A STUDY OF ENERGY MANAGEMENT IN HYBRID CLASS-8 TRUCK PLATOON USING MULTI AGENT OPTIMIZATION

by

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To my beloved parents, my wife and my beautiful daughter

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ABBREVIATIONS

VSM	Vehicle Speed Management
VSCM	Vehicle Speed & Coast Management
VSCGM	Vehicle Speed, Coast & Gear Management
VSCGPM	Vehicle Speed, Coast, Gear & Power Split Management
COM	Control Oriented Model
PS	Power Split
PSC	Pseudo Spectral Collocation
MPC	Model Predictive Control
PID	Proportional Integral Directive
ECM	Electronic Control Module
SCM	Supervisory Control Module
TCM	Transmission Control Module
LQR	Linear Quadratic Regulator
LQG	Linear Quadratic Gaussian
AC	Air-conditioning
Ah	Ampere-hour
AM	Active Material
AVMT	Annual Vehicle Miles Traveled
BEB	Battery Electric Bus
BEV	Battery Electric Vehicle
CA3EM	China Automotive Energy, Environment and Economy Model
CC-CV	Constant current - constant voltage
CD	Charge Depleting Mode
CIDI	Compression Ignition Direct Injection
CNG	Compressed Natural Gas
CO	Carbon monoxide
CO2	Carbon dioxide
COP	Coefficient of Performance

CS	Charge Sustaining Mode
DOD	Depth of Discharge
DOE	Design of Experiments
DP	Dynamic Programming
EC/DMC	Ethylene carbonate and Dimethyl carbonate
ECMS	Equivalent Consumption Minimization Strategy
EPA	Environmental Protection Agency
ESS	Energy Storage System
EV	Electric Vehicle
FCV	Fuel Cell Vehicle
GHG	Greenhouse Gases
GREET	Greenhouse Gases, Regulated Emissions and Energy Use in Transportation
HD	Heavy-Duty
HEV	Hybrid Electric Vehicle
ICE	Internal Combustion Engine
ICEV	Internal Combustion Engine Vehicle
IEA	International Energy Agency
INDC	Intended Nationally Determined Contributions
IPCC	Intergovernmental Panel on Climate Change
IRR	Internal Rate of Return
ISC	Initial System Cost
LFP	Lithium Ferrous Phosphate
LiPF6	Lithium hexa-fluorophosphate
LPG	Liquefied Petroleum Gas
MCMC	Markov Chain Monte Carlo
MD	Medium-Duty
M/G	Motor and Generator
NBIR	Net Benefit Investment Ratio

NHTSA	National Highway Traffic Safety Administration
NMC	Nickel Manganese Cobalt Oxide
NMC+LMO	Nickel Manganese Cobalt Oxide and Lithium Manganese Oxide
NPV	Net Present Value
NOx	Nitrogen oxides
OECD	The Organization for Economic Co-operation and Development
OEM	Original Equipment Manufacturer
PBP	Payback Period
PCOA	Parallel Chaos Optimization Algorithm
PDF	Probability Distribution Function
PHEV	Plug-in Hybrid Electric Vehicle
PM2.5	Particulate Matter with diameter less than 2.5 microns
PM10	Particulate Matter with diameter less than 10 microns
PNGV	Partnership for a New Generation of Vehicles
RC	Resistance Capacitance branch
SEI	Solid Electrolyte Interface
SOC	State of Charge
SOx	Sulfur oxides
SUV	Sport Utility Vehicle
VMT	Vehicle Miles Traveled
ARPA-E	Advanced Research Project Agency - Energy

ABSTRACT

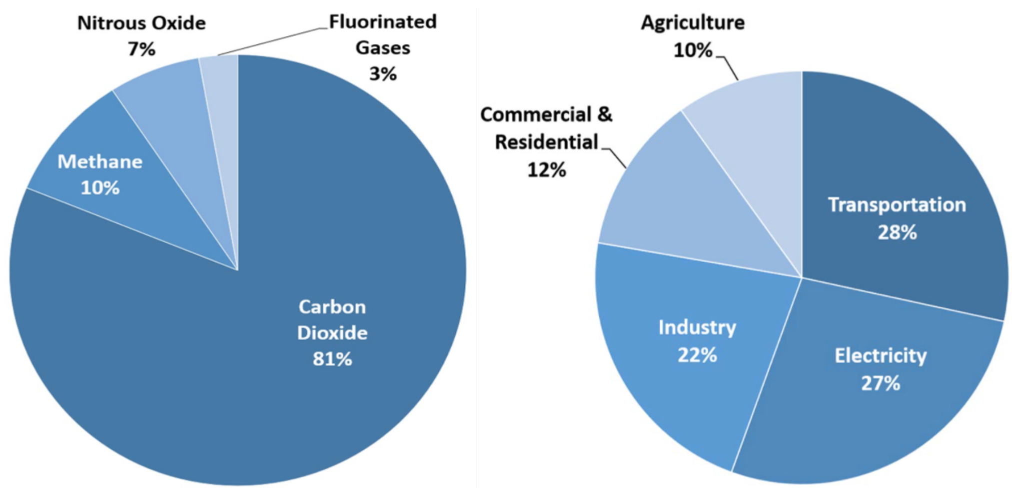
Alternate power sources in automotive class-8 trucking industry is a major focus of research in recent days. Green house gasses, oxides of Nitrogen(NOx), Oxides of Sulphur(SOx), hydrocarbons and particulate matter are major concerns contributing to the shift in alternate fuel strategies. Another direct relation to move to an alternate power strategy is the reduction in net fuel consumption which in turn implicitly improves the emission components. A holistic approach is needed while designing a modern class-8 vehicle. A variety of system architecture, control algorithms, diagnostic levers are needed to be manipulated to achieve the best of blends amongst Total Cost of Ownership (TCO), Drivability, Fuel Economy, Emissions Compliant, Hauling Capacity, etc. The control and system levers are not mutually exclusive and there is a strong correlation amongst all these control and system components. In order to achieve a consensus amongst all these levers to achieve a common objective, is a challenging and complex problem to solve. It is often required to shift the algorithm strategy to predictive information based rather than reactive logic. Predictively modulating and manipulating control logic can help with better fuel efficient solution along with emissions improvement. A further addition to the above challenge is when we add a fleet of vehicle to the problem. So, the problem now is to optimize a control action for a fleet of vehicles and design/select the correct component size. A lot of research has been done and is still underway to use a 48V hybrid system with a small battery using a simple charge sustaining SOC control strategy. This will make the system light enough not to compromise on the freight carrying capacity as well as give some extra boost during the high torque requirement sections in the route for a better fuel and emissions efficient solution. In this work a P2 type 48V hybrid system is used which is side mounted to the transmission via a gear system. The selection of the system and components enables the usage of different control strategies such as neutral coasting and Engine off coasting. This architecture with a traditional 12-15L Internal combustion engine along with a mild 48V hybrid system provides the most viable selection for a long haul class-8 application and is used in this work. It is also possible to identify other component sizes along with architectures for new configurations. The framework in this research work can help develop the study for different component

