in this range of Braess demand the percentual difference of the UEC with respect to SO was smaller than 6% for any demand. This is critically important since it shows that although Braess paradox may occur, the difference in system travel time gain may not be significant, especially if the network is congested. Also, the addition of the link can have practical importance since it can increase the capacity of the given OD flow, so even if Braess may occur, the addition of the link can have other positive effect to have into account. Also in real networks where the travel time function are function of capacity generally under the BPR [67] which takes the form of $t_a = FF(1 + \alpha \left(\frac{x}{c}\right)^4)$. This function implies that the travel time increase from free flow to congested (x=C) has a maximum factor α which ranges from 0.3 to 0.86. Thus, when these BPR are used as link performance function the occurrence of Braess paradox may be even smaller.

3.2.4 Network Capacity based on TAPC

The addition of link capacity to the modelling perspective brings a lot of challenges in terms of the solution algorithm and results interpretation. One of the most challenging aspects is the fact that solutions are no longer guaranteed to exist for any OD matrix but feasibility must be checked. The reason for this is that the new capacity constrains make the problem to be bounded with respect to the link flows, thus the OD demand matrix must satisfy some requirements. These requirements are encapsulated under the umbrella term of network capacity.

The study of network capacity has been a low important subject in transportation networks, although a highly useful aspect of telecommunications and electric networks [61]. Maximum flow problem is a type of such problems where the maximum achievable flow between a source and target node is analyzed and polynomial time algorithms exists [68] for its solution. In the case of traffic flow problems, the network capacity is not as clearly design as the flow in the network should be guided under the User equilibrium principle. Nonetheless, when capacity is achieved and as shown before in this chapter TAPC does not follow classical UE. Initial attempts to quantify this network capacity under TAPC were researched by [69], in which network capacity formulation is presented as a bilevel optimization problem with maximum achievable demand as objective function.

$$\max_{Q} |Q|$$

$$\min_{f} \sum_{i=0}^{n} \int_{0}^{x_i} f_i(s)ds \ (1)$$

$$st. \quad \begin{gathered} Af = x \ (2) \\ Bf = Q \ (3) \\ Af \leq HC \ (4) \\ f \geq 0 \\ Q \geq 0 \end{gathered}$$

Equation 11 Bilevel Network Capacity formulation

The formulation is shown in Equation 11 which is max-min bi level program where the upper level is the maximization of the used OD flows Q and the lower level is the solution of the corresponding TAPC problem. In this case the objective function of the subproblem is not modified but the feasible region of it is. It is important to notice that this problem is highly similar to a primal-dual formulation, although in this case the primal is a nonlinear program. Nevertheless, it is important to notice that an any moment the change on the lower program occurs on the intersection of the facets described by the link flows and capacity. At any TAPC the network capacity is defined as the maximum OD matrix that can be imposed on the system while conserving feasibility. Also as seen before in TAPC any feasible problem will have a unique solution in terms of link flows. Therefore, an equivalent formulation using maximum multicommodity flow formulation is proposed using a node-link incidence formulation which preserves constrains of OD pairs utilized.

$$\max_{x_i} \sum_{i=0}^{n} \bar{X}$$

$$Mx_i \leq 0 \ \forall \ OD_i^-$$

$$Mx_i \geq 0 \ \forall \ OD_i^+$$

$$Mx_i = 0 \ \ \forall \ OD_i^{\emptyset}$$

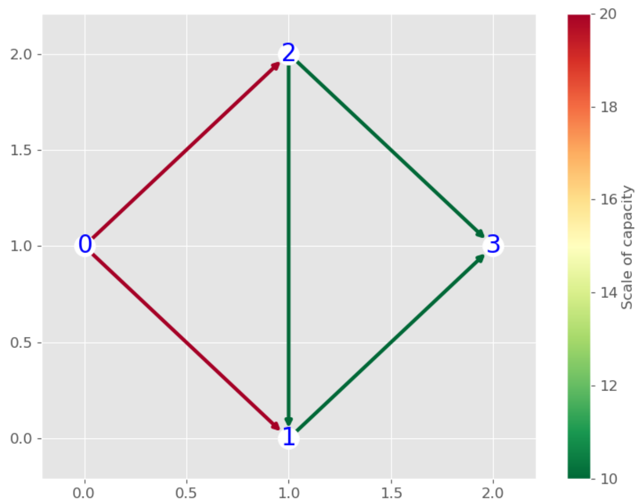
$$\bar{X} = \sum_{i=0}^{r} x_i$$

$$\bar{X} \leq HC$$

$$x_i \geq 0$$

Equation 12 Link flow formulation for Network Capacity

The solution of such problem would be the flows of each link when capacity is achieved, although the relevant information is the sum of such flows which would represent the maximum achievable flow under a given OD pairs utilization. This is because when a solution is reached the link-flows solved will not be unique. In order to better illustrate both the simulation and the fact that the solution may not be unique let us consider the same Braess network in Figure 3 also assume that the only used od pair is 0-3. The math formulation is shown in Figure 10 where there are two possible solution for this problem in terms of maximum links flows, although in both cases the OD flow is equivalent to 20, also the links which get congested are the same for both solutions and they correspond to bottleneck links.



$$\max_{X_o} x_{0-1} + x_{0-2} + x_{2-1} + x_{2-3} + x_{1-3}$$

$$\begin{matrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 & 0 \end{matrix} * \begin{matrix} x_{0-1} \\ x_{0-2} \\ x_{2-1} \\ x_{2-3} \\ x_{1-3} \end{matrix} \begin{matrix} \geq \\ = \\ = \\ \leq \end{matrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix}$$

$$0 \leq X_0 \leq HC$$

$$solution\ X_0 = [10\ 10\ 0\ 10\ 10], [0\ 20\ 10\ 10\ 10]$$

Figure 10 Example of formulation of network capacity problem

This equivalent formulation makes much easier the calculation of the capacity, and the importance of this formulation is that it can help to understand the feasibility of any TAPC problem. But more importantly its practical use could have great implication for planning analysis, since the network capacity defined as the sum of all origin flows of all used OD pairs after this analysis can easily describe the network performance. One simple upper bound for the network capacity is just the sum of all capacities, nevertheless the network capacity is clearly a function of which OD pairs are used and therefore which links start to become bottlenecks. Thus an easy measurement of network capacity performance is $r_p = \frac{NC}{\sum HC}$ where NC is the Network capacity as described in this chapter and the sum of HC is the sum of all hard capacities in the network. As the r_p(ratio-performance) approaches 1 the network is being better utilized by the OD demand. This is especially important in cases such as in Medellin city, where the network capacity of the metro system is performed under simplistic assumptions of worst performance link capacity of network according to [70]. This has made the municipality invest various million dollars in buying more trains to reduce frequency, while the bottleneck of the system is present due to an inefficient use of long OD pairs between its North and South extremes making intermediate Shortest OD pairs have long waiting times. In terms of parking analysis this becomes an essential term since, for a parking network with traffic capacity, an analysis can be done over the network to check if the bottlenecks in the networks correspond to parking links allowing the planners to understand which ones to intervene.

3.2.5 Application of TAPC to Parking Modelling

The usage of side capacity constrains to the parking problem has been broad and is one of the most common practices when static modelling is used. The usage of these constrains is given since the amount of parking cells is well known so the available parking capacity is easily estimated when compared to the road capacity. For the usage of this method a super network representation is generally necessary as 3 networks types must be integrated including traffic and pedestrian links with parking links as connector among them. The usage of such multi-Layer Network is a common standard in both research and Academia alike. This representation implies that multiple networks which belong to different traffic classes may coexist in the same modelling scenario. For example, in the case of Figure 5 if some parking links were joined between a pedestrian and traffic network an equivalent network could be formulated as presented in Figure 11. Thus, the equivalent formulation of the problem including parking would modify the original destinations of the OD matrix by using their equivalent nodes in the pedestrian Network. In this way all traffic would need to park since parking links are the only link between these networks. Also, the parking delay function could take the form of estimated average attendance time from queuing theory and an additional price addition to consider the price charge of each parking slot. Meanwhile the pedestrian network delay functions could be considered constant assuming classical walking speeds and calculating the respective travel time as function of link length which would be constant for any number of users. This would imply that all shortest paths could be precomputed before execution for every used OD pair and then from each parking place a direct link for each possible destination created reducing the increase of complexity by the addition of the new supporting links in the pedestrian network.

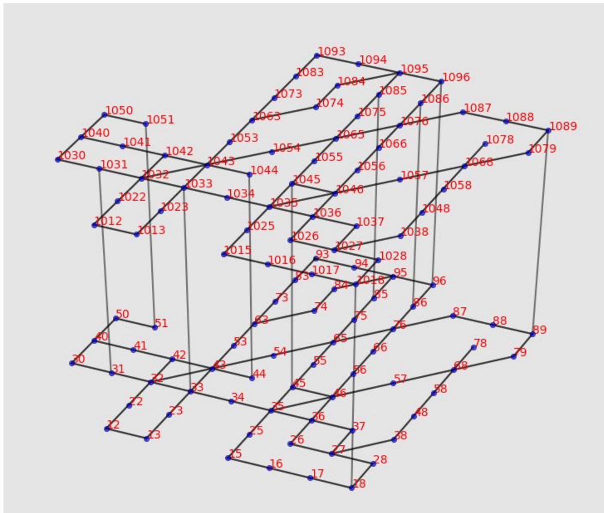


Figure 11 Multi layer Parking network.

There are some drawbacks in this formulation, one of them is the real heterogeneity of users, which perceive the performance function in different manner and have certain characteristics such as different parking times which can be not uniformly distributed across the network. This is important since in this modelling alternative the modeling period which is usually an hour to 3 hours (Peak periods) has the problem that users with different parking times could not be modelled. Then the results will not reflect the election of users based on their own parking time, which has been shown to be an important aspect in parking choice as users are sensitive to it. Also, for any parking cell during a given study period the number of vehicles that can park is equal to the sum of all parking times, this implies that for example a parking cell could park 2 vehicles each one for 20 minutes and then 1 vehicle for 80 minutes or 6 vehicles with 20 minutes each during a 2 hour study period. This equivalence is also related with traffic networks capacity where users' characteristics such as safe distance can determine the spacing between vehicles which in combination o velocity defines the road capacity. Thus, when combined traffic and parking links in a safe environment a right interpretation of the problem characteristics is required. Classical formulation and analysis of parking using hard capacity constrains directly consider the physical space as the hard constrains of these links. Which as discussed is false and a better estimation of them could be the one presented before where the parking capacity of a link is defined as $C_p = \#cells * \frac{SP}{PD}$ with SP being the study period in minutes, PD being the average parking duration of users in that parking slot and the number of cells being the count of number of parking cells in that location. Thus, the TAPC formulation results in an important formulation tool for parking analysis, further improve needs to be done in terms of modelling multiple users in the same network as each user parking duration will influence final results. This aspect will be further analyzed in next chapter comparing results when one or multiple users with different parking duration interact in a network.

4. MULTIUSER TAPC FORMULATION PROPOSED FOR PARKING PROBLEM

In this chapter a multi user TAPC formulation will be proposed which includes network modification, mathematical formulation, solution algorithms, numerical examples and a case study. Multiple author have worked the multi-use traffic assignment [71], [72] generally speaking the problem can be modeled as the addition of flow variables on every link which describe the link flows of each user that might or might not have unique and separate travel cost functions, in the case that the function are independent the problem becomes a set of separable classic traffic assignment problem. In the other case the problem becomes a non-symmetrical cost dependent traffic assignment. In this case the well know method of successive averages becomes the favorite solution methodology because of simplicity, good performance and well stablish demonstration of convergence techniques categorized as fix point methods. In this work the proposed method will create parallel not interconnected networks for each user which be linked between them by the capacity constraints and non-symmetric cost travel functions. Generally speaking, the problem will not have an equivalent minimization program equivalence, but can be formulated as a set of self-dependent optimization problem in a Cournot Game setting which is equivalent to a Variational Inequality. In algorithmic term the classical formulation is hard to solve and generally speaking requires path enumeration to use classical Variational Inequality algorithm or classical Fran wolf iterations with iterative cost performance functions. In this work two algorithm are shown the first used the classical diagonalization approach which in this case consist of solving each user independently using the flows of others users to self-calculate their cost. Also, the network capacity concept is studied in terms of formulation and algorithmic solution. Finally, numerical examples and a study case is presented in order to visualize the outcomes of the model.

4.1 Consideration and Mathematical Formulation.

For the problem formulation it will be necessary to understand the assumptions of the model. In this case the multiple users will be described by a unique perceived travel cost, although for most of the cases the function form of link cost will be the same for all users among same link. These functions will be applied to each network and each of them will be a function of the link flow of the user and the sum of the link flows of the other users. Also, the combined flow of all users cannot exceed specific and common capacity constraints, which make the problem hardly constrain and feasibility will need to be ensured before start. In the case of traffic links, the well-known BPR function will be used, although any other function as long as convex could be used. For the case of parking links their travel time will be a function of the user average parking time and the price of parking of each cell plus a wait time which describes the attention rate of each parking slot as a function of demand and its serving speed. The User average parking time must come from some source which divides user in homogenous groups of people in terms of parking behavior and equal parking time. Therefore, each user will have a given OD demand matrix and parking travel time, plus the perceived cost functions. The proposed formulation consist of a set of n stacked TAPC problems, one for each user type in a Cournot Setting [73] which will be refereed as Multi User Traffic Assignment Problem Capacitated (MUTAPC)

$$\min_{f^v,} \sum_{i=0}^{n} \int_0^{x_i^v} TF_i^v \left(s + \sum_{V \neq v} x_i^v \right) ds \; (1)$$

$$A^v f^v = X^v \; (2)$$

$$B^v f^v = D^v \; (3)$$

$$A^v f^v \leq HC^v \; (4)$$

$$\sum_{v \in V} A^v f^v \leq HC^v \; (5)$$

$$f^v \geq 0$$

$$\sum_{v \in V} PT_v x_p^v \leq HCP$$

$$\forall \, v \; \in V$$

Equation 13 MUTAPC mathematical formulation for UE equilibrium

$$f^v = flow\ variables\ associated\ with\ player\ v\ for\ all\ used\ od\ pairs\ (Decision\ variable)$$

$$x_i^v = flow\ of\ link\ i\ with\ respect\ to\ player\ v$$

$$X^v = link - flow\ vector\ of\ player\ i$$

$$A^v = flow - link\ incidence\ matrix\ for\ player\ v$$

$$B^v = flow - OD\ vector\ matrix\ for\ player\ v$$

$$D^v = Demand\ vector\ for\ player\ v$$

$$TF_i^v = Cost\ function\ of\ link\ i\ for\ player\ v$$

$$PT_v = Parking\ time\ of\ player\ v$$

$$x_p^v = link\ flows\ of\ parking\ links\ for\ player\ v$$

$$HCP^v = Total\ parking\ available\ time\ vector$$

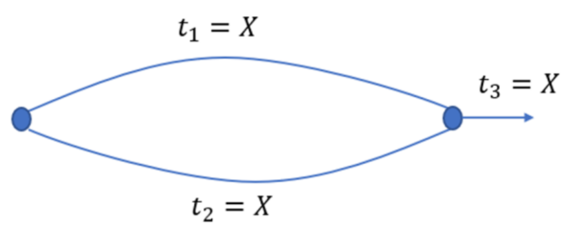
$$V = \ set\ of\ players$$

A detail analysis of the formulation implies that there are V players and each has its own separate TAPC formulation, nevertheless each of them depend on the link flow of the other players link flow solution. This dependency exists for the objective function and the feasible set. On the other hand, a VI formulation of this problem would imply the KKT condition of this equivalent problem to be formulated. Each player KKT would be declared with respect to their own variables, the results of such operation is given in Equation 14.

$$f^v(C^v - U^v + \alpha^v + \alpha_P^v) = 0 \ (1)$$
$$\alpha^v \left(HC - \sum_{v \in V} A^v f^v \right) = 0 \ (2)$$
$$\alpha_p^v \left(HCP - \sum_{v \in V} PT_v x_p^v \right) = 0 \ (3)$$
$$(C^v - U^v + \alpha^v + \alpha_P^v) \geq 0 \ (4)$$
$$A^v f^v = x^v \ (2)$$
$$B^v f^v = D^v \ (3)$$
$$\sum_{v \in V} A^v f^v \leq HC$$
$$\sum_{v \in V} PT_v x_p^v \leq HCP$$
$$f \geq 0 \ (4)$$
$$C^v = \sum_{i=0}^{n} \frac{\partial \int_0^{x_i^v} TF_i^v(s + \sum_{V \neq v} x_i^v) ds}{\partial x_i^v} = \sum_{i=0}^{n} TF_i^v \left(s + \sum_{V \neq v} x_i^v \right)$$

Equation 14 KKT conditions for SITAPC

Analyzing the first condition makes it clear that the resulting optimal flows are optimal with respect to the generalized travel cost function including the LaGrange multipliers of the link constrain of traffic and parking links for each player. Now in terms of uniqueness, it is important to note that the problem may or may not be unique with respect to link flows of each user. This given that this problem is equivalent to the solution of a unique network equilibrium problem with asymmetrical cost functions. In order to have a uniqueness property for such a model the link performance function would need to be symmetrical and have a bigger partial derivative with respect to their own link, with respect to the derivative of the other users or links related with it [74]. This can be rationalized by the undifferentiability test, which could be understood as follows, “if two identical users are assigned to a network with the same OD, then the link flow generated by one could be replaced by the other since in principle they are interchangeable”. This logic implies that in some cases users link flow could be changed and since equilibrium would still hold for both users with respect to each other, another possible solution may emerge in term of link flows of each user. An example of this formulation is given next in order to clarify these concepts.



