TWO-PHASE FLOW INSTABILITY INDUCED BY FLASHING IN NATURAL CIRCULATION SYSTEMS: AN ANALYTICAL APPROACH

by

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To my Grandmother, Sunheri Devi...

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LIST OF SYMBOLS

Latin

A_o, A_c, A_e	cross-sectional area at (A), (B), and (D)
c	mass concentration
C_k	kinematic wave velocity
$C_{k,fl}$	kinematic wave velocity for flashing
C_r^*	ratio of C_k at $z = \lambda$ and l_c
$C_{r,fl}^*$	ratio of $C_{k,fl}$ at $z = \lambda_{fl}$ and l_t
$C_{1,\ldots,7}$	function defined in Sec. 8.4
D_o, D, D_e	hydraulic diameters at (A), (B), and (D)
D_c	diffusion coefficient
$E(z)$	defined by Eq. 6.62
$E_{fl}(z)$	defined by Eq. 6.129
f_m, f_{me}	mixture friction factor in (C) and (D)
f_σ	body force
f_o, f_s	liquid friction factor in (A) and (B)
$f(z)$	heat flux profile/shape function
$f_{fl}(z)$	flashing profile/shape function
$F(z)$	integral of $f(z)$
$F_{fl}(z)$	integral of $f_{fl}(z)$
$\mathfrak{F}_{1,\ldots,9}$	constitutive relationships
g_o, g, g_e	axial gravitational forces in (A), (B), and (D)
$g(\eta, s)$	function defined by Eq. 6.22
$H(z, s)$	function defined by Eq. 6.66
$H_{fl}(z, s)$	function defined by Eq. 6.133
i	enthalapy
Δi_{12}	inlet subcooling
Δi_{fg}	latent heat
j	volumetric flux

k_i, k_e	inlet and exit restriction coefficients
l_o, l_c, l_t	length of inlet, core, and core + chimney channels
I^*	defined by Eq. 8.73
N_{pch}	phase change number
N_d	drift number
N_ρ	density ratio
N_f	friction number
N_{Fr}, N_{Fre}	Froude number
N_{Re}	Reynolds number
N_{sub}	subcooling number
P	pressure
P_{local}	local pressure
P_{sys}	system pressure
ΔP_{ij}	pressure drop from i to j
$\Delta P_{A,\ldots,E}$	pressure drop in region (A), ..., (E)
$Q(s)$	characteristic function
$q_w(z)$	wall heat flux
q_0	average heat flux
$s = a + j\omega$	perturbation variables ($j = \sqrt{-1}$)
a	amplification factor
ω	frequency
T	temperature
t	time
u	velocity
u_r	relative velocity
u_{fi}	velocity at inlet of heated region
u_{gm}, u_{fm}	diffusion velocity of each phase
Δv_{fg}	difference of specific volume of phases
z	axial coordinate

$Z^*(s^*)$	shifted characteristic functions

<u>Greek</u>

α	vapor void fraction
Γ_g, Γ_f	mass generation for each phase
$\Gamma_{g,fl}$	gas generation due to flashing
δu	inlet perturbation
ϵ	perturbation magnitude
θ	angle of chimney with vertical
λ	heated region non-boiling length
$\Lambda_{1,\ldots,24}$	various transfer functions
$\Lambda_{A,\ldots,E}$	transfer functions for various regions
μ	viscosity
ξ	heated perimeter
ρ	density
$\Delta\rho$	density difference between two phases
σ	surface tension
τ_{ij}	residence time between i and j
ϕ_{ce}	compressibility effect on enthalapy change
ϕ_{de}	dissipation effect on enthalapy change
ϕ_k	viscous dissipation on enthalapy change
Ω	local reaction frequency
Ω_0	average reaction frequency
Ω_∞	equivalent reaction frequency in the mixture
Ω_{fl}	reaction frequency for flashing

Subscript

e	exit
f	liquid
fo	liquid in (A)
g	vapor
i	inlet
m	mixture
me	mixture in (D)
s	saturation
12	region (B)
23	region (C)
34	region (D)
45	region (E)

Symbols

$*$	dimensionless
$\frac{D_g}{Dt}$	$\frac{\partial}{\partial t} + u_g \frac{\partial}{\partial z}$
$\frac{D_f}{Dt}$	$\frac{\partial}{\partial t} + u_f \frac{\partial}{\partial z}$
δG	perturbed part of variable G
$\overline{G}$	steady state part of variable G
(A)	upstream un-heated region
(B)	liquid heated region
(C)	two-phase heated region
(D)	two-phase un-heated region
(E)	two-phase region after flashing

ABBREVIATIONS

BWR	Boiling Water Reactor
DWO	Density Wave Oscillations
MSLWR	Multi-application Small Light Water Reactor
PWR	Pressurized Water Reactor
RPV	Reactor Pressure Vessel
SMR	Small Modular Reactor
TRSL	Thermal-Hydraulics and Reactor Safety Laboratory

ABSTRACT

Many two-phase flow systems might undergo flow instabilities even if the system is adiabatic but operates near the saturation conditions, especially in vertical flow conditions. Such instabilities are caused by *flashing* of the fluid in flow. Flashing is a sudden phase change in the fluid caused when local saturation enthalpy falls below the fluid enthalpy and the excess energy is used as latent heat for gas generation.

In the current analysis, a mathematical model is presented for analysis of such instability analytically. The conservation equations have been obtained by statistical averaging in time and space. Then, the concerned system is divided into various regions based on flow conditions, and these averaged equations are used to describe the flow. For flashing-based instability, two parameters are derived from constitutive relationships for the fluid. These two parameters are *Flashing Boundary*, λ_{fl} and *Gas Generation due to Flashing*, $\Gamma_{g,fl}$. These parameters provide for the closure of the mathematical model. Some simple models for flashing have been developed and discussed.

The mathematical model is then solved analytically for *Uniform Heat* and *Flat Model* for the heater and flashing region respectively. The solution is in terms of the characteristic equation which is used to predict the onset of instability caused by flashing. The results are then plotted on the Subcooling-Phase Change number plane. It is observed that inlet and outlet restrictions in the flow does **not** affect the onset of flashing induced instability as the flow rate is coupled with the pressure drop of the system. This is important as these restrictions play a major role in other two-phase flow instabilities such as *Density Wave Oscillations*.

Finally, the stability boundary in the stability plane is compared to experimental data present for flashing. The comparison was made with data of S. Shi, A. Dixit, and F. Inada. The stability boundary satisfactorily agrees with the experimental data thus corroborating the present mathematical model and analysis.

1. INTRODUCTION

1.1 Relevance of the problem

Operational and safety problems in nuclear power plants and some chemical process units and evaporators can arise due to thermally induced flow instabilities. These so-called flow instabilities are generally referred to as irregularities in the flow of a fluid. This fluid can be a coolant for nuclear power plants and evaporators or a chemical mixture in chemical process units.

These flow instabilities can be categorized broadly based on the dynamics of the problem into *static* and *dynamic* flow instabilities [1]. Static flow instabilities can be predicted by using basic conservation laws whereas dynamic flow instabilities need a description of various dynamic effects of the system as a whole such as propagation time, compressibility, etc. [2].

Flashing induced instability is one such instability that is categorized under dynamic flow instabilities. Flashing occurs when superheat of a fluid in flow, suddenly and violently, causes it to undergo phase change. Such types of instabilities are more common in low pressure, gravity dominant flows. Thus making modern natural circulation nuclear power units more vulnerable to such failures.

1.2 Objectives

The current work aims towards providing the analytical results for predicting the onset of Flashing Induced Instability in gravity-dominated flows. The particular objectives of this research investigation are the following:

1. Discuss the relevant two-phase flow field equations and necessary constitutive relations.

2. Obtain correct and important governing equations of the system. Furthermore, a simple derivation for the flashing boundary and gas generation due to flashing.

3. Develop flashing models.

4. Derive the response functions of various parameters to inlet flow perturbation.

5. Derive a characteristic equation that describes the dynamic response of the function.

6. Numerical solution of the first-order terms for dynamic instability boundary.

7. To present the theoretical predictions of the flashing instability in an appropriate parametric domain and to corroborate them with experimental data.

1.3 Outline of the thesis

It is well known about the several types of instabilities and the physics of the mechanisms involved have been theorized by different scientists and peers in the scientific community. This thesis is an attempt to describe instability induced by flashing in terms of mathematics as laid down by N. Zuber [3] in the 1960s in terms of the Drift Flux model. *This model is still used in numerous two-phase fluid problems where an analytical solution is required.*

It is advantageous to describe here the brief outline and flow of the thesis for a better understanding of the material present herein.

In chapter 2, the historical studies of instabilities are discussed and the discovery of flashing has been touched upon. Also, contemporary studies about the phenomenon have been discussed and works of peers have been mentioned where they tried to solve this instability with the help of computer codes.

Chapter 3 discusses experimental studies where we found flashing instability in tandem with density wave oscillations. This chapter describes the experimental setup, the model, and the prototype specifications and discusses in detail several experimental results.

Chapter 4 to 7 describe the basics of two-phase flows, the physics of the flashing induced instability, and the mathematics of the relevant problem. Chapter 4 discusses the basics of two-phase flow, time and area averaging in such flows, and basics of drift flux model. Chapter 5 develops the model structure and solution strategy for the relevant problem. Chapter 6 and 7 discusses the Kinematics and Dynamics of the system respectively.

Chapter 8 and 9 are dedicated to stability analysis of the system, the solution of the characteristic equation, and comparison of the findings with the contemporary data.

2. STATE OF ART

2.1 Mechanism of Instabilities

Thermally-induced flow instabilities can be categorized into *excursive instability* and *oscillatory instabilities* due to propagation phenomena.

Excursive instabilities were first analyzed in 1938 by Ledinegg [4]. For linear heat-flux cases it is shown that under certain conditions, the steady-state system pressure drop vs. flow curve has a negative slope. Hence, as the flow rate is not a linear function of the pressure drop and is also dependent on other system parameters, a flow excursion may occur. Later many researchers [5], [6] extended Ledinegg's analysis to non-uniform heat flux cases.

The oscillatory type of instabilities can be subdivided into the following prominent sub-categories[2]:

1. Pressure Drop Oscillations
2. Density Wave Oscillations
3. Flashing Induced Instability
4. Thermal Oscillations
5. Geysering

Most of the oscillatory types of instabilities encountered in heated systems are low frequency. The frequency of the oscillations is related to the residence time of the particle in channel [7]. Thus, various analysis has been formulated considering the propagation of kinematic wave and time-lag effects.

Although Flashing Instability is not certainly periodic but is considered oscillatory in nature. The name is derived from the fact that such instabilities are caused by sudden vaporization of the liquid phase with a resultant rapid increase in the specific volume of the mixture. It was first observed at the SPERT I-A reactor at the Space Technology Laboratories [8]. Jeglic [9] observed, in experiments conducted with water at very low pressures flowing through a smooth pipe, a flow instability which was described to be due to thermodynamic non-equilibrium. Under such conditions, fluid tends to become superheated due to the absence of nucleation sites. After nucleation, the void grows violently increasing pressure and pushing the liquid out of the pipe. Once the liquid is ejected, pressure decreases, new fluid enters, and the process is repeated.

This phenomenon is mostly described in natural circulation systems in which a chimney is placed downstream of the boiler. As explained by Furuya [10], the process of this Instability is as follows:

a) The heated fluid in the heater section reaches the chimney.

b) The fluid starts boiling where the bulk temperature is higher than the local saturation temperature of the fluid.

c) A further decrease in static pressure promotes further phase change.

d) Increased vapor volume makes the circulation rate faster which in turn makes the vapor in the chimney to be ejected to steam-dome and colder liquid to move to the chimney.

e) After the chimney is filled with this colder liquid, the flow rate decreases and the process restarts.

As described above, the period of this instability is in agreement to residence time of the fluid in the unheated (chimney) region. For the same reason, it is sometimes considered as a *density wave* phenomenon.

2.2 History on Analytical Work

Numerous analytical studies have been performed directed at obtaining a better understanding of thermally induced flow oscillations, developing stability criteria, and determining the mechanisms. The first such analysis was performed by Teletov and Serov [11] in the 1950s. This work entailed obtaining transfer functions for oscillations in heat flux using homogeneous flow model. This analysis became the basis of similar works by N. Zuber

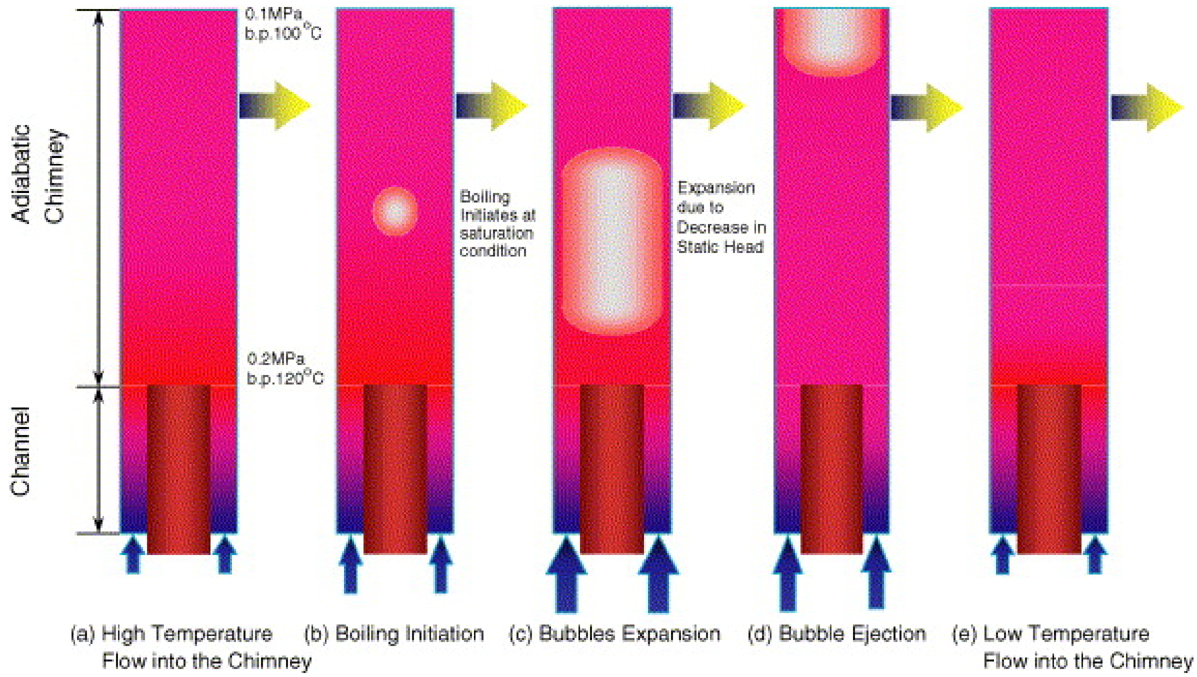


Figure 2.1. Mechanism of Flashing, [10].

[3]. Zuber extended the work of Serov by including slip velocity between the phases and thermal non-equilibrium followed by the development of *Drift-Flux Model* which is used in the present formulation. Following the same formulation structure Ishii [12], [7] extended Zuber's work to include effects of downstream flow on flow instabilities. This formulation also included the development of a methodology to solve the characteristic equations into meaningful results in the Sub-cooling and Phase Change number plane. This was the first solution of its kind and needed the help of early computers for a complete solution.

On the other hand, the problem of flashing was analyzed for rocket engine combustion instabilities. Many early studies which try to describe flashing regarded it as an instability which is caused when the fuel tank is smooth, thus lacking nucleation sites [8], [13]. It was in the 1990s when Furuya [10] systematically described the physics of flashing induced instability for vertical flows of water. He conducted several experiments and described the behavior of flashing with the change of system parameters. Finally, he develops an analytical solution strategy to describe the instability on Sub-cooling and Heater Power plane. The downside of this analysis was that it did not span the whole stability plane *(Sub-cooling and*

Phase Change Number plane). Kuran [14] in the early 2000s studied flashing-based instabilities for their effects on coupled flow/power behavior in Boiling Water Reactors (BWRs). He developed some very important techniques which served as a skeleton for the solution strategy used in the present work. During the same time, Hu [15], Dixit [16], and Manera [17] studied the effect of flashing on start-up characteristics of BWRs. Although Manera's work was experimental, it provided insights on the important characteristics to focus on. She argued that the location of flashing boundary has little to no effect on the occurrence of flashing-based instability. This conclusion is used as one of the bases for the development of *Flashing Models* in current work. The downside of this study is its lack of physical description and analytical study supplementing the experimental data. Also, the frequency of instability is described to be related to the residence time of fluid in the adiabatic chimney. *It can be shown by mathematical analysis that the frequency is tied to the residence time of the gas phase in the chimney section.*

Most recent works on Flashing have been conducted by Shi [18] and Zhang [19]. The first work deals with the onset of flashing when there is bulk boiling in the heater region and the voids facilitate the process of gas generation in the chimney region. This work tries to answer the flashing phenomenon tied to fluid in two-phase and cannot hold superheat. The second work deals with flashing-based instability in the Homogeneous Equilibrium Model. Because this model assumes the two phases are homogeneous, it carries the assumption of constant enthalpy for adiabatic regions of the flow system, although in reality it is well known that gas-phase moves faster under buoyancy, thus taking away the excess heat from the fluid. This analysis is a good first step in the complete description of Flashing induced Instabilities in the two-phase flow of natural circulation systems.

3. EXPERIMENTAL STUDY

3.1 Description of the Prototype

The experimental facility is designed based on a typical PWR-type SMR reactor designed by NuScale. The design parameters for the NuScale reactor are not available to the public thus for the design of the facility, various design parameters were based on the design of the Multi-application Small Light Water Reactor (MSLWR) [20]. The NuScale schematic in Fig. 3.1 shows that the primary loop integrated with the Reactor Pressure Vessel (RPV) and the forced convection pumps have been eliminated in the loop thus, single-phase natural circulation drives the coolant. Compared to BWR-type SMRs, the PWR-type SMRs have

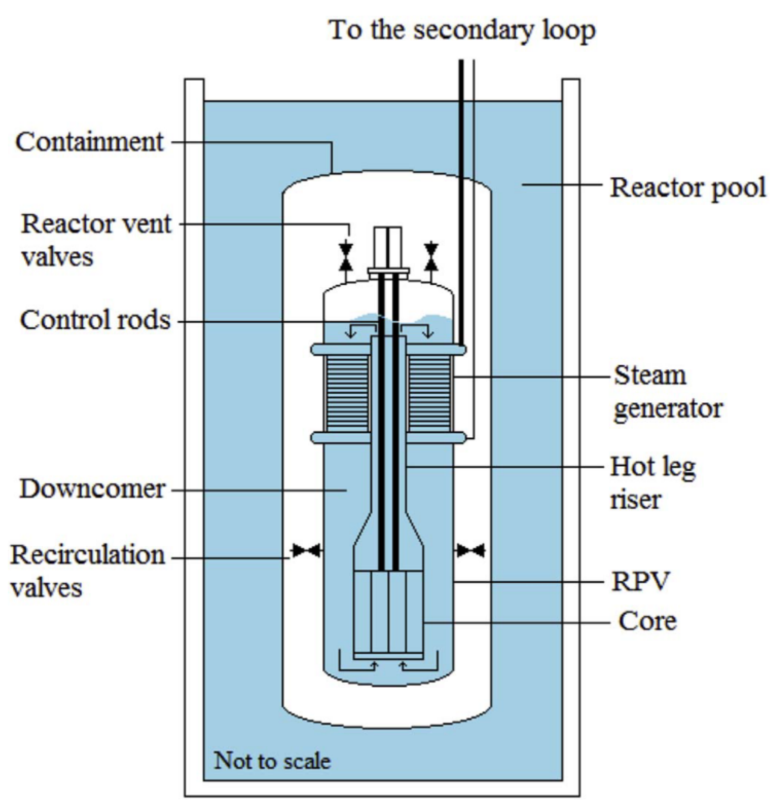


Figure 3.1. Schematic of NuScale module installed underwater. [21]

longer hot and cold legs because of less change in density through the riser. A helical coil steam generator is situated at the top of the RPV near the end of the riser. The coolant fluid then flows through the downcomer after it cools down at the steam generator. All the components are contained by a steel containment which is then submerged into a large reactor pool. This containment serves as a radiation shield and the water in the reactor pool is serves as a heat sink for decay heat in accident scenarios. At the top of the RPV, there are two vent valves to avoid excessive pressure build-up in the RPV. Additionally, two valves are also installed at the bottom of the RPV above the core section to allow coolant from the containment to flow to the reactor core. During normal operation, the space between RPV and containment is in a vacuum for thermal insulation.

3.2 Test Facility Scaling

The scaling laws derived by Ishii and Kataoka [22] for thermal-hydraulic systems operating in single-phase and two-phase natural circulation has been used to scale the experimental facility from the MSLWR specifications. For flow in a single phase, the similarity laws are based on one-dimensional Conservation Equations. For flow in two-phase, a one-dimensional perturbation analysis based on the drift flux model is used to derive the similarity groups. Refer to table 3.1 for a detailed summary of the similarity groups used in the modeling. As per the scaling theory, the non-dimensional similarity groups should be the same for prototype and experimental facility to ensure the similarity of the simulated key phenomenon. As the two-phase phenomenon is more critical for the analysis, the two-phase scaling criteria are applied first. (Also, these criteria are more restrictive than those for single-phase). The fluids used in the prototype and the model are the same and the pressure and temperature scales are the same meaning that the fluid properties are identical between prototype and model. The general scaling ratio is given as per Eq 3.1. The scaling criteria should be as per Eq. 3.2.

$$(\psi)_R = \frac{\psi_m}{\psi_p} = \frac{\psi \text{ in model}}{\psi \text{ in prototype}} \tag{3.1}$$

$$(\psi_{Pch})_R = (\psi_{Fr})_R = (\psi_{th})_R = (\psi_d)_R = (\psi_{sub})_R = 1 \tag{3.2}$$

Table 3.1. Some important similarity groups used for scaling the experimental facility.

Single Phase Flow		**Two Phase Flow**	
Heat Source Number	$Q = \dfrac{q_s'' l_0}{\rho_s c_p u_0 \Delta T_0}$	Drift Number	$N_d = \dfrac{u_{gj}}{u_0}$
Biot Number	$Bi = \dfrac{\bar{h}_\infty \delta_s}{k_s}$	Froude Number	$N_{Fr} = \dfrac{u_0^2}{g l_0 \alpha_0} \dfrac{\rho_f}{\Delta\rho}$
Friction Factor	$\mathrm{F} = \left(\dfrac{f l_0}{D} + K \right)$	Thermal Inertia Number	$N_{th} = \dfrac{\rho_s c_{ps} A_s}{\rho_f c_{pf} A_0}$
Richardson Number	$R = \dfrac{g \beta \Delta T_0}{u_0^2}$	Subcooling Number	$N_{sub} = \dfrac{i_{sub}}{i_{fg}} \dfrac{\Delta\rho}{\rho_g}$
Stanton Number	$St = \dfrac{\bar{h}_\infty \xi_h l_0}{\rho_f c_{pf} A_0 u_0}$	Phase Change Number	$N_{pch} = \dfrac{Q}{m_0 i_{fg}} \dfrac{\Delta\rho}{\rho_g}$

Thus, the dimensional requirements for the similarity is obtained by solving Eq. 3.2. The designing of the test facility is divided into two crucial steps. First the test facility is designed as per the *ideally scaled facility* (ISF) based on the designated space and size for the model without taking into consideration the engineering constraints. The second step is to design the *engineering scaled facility* (ESF) taking into account the practical limitation and cost. The ISF scaling rations are mentioned in the Table 3.2. The ESF is designed taking into consideration the availability of commercially available materials thus bringing in some disparity within the scaling ratios for ISF and ESF. Table 3.3 shows the comparison between the two. Other than the scaling ratios listed some finer details needs to be scaled separately to correctly model local phenomenon such as choking, flashing etc. The verification of this scaling analysis is performed by RELAP5 code simulation which is reported by G. Wang [23]. He verifies the steady and transient state response of the ISF with MASLWR and then compares the responses of blowdown event for ISF and ESF. More details of scaling of this facility is reported by Yan et al. [24]

3.3 Test Facility and Instrumentation

Test loop schematic is shown in Fig. 3.2. The facility has 3 major components: 1) primary loop, 2) containment, and 3) auxiliary loop. The primary loop core consists of 6 heaters arranged in a hexagonal manner with a cumulative maximum power of 20 kW. The

Table 3.2. Scaling ratios for ISF [23]

Parameters	**Ratio** $\left(\psi_{ISF}/\psi_{MASLWR}\right)$
Length	1:4
Volume	1:400
Area	1:100
Velocity	1:2
Mass Flow Rate	1:200
Power	1:200
Hydraulic Diameter	1:$\sqrt{2}$
Time	1:2
Pressure	1:1
Fluid Physical Properties	1:1

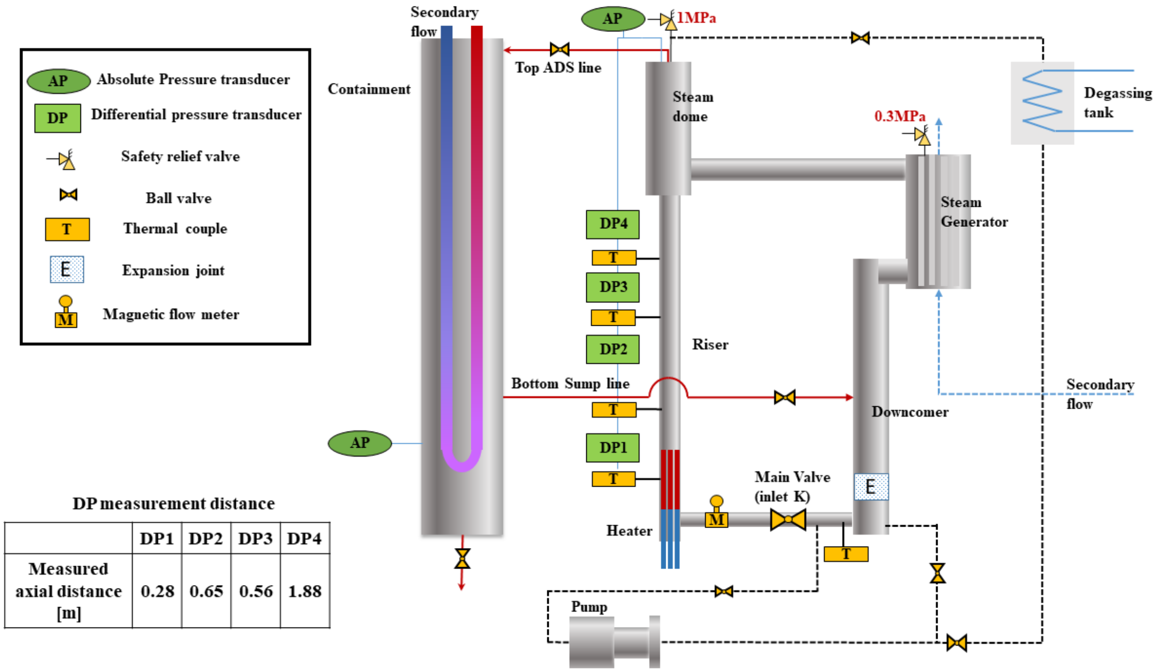
steam generator (SG) is a straight-tubes-type counter-current heat exchanger with secondary flow passing through the tube's side. The containment in this study is used as a large compressible volume. The outer walls are insulated to prevent large heat convection to the environment. However, the insulation cannot be perfect and hence there will still be some heat transfer. The containment is connected to the primary loop with an ADS ball valve. This ADS ball valve is used to simulate Loss of Coolant Accidents (LOCA). The opening of the valve can be controlled to a calculated K-factor to simulate a small range of break sizes encountered during LOCA. In comparison to MALSWR, this test facility is has been simplified in design. The riser is surrounded by an annulus downcomer in MALSWR but in ESF the downcomer is separate and is simply a straight pipe connecting to the bottom of the core. The SG is also simplified as described above rather than a helical tube in MALSWR. Although the heat transfer coefficient in the SG will be different for prototype and ESF the overall effect should not be affected as the phenomenon observed in this formulation doesn't depend on the local heat transfer in the SG. It depends on the total heat taken out from the primary fluid by the secondary flow through SG which should approximately equal to the heat input through the heater minus the energy lost with the primary fluid lost through LOCA. It is necessary to note that the secondary side of the SG is consistent with MALSWR as this is important for experiments involving decay heat removal and Natural Circulation. The auxiliary loop is used for setting up the experimental conditions and is not a part of

Table 3.3. Geometrical Design Parameters of the test facility [23]

Parameters	Specification	Test Facility	ISF Ratio	ESF Ratio
Core	Area(m^2)	0.0064	1:100	1:92.15
	Length(m)	0.33	1:4	1:4.09
RPV	Height(m)	3.44	1:4	1:3.95
	Volume(m^3)	0.126	1:400	1:417.78
Riser	Area(m^2)	0.0054	1:100	1:113.44
	Length(m)	2.01	1:4	1:4.06
Steam Generator	Area(m^2)	0.0564	1:100	1:37.76
	Length(m)	0.635	1:4	1:4.08
Downcomer	Area(m^2)	0.0021	1:100	1:117
	Length(m)	1.626	1:4	1:3.99
Pressurizer	Area(m^2)	0.0351	1:100	1:116.20
	Length(m)	0.483	1:4	1:3.94
Containment	Height(m)	3.61	1:4	1:4.90
	Volume(m^3)	0.350	1:400	1:381

the experiments itself. It is used to adjust various parameters and used in tandem with the pump. The degassing tank in the auxiliary loop is used to remove non-condensable gas initially dissolved in the primary coolant. The degassing process is initiated by heating the primary coolant and making it flow through the auxiliary line till all the gas is removed and, pressure and temperature at the top of the steamdome match the saturation line for the fluid.

The test facility houses various instrumentation including K-type thermocouples, magnetic flow meters, pressure transducers, and impedance void meters. The impedance void meters are produced in-house and are shown to have a relative error of less than 10% for void fractions between 0.2 to 0.4 and less than 5% for void fractions more than 0.4 [25]. The pressure transducers are ST3000 Smart Pressure Transmitters 100 series have an accuracy of 0.0375% of the range of measurement. The flow meter Honeywell MagneW 3000 Plus Smart has an accuracy of 0.5% of the range. The inaccuracy in the readings from the K-type TCs is less than 2.2°C. There are 4 measurement ports in the core and riser section of the experimental facility. Each measurement port measures the temperature, differential



	DP1	DP2	DP3	DP4
Measured axial distance [m]	0.28	0.65	0.56	1.88

Figure 3.2. Schematic of the Experimental Test Facility

pressure between the ports, and void fraction. Also, two absolute pressure transducers are installed in steamdome and containment to measure absolute pressure transient.

The following section is published as a research paper in ANS Winter Meeting-2019 [26] by the author of this thesis and his peers at TRSL.

3.4 Experimental Tests

The current experimental study comprises of two experiments: 1) small break to containment: k-factor of 80, and 2) miniature break to containment: k-factor of 310. For the first experiment, the heater power is set to 18kW, the steam dome is pressurized to 200kPa, and the ADS valve is opened to a k-factor of 80. For the second experiment, the heater power is set to 15kW, the steam dome is pressurized to 150kPa, and the ADS valve is opened to a k-factor of 310. For both experiments, when the system behaves as a quasi-steady state, the data is recorded. The valve at the bottom of the containment and the valve on the bottom sump line are always closed during the experiments.

3.4.1 Experiment 1

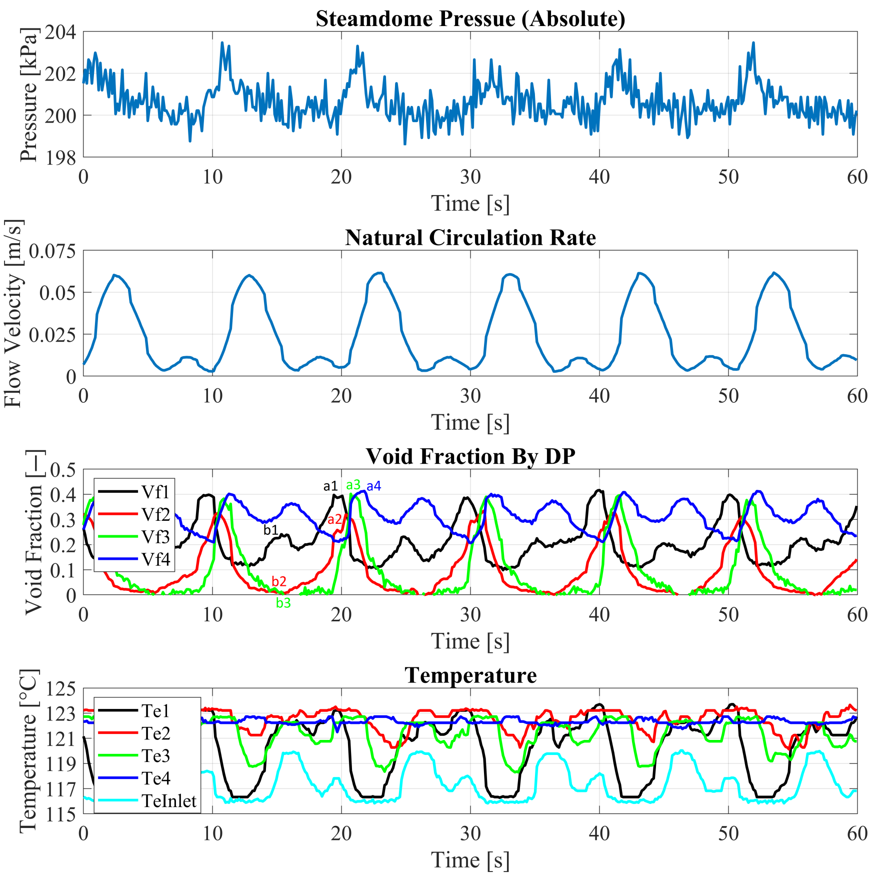


Figure 3.3. Instability of pressure, flow rate, void fraction, and temperature for k-factor of 80, [26]

Fig. 3.3 shows the results for 60 seconds for a k-factor of 80. From the natural circulation rate and the void fraction, two kinds of flashing processes can be observed which give rise to the peaks of different amplitudes on the natural circulation rate chart. The first kind: The higher amplitude peak is due to flashing which commences in the heater section and then the gas phase propagates through the riser section, which is demonstrated by the void fraction propagation from a1 to a4. The propagation velocity of the gas phase is higher (about 0.7 m/s) in comparison to the slower flow rate of the liquid phase (about 0.06 m/s at the peak value). The second kind: The smaller amplitude peak is due to flashing in the heater section, but the bubble collapses due to condensation after the heater section (b2 to b3). Thus, this only gives rise to a slight increase in flow rate. The steam dome pressure and the void fraction show that the first kind of flashing process will always come with an increase of the pressure (about 3 kPa). Thereafter, the pressure decreases because of the steam leakage through the break.