sizing. While this research work is focused towards building a framework for achieving predictive control in a 3 truck platooning system using multi-agent based control, the other supporting work done also helps understand the optimal behavior of the interacting multiple controls when the corridor information such as road grade and route speed limit are known a-priori, in a single vehicle. The build up of this work analyzes an offline simulation of a 4 control optimal solution for a single hybrid truck and then extend the optimal controls to a 3 truck platoon. In the single truck, this research will help identify the interacting zones in the route where the various control actions will provide the best cost benefits which is fuel economy. These benefits are associated as a function of exogenous look ahead information such as grade and speed limit. Further it is also possible to identify the optimal behavior and the look ahead horizon required for achieving that. In other words the optimal behavior and benefits associated with the global solution can be accomplished by implementing rule based control system with a look ahead horizon of 2-5 km. If this would not have been the case then it is almost impossible to design a predictive controller based on the entire route information which can stretch up to hundreds of kilometers. Optimal algorithms of such prediction horizon are not feasible to be implemented in real time controllers. This research work will also help understand the interaction between different active control actions such as predictive speed modulation, gear shift, coasting and power split with passive control levers such as slow down due to hybrid regeneration, hybrid boost during coasting, etc. This will help in architecting a system involving component specifications, active optimal control, look ahead information, hybrid system strength, etc, working in close interaction with each other. Though we analyze these predictive behavior for a single vehicle as a supporting work the prime objective is to include these predictive levers in a platooning system using an agent based method. This multi-agent based technique will help analyze the behavior of multiple trucks in a platoon in terms of fuel efficient safe operation. The focus of this research work is to not directly come up with a controller or strategy but rather to understand the optimality of this control levers for a multi-vehicle platoon system given a look ahead information is available. The research shows that predictive information will help in gaining fuel economy for a platoon of class-8 mild hybrid trucks. It also highlights the challenges in doing so and what needs to be traded off in order to achieve the net fuel benefit.

1. INTRODUCTION

1.1 Motivation

In the context of global warming and its effect on climate change that the world is going through in present days at an alarming rate it is obvious to reduce the dependence on fossil fuels and there by reduce the overall carbon footprint as well as other harmful oxides of nitrogen (NOx) and sulphur (SOx) [1]. The surface temperature of the Earth is rising at an alarming rate and has risen approximately $1.5 \deg F$ since the 1970s [2]. This report [2] also lists the significant threats to weather changes, ecosystems and human health. As shown in Figure 1.1a, Oxides of Carbon is the major contributor in all forms of Green House Gas (GHG). The transportation sector significantly contribute to green house gas emissions (28%)1.1b, with the medium and heavy duty vehicles contributing significantly.



(a) Overview of Greenhouse Gas Emissions in 2018

(b) Total U.S. Greenhouse Gas Emissions by Economic Sector in 2018

Figure 1.1. Total Emissions in 2018 = 6,677 Million Metric Tons of CO2 equivalent. Percentages may not add up to 100% due to independent rounding.

Passenger and freight travel is projected to increase at a steady pace as shown in Figure 1.2, which indicates the dependency on fossil fuels or other efficient alternate fuels to cater to the demand. If the demand is supplied by fossil fuel alone that increases the threat to global warming and drastically will increase the carbon footprint. Additionally it will put a tremendous stress on a better emissions control strategy to control the harmful by-products of combustion such as NOx, SOx, Hydrocarbons, Particulate Matter, etc. Airline industry will see the major thrust in passenger carrying segment while heavy duty line haul industry will be the prime mover in the freight carrying segment. This indicates that a better clean energy solution is of paramount importance for a better future mobility.

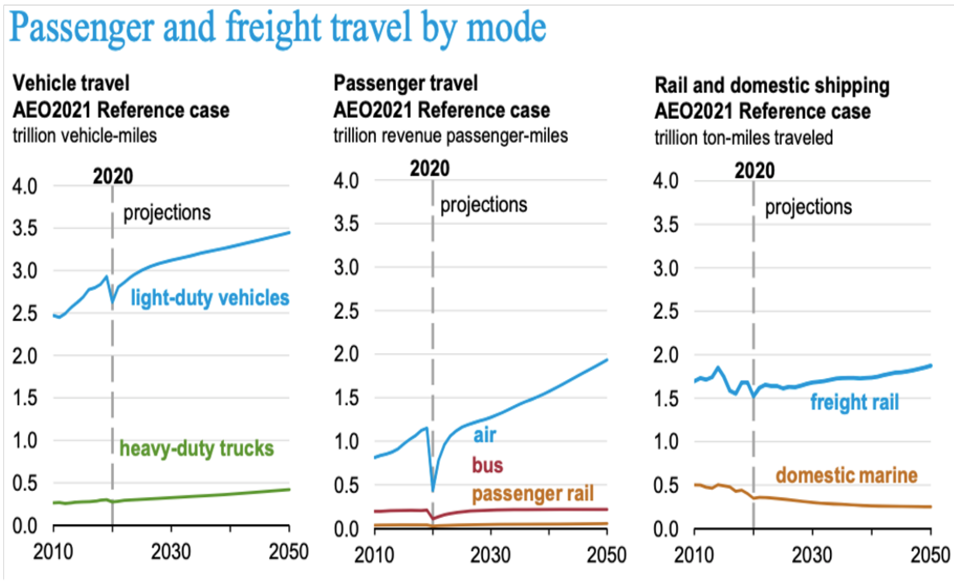


Figure 1.2. Past and Projected comparison of passenger and freight travel by different modes. A significant increase in light duty segment as well as marginal increase in heavy duty segment which will put tremendous pressure of natural resources and environment pollution

A tremendous amount of research is done as well as there are different sectors individually working on a better, cleaner combustion technique by increasing Engine Brake Thermal Efficiency (BTE) as well as fuel economy. Figure 1.31.4 depicts a very nice information about the past trends in fuel economy and energy intensity improvements. The projected improvement is also significant. Particularly for the class 7-8 line haul segment which is the prime focus for this work we see a better trend in fuel economy as well as energy intensity. Department of Energy funded Supertruck-I & Supertruck-II projects are such an example of engine and powertrain improvement projects involving a consortium of OEMs and Tier-1s to achieve a better BTE and engine performance overall. Figure 1.5, shows the achievement so far by Daimler Trucks, in the BTE space for Supertruck-II. There are other such key players

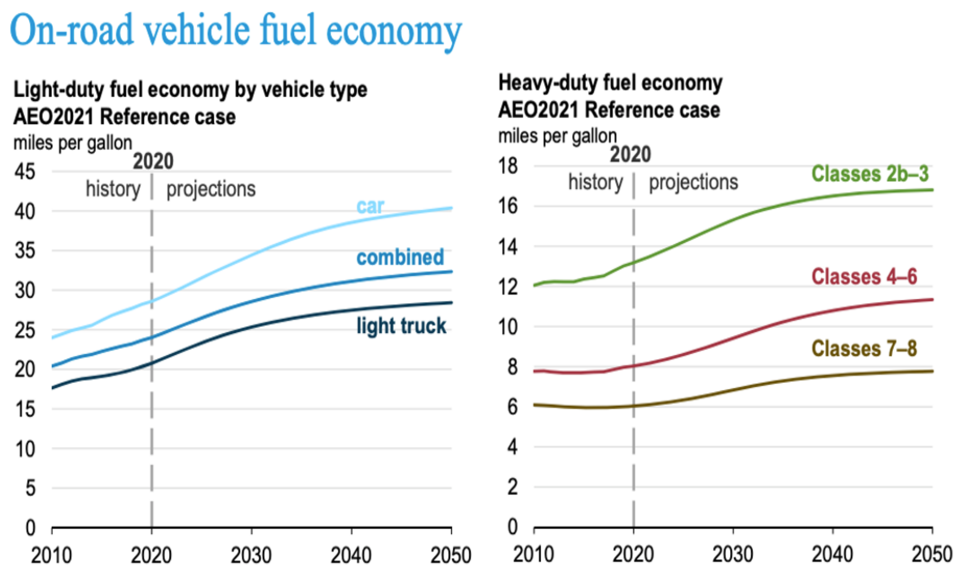


Figure 1.3. Comparison of Fuel Economy for Light Duty Passenger Cars and Heavy Duty Line Haul Applications. The comparison is for past data as well as for projected data up to 2050.