$OD^1 = 10, OD^2 = 10; HC = [20,20] ; HCP = [600]; PT = [10,30]$

Figure 12. Example of Parking problem

In this example the travel performance function is perceived the same by both users and are the same for all links. The OD flows for both users are 10 and capacities are the same. The equivalent mathematical formulation of the problem is given in Figure 12. Given these conditions a solution for the problem is not unique since both players OD can be replaced even if their travel time function is different given that the parking or final link controls the behavior of the system.

$$Min \int f_{1,p1} + f_{1,p2} + \int f_{2,p1} + f_{2,p2} + \int (f_{1,p1} + f_{1,p2} + f_{2,p1} + f_{2,p2} + 10)$$
$$st$$
$$f_{1,p1} + f_{2,p1} = 10$$
$$[f_{1,p1} + f_{1,p2}, f_{2,p1} + f_{2,p2}] \leq 20$$
$$10 * (f_{1,p1} + f_{2,p1}) + 30(f_{1,p2} + f_{2,p2}) \leq 600$$
$$Min \int f_{1,p1} + f_{1,p2} + \int f_{2,p1} + f_{2,p2} + \int (f_{1,p1} + f_{1,p2} + f_{2,p1} + f_{2,p2} + 30)$$
$$st$$
$$f_{1,p2} + f_{2,p2} = 10$$
$$[f_{1,p1} + f_{1,p2}, f_{2,p1} + f_{2,p2}] \leq 20$$
$$10 * (f_{1,p1} + f_{2,p1}) + 30(f_{1,p2} + f_{2,p2}) \leq 600$$

Equation 15. Equivalent mathematical formulation of problem

When solving this problem, it becomes apparent that for symmetry the flows of player one is interchangeable with player 2. Thus, the problem can have multiple solution which still yield the same overall links flows.

4.2 Solution Algorithm

As explained before this problem incorporates a set of multiple users trying to play a simultaneous game with shared resources (Capacity) and unique perception, which although makes the problem suitable for realistic simulation of certain aspects, makes it hard for finding solution algorithms as the mathematical solution does not have advantageous properties such as convexity. This is a recurrent problem of any algorithm or mathematical problem where the problem relaxes assumption trading off complexity with more realistic behaviors. In general, the complexity of solution algorithms increases as algorithm tend to center on the recursion of simplest algorithm which solve instances or subproblems. In this case there are two proposed solution algorithms based on the interpretation of the problem. The first solution algorithm is inspired by the stacked game formulation and solves the problem by a successive solution of parallel and independent TAPC problems which are updated at each iteration.

Algorithm 4 Sequential Independent TAPC solution

Input:
$G = (V, N), X_0, tol$ V=Vertexes,N=Nodes
Output: x_k^v
Update Overall link flow and update $k = k + 1$ $X_k = \sum_{v=1}^{V} x_k^v$
Calculate overall parked travel time vector $PX_k = \sum_{v=1}^{V} PT^v * x_k^v$
for v=1,2....,V **do**
    Update travel link functions vector $F_k^v = F^v(s + x_k)$
    Update Traffic link remaining capacity vector $C_k^v = HC - X_k + x_k^v$
    Update Parking link remaining capacity $PC_k^v = \frac{HCP - PX_k + PT^v * x_k}{PT^v}$
    Solve $x_k^v = TAPC(G, F_k^v, C_k^v, PC_k^v)$
end for
Calculate $X_{k+1} = \sum_{v=1}^{V} x_k^v$
if $||X_{k+1} - X_k|| <= tol$ **then return** x_k^v for all $v \in V$
end if

Figure 13 MUTAPC (Sequential Independent TAPC solution algorithm)

Figure 13 Presents the pseudo code for the solution algorithm proposed, in the case of the TAPC the subproblem may be solved using the CCFW algorithm with a high tolerance standard and a high number of max iterations. This given that the dependence of each players makes the algorithm to have bad convergence if precision of flows is low and in general the number of iterations to reach the equilibrium becomes lower. At convergence the link flows of each user do not change relative to the other users which imply that equilibrium was reached. However, convergence of the algorithm is just assured under strong properties of the link flow functions. The philosophy of the independence of the V players TAPC comes from the multinetwork equivalent formulation which would have independence and could be seen as one TAPC with more links. On the other hand, this problem requires an initial feasible solution, thus a link-node incidence formulation will be created. The equivalent formulation would have number of variables equal to $N * V * Z$ where N would be the number of used origins, V the number of vertexes and Z the number of players. Given that in reality the implementation of multiple players generally would have small values the increase in decision variables would not be intractable in computation terms. The second solution algorithm will be explained in detail in the next chapter when AV are included and will take advantage of the link-node incidence formulation where in each subproblem a Multicommodity flow problem is solved being formulated as LP in the link node space variables.

4.3 Formulation of Network capacity in Parking Problem

In the case of the described problem the network capacity definition keeps being the same, and in the more complex form would be a bilevel program with the upper level being the demand flows and in the lower level the set of Variational inequalities described by the KKT condition of the equivalent mathematical formulation. However as seen before, this problem can be formulated as a multi flow commodity problem with an increased set of link flows for each different user to be simulated, the formulation of one subproblem assignment based on constant link flow times is given below.

$$\min_{x_i^v} \sum_{i=0}^{n} \sum_{v=0}^{V} x_i^v F_i^v(0) \dots (1)$$
$$M^v x_i^v = OD_i^v \ \forall\, i, v \,..\, (2)$$
$$\bar{X} = \sum_{v=0}^{V} x_i^v \ \ (3)$$
$$\bar{X} \leq HC$$
$$\sum_{v=0}^{V} PT^v x_{ip}^v \leq HCP$$
$$x_i^v \geq 0$$

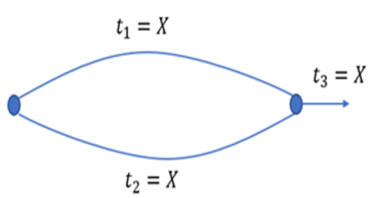
Equation 16. General mathematical formulation for SITAPC subproblem formulation

The formulation is equivalent to the one previously stated with the difference of the additional index for every player and the addition of parking slots conditions which will be the product of a matrix with a vector of players Parking time. This formulation increases the complexity of the overall problem by adding as much variables as users. Now the main advantage of this formulation is that it can be used in order to stablish the network capacity. For this purpose, the dual LP of the link node formulation is considered, although just the LaGrange Multiplier associated with the inequality constrains are taken into account. For the used od pairs the condition is that the corresponding nodes must be greater or equal than zero and the OD flows must be conserved. This construction is based on the maximum flow problem, which in the literature is generally defined as finding the maximum flow between an OD pair.

$$\max_{x_i^v} \sum_{i=0}^{n} \sum_{v=0}^{V} x_i^v \ (1)$$
$$M^v x_i^v = 0 \ \forall\, j \neq i, v$$
$$M^v x_i^v \geq 0 \ \forall\, j = i, v$$
$$M^v x_i^v \leq 0 \ \forall\, j \ with\ nonzero\ destination$$
$$\bar{X} = \sum_{v=0}^{V} x_i^v \ \ (3)$$
$$\bar{X} \leq HC$$
$$\sum_{v=0}^{V} PT^v x_{ip}^v \leq HCP$$
$$x_i^v \geq 0$$

Equation 17 Link flow formulation of network capacity in link flow space

The formulation is equivalent to the one proposed for TAPC, with the addition of the new conditions for hard capacity constraints of the parking links and an increase in the flow variables. In order to make the introduced formulations clearer a simple example formulation is presented as before, in this case the example is the same used for the path flow formulation given in this chapter but in the link flow space.



$OD^1 = 10, OD^2 = 10; HC = [20,20]; HCP = [600]; PT = [10,30]$

$$M = \begin{matrix} 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{matrix} ; OD = \begin{matrix} 10 \\ 0 \\ -10 \\ 10 \\ 0 \\ -10 \end{matrix} ; E = \begin{matrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 10 & 0 & 0 & 10 \end{matrix} \ HC = \begin{matrix} 20 \\ 20 \\ 600 \end{matrix} ; PT = \begin{matrix} 10 \\ 30 \end{matrix} ; X = \begin{matrix} x_1^1 \\ x_2^1 \\ x_3^1 \\ x_1^2 \\ x_2^2 \\ x_3^2 \end{matrix} ;$$

Figure 14. Example of link-flow formulation for SITAPC subproblems

$$\min_{x} 0$$
$$M * X = OD$$
$$\bar{X} = E * X$$
$$\bar{X} \leq HC$$
$$x_i^v \geq 0$$

Equation 18 Mathematical formulation of SITAPC link-flow subproblem

This formulation as seen before differs from the one planted by the equations; this is given by the simplification process that the matrix expressions allow which makes the problem suitable for usage in any linear program solver. In terms of the maximum flow formulation, the problem remains almost the same but the MX matrix multiplication is no longer all equal to zero but the M matrix is divided into block matrix each one corresponding to the columns of the matrix B which are either an origin, destination or none.

$$\max_{x} x_1^1 + x_2^1 + x_3^1 + x_1^2 + x_2^2 + x_3^2$$

$$\begin{aligned} M_a &\geq 0 \\ M_b &= 0 \\ M_c &\leq 0 \\ \bar{X} &= E * X \\ \bar{X} &\leq HC \\ x_i^v &\geq 0 \end{aligned} \qquad \begin{aligned} M_a &= \begin{matrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{matrix} \\ ;\ M_b &= \begin{matrix} -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{matrix} \\ M_c &= \begin{matrix} 0 & 0 & 0 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{matrix} \end{aligned}$$

Equation 19. Formulation of Capacity problem in link-node form

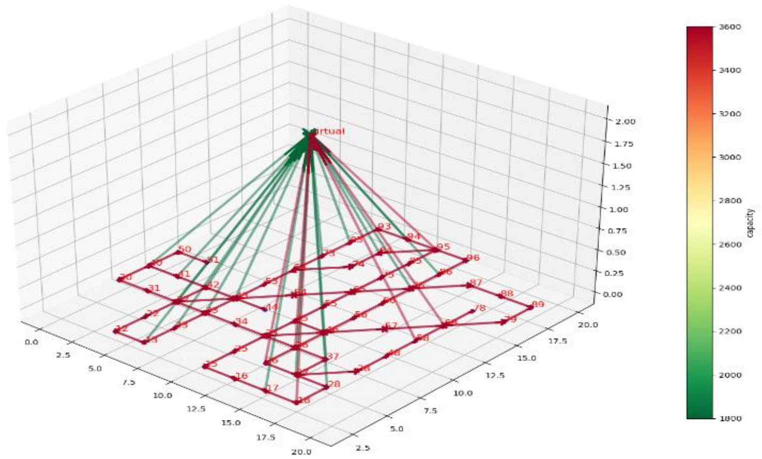
This capacity formulation would give the same answers as before, which can be defined as the maximum total amount of combined flow that under a given OD matrix pattern can be assigned. This brings a huge advantage for planning since as discussed before this network capacity is a number by unit of time that can easily describe the network performance. Which could be used as a way to incentivize mode changing or even demand shifting with urban planning based on the search of optimal usage of available network capacity. Also, with the help of the detection of the bottleneck links a network design problem could be formulated in search of optimal resource allocation to increase Network capacity. This topic has been lightly researched before [75] given that the classical by level formulation of network capacity made it prohibitive harder. However, with the Linear programming formulation given in this work, the problem would become tractable as bi-level program with upper level objective being the optimization of the network capacity, and the lower level the definition of network capacity as a function of the decision variables which can be amount invested on a given link or the addition of new links.

4.4 Discussion of practical importance of proposed Formulation

One of the main goals of this work is the development of a useful formulation for a variety of problems in terms of parking planning and modelling. In this case the aforementioned formulation is able to capture multiple important characteristics such as interaction among different users, hard capacity constraints on links and analysis of network capacity. Therefore, planners can include variables such as parking prices and walking distances in the cost function which could help them to better understand their effect in people choices in an equilibrium setting. On other hand the algorithm capabilities although will not be as fast as classical TAPC, can still be tractable, since each TAPC is around 4 times slower than a normal TAP and the number of players is generally low. The overall added complexity of the capacity constrain and multiuser setting could imply increases in calculation times of around 20. Which although sounds bad would still be tractable for medium size network with a number of links in the order of 1000. In the case of bigger cities, the algorithm as presented would be prohibited, nevertheless in these cases the usage of network partition schemes could be useful. These schemes work by dividing the network in independent subnetwork that can be solved independently making the problem faster while trying to reduce the overall lost in accuracy. These methodologies have been studied in the literature [76], [77] which have gain increased popularity as new models make complexity increases. This can have huge performance implications since the complexity of these assignment problem is polynomial of degree higher than 2 in the best-case scenario with respect to the number of links. Therefore solving 2 networks of the same size is not equal in time complexity than solving a network of twice the size. Thus, practitioners could still gain the benefits of the proposed model in these scenarios for big scale cities while reducing the precision in a controlled manner. On the other hand, one of the biggest contributions in this work is the reformulation of the network capacity as an LP for both TAPC and MUTAPC which can easily be calculated and scales well with number of links even in the order of magnitude of a few thousands. This easy and tractable capacity network definition can help to understand the existence of bottlenecks links and help with the high variability of OD demands as network capacity does not require a detail definition of it in order to be stablished. This formulation could help in the design process where the objective of the network design problem could be replaced by the maximization of network capacity.