3.4.2 Experiment 2

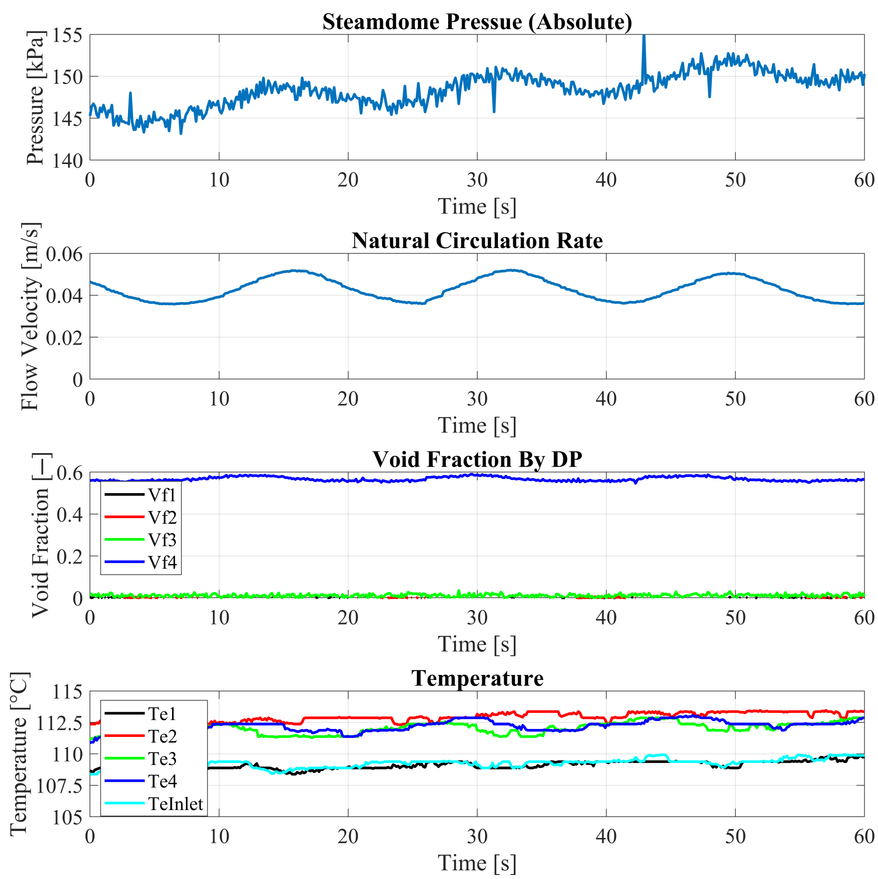


Figure 3.4. Instability of pressure, flow rate, void fraction, and temperature for k-factor of 310, [26]

Fig. 3.4 shows the results for 60 seconds for a k-factor of 310. The flow rate has only one kind of quasi-steady oscillation. The void fraction curves show little to no oscillations which indicates that this instability in the flow rate is not induced by flashing but density fluctuations. Vf4 displays a high mean value because the steam dome is partially filled with steam. The fluctuation in Vf4 indicates that the leakage of the steam through the break will lead to perturbation on the steam dome pressure and hence perturbation on the density difference, which drives the natural circulation rate. When the break k-factor is larger, the restriction on the steam leakage is larger compared to the small k-factor case. As a result, the pressure accumulation will be larger (about 5 kPa at increasing phase) but the pressure release will be smaller (about 3 kPa at decreasing phase). The net effect is that the flashing-induced instability is suppressed while the density wave oscillation is induced.

3.4.3 Conclusion

To investigate the effect of break size (k-factor) at the top of the steam dome with a large compressible volume of the containment on the flow instabilities, two experiments were performed. The results show a distinction between mechanisms of flow instabilities for two different break sizes. Flashing-induced instability occurs in small break cases. On the other hand, density wave oscillation occurs in the miniature break case. It is found that the restriction on the steam leakage by the break size will affect the occurrence of flashing in the heater section. Further studies are needed to investigate the influences due to the heater power and the system pressure on flow instabilities.

4. DYNAMICS OF TWO PHASE FLOW

4.1 Introduction

In continuum mechanics, all the conceptual models are based on conservation principles of mass, momentum, energy, etc. These conservation equations are then closed for a system by the use of constitutive relationships between physical quantities which describe thermodynamic, transport, or chemical properties and boundary conditions.

The same can be expected from the formulation used here. The conceptual model which describes the steady-state and dynamic characteristics of multi-phase media is formulated in the form of appropriate conservation principles and constitutive relations. It's worth noting that the formulation of a two-phase phenomenon is much more complicated than single-phase continuous media.

As described by Ishii [7], the primary difficulty in describing the above principles is the discontinuity between the phases and the presence of the interfacial area which provides a gateway of interactions between the phases. It is not difficult to imagine that for the same void fraction, there can be a range of total interfacial area in the flow. Only for this reason interfacial area and its concentration are very important parameters to measure. This physical separation of the continuum of one phase from another makes it mathematically difficult to define the boundary conditions for that phase. From the point of view of physics, the accuracy of the formulation depends on the structure of the flow.

To circumvent this difficulty, the local properties are described first and then morphed into useful macroscopic descriptions by the use of appropriate averaging procedures. The designing, operating conditions, performance, and safe operation of a large number of engineering systems depend on the availability of accurate and realistic governing equations.

As the present problem of flashing consists of two-phase flow in various regimes, thus it is best to assume the two phases as two continuous phases by the means of area averaging. This way is best suited for the analysis of thermally induced flow instability. Also, this approach can be directly used for determining the mixture properties and determination of similarity groups. *A complete description of the following equations is given by Ishii [7].*

4.2 Averaging

4.2.1 Time Averaging

As the two-phase flow discussed here has a different composition in space and time, it becomes important to consider that the governing conservation equations need to be averaged for time and space to be applicable in the analytical analysis of the system as a whole. Without these averaging the process to solve these conservation equations become taxing and impossible without a computer.

Two-phase flow can be classified into three classes, i.e., separated flows, transitional or mixed flows, and dispersed flows. For thermally induced flow instabilities, the above three classes usually coexist under turbulent flow conditions thus it is appropriate to use statistical averaging rather than formulating a model based on a specific flow class.

To average based on time, let us assume a time interval Δt such that it is sufficiently large to smooth out the local variation of properties but small enough to preserve the macroscopic change of fluid properties for the system. Assuming that the interface is not stationary and for a fixed point, there exist either of the two phases, it is easy to say that $\Delta t = \Delta t_f + \Delta t_g$. *Subscripts refer to liquid and gas phase.*

Now for a function $F = F(\vec{X}, t)$ the time averages will be given as the follows:

$$\overline{F}(\vec{X}, t) \equiv \frac{1}{\Delta t} \int_{\Delta t} F(\vec{X}, t) dt \tag{4.1}$$

For example, statistical gas concentration can be derived using $F(\vec{X}, t) = 0$ for liquid and $F(\vec{X}, t) = 1$ for gas, thus:

$$\alpha = \frac{\Delta t_g}{\Delta t} \tag{4.2}$$

For averaging of a fluid property for the mixture, from Eq. 4.1, we have:

$$\overline{F} = \frac{1}{\Delta t} \int_{\Delta t} F_g(\vec{X}, t) dt + \frac{1}{\Delta t} \int_{\Delta t} F_f(\vec{X}, t) dt = \overline{F}_g + \overline{F}_f \tag{4.3}$$

Now for an Intrinsic flow property (G) such as pressure or temperature, weighted mean value can be defined as:

$$\overline{\overline{G}}_k = \frac{1}{\Delta t_k} \int_{\Delta t_k} G_k dt = \frac{\overline{G}_k}{\alpha_k}, \qquad k = g, f \tag{4.4}$$

Thus it can be shown that:

$$\overline{G} = \overline{G}_f + \overline{G}_g = \alpha \overline{\overline{G}}_g + (1-\alpha)\overline{\overline{G}}_f \tag{4.5}$$

Similarly for Extrinsic properties (F), such that $F_k = \rho_k \psi_k$, where ψ_k is variable per unit mass. It is easy to show that:

$$\overline{\overline{\psi}} = \frac{\overline{\rho\psi}}{\overline{\rho}} = \frac{\Sigma \alpha_k \overline{\overline{\rho}}_k \overline{\overline{\psi}}_k}{\Sigma \alpha_k \overline{\overline{\rho}}_k} \tag{4.6}$$

Note that this formulation does not take into account the effect of the interfacial boundary between the two phases.

4.2.2 Area Averaging

For thermally induced instabilities in two-phase flow, it is easier to formulate the problem in 1-D where the characteristic length in the main flow direction is significantly larger than the one in its perpendicular direction. Let us define the area average of a function G by:

$$\langle G \rangle \equiv \frac{1}{A} \int_A G \, dA \tag{4.7}$$

Thus average of statistical gas concentration can be given as:

$$\langle \alpha \rangle = \frac{1}{A} \int_A \alpha \, dA \tag{4.8}$$

This average is called void fraction.
Similarly for mixture density:

$$\langle \rho_m \rangle = \frac{1}{A} \int_A \rho_m dA = \frac{1}{A} \int_A \left[\alpha \overline{\overline{\rho}}_g + (1-\alpha)\overline{\overline{\rho}}_f \right] dA \tag{4.9}$$

Thus, weighted mean density of each phase is given by:

$$\langle\langle \rho_k \rangle\rangle = \frac{\langle \alpha_k \overline{\overline{\rho_k}} \rangle}{\alpha_k} \tag{4.10}$$

From Eq. 4.9 and Eq. 4.10, we can conclude that:

$$\langle \rho_m \rangle = \langle \alpha \rangle \langle\langle \rho_g \rangle\rangle + \langle 1-\alpha \rangle \langle\langle \rho_f \rangle\rangle \tag{4.11}$$

For Intrinsic properties, we have:

$$\langle G_m \rangle \ = \ \langle \alpha \overline{\overline{G}}_g + (1-\alpha)\overline{\overline{G}}_f \rangle \tag{4.12}$$

Let's define weighted average $\langle\langle G_k \rangle\rangle$ as follows:

$$\langle\langle G_k \rangle\rangle = \frac{\langle \alpha G_k \rangle}{\langle \alpha_k \rangle} \tag{4.13}$$

Thus, Eq. 4.12 can be rewritten as follows:

$$\langle G_m \rangle = \langle \alpha \rangle \langle\langle G_g \rangle\rangle + \langle 1-\alpha \rangle \langle\langle G_f \rangle\rangle \tag{4.14}$$

Similarly for Extrinsic properties, we have $G_m = \rho_m \psi_m$, thus by defining the following:

$$\langle\langle \psi_k \rangle\rangle \equiv \frac{\langle \alpha_k \overline{\overline{\rho}}_k \overline{\overline{\psi}}_k \rangle\rangle}{\langle \alpha_k \rho_k \rangle} \tag{4.15}$$

and,

$$\langle\langle \psi_m \rangle\rangle \equiv \frac{\langle \rho_m \psi_m \rangle}{\langle \rho_m \rangle} \tag{4.16}$$

We obtain the following equation:

$$\langle G_m \rangle = \langle \rho_m \psi_m \rangle = \langle \rho_m \rangle \langle\langle \psi_m \rangle\rangle = \langle \alpha \rangle \langle\langle \rho_g \rangle\rangle \langle\langle \psi_g \rangle\rangle + \langle 1-\alpha \rangle \langle\langle \rho_f \rangle\rangle \langle\langle \psi_f \rangle\rangle \tag{4.17}$$

4.3 Field Equations

Now using the knowledge of time and area averaging from above, the conservation equations for two-phase flow are described in this section.

4.3.1 Mixture Properties

Using the time averaging described above, the different phase and mixture properties can be described as follows:

Density of each phase is defined by:

$$\overline{\overline{\rho}}_k = \frac{\overline{\rho}_k}{\alpha_k} \tag{4.18}$$

Density of the mixture:

$$\rho_m = \overline{\rho}_g + \overline{\rho}_f = \alpha\overline{\overline{\rho}}_g + (1-\alpha)\overline{\overline{\rho}}_g \tag{4.19}$$

Velocity of each phase:

$$u_k = \frac{\overline{\rho_k u_k}}{\overline{\rho}_k} \tag{4.20}$$

Mixture Velocity:

$$u_m = \frac{\overline{\rho u}}{\overline{\rho}} = \frac{\alpha\overline{\overline{\rho}}_g\overline{\overline{u}}_g + (1-\alpha)\overline{\overline{\rho}}_f\overline{\overline{u}}_f}{\rho_m} \tag{4.21}$$

Enthalpy of each phase:

$$i_k = \frac{\overline{\rho_k i_k}}{\overline{\rho_k}} \tag{4.22}$$

Mixture Enthalpy:

$$i_m = \frac{\overline{\overline{\rho i}}}{\overline{\rho}} = \frac{\alpha\overline{\overline{\rho}}_g\overline{\overline{i}}_g + (1-\alpha)\overline{\overline{\rho}}_f\overline{\overline{i}}_f}{\rho_m} \tag{4.23}$$

Also the convective term can be written in the following form:

$$\overline{\rho\psi u} = \overline{\rho_g\psi_g u_g} + \overline{\rho_f\psi_f u_f} \tag{4.24}$$

4.3.2 General Conservation Equation

For an Intrinsic property ψ per unit mass, the conservation equation can be written as the following:

$$\frac{\partial \rho\psi}{\partial t} + \nabla \cdot \rho\psi u + \nabla \cdot J - \phi_b = \phi_s \tag{4.25}$$

Where J is the generalized tensor efflux, ϕ_b is the body term, and ϕ_s is the source term. For different conservation equations the expressions for ψ, J, ϕ_b, and ϕ_s Eq. 4.25 can be written to express conservation of mass, enthalpy, and momentum.

Now if the surface capacity of the property ψ is neglected, the averaged general conservation equation can be written in the following form for phase and mixture:

$$\frac{\partial \overline{\rho_k \psi_k}}{\partial t} + \nabla \cdot \overline{\rho_k \psi_k u_k} + \nabla \cdot \overline{J}_k - \overline{\phi}_{bk} = \overline{\phi}_{sk}, \qquad k = f, g \tag{4.26}$$

and,

$$\frac{\partial \overline{\rho\psi}}{\partial t} + \nabla \cdot \overline{\rho\psi u} + \nabla \cdot \overline{J} - \overline{\phi}_b = \overline{\phi}_s \tag{4.27}$$

also constitutive relation for the source term is given as:

$$\sum_{k=f,g} \overline{\phi}_{sk} + \overline{\phi}_s = 0 \tag{4.28}$$

4.3.3 Time averaged Conservation Equations

Using Eq. 4.26 to 4.28 and appropriate values for ψ, J, ϕ_b and ϕ_s we can write balance equations for mass, enthalpy and momentum.

Conservation of Mass

Using the above referred equations with

$$\overline{\psi} = 1, \quad \overline{J} = 0, \quad \overline{\phi}_b = 0, \quad \overline{\phi}_s = 0, \quad \overline{\phi}_{sk} = \Gamma_k \tag{4.29}$$

we get the following mass balance equations:

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot \rho_m u_m = 0 \tag{4.30}$$

$$\frac{\partial \alpha \overline{\overline{\rho}}_g}{\partial t} + \nabla \cdot \left[\alpha \overline{\overline{\rho}}_g \overline{\overline{u}}_g\right] = \Gamma_g \tag{4.31}$$

$$\frac{\partial (1-\alpha) \overline{\overline{\rho}}_g}{\partial t} + \nabla \cdot \left[(1-\alpha) \overline{\overline{\rho}}_g \overline{\overline{u}}_g\right] = \Gamma_f \tag{4.32}$$

$$\Gamma_g + \Gamma_f = 0 \tag{4.33}$$

Eq. 4.30 is mass balance for the time averaged mixture, Eq. 4.31 and 4.32 are time averaged mass balance for each phase, and Eq. 4.33 states the conservation of mass at the interface.

Conservation of Mixture Momentum

Similarly as above, by taking,

$$\overline{\psi} = u, \quad \overline{J} = -\mathfrak{T} = PI - \boldsymbol{\tau}^{\mu}, \quad \overline{\phi}_b = \rho g, \quad \overline{\phi}_s = (t^j_{\alpha} \sigma a^{\alpha\beta})_{,\beta} \tag{4.34}$$

We get the following mixture momentum balance equation:

$$\frac{\partial \rho_m u_m}{\partial t} + \nabla \cdot \rho_m u_m u_m = -\nabla P_m + \nabla \cdot (\overline{\boldsymbol{\tau}}^{\mu} + \boldsymbol{\tau}^{T}) + \rho_m g_m + \nabla \cdot \boldsymbol{\tau}^{D} + f_{\sigma} \tag{4.35}$$

Where $\boldsymbol{\tau}^{\mu}$ and $\boldsymbol{\tau}^{T}$ are viscous and turbulent stress, $\boldsymbol{\tau}^{D}$ is diffusion stress, and f_{σ} is surface force.

Conservation of Enthalpy

Again, by taking:

$$\overline{\psi} = i, \quad \overline{J} = q, \quad \overline{\phi}_b = -P \nabla \cdot u + \boldsymbol{\tau}^{\mu} : \nabla u, \quad \overline{\phi}_s = 0 \tag{4.36}$$

We get the following mixture enthalapy balance equation:

$$\frac{\partial \rho_m i_m}{\partial t} + \nabla \cdot \rho_m i_m u_m = -(\nabla \cdot \overline{q} + \nabla \cdot q^T) + \overline{\phi}_{ce} + \overline{\phi}_{de} - \nabla \cdot q^D \tag{4.37}$$

Where q^T is the turbulent heat flux, $\overline{\phi}_{ce}$ is the compressibility effect on enthalpy, $\overline{\phi}_{de}$ is the energy dissipation term, and q^D is the diffusion heat flux. Eq. 4.37 is analogous to the single-phase energy equation, just that compressibility effect and dissipation term are modified to account for the effect of the interface, and an additional dissipation term.

4.3.4 Area averaged Conservation Equations

Time averaged conservation equations are given in the previous section. Now for 1-D formulation, these equations are area averaged. Area averaging of Eq. 4.30 yields 1-D mixture continuity equation.

$$\frac{\partial}{\partial t}\langle \rho_m \rangle + \frac{\partial}{\partial z}\langle \rho_m \rangle \langle\langle u_{mz} \rangle\rangle = 0 \tag{4.38}$$

Similarly, the continuity equations for each phase can be obtained from Eq. 4.31 and 4.32:

$$\frac{\partial}{\partial t}\langle \alpha \rangle \langle\langle \rho_g \rangle\rangle + \frac{\partial}{\partial z}\langle \alpha \rangle \langle\langle \rho_g \rangle\rangle \langle\langle u_g \rangle\rangle = \langle \Gamma_g \rangle \tag{4.39}$$

$$\frac{\partial}{\partial t}\langle 1-\alpha \rangle \langle\langle \rho_f \rangle\rangle + \frac{\partial}{\partial z}\langle 1-\alpha \rangle \langle\langle \rho_f \rangle\rangle \langle\langle u_f \rangle\rangle = \langle \Gamma_f \rangle \tag{4.40}$$

Similarly, from Eq. 4.35 we can conclude the following by neglecting the normal stresses and the covariant term:

$$\begin{aligned}\frac{\partial}{\partial t}\langle \rho_m \rangle \langle\langle u_{mz} \rangle\rangle + \frac{\partial}{\partial z}\langle \rho_m \rangle \langle\langle u_{mz} \rangle\rangle \langle\langle u_{mz} \rangle\rangle = \frac{\partial}{\partial z}\langle P_m \rangle - \frac{f_m}{2D}\langle \rho_m \rangle \langle\langle u_{mz} \rangle\rangle^2 + \\ \langle \rho_m \rangle g_z + \langle f_{\sigma z} \rangle - \frac{\partial}{\partial z}\sum_{k=g,f}\langle \alpha \rangle \langle\langle \rho_k \rangle\rangle \langle\langle u_{kz} \rangle\rangle \langle\langle u_{kmz} \rangle\rangle\end{aligned} \tag{4.41}$$

Again, from Eq. 4.37, we can find 1-D Enthalpy Equation as follows after neglecting axial conduction due to molecular and turbulent diffusion:

$$\frac{\partial}{\partial t}\langle\rho_m\rangle\langle\langle i_m\rangle\rangle + \frac{\partial}{\partial z}\langle\rho_m\rangle\langle\langle i_m\rangle\rangle\langle\langle u_{mz}\rangle\rangle = \frac{q_w^{''}\xi_h}{A} + \langle\overline{\Phi}_{ce}\rangle + \langle\overline{\Phi}_{de}\rangle - \frac{\partial}{\partial z}\left\{\langle\overline{q}_z\rangle + \langle q_z^T\rangle\right\} \quad (4.42)$$

Where $\overline{\Phi}_{ce}$ is the effect of compressibility on enthalpy and $\overline{\Phi}_{de}$ is the dissipation due to irreversible work. Now, Equations 4.38 to 4.42 will be used in the formulation of the flashing phenomenon without the average symbols for the ease of writing and understanding of the equations.

4.4 Constitutive Relationships

In addition to all the field equations described above, we need additional information to form a closed system for solution of the flashing problem in two phase flow mixtures. The following are the constitutive relationships needed to close the system:

Thermal Equation of State

As described above in Eq. 4.19 mixture density is given as,

$$\rho_m = \alpha\rho_g + (1-\alpha)\rho_f \quad (4.43)$$

with the thermal equation of state for each phase as:

$$\rho_g = \rho_g(P_g, T_g) \quad (4.44)$$

$$\rho_f = \rho_f(P_f, T_f) \quad (4.45)$$

Caloric Equation of State

From Eq. 4.23, the mixture enthalapy is given as:

$$i_m = \frac{\alpha \rho_g i_g + (1-\alpha)\rho_f i_f}{\rho_m} \tag{4.46}$$

with the caloric state for each phase as:

$$i_g = i_g(P_g, T_g) \tag{4.47}$$

$$i_f = i_f(P_f, T_f) \tag{4.48}$$

Constitutive Equation for Phase Change

As described in Eq. 4.33:

$$\sum_{k=g,f} \Gamma_k = 0, \qquad \Gamma_g = \boldsymbol{\mathfrak{F}}_1 \tag{4.49}$$

Kinematic Constitutive Equation

The relationship for relative motion of Phase, vapor drift velocity is expressed as:

$$u_{gj} = (1-\alpha)(u_g - u_f) = \boldsymbol{\mathfrak{F}}_2 \tag{4.50}$$

Rheological Constitutive Equation

The relationship for Friction Factor:

$$f_m = \boldsymbol{\mathfrak{F}}_3 \tag{4.51}$$

Capillary Force

$$f_\sigma = \mathfrak{F}_4 \tag{4.52}$$

Compressibility Effect

The relationship for Φ_{ce} in Eq. 4.42:

$$\Phi_{ce} = \mathfrak{F}_5 \tag{4.53}$$

Dissipation Effect

The relationship for Φ_{de} in Eq. 4.42:

$$\Phi_{de} = \mathfrak{F}_6 \tag{4.54}$$

Mixture Velocity as Phasic Velocities

Eq. 4.21 defines the mixture velocities as:

$$u_m = \frac{\sum_{k=g,f} \alpha_k \rho_k u_k}{\rho_m} \tag{4.55}$$

Mixture Pressure

The definition of mixture pressure can be described as follows:

$$P_m = \sum_{k=g,f} \alpha_k P_k \tag{4.56}$$

Mechanical State between Phases

For some flows with extremely small radii for dispersed phase

$$P_g - P_f = \mathfrak{F}_7 \tag{4.57}$$

Thermal State between Phases

$$i_g - i_f = \Delta i_{fg} = \boldsymbol{\mathfrak{F}}_8 \tag{4.58}$$

Thermal Boundary Condition

Heat Flux term in Eq. 4.42 is given as:

$$q_w^{"} = \boldsymbol{\mathfrak{F}}_9 \tag{4.59}$$

Geometric Parameters

Constant for a system and are known

$$D = \text{Constant} \tag{4.60}$$

$$\xi / A = \text{Constant} \tag{4.61}$$

$$g = \text{Constant} \tag{4.62}$$

For all the constitutive relationships described here in explicit form, there exists some relationship in terms of instant local variables and they are not completely independent, which can be averaged as per the averaging rules described in this chapter. For thermal and mechanical equilibrium case as in the case of gravity induced flashing, we can conclude that:

$$P_g = P_f = P_s \tag{4.63}$$

$$T_g = T_f = T_s \tag{4.64}$$

Although all the equations used here are quite complex and there are various variables which are interdependent on each other, the 1-D generalization is quite simple. Though, it should be kept in mind that:

- The normal stress and the velocity covariant terms in the mixture momentum equation, Eq. 4.41 has been neglected.
- The axial condition and the enthalpy-velocity covariant in the mixture energy equation, Eq 4.42 has been neglected.

Generally, for boiling turbulent flows, the above two assumptions can be made almost always, thus the resultant model from these equations should agree well with the experimental results within reasonable error bounds.
Now that the necessary information about the governing equations has been obtained, they can be used to develop a mathematical model which describes the static and dynamic stability criterion for gravity-induced flashing instability in natural circulation systems as was used in performing the experiments.

5. FLASHING FORMULATION

5.1 Introduction

To understand the mechanism of the flashing phenomenon in gravity-dominated flows and describe its physics, it is necessary to examine thermodynamic processes and flow characteristics of the system. The system of interest is divided into five different regions (A), (B), (C), (D), and (E) similar to [7]

(A) Upstream Un-heated Region

(B) Single Phase Heated Region

(C) Two-Phase Heated Region

(D) Chimney Region before Flashing

(E) Chimney Region after Flashing

It is necessary to describe all the components of the system which can potentially affect the stability of the system such as an array of heated sections in parallel or a sufficiently large volume after the heating section with enough thermal capacitance to effectively eliminate any oscillations which occur during heating. For a natural circulation PWR-Type SMR, reactor core and hot leg are directly in line with each other and thus such a system can be modeled as a single heating region followed by a downstream chimney section with a change in the cross-sectional area in the core and chimney. A simple model can be described as in Fig. 5.1.

The thermodynamic formulation of the system starts with the fluid entering the system with initial enthalpy i_1 in the region (A) and then subsequently region (B) with the flow velocity u_{fi}. As the fluid flows through the vertical heated region, the transfer of heat will change the enthalpy of the fluid. In this formulation, we shall assume thermal equilibrium between the phases of the fluid if it boils or flashes. Thus, if the fluid reaches saturation enthalpy in the heated region (B), that point is called boiling boundary, and region (C) starts from the boiling boundary and thus the fluid flows through the subsequent regions

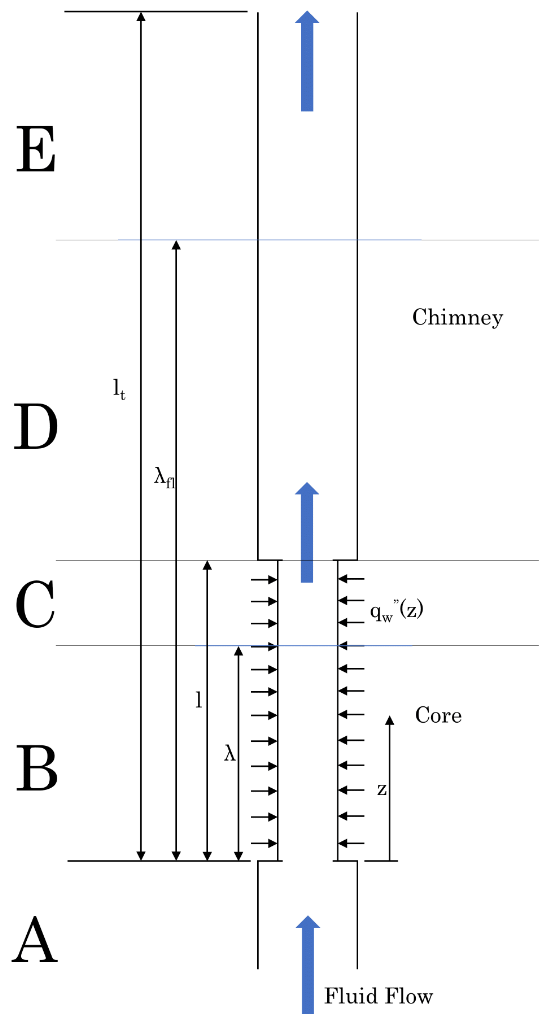


Figure 5.1. Simple Model for Flashing Formulation

in thermal equilibrium between the phases. However, if the fluid did not boil and go to chimney section (D), because being vertical flow the local pressure starts dropping and thus saturation enthalpy starts dropping according to the Clausius-Clapeyron equation. Thus, at a certain distance in the chimney, the fluid violently changes its phase from liquid to gas, and that length is called the *Flashing Boundary* λ_{fl}. Fluid enthalpy at this point is assumed to be saturation enthalpy at local pressure i.e. $i_f = i_{f,sat}(P_{local})$. *This is explored in detail later in the chapter.* After the flashing boundary, two-phase fluid enters the region (E).

For the whole formulation, we shall assume the following:

1. Liquid Phase density is a function of System Pressure, P_{sys}, and is constant thru the different regions.

2. Both the phases are in thermal equilibrium.

3. The process is always isochoric in all regions.

5.2 Physical Interpretation of the Problem

The local pressure P_{local} changes with the change in position in the Chimney Region (D) and (E). The saturation enthalpy drops with the drop in local pressure. As shown in Fig. 5.2, with an increase in z (drop in P_{local}), the saturation enthalpy for the fluid drops, and thus at some point saturation enthalpy and fluid enthalpy are the same. We call this point in z as Flashing Boundary λ_{fl}. Beyond λ_{fl}, the fluid is super-heated until *Nucleation* happens. Nucleation is the process of generation of the first void in the flow at special Nucleation sites. These sites can be any irregularities in the concerned system such as welds, rivets, joints, surface roughness, or fouling. Once nucleation initiates the voids, these voids serve as additional nucleation sites which facilitate further void generation thus releasing the super-heat in form of latent heat for vapor generation.

Once void generation happens, the flow rate in the concerned system will suddenly increase pertaining to the reduction in density of the fluid in flow. Increased flow rate, in turn, will allow the fluid to carry less heat from the heated regions (B) and (C). Thus suppressing subsequent flashing. Once the normal flow rate is established, flashing happens again, thus causing an increase in flow rate again, which makes it a self-sustaining flow instability.

Now that the basic physics for the instability is explained, the later sections explore the mathematical formulation for Two-Phase flow instability induced by flashing.

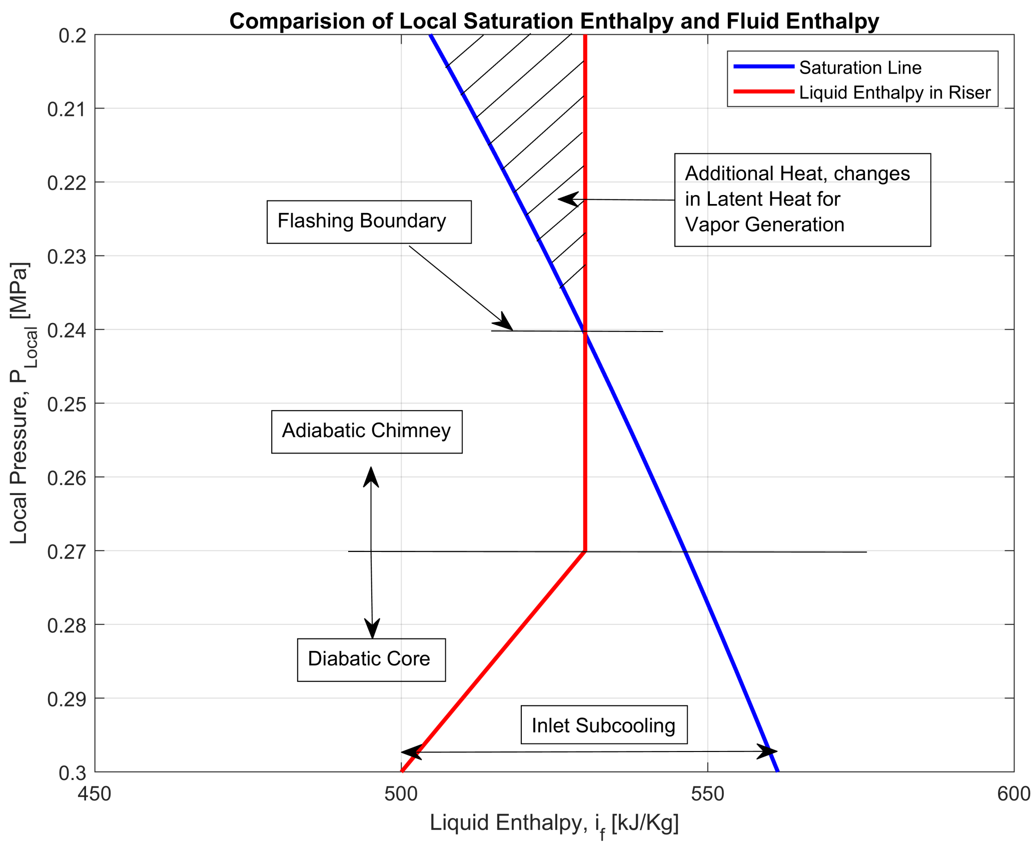


Figure 5.2. Comparison of Local Saturation Enthalpy and Fluid Enthalpy

5.3 Governing Equations

As we have explored in Chapter 4 the different Field Equations and Constitutive Relationships for Two-Phase Flow shall be used here to describe the physics of the fluid flow in the above-described regions.