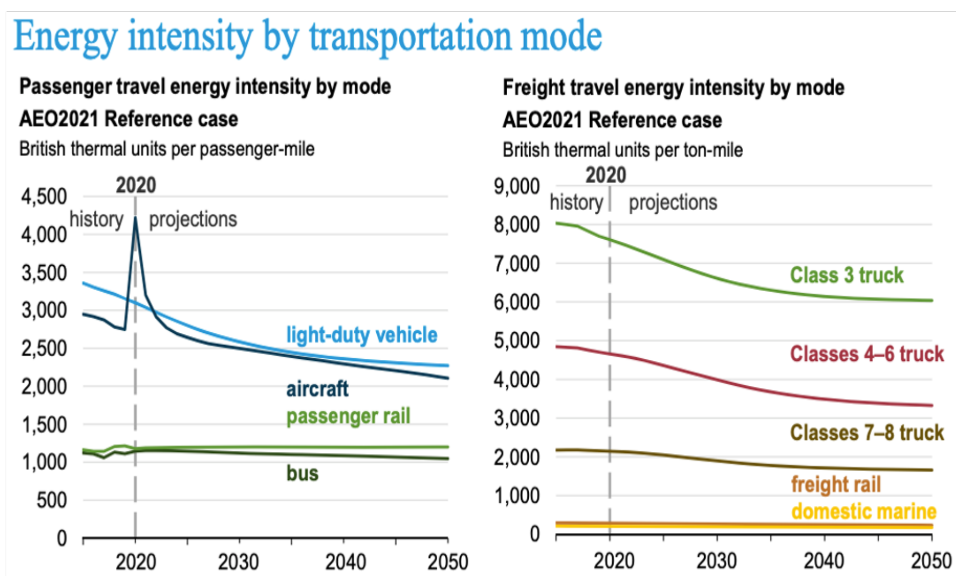


Figure 1.4. Past and Projected Energy Intensity comparison between Passenger Travel Modes and Freight Travel Modes. Class 7-8 cargo hauling modes are expected to have a projected improvement in energy consumption.

as Cummins Inc, Peterbuilt, Volvo, Daimler along with academic institutions working on demonstrating a better powertrain capability. While this is a positive element in the engine

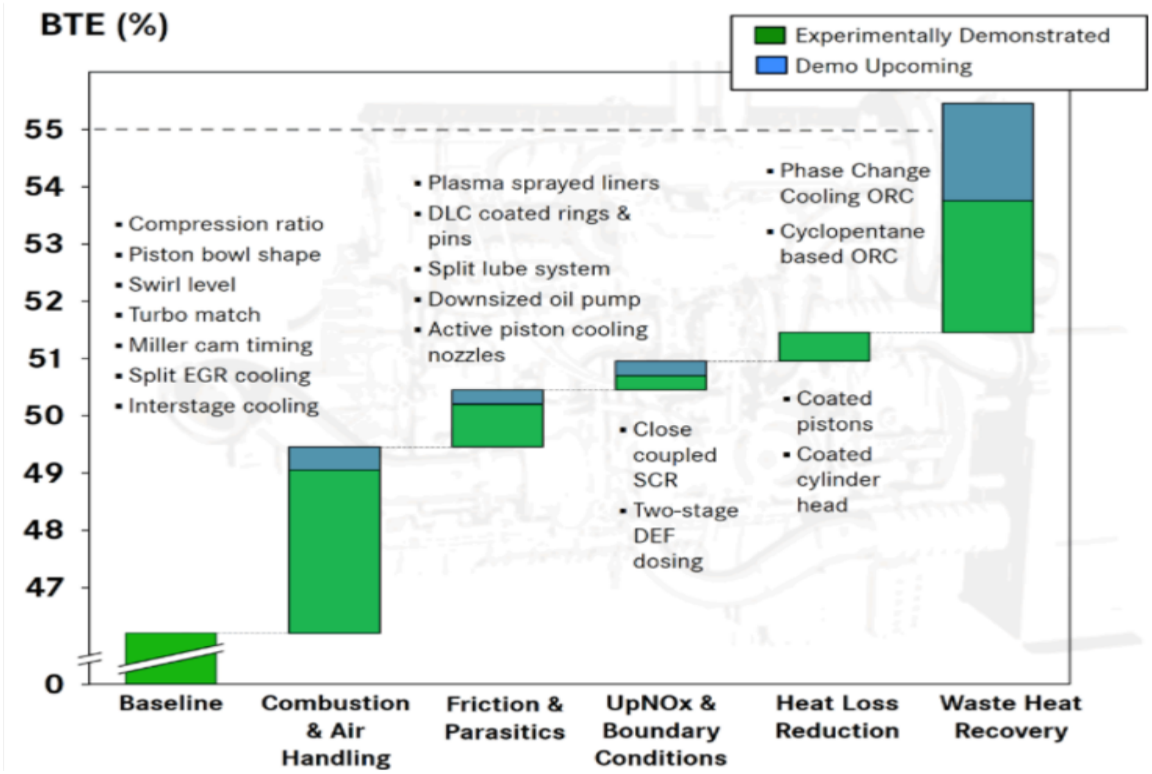


Figure 1.5. Department of Energy(DOE) Sponsored Super Truck II, Engine Brake Thermal Efficiency(BTE) Achievement

development side, there is still a huge scope in improving the overall scenario by compiling everything together as one package which will work together holistically to achieve a better fuel efficient, low carbon, low emissions solution. As an example while Supertruck-I was more towards conventional vehicle performance improvement, Supertruck-II brought in the concept of hybrid electrified powertrain to better augment the powertrain performance. It is quite intellectual to assume that Supertruck-III would definitely bring in the flavors of autonomy and fleet management to augment the hybrid powertrain for a holistic performance achievement. There are many such programs investigating the need to hybridize the conventional powertrain in a variety of architecture. Figure 1.6, shows the fuel economy, Distance travelled and energy consumption metrics as a function of speed, kinetic energy and distance. This data as reported by NREL showcases a good potential case study for hybridization benefits in a heavy duty and medium duty segment. While the passenger light duty vehicle segment is quite easy to hybridize due to its low profile in cargo carrying requirements, it is a challenging task to hybridize a heavy duty vehicle.