4.5 Numerical Example and sensitivity analysis

In order to better summarize some of the formulations and algorithms illustrated in this chapter a simple numerical example will be made in a parking and traffic network. In this network the walking nodes will be removed and just a virtual node of parking will be considered. The example will contemplate what happens in terms of run time and overall traffic distribution and congestion of the network. In this example all traffic link capacity will be set at 1800 veh/h, the study period will be 1 hour and the OD pairs to use consist of an origin in node 40 and destination in node 89. Also parking vehicles going from this target to the virtual parking node will be considered. A fifth of the total demand generated will consist of parking players and the rest will be traffic players. The BPR functions used will have the form $f(x) = t_f * (1 + 0.8\left(\frac{x}{0.8c}\right)^{3.5})$ where $t_f = \frac{length(m)}{600\ m/min}$ so travel time will be given in minutes. The parking cost of the links will be equal to 1 USD/hour or 0.16 USD/minute. Three user types will be used based on parking times of 10 minutes, 30 minutes and 60 minutes. The original distribution of players will consist of 60 % of players parking 10 minutes, 30 % parking 30 minutes and the rest 10 % parking 60 minutes. So, the average parking time of all users is equal to 21 minutes. The comparison will be made between running the model following the original distribution (3 type of players) and running the model with just 1 player having the average parking time of 21 minutes. The total number of players to simulate will be 3000 in 1 hour and their finals OD matrixes will be described below.



$$f(x) = \frac{l * \left(1 + 0.8 * \left(\frac{x}{0.8(3600)}\right)^{3.5}\right)}{10}; cap = 3600; \# \, cells = 30; study \ period = 60 \ min$$

$$Player1, PT = 10 \ min \ , OD = \{[40{,}89]: 1500, [89, virtual]: 300\}$$

$$Player2, PT = 30 \ min \ , OD = \{[40{,}89]: 750, [89, virtual]: 150\}$$

$$Player3, PT = 60 \ min \ , OD = \{[40{,}89]: 250, [89, virtual]: 50\}$$

$$AVG \ player, PT = 21 \ min \ , OD = \{[40{,}89]: 2500, [89, virtual]: 500\}$$

Figure 15. Numeric example MUTAPC

An analysis of the convergence for the 1 player case was made showing the required time and change in norm among solution for all 30 iterations. The results are shown in Figure 16, the convergence of the algorithm is achieved in the first five iterations, which implies that overall, the algorithm convergence is fast. The spent time per iteration shows that the linear program takes more time that the line search, in this case since the system has a low number of players and links the difference is not extreme. Although in realistic cases it is expected that the solution of the LP takes around 90% of overall running time.

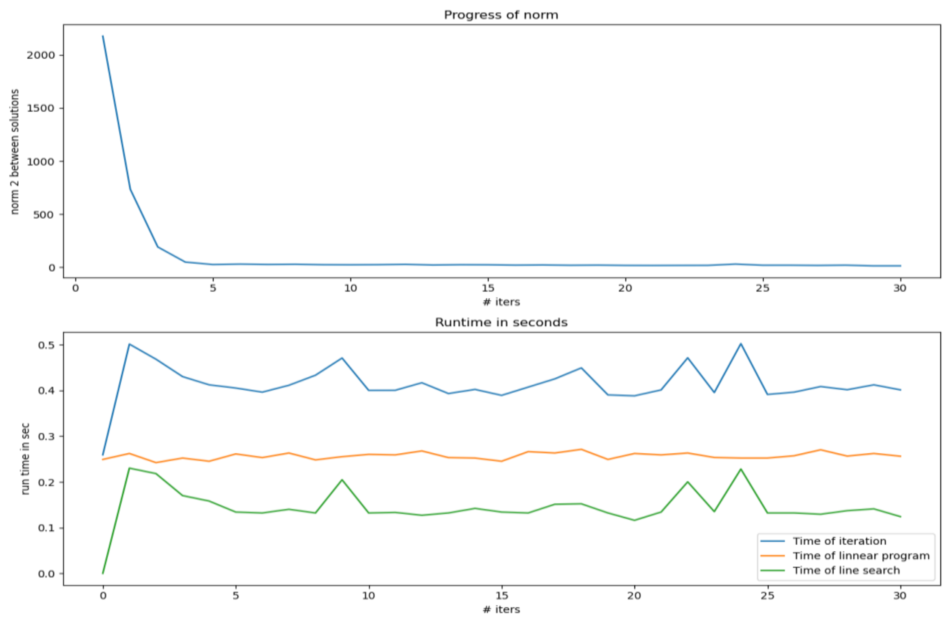


Figure 16. Analysis of convergence of algorithm

The solutions of the problem show that for the case in of the only player the parking links are occupied based on capacity, which implies that the closer links are occupied faster. In the case of the multiplayer scenario the solution shows that players with shorter parking time tend to park closer this is given by the fact that the total cost of parking is smaller so tradeoff of traffic distance and parking destination is made.

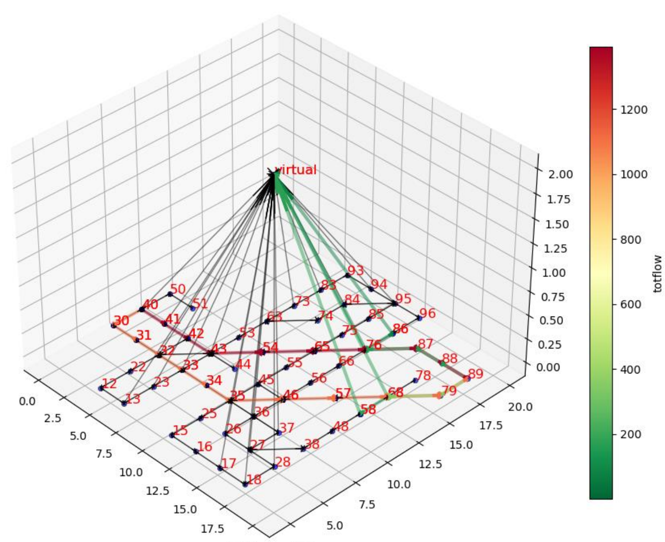


Figure 17. Average player solution

In the case of the multiplayer traffic assignment the solution is not unique which implies that in the traffic network the distribution of players could have a multiplicity of solution for each link player traffic. One important point about the multiplicity of the solution is that predicting reality becomes much harder as there are multiple outcome states that could happen in reality. In general, the bigger the number of players the highest number of possibilities for flow player exists.

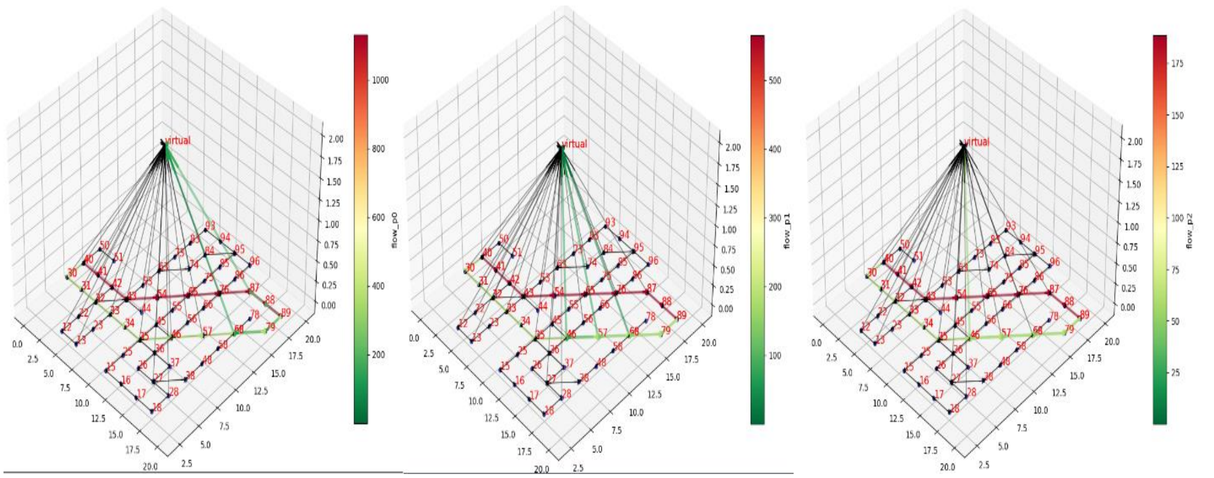


Figure 18 Network solution for the 3 player MUTAPC formulation

When the solutions of both problems are compared it becomes apparent that traffic and parking flows become different as players with different parking locations distribute themselves in different manner forcing them to have different to have different traffic paths. Overall, when just one player is considered the overall time spent in the traffic network is 20% smaller than when the three players are considered separately. The parking cost difference among each solution is also notorious and the total collection of the system when just one player is considered is around 10 % higher when one unique user is considered. This implies that the inclusion of more players makes their selection to be more constrained by their parking selection which affect the overall traffic cost, although reducing parking cost. In reality this implies that the multiplayer formulation can have an important impact on cost or benefit analysis of new fees or structural decisions. Also, this becomes more important in the case where players differences exist in both parking duration, destinations and type of vehicles.

As part of the study of the described model a sensitivity analysis of the numerical example is proposed. For this a variation of some of the parameters of the model is performed, in this case the parameters to be analyzed are the total number of cells and average parking time. The main reason for this analysis is to understand the influence of this variables over the network level properties. The first sensitivity analysis is made with respect to the total number of cells and the results show a decrease in cost and the overall relationship of flow/cap which are metrics related with congestion. Close to the 27000 number of cells the reduction of outputs variables is stopped which implies that not benefits are longer obtained. Thus, the traffic network congestion is heavily influenced by the availability of parking as cruising for parking becomes harder when scarcity is present. This result is several importance as shows the influence of the parking layer of model into the traffic network which is one of the central points of this work, as classical models separate different transportation layers although in reality they have influences among them.

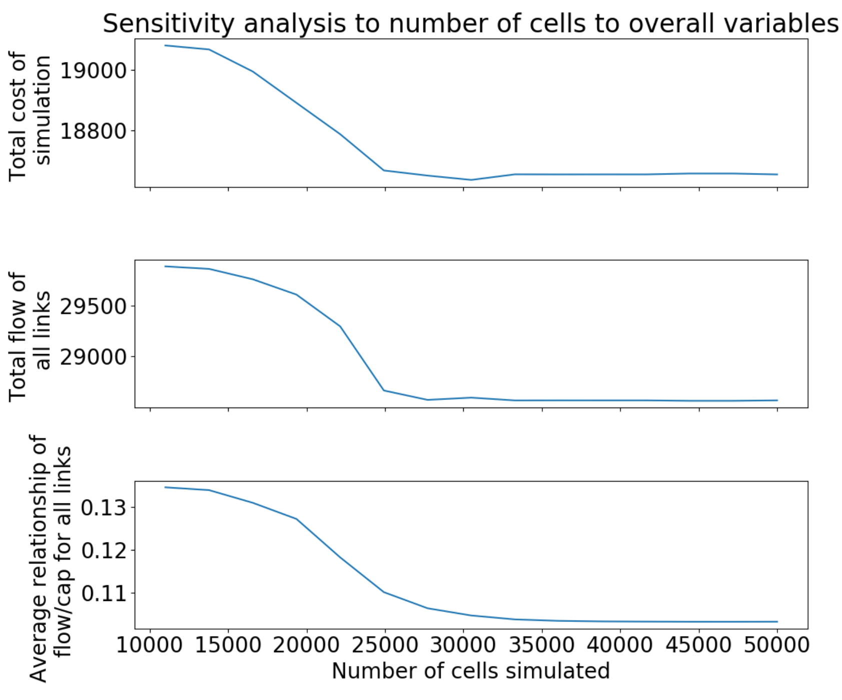


Figure 19 Sensitivity analysis of number of parking cells

On the other hand, the sensitivity analysis of the average parking duration is performed, which is of essential importance as different zones of the city can have different parking durations and one of the strengths of the developed model is the inclusion of parking duration as a analysis variable. Overall, the effect of the variable repercusses in the route decision, network capacity and parking costs. In this case the sensitivity analysis was used using the same default values for network properties, results are shown in Figure 20. The analysis show that there is highly nonlinear behavior with respect to parking duration where after the average parking duration reaches a value higher than 55 minutes then the system starts to experience an increase in system cost. Nevertheless overall, these changes are small in properties, this is due to the low sensitivity of the MUTAPC to variables related to the objective function.

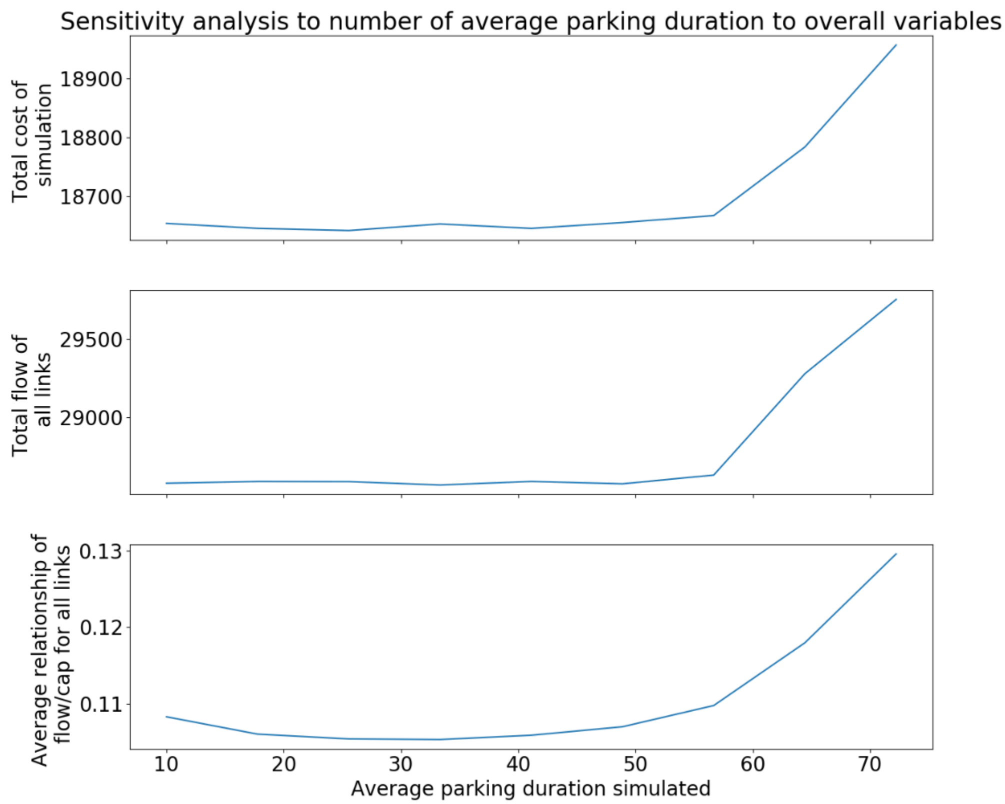


Figure 20 Sensitivity to average parking duration

Overall, the sensitivity analysis performed shows small changes in the output properties this is due to the small sensitivity to this problem to the objective function. Generally speaking, Traffic assignment is more sensitive to the constrain set than to the decision function. The main reason behind this is that generally speaking the functions used have not much variation over the possible traffic flows that could come out of the problem. In the case of MUTAPC the problem becomes even stronger as the hard capacity constrain limit the decision set of users in an important manner. Nevertheless, the main change of this input to the overall problem properties come to the actual distribution of flows which change based on the inputs. Overall, the developed model is able to introduce different characteristics and the effects among them into account as for example user characteristics, parking capacities, traffic conditions or network structure. In this case the main variables tested were the one related with parking as is one of the main subjects of analysis in this work.

4.6 Study case in Bello City, Colombia

In order to make a comparable scenario of real modelling, a real case in the city of Bello Colombia is chosen, this case was taken from a study made in this city where dynamic models were used. In the case of static models, the study period should be of short duration to reduce the possible violations of densities which happen when flow of links is not uniformly distributed over the study period. This implies that static modelling works best when the study period can be considered to have a constant behavior during its duration, so shorter study periods are desired. In the case of users, the different users to be modelled must be represented by am OD matrix, type of equilibrium to search and parking duration. Also, if required each user could have a different perspective of the travel cost functions of each segment, although in this case it will remain constant among users. In this chapter a general introduction of all data will be given, then the demand table procedure will be explained and the results for the model will be presented using the aforementioned modelling.

4.6.1 General description of the Data

The data chosen for analysis comes from a study developed in 2017 in Bello, a Colombian city located at the north of Medellin Metropolitan area. Bello has near 0.5 Million inhabitants. This data consists of three datasets. The first dataset is a parking plate rotation study in which for one day from 7:00 AM to 7:00 PM, approximately 60% of all the parking spots were tracked in a sector of 2.1 km2 accounting for 15% of the city urban area. The total number of segments add up to 30 km in length, which is the 8% of the network size. The information recorded for 13500 vehicles includes initial node, parking destination, initial park time, end park time. For the out of street parking lots, a dataset with the total demand of 8000 vehicles per day was used, which contained their typology and mean parking durations for each parking lot, a description of this network is shown in Figure 21. The second dataset consist of the OD matrix of the background traffic, which consists of one OD matrix per hour of all the background traffic which will not park. The last dataset consists of the road and parking network information, containing the length, geometric characteristics, number of cells, price of parking and type of parking of all parking zones in the study area. This dataset includes parking in and out of street and contain a total of 340 road segments and 40 out of street parking locations. The combination of the datasets brings some challenges, one of the most important is the fact that the original OD matrix did not have information of the arrival time and parking duration of each OD. Therefore, a random model was required to be created using the first and third dataset which is able to create a random demand table which includes all the users to be simulated with the corresponding origin node, destination node, start time and parking duration. With this random function created, multiple experiment setups of demand could be sampled and then applied to the desired model either static or dynamic, taking into account that in the static model the parking duration time would be reclassified into the desired division bins predefining the different type of users with respect to parking time.