5.3.1 Upstream Un-heated Region (A)

As this section has no heat input and significantly small length such that density of the liquid phase can be assumed a function of system pressure only, and thus, $\rho_f = \rho_f(P_s) = const.$ Also, we assume that this region consists of constant area duct and some inlet constriction which can be modeled with the help of a pressure drop device like an orifice. Thus, from Eq. 4.40 for single phase, we can conclude that:

$$u_{fo} = \frac{A_c}{A_o} u_{fi}(t) \tag{5.1}$$

Here A_c and A_o are flow areas of Heated and Up-stream region respectively. Also, from Eq. 4.41 for single phase, we can find Momentum Equation for region (A):

$$-\frac{dP}{dz} = \rho \left\{ \frac{\partial u_{fo}}{\partial t} + u_{fo} \frac{\partial u_{fo}}{\partial z} + g_o + \left(\frac{f_o}{2D_o} + k_i \right) u_{fo}^2 \right\} \tag{5.2}$$

Also, as this region is isenthalpic, we can write Energy Equation for region (A):

$$\frac{Di_{fo}}{Dt} = 0 \tag{5.3}$$

5.3.2 Single Phase Heated region (B)

Fluid from Region (A) enters region (B) where it is heated with the help of the core. Thus for single phase, Eq. 4.40, Eq. 4.41 and 4.42 can be written as follows:

Continuity Equation

$$\frac{\partial \rho_f}{\partial t} + \frac{\partial \rho_f u_f}{\partial z} = 0 \tag{5.4}$$

Momentum Equation

$$-\frac{\partial P}{\partial z} = \rho_f \left\{ \frac{\partial u_f}{\partial t} + u_f \frac{\partial u_f}{\partial z} + g + \frac{f_s}{2D} u_{fo}^2 \right\} \tag{5.5}$$

Energy Equation

$$\frac{\partial i_f}{\partial t} + u_f \frac{\partial u_f}{\partial z} = \frac{q_w \xi}{\rho_f A_c} \equiv q_w \tag{5.6}$$

5.3.3 Two Phase Heated Region (C)

For taking into account the relative velocity between the two phases it is useful to formulate the governing equations in terms of mixture field equations with some diffusion equations thus using Eq. 4.38 to Eq. 4.42, we get the following:

Mixture Continuity Equation

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial \rho_m u_m}{\partial z} = 0 \tag{5.7}$$

Gas Continuity Equation

$$\frac{\partial \alpha \rho_g}{\partial t} + \frac{\partial}{\partial z}\Big(\alpha \rho_g u_m\Big) = \Gamma_g - \frac{\partial}{\partial z}\left\{\frac{\alpha \rho_g \rho_f}{\rho_m} u_{gj}\right\} \tag{5.8}$$

Mixture Momentum Equation

$$-\frac{\partial P}{\partial z} = \rho_m \left\{\frac{\partial u_m}{\partial t} + u_m \frac{\partial u_m}{\partial z}\right\} + g\rho_m + \frac{f_m}{2D}\rho_m u_m^2 + \frac{\partial}{\partial z}\left\{\frac{\rho_f - \rho_m}{\rho_m - \rho_g}\frac{\rho_g \rho_f}{\rho_m} u_{gj}^2\right\} \tag{5.9}$$

Mixture Energy Equation

$$\rho_m \left\{\frac{\partial i_m}{\partial t} + u_m \frac{\partial i_m}{\partial z}\right\} = \frac{q_w \xi}{A_c} - \frac{\partial}{\partial z}\left\{\frac{\alpha \rho_g \rho_f}{\rho_m} u_{gj} \Delta i_{fg}\right\} \tag{5.10}$$

As explained in Chapter 4, it is realized that $\rho_m = \rho_m(P_{sys}, \alpha)$, $i_m = i_m(P_{sys}, \alpha)$, and gas drift flux velocity u_{gj} can be defined with system parameters. *Although it is necessary to decouple these equations, thus u_{gj} should be treated as constant* [7].

5.3.4 Chimney Region before Flashing (D)

For taking into account the generality of the problem it is assumed that this region has a different cross-sectional area than the heated region but is constant throughout the chimney and thus the governing equations scale differently based on the area ratio. This region consists of a pressure drop device which can be modeled as an orifice. Also, as this region is Un-heated, there is no change of fluid enthalpy in the chimney. The local pressure drops as the flow direction is opposite of the gravitational force. *Flashing is prevalent in gravity-dominated flows.*

Continuity Equation

$$\frac{\partial u_{me}}{\partial z} = 0 \tag{5.11}$$

Thus the velocity does not change in the flow direction. The solution of the above equation can be given as:

$$u_{me} = \frac{A_c}{A_e} u_m(l,t) \tag{5.12}$$

Energy Equation

$$\frac{D i_{me}}{Dt} = 0 \tag{5.13}$$

Where D is the material derivative and standard definition of material derivative is used with $u = u_{me}$

Momentum Equation

$$-\frac{\partial P}{\partial z} = \rho_{me}\left\{\frac{\partial u_{me}}{\partial t} + u_{me}\frac{\partial u_{me}}{\partial z}\right\} + g_e\rho_{me} + \\ + \left(\frac{f_{me}}{2D_e}\rho_{me} + k_e\right)u_{me}^2 + \frac{\partial}{\partial z}\left\{\frac{\rho_f - \rho_{me}}{\rho_{me} - \rho_g}\frac{\rho_g\rho_f}{\rho_m}u_{gj}^2\right\} \tag{5.14}$$

5.3.5 Chimney Region after Flashing (E)

After the flashing boundary, the fluid suddenly starts to vaporise giving rise to flow oscillations based on flashing. This phenomenon happens when the Local Saturation Temperature, $T_{sat}(P_{local})$, falls below the fluid temperature in the chimney and thus, the super-heat of the fluid goes into latent heat for phase change to gas phase. Thus the governing equations can be described as follows based on Eq. 4.38 to 4.42:

Mixture Continuity Equation

$$\frac{\partial \rho_{me}}{\partial t} + \frac{\partial \rho_{me} u_{me}}{\partial z} = 0 \tag{5.15}$$

Gas Continuity Equation

$$\frac{\partial \alpha \rho_g}{\partial t} + \frac{\partial}{\partial z}\Big(\alpha \rho_g u_{me}\Big) = \Gamma_{g,fl} - \frac{\partial}{\partial z}\left\{\frac{\alpha \rho_g \rho_f}{\rho_{me}} u_{gj,fl}\right\} \tag{5.16}$$

Mixture Momentum Equation

$$-\frac{\partial P}{\partial z} = \rho_{me}\left\{\frac{\partial u_{me}}{\partial t} + u_{me}\frac{\partial u_{me}}{\partial z}\right\} + g\rho_{me} + \\ + \frac{f_{me}}{2D_e}\rho_{me}u_{me}^2 + \frac{\partial}{\partial z}\left\{\frac{\rho_f - \rho_{me}}{\rho_{me} - \rho_g}\frac{\rho_g \rho_f}{\rho_{me}} u_{gj,fl}^2\right\} \tag{5.17}$$

For this region, as we know that the superheat of the fluid generated due to pressure gradient is changed to latent heat for the phase change to the gas phase. Thus, the local enthalpy can be described as the local mixture enthalpy at saturation.

Mixture Energy Equation

$$i_{me}(z,t) = i_{me,sat}(P_{local}, \alpha) \tag{5.18}$$

Exact computation of local enthalpy of the mixture is insignificant from the viewpoint of solution of instability, however, $\Gamma_{g,fl}$ *needs to be evaluated with high fidelity.*

Thus with the help of the above described governing equations and constitutive relationships between the different parameters, the whole system can be described, and thus pressure response can be assessed based on the inlet velocity.

5.4 Estimation of Gas Generation due to Flashing, $\Gamma_{g,fl}$

As it has been stated before, the Gas Generation happens after the flashing boundary in the adiabatic chimney section where the gradient in pressure of the fluid results in a change of saturation temperature of the fluid. Thus, the superheat goes into latent heat resulting in a phase change to the gas phase. Although fluids can sustain a certain amount of superheat before nucleation, it highly depends on the local conditions such as surface roughness of the pipe, or certain abnormalities such as welding, rivets, chips, and fouling. All engineering systems, in general, have one or more of the above-mentioned irregularities and thus, simultaneous nucleation is a rare occurrence. Also, as the flashing-based void generates on the chimney wall, these voids themselves act as nucleation sites. Thus, the prediction of $\Gamma_{g,fl}$ is a challenge.

Thus for this formulation, we shall assume that the flashing boundary occurs at the point where local saturation temperature equals the fluid temperature and both the phases are in thermal equilibrium and follows the saturation curve eliminating the heat transfer terms between the phases. *Although it shall be noted that mere changing of the boundary with an empirical parameter will not change the formulation or the solution method itself and this formulation can still be applied.*

In order to estimate gas generation due to flashing, we need to modify Eq. 4.26 for enthalapy of a phase in the following manner for area averaged equation as described in [14]:

$$\left\{\frac{\partial i_k \alpha_k \rho_k}{\partial t} + \frac{\partial i_k \alpha_k \rho_k u_k}{\partial z}\right\} = -\frac{\partial}{\partial z}\Big(\alpha_k q_k\Big) + \alpha_k \frac{D_k P}{Dt} + \Gamma_k i_{ki} + a_i q_{ki} + \phi_k \tag{5.19}$$

where a_i is interfacial area concentration, i_{ki} is the enthalapy of k-phase at interface, Γ_k is the k-phase generation term and ϕ_k is the viscous dissipation term for k-phase. This form of phasic energy equation is explained in detail in [27], Eq. 9.28. If we assume thermal equilibrium, and $\phi_k = 0$, we get:

$$\left\{\frac{\partial i_k \alpha_k \rho_k}{\partial t} + \frac{\partial i_k \alpha_k \rho_k u_k}{\partial z}\right\} = -\frac{\partial}{\partial z}\Big(\alpha_k q_k\Big) + \alpha_k \frac{D_k P}{Dt} \tag{5.20}$$

This equation can be rewritten in the following form:

$$\alpha_k \rho_k \left\{\frac{\partial i_k}{\partial t} + u_k \frac{\partial i_k}{\partial z}\right\} + i_k \left\{\frac{\partial \alpha_k \rho_k}{\partial t} + \frac{\partial \alpha_k \rho_k u_k}{\partial z}\right\} = -\frac{\partial}{\partial z}\Big(\alpha_k q_k\Big) + \alpha_k \frac{D_k P}{Dt} \tag{5.21}$$

If we look closely, this equation can be written in the form of phasic continuity equation in the following way:

$$i_k \left\{\frac{\partial \rho_k \alpha_k}{\partial t} + \frac{\partial \alpha_k \rho_k u_k}{\partial z}\right\} = -\frac{\partial}{\partial z}\Big(\alpha_k q_k\Big) + \alpha_k \frac{D_k P}{Dt} - \alpha_k \rho_k \frac{D_k i_k}{Dt} \tag{5.22}$$

Thus, we can rewrite the above equation using Eq. 4.31 and Eq. 4.32 as follows:

$$i_k \Gamma_k = -\frac{\partial}{\partial z}\Big(\alpha_k q_k\Big) + \alpha_k \frac{D_k P}{Dt} - \alpha_k \rho_k \frac{D_k i_k}{Dt} \tag{5.23}$$

This equation describes the k-phase generation term from continuity and energy equation. The terms on the right hand side describes the mode of generation of that phase. First term on the right hand side describes the gas generation by external heat source, second and third term describes the mode of generation by change of pressure (compressibility effect). *Its useful to note here that as fluid enthalapy also changes with pressure, the two terms do not indicate two separate mechanisms.* Thus in the above equation, we can set $\Gamma_k = \Gamma_{k,q} + \Gamma_{k,P}$, we get:

$$i_k \Big(\Gamma_{k,q} + \Gamma_{k,P}\Big) = -\frac{\partial}{\partial z}\Big(\alpha_k q_k\Big) + \alpha_k \frac{D_k P}{Dt} - \alpha_k \rho_k \frac{D_k i_k}{Dt} \tag{5.24}$$

As it is described before that the phenomenon of flashing happens due to drop in pressure, we can say that $\Gamma_{k,fl} = \Gamma_{k,P}$. Thus from the above equation, we find:

$$i_k \Gamma_{k,fl} = \alpha_k \frac{D_k P}{Dt} - \alpha_k \rho_k \frac{D_k i_k}{Dt} \tag{5.25}$$

But the above equation is not complete representation of gas phase generation as gas is generated from liquid phase, and thus equations for both phases should be used using constitutive relationship given by Eq. 4.33. We conclude as follows:

$$\Delta i_{fg} \Gamma_g = \sum_{k=f,g} \alpha_k \left\{ \frac{D_k P}{Dt} - \rho_k \frac{D_k i_k}{Dt} \right\} \tag{5.26}$$

As we have assumed in the beginning of this formulation that after the flashing boundary, the phases are in thermal equilibrium and follows the saturation curve, we can use Clausius-Clapeyron equation given as follows:

$$di_{k,sat} = \frac{c_{pk} T_{k,sat} \Delta v_{fg}}{\Delta i_{fg}} dP \tag{5.27}$$

Using Eq. 5.26 and Eq. 5.27 we can find that:

$$\Gamma_{g,fl} = \frac{1}{\Delta i_{fg}} \sum_{k=f,g} \alpha_k \left\{ 1 - \rho_k c_{pk} T_{k,sat} \frac{\Delta v_{fg}}{\Delta i_{fg}} \right\} \frac{D_k P}{Dt} \tag{5.28}$$

For a Natural Circulation PWR-type system, which operates at very low quality flow conditions, the following assumptions can be made.

$$\alpha \rho_g c_{pg} \ll (1-\alpha) \rho_f c_{pf} \tag{5.29}$$

$$(1-\alpha)\rho_f u_f = \rho_f u_f \equiv G \tag{5.30}$$

The above equations say that the heat-carrying capacity of the liquid phase is much greater than the gas phase and most of the mass transport is done by the liquid phase. Both the assumptions will hold for most natural circulation flows seen in nuclear reactors. Also if we assume that for such low-quality flow, the pressure drop is mostly hydrostatic, we can get a very simple expression for $\Gamma_{g,fl}$:

$$\Gamma_{g,fl} = G c_{pf} T_{sat} \frac{\Delta v_{fg}}{\Delta i_{fg}^2} \rho_f g \cos\theta \tag{5.31}$$

Where $\cos\theta$ represents the orientation of the chimney section. For vertical flow formulation, $\cos\theta = 1$

5.5 Estimation of Flashing Boundary λ_{fl}

We have been discussing the flashing boundary but it is not a geometric parameter and it needs to be calculated for a certain system given initial and/or boundary conditions. As for the current formulation, we have assumed that the fluid properties depend on the system pressure P_{sys} only. For finding the flashing boundary, however, we need to take into account the change of local saturation enthalpy as Flashing depends on this change to occur. Furthermore, it is to be noted that this boundary is assumed to to static at the location where steady-state Fluid Enthalpy $\bar{i}_f(\lambda_{fl})$ equals saturation enthalpy at that Local

Pressure $i_{f,sat}(P_{local})$ Thus, if we assume that the fluid enters the heated section (B) at an inlet enthalpy $i_{f,in}$ with a steady-state inlet velocity $\overline{u}_f$, inlet subcooling can be defined as $i_{sub} = i_{f,sat,in} - i_{f,in}$. While going thru the Heated Regions (B) and (C), the steady-state increase in fluid enthalpy can be given as:

$$\bar{i}_f(l_{core}) = i_{f,in} + q_w \frac{l_{core}}{\overline{u}_f} \tag{5.32}$$

As the Chimney Region is adiabatic, the enthalpy does not change with the change of z, however, the local saturation enthalpy is decreasing with the drop of hydrostatic head. Thus, at the point of Flashing Boundary, we can say the following:

$$\bar{i}_f(l_{core}) = i_f(\lambda_{fl}) = i_{f,sat}\Big(P(\lambda_{fl})\Big) \tag{5.33}$$

Integrating the Clausius Clapeyron Equation, Eq. 5.27 from inlet to Flashing Boundary λ_{fl}, we find that:

$$i_{f,sat}(\lambda_{fl}) - i_{f,sat,in} = \frac{c_{pf} T_{sat} \Delta v_{fg}}{\Delta i_{fg}} \Big[P(\lambda_{fl}) - P_{in}\Big] \tag{5.34}$$

Assuming the pressure drop is hydrostatic as in Section 5.4, we get:

$$P(\lambda_{fl}) - P_{in} = -\rho_f g \lambda_{fl} \tag{5.35}$$

Thus, combining above four equations, we conclude:

$$\lambda_{fl} = \Delta i_{fg} \frac{i_{sub} - q_w \dfrac{l_{core}}{\overline{u}_f}}{\rho_f g c_{pf} T_{sat} \Delta v_{fg}} \tag{5.36}$$

It is important to note here that λ_{fl} is measured vertically, not along the orientation of the fluid flow. Also, it is clear from the above formulation that λ_{fl} is only a function of fluid properties and system initial and boundary conditions. As will be defined later, for both, fluid properties and boundary conditions defined as constant, we fix Flashing Boundary λ_{fl} too.

5.6 Solution Technique

The system has been formulated by considering five different regions as mentioned before in this chapter. The governing equations can now be solved with the help of boundary and/or initial conditions. For the objective of determining the stability criteria of the system, many methods can be used. Ishii in his thesis [7] used the method of dynamic stability described by poles and zeros of a characteristic equation written with the help of a perturbation parameter and its subsequent response. This theory was first developed by Mikhailov for linear systems with no time delays and then modified by Nyquist using Cauchy's argument principle. Y. I. Neimark [28], later extended this theory to systems with variable parameters and thus in his theorem stating that in this parameter plane, there exists a sub-plane for which the system behaves the same for various values of the parameters. (*i.e. Characteristic equation has the same number of poles and zeros on the right-hand side of the s-plane in that sub-plane.*)

In this analysis the disturbance is added to the system is in the form of perturbations of inlet velocity. This is done following Ishii's [7] formulation and N. Zuber's formulation before him. It is to be noted here that perturbations in inlet enthalpy are also permissible, just that the characteristic equation will change accordingly. Thus, if we assume the origin at the inlet of the heater section, we can write the following boundary and initial conditions at $z = 0$ and $t \geq 0$.

$$\rho_f = \rho_f(P_{sys}) \equiv const. \tag{5.37}$$

$$i_f = i_1 \equiv const. \tag{5.38}$$

$$u_f = u_{fi}(t) = \overline{u}_{fi} + \delta u(t) \tag{5.39}$$

The steady state inlet velocity is given by $\overline{u}_{fi}$ and the perturbation on the velocity is given by $\delta u(t)$. In this analysis, a frequency response method is used and thus the velocity perturbation can be given by an exponential function as follows:

$$\delta u(t) = \epsilon e^{st} \tag{5.40}$$

where,

$$s = a + j\omega \tag{5.41}$$

It is worth noting here that 's' is a complex variable and the real part of 's' gives the amplification coefficient of that oscillation mode, whereas imaginary part ω represents the angular frequency of that mode. For this analysis, we assume that the perturbations are much smaller than the steady velocity and thus $\epsilon \ll \overline{u}_{fi}$, so that we can neglect higher orders of ϵ making it a linear analysis in ϵ.

Thus, if we substitute Eq. 5.40 into Eq. 5.39, we get:

$$u_{fi} = \overline{u}_{fi} + \epsilon e^{st} \tag{5.42}$$

The procedure for solving the governing equations involves the decoupling of the conservation equations with the help of meaningful assumptions and some systematically developed relations. The solution strategy can be summarised by Fig. 5.3. It is notable here that the potential flow theory can be applied to a two-phase system, the only difference being the divergence of the velocity of the center of mass is zero in potential flow, but in two-phase boiling systems, this term equals volume source due to phase change. [7]

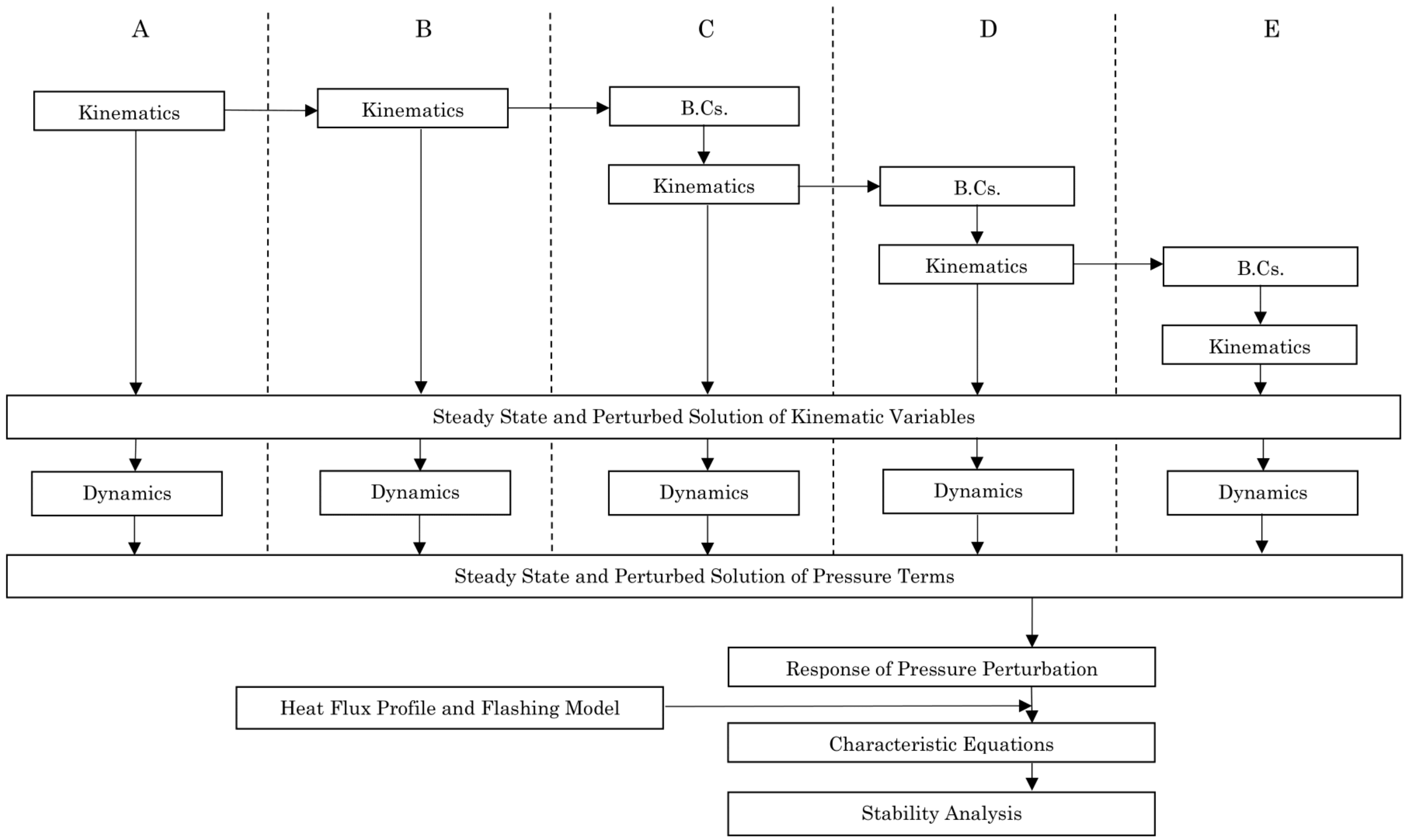


Figure 5.3. Strategy for analytical solution employed in the formulation

6. KINEMATICS OF FLUID

6.1 Introduction

The governing equations for different regions have been derived from conservation equations in Chapter 5. In this chapter, we shall solve the mass and energy conservation equations in a decoupled fashion from the momentum conservation equations which shall be tackled later on. Along the way during the chapter, various salient features of the kinematic equations and solutions will be discussed.

6.2 Kinematics of Liquid Regions (A) and (B)

In the liquid region, the density is constant, thus the continuity equations can be readily solved for velocity fields. From Eq. 5.1 and 5.42:

$$u_{fo} = \frac{A_c}{A_o}\left(\overline{u}_{fi} + \epsilon e^{st}\right) \tag{6.1}$$

Where the steady state and perturbed parts are give as follows:

$$\overline{u}_{fo} = \frac{A_c}{A_o}\overline{u}_{fi} \tag{6.2}$$

$$\delta u_{fo}(t) = \frac{A_c}{A_o}\epsilon e^{st} \tag{6.3}$$

As this region is isenthalpic, thus solution of Eq. 5.3 with Initial Condition from Eq. 5.38 can be given as:

$$i_{fo} = i_1 \tag{6.4}$$

The above results for region (A) shows that the density and enthalpy are constant and velocity is a function of time only.

$$\rho_f(z) = \rho_f(P_{sys}) \equiv const. \tag{6.5}$$

For region (B) the boiling boundary (λ) is unknown, thus when solving for velocity field and enthalpy, it needs to be taken into account that the boiling boundary occurs when $i_f = i_{sat}$, that is, when the fluid enthalpy reaches the saturation enthalpy based on the system pressure. The form of Eq. 5.4 and 5.6 allows us to solve this problem and find $\lambda(t)$. This can be done as follows, assuming liquid density doesn't change with heat addition:

$$\frac{\partial u_f}{\partial z} = 0 \tag{6.6}$$

Integrating it with the initial condition given by Eq. 5.42, we get:

$$u_f(t) = u_{fi}(t) = \overline{u}_{fi} + \epsilon e^{st} \tag{6.7}$$

This means velocity field in region (B) is a function of time only. With known velocity field, Eq. 5.6 becomes:

$$\frac{\partial i_f}{\partial t} + u_f(t)\frac{\partial i_f}{\partial z} = q_w \tag{6.8}$$

Now if the heat generated by the heating section is not uniform and a function of 'z', the above equation can be solved in the following manner by average heat flux q_o defined by:

$$q_o = \frac{1}{l_c}\int_0^{l_c} q_w(z)dz \tag{6.9}$$

As per the above equation:

$$q_w(z) = q_o f(z) \tag{6.10}$$

Which defines the dimensionless heat flux distribution $f(z)$. If we assume that this function is continuous, differential, and positive function, we can define the integral of $f(z)$ as:

$$F(z) = \int_0^{z} f(z)dz \tag{6.11}$$

With use of this definition, Eq. 6.8 can be solved as:

$$\frac{D_{v_f} i_f}{Dt} = \frac{q_o \xi}{A_c \rho_f} f(z) \tag{6.12}$$

This equation can be solved in the following form:

$$dt = \frac{dz}{v_f(t)} = \frac{di_f}{q_o f(z)} \tag{6.13}$$

The boundary conditions for region (B) are given by exit conditions of (A) as follows:

$$t = \tau_1 \quad \text{and} \quad i_f = i_1 \quad \text{and} \quad z = 0 \tag{6.14}$$

Thus, the solution can be given as following:

$$z = \overline{u}_{fi}(t - \tau_1) + \epsilon e^{st}\frac{1 - e^{-s(t-\tau_1)}}{s} \quad \text{or} \quad z = \overline{z} + \delta z \tag{6.15}$$

and

$$di_f = q_o f\Big(z(t)\Big)dt \tag{6.16}$$

As we know that region (B) ends when bulk boiling starts in the heated section, thus:

$$\lambda = \overline{u}_{fi}\tau_{12} + \epsilon e^{st}\frac{1 - e^{-s\tau_{12}}}{s} \tag{6.17}$$

Where time lag $\tau_{12} = \tau_2 - \tau_1$ is given by the following equation:

$$\Delta i_{12} = i_{f,sat} - i_1 = q_o \int_{\tau_1}^{\tau_2} f\Big(z(t)\Big)dt \tag{6.18}$$

Above equation can also be solved for steady state in the following manner:

$$\Delta i_{12} = q_o \frac{1}{\overline{u}_{fi}} \int_0^{\lambda} f(z)dz = q_o \frac{1}{\overline{u}_{fi}} F(\overline{\lambda}) \tag{6.19}$$

First order solution is given by:

$$\Delta i_{12}\frac{1}{q_o} = \int_{\tau_1}^{\tau_2} f\Big(u_{fi}[t - \tau_1]\Big)dt + \int_{\tau_1}^{\tau_2} f\Big(u_{fi}[t - \tau_1]\Big)\delta z dt \tag{6.20}$$

by defining $\tau_{12} = \overline{\tau}_{12} + \delta\tau_{12}$, and using Taylor expansion, we can show that the two integrals have the form:

$$I_1 = \int_{\tau_1}^{\tau_2} f\Big(u_{fi}[t-\tau_1]\Big)dt = \frac{1}{\overline{u}_{fi}} F(\overline{u}_{fi}, \tau_{12}) = \frac{F(\overline{\lambda})}{\overline{u}_{fi}} + f(\overline{\lambda})\delta\tau_{12} \tag{6.21}$$

For easiness of solving the second integral, lets define a function g as:

$$g(\eta, s) = \int_0^{\eta} f(\eta) e^{\frac{s\eta}{\overline{u}_{fi}}} d\eta \tag{6.22}$$

Thus, second integral has the form:

$$I_2 = \int_{\tau_1}^{\tau_2} f\Big(u_{fi}[t-\tau_1]\Big)\delta z dt = \frac{\epsilon e^{s\tau_1}}{s\overline{u}_{fi}} \left\{ g(\overline{\lambda}, s) - \Big[f(\overline{\lambda}) - f(0)\Big] \right\} \tag{6.23}$$

So, complete first order solution is given by:

$$\delta\tau_{12}(t) = -\epsilon e^{st} \frac{1}{f(\overline{\lambda})\overline{u}_{fi}} \left\{ g(\overline{\lambda}, s) - \Big[f(\overline{\lambda}) - f(0)\Big] \right\} \frac{e^{-s\tau_{12}}}{s} \tag{6.24}$$

This suggests that the residence time in the heater region is a transfer function and we can simplify by defining:

$$\frac{\delta\tau_{12}}{\epsilon e^{st}} \equiv \Lambda_1(s) = -\frac{1}{f(\overline{\lambda})\overline{u}_{fi}} \left\{ g(\overline{\lambda}, s) - \Big[f(\overline{\lambda}) - f(0)\Big] \right\} \frac{e^{-s\tau_{12}}}{s} \tag{6.25}$$

Now that we know τ_{12}, the boiling length λ, can be calculated as follows from Eq. 6.17:

$$\lambda(t) = \lambda = \overline{u}_{fi}\tau_{12} + \epsilon e^{st} \left\{ \frac{1 - e^{-s\overline{\tau}_{12}}}{s} + \overline{u}_{fi}\Lambda_1(s) \right\} \tag{6.26}$$

Thus, we find another transfer function and we define:

$$\frac{\delta\lambda}{\epsilon e^{st}} \equiv \Lambda_2(s) = \frac{1 - e^{-s\overline{\tau}_{12}}}{s} + \overline{u}_{fi}\Lambda_1(s) \tag{6.27}$$

For example, for a uniform heat flux condition, which is used in this formulation, we find:

$$\Lambda_1(s) = 0 \tag{6.28}$$

and,

$$\Lambda_2(s) = \frac{1 - e^{-s\overline{\tau}_{12}}}{s} \tag{6.29}$$

Although, it is not important from the point of stability problem to know the enthalapy field, it can come in handy to know energy wave propagation for future analysis, thus from Eq. 6.16:

$$i_f(z,t) - i_1 = \frac{q_o}{\overline{u}_{fi}}\Bigg\{F(z) + \epsilon e^{st}\Big[-f(z)\frac{1 - e^{-\frac{sz}{\overline{u}_{fi}}}}{s} + \\ + \Big(g(z,s) - f(z) + f(0)\Big)e^{-\frac{sz}{\overline{u}_{fi}}}\Big]\Bigg\} \tag{6.30}$$

6.3 Kinematics of the Heated Mixture Region (C)

The governing equations for region (C) have been developed in chapter 5. Recalling constant fluid properties are valid for thermal equilibrium assumptions. Also as discussed before the gas drift flux velocity u_{gj} is assumed to be constant, can transform the two continuity equations 5.7 and 5.8 into the following form:

$$\frac{\partial j}{\partial z} = \Gamma_g \frac{\Delta\rho}{\rho_g \rho_f} \tag{6.31}$$

and:

$$\frac{\partial \rho_m}{\partial t} + C_k \frac{\partial \rho_m}{\partial z} = -\Gamma_g \frac{\Delta\rho\rho_m}{\rho_g \rho_f} \tag{6.32}$$

where C_k is given by:

$$C_k \doteq j + u_{gj} \tag{6.33}$$

The first equation describes the increase of j due to phase change. The second equation has a form of the wave equation and can be called density propagation equation [7]. From constitutive relationships developed in chapter 4, namely Caloric Equation of State, we can find:

$$di_m = -\frac{\Delta i_{fg}\rho_g\rho_f}{\Delta\rho}\frac{d\rho_m}{\rho_m^2} \tag{6.34}$$

The above equations needs another relationship to be solved completely which will be given by energy equation 5.10.

$$\frac{\partial\rho_m}{\partial t} + C_k\frac{\partial\rho_m}{\partial z} = -\left[\frac{q_w\xi}{A_c}\frac{1}{\Delta i_{fg}}\right]\frac{\Delta\rho\rho_m}{\rho_g\rho_f} \tag{6.35}$$

Thus, from case of thermal equilibrium, we conclude:

$$\Gamma_g = \frac{q_w\xi}{A_c}\frac{1}{\Delta i_{fg}} \tag{6.36}$$

This equation relates the vapor generation Γ_g to heat flux and the latent heat Δi_{fg}. *This result is not a consequence of the formulation in term of C_k, but of the constants i_f, i_g, ρ_f, and ρ_g with thermal equilibrium condition.*

Thus, Eq. 6.31 can be used to define characteristic frequency of phase change $\Omega(z)$:

$$\frac{\partial j}{\partial z} = \Gamma_g\frac{\Delta\rho}{\rho_g\rho_f} \equiv \Omega(z) \tag{6.37}$$

In terms of average heat flux, $\Omega(z)$ can be expressed similarly:

$$\Omega(z) = \Omega_0 f(z) \tag{6.38}$$

Now, Eq. 6.31 and 6.32 can be rewritten by using above definition of $\Omega(z)$ as:

$$\frac{\partial j}{\partial z} = \Omega_0 f(z) \tag{6.39}$$

$$\frac{\partial\rho_m}{\partial t} + C_k\frac{\partial\rho_m}{\partial z} = -\Omega_0 f(z)\rho_m \tag{6.40}$$

and we know from Eq. 6.33, $C_k = j + u_{gj}$, and we will treat u_{gj} as constant. *Ishii in his thesis justified this assumption in Chapter 5 [7]*

Kinematic Wave Velocity C_k

Integration of the volumetric flux equation 6.31 with boundary conditions at the end of region (B) can be integrated directly and yields:

$$j(z,t) = u_{fi} + \Omega_0 \int_{\lambda(t)}^{z} f(z)dz \tag{6.41}$$

Defining $F(z)$ as integral of $f(z)dz$, we get:

$$j(z,t) = u_{fi} + \Omega_0 \Big[F(z) - F\Big(\lambda(t)\Big) \Big] \tag{6.42}$$

Now, using Eq. 6.33 we get:

$$C_k = u_{fi} + u_{gj} + \Omega_0 \Big[F(z) - F\Big(\lambda(t)\Big) \Big] \tag{6.43}$$

The steady state solution of the above equation is given as:

$$\overline{C}_k(z) = \overline{u}_{fi} + u_{gj} + \Omega_0 \Big[F(z) - F\Big(\overline{\lambda}\Big) \Big] \tag{6.44}$$

and the first order solution is given as:

$$\delta C_k(t) = \delta u_f(t) - \Omega_0 f(\overline{\lambda}) \delta\lambda(t) \tag{6.45}$$

Which is another transfer function and similarly as before, we define:

$$\frac{\delta C_k(t)}{\epsilon e^{st}} \equiv \Lambda_3(s) = 1 - \Omega_0 f(\overline{\lambda}) \Lambda_2(s) \tag{6.46}$$

Density Wave Propagation

Now we know the solution of kinematic wave velocity C_k, and we know gas generation due to heat flux Γ_g, thus, density propagation equation can be solved by introducing a new variable ϕ which is defined as follows:

$$\phi(z,t) \equiv \ln\left[\frac{\rho_m(z,t)}{\rho_f}\right] \tag{6.47}$$

Thus, Eq. 6.40 can be morphed to the following form:

$$\frac{\partial \phi}{\partial t} + C_k(z,t)\frac{\partial \phi}{\partial z} = -\Omega_0 f(z) \tag{6.48}$$