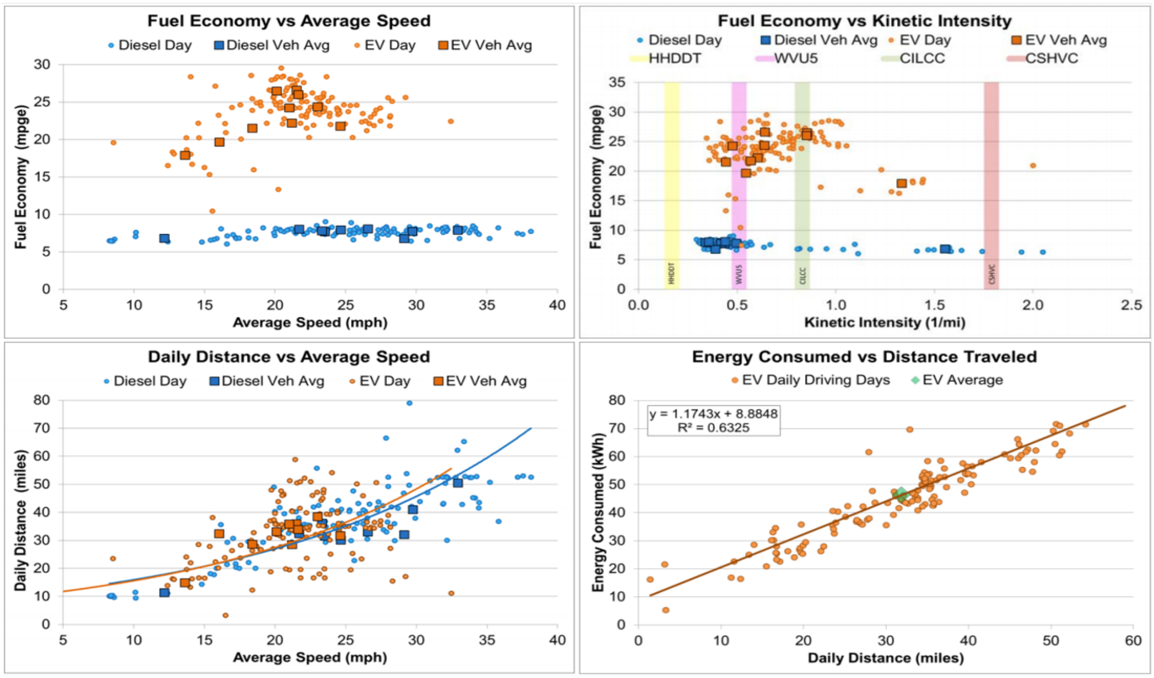


Figure 1.6. National Renewable Energy Laboratory (NREL) Data showing Fuel Economy Comparison data showing

The mandated percentage improvement of fuel economy for various types of vehicles is summarized in Figure 1.7. It can be seen that these regulations mandate significant reduction in CO2 emissions and fuel consumption for heavy-duty vehicles, e.g. about 15% for Class 8 heavy haul, and 20-30% for heavyduty pickups and vans by 2030.

All these numbers and reports indicates the need to have an overall better transportation methodology using an improved powertrain running an advanced control scheme. This also calls for introducing a more better platooning system. Predictive interactive controls in platoon is one such technology which is most researched these days.

1.2 Background

While passenger & light duty segment has seen a rapid growth towards chassis dynamics, infotainment, autonomy the heavy duty line-haul segment has seen tremendous effort towards

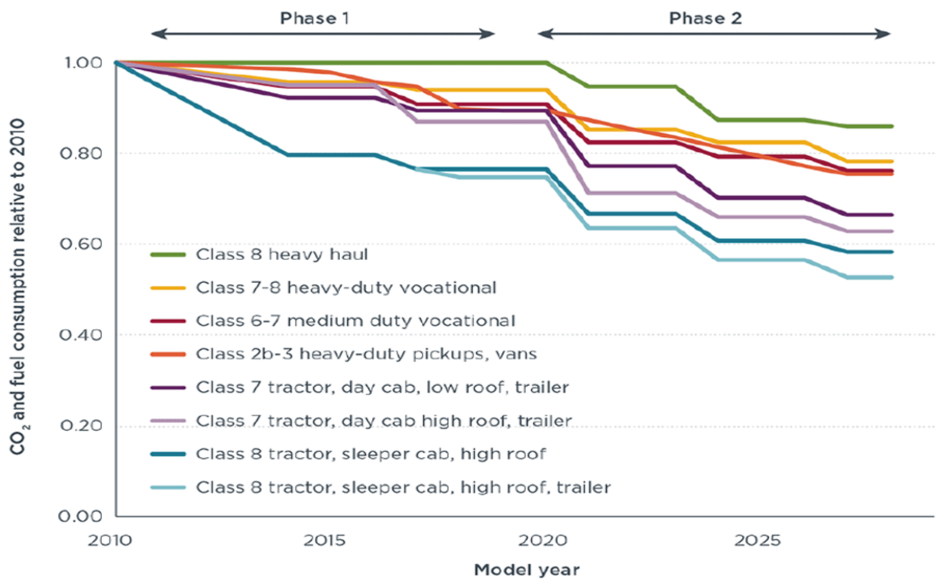


Figure 1.7. Summary of CO2 and fuel consumption reduction from adopted Phase 1 and proposed Phase 2 heavy-duty vehicle standards for selected vehicle categories [3]

a more cleaner combustion process. While a more cleaner combustion is achieved as a result of unprecedented improvement in the traditional internal combustion engine (ICE), there is also a holistic improvement being made each year to the complete powertrain system. When it comes to the powertrain, hybrid technology plays a pivotal role in defining the technology of the future towards a green sustainable solution for freight carriers. Heavy duty line-haul vehicles are focused towards meeting timely delivery of required cargo between destinations. This means that these segment is limited by payload bearing capacity as well maintaining a given road speed while hauling goods. These make any control formulation, around these applications along with component sizing, a constrained optimization problem, which depends heavily on load and route vehicle speed. Class 8 vehicle platoon is also studied as a viable solution for long distance freight carriers. Taylor, et al, [4] demonstrated a comprehensive analysis of Class 8 vehicle platoon with respected to corridor information such as road grade, traffic speed, road speed limit. This analysis shows the potential benefits of such systems when platooning as a function of grade and traffic flow. Emissions impact for different engine operating surfaces are also of prime importance while designing a global optimal solution. In order to achieve performance, engine operating points may shift and thereby adversely affect the carbon footprint as well as the tailpipe components. Hence it is often required to strike a balance between the engine operating points and the tailpipe emission limits. Though it can be argued that the after treatment systems such as diesel particulate filters, diesel oxidation catalyst and selective catalytic reductant will take care of the harmful elements but it is always not possible to decompose the elements in the after treatment system if they are not limited by the engine operation to a safe value.