Figure 21. Traffic network of Bello city

4.6.2 Demand Table Generation

The demand table must specify the characteristics of all the users to be generated in the model which includes 5 basic fields: origin, destination, arrival time, parking duration, type of user (Human or autonomous). As explained before the input datasets did not have the information in this detail so a model was created in order to randomly sample such a table for any specific scenario of number of vehicles, OD pairs to use and number of autonomous vehicles. The construction of such a function consisted of two stages which will be detailed and the overall way of using the model to generate any demand table will be described. In the first stage a simple statistical analysis was carried out, including a distribution of the parking duration for the whole city. For each spatial segment the mean parking time was obtained. In this case the result of this distribution is shown below in Figure 22.

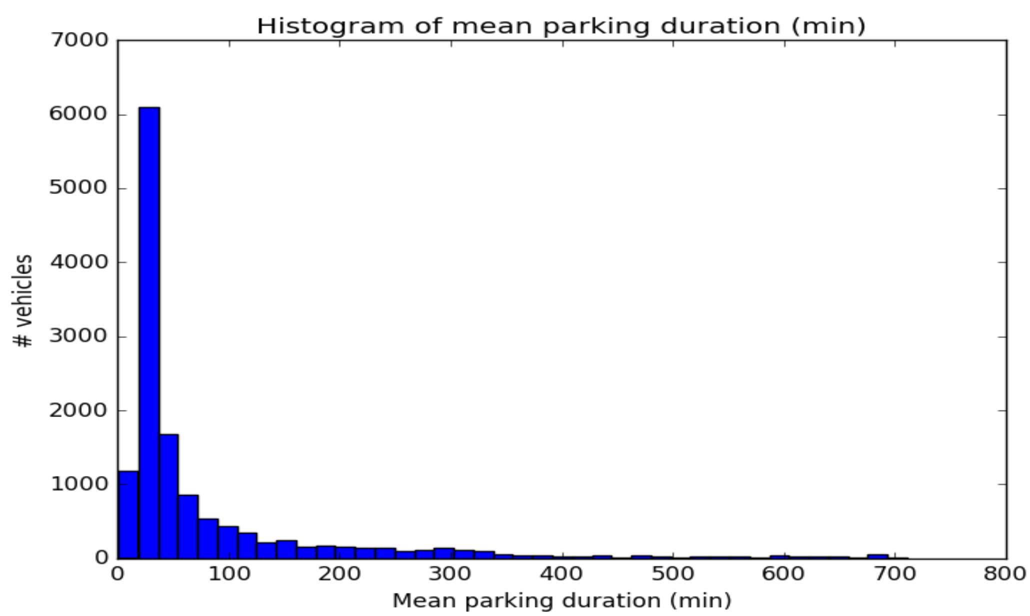


Figure 22 Histogram of mean parking duration

For the second stage a plot of parking duration vs initial parking time is shown in Figure 23 The “triangular behavior” in the data points is explained by the study period, which means that for a given time the maximum time that a car parking duration can be recorded is the last time of the time period, which in this case is at 8pm (1200 mins). This implies that the information has a bias towards short parking duration. Due to this the interpolation was done with the data from 7 AM to 2 PM, to overcome the lack of data on the evening period.

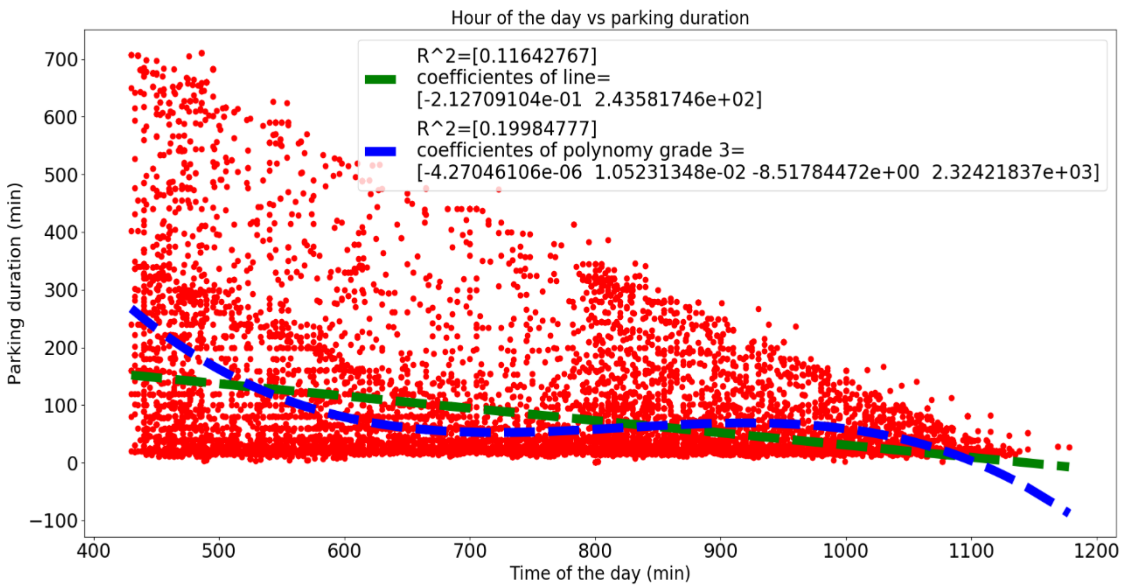


Figure 23. Parking duration vs hour of the day.

Based on these two inputs a random procedure is created which replicates the overall data obtained in the study by the usage of a random sample from a distribution whose parameters are function of departure time and location. The final spatial temporal function is described in the equation below:

$$F(\overline{PD}_s, t) = \overline{PD}_s * \left(-\frac{427t^3}{5949000000} + \frac{263t^2}{1487250} - \frac{851t}{5949} + \frac{232000}{5949}\right) \quad (6)$$

Where:

$\overline{PD}_s$ is the mean parking duration of the segment s

t is the time of the day in minutes

This function F gives the mean parking time of any vehicle in that segment, then with information a real value of parking duration is sampled from the general parking duration distribution of Figure 22. Therefore, to create the demand table of users the following two step are required.

1. Stablish the number of users to create for each OD pair, also assign the initial or departure time, either from outsources or from a random sample of the arrival time distribution of the data.
2. For each user in each destination apply the $F(\overline{PD}_s, t)$ with the mean parking duration of the segment and the arrival time, then with this value normalize the parking duration distribution and sample the parking durations from it for each user.

4.6.3 Setup and Results

Base on the general description of the data and the process for creating the players demand table a simple loading of the model was executed in order to obtain the static model results. As explained before for this study a period of 120 minutes was used which cover the time period of 12 PM to 2 PM, selected based on the peak parking demand that in the zone occurs in this moment. The total demand reported from the OD matrix of this period is about 25000 vehicles with a particular OD matrix distribution for the whole period. The 3 % more important od pairs are shown in Figure 24, where long od pairs can be checked implying a big amount of passing traffic. From the overall demand around 15000 trips have as final destination the parking in the study zone. As described before all traffic links are considered to have an 1800 veh/h capacity which becomes 3600veh/p where p means the study period, the total number of directed traffic links is 223. Considering this we have a maximum or brute network capacity of 802000 vehicle in the study period which will be drastically lower when the actual matrix distribution of players is taken into account in the network capacity estimation.

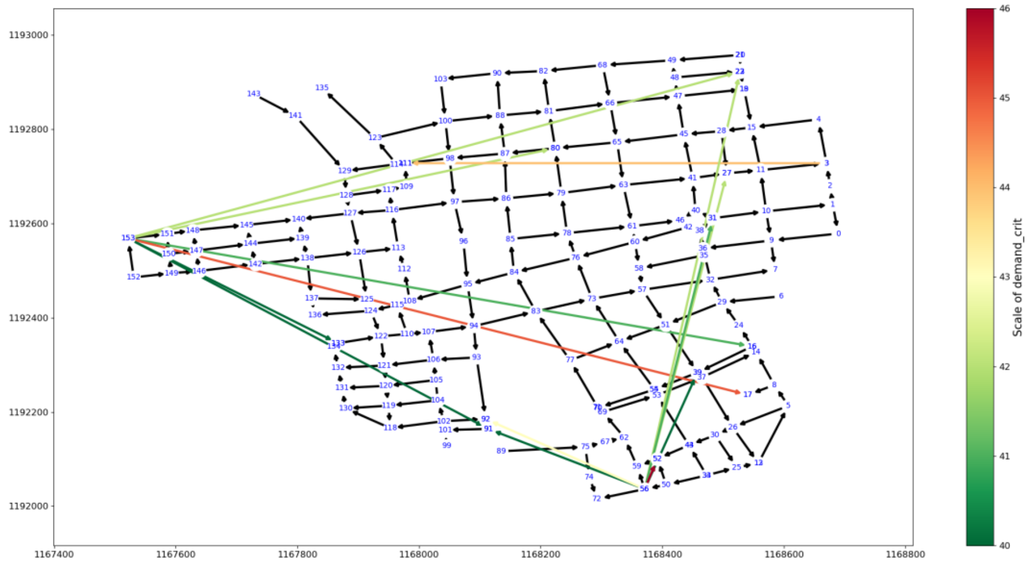


Figure 24. 3 percent most important OD pairs in study case from 12 PM to 2 PM

Parking wise the network has a total amount of 4683 cells distributed among out of street and curbside parking, considering that the average parking duration is 30 minutes, the estimated cell rotation would be of 4 veh/p so the average capacity of the parking system would be of an estimated 18732 vehicles which in theory should be enough to accommodate the input demand. In terms of users the parking duration distribution was divided into three users which will have the following parking duration: player 1 will contains all duration from 0 to 20 minutes with an average parking duration of 10 minutes and covering 46% of all demand; second player will cover duration from 20 to 60 minutes having an average parking duration of 40 minutes and covering 32 % of all demand and player 3 will cover all durations higher than 60 minutes with an average parking duration of 90 minutes and covering the 22% of all demand. The reason why three player were chosen was based on numerical examples experiences where increase in users have a highly significant complexity gain, while poor converging behavior given that players perceive and behave to similarly. The final difficulty in modelling is the stochastic behavior of the table generation function which assign players a random duration implying that for each run the player demand table is different although overall average of parking durations and od matrixes are conserved. Thus, a simple Montecarlo sampling was made were the od demand matrix was called a total of 50 times and the results of total link flows and players link flows averaged for all cases.

The run time of the algorithm per iteration was around 20 minutes implying a total run time of around 17 hours on a 8 core 3.4 GHz processor with 64 Gb of ram.

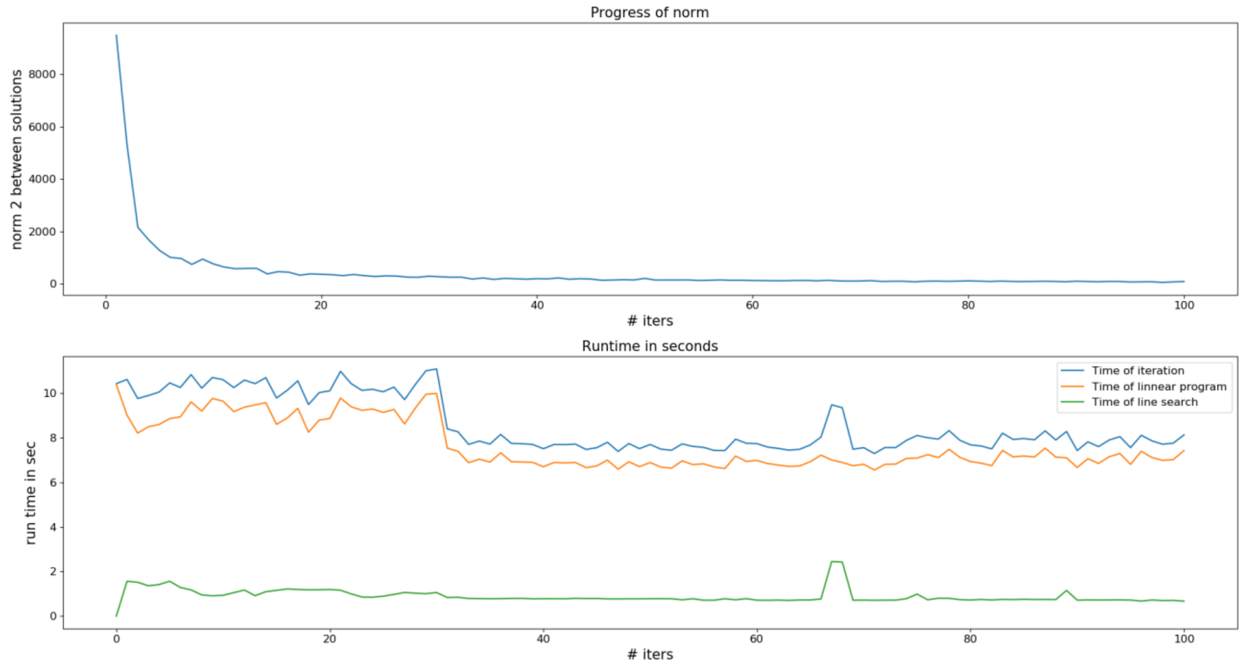


Figure 25. Algorithm convergence analysis for one run

Convergence results for one of the fifty runs is shown in Figure 25, where most of the norm difference is achieves in the main 20 iterations, compared to past numerical examples the LP solution starts to be more than 85% of total run time, given the increase in size and number of OD pairs. On the other hand, the results of the aggregated link flows are shown in Figure 26, where it becomes clear that the traffic flow congestion is quite low, although parking links start to get congested. In fact, the average occupation rate which is defined as the average of the relationship between flow and their capacity is 0.33 for all the network, while being 0.21 for traffic network and 0.42 for parking links. Now based on this it would be easy to assume that the leftover capacity could be obtained from these leftovers' capacity, but again this relationship fails to take into account bottlenecks and moving patterns. In order to analyze the capacity of the network 2 analysis are made.