For applying the perturbation, we need to define the perturbation variables as follows:

$$\phi(z,t) = \overline{\phi}(z) + \delta\phi(z,t) \tag{6.49}$$

and

$$\rho_m(z,t) = \overline{\rho}_m(z) + \delta\rho_m(z,t) \tag{6.50}$$

From the definition of ϕ, the relationship between $\overline{\phi}$ and $\overline{\rho}_m$, $\delta\phi$ and $\delta\rho_m$ can be obtained using order of magnitude analogy. Thus, we find:

$$\overline{\phi}(z) = \ln\left[\frac{\overline{\rho}_m(z)}{\rho_f}\right] \tag{6.51}$$

and

$$\delta\phi(z,t) = \frac{\delta\rho_m(z,t)}{\overline{\rho}_m(z)} \tag{6.52}$$

Now using Eq. 6.43 with Eq. 6.48 and Eq. 6.49, we get

for steady state:

$$\overline{C}_k(z)\frac{d\overline{\phi}}{dz} = -\Omega_0 f(z) \tag{6.53}$$

and,

for first order:

$$\frac{\partial \delta\phi}{\partial t} + \overline{C}_k(z)\frac{\partial \delta\phi}{\partial z} = \delta C_k(t)\frac{\Omega_0 f(z)}{\overline{C}_k(z)} \tag{6.54}$$

For steady state solution, we integrate Eq. 6.51 from $\overline{\lambda}$ to z, we get:

$$\overline{\phi}(z) - \overline{\phi}(\overline{\lambda}) = -\int_{\overline{\lambda}}^{z} \frac{\Omega_0 f(z)}{\overline{C}_k(z)} dz \tag{6.55}$$

From steady state solution of $\overline{C}_k$, we know that:

$$\frac{d}{dz}\overline{C}_k(z) = \Omega_0 f(z) \tag{6.56}$$

Thus, from above 2 equations,

$$\overline{\phi}(z) - \overline{\phi}(\overline{\lambda}) = -\int_{\overline{\lambda}}^{z} \frac{d\overline{C}_k(z)}{\overline{C}_k(z)} = -\ln\left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})}\right] \tag{6.57}$$

Thus, using Eq. 6.57 and Eq. 6.51, we conclude:

$$\frac{\overline{\rho}_m(z)}{\rho_f} = \frac{\overline{C}_k(\overline{\lambda})}{\overline{C}_k(z)} = \frac{\overline{u}_{fi} + u_{gj}}{\overline{u}_{fi} + u_{gj} + \Omega_0\Big[F(z) - F(\overline{\lambda})\Big]} \tag{6.58}$$

For first order solution, we rewrite Eq. 6.54 in the following form:

$$\frac{D_{\overline{C}_k}\delta\phi}{Dt} = \delta C_k(t)\frac{\Omega_0 f(z)}{\overline{C}_k(z)} \tag{6.59}$$

Which directly translates into the following form:

$$dt = \frac{dz}{\overline{C}_k(z)} = \frac{\overline{C}_k(z)}{\delta C_k(t)\Omega_0 f(z)} d\delta\phi \tag{6.60}$$

If we integrate the first equality in the above equation,

$$t - \tau_2 = \int_{\lambda(\tau_2)}^{z} \frac{dz}{\overline{C}_k} \equiv \Big[E(z) - E\Big(\lambda(\tau_2)\Big)\Big] \tag{6.61}$$

Where $E(z)$ is a new function defined by:

$$E(z) = \int \frac{1}{\overline{C}_k} dz \tag{6.62}$$

Thus, time lag can be solved by expanding $\lambda(\tau_2) = \overline{\lambda} + \delta\lambda(\tau_2)$ and retaining only first order terms of Taylor's expansion as follows:

$$t = \tau_2 + \Big[E(z) - E(\overline{\lambda})\Big] - \frac{1}{\overline{C}_k(\overline{\lambda})}\delta\lambda(\tau_2) \tag{6.63}$$

The second equality of Eq. 6.60 can be solved for the following result:

$$d\delta\phi(z,t) = \frac{1}{\overline{C}_k^2(z)} \frac{d\overline{C}_k(z)}{dz} \delta C_k(t) dz \tag{6.64}$$

We have already obtained the expressions for $\delta C_k(t)$ and relationship between t and z, thus, substituting Eq. 6.46 and Eq. 6.63 in the above equation and retaining only first orders of ϵ, we get:

$$d\delta\phi = \epsilon e^{s\tau_2} \frac{1}{\overline{C}_k^2(z)} e^{s[E(z)-E(\overline{\lambda})]} \Lambda_3(s) d\overline{C}_k(z) \tag{6.65}$$

For the ease of understanding, let us define another function such that:

$$H(z,s) = \int \frac{1}{\overline{C}_k^2(z)} e^{s[E(z)-E(\overline{\lambda})]} d\overline{C}_k(z) \tag{6.66}$$

Now, integrating Eq.6.65 from $\lambda(\tau_2)$ to z and retaining only first orders of ϵ, we get:

$$\delta\phi(z,\tau_2) - \delta\phi\Big(\lambda(\tau_2)\Big) = \epsilon e^{s\tau_2} \Lambda_3(s) \Big[H(z,s) - H(\overline{\lambda}, s)\Big] \tag{6.67}$$

For finding the complete solution of the density wave propagation, the above equation needs to be solved for boundary condition in tandem with steady state solution given by Eq. 6.58. Thus:

$$\phi(z,\tau_2) = \overline{\phi}(z) + \delta\phi(z,\tau_2)$$

$$= \ln\left[\frac{\overline{C}_k(\overline{\lambda})}{\overline{C}_k(z)}\right] + \left\{\delta\phi\Big(\lambda(\tau_2)\Big) + \epsilon e^{s\tau_2}\Lambda_3(s)\Big[H(z,s) - H(\overline{\lambda},s)\Big]\right\} \tag{6.68}$$

and the boundary condition for ϕ is given by:

$$\phi\Big(\lambda(\tau_2), \tau_2\Big) = 0 \tag{6.69}$$

Above boundary condition just suggests that the entering fluid in region (C) is in liquid-phase. Applying this boundary condition to equation above, we get:

$$\delta\phi(z,\tau_2) = \epsilon e^{s\tau_2}\left\{\frac{\Omega_0 f(\overline{\lambda})\Lambda_2}{\overline{C}_k(\overline{\lambda})} + \Lambda_3(s)\Big[H(z,s) - H(\overline{\lambda},s)\Big]\right\} \tag{6.70}$$

Invoking Eq. 6.63 in exponential form for the above equation, we find:

$$\epsilon e^{s\tau_2} \doteq \epsilon e^{st} e^{-s[E(z)-E(\overline{\lambda})]} \tag{6.71}$$

Using the above 2 equations, complete solution for perturbed part of density wave propagation equation can be given as:

$$\delta\phi(z,t) = \epsilon e^{st} e^{-s[E(z)-E(\overline{\lambda})]}\left\{\frac{\Omega_0 f(\overline{\lambda})\Lambda_2}{\overline{C}_k(\overline{\lambda})} + \Lambda_3(s)\Big[H(z,s) - H(\overline{\lambda},s)\Big]\right\} \tag{6.72}$$

Although this solution is in terms of ϕ, its easy to convert it to ρ_m, we invoke Eq. 6.50:

$$\frac{\rho_m}{\rho_f} = \frac{\overline{\rho}_m(z)}{\rho_f} + \frac{\delta\rho_m(z,t)}{\rho_f} \tag{6.73}$$

Thus, steady state part is given by:

$$\frac{\overline{\rho}_m(z)}{\rho_f} = \frac{\overline{C}_k(\overline{\lambda})}{\overline{C}_k(z)} \tag{6.74}$$

and for first order,

$$\frac{\delta\rho_m}{\rho_f} = \epsilon e^{st}\Lambda_4(z,s) \tag{6.75}$$

where $\Lambda_4(z,s)$ is given by:

$$\frac{1}{\epsilon e^{st}}\frac{\delta\rho_m}{\rho_f} \equiv \Lambda_4(z,s) =$$
$$= \left[\frac{\overline{C}_k(\overline{\lambda})}{\overline{C}_k(z)}\right] e^{-s[E(z)-E(\overline{\lambda})]} \left\{\frac{\Omega_0 f(\overline{\lambda})\Lambda_2}{\overline{C}_k(\overline{\lambda})} + \Lambda_3(s)\Big[H(z,s) - H(\overline{\lambda},s)\Big]\right\} \quad (6.76)$$

Mixture Velocity Field u_m

We have derived solution for volumetric flux, j and mixture density ρ_m, thus, evaluating mixture velocity u_m is merely an arithmetic task as u_m can be expressed in the following was in terms of j and ρ_m:

$$u_m = j - \left[\frac{\rho_f}{\rho_m} - 1\right] u_{gj} \quad (6.77)$$

Recalling the definition of kinematic wave velocity, $C_k = j + u_{gj}$, thus mixture velocity is given as follows:

$$u_m(z,t) \doteq \left\{\overline{C}_k(z) - \frac{\rho_f}{\overline{\rho}_m} u_{gj}\right\} + \left\{\delta C_k(t) + \frac{\rho_f}{\overline{\rho}_m}\frac{\delta\rho_m}{\overline{\rho}_m} u_{gj}\right\} \quad (6.78)$$

Invoking Eq. 6.74 for above equation, we find:

$$\frac{u_m(z,t)}{\overline{u}_{fi}} = \left\{\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})}\right\} + \left\{\frac{\delta C_k(t)}{\overline{u}_{fi}} + \left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})}\right]^2 \frac{u_{gj}}{\overline{u}_{fi}}\frac{\delta\rho_m}{\rho_f}\right\} \quad (6.79)$$

Using the perturbed response of the kinematic wave velocity δC_k and Mixture Density ρ_m, we can rewrite the above equation in the following form:

$$\frac{u_m(z,t)}{\overline{u}_{fi}} = \left\{\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})}\right\} + \frac{\epsilon e^{st}}{\overline{u}_{fi}}\left\{\Lambda_3(s) + \left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})}\right]^2 u_{gj}\Lambda_4(z,s)\right\} \quad (6.80)$$

Thus, steady state response is given by:

$$\frac{\overline{u}_m(z)}{\overline{u}_{fi}} = \frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})} \quad (6.81)$$

and, perturbed part, which is also a perturbation variable:

$$\frac{\delta u_m(z,t)}{\overline{u}_{fi}} = \frac{\epsilon e^{st}}{\overline{u}_{fi}} \Lambda_5(z,s) \tag{6.82}$$

where Λ_5 is given as:

$$\frac{\delta u_m(z,t)}{\epsilon e^{st}} \equiv \Lambda_5(z,s) = \Lambda_3(s) + \left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})}\right]^2 u_{gj} \Lambda_4(z,s) \tag{6.83}$$

Mixture Enthalpy i_m

As discussed before, the enthalpy response does not directly effect the dynamic nature of the stability boundary, however, it is easy to calculate here for the purpose of future investigation. From Eq. 6.34 the enthalapy field is given as follows:

$$\frac{i_m(z,t)}{i_{f,sat}} = 1 + \frac{\rho_g \Delta i_{fg}}{\Delta \rho i_{f,sat}} \frac{\Omega_0 \left[F(z) - F(\overline{\lambda})\right]}{\overline{C}_k(\overline{\lambda})} - \left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})}\right]^2 \frac{\rho_g \Delta i_{fg}}{\Delta \rho i_{f,sat}} \frac{\delta \rho_m}{\rho_f} \tag{6.84}$$

Residence Time in Region (C)

For the complete description of the system, it is necessary to know the residence time, or the time for which fluid particle remains in the region (C) which is given by τ_{23}. The Eq. 6.63 describes the residence time. For $z = l_c$, we can evaluate τ_{23} as:

$$\tau_{23} = E(l_c) - E(\overline{\lambda}) - \frac{1}{\overline{C}_k(\overline{\lambda})} \delta\lambda(\tau_2) \tag{6.85}$$

From the Eq. 6.27, we can substitute $\delta\lambda(\tau_2)$ as follows in steady and perturbed part:

$$\overline{\tau}_{23} = E(l_c) - E(\overline{\lambda}) \tag{6.86}$$

and

$$\delta\tau_{23}(t) = -\frac{\epsilon e^{st}}{\overline{C}_k(\overline{\lambda})} e^{-s\left[E(l_c) - E(\overline{\lambda})\right]} \Lambda_2(s) \tag{6.87}$$

6.4 Kinematics of Chimney Region before Flashing (D)

As this is an adiabatic region, the velocity field is independent of the position and is just a function of time, the density field however is dependent on both time and position. From the governing equations for region (D) derived in chapter 5, we can express the following, knowing the boundary conditions from exit of Region (C):

$$\frac{u_{me}}{\overline{u}_{fi}} = \frac{\overline{u}_{me}}{\overline{u}_{fi}} + \frac{\delta u_{me}(t)}{\overline{u}_{fi}} = \left[\frac{A_c}{A_e}\right]\left\{\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})} + \frac{\epsilon e^{st}}{\overline{u}_{fi}}\Lambda_5(l_c, s)\right\} \tag{6.88}$$

On the other hand, from the velocity field, we can find the position of the particle with respect to time as follows:

$$z - l_c = \overline{u}_{fi}\left[\frac{A_c}{A_e}\right]\left\{\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}(t - \tau_3) + \frac{\Lambda_5(l_c, s)}{\overline{u}_{fi}}\frac{\epsilon\left[e^{st} - e^{s\tau_3}\right]}{s}\right\} \tag{6.89}$$

The boundary condition for density is given as:

$$\frac{\rho_{me}(z, \tau_3)}{\rho_f} = \frac{\rho_m(l_c, \tau_3)}{\rho_f} = \left[\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\right]^{-1} + \epsilon e^{s\tau_3}\Lambda_4(l_c, s) \tag{6.90}$$

From the above 2 equations, if we eliminate the parameter τ_3, and retain only first powers of ϵ, we get:

$$\frac{\rho_{me}(z, t)}{\rho_f} = \left[\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\right]^{-1} + \epsilon e^{st}\left\{\exp\left[-s\left[\frac{z - l_c}{\overline{u}_{fi}}\right]\left[\frac{\overline{C}_k(\overline{\lambda})}{\overline{C}_k(l_c)}\right]\left[\frac{A_e}{A_c}\right]\right]\Lambda_4(l_c, s)\right\} \tag{6.91}$$

Which includes another transfer function, thus we will define it as:

$$\frac{1}{\epsilon e^{st}}\frac{\delta\rho_{me}(z, t)}{\rho_f} \equiv \Lambda_6(z, s) = \exp\left[-s\left[\frac{z - l_c}{\overline{u}_{fi}}\right]\left[\frac{\overline{C}_k(\overline{\lambda})}{\overline{C}_k(l_c)}\right]\left[\frac{A_e}{A_c}\right]\right]\Lambda_4(l_c, s) \tag{6.92}$$

From Section 5.5, we can calculate $\lambda_{fl}$and thus, we can calculate the residence time for fluid in section (D), which is given by:

$$\tau_{34} = \frac{\lambda_{fl} - l_c}{\overline{u}_{fi}} \left[\frac{A_e}{A_c}\right] \left[\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\right]^{-1} - \epsilon e^{st} \left\{ \left[\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\right]^{-1} \frac{1}{\overline{u}_{fi}} \frac{\left[1 - e^{-s\overline{\tau}_{34}}\right]}{s} \right\} \tag{6.93}$$

Thus, steady and perturbed parts are given by:

$$\overline{\tau}_{34} = \frac{\lambda_{fl} - l_c}{\overline{u}_{fi}} \left[\frac{A_e}{A_c}\right] \left[\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\right]^{-1} \tag{6.94}$$

and,

$$\delta\tau_{34} = -\epsilon e^{st} \left\{ \left[\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\right]^{-1} \frac{1}{\overline{u}_{fi}} \frac{\left[1 - e^{-s\overline{\tau}_{34}}\right]}{s} \right\} \tag{6.95}$$

6.5 Kinematics of Chimney Region after Flashing (E)

From Sections 5.4 and 5.5, we know two very important parameters for the solution of the governing equations for the region (E). The final approximations for $\Gamma_{g,fl}$ and λ_{fl} are very simple in nature and can be approximated as a constant. This approximation is very important to decouple the governing equations to analytically solve them. Thus for the matter of this formulation, both of them should be assumed constant.

However, it should be noted that, if for future investigation one needs more accurate models for $\Gamma_{g,fl}$ and λ_{fl}, it can be easily derived from the formulation by relaxing the assumptions one by one. Such an investigation should be aimed to solve with the help of numerical methods as an analytical solution would not be possible for coupled governing equations.

Thus, with the above assumptions, the kinematics of region (E) looks exactly like kinematics of region (C) but for the boundary conditions, which is given by the exit of the region (D) for the former and by the exit of the region (B) for later. However, as the kinematics of region (D) has been solved in the previous section, a systematic approach is easy to follow and meaningful results could be found as follows:

We assume gas drift flux velocity in this region as constant but scaled by the are ratio to maintain uniformity across the formulation. Thus $u_{gj,fl} = {}^{A_c}/_{A_e} u_{gj}$. So, from Eq. 5.15 and 5.16:

$$\frac{\partial j_{fl}}{\partial z} = \Gamma_{g,fl}\frac{\Delta\rho}{\rho_g \rho_f} \tag{6.96}$$

and:

$$\frac{\partial \rho_{me}}{\partial t} + C_{k,fl}\frac{\partial \rho_{me}}{\partial z} = -\Gamma_{g,fl}\frac{\Delta\rho\rho_{me}}{\rho_g \rho_f} \tag{6.97}$$

where $C_{k,fl}$ is given by:

$$C_{k,fl} \doteq j_{fl} + u_{gj,fl} \tag{6.98}$$

The first equation describes the increase of 'j' due to flashing. The second equation has a form of the wave equation and can be called flashing propagation equation.

Thus, Eq. 6.96 can be used to define characteristic frequency for flashing $\Omega_{fl}(z)$:

$$\frac{\partial j_{fl}}{\partial z} = \Gamma_{g,fl}\frac{\Delta\rho}{\rho_g \rho_f} \equiv \Omega_{fl}(z) \tag{6.99}$$

In terms of average heat flux, $\Omega(z)$ can be expressed similarly:

$$\Omega_{fl}(z) = \Omega_{0,fl} f_{fl}(z) \tag{6.100}$$

Now, Eq. 6.96 and 6.97 can be rewritten by using above definition of $\Omega(z)$ as:

$$\frac{\partial j_{fl}}{\partial z} = \Omega_{0,fl} f_{fl}(z) \tag{6.101}$$

$$\frac{\partial \rho_{me}}{\partial t} + C_{k,fl}\frac{\partial \rho_{me}}{\partial z} = -\Omega_{0,fl} f_{fl}(z)\rho_{me} \tag{6.102}$$

A small note on Function $f_{fl}(z)$

Function $f_{fl}(z)$ is a non-negative continuous function which defines the shape of characteristic frequency for flashing Ω_{fl}, which consequently defines the shape of the $\Gamma_{g,fl}$. Although we have mentioned before that $\Gamma_{g,fl}$ is constant, this formulation gives another way to solve the governing equations without using the coupled version of governing equations. This form of intervention requires one to model the shape of characteristic frequency in a reasonable manner. A few models are discussed below and the reasoning behind it is also stated. *These models were developed by discourses with Prof. Ishii.* We need to take into account some constraints before discussing theses models.

As $f_{fl}(z)$ modifies the shape of $\Gamma_{g,fl}$, we have to make sure that it doesn't change the total gas generation, thus leading up to our first constraint:

$$\frac{1}{l_t - \lambda_{fl}} \int_{\lambda_{fl}}^{l_t} f_{fl}(z) dz = 1 \tag{6.103}$$

The gas generated in a flashing phenomenon does not change back to liquid phase, thus our second constraint:

$$f_{fl}(z) \geq 0 \quad \forall \quad \lambda_{fl} \leq z \leq l_t \tag{6.104}$$

The following models can be used for flashing formulation and can be visualised as Fig. 6.1:

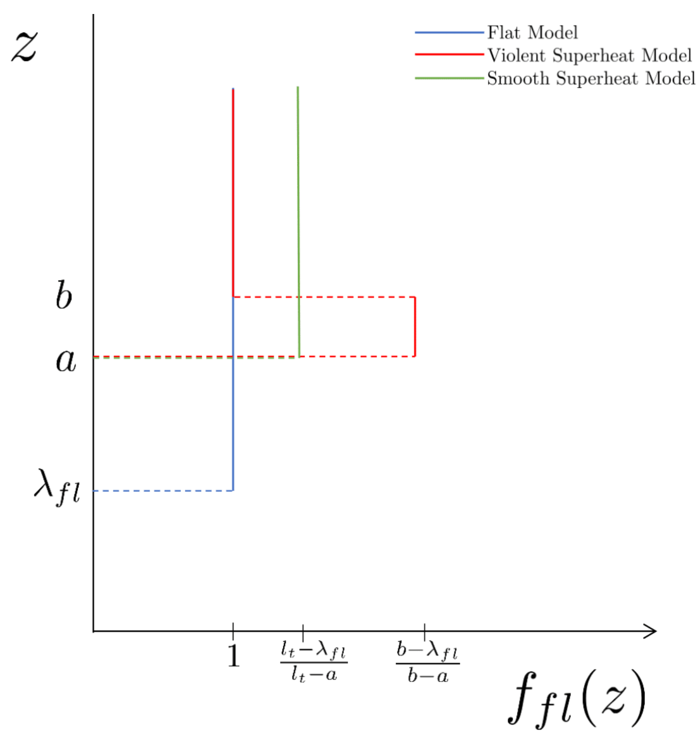


Figure 6.1. Flashing Models

Flat Model

$$f_{fl}(z) = 1 \tag{6.105}$$

This model is according to the theory that nucleation happens as soon as we have a small amount of superheat in the liquid phase, thus suggesting that liquid phase can not hold any superheat. Such an assumption will hold true for almost all engineering systems which have plenty of nucleation sites in the form of welding joint, rivets, bolts, couplings, and fouling. *This model has been used in the formulation*

Violent Superheat Model

$$\begin{aligned}
f_{fl}(z) &= 0 && \text{for} \quad \lambda_{fl} \leq z \leq a \\
f_{fl}(z) &= \frac{b - \lambda_{fl}}{b - a} && \text{for} \quad a < z \leq b \\
f_{fl}(z) &= 1 && \text{for} \quad b < z \leq l_t
\end{aligned} \tag{6.106}$$

Where $\lambda_{fl} < a < b < l_t$. This model allows the fluid to hold slight amount of superheat and release it suddenly in a small time lag and then the gas phase provides for the nucleation sites for further nucleation to happen as per the thermal equilibrium condition between the phases. Parameters a and b can be chosen freely under the constraint mentioned above. Parameter a describes the amount of superheat the fluid can hold. Parameter b dictates how quickly the superheat is released from the liquid phase in terms of latent heat to convert the fluid to gas phase in a violent fashion.

Smooth Superheat Model

$$\begin{aligned}
f_{fl}(z) &= 0 && \text{for} \quad \lambda_{fl} \leq z \leq a \\
f_{fl}(z) &= \frac{l_t - \lambda_{fl}}{l_t - a} && \text{for} \quad a < z \leq l_t
\end{aligned} \tag{6.107}$$

Where $\lambda_{fl} < a < l_t$. As the model described above, this model does also allows some superheat to accumulate in the fluid before flashing, but the gas generation is not violent and happens at a constant rate after nucleation starts. This type of model can be justified for systems with relatively high flow rates where not enough time is given to the fluid to get rid of all of its superheat. *It should also be noted that this model has only one parameter and can be fine-tuned relatively easier than the model discussed above.*

Kinematic Wave Velocity for Flashing $C_{k,fl}$

Integration of the volumetric flux equation 6.96 with boundary conditions at the end of region (D) can be integrated directly and yields:

$$j_{fl}(z,t) = u_{me}(\lambda_{fl},t) + \Omega_{0,fl}\int_{\lambda_{fl}}^{z} f_{fl}(z)dz \tag{6.108}$$

Defining $F_{fl}(z)$ as integral of $f_{fl}(z)dz$, we get:

$$j_{fl}(z,t) = u_{me}(\lambda_{fl},t) + \Omega_0\Big[F_{fl}(z) - F_{fl}(\lambda_{fl})\Big] \tag{6.109}$$

Now, using Eq. 6.98 we get:

$$C_{k,fl}(z,t) = u_{me}(\lambda_{fl},t) + u_{gj,fl} + \Omega_{0,fl}\Big[F_{fl}(z) - F_{fl}(\lambda_{fl})\Big] \tag{6.110}$$

The steady state solution of the above equation is given as:

$$\overline{C}_{k,fl}(z) = \overline{u}_{me}(\lambda_{fl}) + u_{gj,fl} + \Omega_{0,fl}\Big[F_{fl}(z) - F_{fl}\Big(\lambda_{fl}\Big)\Big] \tag{6.111}$$

and the first order solution is given as:

$$\delta C_{k,fl}(t) = \delta u_{me}(t) \tag{6.112}$$

Thus another Transfer Function is defined as:

$$\frac{\delta C_{k,fl}(t)}{\epsilon e^{st}} \equiv \Lambda_7(s) = \left[\frac{A_c}{A_e}\right]\Lambda_5(l_c,s) \tag{6.113}$$

Density Wave Propagation

Now we know the solution of kinematic wave velocity for flashing $C_{k,fl}$, and we know gas generation due to flashing $\Gamma_{g,fl}$, thus, density propagation equation can be solved by introducing a new variable ϕ_{fl} which is defined as follows:

$$\phi_{fl}(z,t) \equiv \ln\left[\frac{\rho_{me,fl}(z,t)}{\overline{\rho}_{me,fl}(\lambda_{fl})}\right] \tag{6.114}$$

Thus, Eq. 6.102 can be morphed to the following form:

$$\frac{\partial \phi_{fl}}{\partial t} + C_{k,fl}(z,t)\frac{\partial \phi_{fl}}{\partial z} = -\Omega_{0,fl} f_{fl}(z) \tag{6.115}$$

For applying the perturbation, we need to define the perturbation variables as follows:

$$\phi_{fl}(z,t) = \overline{\phi}_{fl}(z) + \delta\phi_{fl}(z,t) \tag{6.116}$$

and

$$\rho_{me,fl}(z,t) = \overline{\rho}_{me,fl}(z) + \delta\rho_{me,fl}(z,t) \tag{6.117}$$

From the definition of ϕ_{fl}, the relationship between $\overline{\phi}_{fl}$ and $\overline{\rho}_{me}$, $\delta\phi_{fl}$ and $\delta\rho_{me}$ can be obtained using order of magnitude analogy. Thus, we find:

$$\overline{\phi}_{fl}(z) = \ln\left[\frac{\overline{\rho}_{me,fl}(z)}{\overline{\rho}_{me,fl}(\lambda_{fl})}\right] \tag{6.118}$$

and

$$\delta\phi_{fl}(z,t) = \frac{\delta\rho_{me,fl}(z,t)}{\overline{\rho}_{me,fl}(z)} \tag{6.119}$$

Now using Eq. 6.110 with Eq. 6.115 and Eq. 6.116, we get

for steady state:

$$\overline{C}_{k,fl}(z)\frac{d\overline{\phi}_{fl}}{dz} = -\Omega_{0,fl} f_{fl}(z) \tag{6.120}$$

and,

for first order:

$$\frac{\partial \delta\phi_{fl}}{\partial t} + \overline{C}_{k,fl}(z)\frac{\partial \delta\phi_{fl}}{\partial z} = \delta C_{k,fl}(t)\frac{\Omega_{0,fl} f_{fl}(z)}{\overline{C}_{k,fl}(z)} \tag{6.121}$$

For steady state solution, we integrate Eq. 6.118 from λ_{fl} to z, we get:

$$\overline{\phi_{fl}}(z) - \overline{\phi_{fl}}(\lambda_{fl}) = -\int_{\lambda_{fl}}^{z} \frac{\Omega_{0,fl} f_{fl}(z)}{\overline{C}_{k,fl}(z)} dz \tag{6.122}$$

From steady state solution of $\overline{C}_{k,fl}$, we know that:

$$\frac{d}{dz}\overline{C}_{k,fl}(z) = \Omega_{0,fl} f_{fl}(z) \tag{6.123}$$

Thus, from above 2 equations,

$$\overline{\phi}_{fl}(z) - \overline{\phi}_{fl}(\lambda_{fl}) = -\int_{\lambda_{fl}}^{z} \frac{d\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(z)} = -\ln\left[\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})}\right] \tag{6.124}$$

Thus, using Eq. 6.124 and Eq. 6.118, we conclude:

$$\frac{\overline{\rho}_{me,fl}(z)}{\overline{\rho}_{me,fl}(\lambda_{fl})} = \frac{\overline{C}_{k,fl}(\lambda_{fl})}{\overline{C}_{k,fl}(z)} = \frac{\overline{u}_{me}(\lambda_{fl}) + u_{gj,fl}}{\overline{u}_{me}(\lambda_{fl}) + u_{gj,fl} + \Omega_{0,fl}\Big[F_{fl}(z) - F_{fl}(\lambda_{fl})\Big]} \tag{6.125}$$

For first order solution, we rewrite Eq. 6.121 in the following form:

$$\frac{D_{\overline{C}_{k,fl}}\delta\phi_{fl}}{Dt} = \delta C_{k,fl}(t)\frac{\Omega_{0,fl} f_{fl}(z)}{\overline{C}_{k,fl}(z)} \tag{6.126}$$

Which directly translates into the following form:

$$dt = \frac{dz}{\overline{C}_{k,fl}(z)} = \frac{\overline{C}_{k,fl}(z)}{\delta C_{k,fl}(t)\Omega_{0,fl} f_{fl}(z)} d\delta\phi_{fl} \tag{6.127}$$

If we integrate the first equality in the above equation,

$$t - \tau_4 = \int_{\lambda_{fl}}^{z} \frac{dz}{\overline{C}_{k,fl}} \equiv \Big[E_{fl}(z) - E_{fl}(\lambda_{fl})\Big] \tag{6.128}$$

Where $E_{fl}(z)$ is a new function defined by:

$$E_{fl}(z) = \int \frac{1}{\overline{C}_{k,fl}} dz \tag{6.129}$$

Thus, time lag can be solved as follows:

$$t = \tau_4 + \Big[E(z) - E(\lambda_{fl})\Big] \tag{6.130}$$

The second equality of Eq. 6.127 can be solved for the following result:

$$d\delta\phi_{fl} = \frac{1}{\overline{C}^2_{k,fl}(z)} \frac{d\overline{C}_{k,fl}(z)}{dz} \delta C_{k,fl}(t) dz \tag{6.131}$$

We have already obtained the expressions for $\delta C_{k,fl}(t)$ and relationship between t and z, thus, substituting Eq. 6.113 and Eq. 6.130 in the above equation and retaining only first orders of ϵ, we get:

$$d\delta\phi_{fl} = \epsilon e^{s\tau_4} \frac{1}{\overline{C}^2_{k,fl}(z)} e^{s[E_{fl}(z) - E_{fl}(\lambda_{fl})]} \Lambda_7(s) d\overline{C}_{k,fl}(z) \tag{6.132}$$

For the ease of understanding, let us define another function such that:

$$H_{fl}(z,s) = \int \frac{1}{\overline{C}^2_{k,fl}(z)} e^{s[E_{fl}(z) - E_{fl}(\lambda_{fl})]} d\overline{C}_{k,fl}(z) \tag{6.133}$$

Now, integrating Eq.6.132 from λ_{fl} to z and retaining only first orders of ϵ, we get:

$$\delta\phi(z,\tau_4) - \delta\phi(\lambda_{fl}) = \epsilon e^{s\tau_4} \Lambda_7(s) \Big[H_{fl}(z,s) - H(\lambda_{fl}, s)\Big] \tag{6.134}$$

For finding the complete solution of the density wave propagation, the above equation needs to be solved for boundary condition in tandem with steady state solution given by Eq. 6.125. Thus:

$$\phi_{fl}(z,\tau_4) = \overline{\phi}_{fl}(z) + \delta\phi(z,\tau_4)$$

$$= \ln\left[\frac{\overline{C}_{k,fl}(\lambda_{fl})}{\overline{C}_{k,fl}(z)}\right] + \left\{\delta\phi_{fl}\Big(\lambda_{fl}\Big) + \epsilon e^{s\tau_4}\Lambda_7(s)\Big[H_{fl}(z,s) - H_{fl}(\lambda_{fl},s)\Big]\right\} \quad (6.135)$$

and the boundary condition for ϕ is given by:

$$\phi\Big(\lambda_{fl},\tau_4\Big) = 0 \quad (6.136)$$

Using this boundary condition, we get:

$$\delta\phi_{fl}(z,\tau_4) = \epsilon e^{s\tau_4}\left\{\Lambda_7(s)\Big[H_{fl}(z,s) - H_{fl}(\overline{\lambda},s)\Big]\right\} \quad (6.137)$$

Invoking Eq. 6.130 in exponential form for the above equation, we find:

$$\epsilon e^{s\tau_4} \doteq \epsilon e^{st} e^{-s[E_{fl}(z) - E_{fl}(\lambda_{fl})]} \quad (6.138)$$

Using the above 2 Equations, complete solution for perturbed part of density wave propagation equation can be given as:

$$\delta\phi_{fl}(z,t) = \epsilon e^{st} e^{-s[E_{fl}(z) - E_{fl}(\overline{\lambda})]}\left\{\Lambda_7(s)\Big[H_{fl}(z,s) - H_{fl}(\lambda_{fl},s)\Big]\right\} \quad (6.139)$$

Although this solution is in terms of ϕ_{fl}, its easy to convert it to $\rho_{me,fl}$, we invoke Eq. 6.117:

$$\frac{\rho_{me,fl}(z)}{\overline{\rho}_{me,fl}(\lambda_{fl})} = \frac{\overline{\rho}_{me,fl}(z)}{\overline{\rho}_{me,fl}(\lambda_{fl})} + \frac{\delta\rho_{me,fl}(z,t)}{\overline{\rho}_{me,fl}(\lambda_{fl})} \quad (6.140)$$

Thus, steady state part is given by:

$$\frac{\overline{\rho}_{me,fl}(z)}{\overline{\rho}_{me,fl}(\lambda_{fl})} = \frac{\overline{C}_{k,fl}(\lambda_{fl})}{\overline{C}_{k,fl}(z)} \quad (6.141)$$

and for first order,

$$\frac{\delta\rho_{me,fl}(z,t)}{\overline{\rho}_{me,fl}(\lambda_{fl})} = \epsilon e^{st}\Lambda_8(z,s) \quad (6.142)$$

where $\Lambda_8(z,s)$ is given by:

$$\frac{1}{\epsilon e^{st}} \frac{\delta \rho_{me,fl}}{\overline{\rho}_{me,fl}(\lambda_{fl})} \equiv \Lambda_8(z,s) =$$

$$= \left[\frac{\overline{C}_{k,fl}(\lambda_{fl})}{\overline{C}_{k,fl}(z)}\right] e^{-s[E_{fl}(z)-E_{fl}(\lambda_{fl})]} \left\{\Lambda_7(s)\Big[H_{fl}(z,s) - H_{fl}(\lambda_{fl},s)\Big]\right\} \quad (6.143)$$

Mixture Velocity Field u_m

We have derived solution for volumetric flux due to flashing, j_{fl} and mixture density in flashing region $\rho_{me,fl}$, thus, evaluating mixture velocity in flashing region $u_{me,fl}$ is merely an arithmetic task as u_m can be expressed in the following way in terms of j_{fl} and $\rho_{me,fl}$:

$$u_{me,fl}(z,t) = j_{fl}(z,t) - \left[\frac{\overline{\rho}_{me,fl}(\lambda_{fl})}{\rho_{me,fl}(z,t)} - 1\right] u_{gj,fl} \quad (6.144)$$

Recalling the definition of kinematic wave velocity for flashing, $C_{k,fl} = j_{fl} + u_{gj,fl}$, thus mixture velocity is given as follows:

$$u_{me,fl}(z,t) \doteq \left\{\overline{C}_{k,fl}(z) - \frac{\overline{\rho}_{me,fl}(\lambda_{fl})}{\overline{\rho}_{me,fl}} u_{gj,fl}\right\} + \left\{\delta C_{k,fl}(t) + \frac{\rho_f}{\overline{\rho}_{me,fl}} \frac{\delta \rho_{me,fl}}{\overline{\rho}_{me,fl}} u_{gj,fl}\right\} \quad (6.145)$$

Invoking Eq. 6.141 for above equation, we find:

$$\frac{u_{me,fl}(z,t)}{\overline{u}_{me,fl}(\lambda_{fl})} = \left\{\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})}\right\} + \left\{\frac{\delta C_{k,fl}(t)}{\overline{u}_{fi}} + \left[\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})}\right]^2 \frac{u_{gj,fl}}{\overline{u}_{fi}} \frac{\delta \rho_{me,fl}}{\rho_f}\right\} \quad (6.146)$$

Using the perturbed response of the kinematic wave velocity due to flashing $\delta C_{k,fl}$ and mixture density in flashing region, $\rho_{me,fl}$, we can rewrite the above equation in the following form:

$$\frac{u_{me,fl}(z,t)}{\overline{u}_{me,fl}(\lambda_{fl})} = \left\{\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})}\right\} + \frac{\epsilon e^{st}}{\overline{u}_{me,fl}(\lambda_{fl})} \left\{\Lambda_7(s) + \left[\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\overline{\lambda})}\right]^2 u_{gj,fl} \Lambda_8(z,s)\right\} \quad (6.147)$$

Thus, steady state response is given by:

$$\frac{\overline{u}_{me,fl}(z)}{\overline{u}_{me,fl}(\lambda_{fl})} = \frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})} \quad (6.148)$$

and, perturbed part, which is also a perturbation variable:

$$\frac{\delta u_{me,fl}(z,t)}{\overline{u}_{me,fl}(\lambda_{fl})} = \frac{\epsilon e^{st}}{\overline{u}_{me,fl}(\lambda_{fl})}\Lambda_9(z,s) \tag{6.149}$$

where Λ_9 is given as:

$$\frac{\delta u_{me,fl}(z,t)}{\epsilon e^{st}} \equiv \Lambda_9(z,s) = \Lambda_7(s) + \left[\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})}\right]^2 u_{gj,fl}\Lambda_8(z,s) \tag{6.150}$$

Mixture Enthalpy i_m

It has been previously discussed in Chapter 5 that the enthalpy of the two-phase mixture in the chimney region after flashing is insignificant to the dynamic problem, however, it is easy to evaluate and is equal to mixture saturation enthalpy at local pressure. Although another direct approach can be applied to find the enthalpy field. As the Chimney Regions (D) and (E) are both adiabatic, the enthalpy is given as the exit enthalpy from region C. This statement is slightly inaccurate as a result of Gas Drift Flux Velocity due to Flashing $u_{gj,fl}$. Because of this, the gas phase moves faster than the liquid phase, taking away heat faster from the fluid than the liquid phase. Thus, it should be pondered upon before formulating flashing phenomena by Numerical Methods.