In order to achieve a better overall powertrain efficiency and obtain a cleaner solution, hybridization of powertrain is of paramount interest. There are different configurations of hybrid powertrain architectures that are explored over the last decade. Each architecture has its own benefit and challenges and the selection of an architecture is heavily dependent on the application and the requirements. In heavy duty segment where freight carrying capacity is of utmost importance it is very essential to keep the weight of the hybrid system low in order to not compromise the freight carrying limit. Hence a small 48V hybrid system is often, a thought of solution for these applications. A very potential solution which is well explored in literature as well as in industry is the 48V parallel P2 architecture which offers a light weight torque assist on top of the internal combustion engine's (ICE) torque curve. This augmented torque when required provides a better and cleaner powertrain without compromising a lot on the net freight weight. Another architecture which has a lot of potential in the class 6 segment is the strong hybrid solution with a series architecture. Though this architecture gives a much better fuel economy there is a tradeoff to the overall system weight which reduces the net freight carrying capacity. In this architecture a downsized engine is used to charge the strong hybrid system. Another downside to this system is the occasional cooling of the after treatment system when the engine is off. Since the after-treatment system has to be at a certain temperature in order to convert the harmful engine emissions, this causes the NOx conversion to be less effective when the engine turns on and produces pollutants to a cold after-treatment system. All these constraints led to many academic and industry researchers to explore a variety of hybrid powertrain architectures along with engine optimization strategies to achieve a better, cleaner, efficient and reliable application. While a lot of work has been done in this domain but its often done in buckets without considering the complete system as a single unit. There is good amount of scope to have a optimal solution design including engine and after-treatment dynamics along with powertrain architecture. Now that we understand the vehicle dynamics and systems interaction the next level is to include the controls dimension. Most often control systems are based on reactive control strategies. As an example when cresting a hill the vehicle will only downshift based on the reactive torque requirement when the torque demand is not able to meet the vehicle acceleration/speed. This is pure reactive control. Now if the predictive element is introduced which can be road grade ahead, traffic information, weather information, etc, we get the opportunity to implement a much better predictive controller. The key element to this predictive controller is to understand what should be the predictive information and for how much ahead in time the information is needed. Once the predictive lever is introduced the control cannot be the regular reactive control but rather has to be some sort of predictive control with a look ahead horizon. This poses a challenge to implement a global optimal predictive controller capable of running in a modern controller hardware.

1.3 Contribution

This research work has two prime components. A simulation based class 8 mild hybrid truck is designed to formulate a complex multi-control multi-objective optimization framework. A true global optimality is studied in this step for a single hybrid vehicle The various interactive controls scheme that can be deployed in a predictive way for a mild hybrid class 8 application are proposed based on the results from this step. The second proposal is made for a predictive control approach of a platoon of 3 trucks using multi-agent based solver.

1.3.1 Development of a multi-objective predictive optimal control formulation for a hybrid class-8 truck

A true global optimal control strategy is developed and analyzed for a 48V mild hybrid class-8 truck. This part of the research work is developed as the foundation for the multi-agent framework. Chapter 5 discusses this formulation in detail. To the best of the author's knowledge a predictive multi-objective control strategy for an interactive hybrid system in a class-8 application is not studied well in literature. To address this gap and help understand the possibility of control system design this framework will be of paramount importance. The optimal solution for the entire route with the knowledge of predictive information can help design more robust energy efficient control strategy for a class-8 mild hybrid application. This framework also discusses the interaction of hybrid system with predictive controls. A list of predictive control strategies is prescribed as an outcome of the optimal study done in this chapter. The control strategies are prescribed as a function of road gradability.

1.3.2 Development of a multi-agent based predictive control strategy for a platoon of class-8 trucks

This is the prime objective of the research where the predictive optimal control strategy obtained from the previous analysis is fed to a simulation framework of 3 platooning trucks. A agent based controller is proposed using the modified Metropolis Algorithm to solve a optimization objective involving 3 trucks together in the platoon. The objective in this formulation is to arrive to an energy domain consensus between the trucks in the platoon following a state update mechanism to minimize an objective cost. This formulation is defined and studied in chapter 6. A list of control strategies is prescribed at the end of this chapter as a function of road gradability.

1.4 Thesis Outline

Chapter 2: Present Technology & Research Objective - This chapter provides an assessment of the present technology of predictive interactive control strategies for platooning class 8 trucks. The different control actions interacting with the mild hybrid system for a class 8 truck is studied. Predictive platoon with multiple interactive control levers are not studied well in literature for their energy domain optimization. Further the application an agent based approach for platooning consensus is not available in literature to the best of the author's knowledge. The lack of an integrated multi-objective approach for predictive energy optimization in mild hybrid trucks motivated the author for this work which is discussed in this chapter. The analytical objectives are discussed along with the hypothesis of energy saving is claimed in this chapter.

Chapter 3: System Design - A detailed design of a control oriented model is presented in this chapter. The dynamics of individual components along with the requirement of map based transfer functions are discussed in this chapter. A 48V mild hybrid with P2 configuration is also discussed in detail along with the component specification. The battery is designed as Coulomb counting SOC based structure but a detailed 1D electrochemical model is used to tune the parameters for a battery cooling system. This model is not used in the control oriented model but is used for tuning the parameters needed for the battery thermal management. Additionally in this section the route information is also discussed and the baseline results are portrayed. This result is used to estimate any benefit associated with the single vehicle offline global optimal solution as well as the multi-vehicle platoon configuration.

Chapter 4: Problem Formulation & Approach - This chapter discusses the high level problem. The details of the problem is defined and the objectives are laid out. Though the detailed analysis of the problem is done sequentially in the following chapters this chapter sets the prelude to the objectives in hand and how the complete problem in going to be solved.

Chapter 5: Energy Optimal Predictive Control of a single Class-8 truck - The final optimal solution for the 3 truck platoon system requires the problem to be solved for the single truck and then the optimal solution to be fed to the platooning system. This chapter proposes a framework and control strategy to be deployed to a hybrid vehicle for absolute global optimality. The proposed framework analyzes the true optimality between interactive control levers when look ahead information is available for the entire route.

Chapter 6: Energy Optimal Predictive Control of a Platoon - This chapter discusses the method and the objectives behind how the predictive optimal control strategy is used in a 3 truck platoon using the multi-agent based control structure. The proposed framework augment the optimal solution from chapter 5 and proposes the platooning system. The proposed framework shows better fuel benefits based on simulation results.

Chapter 7: Summary & Conclusion - This chapter discusses the major outcome of the work and highlights the benefits associated with it. This chapter also summarizes the understanding which is gained from the optimal behavior in multiple platooning trucks. It also discusses the benefits associated with control levers which are of prime importance and also talks about the passive benefits which are gained based in implicit behavior of the control levers.

Chapter 7: Recommendation & Future Work - Finally the scope of improvement is discussed in this chapter. It also states the necessary recommendations which are needed to achieve a more holistic optimal control understanding of the complete powertrain. As an example the after treatment parameters such as NOx, SOx, HC, system temperatures, etc are not used as an active element in the optimization process. Nor in the cost function neither as constraints.

2. PRESENT TECHNOLOGY & RESEARCH OBJECTIVE

Hybridization of heavy duty vehicle's powertrain has gained lot of importance due to their potential fuel saving capability. A number of different architectures and strategies have been explored by different researchers. [5],[6],[7],[8] studied the importance of hybridization of the powertrain and the respective energy savings associated with it. A few alternate fuel options are also explored as major power producing unit in line haul trucks[9],[10],[11],[12],[13]. These alternate power technologies are predominantly being sought of because of their high energy density. Various hybrid architectures and topology are available depending on application. A very detailed study on these component sizing, positioning, and advantages, disadvantages are studied in [14],[15], [16].