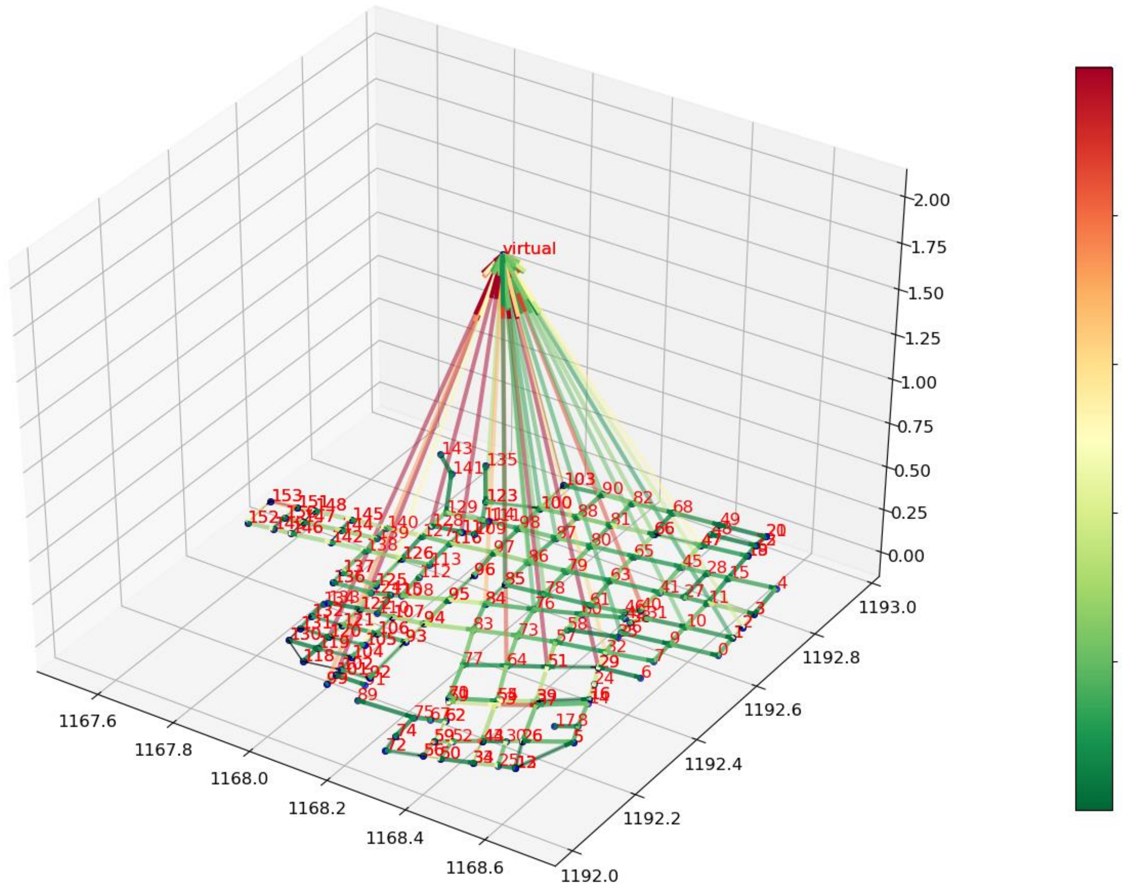


Figure 26. Results of study case aggregated flows using MUTAPC

The total travel time of the system based on the link flows is 4.5 million minutes and the total collected money by the parking places is estimated to be 3.3 thousand dollars both of these measure in the study period of 120 minutes. Is important to notice that the monetary money found is similar to the ones estimated from the original Bello study, where the calculated earning of the noon parking period rounded 5 thousand dollars although in the original study a dynamic methodology was used. In terms of capacity the estimation was divided in two part for the first capacity estimation all background traffic was removed and only parking destination flows were taken into account, giving as result a network capacity of 18096 vehicles in the study period. This value is around 96% of the average network capacity, implying that the parking distribution of the current parking places is able to attend the OD matrix pattern accordingly and that the traffic network does not have hard bottlenecks for the parking network. On the other hand, the network capacity for the traffic network is about 43200 vehicles, while the sum of all the capacities in the traffic network gives an estimated capacity of 1.6 million vehicles. In here the difference among the capacities is highly clear and shows that the traffic networks have much higher reserve capacity, since the relationship in this case is just about 2% of the sum of the upper bound for the network capacity. This is a normal fact an there are multiple reasons why this happens. The first one is that traffic trips generally cover more than 1 link why imply than one od flow can occupy the capacity of multiple links at once, the second reason is that traffic network can serve multiple purpose including connectivity which might not be efficient for most of the time but that even on the minimal requirements make the link have a high capacity which will not be utilized. Thus, redundancy is a common part of transportation network which also gives the network resilience to link failure which frequently occur in the forms of repairs and accidents. The overall results for each player are different and give a more detail distribution of players choosing parking based on a variety of decisions. The presented model has the power to be able to incorporate individual characteristics aggregated while still conserving fast convergence results and equilibrium considerations. Also, the network capacity calculation allows the planer to check how efficient the parking and traffic network are and could be used in stochastic or robust optimization frameworks where varying od patterns can be used in order to better understand if the network could accommodate a future or peak demand that can happen during events. Also, the formulated link node formulation allows the model to be flexible when certain conditions are to be taking into account, for example if a given link or set of links must preserve certain conditions of flows or follows rules as certain maximum allowable number of users by type as can happen when modelling users of different vehicle types.

4.7 Final Model Remarks and possible impacts on policy design

The proposed model in this chapter is a combination of multiple characteristics that play a central role in parking and traffic modelling, for one side the multiuser traffic assignment allows planner to take into account user and vehicle characteristics that play a role in the perceived travel time and capacities of different categories of vehicles in a given link. From the other side the link node formulation allows for the development of new algorithm approaches based on efficient subroutines which can easily be adapted to new conditions., This differs from traditional traffic assignment as generally speaking each new formulation requires the development of specialized algorithms crafter in adjustment to the different assumptions, nevertheless since the problem can actually be enumerated in polynomial time, then the problem be solved using efficient LP subroutines which can adjust to function changes or new constraints related to flows. On the other hand, this formulation allows for a simpler formulation of the network capacity problem as a partial dual of this formulation and resulting in another LP. This brings huge winning over previous attempts for the network capacity definition which were based on bi level programming, thus making calculation a tedious process. With this new formulation future works of network design could focus on maximization of network capacity, which has the potential of being a network property that can be easy to express, as well as be a way to understand whether a given network could withstand a given increase in demand. The numerical tests presented give evidence of the different that a model with one average user vs multiple users with different parking durations can have over the cost results, which is especially true for the parking links. This difference is expected to go stronger if different users include different parking times or vehicle characteristics such as motorcycles, trucks or cars, thus in practical terms it is recommended that a sensitivity analysis of the model on the number of users is presented.

In terms of practical usage, the presented model has multiple advantages over the classical models presented in the literature. For example, when compared to classical Traffic Assignment the described model is able to capture hard capacity constraints, different users and parking conditions. With respect to the last models in the literature most of them implement multi user with some capacity constraints. Nevertheless, the parking durations aspect is not taking into account, and the actual parking capacity division among users not explicitly taken into account. Overall the developed method has the capacity to take into account different modelling aspects which previous method would not, for example what happens if payment increases for the parking locations, which implies that users with different parking durations will have different costs. Also, for example what would happen if you force or create certain policies which can reduce or alter the parking duration of users. The network capacity on other hand gives a completely different perspective to the problem as it enables policy makers to test the effectiveness of policies such as parking restriction or increase in fees in terms of overall network capacity, instead of actual network cost. This is extremely important as the sensitivity analysis showed that the cost variables were not sensitive to the parking characteristics. Nevertheless, capacity is highly dependent upon these variables.

On other hand the developed model and the possible policies that could be evaluated with it can have a big impact on common social dilemmas that arise in the context of parking for example the shared resource dilemma that will exist when parking is shared or managed in the perspective of AV. The social dilemmas problems are of highly importance and have made a resurgence in this century as more data and ways of modelling social behavior are available. This in the context of parking for example arises when free parking is available, which for example incentivizes users to have long durations parking which decreases the network capacity and overall social benefit. Thus, special policies or fees that tackle such users is of absolute importance, as they can have an important impact of network efficiency if not tackled. In fact, this dilemma of the common is one of the reasons why so many countries are promoting policies such as congestion charge in some places in cities. As cost makes users to realize their actions and to change behaviors which in turn benefit overall society in terms of congestion and pollution. Parking for example can be used in a similar manner but in a easier way by increasing cost of parking in some places to incentivize the usage of certain transportation modes. The present model has the power to be able to take into account a lot of characteristics and variables which were not able to be tackled efficiently or at all by previous methods.

5. STATIC MODELLING OF PARKING INCLUDING AUTONOMOUS VEHICLES

In this chapter autonomous vehicle s(AV) will be included in the modeling scenarios, for this some assumptions on the changes that they will bring will be discussed. The same multiuser formulation will be used although some changes will become necessary in order to make the resulting flows to follow SO instead of UE. This chapter will be divided in five main parts, the first will be an overall view of the inclusion of autonomous vehicles and related literature, the second one will give the mathematical formulation for the problem when including the autonomous vehicles and its implications on uniqueness of solution, the third part will give an overview of the solution algorithm and network capacity, the forth chapter will give some small numerical examples to exemplify the explain model and the final subchapter will have the same numerical example of Bello city, including the difference in solution as autonomous vehicles start to appear in the market and a share of total users have this property.

5.1 General description of implications of Autonomous Vehicles in Static Assignment and Parking Modeling

Autonomous vehicles are one of the main technologies that are expected to highly change the driving dynamics and transportation usage in the entire world. Right now, the technology is on a rapid development and is able to do automatic cruising in highways or non-urban roads where intersections are scarce. In terms of transportation modelling the last two years have seen a rise in the number of researches related with the topic. One of the first researches of the topic was given by [78] where the author introduced a cell transmission model in a dynamic case where the autonomous vehicles affected different variables related to the vehicle movement. Is important to notice that in general autonomous vehicles (AV) are expected to bring a variety of different conditions which vary greatly by the perspective of experts in the field. Nevertheless in general Autonomous Vehicles (AV) are expected to change, demand or number of trips, capacity of roads, travel time of vehicles, parking patterns and mode choice [79]. In this work a framework for modelling the interactions among these two types of users will be explored, and the possible outcomes as the technology is adopted will be discussed. In terms of demand change brought by AV, the literature can be divided in two categories, the first one which proposes a long-term future where all vehicles are autonomous and thus not human driver is allowed or a future with mixed traffic of AV and NV. In the case of this work a mixed traffic will be considered, parking wise this would bring an important change in possibility of empty trips of AV searching for parking. Thereby any of three behavior of users could happen, the first one would imply coming back to the original source of the trip, the second one is to search available and cheap parking far away from the origin, and the third possibility is that the AV try to serve other users trips reducing parking needs and empty trips. In general, each of these scenarios would imply completely different things in terms of modelling, in this work the adopted choice will be the possibility of vehicles dropping passengers and then searching for parking with a special importance of price of parking over travel time. In terms of travel time and capacity, it is expected that as AV are less prone to error the acceptance rates of velocities could improve, thus increasing roads capacity and reducing travel time. In this aspect the literature is filled with multiple suppositions of which aspects of the driver following model would be impacted and what could be the resulting conditions. In this regard [80] present a cell transmission model using a two state speed model including a percentage rate of CAV (Connected Autonomous Vehicles) with respect to NV, their results suggest that the main aspect of the model id the T_{ACC} which describes the gap time acceptance of CAV, varying from 1.1 seconds to 2 seconds. According to this work the resulting capacity of a 2-lane road with CAVS would follow a linear regime until CAV reach 40% and then a second regime which depends on TACC as shown in Figure 27. Now in general is important to notice that these results show a high susceptibility to the TACC and in general capacity increases even with a small percentage of CAV involved in the system. Nevertheless, the assumptions made in the paper are quite conservative since they assume no response time of CAV, which in reality means no delay of processing time and decision making of the vehicle. Also the model does not take into account outside factors such as pedestrian movements and other interaction which become highly important in urban context while the work focused on rural roads. In general, the results show that the capacity of the road can be considered as a linear function of percentage of CAV in reasonable conditions of acceptance gaps times.

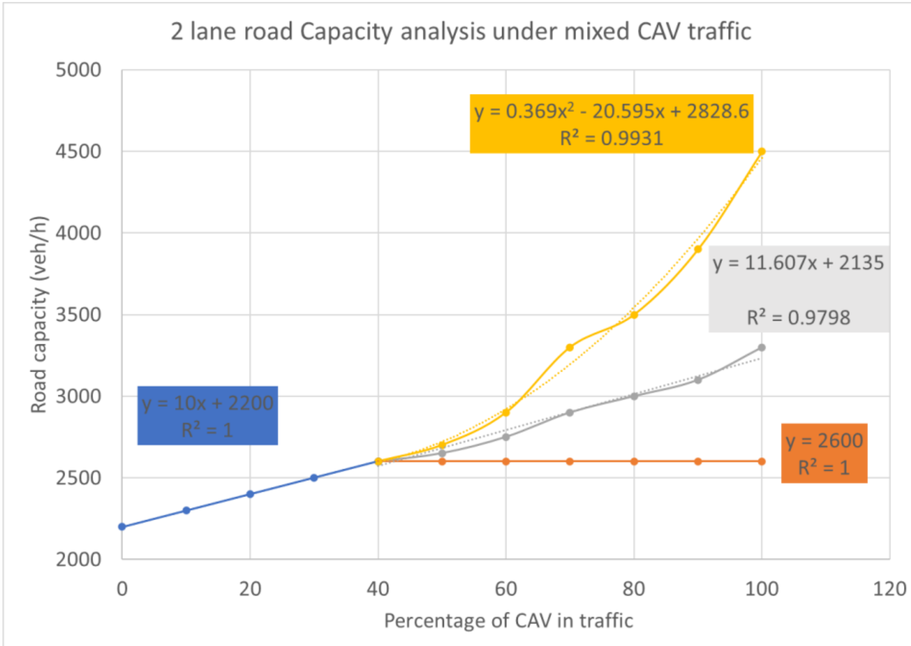


Figure 27 lane road capacity analysis with CAV mixed traffic

The BPR function is one of the generally used function for modelling influence of congestion on travel time and has been widely used and calibrated for different condition in both urban and rural roads [81]. When AV are considered authors have proposed multiple modifications, one of them is just the change of inputs while conserving the overall form. In this aspect the two main changes of the function are the fact that the flee flow time and capacity become a function of the percentage of CAV or AVS. As discussed before the capacity is clear function of this percentage, although the freeflow time is generally considered as constant with respect to this variable. Nevertheless, some authors [82], [83] point out that in fully autonomy free flow travel time could increase as intersection would not become a hard constrain of the velocity and vehicles could accept higher velocities. But in this work the accepted hypothesis will be the that just capacity would change and other parameters would remain constant as assumed in [84], where the assumed new BPR would become:

$$f(x) = t_f(1{+}\propto \left(\frac{X}{(C_{AV} - C_{NV}) * (\gamma) + C_{NV}}\right)^{\beta})$$

Where $X = total\ flow\ in\ link, \gamma = proportion\ of\ AV\ related\ to\ total\ flow$

$$C_{AV} = Capacity\ of\ road\ when\ all\ vehicles\ are\ AV$$

$$C_{NV} = Capacity\ of\ road\ when\ all\ vehicles\ are\ NV$$

In this modification the main assumption is that capacity of mixed traffic is a linear combination of both capacities which as seen before is not a bad assumption in the case of CAV, nevertheless in the case of AV if the technology is primitive and the AV driving characteristics stay low to keep safe distances then such assumption will not hold. Under such conditions a sensitivity analysis of the function shows that the maximum ranges of variation of such a function are defined, and are bounded by $1+\propto$ times the free flow time as both flows are bounded by the capacity. In terms of macro model or network models considering Autonomous Vehicles and parking behavior, is possible to check that the literature is still new. One of the first studies in the topic was given in [85] where the authors took into account a multi class traffic assignment such that all users follow the UE principle and the generalized cost of trip was to be minimized. This cost included the travel time cost plus fuel consumption cost and parking fee, the main differences between AV(Autonomous Vehicles) and NV (Normal vehicles) consisted in the fact that AV included routes where the vehicles first went to the destination and later parked while NV routes included the users parking and then walking to final destination. In here a tradeoff is made in AV since travel time can be decrease, parking costs can decrease but fuel costs can increase since vehicle needs to go farther away to park.

In this work the approach for AV and parking will include CAV which implies that capacity of network will vary in small ranges as initially the technology is expected to perform in similar fashion to NV. The parking behavior of NV will be assumed to have as final destination a parking node, while CAV will be supposed to drive users in UE to the original destination and then create a new trip from this node to any available parking while decreasing the transformation factor of travel time of links and following the SO. Also, hard capacity constraints of flows will be considered on all links and all users will be identified by their type and parking duration which will make them choose different parking places as the cost of parking is a function of parking duration and price of parking per time unit. Network capacity will be discussed which in this case will be different from the previous chapter by the fact that capacity becomes a function of the percentage of autonomous vehicles, nevertheless since such change make the feasible region nonlinear a simplification will made making the feasible region linear but taking into account the proportion of AV and NV.

5.2 Mathematical Formulation of Traffic Assignment with Autonomous Vehicles in Parking context

As in the chapter 3.1.1, the formulation for this problem is equivalent where the overall problem could be considered as a MUTAPC, but a part of the players will be AV that will make their objective function to be different and since the capacity is a function of the relationship of AV and total traffic then the capacity will be modified as well. The main differences between the models are listed below.