Residence Time in Region (E)

For the complete description of the system, it is necessary to know the residence time, or the time for which fluid particle remains in the region (E) which is given by τ_{45}. The Eq. 6.130 describes the residence time. For $z = l_t$, we can evaluate τ_{45} as:

$$\tau_{45} = E_{fl}(l_t) - E_{fl}(\lambda_{fl}) \equiv \overline{\tau}_{45} \tag{6.151}$$

7. DYNAMICS OF FLUID

7.1 Introduction

In the previous chapters 5 and 6, the field equations have been described from the conservation equations, and kinematics of the fluid has been solved to find velocity and density response of the system. These responses are particularly important to solve the momentum conservation equations to find the pressure response of the system. As pressure response is a function of velocity perturbation, a transfer function can describe the dynamic behavior of the system.

In this chapter, we shall continue to follow the compartmentalized approach described in Chapter 5 where we divided the system into five regions and then solved the governing equations for those regions.

As per our formulation, we have assumed velocity perturbations to be comparatively much smaller than fluid velocity itself, thus friction factor f_s, f_m are to be assumed constant too. For the single-phase friction factor, we reach such a conclusion because of two reasons. One, friction factor is inversely proportional to one-fourth power of Reynolds Number, thus mathematically it is not a significant term. Two, the large eddies are not sensitive to small changes in velocity. For the two-phase friction factor, we can say that for natural circulation gravity-dominated flow, flow quality is low, thus the above-described reasons still apply.

7.2 Pressure Drop in the Upstream Un-heated Region (A)

For region (A), momentum equation is given by Eq. 5.2 and to solve it, velocity and density fields are given by Eq. 6.1 and 6.5. Integrating the momentum equation from $-l_o$ to 0 we get:

$$\Delta P_{01} = \int_{-l_o}^{0} \rho_f \left\{ \frac{\partial u_{fo}}{\partial t} + u_{fo}\frac{\partial u_{fo}}{\partial z} + g_o + \left(\frac{f_o}{2D_o}\right) u_{fo}^2 \right\} dz + k_i \rho_f u_{fi}^2 \tag{7.1}$$

Integrating above equation and retaining only first order terms in ϵ, we get:

$$\Delta P_{01} = \rho_f \left\{ g_o l_o + \frac{f_o l_o}{2D_o} \left[\frac{A_c}{A_o} \right]^2 \overline{u}_{fi}^2 + k_i \overline{u}_{fi}^2 \right\}$$
$$+ \rho_f \left\{ \left[\frac{A_c}{A_o} \right] s l_o + \frac{f_o l_o}{2D_o} 2 \left[\frac{A_c}{A_o} \right]^2 \overline{u}_{fi} + 2k_i \overline{u}_{fi} \right\} \delta u_{fi} \equiv \overline{\Delta P}_{01} + \delta \Delta P_{01} \quad (7.2)$$

Which includes a perturbation parameter which describes the change of pressure drop in region (A) with perturbation in velocity, thus we define:

$$\frac{\delta \Delta P_{01}}{\delta u_{fi}} \equiv \Lambda_A(s) = \rho_f \left\{ \left[\frac{A_c}{A_o} \right] s l_o + \frac{f_o l_o}{2D_o} 2 \left[\frac{A_c}{A_o} \right]^2 \overline{u}_{fi} + 2k_i \overline{u}_{fi} \right\} \quad (7.3)$$

7.3 Pressure Drop in Single Phase Heated Region (B)

As pressure drop response is calculated in the above section, pressure drop in region (B), ΔP_{12}, is found in a similar way by integrating Eq. 5.5 from 0 to λ using velocity field given by Eq. 6.7 and constant density assumption. The final expression for pressure drop can be given as follows:

$$\Delta P_{12} = \rho_f \left[g + \frac{f_s}{2D} \overline{u}_{fi}^2 \right] \overline{\lambda}$$
$$+ \rho_f \left[s + \frac{f_s}{2D} 2\overline{u}_{fi} \right] \overline{\lambda} \delta u_{fi} + \rho_f \left[g + \frac{f_s}{2D} \overline{u}_{fi}^2 \right] \delta \lambda \equiv \overline{\Delta P}_{12} + \delta \Delta P_{12} \quad (7.4)$$

This equation can be simplified with the help of Eq. 6.27 and the perturbed part can be written in the form of perturbation parameter which is described as:

$$\frac{\delta \Delta P_{12}}{\delta u_{fi}} \equiv \Lambda_B(s) = \rho_f \left\{ s\overline{\lambda} + \frac{f_s}{2D} 2\overline{u}_{fi} \overline{\lambda} + \left[g + \frac{f_s}{2D} \overline{u}_{fi}^2 \right] \Lambda_2(s) \right\} \quad (7.5)$$

The right-hand side of the above equation describes the various effects on the pressure response. The first term describes the effect of inertia, the second term is the effect of perturbation of velocity on the frictional pressure drop, the third term describes the effect of perturbing boiling boundary.

7.4 Pressure Drop in Two-Phase Heated Region (C)

The mixture momentum equation, Eq. 5.9 is integrated similarly as above but as the Velocity and Density fields given by Eq. 6.80 and 6.73 are a function of z as well, it is somewhat complicated and the solution will be described step wise in this section.

Integration of 5.9 is done from boiling boundary $\lambda(t)$ to l_c.

$$\Delta P_{23} = \int_{\lambda}^{l_c} \left\{ \rho_m \left[\frac{\partial u_m}{\partial t} + u_m \frac{\partial u_m}{\partial z} \right] + g\rho_m + \frac{f_m}{2D}\rho_m u_m^2 + \frac{\partial}{\partial z}\left[\frac{\rho_f - \rho_m}{\rho_m - \rho_g} \frac{\rho_g \rho_f}{\rho_m} u_{gj}^2 \right] \right\} dz \tag{7.6}$$

Each term from the above equation is solved individually.

7.4.1 The Inertial Term

The acceleration pressure drop in the momentum equation is given by:

$$\Delta P_{23,a} = \int_{\lambda}^{l_c} \rho_m \frac{\partial u_m}{\partial t} dz \tag{7.7}$$

Substituting Eq. 6.73 and 6.80 into the above equation, we get:

$$\Delta P_{23,a} = \epsilon e^{st} s \rho_f \int_{\overline{\lambda}}^{l_c} \left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})} \right]^{-1} \Lambda_5(z,s) dz \equiv \delta \Delta P_{23,a} \tag{7.8}$$

Which is a transfer function and thus we define:

$$\frac{\delta \Delta P_{23,a}}{\delta u_{fi}} \equiv \Lambda_{10} = s\rho_f \int_{\overline{\lambda}}^{l_c} \left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})} \right]^{-1} \Lambda_5(z,s) dz \tag{7.9}$$

It should be noted that Inertial Term's steady state part is zero.

7.4.2 Convective Acceleration Term

The convective acceleration term is given by:

$$\Delta_{23,c} = \int_{\lambda}^{l_c} \rho_m u_m \frac{\partial u_m}{\partial z} dz \tag{7.10}$$

Substituting Eq. 6.73 and 6.80 into the above equation, we get:

$$\Delta P_{23,c} = \rho_f \overline{u}_{fi}^2 \left[\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})} - 1 \right] + \epsilon e^{st} \Lambda_{11}(s) \tag{7.11}$$

Where $\Lambda_{11}(s)$ is a transfer function and is defined as follows:

$$\begin{aligned} \lambda_{11}(s) = \rho_f \overline{u}_{fi}^2 \Bigg\{ & \left[\left(\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})} \right)^2 \Lambda_4(l_c, s) - \Lambda_4(\overline{\lambda}, s) \right] \\ & + \frac{2}{\overline{u}_{fi}} \left[\Lambda_5(l, s) - \Lambda_5(\overline{\lambda}, s) \right] - \frac{\Omega_0}{\overline{C}_k(\overline{\lambda})} f(\overline{\lambda}) \lambda_2(s) \\ & + \frac{1}{\overline{u}_{fi}} s \int_{\overline{\lambda}}^{l_c} \left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})} \right] \Lambda_4(z, s) dz \Bigg\} \end{aligned} \tag{7.12}$$

7.4.3 The Gravitational Term

The gravitational term is given by:

$$\Delta P_{23,g} = \int_{\lambda}^{l_c} g \rho_m dz \tag{7.13}$$

Substituting Eq. 6.73 into the above equation, we get:

$$\begin{aligned} \Delta P_{23,g} = g \rho_f \Bigg\{ \int_{\overline{\lambda}}^{l_c} \left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})} \right]^{-1} dz + \epsilon e^{st} \left[\int_{\overline{\lambda}}^{l_c} \Lambda_4(z, s) dz - \Lambda_2(s) \right] \Bigg\} \\ \equiv \overline{\Delta P}_{23,g} + \delta \Delta P_{23,g} \end{aligned} \tag{7.14}$$

Which contains another transfer function given by:

$$\frac{\delta \Delta P_{23,g}}{\delta u_{fi}} + g \rho_f \Lambda_2(s) \equiv \Lambda_{12}(s) = g \rho_f \int_{\overline{\lambda}}^{l_c} \Lambda_4(z, s) dz \tag{7.15}$$

7.4.4 The Frictional Term

The frictional term is given by:

$$\Delta P_{23,f} = \int_{\lambda}^{l_c} \frac{f_m}{2D}\rho_m u_m^2 dz = \int_{\overline{\lambda}}^{l_c} \frac{f_m}{2D}\rho_m u_m^2 dz - \frac{f_s}{2D}\rho_f \overline{u}_{fi}^2 \delta\lambda \tag{7.16}$$

From Eq. 6.80 and 6.73, retaining only first powers of ϵ, we get Steady State and Fisrt Order terms are given as:

$$\overline{\Delta P}_{23,f} = \frac{f_m}{2D}\int_{\overline{\lambda}}^{l_c} \rho_f \overline{u}_{fi}^2 \left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})}\right] dz \tag{7.17}$$

and,

$$\delta P_{23,f} = \epsilon e^{st}\left\{\Lambda_{13}(s) - \frac{f_s}{2D}\overline{u}_{fi}^2 \Lambda_2(s)\right\} \tag{7.18}$$

Where Λ_{13} is a transfer function and is given as:

$$\frac{\delta P_{23,f}}{\delta u_{fi}} + \frac{f_s}{2D}\overline{u}_{fi}^2\Lambda_2(s) \equiv \Lambda_{13}(s) = \frac{f_m}{2D}\rho_f \overline{u}_{fi}^2 \int_{\overline{\lambda}}^{l_c} \left\{\left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})}\right]^2 \Lambda_4(z,s) + \frac{2}{\overline{u}_{fi}}\Lambda_5(z,s)\right\} dz \tag{7.19}$$

7.4.5 The Drift Stress Term

The drift stress term is given as:

$$\Delta P_{23,d} = \int_{\overline{\lambda}}^{l_c} \frac{\partial}{\partial z}\left\{\frac{\rho_f - \rho_m}{\rho_m - \rho_g}\frac{\rho_g \rho_f}{\rho_m} u_{gj}^2\right\} dz \tag{7.20}$$

As we know gas drift flux velocity u_{gj} is assumed to be constant, the above equation is easy to integrate and for proper limits, we get:

$$\Delta P_{23,d} = \frac{\rho_f - \rho_m(l_c,t)}{\rho_m(l_c,t) - \rho_g}\frac{\rho_g \rho_f}{\rho_m(l_c,t)} u_{gj}^2 \tag{7.21}$$

The above equation contains a singularity where $\rho_m \to \rho_g$. This is the state where gas phase is significantly larger than liquid phase and flow quality, $x \to 1$. This case is insignificant for the current problem as the system will never reach this condition. Thus, retaining only first powers of ϵ, we get:

$$\Delta P_{23,d} = \left[\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})} - 1\right]\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\rho_g u_{gj}^2 + \epsilon e^{st}\left[1 - 2\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\right]\left(\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\right)^2 \rho_g u_{gj}^2 \Lambda_4(l_c, s) \quad (7.22)$$

Where Steady State and First Order terms are given by:

$$\overline{\Delta P}_{23,d} = \left[\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})} - 1\right]\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\rho_g u_{gj}^2 \quad (7.23)$$

and,

$$\frac{\delta \Delta P_{23,d}}{\partial u_{fi}} \equiv \Lambda_{14}(s) = \left[1 - 2\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\right]\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}^2 \rho_g u_{gj}^2 \Lambda_4(l_c, s) \quad (7.24)$$

7.4.6 Total Pressure Drop in the Heated Mixture Region (C)

From all the above results of different terms of the pressure response, net pressure response can be evaluated by adding all the terms, thus Steady State and First Order terms for pressure drop in region (C) ΔP_{23} is given as:

$$\begin{aligned}\Delta P_{23} = \overline{\Delta P}_{23,c} + \overline{\Delta P}_{23,g} + \overline{\Delta P}_{23,f} + \overline{\Delta P}_{23,d} \\ + \delta\Delta P_{23,a} + \delta\Delta P_{23,c} + \delta\Delta P_{23,g} + \delta\Delta P_{23,f} + \delta\Delta P_{23,d} \\ \equiv \overline{\Delta P}_{23} + \delta\Delta P_{23} \quad (7.25)\end{aligned}$$

The perturbed pressure drop can also be expressed in terms of perturbation parameters as:

$$\begin{aligned}\delta\Delta P_{23} = \epsilon e^{st}\Big\{\Lambda_{10}(s) + \Lambda_{11}(s) + \Lambda_{12}(s) + \Lambda_{13}(s) + \Lambda_{14}(s)\Big\} \\ - \epsilon e^{st}\Big\{g\rho_f + \frac{f_s}{2D}\rho_f \overline{u}_{fi}^2\Big\}\Lambda_2(s) \equiv \epsilon e^{st}\Lambda_C \quad (7.26)\end{aligned}$$

7.5 Pressure Drop in Chimney Region before Flashing (D)

The mixture momentum equation, Eq. 5.14 is integrated in a similar manner as above as the velocity and density fields given by Eq. 6.88 and 6.91 are a function of z as well, it is somewhat complicated and the solution will be described step wise in this section.

Integration of 5.14 is done from exit of heater section l_c to Flashing Boundary λ_{fl}.

$$\Delta P_{34} = \int_{l_c}^{\lambda_{fl}} \rho_{me} \left\{ \frac{\partial u_{me}}{\partial t} + u_{me} \frac{\partial u_{me}}{\partial z} \right\} + g_e \rho_{me} + \left(\frac{f_{me}}{2D_e} \rho_{me} + k_e \right) u_{me}^2 + \frac{\partial}{\partial z} \left\{ \frac{\rho_f - \rho_{me}}{\rho_{me} - \rho_g} \frac{\rho_g \rho_f}{\rho_{me}} u_{gj}^2 \right\} dz \quad (7.27)$$

Each term from the above equation is solved individually.

7.5.1 The Exit Pressure Drop

For exit pressure drop we have:

$$\Delta P_{34,e} = k_e \rho_m u_m^2 = k_e \rho_f \overline{u}_{fi}^2 \frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})} + \epsilon e^{st} \Lambda_{16}(s) \equiv \overline{\Delta P}_{34} + \delta \Delta P_{34} \quad (7.28)$$

Where the transfer function Λ_{15} is given by:

$$\frac{\delta \Delta P_{34,e}}{\delta u_{fi}} \equiv \Lambda_{15}(s) = k_e \rho_f \overline{u}_{fi}^2 \left[\frac{2}{\overline{u}_{fi}} \Lambda_5(l_c, s) + \frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})} \Lambda_4(l_c, s) \right] \quad (7.29)$$

7.5.2 The Inertial Term

The acceleration pressure drop in the momentum equation is given by:

$$\Delta P_{34,a} = \int_{l_c}^{\lambda_{fl}} \rho_{me} \frac{\partial u_{me}}{\partial t} dz \quad (7.30)$$

Substituting Eq. 6.91 and 6.88 into the above equation, we get:

$$\Delta P_{34,a} = \epsilon e^{st} s \rho_f \left[\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})} \right]^{-1} \left[\frac{A_c}{A_e} \right] \Lambda_5(l_c, s) dz \equiv \delta \Delta P_{23,a} \quad (7.31)$$

Which is a transfer function and have a similar form to Λ_7, so from Eq. 6.113, we define:

$$\frac{\delta\Delta P_{34,a}}{\delta u_{fi}} \equiv \Lambda_{16}(s) = s\rho_f \left[\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\right]^{-1} \Lambda_7(s) \tag{7.32}$$

It should be noted that inertial term's steady state part is zero.

7.5.3 Convective Acceleration Term

The convective acceleration term is given by:

$$\Delta P_{34,c} = \int_{l_c}^{\lambda_{fl}} \rho_{me} u_{me} \frac{\partial u_{me}}{\partial z} dz = 0 \tag{7.33}$$

As velocity field is not a function of z in Region (D)

7.5.4 The Gravitational Term

The gravitational term is given by:

$$\Delta P_{34,g} = \int_{l_c}^{\lambda_{fl}} g\rho_{me} dz \tag{7.34}$$

Substituting Eq. 6.91 into the above equation, we get:

$$\begin{aligned}\Delta P_{34,g} = g_e\rho_f \left\{ \left[\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\right]^{-1} (\lambda_{fl} - l_c) + \epsilon e^{st} \left[\overline{u}_{fi} \frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})} \left[\frac{A_c}{A_e}\right] \frac{1 - e^{-s\overline{\tau}_{34}}}{s} \Lambda_4(l_c, s)\right]\right\} \\ \equiv \overline{\Delta P}_{34,g} + \delta\Delta P_{34,g}\end{aligned} \tag{7.35}$$

Which contains another transfer function given by:

$$\frac{\delta\Delta P_{34,g}}{\delta u_{fi}} \equiv \Lambda_{17}(s) = g\rho_f \left[\overline{u}_{fi} \frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})} \left[\frac{A_c}{A_e}\right] \frac{1 - e^{-s\overline{\tau}_{34}}}{s} \Lambda_4(l_c, s)\right] \tag{7.36}$$

7.5.5 The Frictional Term

The frictional term is given by:

$$\Delta P_{34,f} = \int_{l_c}^{\lambda_{fl}} \frac{f_{me}}{2D_e} \rho_{me} u_{me}^2 dz \tag{7.37}$$

From Eq. 6.88 and 6.91, retaining only first powers of ϵ, we get Steady State and Fisrt Order terms are given as:

$$\overline{\Delta P}_{34,f} = \frac{f_{me}}{2D_e} \rho_f \overline{u}_{fi}^2 \left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})} \right] \left[\frac{A_c}{A_e} \right]^2 (\lambda_{fl} - l_c) \tag{7.38}$$

and,

$$\delta P_{34,f} = \epsilon e^{st} \Lambda_{18}(s) \tag{7.39}$$

Where Λ_{18} is a transfer function and is given as:

$$\frac{\delta P_{34,f}}{\delta u_{fi}} \equiv \Lambda_{18}(s) = \frac{f_{me}}{2D_e} \rho_f \overline{u}_{fi}^2 \left\{ \overline{u}_{fi} \left[\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})} \right]^3 \frac{1 - e^{-s\overline{\tau}_{34}}}{s} \Lambda_4(l_c, s) + \frac{2}{\overline{u}_{fi}} \left[\frac{A_c}{A_e} \right]^2 \Lambda_5(z, s) \right\} dz \tag{7.40}$$

7.5.6 The Drift Stress Term

The drift stress term is given as:

$$\Delta P_{34,d} = \int_{l_c}^{\lambda_{fl}} \frac{\partial}{\partial z} \left\{ \frac{\rho_f - \rho_m}{\rho_m - \rho_g} \frac{\rho_g \rho_f}{\rho_m} u_{gj}^2 \right\} dz \tag{7.41}$$

Which can be approximated by

$$\Delta P_{34,d} \approx \frac{\rho_f - \rho_{me}}{\rho_{me}} \frac{\rho_g \rho_f}{\rho_{me}} u_{gj}^2 \Big|_{l_c}^{\lambda_{fl}} \tag{7.42}$$

Where Steady State and First Order terms are given by:

$$\overline{\Delta P}_{34,d} = 0 \tag{7.43}$$

and,

$$\frac{\delta \Delta P_{34,d}}{\partial u_{fi}} \equiv \Lambda_{19}(s) = \Lambda_{12}(s)[e^{-s\overline{\tau}_{34}} - 1] \tag{7.44}$$

7.5.7 Total Pressure Drop in the Chimney Region before Flashing (D)

From all the above results of different terms of the pressure response, net pressure response can be evaluated by adding all the terms, thus Steady State and First Order terms for pressure drop in region (D) ΔP_{34} is given as:

$$\begin{aligned}\Delta P_{34} = \overline{\Delta P}_{34,e} + \overline{\Delta P}_{34,g} + \overline{\Delta P}_{34,f} + \overline{\Delta P}_{34,e} \\ + \delta\Delta P_{34,a} + \delta\Delta P_{34,c} + \delta\Delta P_{34,g} + \delta\Delta P_{34,f} + \delta\Delta P_{34,d} \\ \equiv \overline{\Delta P}_{23} + \delta\Delta P_{23}\end{aligned} \tag{7.45}$$

The perturbed pressure drop can also be expressed in terms of perturbation parameters as:

$$\delta\Delta P_{34} = \epsilon e^{st}\left\{\Lambda_{15}(s) + \Lambda_{16}(s) + \Lambda_{17}(s) + \Lambda_{18}(s) + \Lambda_{19}(s)\right\} \equiv \epsilon e^{st}\Lambda_D \tag{7.46}$$

7.6 Pressure Drop in Chimney Region after Flashing (E)

The mixture momentum equation, Eq. 5.17 is integrated similarly as above as the velocity and density fields are given by Eq. 6.147 and 6.140. Integration of 5.17 is done from flashing boundary λ_{fl} to end of the chimney l_t.

$$\begin{aligned}\Delta P_{45} = \int_{\lambda_{fl}}^{l_t} \rho_{me,fl}\left\{\frac{\partial u_{me,fl}}{\partial t} + u_{me,fl}\frac{\partial u_{me,fl}}{\partial z}\right\} + g_e\rho_{me,fl} \\ + \frac{f_{me}}{2D_e}\rho_{me,fl}u_{me,fl}^2 + \frac{\partial}{\partial z}\left\{\frac{\rho_f - \rho_{me,fl}}{\rho_{me,fl} - \rho_g}\frac{\rho_g\rho_f}{\rho_{me,fl}}u_{gj}^2\right\}dz\end{aligned} \tag{7.47}$$

Each term from the above equation is solved individually.

7.6.1 The Inertial Term

The acceleration pressure drop in the momentum equation is given by:

$$\Delta P_{45,a} = \int_{\lambda_{fl}}^{l_t} \rho_{me,fl} \frac{\partial u_{me,fl}}{\partial t} dz \tag{7.48}$$

Substituting Eq. 6.140 and 6.147 into the above equation, we get:

$$\Delta P_{45,a} = \epsilon e^{st} s \rho_f \int_{\lambda_{fl}}^{l_t} \left[\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})} \right]^{-1} \Lambda_9(z,s) dz \equiv \delta \Delta P_{23,a} \tag{7.49}$$

Which is a transfer function, we define:

$$\frac{\delta \Delta P_{45,a}}{\delta u_{fi}} \equiv \Lambda_{20}(s) = s\rho_f \int_{\lambda_{fl}}^{l_t} \left[\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})} \right]^{-1} \Lambda_9(z,s) dz \tag{7.50}$$

It should be noted that Inertial Term's steady state part is zero.

7.6.2 Convective Acceleration Term

The convective acceleration term is given by:

$$\Delta P_{34,c} = \int_{\lambda_{fl}}^{l_t} \rho_{me,fl} u_{me,fl} \frac{\partial u_{me,fl}}{\partial z} dz \tag{7.51}$$

Steady State and First Order terms for the above equation is given as follows:

$$\overline{\Delta P}_{45,c} = \rho_f \overline{u}_{fi}^2 \left[\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})} - 1 \right] \tag{7.52}$$

and,

$$\delta \Delta P_{45,c} = \epsilon e^{st} \Lambda_{22}(s) \tag{7.53}$$

Where $\Lambda_{21}(s)$ is a transfer function and is defined as follows:

$$\lambda_{21}(s) = \rho_f \overline{u}_{fi}^2 \Bigg\{ \left[\left(\frac{\overline{C}_{k,fl}(l_t)}{\overline{C}_{k,fl}(\lambda_{fl})} \right)^2 \Lambda_8(l_t, s) - \Lambda_8(\lambda_{fl}, s) \right]$$

$$+\frac{2}{\overline{u}_{fi}}\Big[\Lambda_9(l_t,s)-\Lambda_9(\lambda_{fl},s)\Big]$$
$$\left.+\frac{1}{\overline{u}_{fi}}s\int_{\lambda_{fl}}^{l_t}\left[\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})}\right]\Lambda_8(z,s)dz\right\} \quad (7.54)$$

7.6.3 The Gravitational Term

The gravitational term is given by:

$$\Delta P_{45,g}=\int_{\lambda_{fl}}^{l_t} g\rho_{me,fl}dz \quad (7.55)$$

Substituting Eq. 6.140 into the above equation, we get:

$$\Delta P_{45,g}=g_e\rho_f\left\{\int_{\lambda_{fl}}^{l_t}\left[\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})}\right]^{-1}dz+\epsilon e^{st}\left[\int_{\lambda_{fl}}^{l_c}\Lambda_8(z,s)dz\right]\right\}$$
$$\equiv\overline{\Delta P}_{34,g}+\delta\Delta P_{34,g} \quad (7.56)$$

Which contains another transfer function given by:

$$\frac{\delta\Delta P_{34,g}}{\delta u_{fi}}\equiv\Lambda_{22}(s)=g_e\rho_f\int_{\lambda_{fl}}^{l_c}\Lambda_8(z,s)dz \quad (7.57)$$

7.6.4 The Frictional Term

The frictional term is given by:

$$\Delta P_{45,f}=\int_{\lambda_{fl}}^{l_t}\frac{f_{me}}{2D_e}\rho_{me,fl}u_{me,fl}^2dz \quad (7.58)$$

From Eq. 6.147 and 6.140, retaining only first powers of ϵ, we get Steady State and Fisrt Order terms are given as:

$$\overline{\Delta P}_{45,f}=\frac{f_{me}}{2D_e}\rho_f\overline{u}_{fi}^2\int_{\lambda_{fl}}^{l_t}\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})}dz \quad (7.59)$$

and,

$$\delta P_{45,f} = \epsilon e^{st}\Lambda_{24}(s) \tag{7.60}$$

Where Λ_{23} is a transfer function and is given as:

$$\frac{\delta P_{45,f}}{\delta u_{fi}} \equiv \Lambda_{23}(s) = \frac{f_{me}}{2D_e}\rho_f \overline{u}_{fi}^2 \int_{\lambda_{fl}}^{l_t} \left\{ \left[\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})} \right]^2 \Lambda_8(z,s) + \frac{2}{\overline{u}_{fi}}\Lambda_9(z,s) \right\} dz \tag{7.61}$$

7.6.5 The Drift Stress Term

The drift stress term is given as:

$$\Delta P_{45,d} = \int_{\lambda_{fl}}^{l_t} \frac{\partial}{\partial z} \left\{ \frac{\rho_f - \rho_{me,fl}}{\rho_{me,fl} - \rho_g} \frac{\rho_g \rho_f}{\rho_{me,fl}} u_{gj,fl}^2 \right\} dz \tag{7.62}$$

Which can be approximated by

$$\Delta P_{45,d} \approx \frac{\rho_f - \rho_{me,fl}}{\rho_{me,fl}} \frac{\rho_g \rho_f}{\rho_{me,fl}} u_{gj,fl}^2 \Bigg|_{\lambda_{fl}}^{l_t} \tag{7.63}$$

Where Steady State and First Order terms are given by:

$$\overline{\Delta P}_{45,d} = \left[\frac{\overline{C}_{k,fl}(l_t)}{\overline{C}_{k,fl}(\lambda_{fl})} - 1 \right] \frac{\overline{C}_{k,fl}(l_t)}{\overline{C}_{k,fl}(\lambda_{fl})} \rho_g u_{gj,fl}^2 \tag{7.64}$$

and,

$$\frac{\delta \Delta P_{45,d}}{\delta u_{fi}} \equiv \Lambda_{24}(s) = \left[1 - 2\frac{\overline{C}_{k,fl}(l_t)}{\overline{C}_{k,fl}(\lambda_{fl})} \right] \left(\frac{\overline{C}_{k,fl}(l_t)}{\overline{C}_{k,fl}(\lambda_{fl})} \right)^2 \rho_g u_{gj,fl}^2 \Lambda_8(l_t,s) \tag{7.65}$$

7.6.6 Total Pressure Drop in the Chimney Region after Flashing (E)

From all the above results of different terms of the pressure response, net pressure response can be evaluated by adding all the terms, thus Steady State and First Order terms for pressure drop in region (E) ΔP_{45} is given as:

$$\Delta P_{45} = \overline{\Delta P}_{45,c} + \overline{\Delta P}_{45,g} + \overline{\Delta P}_{45,f} + \overline{\Delta P}_{45,d}$$

$$+ \delta\Delta P_{45,a} + \delta\Delta P_{45,c} + \delta\Delta P_{45,g} + \delta\Delta P_{45,f} + \delta\Delta P_{45,d}$$
$$\equiv \overline{\Delta P}_{45} + \delta\Delta P_{45} \quad (7.66)$$

The perturbed pressure drop can also be expressed in terms of perturbation parameters as:

$$\delta\Delta P_{45} = \epsilon e^{st}\Big\{\Lambda_{20}(s) + \Lambda_{21}(s) + \Lambda_{22}(s) + \Lambda_{23}(s) + \Lambda_{24}(s)\Big\} \equiv \epsilon e^{st}\Lambda_D \quad (7.67)$$

7.7 Pressure Response of System

In this chapter, steady-state and perturbed pressure drops have been derived separately. The total of all these pressure responses should be equal to external pressure drop ΔP_{ex} imposed at the boundary. *For natural circulation flows, it should be approximately equal to the difference of hydrostatic head of hot and cold leg.* External pressure drop should have a form:

$$\Delta P_{ex} \equiv \overline{\Delta P}_{ex} + \delta\Delta P_{ex} \quad (7.68)$$

Which should in turn be equal to arithmetic sum of all the pressure drops found in this chapter given as follows:

$$\Delta P_{ex} = \Big\{\overline{\Delta P}_{01} + \overline{\Delta P}_{12} + \overline{\Delta P}_{23} + \overline{\Delta P}_{34} + \overline{\Delta P}_{45}\Big\}$$
$$+ \Big\{\delta\Delta P_{01} + \delta\Delta P_{12} + \delta\Delta P_{23} + \delta\Delta P_{34} + \delta\Delta P_{45}\Big\} \quad (7.69)$$

For Steady State operational conditions, $\delta\Delta P_{ex}$ should become zero.

When the system is unstable, $\delta\Delta P_{ex}$ should be arithmetic sum of all the perturbations of the system pressure drop. Adding all the system pressure perturbations, we get:

$$\delta\Delta P_{ex} = \epsilon e^{st}\left\{\Lambda_A(s) + \Lambda_B(s) + \Lambda_C(s) + \Lambda_D(s) + \Lambda_E(s)\right\} \quad (7.70)$$

Where $\Lambda_A(s)$, $\Lambda_B(s)$, $\Lambda_C(s)$, $\Lambda_D(s)$, and $\Lambda_E(s)$ are given by Eq. 7.3, 7.5, 7.26, 7.46, and 7.67 respectively. Eq. 7.70 is the characteristic equation for the system as it provides the pressure response with perturbations in the inlet velocity as, $\delta u_{fi} = \epsilon e^{st}$.