[17] presents a simulation study of various Battery Electric Vehicle (BEV) types to compare their performance when driving on real-road drive cycles to highly optimized eco-driving cycles. The results of the simulation confirmed that eco-driving has a high potential to reduce energy consumption for all types of BEVs. This study also compares the impact of eco-driving on conventional vehicles to comparable BEVs. An evaluation of eco-driving for two very different service types - local urban service and express service is conducted by the authors in [18], they implemented strategies to minimize fuel consumption by limiting instantaneous vehicle specific power while maintaining average speed and conserving total distance. Studies of platooning trucks in literature are largely experimental-based, and not simulation-based. Of the simulation-based platooning truck studies, Siemon et al. simulated Peterbilt 579 trucks in 4 truck platoons at spacing of 30, 50 and 100 ft gaps at 24.6 m/s (55 MPH) with different trailer configurations (box, shipping container, and flatbed trailers) found fuel savings of 2.5% for the lead, 9.5% for the second, 11.5% for the third, and 13% for the fourth truck [19]. The inter-vehicle dynamics, grade and speed effect, and shifting are not considered as there is no true platooning controller used. Johansson et al. simulates two platooning Class 8 Trucks on a 2 km stretch of flat highway, initially traveling at 90 km/hr ultimately slowing to 60 km/hr [20]. Experimental two-truck platooning results have previously demonstrated platoon-averaged fuel savings of 2.7-9.7% in Class 8 trucks traveling at highway speeds [21]. Platoons consisting of three Class 8 trucks operating at steady-state,

on flat ground, at 85 KPH (52.8 MPH), with a gap of 6 m (19.7 feet), demonstrated 4-5% fuel savings for the lead truck, 10% for the second following truck, and 14% for the third following truck at an altitude of 1,800 m (6,000 ft) where the air density is 80% of that at sea level [22]. Flat-ground test track experiments of three platooning heavy trucks at 80 km/hr (49.7 MPH) with a gap of 10 m (32.8 feet) showed fuel savings of 4% for the lead truck, 19% for the second truck, and 17% for the third truck. Fuel savings reduced to 1% (lead), 15% (second), and 16% (third) when the gap was increased to 15 m (49.2 feet) [23]. Peloton Technology experimentally demonstrated that a production intent platooning controller is capable of saving 7.25% on flat ground [24], [25]. A comparison of predicted fuel savings from platooning in simulation is made to this experimental data. Tsugawa et al, [26] showed that an automated truck platoon of 3 fully-automated trucks driving at 80km/h with a gap of 10m is capable of steady state driving and lane changing.The lateral control is based on the lane marker detection and the longitudinal control is based on the gap measurement and inter-vehicle communications. Fuel saving of 14 percent can be achieved on a test track and along the expressway using this feature. Traffic flow has important decision making aspects in truck platooning as discussed by Calvert et al, [27]. As discussed by them truck platooning has significant effects on traffic flow performance. These effects were tested for the influence of traffic states, truck gap settings, platoon sizes, and the share of equipped trucks. The results showed truck platooning to have slight negative effect on non-saturated traffic flow in contrast to having a large negative effect on saturated traffic flow. This paper recommends to improve platoon strategies and for policymakers to only allow truck platooning outside of busy saturated traffic areas. [28], presents truck platooning in autonomous heterogeneous trucks. As per the paper, every autonomous truck should keep following the leader truck's way-points while maintaining a designated distance from the truck ahead. [29], presents a flexible agent-based simulations model to serve as a matchmaking system for truck platooning. In contrast to centralized systems, this matchmaking is done locally among trucks using real-time data. Since this type of matchmaking is done spontaneously, this type of platoon matching is denoted as real-time matchmaking. Truck platooning attempts to significantly reduce fuel consumption for the follower truck as air drag and inter-vehicle gap is reduced. As per [30], platoon formation changes based on the start and end destinations for each truck and is also affected by other road users. This papers investigates how traffic may affect a merging maneuver of two trucks trying to form a platoon and observed that there could be a merge delay of over 10 percent when compared to the ideal case with the absence of traffic. As per [31], the efficiency of platooning is not only dependant on aerodynamic drag but also by the diffusion of platooning technology, the maximum platoon length and the tightness of time windows. The research in this paper shows that these factors can considerably reduce the positive effects of truck platooning. Guo et al, [32], investigates the problem of speed planning and tracking control of a platoon of trucks on highways. The speed planning algorithm uses average vehicle instead of platoon leader, thus making speed profile more fuel-efficient for platoons with vehicles if different weights and sizes. The vehicle controller is designed considering road slope and heterogeneity of vehicles. [33] proposes a cooperative distributed approach for forming/modifying platoons of trucks based on real time consensus algorithm. This approach when compared with a centralized optimization-based algorithm, proved to be a more general scheme that is able to form platoons even in cases with large initial separation of trucks and is capable of handling complex situations using its capability to form partial platoons. Zhang et al,[34] discussed that most literature only provides scattered pieces of information regarding fuel economy in truck platoons. This paper summarizes the methodologies, the fuel consumption contributing factors, methods to improve platooning rate, and future control strategies to generate fuel-efficient speed profiles for each vehicle driving in a platoon. [35] proposes a two-layer control architecture to safely and fuel-efficiently coordinate the vehicles in the platoon. The layers contain information on road topography and the real-time control of the vehicles using dynamic programming to compute fuel-optimal speed profile and a distributed model predictive control framework for real-time control of vehicles. Kaluva et al, [36] analyses the impact of platooning in urban environments by studying the influence of inter vehicle distance, platoon size and vehicle speed on the drag coefficients of the vehicles in a platoon. This study utilized two vehicle models, a minibus and a passenger car are analysed to characterize the drag coefficients. Muratori et al, [37] statistically analyses a large collection of real-world US truck usage data to estimate the fraction of total miles that are technically suitable for platooning. This paper focuses on estimating ”Platoonable” mileage based on overall highway vehicle use and prolonged high-velocity travelling and established that about 65 percent of the total miles driven could be driven in a platoon formation, leading to a 4 percent reduction in total truck fuel consumption. [38] assesses the impact of an eco-driving training program on fuel savings and reduction of CO_2 emissions in a well-designed field trial. This methodology proposed by Wang et al, includes different types of road sections under various traffic conditions and a systematic method to evaluate the overall and specific impacts of eco-driving. this paper offers great insights for policymakers in road transport planning and for drivers when applying eco-driving techniques. [39] explains how a truck driver controls his vehicle with the motive of maintaining a desired velocity while keeping the fuel consumption as low as possible. This is achieved by estimating oncoming operation points of the powertrain and optimal choice of inputs.This information is used as an input in an algorithm for the implementation of a predictive gearshift program and predictive cruise controller. In the paper [40] a novel predictive technology is used to incorporate the cruise set speed along with a gear shift point. The numerical based algorithm used a combination of nonlinear dynamics constraint and objective cost. The mixed integer problem due to the gear choice is solved partially by the outer convexification process. Benefits are shown on real world and artificial routes. Hellstrom et al, [41] explores how information about future road slopes can be used in a heavy truck with an aim of reducing fuel consumption without increasing total travel time. The longitudinal behavior of the vehicle is controlled by determining accelerator and brake levels and also which gear to engage. Paper [42] presents a novel predictive control scheme is used for energy management in hybrid trucks driving autonomously on the highway. This scheme uses information from GPS together with speed limits along the planned route to schedule charging and discharging of the battery, the vehicle speed, the gear and decision of when to turn off the engine and drive electrically. Borek et al, [43] presents an optimal strategy for heavy-duty trucks that minimizes fuel consumption in urban ares. This strategy uses an online convex model predictive control strategy that balances a trade-off between reducing braking effort and tracking optimal velocity. [44] introduces a model predictive control algorithm which attempts to reduce the cost of operation of heavy trucks with cruise control application based on road topology information obtained through GPS positioning and 3D maps. [45] attempts to solve one prominent challenge of truck platooning which is the safe and efficient interaction of the trucks with surrounding traffic, mainly in cases of lance changes that may lead to the decoupling of truck platoons. This paper proposes a supervisory tactical strategy based on a first-order car-following model with bounded acceleration is designed to maximize the flow at merge discontinuities. Kock et al, [44] proposes implementation of predictive optimal algorithms operating the truck at economically favourable operation points by considering the costs of operation and dynamics of the vehicle. This approach considers GPS positioning and 3D maps for slope, curve and speed limit information of future road segments. The paper [46] proposes a model predictive control method to control the clutch engagement process effectively shorten the torque interruption, thus enhancing the gear downshift quality. Huang et al, [47] explains an anti-idling regenerative auxiliary power system for service vehicles. Service vehicles generally have predetermined routes but the mass/load of such vehicles vary during drive cycle. Therefore, this paper recommends using an adaptive model predictive controller designed to account for this variation. The paper [48] explains a way of exploiting vehicular on board prediction for a limited time horizon and minimizing the auxiliary energy consumption of the electric cooling system through real-time optimization. The paper [49] provides a comparison of three strategies using model predictive control in with the objective of minimizing fuel consumption for a heavy-duty truck. The three strategies are; a time-based formulation that penalizes braking effort in place of fuel consumption, a simplified approach to the first strategy, and a distance-based convex formulation that maintains a tradeoff between energy expenditure and tracking of the coarsely optimized velocity. In the operation of long-haul trucks, fuel costs have a large impact on total cost of ownership. This paper [50] attempts to solve the problem of obtaining a trade-off between minimizing the fuel consumption and simultaneously maximizing the vehicle speed thus eventually decreasing time-related fixed costs. The paper [51] explores learning-based predictive cruise control and the impact of this technology on increasing fuel efficiency for commercial trucks by implementing predictive cruise control model which uses future road conditions and solves for cost-effective course of action. The paper [52] provides a comparison of three strategies using model predictive control in with the objective of minimizing fuel consumption for a heavy-duty truck. Two of these three strategies can then be adapted to accommodate the presence of traffic and optimally navigate signalized intersections using infrastructure-to-vehicular communication. Szabo et al,[53] presents a cascaded control system with a discrete-time model-predictive control in the outer loop and a flatness-based 2-DOF controller in the inner loop. The paper [54] illustrates how optimizing the power split among different energy sources in electric trucks and following distance should be performed to ensure safety, drag reduction and energy consumption. Earnhardt et al, [55] introduces two controllers capable of alternating between independent vehicle velocity trajectory optimization and a collaborative platooning approach based on the aggregate fuel savings of all the vehicles in the platoon. [56] investigates the fuel saving potential of predictive optimal control methods for the engine cooling system in conventional trucks. The advantages of this approach are the recovery of brake energy and the balance of energy sources in order to minimize total energy. As energy management strategy is crucial in improving the fuel economy of hybrid electric vehicles, this paper [57] targets at evaluating the role of velocity forecast in the adaptive equivalent consumption minimization strategies. It is a challenge to implement Equivalent Consumption Minimum Strategy online for real-time control due to the complex calculations needed. This paper [58] attempts to reduce ECMS's calculation load by proposing an adaptive Simplied-ECMS-based strategy for a parallel plug-in hybrid electric vehicle. [59] proposes a novel real-time energy management strategy for parallel hybrid electric vehicles. This approach uses adaptive ECMS which sets the time-varying equivalent factor. Hybrid electric vehicles have been known to be a feasible option to reduce fuel consumption and emissions. This paper [60] proposes a fuzzy logic controller adjusting the equivalent factor based on the deviation between reference state of charge and actual state of charge for better trajectory. Trian et al, [61] proposes an adaptive energy management system consisting of off-line and online parts to improve the energy efficiency of a parallel hybrid electric bus. The offline part focuses on the recognizing the precision of driver's driving style based on the hybrid algorithm. The online part incorporates driver's driving style into equivalent consumption minimization strategy.