- Players are known and identified by two characteristics. The first one being their parking travel function and second one being whether they are AV, NV or CAV which will influence their objective function, since NV will follow UE but will be included in X_{NV}, AV will follow UE but will count as X_{AV} and CAV which will follow SO and will count as X_{AV}.
- The BPR of all links will be a function of the total flow AV and NV in each link, also the capacity of all links will be a linear function of the relationship between the number of AV over the total flow of the link. However, this would make the problem much harder since the feasible region of the overall problem would be nonlinear and probably non convex, this would imply that for a given link:

  $$X_{AV} + X_{NV} \leq \frac{(C_{AV} - C_{NV})(X_{AV})}{X_{AV} + X_{NV}} + C_{NV}$$

  Which is a clear nonlinear constrain, as solution the following modification will be made to keep the problem simple enough:

  $$C_{NV}X_{AV} + C_{AV}X_{NV} \leq (C_{AV} * C_{NV})$$

  This simplification makes the hard constrain to be linear in the link flow space and the feasible region would still be a polytope which would imply that it would be convex as long as it is feasible.
- NV will have as final destination a parking node implying that users need to walk to the actual final destination, while for the CAV two trips will be created for each real trip. The first trip would be normal UE trip although the flow would still count as AV and a second trip from the actual traffic destination node to the virtual parking node connecting all parking links.

The final formulation could be seen as a stacked Cournot game with 3 main type of players being NV AV and CV, and each of them with different players in terms of parking time.

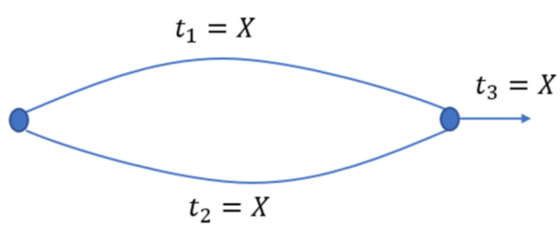
$$\min_{f_{NV}^v,} \sum_{i=0}^{n} \int_0^{x_i^v} TF_i^v \left(s; \sum_{V \neq v} x_{CAV}{}_i^v + \sum_{V \neq v} x_{AV}{}_i^v \right) ds \ (1) \ \forall\, v \ \in V_{NV}$$
$$\min_{f_{AV}^v,} \sum_{i=0}^{n} \int_0^{x_i^v} TF_i^v \left(\sum_{V \neq v} x_{NV}{}_i^v ; s + \sum_{V \neq v} x_{AV}{}_i^v \right) ds \ (1) \ \forall\, v \ \in V_{AV}$$
$$\min_{f_{CAV}^v,} \sum_{i=0}^{n} x_{CAV} * TF_i^v \left(\sum_{V \neq v} x_{NV}{}_i^v ; X_{CAV} + \sum_{V \neq v} x_{AV}{}_i^v \right) \ \forall\, v \ \in V_{CAV}$$
$$A_{NV}^v f_{NV}^v + A_{AV}^v f_{AV}^v + A_{CAV}^v f_{CAV}^v = X^v \ (2)$$
$$B_{NV}^v f_{NV}^v + B_{AV}^v f_{AV}^v + B_{CAV}^v f_{CAV}^v = D^v \ (2)$$
$$C_{AV}(A_{NV}^v f_{NV}^v) + C_{NV}(A_{AV}^v f_{AV}^v + A_{CAV}^v f_{CAV}^v) \leq (C_{AV} * C_{NV}) \ (4)$$
$$\sum_{v \in V} PT_v x_p^v \leq HCP$$
$$f_{NV}^v, f_{AV}^v, f_{CAV}^v \geq 0$$
$$\forall\, v \ \in V_{CAV}, V_{AV}, V_{NV}$$

Equation 20 SITAPC mathematical formulation for MIXED equilibrium

$$f^v = flow\ variables\ associated\ with\ player\ v\ for\ all\ used\ od\ pairs\ (Decision\ variable)$$
$$x_i^v = flow\ of\ link\ i\ with\ respect\ to\ player\ v$$
$$X^v = link - flow\ vector\ of\ player\ i$$
$$A^v = flow - link\ incidence\ matrix\ for\ player\ v$$
$$B^v = flow - OD\ vector\ matrix\ for\ player\ v$$
$$D^v = Demand\ vector\ for\ player\ v$$
$$TF_i^v = Cost\ function\ of\ link\ i\ for\ player\ v$$
$$PT_v = Parking\ time\ of\ player\ v$$
$$x_p^v = link\ flows\ of\ parking\ links\ for\ player\ v$$
$$HCP^v = Total\ parking\ available\ time\ vector\ (\#cells * simulation\ time)$$
$$V\ set\ of\ players$$
$$C_{NV} = Normal\ Capacity\ vector\ of\ all\ links$$
$$C_{AV} = Normal\ Capacity\ vector\ of\ all\ links\ assuming\ 100\%\ AV$$

In this formulation there would be a total of N players, each player would be constrained by the same feasible region. The input of the users could be summarized in a trip table containing the respective origin, demand, type of user, parking time of each users and trip. Then the table would be aggregated based on type of vehicle (AV,NV,CAV) and parking time which describes a player, and its OD matrix would be defined by aggregating the number of trips for each common OD pair of each user. In terms of uniqueness and existence of the solution it can be noted that since the feasible region is a polytope then if it is feasible the problem will be bounded. Nevertheless the solution would not be unique, since the problem corresponds to a nonsymmetrical cost functions given that the function of each link is equal to $F(X+Y)$ which in general means that $\frac{\partial F}{\partial x} \neq \frac{\partial F}{\partial y}$ and as mention in the chapter before if not such condition is met then the VI problem or the equivalent Cournot game would not have an unique solution.

In order to better understand the formulation, the same example as before will be made for the formulation but in this case all players will have a parking time of 10 minutes. In this case 3 players will be needed based on the aforementioned model, the first one being the NV, second one being the AV following UE until reaching node 2 and the third player being an extra player going from node 2 to node 3 (end of link 3) following the SO. In total there are 6 flows the first flows for NV players contain link 1 and link 3 and the second one contains link 2 and 3 flows. For the AV players the first flow contains just link 1 and the second flow contains link 2. For the CAV player the flow 1 contains link 3 which corresponds to a parking link.



$OD^1 = 10, OD^2 = 10; HC = [20,20]\ ; HCP = [600]; PT = [10,30]$

Figure 28 Example of Parking problem

$$Min \int (f_{1,pNV} + f_{1,pAV}) + \int (f_{2,pNV} + f_{2,pNV}) + \int (f_{1,pNV} + f_{2,pNV} + f_{1,pCAV} + 10)$$

$$st$$

$$f_{1,pNV} + f_{2,pNV} = 10$$

$$C_{AV}(f_{1,pNV}) + C_{NV} f_{1,pAV} \leq (C_{NV} * C_{AV})$$

$$C_{AV}(f_{2,pNV}) + C_{NV} f_{2,pAV} \leq (C_{NV} * C_{AV})$$

$$10 * (f_{1,pCAV} + f_{1,pNV} + f_{2,pNV}) \leq 600$$

$$Min \int (f_{1,pNV} + f_{1,pAV}) + \int (f_{2,pNV} + f_{2,pNV})$$

$$st$$
$$f_{1,pAV} + f_{2,pAV} = 10$$
$$C_{AV}\left(f_{1,pNV}\right) + C_{NV} f_{1,pAV} \leq (C_{NV} * C_{AV})$$
$$C_{AV}\left(f_{2,pNV}\right) + C_{NV} f_{2,pAV} \leq (C_{NV} * C_{AV})$$
$$Min\left(f_{1,pCAV}\right) * (f_{1,pNV} + f_{2,pNV} + f_{1,pCAV} + 10)$$
$$st$$
$$f_{1,pCAV} = 10$$
$$10 * (f_{1,pCAV} + f_{1,pNV} + f_{2,pNV}) \leq 600$$

Equation 21. Mathematical formulation of numerical example MUTAPC with AV

The problem does not have a unique solution and shows the complexity of the multiuser traffic assignment. Main drawback of the formulation is the linear increase in problem size with respect to number of od and players, given that at each iteration an LP is solved which has a non-linear run time with respect to problem size. Nevertheless, for small instances the problem can be solved using A variational Inequality solver as CPLEX which differs from previous approaches. The main differ comes from the fact that classical Vi formulation of the assignment problem work in path flow space, which cannot be enumerated in polynomial time, thus some heuristics are created to calculate feasible paths before solution time. The main problem with this approach is that limits the available paths for each user and therefore may make the problem infeasible while remaining capacity is still existent.

5.3 Solution Algorithm

As was the case with the basic formulation the solution algorithm for the classical formulation the solution algorithm consists of the solution of a set of independent TAPC problems such that in each iteration each player perspective of the network is modified based on the remaining capacity of traffic and parking links. Figure 29 shows the pseudocode of the algorithm, and the procedure is almost the same as in the case of NV. The only addition is the calculation of the new delay function which results from the replacement of the link numeric values of past iterations and if the player follows SO the replacement with the equivalent function which will solve the TAPC. The reason why the same TAPC problem (which solves for UE) can be used with an equivalent modified function is given by the fact that the solution of a normal TAPC problem with SO is equivalent to solving for UE with marginal cost. Thus, solving SO implies $F^{SO}(s) =$

$F(s;x,y) + s * \frac{\partial F(s;x,y)}{\partial s}$. A formal proof of this is given in [51] where the first order condition of the SO and UE formulation are compared and then it is proven that the SO is equivalent to for solve for UE with a modified delay function. Now the convergence of the overall algorithm is not guaranteed as cycling could occur. Therefore, this algorithm can be considered a heuristic.

Algorithm 5 Sequential Independent TAPC solution

Input:
$G = (V,N), X_0, tol$ V=Vertexes,N=Nodes
Output: x_k^v
Get initial feasible solution
Get total link flow and divide it by AV and NV $k = k+1$ $X_k = \sum_{v=1}^{V} x_k^v$ $X_k^{AV} = \sum_{v \in AV}^{V} x_k^v$ $X_k^{NV} = \sum_{v \in NV}^{V} x_k^v$
Calculate overall parked travel time vector $PX_k = \sum_{v=1}^{V} PT^v * x_k^v$
Calculate overall capacity factor vector $HTC = C_{AV} * (C_{NV} + C_{AV})$
for v=1,2....,V **do**
  Update travel link functions vector
  if $v \in AV$ **then**
    $F_k^v = F^v(X_k^{AV} - x_k^v; X_k^{NV}) + x_k^v * \frac{\partial F^v(X_k^{AV} - x_k^v; X_k^{NV})}{\partial x_k^v}$
  else
    $F_k^v = F^v(X_k^{AV}; X_k^{NV} - x_k^v)$
  end if
  Update Traffic link remaining capacity vector
  if $v \in AV$ **then**
    $C_k^v = \frac{HTC - C_{NV} * X_k^{AV} - C_{AV} * X_k^{NV} + C_{NV} * x_k^v}{C_{AV} + C_{NV}}$
  else
    $C_k^v = \frac{HTC - C_{NV} * X_k^{AV} - C_{AV} * X_k^{NV} + C_{AV} * x_k^v}{C_{NV} + C_{NV}}$
  end if
  Update Parking link remaining capacity $PC_k^v = \frac{HCP - PX_k + PT^v * xk}{PT^v}$
  Solve $x_k^v = TAPC(G, F_k^v, C_k^v, PC_k^v)$
end for
Calculate $X_{k+1} = \sum_{v=1}^{V} x_k^v$
if $||X_{k+1} - X_k|| <= tol$ **then return** x_k^v for all $v \in V$
else
  Return to line 3
end if

Figure 29 MUTAPC (Sequential Independent TAPC solution algorithm) with AV

In general terms the algorithm main iteration in a big network will be in the TAPC solution and the general complexity of the problem in the observed examples seems to be focused on the solution of each TAPC. A good initial feasible solution can greatly improve the convergence of the problem. As was the case before in order to have a good initial feasible solution the link-node formulation is proposed, in this case the formulation is described in Equation 22

$$\min_{x_i^v} \sum_{i=0}^{n} \sum_{v=0}^{V} x_i^v \, F_i^v(0;0) \ldots (1)$$
$$M^v x_i^v = OD_i^v \; \forall \, i, v \ldots (2)$$
$$\bar{X}_{AV} = \sum_{v=0}^{V} x_i^v \;\; \forall \, v \;\in v_{AV} (3)$$
$$\bar{X}_{NV} = \sum_{v=0}^{V} x_i^v \;\; \forall \, v \;\in v_{NV}$$
$$C_{NV} \bar{X}_{AV} + C_{AV} \bar{X}_{NV} \le (C_{AV} * C_{NV})$$
$$\sum_{v=0}^{V} PT^v x_{ip}^v \le HCP$$
$$x_i^v \ge 0$$

Equation 22. AV MUTAPC Link-Node formulation

In this formulation is important to notice that in order to guarantee the OD demand conservation a copy of all links is required for each player and for each O or for each D, the reason for this is that in the link-node formulation when multiple origin and destinations are present then if just the original network is considered then the OD matrix specific values can't be guaranteed but just the sum of rows and columns can be guaranteed. This means that although this will increase the complexity of the problem since more links will be required the OD matrix conservation constrains will be accomplished. One of the reasons why this LP can be so important is that any LP solver can tell if a given problem is feasible or not which is important given the hard capacity constraint on the links flows which can help to determine whether a given OD matrix pattern can be assigned. As before other advantage of the LP formulation is that the solution is well developed and highly optimized and I a basic feasible solution is present then the problem can be solved from there using classical simplex. This formulation takes another importance if the problem is solved using Fran-wolf algorithm with diagonalization where each subproblem is a multiplayer TAPC described by the LP, then the functions are updated based on the flow of other vehicles and the line search performed in terms of their own link flows.

5.4 Network Capacity Definition

It is expected that network capacity under the existence of autonomous vehicle will change, since their presence changes the feasible set by dividing the flow in a linear combination of the capacity given by Autonomous vehicles and normal drives. This in order to measure such change is important to stablish the possible upper bounds for this value. As explained before the network capacity under a given set of OD flows is defined as the maximum amount of flow that can be putted in these OD pairs in order to maximize the throughput of the network. The formulation of the problem can be defined in two ways, the first way is to define the capacity as a multi flow and sinks maximum flow problem. Under this formulation the problem can be seen as the solution of a set of one source and all targets connected to a unique sink. Also, other way to define the problem would be as the solution of a set of 1 source 1 sink maximum flow problems and after each solution iteration the left-over capacity of used links must be considered. This formulation can also be transformed to the maximum multicommodity flow problem which can be formulated as an LP and therefore has Polynomial time complexity [86]. The proposed LP formulation for this problem has the following description.

$$
\begin{gathered}
\max_{x_i^v} \sum x_{virtual}^v \\
M^v x_i^v = 0 \; \forall \; non\ used\ node, v \\
M^v x_{virtual}^v - M^v x_{virtual}^v = 0 \; \forall \; used\ sinks\ and\ nodes, v \\
\bar{X}_{AV} = \sum_{v=0}^{V} x_i^v \;\; \forall \; v \; \in v_{AV} \\
\bar{X}_{NV} = \sum_{v=0}^{V} x_i^v \;\; \forall \; v \; \in v_{NV} \\
C_{NV}\bar{X}_{AV} + C_{AV}\bar{X}_{NV} \leq (C_{AV} * C_{NV}) \\
\sum_{v=0}^{V} PT^v x_{ip}^v \leq HCP \\
x_i^v \geq 0
\end{gathered}
$$

Equation 23 Network capacity link-node formulation

In order to user this formulation is important to create virtual copies of all edges in the network for any non-zero origin id the OD matrix, which guarantee that the od pairs to be used are the ones which were constraint by the problem. Also, in the non-zero origins or destination a virtual link must be created which works as flow measurers and to maintain the constrains for used od pairs. This formulation has the advantage of taken advantage of linear programming as a well stablished subject, although it suffers from the problem that linear programs can have infinite solution when the gradient of the objective function is perpendicular to any of the constrains faces. In this case the feasible set or polytope is described by the adjacency matrix and linear combination of capacity constrains for parking and vehicular flows. So for example if the network has a cycle is possible to put maximum flow on this cycle and not altered the solution given that the flow conservation constrains allow it and the overall solution would not change. The following example shows the setup, formulation and problems.