7.8 General Characteristic Equation

Up to this point, perturbation method has been applied to find system pressure response to change in inlet velocity, although in reality the change of pressure drop of the system demands the flow velocity to change to maintain dynamic equilibrium. For examination of the stability of the system, it is necessary to specify generalized input force and output displacement. For our system, the input force is pressure drop perturbations $\delta\Delta P_{ex}$ and velocity perturbations δu_{fi} is output displacement. This relationship is obtained in the following form:

$$\delta u = \left[\frac{1}{Q(s)}\right]\delta\Delta P_{ex} \tag{7.71}$$

where $Q(s)$ is the Characteristic Equation which can be obtained from Eq. 7.70. Thus:

$$Q(s) = \Lambda_A(s) + \Lambda_B(s) + \Lambda_C(s) + \Lambda_D(s) + \Lambda_E(s) \tag{7.72}$$

It is well known in control theory that the nature of roots of the characteristic equation $Q(s)$ determine the stability of the system. Roots of $Q(s)$ are given by:

$$Q(s) = 0 \tag{7.73}$$

The mathematical modeling of the flashing phenomenon in a natural circulation gravity dominated flow is essentially complete and the problem has now morphed to: Examination of nature of the roots of characteristic equation given as follows in complex 's' plane.

$$Q(s) \equiv \Lambda_A(s) + \Lambda_B(s) + \Lambda_C(s) + \Lambda_D(s) + \Lambda_E(s) = 0 \tag{7.74}$$

It is imperative to note that if all the roots of this equation lie in the left half of the 's' plane, the system is asymptotically stable because as $t \to \infty$, $\delta u \to 0$ attributing to the negative real part of 's'. Also, from the same reasoning it can be inferred that if any root of the said equation is in the right half of the 's' plane, the system is unstable because as $t \to \infty$, $\delta u \to \infty$ attributing to the positive real part of 's'.

7.9 Summary of the Transfer Functions

This is a good checkpoint to summarize all the Transfer Functions and characteristic functions defined and obtained from the theoretical analysis developed in the previous chapters.

Some Essential Definitions

A list of functions defined in the formulation and are used in Transfer Functions:

$$f(z) = \frac{q_w}{{}^1\!/_{l_c} \int_0^{l_t} q_w(z)dz} \tag{7.75}$$

$$F(z) = \int_0^z f(z)dz \tag{7.76}$$

$$g(\eta, s) = \int_0^\eta f(\eta)e^{\frac{s\eta}{\overline{\overline{u}}_{fi}}} d\eta \tag{7.77}$$

$$\overline{C}_k(z) = \overline{u}_{fi} + u_{gj} + \Omega_0 \left[F(z) - F(\overline{\lambda})\right] \tag{7.78}$$

$$E(z) = \int \frac{1}{\overline{C}_k(z)} dz \tag{7.79}$$

$$H(z, s) = \int \frac{1}{\overline{C}_k^2(z)} e^{s[E(z) - E(\overline{\lambda})]} d\overline{C}_k(z) \tag{7.80}$$

$$\overline{\lambda} = F^{-1}\left\{\frac{\Delta i_{12}\overline{u}_{fi}}{q_0}\right\} \tag{7.81}$$

$$\Gamma_{g,fl} = Gc_{pf}T_{sat}\frac{\Delta v_{fg}}{\Delta i_{fg}^2}\rho_f g \cos\theta \tag{7.82}$$

$$f_{fl}(z) = \frac{\Omega_{fl}(z)}{{}^1\!/_{l_t - \lambda_{fl}} \int_{\lambda_{fl}}^{l_t} \Omega_{fl}(z)dz} \tag{7.83}$$

$$F_{fl}(z) = \int_{\lambda_{fl}}^z f_{fl}(z)dz \tag{7.84}$$

$$E_{fl}(z) = \int \frac{1}{\overline{C}_{k,fl}} dz \tag{7.85}$$

$$H_{fl}(z, s) = \int \frac{1}{\overline{C}_{k,fl}^2(z)} e^{s[E_{fl}(z) - E_{fl}(\lambda_{fl})]} d\overline{C}_{k,fl}(z) \tag{7.86}$$

$$\lambda_{fl} = \Delta i_{fg} \frac{i_{sub} - q_w \dfrac{l_{core}}{\overline{u}_f}}{\rho_f g c_{pf} T_{sat} \Delta v_{fg}} \tag{7.87}$$

Various Time Lag Terms

Residence time of the fluid in different regions is given by following:

$$\overline{\tau}_{01} = \left[\frac{A_o}{A_c}\right] \frac{l_o}{\overline{u}_{fi}} \tag{7.88}$$

$$\overline{\tau}_{12} = \frac{\overline{\lambda}}{\overline{u}_{fi}} \tag{7.89}$$

$$\overline{\tau}_{23} = E(l_c) - E(\overline{\lambda}) \tag{7.90}$$

$$\overline{\tau}_{34} = \left[\frac{\lambda_{fl} - l_c}{\overline{u}_{fi}}\right] \left[\frac{A_e}{A_c}\right] \left[\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\right]^{-1} \tag{7.91}$$

$$\overline{\tau}_{45} = E_{fl}(l_t) - E_{fl}(\lambda_{fl}) \tag{7.92}$$

Various Transfer Functions

Different transfer functions defined in the formulation are given below:

$$\frac{\delta\tau_{12}}{\epsilon e^{st}} \equiv \Lambda_1(s) = -\frac{1}{f(\overline{\lambda})\overline{u}_{fi}} \left\{ g(\overline{\lambda}, s) - \Big[f(\overline{\lambda}) - f(0) \Big] \right\} \frac{e^{-s\tau_{12}}}{s} \tag{7.93}$$

$$\frac{\delta\lambda}{\epsilon e^{st}} \equiv \Lambda_2(s) = \frac{1 - e^{-s\overline{\tau}_{12}}}{s} + \overline{u}_{fi}\Lambda_1(s) \tag{7.94}$$

$$\frac{\delta C_k(t)}{\epsilon e^{st}} \equiv \Lambda_3(s) = 1 - \Omega_0 f(\overline{\lambda})\Lambda_2(s) \tag{7.95}$$

$$\begin{aligned} \frac{1}{\epsilon e^{st}} \frac{\delta\rho_m}{\rho_f} &\equiv \Lambda_4(z, s) = \\ &= \left[\frac{\overline{C}_k(\overline{\lambda})}{\overline{C}_k(z)}\right] e^{-s[E(z) - E(\overline{\lambda})]} \left\{ \frac{\Omega_0 f(\overline{\lambda})\Lambda_2}{\overline{C}_k(\overline{\lambda})} + \right. \\ &\quad \left. + \Lambda_3(s) \Big[H(z, s) - H(\overline{\lambda}, s) \Big] \right\} \end{aligned} \tag{7.96}$$

$$\frac{\delta u_m(z, t)}{\epsilon e^{st}} \equiv \Lambda_5(z, s) = \Lambda_3(s) + \left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})}\right]^2 u_{gj}\Lambda_4(z, s) \tag{7.97}$$

$$\frac{1}{\epsilon e^{st}}\frac{\delta\rho_{me}(z,t)}{\rho_f} \equiv \Lambda_6(z,s) = \exp\left[-s\left[\frac{z-l_c}{\overline{u}_{fi}}\right]\left[\frac{\overline{C}_k(\overline{\lambda})}{\overline{C}_k(l_c)}\right]\left[\frac{A_e}{A_c}\right]\right]\Lambda_4(l_c,s) \tag{7.98}$$

$$\frac{\delta C_{k,fl}(t)}{\epsilon e^{st}} \equiv \Lambda_7(s) = \left[\frac{A_c}{A_e}\right]\Lambda_5(l_c,s) \tag{7.99}$$

$$\begin{aligned}\frac{1}{\epsilon e^{st}}\frac{\delta\rho_{me,fl}}{\rho_f} &\equiv \Lambda_8(z,s) = \\ &= \left[\frac{\overline{C}_{k,fl}(\lambda_{fl})}{\overline{C}_{k,fl}(z)}\right]e^{-s[E_{fl}(z)-E_{fl}(\lambda_{fl})]}\times \\ &\times\left\{\Lambda_7(s)\Big[H_{fl}(z,s)-H_{fl}(\lambda_{fl},s)\Big]\right\}\end{aligned} \tag{7.100}$$

$$\frac{\delta u_{me,fl}(z,t)}{\epsilon e^{st}} \equiv \Lambda_9(z,s) = \Lambda_7(s) + \left[\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})}\right]^2 u_{gj,fl}\Lambda_8(z,s) \tag{7.101}$$

$$\frac{\delta\Delta P_{23,a}}{\delta u_{fi}} \equiv \Lambda_{10} = s\rho_f\int_{\overline{\lambda}}^{l_c}\left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})}\right]^{-1}\Lambda_5(z,s)dz \tag{7.102}$$

$$\begin{aligned}\frac{\delta\Delta P_{23,c}}{\delta u} &\equiv \Lambda_{11}(s) = \\ &= \rho_f\overline{u}_{fi}^2\left\{\left[\left(\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\right)^2\Lambda_4(l_c,s)-\Lambda_4(\overline{\lambda},s)\right]+\right. \\ &+\frac{2}{\overline{u}_{fi}}\Big[\Lambda_5(l,s)-\Lambda_5(\overline{\lambda},s)\Big]-\frac{\Omega_0}{\overline{C}_k(\overline{\lambda})}f(\overline{\lambda})\lambda_2(s)+ \\ &\left.+\frac{1}{\overline{u}_{fi}}s\int_{\overline{\lambda}}^{l_c}\left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})}\right]\Lambda_4(z,s)dz\right\}\end{aligned} \tag{7.103}$$

$$\frac{\delta\Delta P_{23,g}}{\delta u_{fi}} + g\rho_f\Lambda_2(s) \equiv \Lambda_{12}(s) = g\rho_f\int_{\overline{\lambda}}^{l_c}\Lambda_4(z,s)dz \tag{7.104}$$

$$\begin{aligned}\frac{\delta P_{23,f}}{\delta u_{fi}} + \frac{f_s}{2D}\overline{u}_{fi}^2\Lambda_2(s) &\equiv \Lambda_{13}(s) \\ &= \frac{f_m}{2D}\rho_f\overline{u}_{fi}^2\int_{\overline{\lambda}}^{l_c}\left\{\left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})}\right]^2\Lambda_4(z,s)+\frac{2}{\overline{u}_{fi}}\Lambda_5(z,s)\right\}dz\end{aligned} \tag{7.105}$$

$$\frac{\delta\Delta P_{23,d}}{\partial u_{fi}} \equiv \Lambda_{14}(s) = \left[1-2\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\right]\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}^2\rho_g u_{gj}^2\Lambda_4(l_c,s) \tag{7.106}$$

$$\frac{\delta\Delta P_{34,e}}{\delta u_{fi}} \equiv \Lambda_{15}(s) = k_e\rho_f\overline{u}_{fi}^2\left[\frac{2}{\overline{u}_{fi}}\Lambda_5(l_c,s)+\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\Lambda_4(l_c,s)\right] \tag{7.107}$$

$$\frac{\delta\Delta P_{34,a}}{\delta u_{fi}} \equiv \Lambda_{16}(s) = s\rho_f\left[\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\right]^{-1}\Lambda_7(s) \tag{7.108}$$

$$\frac{\delta\Delta P_{34,g}}{\delta u_{fi}} \equiv \Lambda_{17}(s) = g\rho_f\left[\overline{u}_{fi}\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})}\left[\frac{A_c}{A_e}\right]\frac{1-e^{-s\overline{\tau}_{34}}}{s}\Lambda_4(l_c,s)\right] \tag{7.109}$$

$$\frac{\delta P_{34,f}}{\delta u_{fi}} \equiv \Lambda_{18}(s) = \\ = \frac{f_{me}}{2D_e}\rho_f \overline{u}_{fi}^2 \left\{ \overline{u}_{fi} \left[\frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})} \right]^3 \frac{1 - e^{-s\overline{\tau}_{34}}}{s} \Lambda_4(l_c, s) + \right. \\ \left. + \frac{2}{\overline{u}_{fi}} \left[\frac{A_c}{A_e} \right]^2 \Lambda_5(z, s) \right\} dz \tag{7.110}$$

$$\frac{\delta \Delta P_{34,d}}{\partial u_{fi}} \equiv \Lambda_{19}(s) = \Lambda_{12}(s)[e^{-s\overline{\tau}_{34}} - 1] \tag{7.111}$$

$$\frac{\delta \Delta P_{45,a}}{\delta u_{fi}} \equiv \Lambda_{20}(s) = s\rho_f \int_{\lambda_{fl}}^{l_t} \left[\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})} \right]^{-1} \Lambda_9(z, s) dz \tag{7.112}$$

$$\frac{\delta \Delta P_{45,c}}{\delta u} \equiv \Lambda_{21}(s) = \\ = \rho_f \overline{u}_{fi}^2 \left\{ \left[\left(\frac{\overline{C}_{k,fl}(l_t)}{\overline{C}_{k,fl}(\lambda_{fl})} \right)^2 \Lambda_8(l_t, s) - \Lambda_8(\lambda_{fl}, s) \right] + \right. \\ + \frac{2}{\overline{u}_{fi}} \left[\Lambda_9(l_t, s) - \Lambda_9(\lambda_{fl}, s) \right] + \\ \left. + \frac{1}{\overline{u}_{fi}} s \int_{\lambda_{fl}}^{l_t} \left[\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})} \right] \Lambda_8(z, s) dz \right\} \tag{7.113}$$

$$\frac{\delta \Delta P_{45,g}}{\delta u_{fi}} \equiv \Lambda_{22}(s) = g_e \rho_f \int_{\lambda_{fl}}^{l_c} \Lambda_8(z, s) dz \tag{7.114}$$

$$\frac{\delta P_{45,f}}{\delta u} \equiv \Lambda_{23}(s) = \\ = \frac{f_{me}}{2D_e} \rho_f \overline{u}_{fi}^2 \int_{\lambda_{fl}}^{l_t} \left\{ \left[\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})} \right]^2 \Lambda_8(z, s) + \right. \\ \left. + \frac{2}{\overline{u}_{fi}} \Lambda_9(z, s) \right\} dz \tag{7.115}$$

$$\frac{\delta \Delta P_{45,d}}{\delta u_{fi}} \equiv \Lambda_{24}(s) = \\ = \left[1 - 2\frac{\overline{C}_{k,fl}(l_t)}{\overline{C}_{k,fl}(\lambda_{fl})} \right] \left(\frac{\overline{C}_{k,fl}(l_t)}{\overline{C}_{k,fl}(\lambda_{fl})} \right)^2 \rho_g u_{gj,fl}^2 \Lambda_8(l_t, s) \tag{7.116}$$

7.10 Some Salient Points on Formulation

Before proceeding with the mathematical solution on the transfer functions described above, it is important to highlight the differences of this formulation from various formulations of the same nature provided by N. Zuber in the 1960s and by M. Ishii in 1971 in his thesis [7]. N. Zuber formulated their system of interest in a similar manner by compartmentalizing different regions and then solving the kinematics and dynamics of the problem by decoupling both governing equations by assuming that the density of the mixture is only a function of mixture enthalpy. Ishii in his thesis takes into account the use of drift flux velocity but he assumes it to be constant to decouple conservation equations. However, later he shows that for different flux velocities, the stability of the system varies slightly. Thus theorizing that effect of this velocity is negligible on system stability. In this formulation, the same analogy is extended to the natural circulation system where exists a large riser (chimney). A method to find the flashing boundary and the gas generation due to flashing has been discussed. Further taking direct inspiration from Ishii's work, the gas generation term is morphed with the help of a shaping function. The effect of that shaping functions translates to modification of the transfer functions. From various experimental data for the flashing phenomenon, the parameters for the shaping function can be defined. *It should be noted here that there is no analytical way to define the parameters of the shaping function '$f_{fl}(z)$'.*

Apart from all the above-mentioned changes, a modification is omitted from Ishii's formulation in terms of the dynamic aspect of friction factor assuming that the fluid flow variations are very small to effectively change the dynamic nature of the system. Finally, it should be noted that the transfer functions and the characteristic equation are for a distributed parameter system since the governing field equations have been formally integrated.

8. FINAL FORM OF CHARACTERISTIC EQUATION

8.1 Introduction

In the previous chapters, the dynamic problem of flashing induced instability has been decomposed and morphed into a simpler problem of finding the nature of roots of the characteristic equation 7.74. The transfer functions obtained till now which are summarized in the previous chapter can be solved for different scenarios such as non-uniform heat flux, or different models of flashing described in Sec. 6.5.

In this Chapter, the Transfer functions are solved for Uniform Heat Flux, and Flat Model for flashing. Thus we assume that $f(z) = 1$, and $f_{fl}(z) = 1$. Also for simplicity we define two new variables as follows:

$$C_r^* = \frac{\overline{C}_k(l_c)}{\overline{C}_k(\overline{\lambda})} \tag{8.1}$$

Which describes the ratio of kinematic wave velocity at the exit to the inlet of the heated region (C). Also,

$$C_{r,fl}^* = \frac{\overline{C}_{k,fl}(l_t)}{\overline{C}_{k,fl}(\lambda_{fl})} \tag{8.2}$$

Which describes the ratio of kinematic wave velocity at exit to inlet of flashing region (E). The symbol '$*$' suggests that the variable is dimensionless.

8.2 Transfer Functions (Calculated)

From the above assumptions and definitions, the following transfer functions are calculated:

$$\frac{\delta\tau_{12}}{\delta u} \equiv \Lambda_1(s) = 0 \tag{8.3}$$

$$\frac{\delta\lambda}{\delta u} \equiv \Lambda_2(s) = \frac{1 - e^{-s\overline{\tau}_{12}}}{s} \tag{8.4}$$

$$\frac{\delta C_k(t)}{\delta u} \equiv \Lambda_3(s) = 1 - \Omega\Lambda_2(s) \tag{8.5}$$

$$\frac{1}{\delta u}\frac{\delta\rho_m}{\rho_f} \equiv \Lambda_4(z,s) =$$

$$= \left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})}\right]^{-2} \frac{1}{\overline{C}_k(\overline{\lambda})}\frac{\Omega}{s-\Omega}\Lambda_3(s)+ \tag{8.6}$$

$$+ \left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})}\right]^{-s/\Omega-1} \frac{1}{\overline{C}_k(\overline{\lambda})}\left\{1-\frac{s}{s-\Omega}\Lambda_3(s)\right\}$$

$$\frac{\delta u_m(z,t)}{\epsilon e^{st}} \equiv \Lambda_5(z,s) =$$

$$= \left[1+\frac{u_{gj}}{\overline{C}_k(\overline{\lambda})}\frac{\Omega}{s-\Omega}\right]\Lambda_3(s) \tag{8.7}$$

$$+ \left[\frac{\overline{C}_k(z)}{\overline{C}_k(\overline{\lambda})}\right]^{1-s/\Omega} \frac{u_{gj}}{\overline{C}_k(\overline{\lambda})}\left\{1-\frac{s}{s-\Omega}\Lambda_3(s)\right\}$$

$$\frac{1}{\epsilon e^{st}}\frac{\delta\rho_{me}(z,t)}{\rho_f} \equiv \Lambda_6(z,s) = \exp\left[-s\left[\frac{z-l_c}{\overline{u}_{fi}}\right]\left[\frac{1}{C_r^*}\right]\left[\frac{A_e}{A_c}\right]\right]\Lambda_4(l_c,s) \tag{8.8}$$

$$\frac{\delta C_{k,fl}(t)}{\epsilon e^{st}} \equiv \Lambda_7(s) = \left[\frac{A_c}{A_e}\right]\Lambda_5(l_c,s) \tag{8.9}$$

$$\frac{1}{\epsilon e^{st}}\frac{\delta\rho_{me,fl}}{\rho_f} \equiv \Lambda_8(z,s) =$$

$$= \left[\frac{1}{\overline{C}_{k,fl}(\lambda_{fl})}\frac{\Omega_{fl}}{s-\Omega_{fl}}\left[\left(\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})}\right)^{-2} - \right.\right. \tag{8.10}$$

$$\left.\left. - \left(\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})}\right)^{-s/\Omega_{fl}-1}\right]\right]\Lambda_7(s)$$

$$\frac{\delta u_{me,fl}(z,t)}{\epsilon e^{st}} \equiv \Lambda_9(z,s) =$$

$$= \left[1+\frac{u_{gj,fl}}{\overline{C}_{k,fl}(\lambda_{fl})}\frac{\Omega_{fl}}{s-\Omega_{fl}}\left[1-\left(\frac{\overline{C}_{k,fl}(z)}{\overline{C}_{k,fl}(\lambda_{fl})}\right)^{-s/\Omega_{fl}+1}\right]\right]\Lambda_7(s) \tag{8.11}$$

$$\frac{\delta\Delta P_{23,a}}{\delta u_{fi}} \equiv \Lambda_{10} = \rho_f\left\{\left(1+\frac{u_{gj}}{\overline{C}_k(\overline{\lambda})}\frac{\Omega}{s-\Omega}\right)\overline{C}_k(\overline{\lambda})\ln\left(C_r^*\right)\frac{s}{\Omega}\Lambda_3(s)+\right. \tag{8.12}$$

$$\left. + u_{gj}\frac{s}{s-\Omega}\left[1-\frac{s}{s-\Omega}\Lambda_3(s)\right]\left(1-C_r^* e^{-s\overline{\tau}_{23}}\right)\right\}$$

$$
\begin{aligned}
\frac{\delta \Delta P_{23,c}}{\delta u} &\equiv \Lambda_{11}(s) = \\
&= \rho_f \overline{u}_{fi}^2 \Bigg\{ \frac{-1}{\overline{C}_k(\overline{\lambda})}\Big(1-\Lambda_3(s)\Big) + \frac{1}{\overline{u}_{fi}} \frac{s}{s-\Omega} \ln C_r^* \Lambda_3(s) + \\
&+ \left(1 - \frac{s}{s-\Omega}\Lambda_3(s)\right)\Big[C_r^* e^{-e\overline{\tau}_{23}} - 1\Big] \times \\
&\times \left(\frac{1}{\overline{C}_k(\overline{\lambda})} + \frac{2}{\overline{C}_k(\overline{\lambda})}\frac{u_{gj}}{\overline{u}_{fi}} - \frac{s}{s-\Omega}\frac{1}{\overline{u}_{fi}}\right)\Bigg\}
\end{aligned} \tag{8.13}
$$

$$
\begin{aligned}
\frac{\delta \Delta P_{23,g}}{\delta u_{fi}} + g\rho_f \Lambda_2(s) &\equiv \Lambda_{12}(s) = g\rho_f \Bigg\{ \frac{1}{\overline{C}_k(\overline{\lambda})} \frac{\Omega}{s-\Omega} \frac{l_c - \overline{\lambda}}{C_r^*} \Lambda_3(s) + \\
&+ \frac{1}{s}\left(1 - \frac{s}{s-\Omega}\Lambda_3(s)\right)\Big[1 - e^{-s\overline{\tau}_{23}}\Big]\Bigg\}
\end{aligned} \tag{8.14}
$$

$$
\begin{aligned}
\frac{\delta P_{23,f}}{\delta u_{fi}} + \frac{f_s}{2D}\overline{u}_{fi}^2 \Lambda_2(s) &\equiv \Lambda_{13}(s) \\
&= \frac{f_m}{2D}\rho_f \overline{u}_{fi}^2 \Bigg\{ \frac{\Omega}{s-\Omega}\frac{l_c-\overline{\lambda}}{\overline{C}_k(\overline{\lambda})}\Lambda_3(s) + \\
&\qquad + \frac{2}{\overline{u}_{fi}}\left(1 + \frac{u_{gj}}{\overline{C}_k(\overline{\lambda})}\frac{\Omega}{s-\Omega}\right)(l_c - \overline{\lambda})\Lambda_3(s) + \\
&\qquad + \left(1 + 2\frac{u_{gj}}{\overline{u}_{fi}}\right)\frac{1}{s-2\Omega}\left(1 - \frac{s}{s-\Omega}\Lambda_3(s)\right) \times \\
&\qquad \times \left[1 - C_r^{*2} e^{-s\overline{\tau}_{23}}\right]\Bigg\}
\end{aligned} \tag{8.15}
$$

$$
\begin{aligned}
\frac{\delta \Delta P_{23,d}}{\partial u_{fi}} &\equiv \Lambda_{14}(s) = \Big(1 - 2C_r^*\Big)\rho_f u_{gj}^2 \Bigg\{ \frac{1}{\overline{C}_k(\overline{\lambda})}\frac{\Omega}{s-\Omega}\Lambda_3(s) + \\
&\qquad + \frac{C_r^*}{\overline{C}_k(\overline{\lambda})}\left[1 - \frac{s}{s-\Omega}\Lambda_3(s)\right] e^{-s\overline{\tau}_{23}}\Bigg\}
\end{aligned} \tag{8.16}
$$

$$
\begin{aligned}
\frac{\delta \Delta P_{34,e}}{\delta u_{fi}} &\equiv \Lambda_{15}(s) = k_e \rho_f \overline{u}_{fi}^2 \Bigg\{ \left[2 + \frac{1}{\overline{C}_k(\overline{\lambda})}\Big(\overline{u}_{fi} + 2u_{gj}\Big)\frac{\Omega}{s-\Omega}\right]\Lambda_3(s) + \\
&\qquad + \frac{C_r^*}{\overline{C}_k(\overline{\lambda})}\Big[\overline{u}_{fi} + 2u_{gj}\Big]\left(1 - \frac{s}{s-\Omega}\right) e^{-s\overline{\tau}_{23}}\Bigg\}
\end{aligned} \tag{8.17}
$$

$$
\begin{aligned}
\frac{\delta \Delta P_{34,a}}{\delta u_{fi}} &\equiv \Lambda_{16}(s) = \frac{\rho_f}{C_r^*}(\lambda_{fl} - l_c)\left[\frac{A_c}{A_e}\right] s \Bigg\{ \left[1 + \frac{u_{gj}}{\overline{C}_k(\overline{\lambda})}\frac{\Omega}{s-\Omega}\right]\Lambda_3(s) + \\
&\qquad + C_r^* \frac{u_{gj}}{\overline{C}_k(\overline{\lambda})}\left[1 - \frac{s}{s-\Omega}\Lambda_3(s)\right] e^{-s\overline{\tau}_{23}}\Bigg\}
\end{aligned} \tag{8.18}
$$

$$
\begin{aligned}
\frac{\delta\Delta P_{34,g}}{\delta u_{fi}} &\equiv \Lambda_{17}(s) = g_e\rho_f\overline{u}_{fi}\left[\frac{A_c}{A_e}\right]\frac{1-e^{-s\overline{\tau}_{34}}}{s}\left\{\frac{1}{C_r^*}\frac{1}{\overline{C}_k(\overline{\lambda})}\frac{\Omega}{s-\Omega}\Lambda_3(s)+\right. \\
&\left.+\frac{1}{\overline{C}_k(\overline{\lambda})}\left[1-\frac{s}{s-\Omega}\Lambda_3(s)\right]e^{-s\overline{\tau}_{23}}\right\}
\end{aligned} \tag{8.19}
$$

$$
\begin{aligned}
\frac{\delta P_{34,f}}{\delta u_{fi}} &\equiv \Lambda_{18}(s) = \\
&= \frac{f_{me}}{2D_e}\rho_f\overline{u}_{fi}\left[\frac{A_c}{A_e}\right]^2\left\{2(\lambda_{fl}-l_c)\left[\left(1+\frac{u_{gj}}{\overline{C}_k(\overline{\lambda})}\frac{\Omega}{s-\Omega}\right)\Lambda_3(s)+\right.\right. \\
&\left.+C_r^*\frac{u_{gj}}{\overline{C}_k(\overline{\lambda})}e^{-s\overline{\tau}_{23}}\left(1-\frac{s}{s-\Omega}\Lambda_3(s)\right)\right]+ \\
&+\overline{u}_{fi}^2\left[\frac{1}{\overline{C}_k(\overline{\lambda})}\frac{\Omega}{s-\Omega}\Lambda_3(s)+\right. \\
&\left.\left.+\frac{C_r^*}{\overline{C}_k(\overline{\lambda})}\left(1-\frac{s}{s-\Omega}\right)e^{-s\overline{\tau}_{23}}\right]\left(\frac{1-e^{-s\overline{\tau}_{34}}}{s}\right)\right\}
\end{aligned} \tag{8.20}
$$

$$
\frac{\delta\Delta P_{34,d}}{\partial u_{fi}} \equiv \Lambda_{19}(s) = \Lambda_{12}(s)[e^{-s\overline{\tau}_{34}}-1] \tag{8.21}
$$

$$
\begin{aligned}
\frac{\delta\Delta P_{45,a}}{\delta u_{fi}} &\equiv \Lambda_{20}(s) = \rho_f\left\{\left(1+\frac{u_{gj,fl}}{\overline{C}_{k,fl}(\lambda_{fl})}\frac{\Omega_{fl}}{s-\Omega_{fl}}\right)\times\right. \\
&\times\overline{C}_{k,fl}(\lambda_{fl})\ln(C_{r,fl}^*)\frac{s}{\Omega_{fl}}\Lambda_7(s)+ \\
&\left.+u_{gj,fl}\frac{s}{s-\Omega_{fl}}\left[\frac{-\Omega_{fl}}{s-\Omega_{fl}}\Lambda_7(s)\right]\left(1-C_r^*e^{-s\overline{\tau}_{45}}\right)\right\}
\end{aligned} \tag{8.22}
$$

$$
\begin{aligned}
\frac{\delta\Delta P_{45,c}}{\delta u} &\equiv \Lambda_{21}(s) = \\
&= \rho_f\overline{u}_{fi}^2\left\{\frac{1}{\overline{u}_{fi}}\frac{s}{s-\Omega_{fl}}\ln C_r^*\Lambda_7(s)+\right. \\
&+\left(\frac{-\Omega_{fl}}{s-\Omega_{fl}}\Lambda_7(s)\right)\left[C_{r,fl}^*e^{-e\overline{\tau}_{45}}-1\right]\times \\
&\left.\times\left(\frac{1}{\overline{C}_{k,fl}(\lambda_{fl})}+\frac{2}{\overline{C}_{k,fl}(\lambda_{fl})}\frac{u_{gj,fl}}{\overline{u}_{fi}}-\frac{s}{s-\Omega_{fl}}\frac{1}{\overline{u}_{fi}}\right)\right\}
\end{aligned} \tag{8.23}
$$

$$
\begin{aligned}
\frac{\delta\Delta P_{45,g}}{\delta u_{fi}} &\equiv \Lambda_{22}(s) = g\rho_f\left\{\frac{1}{\overline{C}_{k,fl}(\lambda_{fl})}\frac{\Omega_{fl}}{s-\Omega_{fl}}\frac{l_t-\lambda_{fl}}{C_{r,fl}^*}\Lambda_7(s)+\right. \\
&\left.+\frac{1}{s}\left(\frac{-\Omega_{fl}}{s-\Omega_{fl}}\Lambda_7(s)\right)\left[1-e^{-s\overline{\tau}_{45}}\right]\right\}
\end{aligned} \tag{8.24}
$$

$$
\begin{aligned}
\frac{\delta P_{45,f}}{\delta u_{fi}} \equiv \Lambda_{23}(s) = \frac{f_{me}}{2D_e}\rho_f \overline{u}_{fi}^2 \Bigg\{ & \frac{\Omega_{fl}}{s-\Omega_{fl}} \frac{l_t - \lambda_{fl}}{\overline{C}_{k,fl}(\lambda_{fl})}\Lambda_7(s) + \\
& + \frac{2}{\overline{u}_{fi}}\left(1 + \frac{u_{gj,fl}}{\overline{C}_{k,fl}(\lambda_{fl})}\frac{\Omega_{fl}}{s-\Omega_{fl}}\right)(l_t - \lambda_{fl})\Lambda_7(s) + \\
& + \left(1 + 2\frac{u_{gj}}{\overline{u}_{fi}}\right)\frac{\Omega_{fl}}{s - 2\Omega_{fl}}\left(\frac{-\Omega_{fl}}{s-\Omega_{fl}}\Lambda_7(s)\right)\times \\
& \times \left[1 - C_{r,fl}^{*2} e^{-s\overline{\tau}_{45}}\right]\Bigg\}
\end{aligned} \tag{8.25}
$$

$$
\begin{aligned}
\frac{\delta \Delta P_{45,d}}{\partial u_{fi}} \equiv \Lambda_{24}(s) = \left(1 - 2C_{r,fl}^{*}\right)\rho_g u_{gj}^2 \Bigg\{ & \frac{1}{\overline{C}_{k,fl}(\lambda_{fl})}\frac{\Omega_{fl}}{s-\Omega_{fl}}\Lambda_7(s) + \\
& + \frac{C_{r,fl}^{*}}{\overline{C}_{k,fl}(\lambda_{fl})}\left[\frac{-\Omega_{fl}}{s-\Omega_{fl}}\Lambda_7(s)\right]e^{-s\overline{\tau}_{45}}\Bigg\}
\end{aligned} \tag{8.26}
$$

The transfer functions can be solved in the form provided here but there are so many variables to get some meaningful results. For simplified characteristic equation, we need to non-dimensionalize the transfer functions in a systematic manner such that results can be presented in a manner which is easy to study and employ in practical applications.

8.3 Similarity Groups Governing the Stability of the System

In accordance to formulation mentioned in [7], we choose the length scale of the system as core length 'l_c', and time scale as reciprocal of characteristic frequency of phase change, '$1/\Omega$'.