There are wide range of controls available and a variety of vehicle models but none solved an energy management strategy for a platoon using detailed optimal behavior for multiple states and controls. The primary objective of this work is to find the best strategy in terms of global optimality with all levers interacting together. This kind of setup is not studied so far to the best of the author's knowledge. There are no solution available for a predictive controller trying to control more than 4 levers using a dynamical system with more than 5 states for a 3 truck platoon. In this work an attempt is made to design, implement, analyze and understand the multi-objective optimization based, true global behavior for a mild hybrid electric class-8 truck and then extend the optimality to solve a problem for the 3 truck platoon. While the single truck optimality help understand the true optimal strategies than can be deployed on a mild hybrid truck based on look ahead knowledge of the route, the multi-agent based method will define the optimal strategy for a platoon of 3 trucks when look ahead information is available.

Table 2.1. Summary of Literature and Contribution

Contribution Summary		
Topics	Literature	Proposed
Predictive Single Control - Class 8	Matured	-
Multiple Interactive Predictive Controls - Class 8	Insufficient	Contribution
Multi-agent predictive platooning - Class 8	Insufficient	Contribution

Table 2.1 summarizes the contribution made through this work. This research was done as part of bridging the gap between existing literature and what the author thinks shall help design predictive platooning system of class-8 trucks.

2.1 Inspiration

This research is inspired from human behavioral science as exhibited while riding a bicycle. Riding a professional road bike is an advanced method of utilizing look ahead perceived information for an energy management solution. Figure 2.1, shows the different control levers used during a ride by a rider. It also shows that the look ahead information is perceived by the eyes and the brain does the rest of the energy management to control the different controls in the right efficient way. An efficient control strategy is of paramount importance while riding on a hilly terrain due to the limitation of energy production. Figure 2.2 shows