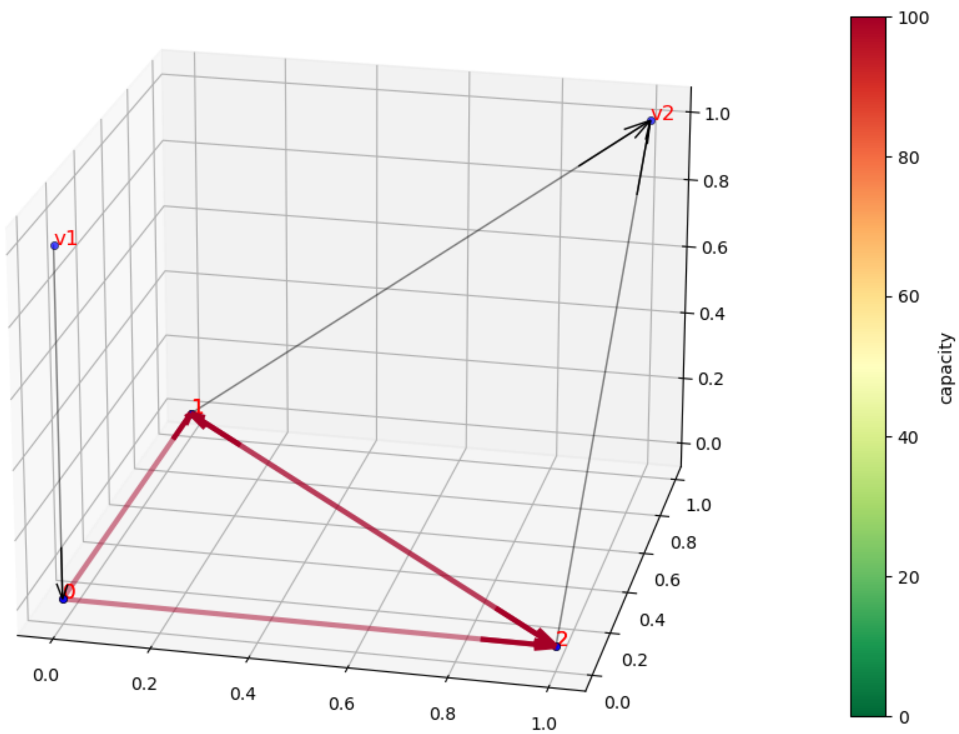


Figure 30 MUTAPC with AV Network capacity example

$$OD_{NV} = [0,1], OD_{AV} = [0,2]; number\ of\ players = 2; capacity\ NV = 100; capacity\ AV = 200$$
$$x_{ij}^{AV} = flow\ link\ i - j\ for\ AV\ player, x_{ij}^{NV} = flow\ link\ i - j\ for\ NV\ player$$
$$\max x_{v1,1}^{AV} + x_{v1,1}^{NV}$$
$$x_{v1,1}^{AV} - x_{2,v2}^{AV} - x_{2,v2}^{AV} = 0$$
$$x_{v1,1}^{NV} - x_{2,v2}^{NV} - x_{2,v2}^{NV} = 0$$
$$x_{v1,1}^{AV} - x_{0,1}^{AV} - x_{0,2}^{AV} = 0$$
$$x_{v1,1}^{NV} - x_{0,1}^{NV} - x_{0,2}^{NV} = 0$$
$$x_{0,1}^{AV} + x_{2,1}^{AV} - x_{1,2}^{AV} - x_{1,v2}^{AV} = 0$$
$$x_{0,2}^{NV} + x_{1,2}^{NV} - x_{2,1}^{NV} - x_{2,v2}^{NV} = 0$$
$$x_{0,1}^{AV} + x_{2,1}^{AV} - x_{1,2}^{AV} - x_{1,v2}^{AV} = 0$$
$$x_{0,1}^{NV} + x_{2,1}^{NV} - x_{1,2}^{NV} - x_{1,v2}^{NV} = 0$$
$$100x_{0,1}^{AV} + 200x_{0,1}^{NV} \leq 200 * 100$$
$$100x_{0,2}^{AV} + 200x_{0,2}^{NV} \leq 200 * 100$$
$$100x_{1,2}^{AV} + 200x_{1,2}^{NV} \leq 200 * 100$$
$$100x_{2,1}^{AV} + 200x_{2,1}^{NV} \leq 200 * 100$$
$$x_{i,j}^{AV}, x_{i,j}^{NV} \forall\ i,j$$

Equation 24 LP formulation for AV and NV example of network capacity

The solution for this problem is not unique as for example the solution of flows $x_{ij}^{AV} = [100,100,0,0], x_{ij}^{NV} = [0,0,0]$ or $x_{ij}^{AV} = [100,100,100,100], x_{ij}^{NV} = [0,0,0]$

The reason for this is that there is a cycle between nodes 1 and 2 so any unitary flow that is smaller than the minimum capacity in the cycle can be put in the cycle and the solution would still be the same as not extra flow is being added to the virtual nodes. The problem with this fact is that given a solution there are two main things that are important to have for analysis, the first one is the network capacity which is just the sum of the virtual nodes and is conserved for any feasible solution. The second one is the flow of edges when capacity is reached which since is not unique can have multiple values but the constrain links must be the same regardless of the solution, this means that at capacity the limiting links are the same although the distribution of flow might be different across the non-hard constrained links. Therefore, this link formulation of the problem can cause the problem of giving false flows of links which are just receiving a cyclic flow which is not relevant for the capacity analysis. In order to solve this problem, there are two main approaches, the first approach is using the first proposed formulation which solves the multi od pair capacity problem as a sequence of one source multiple sinks problem. In this scenario the solution algorithm for the problem is not Linear program but the classical ford-Fulkerson algorithm [87]. In this case because links share capacity across players a modification over the original algorithm is needed and will be described in the next subchapter.

5.5 Solution Algorithm for Network Capacity Problem

In order to solve the network capacity problem two main algorithms are proposed the first algorithm is based on the link-node formulation and uses linear programing for solving the problem. The advantage of this formulation is that is easy to understand, is fast since LP solvers are well developed and is flexible if more linear condition across links must be added. The algorithm consists of a processing stage which takes a base network and a player's table which contains information of thew type of players and od matrix. The network information must contain information of source, links and capacity for AV and capacity for NV. In this processing stage virtual links are added for each player and non-zero origin on its od matrix, also conditions of equality across the virtual sinks and sources edges for each player are created. After this the LP is formulated and solved and the network capacity is reported as the objective function value when the LP is solved. In order to get the actual link flows the problem is resolved with the values of the source and sink edges as constraints, but the objective function is changed to minimize the overall flow of all links while conserving the link-node matrix. This process gives the link flows obtained after the formulation. The second algorithm consists of the sequential maximum flow problem solutions, this algorithm consists of a basic update and solution stage that is repeated for every player and every nonzero row of the OD matrix of each player. The updating stage consist of calculating the remaining capacity of the network based on the past solutions based on the parking duration of each player and its type AV or NV. The order in which the players are solved is important, since each player has characteristics which affect the available capacity for posterior maximum flow solutions. Thus, the order of the players should be one such that maximizes the overall flow. In general, the order problem is hard since, it is not clear which players flow should be maximized first or whether a series of players should be maximized jointly. Nevertheless, for simplicity the heuristic used in the following algorithm assumes that players must be sorted based on their type and then on their parking duration. The pseudocode for this algorithm is shown in Figure 31

Algorithm 6 Network Capacity using sequential max flow

Input:
$G = (V,N), X_0, tol$ V=Vertexes,N=Nodes
Players table
Output: sol, x_k^v
Sort *Players* based on AV,NV and parking time
$x_k^v = 0 \forall v, k$
Calculate overall capacity factor vector for traffic link $HTC = (C_{NV} * C_{AV})$
Calculate overall capacity factor vector for parking links $HPC = cells * studyperiod$
Initiate $sol = empty$
for v=1,2....,V **do**
    Get OD matrix for each non zero origin O =nonzerovector of OD for player v
    $k = 0$
    for O in OD **do**
        Get total link flow and divide it by AV and NV
        $k = k+1$ $X_k = \sum_{v=1}^{V} x_k^v$ $X_k^{AV} = \sum_{v \in AV}^{V} x_k^v$ $X_k^{NV} = \sum_{v \in NV}^{V} x_k^v$
        Calculate overall parked travel time vector $PX_k = \sum_{v=1}^{V} PT^v * x_k^v$
        Update link remaining capacity
        if $v \in AV$ **then**
            $C_k^v = \frac{HTC - C_{NV} * X_k^{AV} - C_{AV} * X_k^{NV} + C_{NV} * x_k^v}{HTC}$
        else
            $C_k^v = \frac{HTC - C_{NV} * X_k^{AV} - C_{AV} * X_k^{NV} + C_{AV} * x_k^v}{HTC}$
        end if
        Update Parking link remaining capacity $PC_k^v = \frac{HPC - PX_k + PT^v * xk}{PT^v}$
        Create virtual links between destination an virtual sink
        max-flow, x_k^v =Solve max-flow problem from origin to sink node in updated network
        sol[v,k]=max-flow
    end for
end for

Figure 31. Pseudocode for Sequential flow solution of network capacity

The advantages of this algorithm are that it needs to solve K independent maximum flow problems which are efficient to solve and we are sure that no cyclic flows will be present since the maximum flow algorithm do not allow them because of their internal algorithm structure. Comparing both solution method is important to notice to use the theoretical algorithm complexities of LP and the desired solver for the maximum flow problem which in this case is the Ford Fulkerson algorithm. Using efficient interior point method of LP is known that their complexity is bounded by $O(n^{2+\frac{1}{6}}\log(\frac{n}{0.01}))$ [88] where n is the number of variables and L is the number of bytes. On the other hand Edmonds karp algorithm has a complexity of $O(V^2 * E)$ where V is the number of vertexes and E is the number of edges in the graph. Using the fact that the number of variables $n = kE$ and the fact that in transportation networks $V \approx \frac{E}{3}$, then in terms of the number of edges the LP algorithm would have a complexity of $O((kE)^{2.2}\log(\frac{kE}{0.01}))$, while using the proposed heuristic $O(k * \frac{E^3}{9})$, where k is the sum of the number of nonzero rows of the OD matrix of all players. This analysis allows to check that the problem with the LP problem is that it grows in more than quadratic terms with respect to k, thus for a big number of players or non-zero od matrixes the heuristic approach will be much better.

In terms of implications of the autonomous vehicles on the capacity is easy to check that the maximum factor of increase in capacity of the network is bounded by the capacity of the network if all flows are AV, which as discussed is in the ranges of 2 to 3 the original capacity of the network. The implication of the network analysis can help planner to better understand bottlenecks and possible earnings in capacity by the usage of spatial districts where all vehicle could be autonomous, which in normal case may be congested or constrain by highly congested links. Also, in general network capacity can be used as simple metrics which allow planners to better communicate the possible outcomes of infrastructure or policy planning, as travel times earnings may not be enough to describe the improvements of changes in the network. Also given that in general the network design problem is a hard problem since generally speaking the lower level is the solution of a nonlinear program then this approach becomes essential for simplification of the solution algorithms. This means that since the network capacity can be casted as a LP then if any network design problem is made in order to maximize the capacity of the network the subproblems to be solved would be much easier to solve.

5.6 Numerical Example

In order to better understand the described models and to check the implications in terms of travel time and capacity when autonomous vehicles are present a small example will be conducted which shows what happens with the network capacity and network travel times as Autonomous vehicles are added to a network. For this example, one OD pair will be used and drivers will be divided in AV and NV, also the parking duration of all players will be set equal to 20 minutes, the used network will be the same as in the past numeric examples and the capacity of all traffic links will be set to 1800 veh/h for NV and 3600 veh/h for AV. For the parking links all parking links will have a cell count of 10 cells and the study period will be 60 minutes. The BPR functions used will be the ones described in this chapter with $f(x) = t_f(1+\propto \left(\frac{X}{(C_{AV}-C_{NV})*(\gamma)+C_{NV}}\right)^{\beta})$. In order to check the implication for the autonomous vehicles 2 main parameters will be analyzed total demand in the od and the percentage of AV vehicles. The total demand will vary from 0 to 3000 vehicles. The results are shown in, where in terms of parking duration it becomes clear that overall network cost does not change drastically for any demand level or amount of AV and NV. These results are aligned with previous results from chapter 3, where the difference of UE and SO was discussed in TAPC. The reason why this happens is given by the upper bound of travel time that the BPR function has given the link flow upper bound. Thus, as these BPR are convex with respect to total link flow, then even for intermediate link flows between 0 and capacity the gains of optimal matching are expected to be low, this is especially true as the network gets closed to its capacity, as in this case the choice of users becomes very limited and therefore the possible cost saving becomes smaller.

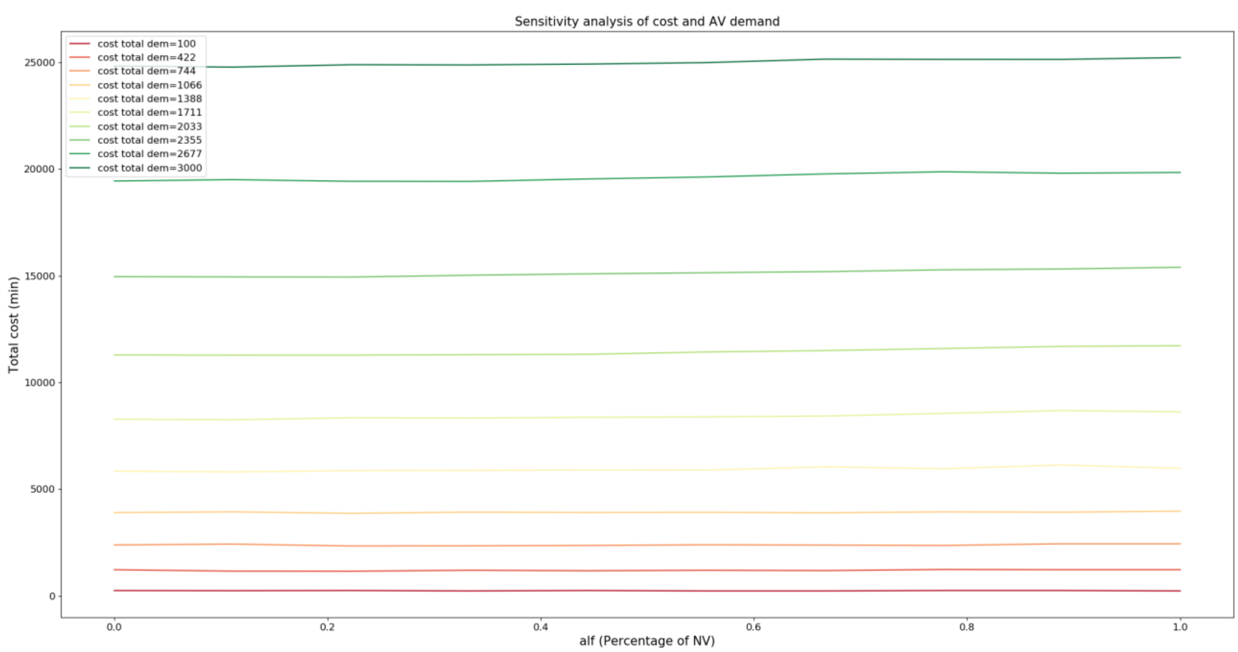


Figure 32. Sensitivity analysis of total network travel time with respect to AV and NV