Based on above defined scales, the following dimensionless parameters can be defined:

Geometric Parameters:

$$
\begin{array}{llll}
z^* = \dfrac{z}{l_c} & l_o^* = \dfrac{l_o}{l_c} & l_c^* = \dfrac{l_c}{l_c} = 1 & l_t^* = \dfrac{l_t}{l_c} \\
D_o^* = \dfrac{D_o}{l_c} & D^* = \dfrac{D}{l_c} & D_e^* = \dfrac{D_e}{l_c} & A_o^* = \dfrac{A_o}{A_c} \\
A_c^* = \dfrac{A_c}{A_c} = 1 & A_e^* = \dfrac{A_e}{A_c} & &
\end{array}
$$

Boiling Length λ^*:

$$\lambda^* = \frac{\overline{\lambda}}{l_c} = \frac{\Delta i_{12}}{q_w}\frac{\overline{u}_{fi}}{l_c} \tag{8.27}$$

Flashing Length λ_{fl}:

$$\lambda_{fl}^* = \frac{\lambda_{fl}}{l_c} = \frac{\Delta i_{fg}}{l_c}\frac{i_{sub} - q_w\frac{l_c}{\overline{u}_{fi}}}{\rho_f g c_{pf} T_{sat} \Delta v_{fg}} \tag{8.28}$$

We should also define dimensionless frequency of phase change in heated region (C):

$$\Omega = \Omega_0 = \frac{q_w \xi}{A_c \Delta i_{fg}}\frac{\Delta\rho}{\rho_g \rho_f} \tag{8.29}$$

and for flashing region (E):

$$\Omega_{fl}^* = \frac{\Omega_{fl}}{\Omega} = \frac{\Gamma_{g,fl}}{\Omega}\frac{\Delta\rho}{\rho_f \rho_g} \tag{8.30}$$

Velocity Field

Inlet velocity:

$$u_{fi}^* = \frac{\overline{u}_{fi}}{\Omega l_c} \tag{8.31}$$

Drift Flux Velocity in Region (C):

$$u_{gj}^* = \frac{u_{gj}}{\overline{u}_{fi}} = \frac{u_{gj}}{u_{fi}^* \Omega l_c} \tag{8.32}$$

Mixture Velocity in Region (C):

$$u_m^* = \frac{u_m}{\overline{u}_{fi}} = \frac{u_m}{u_{fi}^* \Omega l_c} \tag{8.33}$$

Kinematic Wave Velocity in Heater Region (C)

$$C_k^*(z^*) = \frac{\overline{C}_k(z)}{\Omega l_c} \quad \forall \quad z^* < 1 \tag{8.34}$$

Thus, we can compute:

$$C_k^*(l_c^*) = u_{fi}^*(1 + u_{gj}^*) + (1 - \lambda^*) \tag{8.35}$$

and,

$$C_k^*(\lambda^*) = u_{fi}^*(1 + u_{gj}^*) \tag{8.36}$$

Thus,

$$C_r^* = 1 + \frac{1 - \lambda^*}{u_{fi}^*(1 + u_{gj}^*)} \tag{8.37}$$

Drift Flux Velocity in Region (E):

$$u_{gj,fl}^* = \frac{u_{gj,fl}}{\overline{u}_{fi}} = \frac{u_{gj,fl}}{\overline{u}_{fi}^* \Omega l_c} = \frac{u_{gj}^*}{A_e^*} \tag{8.38}$$

Mixture Velocity in Region (D):

$$u_{me}^* = \frac{\overline{u}_{me}}{\Omega l_c} = \left[\frac{1}{A_e^*}\right] C_r^* u_{fi}^* \tag{8.39}$$

Mixture Velocity in Region (E):

$$u_{me,fl}^*(z^*) = \frac{\overline{u}_{me,fl}(z)}{\Omega l_c} \tag{8.40}$$

Kinematic Wave Velocity in Chimney Region after Flashing (E)

$$C_{k,fl}^*(z^*) = \frac{\overline{C}_{k,fl}(z)}{\Omega l_c} \quad \forall \quad \lambda_{fl}^* < z^* < l_t^* \tag{8.41}$$

Thus, we can compute:

$$C_{k,fl}^*(l_t^*) = \frac{u_{fi}^*}{A_e^*}\left(C_r^* + u_{gj}^*\right) + \Omega_{fl}^*(l_t^* - \lambda_{fl}^*) \tag{8.42}$$

and,

$$C_{k,fl}^*(\lambda_{fl}^*) = \frac{u_{fi}^*}{A_e^*}\left(C_r^* + u_{gj}^*\right) \tag{8.43}$$

Thus,

$$C_{r,fl}^* = 1 + \frac{\Omega_{fl}^*(l_t^* - \lambda_{fl}^*)}{{}^{u_{fi}^*}/{}_{A_e^*}\left(C_r^* + u_{gj}^*\right)} \tag{8.44}$$

Residence Time:

$$\tau_{12}^* = \overline{\tau}_{12}\Omega = \frac{\lambda^*}{u_{fi}^*} \tag{8.45}$$

$$\tau_{23}^* = \overline{\tau}_{23}\Omega = \ln C_r^* \tag{8.46}$$

$$\tau_{34}^* = \overline{\tau}_{34}\Omega = (\lambda_{fl}^* - 1)\frac{1}{u_{fi}^* u_{me}^*} \tag{8.47}$$

$$\tau_{45}^* = \overline{\tau}_{45}\Omega = \frac{1}{\Omega_{fl}^*}\ln C_{r,fl}^* \tag{8.48}$$

Independent Variable:

$$s^* = \frac{s}{\Omega} \qquad \text{and} \qquad s_{fl}^* = \frac{s^*}{\Omega_{fl}^*} \tag{8.49}$$

The Pressure Drop Term:

$$\Delta P^* = \frac{\Delta P}{(\Omega l_c)^2 \rho_f} \tag{8.50}$$

Gravitational Term:

$$g^* = \frac{g}{\Omega^2 l_c} \qquad \text{and} \qquad g_e^* = \frac{g_e}{\Omega^2 l_c} \tag{8.51}$$

which can also be defined by Froude Number as:

$$N_{fr} \equiv \frac{u_{fi}^{*2}}{g^*} \qquad \text{and} \qquad N_{fr,e} \equiv \frac{u_{fi}^{*2}}{g_e^*} \tag{8.52}$$

8.4 Dimensionless Characteristic Equation

Using the dimensionless groups defined in Section 8.3, we can rewrite the transfer functions given in Section 8.2. Thus, characteristic equation given by Eq. 7.74 reduces to the form:

$$\frac{Q(s)}{\rho_f(\Omega l_c)u_{fi}^*} \equiv Q^*(s^*) = \frac{\Lambda_A^*}{u_{fi}^*} + \frac{\Lambda_B^*}{u_{fi}^*} + \frac{\Lambda_C^*}{u_{fi}^*} + \frac{\Lambda_D^*}{u_{fi}^*} + \frac{\Lambda_E^*}{u_{fi}^*} \tag{8.53}$$

Where each term represents the pressure drop response in dimensionless form for upstream un-heated region, single phase heated, two phase heated, chimney before flashing, and chimney after flashing respectively. The parametric expressions for the above terms is given as follows:

Upstream Unheated Region (A)

$$\frac{\delta\Delta P^*_{01}}{\delta u^* u^*_{fi}} = \frac{\Lambda^*_A}{u^*_{fi}} = 2k_i + \left(\frac{l^*_o}{A^*_o}\right)\frac{1}{u^*_{fi}}s^* + \frac{f_o}{2D^*_o}2l^*_o\left(\frac{1}{A^*_o}\right)^2 \tag{8.54}$$

Single Phase Heated Region (B)

$$\frac{\delta\Delta P^*_{12}}{\delta u^* u^*_{fi}} = \frac{\Lambda^*_B}{u^*_{fi}} = s^*\tau^*_{12} + \frac{f_s}{2D^*}2\lambda^* + \left[\frac{u^*_{fi}}{N_{Fr}} + \frac{f_s}{2D^*}u^*_{fi}\right](1-\Lambda^*_3) \tag{8.55}$$

Two Phase Heated Region (C)

$$\begin{aligned}\frac{\delta\Delta P^*_{23}}{\delta u^* u^*_{fi}} = \frac{\Lambda^*_C}{u^*_{fi}} = &\left\{\ln C^*_r\left[(1+u^*_{gj}) + u^*_{gj}\frac{1}{s^*-1}\right]s^*\Lambda^*_3 - u^*_{gj}\left(\frac{1}{s^*-1}\right)^2 s^* C^*_2\right\}+\\ &+\left\{\frac{-1}{1+u^*_{gj}}(1-\Lambda^*_3) + \ln C^*_r\left(\frac{s^*}{s^*-1}\right)\Lambda_3 - \left[\left(\frac{1}{s^*-1}\right)^2 - \frac{u^*_{gj}}{1+u^*_{gj}}\frac{1}{s^*-1}\right]\right\}+\\ &+\left(\frac{u^*_{fi}}{N_{Fr}}\right)\left\{\left(1-\frac{1}{C^*_r}\right)\frac{\Lambda_3}{s^*-1} - \frac{1}{s^*}\frac{1}{s^*-1}C^*_1 - (1-\Lambda^*_3)\right\}+\\ &+\frac{f_m}{2D^*}\left\{(1-\lambda^*)\left(2-\frac{1}{1+u^*_{gj}}\right)\frac{\Lambda^*_3}{s^*-1} - u^*_{fi}(1+2u^*_{gj})\left(\frac{1}{s^*-1}\right)\left(\frac{1}{s^*-2}\right)C^*_3\right\}-\\ &-\frac{f_s}{2D^*}u^*_{fi}(1-\Lambda^*_3) - \rho^*_g\frac{u^{*2}_{gj}}{1+u^*_{gj}}\left[2C^*_r - 1\right]\frac{C^*_4}{s^*-1}\end{aligned} \tag{8.56}$$

Chimney Region before Flashing (D)

$$\frac{\delta\Delta P_{34}^*}{\delta u^* u_{fi}^*} = \frac{\Lambda_D^*}{u_{fi}^*} = k_e\left[2\Lambda_3^* + \frac{1+2u_{gj}^*}{1+u_{gj}^*}\frac{1}{s^*-1}C_4^*\right] + \\
+ \frac{1}{u_{fi}^*}\left(\frac{\lambda_{fl}^*-1}{A_e^*}\right)\frac{1}{C_r^*}\left[s^*\Lambda_3^* + \frac{u_{gj}^*}{1+u_{gj}^*}\frac{s^*}{s^*-1}C_4^*\right] + \\
+ \left(\frac{u_{gj}^*}{N_{Fr,e}}\right)\frac{1}{1+u_{gj}^*}\frac{1}{C_r^*}\frac{1}{s^*-1}\left(\frac{1-e^{-s\tau_{34}^*}}{s^*}\right)C_4^* + \\
+ \frac{f_{me}}{2D_e^*}\frac{1}{A_e^{*2}}\left[2l_e^*\left(\Lambda_3^* + \frac{u_{gj}^*}{1+u_{gj}^*}\frac{1}{s^*-1}C_4^*\right) + \frac{u_{gj}^*}{1+u_{gj}^*}\frac{1-e^{-s^*\tau 34^*}}{s^*(s^*-1)}C_4^*\right] - \\
- \rho_g^*\frac{u_{gj}^{*2}}{1+u_{gj}^*}[2C_r^*-1]\frac{1}{s^*-1}C_4^*(e^{-s^*\tau_{34}^*}-1) \quad (8.57)$$

Chimney Region after Flashing (E)

$$\frac{\delta\Delta P_{45}^*}{\delta u^* u_{fi}^*} = \frac{\Lambda_E^*}{u_{fi}^*} = \left\{ \ln C_{r,fl}^*\left[\frac{1}{A_e^*}(C_r^* + u_{gj}^*)\right]\left(1 + \frac{u_{gj}^*}{C_r^*+u_{gj}^*}\frac{1}{s_{fl}^*-1}\right)s_{fl}^*\Lambda_7^* - \right. \\
\left. - \frac{u_{gj}^*}{A_e^*}\left(\frac{1}{s_{fl}^*-1}\right)^2 s_{fl}^*\Lambda_7^* C_6^*\right\} + \\
+ \left\{\ln C_{r,fl}^*\left(\frac{s_{fl}^*}{s_{fl}^*-1}\right)\Lambda_7 + \left[\frac{1}{s_{fl}^*-1}\Lambda_7^* C_6^*\left(\frac{A_e^*+2u_{gj}^*}{C_r^*+u_{gj}^*} - \frac{s_{fl}^*}{s_{fl}^*-1}\right)\right]\right\} + \\
+ \left(\frac{u_{fi}^*}{N_{Fre}}\right)\left\{\left(1 - \frac{1}{C_{r,fl}^*} - \frac{1}{s_{fl}^*}C_5^*\right)\frac{1}{\Omega_{fl}^*}\frac{1}{s_{fl}^*-1}\Lambda_7^*\right\} + \\
+ \frac{f_{me}}{2D_e^*}\left\{\frac{1}{s_{fl}^*}\frac{1}{\Omega_{fl}^*}\left[C_{r,fl}^*-1\right]\Lambda_7^* + 2\left[1 + \frac{u_{gj}^*}{C_k^*+u_{gj}^*}\frac{1}{s_{fl}^*-1}\right](l_t^*-\lambda_{fl}^*)\Lambda_{fl}^* - \right. \\
\left. - \left(1 + 2\frac{u_{gj}^*}{A_e^*}\right)\frac{1}{s_{fl}^*-2}\frac{1}{s_{fl}^*-1}\Lambda_7^* C_7^*\right\} - \\
- \left\{\rho_g^*\frac{u_{gj}^*}{A_e^*}\frac{u_{gj}^*}{C_r^*+u_{gj}^*}\left[2C_{r,fl}^*-1\right]\frac{1}{s_{fl}^*-1}\Lambda_7^* C_6^*\right\} \quad (8.58)$$

Where C_1^*, C_2^*, C_3^*, C_4^*, C_5^*, C_6^*, and C_7^* are given as follows:

$$C_1^* = e^{-s^*\tau_{12}^*} - e^{-s^*\tau 13^*} \tag{8.59}$$

$$C_2^* = e^{-s^*\tau_{12}^*} - C_r^* e^{-s^*\tau 13^*} \tag{8.60}$$

$$C_3^* = e^{-s^*\tau_{12}^*} - C_r^{*2} e^{-s^*\tau 13^*} \tag{8.61}$$

$$C_4^* = \Lambda_3^* - C_r^* e^{-s^*\tau 13^*} \tag{8.62}$$

$$C_5^* = 1 - e^{-s^*\tau_{45}^*} \tag{8.63}$$

$$C_6^* = 1 - C_{r,fl}^* e^{-s^*\tau_{45}^*} \tag{8.64}$$

$$C_7^* = 1 - C_{r,fl}^{*2} e^{-s^*\tau_{45}^*} \tag{8.65}$$

8.5 Some Discussions on Similarity Groups

Froude Number:

$$N_{Fr} = \frac{u_{fi}^2}{g l_c} \qquad \text{and} \qquad N_{Fre} = \frac{u_{fi}^2}{g_e l_c} \tag{8.66}$$

Reynolds Number:

$$N_{Re} = \frac{\rho_f u_{fi} D}{\mu_f} \tag{8.67}$$

Subcooling Number:

$$N_{sub} = \tau_{12}^* = \frac{\Delta i_{12}}{\Delta i_{fg}} \frac{\Delta\rho}{\rho_g} \tag{8.68}$$

Phase Change Number:

$$N_{pch} = \frac{1}{u_{fi}^*} = \frac{q_w \xi l_c}{A_c \overline{u}_{fi} \Delta i_{fg}} \frac{\Delta\rho}{\rho_g \rho_f} \tag{8.69}$$

Flashing Number

$$N_{fl} = \frac{\Omega_{fl}^* l_t^*}{u_{fi}^*} = \Gamma_{g,fl} \frac{l_t}{\overline{u}_{fi}} \frac{\Delta\rho}{\rho_g \rho_f} \tag{8.70}$$

Drift Number:

$$N_d = u_{gj}^* \tag{8.71}$$

Density Number:

$$\rho_g^* = \frac{\rho_g}{\rho_f} \tag{8.72}$$

Flashing Boundary in terms of Phase Change and Subcooling Number:

$$\lambda_{fl}^* = \frac{\Delta i_{fg}^2}{g i_{f,sat}} \left(\frac{\rho_g}{\rho_f - \rho_g} \right)^2 \frac{1}{l_c} \left[N_{sub} - N_{pch} \right] \equiv \left[N_{sub} - N_{pch} \right] I^*(P_{sys}, l_c) \tag{8.73}$$

Where the function 'I^*' depends only on the system pressure and the characteristic length of the system. 'I^*' is also non-negative which is obviously clear from its definition. This function determines how flashing boundary scales with change in difference of N_{sub} and N_{pch}. It should be noted that the flashing boundary can not be negative, thus for flashing to happen:

$$N_{sub} - N_{pch} \geq 0 \tag{8.74}$$

Also, if the flashing boundary does not occur in chimney region, system is stable. Thus, we have another constraint, which is given by:

$$N_{sub} - N_{pch} \leq \frac{l_t^*}{I^*} \tag{8.75}$$

Other than Flashing Boundary λ_{fl}^*, all other similarity groups discussed above are independent of each other and they are basic parameters governing the dynamics of the system.

9. SOLUTION AND DISCUSSIONS

9.1 Introduction

The solution of the system is dependent on the history of the process which is characterized by time-delay terms which are residence time terms. If Kinematic Wave Velocity is only a linear function of the flow direction coordinate z, the characteristic equation can be expressed as an algebraic combination of the transfer functions. In the dimensionless form the Characteristic Equation can be expressed as:

$$Q^*\left(s^*, \frac{1}{s^*}, \frac{1}{s^*-1}, \frac{1}{s^*-2}, s^*_{fl}, \frac{1}{s^*_{fl}}, \frac{1}{s^*_{fl}-1}, \frac{1}{s^*_{fl}-2}, e^{-s^*\tau^*_{12}}, e^{-s^*\tau^*_{23}}, e^{-s^*\tau^*_{34}}, e^{-s^*\tau^*_{45}}\right) = 0 \quad (9.1)$$

Remembering that Flashing terms arise only due to Chimney Region after Flashing (E), we can split the characteristic Equation in the following manner:

$$Q^*_h + Q^*_{fl} = 0 \quad (9.2)$$

From the formulation we know that $s^* = 0, 1, 2$ and $s^*_{fl} = 0, 1, 2$ are not zeros of the above equation, we should define a shifted characteristic equation such that the singularities can be removed keeping in mind that all the terms with $\frac{1}{s^*}$ and $\frac{1}{s^*_{fl}}$ are non-deterministic in itself, (i.e. of the from of $\frac{0}{0}$ for $s^* = 0$ and $s^*_{fl} = 0$). The shifted characteristic equation can be written as follows:

$$Z^* = Z^*_h + Z^*_{fl} \equiv (s^*-1)^2(s^*-2)Q^*_h + (s^*_{fl}-1)^2(s^*_{fl}-2)Q^*_{fl} = 0 \quad (9.3)$$

Now, it should be noted that for flashing to happen $Z^*_h = 0$ as the system is stable before the flashing happens. This arises from the fact that in the formulation, we have rejected the possibility of sub-cooled boiling in the heater section and condensation in the chimney. Thus, no fluid, which has been bulk boiled in heated regions, flashes. Taking into account the above consideration, we can safely say that for the flashing phenomenon, only the nature of roots of $Z^*_{fl} = 0$ are important.

The nature of roots for $Z^*_h = 0$ have been rigorously examined in [7] and can be reproduced with the above formulation. However, for a complete picture, we can solve both the parts of the characteristic equation. The operational plane for both parts is exclusive of each other and thus, numerically it doesn't make sense to solve them together.

9.2 Solution Method

The stability criterion from the encirclement theorem was developed in the 1930s for linear stability which was later extended for closed-loop systems by Nyquist [29] in the early 1930s. In the 1940s, where the development of telecommunication devices was soaring partly because of World War II, Mikhailov [30] extended the theory by Nyquist and others [31] for time-delay systems. These formulations are not restricted to rational algebraic functions. Many contemporary books on control theory [32] develop the understanding up to the point of time-delay systems. To summarise the encirclement method, it was developed on Cauchy's Argument Principle which states that: $2\pi j$ *times the difference of Zeros to Poles of a meromorphic function is equal to the anti-clockwise closed-loop integral of the logarithmic derivative of that function provided that zeros and poles lie inside the contour not on it.*

Let's take a complex function in s plane such that:

$$\mathfrak{G} = \mathfrak{G}(s^*, e^{s^*}) \tag{9.4}$$

Then encirclement theorem gives:

$$\frac{1}{2\pi j} \oint_{C^+} \frac{\mathfrak{G}}{\mathfrak{G}} ds = Z - P \tag{9.5}$$

Where Z and P are zeros and poles in contour C in counter clock wise direction. The integral of the logarithmic derivative of the function $\mathfrak{G}$ can be visualised as $2\pi j$ times the winding of origin by substituting $\mathfrak{G} = \omega$, such that:

$$\oint_{C^+} \frac{\mathfrak{G}}{\mathfrak{G}} ds = \oint_{\mathfrak{G}(C^+)} \frac{1}{\omega} d\omega \tag{9.6}$$

When the characteristic equation does not have roots on the right half of the s^* plane the stability of the system can be studied by the help of the encirclement principle. By taking the contour defined by the Fig. 9.1, we can cover right-hand side for Eq. 9.5. Hence by knowing P, we can find Z. For our application we can list the following salient features for the shifted characteristic equation 9.3:

1. The order of the polynomial is n.
2. No poles
3. No zeros on the contour C and k zeros in C

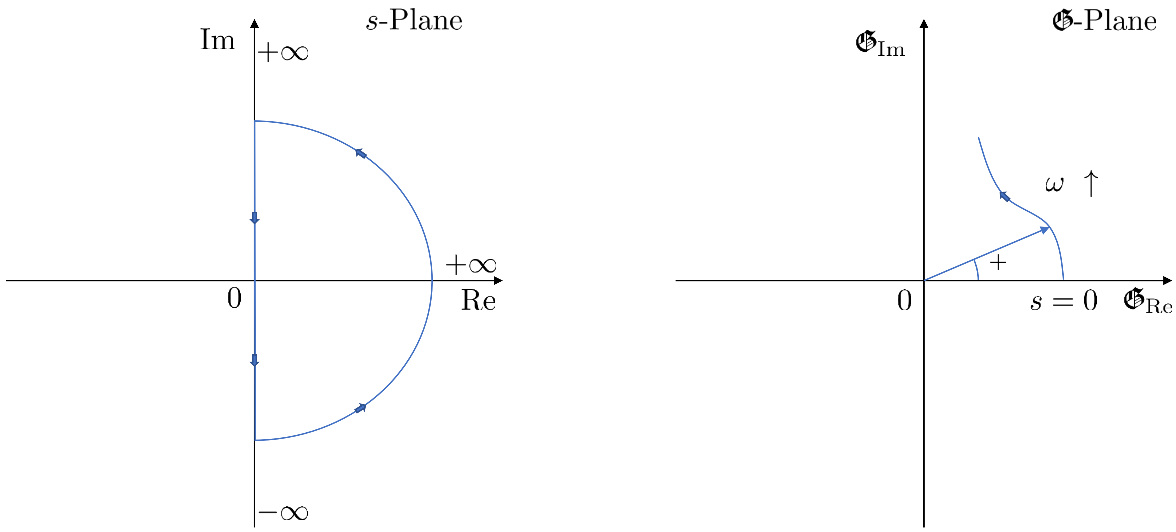


Figure 9.1. Conformal Mapping

Thus, Eq. 9.5 can be morphed into:

$$k = \frac{1}{2}\left\{ n - \frac{1}{\pi j}\oint_{-\infty}^{\infty} \frac{1}{\omega} d\omega \right\} \tag{9.7}$$

For stability of the system, if $\mathfrak{G}$ is replaced with Eq. 9.3, k should be 3 for stability. This theory needs to be extended for the development of a Stability Map. Y. I Neimark in his paper [28] first developed the rigorous algorithm he called 'D-decomposition' (*or D-Partition*). Mitrovic [33], [34], [35] discussed the technique in Western Literature in 1959. Several other mathematicians [36], [37] extended the theory for non-linear parameter dependence.

D-Partition Method

For all polynomials with complex coefficients,

$$P_n = \sum_{0}^{n} c_n s^n \quad \text{where} \quad c_n = a_n + jb_n \tag{9.8}$$

Let us define a complex projective hyperspace R_{2n}. Let $D(k, n-k)$ is a set of all P_n in R_{2n} which have k-roots in left-half of s-plane and $n-k$ roots in right half-plane. Thus D divides the hyperspace into sub-spaces. This partition of R_{2n} is called D-partition. Now let us consider a family of such polynomials with two real parameters τ and ν in the following form:

$$\tau P(s) + \nu Q(s) + R(z) \tag{9.9}$$

If there is a root $j\omega$, we can split the above equation into real and imaginary parts as such:

$$\tau P_1(\omega) + \nu Q_1(\omega) + R_1(\omega) = 0 \tag{9.10}$$

$$\tau P_2(\omega) + \nu Q_2(\omega) + R_2(\omega) = 0 \tag{9.11}$$

This can be solved to obtain a curve Γ in real parameter plane (τ, ν)-plane parameterized by ω which divide the (τ, ν)-plane into subplanes for which the polynomial behavior is uniform.

($i.e.$ *The polynomials belongs to same set* $D(k, n-k)$). Thus for different roots of the polynomial ω_n, we can obtain different curves for Γ_n.

Later this theory was expanded for more intricate family of polynomials parameterized by m real parameters. Such a family of polynomials can be defined as:

$$\mathfrak{G}(s, N_1, N_2, ..., N_m) = 0 \tag{9.12}$$

Where N_1 to N_m are real parameters that are independent of each other.

Like earlier, we can substitute $s = j\omega$ and split the above polynomial such that:

$$\mathfrak{G}_{Re}(\omega, N_1, N_2, ..., N_m) = 0 \tag{9.13}$$

$$\mathfrak{G}_{Im}(\omega, N_1, N_2, ..., N_m) = 0 \tag{9.14}$$

The solution of the above equations divides the m-dimensional parameter space into subspaces for which the nature of roots of the polynomials is uniform.

9.3 Stability Plane

The parametric study for flashing induced stability for the described system can be done with the D-Partition method described above using the non-dimensional parameters/similarity groups obtained in Chapter 8. The similarity groups defined are k_i, k_e, N_g, N_{fr}, N_{Re}, N_{sub}, N_{pch}, N_d, and ρ_g^*. Whereas the perturbation variable s^* is an auxiliary variable. For the sake of representation of results in a usable format, we need to select two parameters such that 2-D parameter space is defined and stability boundaries can be represented. For a fixed geometry, and a defined system pressure all parameters but N_{pch} and N_{sub} are fixed. Thus the stability plane should be consisting of N_{pch} and N_{sub}. We need not analyze for all values of the two parameters as there exist physical constraints binding the parameterized plane. The said constraints are listed below.

Maximum Subcooling: The fluid entering the concerned facility should have enthalapy more than the freezing point i_s and less than the boiling enthalapy, which translates to the following condition:

$$0 \leq N_{sub} \leq \frac{\Delta i_s}{\Delta i_{fg}} \frac{\Delta \rho}{\rho_g} \tag{9.15}$$

Flashing Boundary: For the system to become unstable by flashing, the fluid should flash in the chimney, thus leading up to the constraint:

$$N_{sub} - N_{pch} \leq \frac{l_t^*}{I^*} \tag{9.16}$$

Where I^* has been defined by the Eq. 8.73

Maximum Phase Change: The system is defined for mixture and some assumptions assume that the system steam quality is low, thus there is a constraint on N_{pch} given by:

$$0 \leq N_{pch} \leq N_{sub} + \frac{\Delta \rho}{\rho_g} \tag{9.17}$$

Some system parameters can also be described in simple form such as:

Boiling Length:

$$\lambda^* = \frac{N_{sub}}{N_{pch}} \tag{9.18}$$

Flashing Length

$$\lambda_{fl}^* = [N_{sub} - N_{pch}]\, I^* \tag{9.19}$$

The stability plane with all the constraints can be shown by the Fig. 9.2.

As discussed earlier, the solution of DWO part of the stability plane of Fig. 9.2 has already been discussed earlier in literature leading upto this formulation, thus for the sake of omitting repetition, only the flashing part of the stability plane is solved. The solved stability plane is depicted by Fig. 9.3.

9.4 Application of Solution Method

Description of the solution method has been provided above, however, the application of the provided analysis is done with the help of computers. Although computers are used to

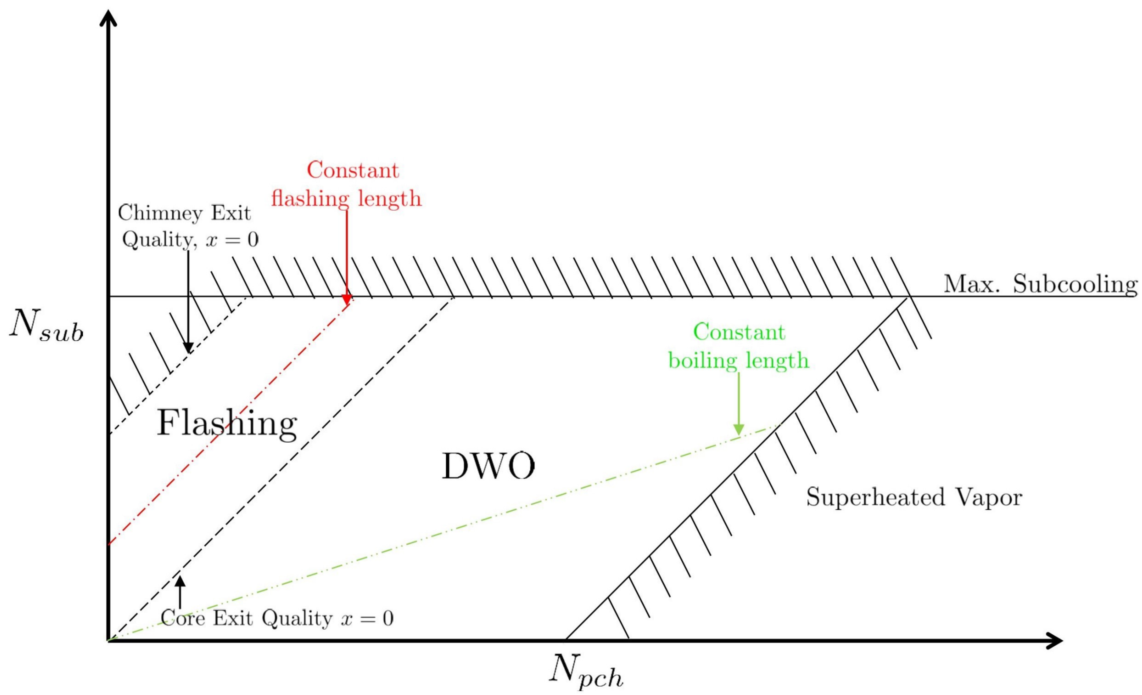


Figure 9.2. Stability plane and salient features

carry out the heavy lifting of Numerical solutions, the formulation is in principle analytical. Here we should develop a numerical strategy for finding the stability maps.

The shifted characteristic equation, Eq. 9.3 is calculated of a given set of parameters (N_{pch}, N_{sub}) and ω^* in real and imaginary parts. For $N_{sub} < N_{pch}$, only Z_h^* is calculated as Z_{fl}^* doesn't exist for such a case. For $N_{sub} > N_{pch}$, only Z_{fl}^* is calculated as we have assumed that no fluid which bulk boils in the heated region, flashes. (*As nucleation sites exist in the form of voids and any superheat in the fluid will cause phase change, consequently flashing boundary does not exist if fluid bulk boils in the heater region.*) Thus Z_h^* has to be 0 for flashing.

The boundary of the maximum value for ω^* can be found by the constraint:

$$\lim_{\omega^* \to \infty} Z_{Re}^* = +\infty \tag{9.20}$$

Thus, for every computed value of Z^*, we can check the value of Z_{Re}^* and use it as the termination condition to terminate computation.

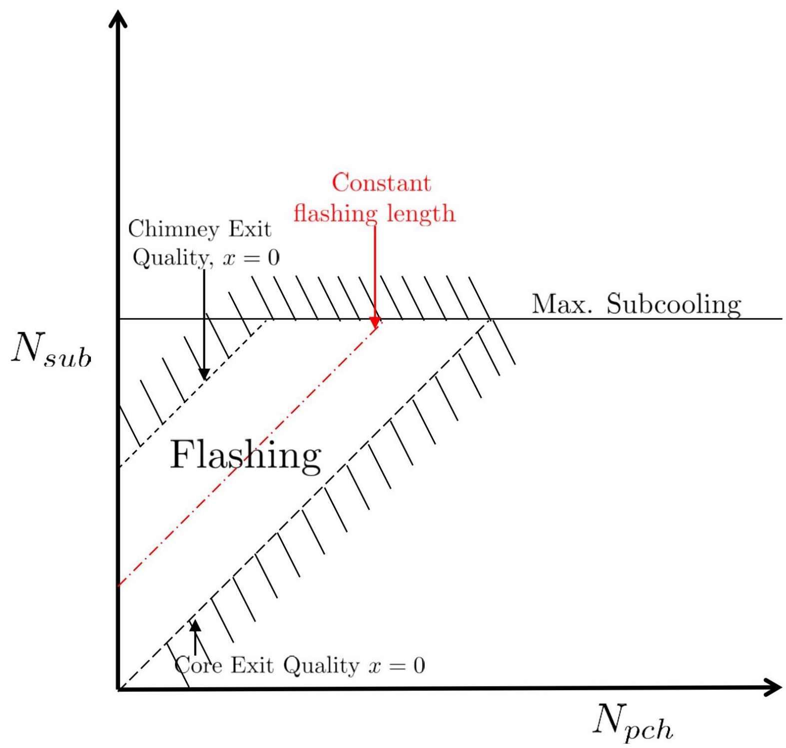


Figure 9.3. Stability plane for flashing

As Eq. 9.3 is non-dimensional in nature, the various parameters other than (N_{pch}, N_{sub}) should be input in non-dimensional form. Some supplemental information is provided as constitutive equations for f_s, f_m, and f_{me}. The following variable should also be parameterized:

$$C_r^* = 1 + \frac{N_{pch} - N_{sub}}{1 + u_{gj}^*} \tag{9.21}$$

Basic Structure of Computation Method

For the stability plane described previously in the chapter, we divide the plane in a mesh for different (N_{pch}, N_{sub}) and thus we can systematically compute Z^* at every point in the mesh. Then for that particular point in the mesh, we find Z^*_{Re} and Z^*_{Im} as a function of ω^*. As discussed in the D-partition method, the boundary exists when:

$$Z_{Re}(\omega_c^*) = Z_{Im}(\omega_c^*) = 0 \tag{9.22}$$

Let us call this frequency ω_c as crossover frequency. To find this frequency, we find the root of the imaginary part of the Eq. 9.3 as follows:

$$Z_{Im}(\omega_c^*) = 0 \tag{9.23}$$

It is recommended here to find roots of the equation by using the Levenberg-Marquardt method to a precision of 10^{-15}. Once the crossover frequency has been found for a point in mesh created above, the value of the real part is checked. If the value of the real part for one point in the mesh switched signs from the one calculated at the previous mesh point, it is an indication that the zero lied somewhere in between the two mesh points just calculated. (*i.e. Check the condition*):

$$Z_{Re}^*(\omega_c^*, N_{pch,1}, N_{sub,1}) \times Z_{Re}^*(\omega_c^*, N_{pch,2}, N_{sub,2}) \leq 0 \tag{9.24}$$

Hence make bifurcation of these two points such that:

$$(N_{pch,i}, N_{sub,i}) = \frac{(N_{pch,1}, N_{sub,1}) + (N_{pch,2}, N_{sub,2})}{2} \tag{9.25}$$

And, now calculate ω_c^* and then, Z_{re}^* for the intermediate point and again repeat this process till a precision of 10^{-2} is reached in the value of (N_{pch}, N_{sub}). This point is the solution for the boundary of a sub-plane in the solution plane.

Following the above said steps we find several points where a boundary exist, which means we have divided the plane into several sub-planes where the nature of roots of the characteristic equation is uniform. To check the nature in that sub-plane, check the stability criteria given by Eq: 9.7 for one point in the sub-plane. Hence the various sub-planes can be labelled as <u>stable</u> or <u>unstable</u>.

9.5 Results

The results of the analysis are presented in this section with some discussion of salient points and comparison with experimental data. Effects of change of inlet and exit restrictions have also been studied and discussed later in the section.

In Fig. 9.4 the stability boundary for flashing is shown. the domain of the solution is bound by the two exit quality lines in accordance to Sec. 9.3. It should be noted that there exist many curves for unstable sub-planes but the most important one is the first boundary which is shown in the figure since it defines the region for stability. For the purpose of flashing, the sub-plane lower than the stability boundary is unstable, and the sub-plane above the stability boundary is stable as indicated by the arrows.