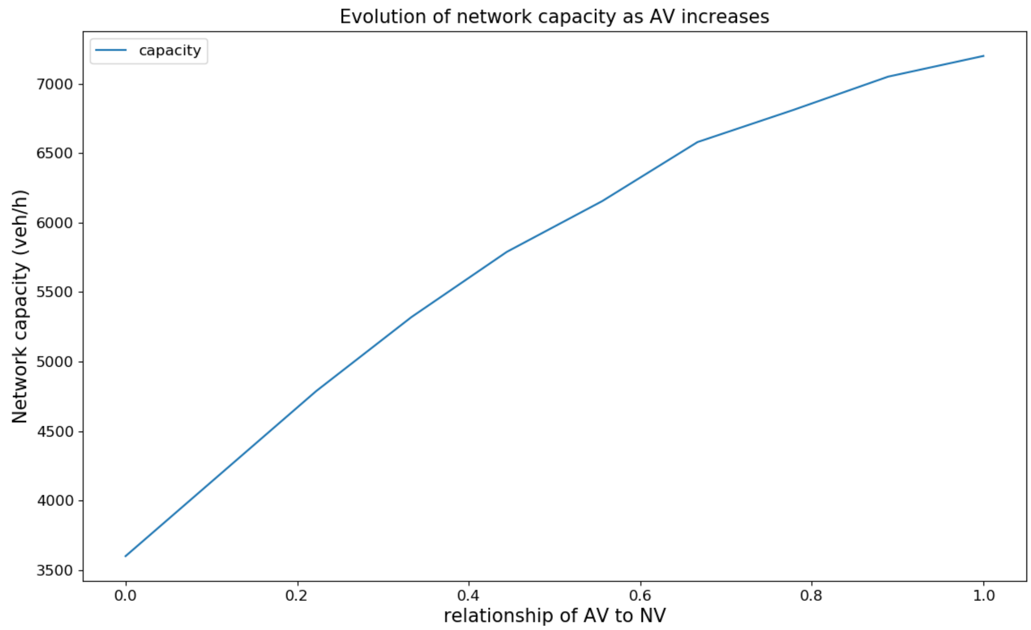


Figure 33 Network capacity evolution as AV percentage increases

Nevertheless, in terms of network capacity the results show a different behavior were the increased if AV have a significant impact on the overall network capacity. These results were expected as capacity has bigger room of improvement as Av are incorporated into the traffic streams. In this case the bottleneck links correspond to traffic links given that the used od pair is connected by only 2 vertexes whose capacity in the study period is equal to 1800 each. The overall behavior of the capacity curve seems to be nonlinear, and concave. The overall numerical results of this numerical example are similar to related results in the literature [89], [90], where a comparison of the incorporation of AV was made in terms of game theoretic formulation. From this numerical example it becomes important to notice that results are consistent with the expectations and last results, also it shows the potential power of the presented methodology which enables practitioner with tools to justify and predict what would happen as AV enter the market. In general, it should be expected that overall travel time would not be greatly impacted although network capacity would be a major change. Broadly speaking AV penetration in the market would have two main effects. The main one being the increase in network capacity and the second one being the reduction of network travel time which is given by the SO pattern of these vehicles. However, this result is based upon the fact that AV would behave as CAV when searching for parking which could not be true in reality so modification of this behavior should be taken carefully into account

5.7 Case Study in Bello City, Colombia

As was the case in last chapter a more profound implementation of the devolved model will be used in a real study case in order to understand the convergence properties of the model in more realistic scenarios and to analyze the possible effects of it on real analysis cases. Overall, the data used for this analysis is the same as described in chapter 4.6 and can be referenced in there. The main difference will be the presence of AV and CAV, which as described in this section will be based on the type of user their represent. The variation of all AVs will go from 0 to 1 and will include the original OD matrix and the variation as the AV proportion increases. A comparison of network capacity will also be made. In this case the capacity of the road when all vehicles are AV will be equal to 3600 veh/h, so in theory when all vehicles become AV the capacity of the network should double. The overall setup begins with a variation from demand going from 1000 vehicles to 13000 vehicles and for each demand size the overall demand is repeated based on the proportion of AV and NV. In total 25 scenarios were run in order to obtain the total cost and capacity of each run case. The results of the overall network are similar to the ones obtain in the simple numerical results, were the possible saving in system travel time are smaller than 10% of the UE solution.

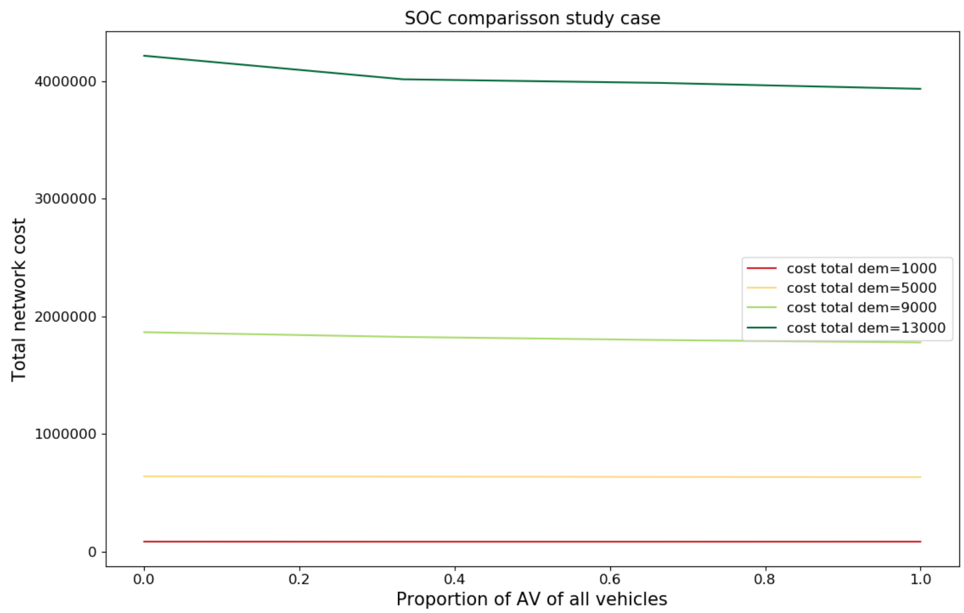


Figure 34. Study case comparison of AV ns NV

Although in general the difference among pure UE and SO solution becomes larger as demand increases. Is important to notice that the network capacity of network under the given od pattern demand is not 13000, but instead that is the maximum amount of demand until one of the od pairs can't handle more demand. This is due to the fact that demand is increasingly linearly and equally among all ODs, so once one OD reaches capacity no more increase on the overall demand can be made unless this od remains unmodified, which would make the results not comparable. On the other hand, the network capacity result shows a similar result than the numerical example, although the capacity seems to be reached at a lower level of AV penetration. These results indicate the potential improvement of capacity as Av enter the market, although the gain in travel time is small. One of the main reasons why this happens is that the BPR functions changes considered in this work are just related with an increase in capacity, although the main factor of these functions are the α, β parameters. Since these parameters define the upper bounds of travel time for each links based on capacity. In comparison the literature in the dynamic case report possible gains in function travel time much more favorable [85], [91].

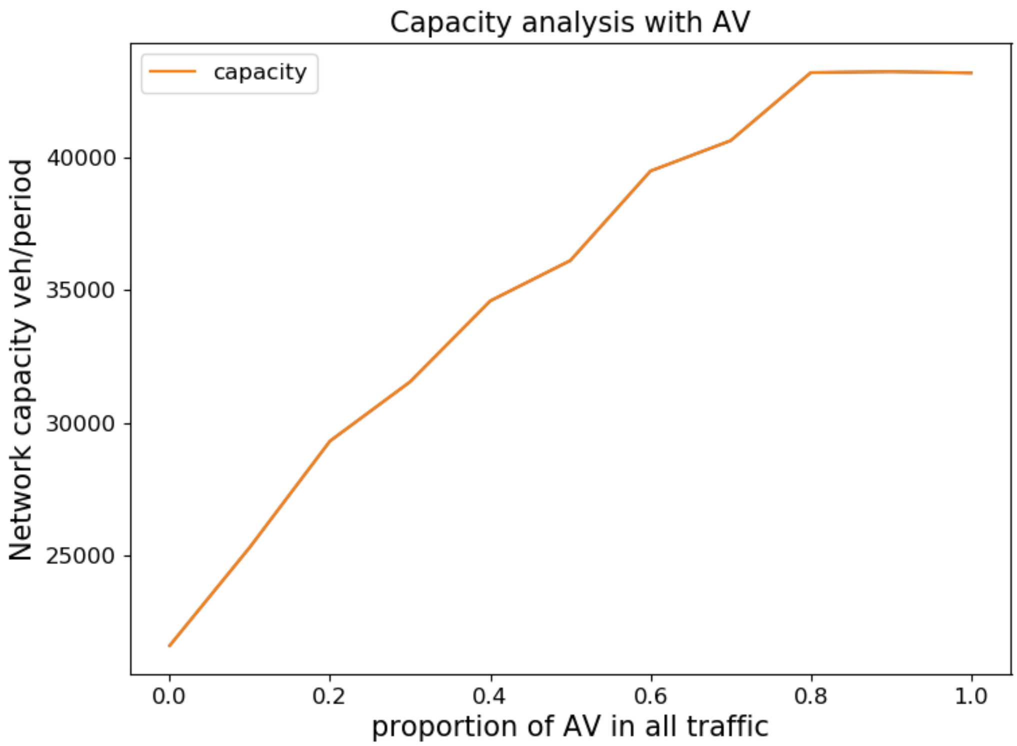


Figure 35. Network analysis for study case with AV

Overall thew results from this study case show the capacity of the model to capture capacity and importance of suers decisions among these characteristics. Overall, the proposed model shows that for the city the main gain with the implementation of AV rely on an increase on network capacity. Nevertheless, these results are given by the way which the BPR function was modified with respect to just NV.

5.8 Final Model Remarks

A multiuser model taking into account parking characteristics and autonomous vehicles was presented based on the developed multi user framework. The presented model present AV as a percentage of overall demand which has the c=characteristic of act as AV while driving to the final destination and CAV when search parking, which implies that vehicles follow the UE while driving and SO when doing parking search. The main change over the network characteristics considered by AV in this model is over link capacity, which may under estimate possible time saving as free flow times could be expected to decrease as well in the presence of AV. Nonetheless such changes could easily be implemented in the existing network if strong evidence supporting these changes on the cost delay functions are found. As was the case with MUTAPC a node link formulation of the problem is presented and associated algorithm developed. The network capacity definition remains the same, although a further exploration of alternative solution algorithms is explored by using multiple one maximum flow problems. The developed algorithm allows for a sensitivity analysis of percentage of AVs over network capacity, showing a nonlinear behavior between NV capacity and AV capacity. The numerical tests developed support this remark and showed a marginal change in overall system travel time as the number of AV increases in the network, this small difference has been also predicted in some more detail dynamic studied. Nevertheless, in this case the main reason for this small difference is the delay cost function modification which as discussed did not take into account changes in free flow time or alfa parameters which are the two main parameters in the BPR function. Overall, the results of this study open the door for more in-depth practical analysis of implementation schemes of AV and its direct impact of parking and traffic which can shape future policies such as charging parking, parking fees, among others which are in a critical point of discussing as society starts to adopt these technologies.

6. CONCLUSSION AND FUTURE WORK

6.1 Conclusions

The main objective of this work as described before was the development of a more precise methodology for modelling traffic and parking coexisting in the same scenarios under static assignment including efficient algorithms for its solution. The second main goal was the clear definition of the network capacity including a new mathematical formulation and solution algorithm. Both of these objectives were achieved by the present work and were carefully described and exemplified. The proposed model is a combination of hard capacity constraints, multiuser traffic assignment and parking and traffic modelling which as discussed have been worked extensively in the literature in an independent manner. Nevertheless, in this work a full integration of these characteristics is proposed which has the advantage of having a mathematical formulation based on game theory which could be used for theoretical results. Although at the same time the presented algorithms in this work allow the problem to be tractable in reasonable computation time based on Linear programs solved at each iteration. The presented model has the power of the inclusion of the parking duration and the multiplayer perspective which play a central role when modeling realistic scenarios. The first results made on chapter three pointed out to the importance of the hard capacity constrains in the problem which for example completely change the concept of price of Anarchy which is an important factor considered in traffic assignment and that has traditionally been used as a way of bounding the possible advantages of AV. Results from the study case and numerical example show that possible travel time reduction are small in the order of 10% and as network get closes to capacity difference among UE and SO becomes smaller as possible routing option shrinks. One of the most important contribution of this work comes from the link node formulation of the traffic assignment problem which although do not show a much faster algorithm convergence than classic traffic assignment, allows for an easier definition of the network capacity problem. Also, this formulation allows to have an initial feasible solution for the MUTAPC algorithm which is not trivial as the presence of capacity contains make hard to check feasibility. On the other hand, this formulation allows to redefine the network capacity problem as an LP which has enormous algorithm and theoretical importance. Considering practical implications, the present work allow planner to take more characteristic of users into account for example vehicle characteristics, perception of travel time cost, non-symmetrical cost function, among others. This can allow planners to have a holistic view when considering scenarios testing and have numerical realizations of their repercussion. Also, the network capacity definition expands the concept of link capacity which is highly important in traffic engineering to the network space which can be used as reference for explanation purposes. Also, in terms of network design classic approaches for the problem take into account minimization or reduction of travel time cost. This new formulation allows the network design to be formulated in easily manner which allow solution algorithm to work better, so new network design problem could be formulated in order to maximize network capacity.

The AV formulation presented in this work presented some limitations in modeling cases, as the cost delay function was modified by capacity only which results in small changes in network travel time that are conservative considering previous literature results. Nevertheless, the presented model has the advantage of taking into account parking characteristics, user perception and routing choice, and can be modified on arbitrary delay cost function which could more realistically express the time wins. The objective with the developed algorithm is to develop a python package and make it openly available which can have impacts on the scientific network as further improvements and applications of the model can be used.

6.2 Limitations

The presented model was design in the realm of static model which implies that during the study period all flow variables are considered to be uniform and constant. Generally speaking, this modelling scenarios are used in peak hours and some previous authors have stablish the necessity for further analysis on model performance to predict user behavior and the strong stationarity assumptions. Static assignment keeps being one of the most useful tools for traffic engineering, since it counts with a strong mathematical background and some important theoretical results. Also, the speed of solution algorithm, scalability and low input data requirements make it quite competitive with respect to more robust models as dynamic models or ABS. In this regard the presented model although take into account capacity constrains do not take into account spatiotemporal characteristics such as densities, so the selected study period should be uniform and short such that the required stationary assumptions hold. On the other hand, the presented model does not have theoretical guarantees of the existence of solution or convergence of the algorithm as the input delay cost function and constrain set has a more complex form that in the classic TAP formulation. This implies that the usage of this model must include sensitivity analysis and multiple runs which confirm convergence of algorithm and consistency among solutions. In regards to the AV modeling the present work focus the attention of AV on its capacity constraint changes and behavior model of vehicles. Thus, the network cost or travel time is not expected to change which remains on conservative side of AV, as some other studies point out to bigger benefits on its implementations. Nevertheless, there is not unified acceptance among researches on the possible changes that this technology will imply, so a conservative modification was used, although other ones could still be used under same formulation and solution algorithms.

6.3 Future Work

Future work must be focus on the validation of the model with real data and exploring the importance and usage of dynamic models in the evaluation of parking measurement with and without AV. One way of doing this simulation involves the usage of commercial software such as SUMO, VISSIM and TRANSCAD. These models are based on ABS or simulation, although they are not equilibrium based, so software such as DYNASMART. The overall objective of this comparison is to test at which point the hypothesis of the static model start to fail and how well the model predicts. An implementation of AV will be required and overall, this is still a hot research topic, as dynamic models that track AV and parking are still on the rise. A second extension of this work is the usage of the network capacity as input for network design problems which can have big implications in practical terms. As discussed, the main advantage of this formulation is the simplicity of the lower level as being described by an LP. On the practical side the usage of this type of network design has the advantage of being less limited by the estimated OD which players a huge role as real planning take place in long term planning periods which imply that precision of estimated future OD matrix will be low. But the network capacity problem allows planners to give estimates of how much total demand can the network serve and when combined with actual travel times could help to developed city wise network levels. This type of work has been in a sense been studied from the perspective of city-wide fundamental histograms, although that approach does not have a simple mathematical formulation as the proposed in this work which could imply easier calculations and better possibilities for network design.

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VITA

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