From the stability map, we can also find a critical Phase Change Number $N_{pch,c}$, such that all flows with less than this critical value is all Stable. It originates from the fact that $N_{pch} = {}^{1}/_{u^*_{fi}}$, which is a direct indication that the flow velocity is too high to cause flashing. The map is almost a straight line which very nearly parallel to the exit quality lines. Thus, simple criteria can be developed and thus used in Numerical Solvers for further study and modeling of the phenomenon. In terms of the parameters, it is notable that for higher Sub-cooling Number, the system tends to be stable than lower Sub-cooling Number. The given map is for the system parameters given by Table 9.1, which are typical for a PWR-Type SMR.

Table 9.1. Typical Values for Stability Map

Parameters	**Values**
Core Length	1.3 m
Core Diameter	45.1 mm
Chimney Length	4.0 m
Chimney Diameter	45.1 mm
N_{Re}	4.5×10^4
f_s	0.03
f_m	$2 \times f_s$
N_{Fr}, N_{Fre}	0.0025
I^*	0.33

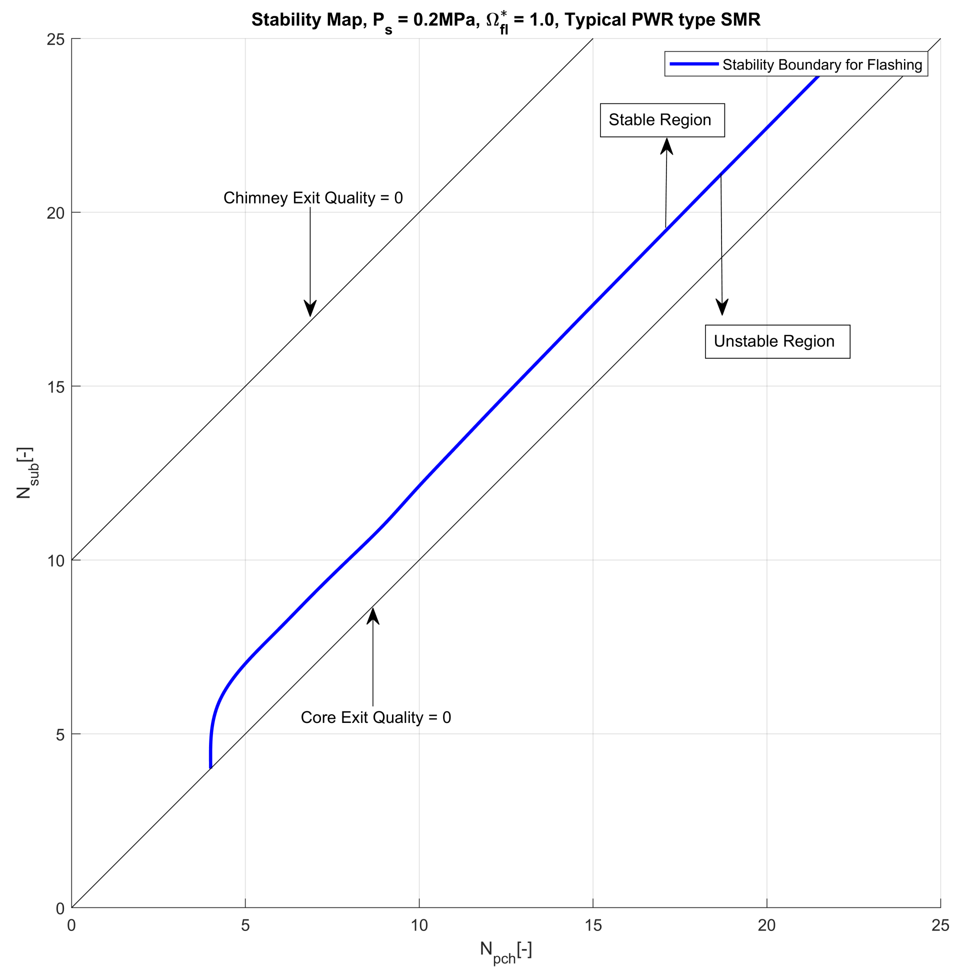


Figure 9.4. Instability Map for $\Omega^*_{fl} = 1.0$ and $P_{sys} = 0.2$ MPa

Effects of Inlet and Exit Restrictions

Fig. 9.5 to Fig. 9.8 shows the effects of Inlet and Exit Restrictions k_i and k_e on the stability map and dimensionless frequency ω^*. It can be concluded that k_i and k_e does **not** affect the stability boundary. Ishii [7] concluded that k_i and k_e have tremendous stabilizing and destabilizing effects respectively for increasing restriction. This statement is particularly true for almost all density wave induced instability phenomenon. One of the reasons for this strong dependence in DWO in contrast to flashing induced instability is that the fluid velocity is independent of the pressure drop in DWO based instabilities. For flashing in natural circulation systems, the flow velocity is coupled with pressure drop. Thus any change in pressure drop will result in a change in flow velocity. It is theorized here following the above-given argument that the coupling of pressure drop with flow velocity *(phase change number)* is the main factor for flashing induced instability to be almost free from any effect of inlet and exit restrictions.

Comparison with Experimental Data

It is necessary to compare the given stability map with contemporary experimental observations. Here, the comparison is made with three independent studies by S. Shi [18], A. Dixit [16], and F. Inada [38]. Fig. 9.9 to 9.11 compares the analytical boundary to the experiments performed by the researchers respectively. The comparison in Fig. 9.9 is loosely bound to the analytical study as the experimental facility on which the experiment was performed lacked a pressure control device such as a pressurizer, and thus, any fluctuations in system operating conditions indicate fluctuations in system pressure which makes classification of data difficult.

Fig. 9.10 co-relates very tightly to the analytical predictions. This data was produced in such a manner to span the (N_{pch}, N_{sub}) plane, hence for the whole plane the effect of the stability boundary is easy to spot. All experimental conditions match with the boundary but a few test cases. A similar trend can be seen in Fig. 9.11 also shows close co-relation but this data was collected in (Sub-Cooling $°C$, Heat Flux kW) plane. When converted to the parameter plane the data does not span the plane completely, thus a very small region

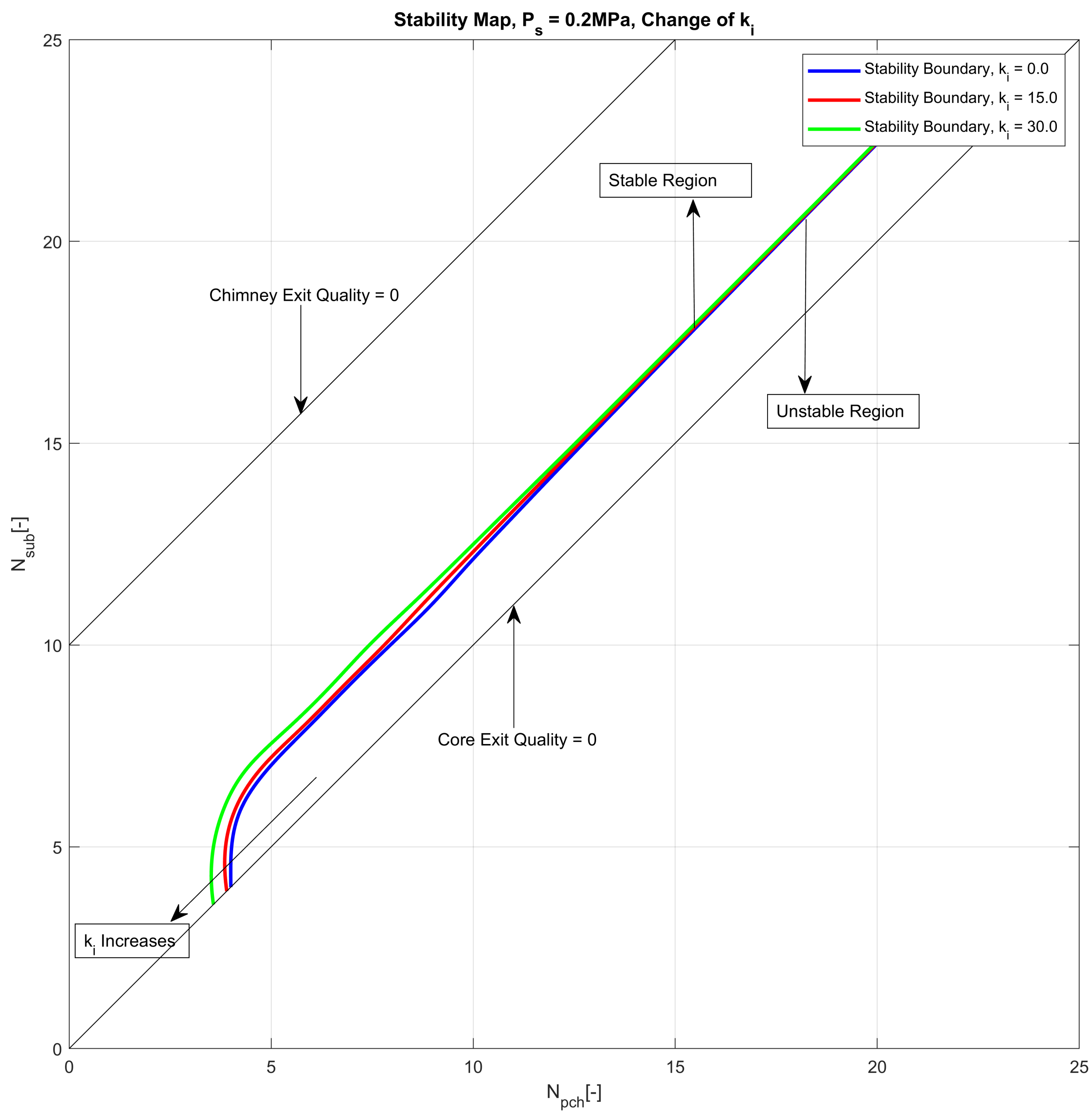


Figure 9.5. Effect of Inlet Restriction, $P_{sys} = 0.2$ MPa, $k_i = 0.0, 15.0, 30.0$

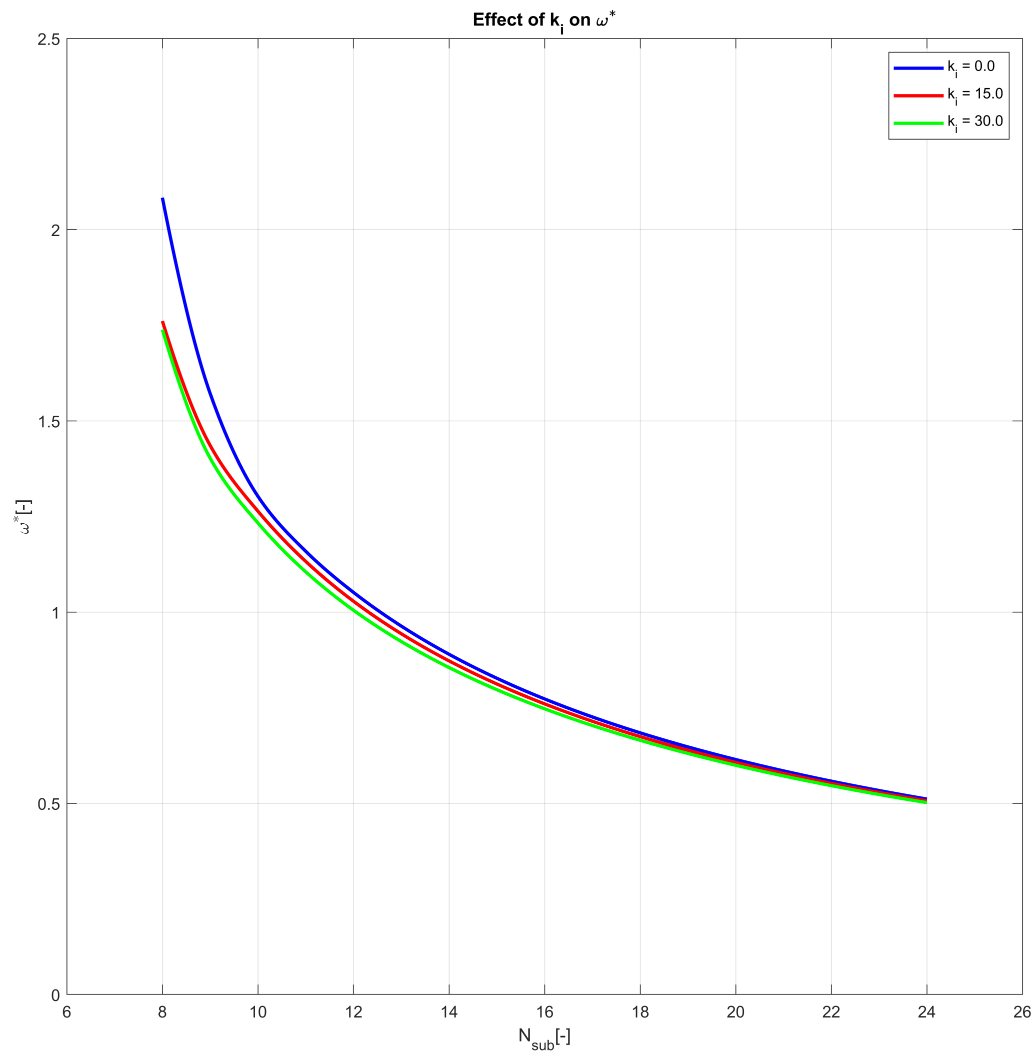


Figure 9.6. Effect of Inlet Restriction on dimensionless Frequency, $P_{sys} = 0.2$ MPa, $k_i = 0.0, 15.0, 30.0$

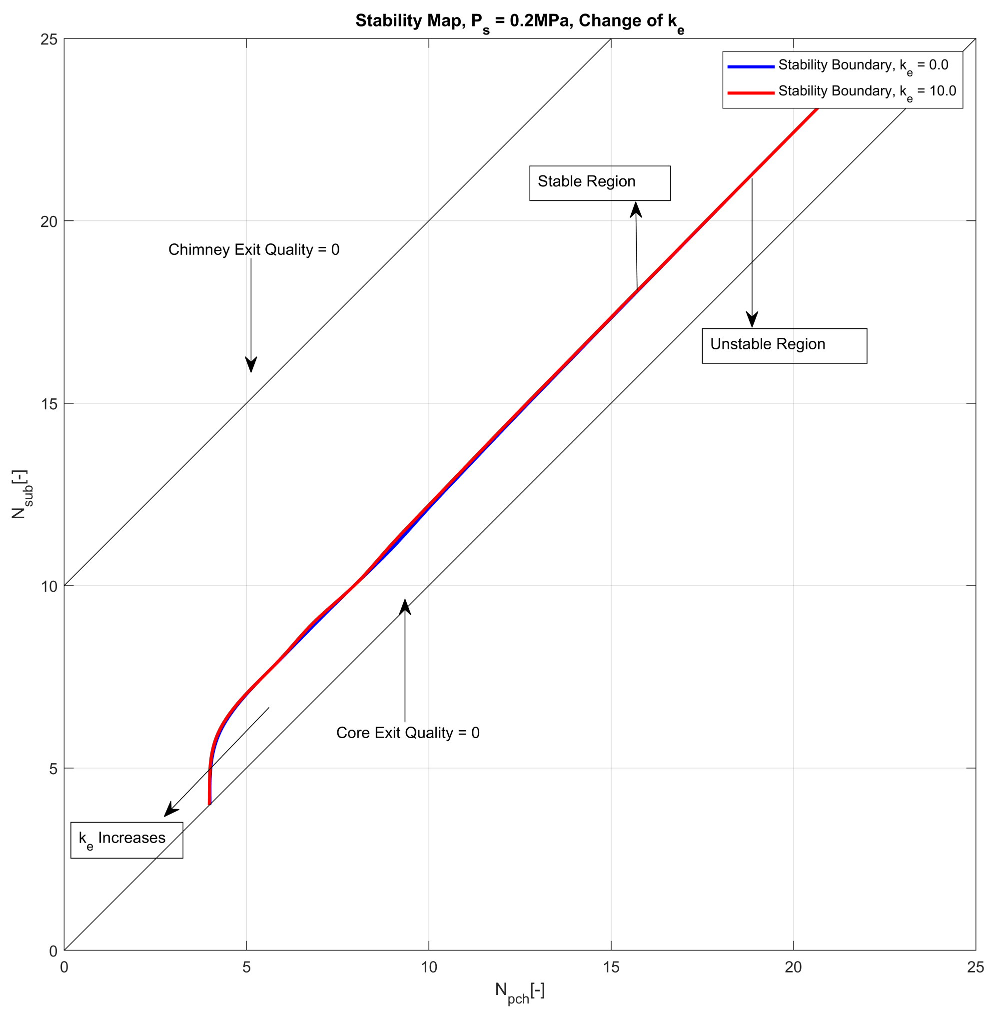


Figure 9.7. Effect of Exit Restriction, $P_{sys} = 0.2$ MPa, $k_e = 0.0, 10.0$

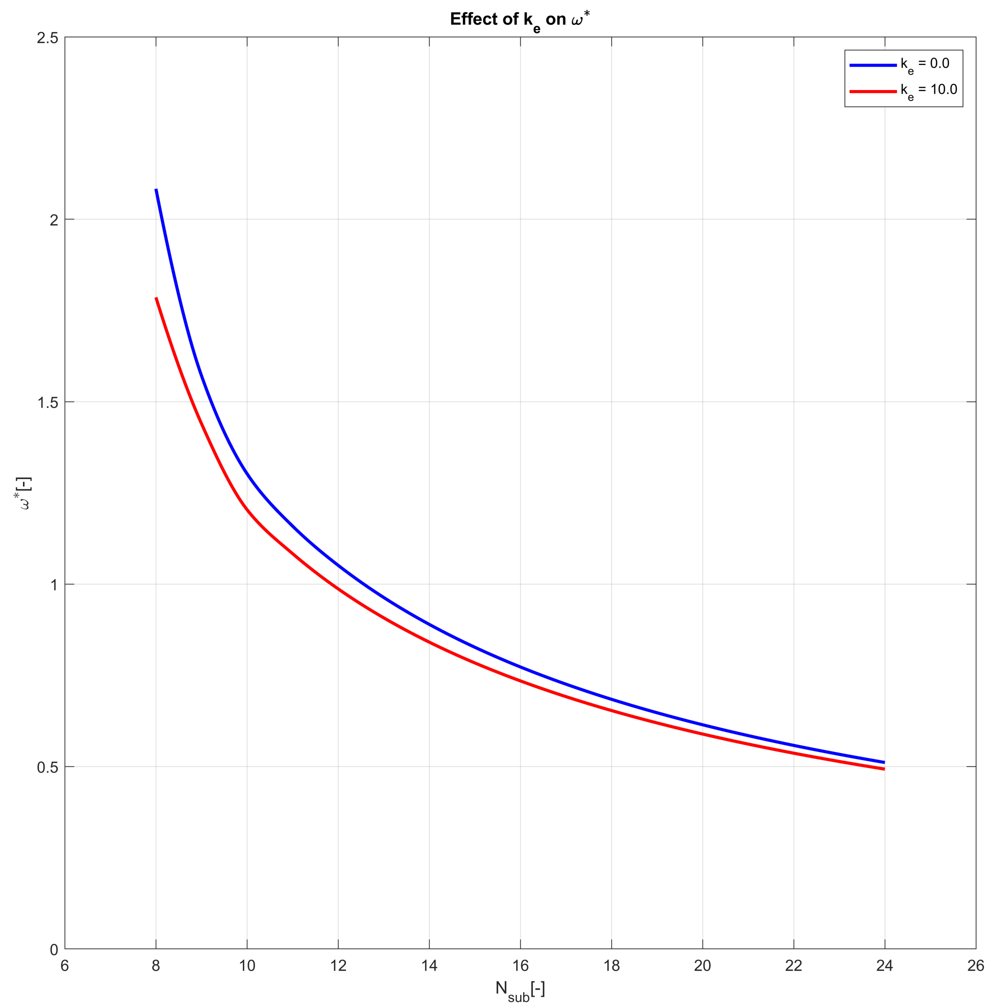


Figure 9.8. Effect of Exit Restriction on dimensionless Frequency, $P_{sys} = 0.2$ MPa, $k_i = 0.0, 10.0$

of the experimental observations can be confirmed to be in accordance with the presented analysis.

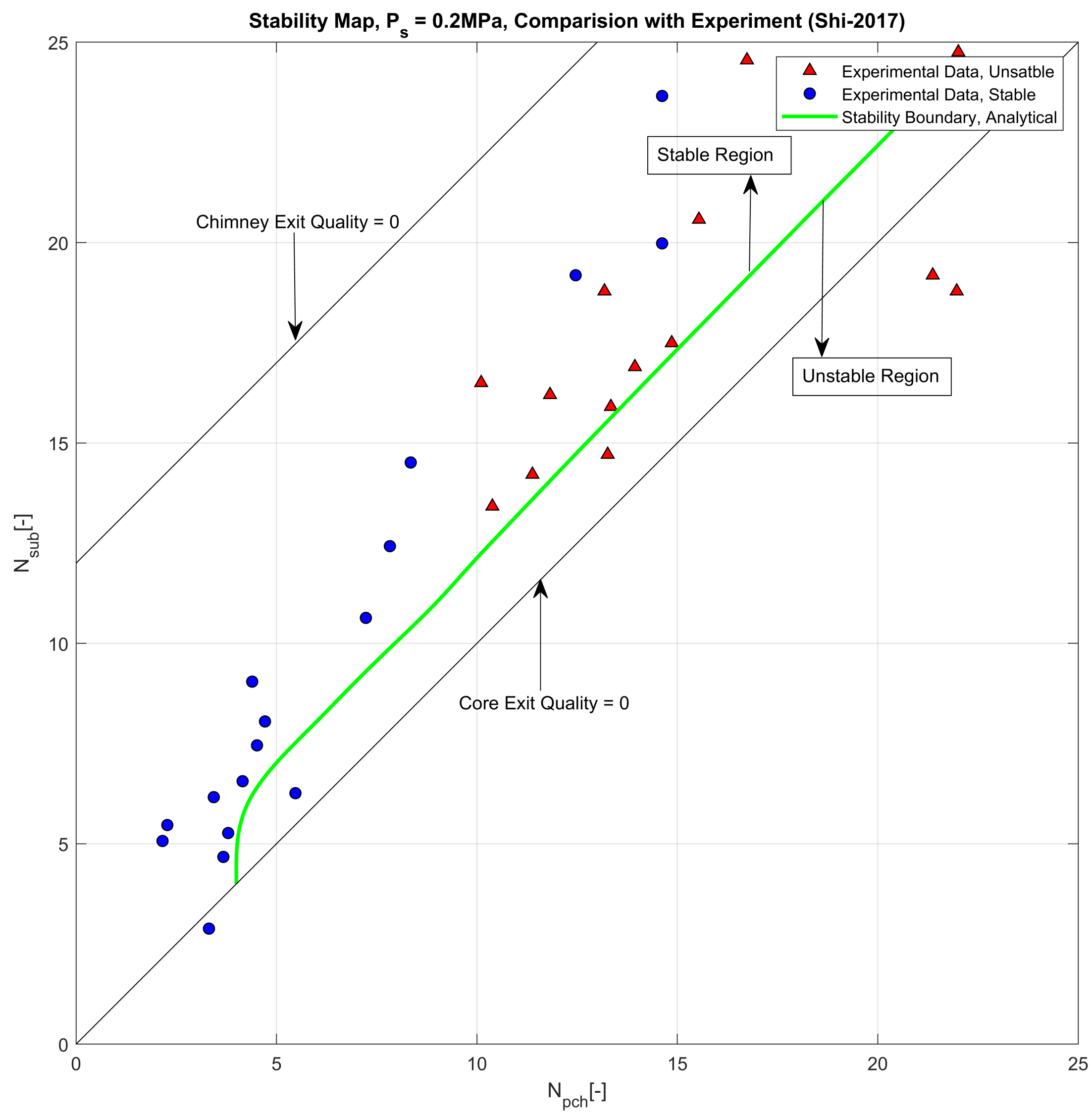


Figure 9.9. Comparison with Experimental Data, Shi-2017 [18], $P_{sys} = 0.2$ MPa

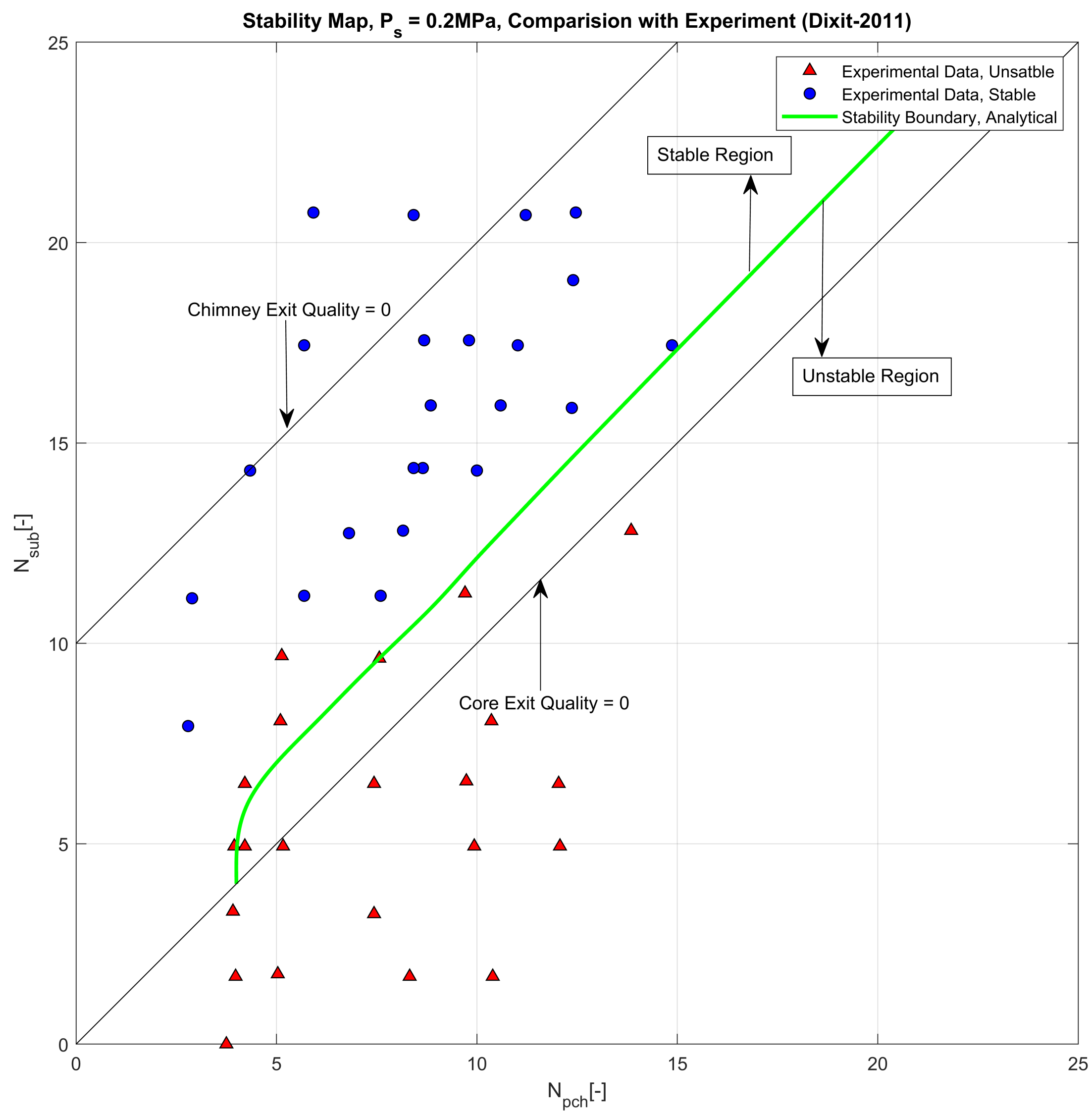


Figure 9.10. Comparison with Experimental Data, Dixit-2011 [16], $P_{sys} = 0.2$ MPa

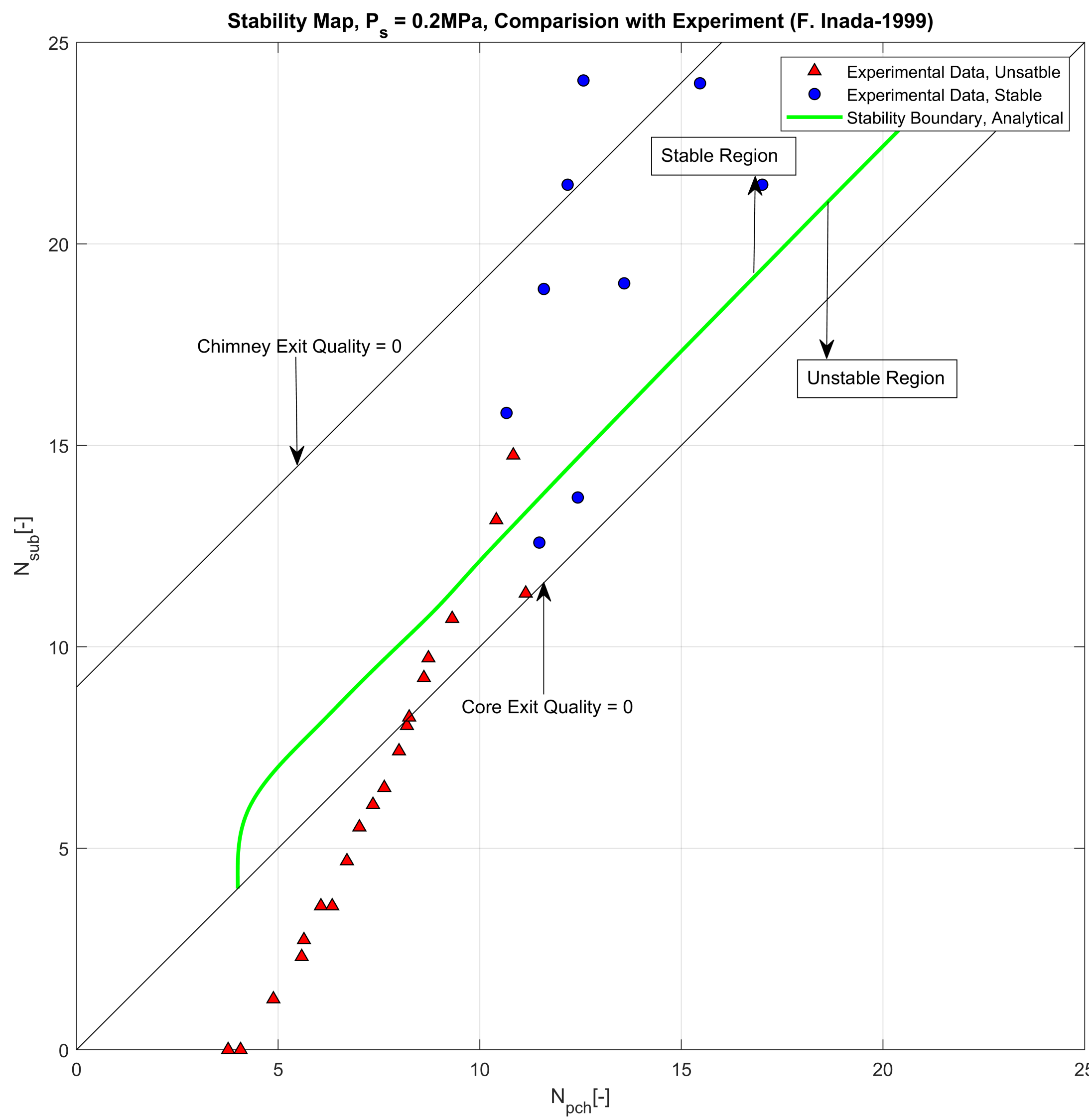


Figure 9.11. Comparison with Experimental Data, Inada-1999 [38], $P_{sys} = 0.2$ MPa

10. SUMMARY

The current analysis starts with a description of experiments performed on PWR-Type SMR scaled facility in TRSL. The need to express the phenomenon of flashing in terms of physics and mathematics is discussed in earlier chapters of this work. The conservation equations are thus discussed and transformed in Time Averaged and Area Averaged forms. These forms are widely used in the analytical solution of two-phase systems or two component systems. The statistically averaged equations contain the terms which describe the behavior of interaction of the fluids in concern. From the conservation relationships described in Chapter 4, it is evident that the different relationships are coupled, and solving them in the original form is impossible analytically, thus certain assumptions are needed to decouple these equations. First, it is assumed that the enthalpy is only a function of temperature, thus decoupling the Continuity and Energy Equations. Later the interaction term between the phases also was assumed constant, so that the derivative of that term becomes zero, thus decoupling Momentum and Energy conservation equations.

In Chapter 5 a simple compartmentalized approach is used to divide the system into five zones of similar flow conditions and labeled as (A), (B), (C), (D), and (E). For each zone/region, the conservation equations are formulated and discussed. Later in the chapter, the parameters used for flashing, namely Flashing Boundary λ_{fl} and Gas generation due to Flashing $\Gamma_{g,fl}$ are derived from Constitutive Relationships and conservation equations respectively. All the assumptions and simplifications are discussed and justified. It is advised in the chapter that for more realistic results, the assumptions can be one-by-one relaxed and a complex form for these parameters can be obtained.

The formulated governing equations are thus solved for kinematics and dynamics for the five regions in Chapter 6 and 7. Here, a systematic approach to finding the velocity and density field is described in detail. The transfer functions for small perturbation in velocity are derived and labeled at Λ's. Equivalence to wave equations and wave velocity are used, and Kinematic Wave Velocity for Heater (C_k) and Flashing ($C_{k,fl}$) Regions are defined. Flashing is modeled equivalent to phase change by heating, the difference being the change of phase is through the superheat of the fluid converting to latent heat. This equivalence leads to the necessity of modeling the shape of this heat flux equivalent gas generation. A note on the various shapes is provided explaining the physics behind each of the models. The different pressure drops in the five regions are calculated here and a general expression is stated which can be substituted with appropriate transfer functions for different situations. Pressure response of the system and general characteristic equation have been summarized here as well.

The following Chapters 8 and 9 discuss the final form of the characteristic equation for a simple case of uniform heat flux and a flat model for flashing. All transfer functions are calculated and listed here. Also, system similarity groups are discussed here. Thus, the characteristic equation is modified in a dimensionless form to describe the system in the simplest manner possible. It is found here that the most important similarity groups are Phase Change Number and Sub-cooling Number. The characteristic equation is solved with the help of the D-partition method and the solution strategy is discussed. Finally, the results of the analysis are discussed and compared with contemporary experimental data. It is found that the Instability map found by the current method matches very closely to the experimental results and thus confirming the viability of the current work. Lastly, the change of inlet and exit restrictions, k_i and k_e is discussed. Contrary to other instabilities, it is found that flashing induced instabilities are not affected by inlet and exit restrictions. This is theorized to be such because of the strong coupling of flow velocity with pressure drop in the system as this is the driving force for natural circulation.

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