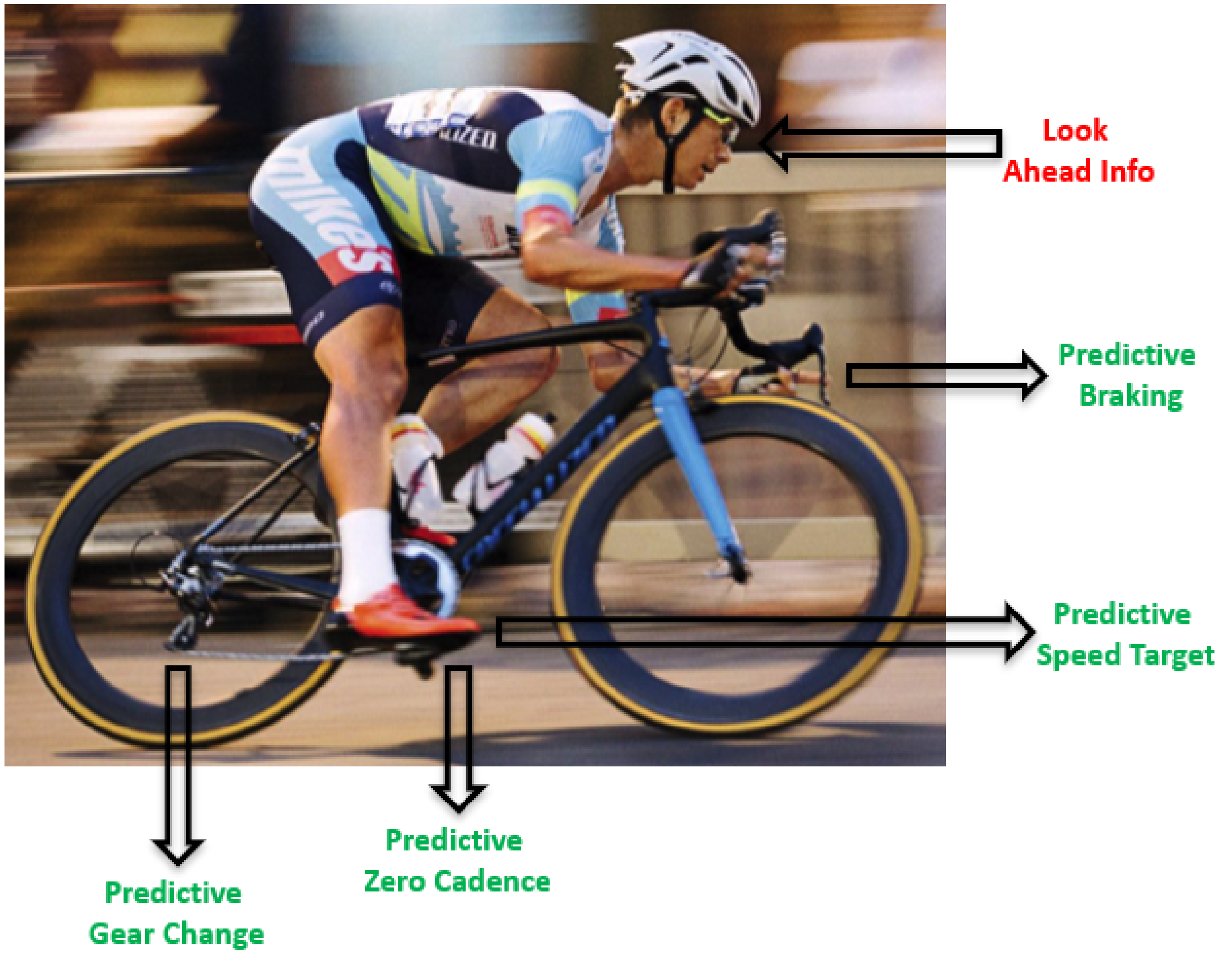


Figure 2.1. Predictive control analogy derived from bicycling case

the different predictive controls in an illustrative way for various section of the route with different grade profile. To understand better the control actions we will look at the cyclist at different part of the route with different gradients.

- At point **A**, the cyclist sees a hill coming up. While still at a milder grade and mostly flat section the cyclist uses the look ahead road grade information for the entire line of sight which is shown by the red arrow. Based on that grade information the cyclist predicts that to crest the hill the better strategy is to speed up during the flat section where it will need less energy to attain a greater speed and then utilize that kinetic energy to help crest the hill. It is important to note that the range of speed increase has to to be judiciously done so that it is not required to brake at the end of the

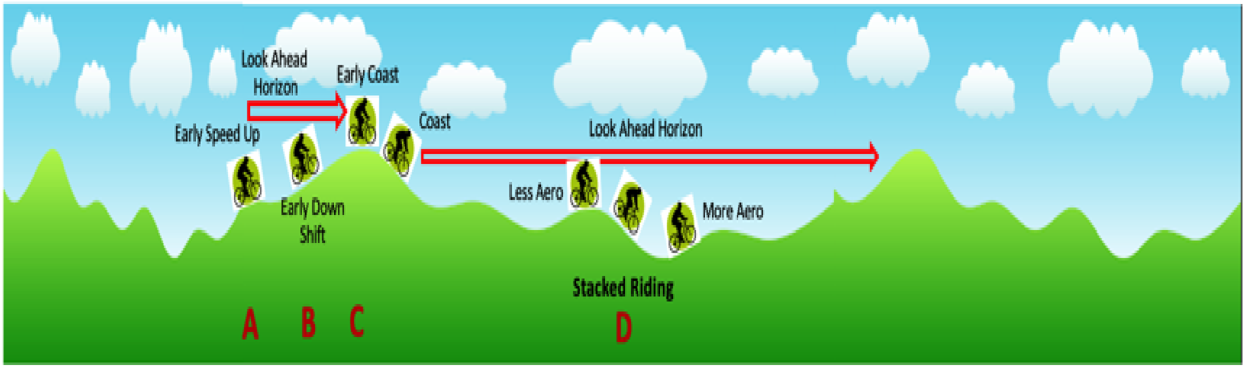


Figure 2.2. Short horizon predictive controls as exhibited in professional bicycling.

hill. Essentially it is not required to gain kinetic energy more than what is required and then waste the energy gained by braking which is basically loss of energy at the expense of rider power/calorie. This is termed as predictive speed management wherein instead of targeting a constant cruise speed the rider can decide to increase the target dynamically depending on the need, based on look ahead road grade information.

- The second part of the route is at point **B**, where the rider is on the uphill and has attained enough kinetic energy to crest the hill. At this point the rider predictively and progressively down shift to help crest the hill with less effort and thereby compromising a bit on the speed. Depending on individual rider power this down shift will be decided. Some riders can crest the hill with out down shift if the speed gained before the hill is sufficient. It is worth noting that different levels of complex optimization process based on different objectives are performed by different riders.

- The third part of the route is at point **C**, where the rider is at the apex of the hill. At this point the rider get a long look ahead information for the grade ahead. The efficient control is to pulse and glide throughout the down grade and also the flat section following a steep down hill. The rider can optimize a best speed target plan by manipulating cadence and cycle speed.

- The last point **D**, is not a unique point from the previous three points but rather a situation which can occur in conjunction to the previous 3 points. The riders form a stacked up profile to reduce aerodynamic drag loss. The individual riders in the stack can determine their controls based on their look ahead knowledge for the most efficient energy management of the entire group.

2.2 Extension to Automobile Application

The high level primary objective for a class 8 platoon of 3 trucks is very similar to the biking scenario. It is to formulate a complex multi-objective optimization method which will provide us with a global optimal profile for all the control levers we have discussed so far. The problem is solved in a sequential method by bringing in one control lever at a time. First the speed management problem is solved then the coast management and then the two of those together to see the interaction between the two. The final solution is where all the 4 control levers **Speed, Coast (Neutral and Engine Off), Gear and Power Split** are solved together. The solution is done for a chosen route. So, for each vehicle configuration change and route change we need to get the optimal solution. The method used to solve this non-linear global optimal problem is by a computationally challenging dynamic programming algorithm. Hence, this part of the problem is done offline and is referred to as a offline optimizer. This is also done individually for each truck. So if two trucks have two different configuration they will have two separate optimal profiles for the control levers.

The next big part of the work is when multiple such trucks form a platoon and cooperatively drives through a route following the optimal profile for each truck. A multi-agent based control strategy is developed to achieve better fuel efficiency in a platoon operation with the optimal control profile. Each agent runs its own state updates based on the designed metropolis algorithm and then solve the cost function to find the minimum objective value. A detailed objective for each problem formulation is defined in this section. *The primary objective of this research is to understand the potential of* ***Cycle Efficiency Management*** *solution in term of* ***Absolute Fuel Economy*** *numbers and the impact of this optimal solution to* ***Emissions***. In this research a complex holistic study is attempted to see the interactions between different control levers and how they perform. The research will answer some of the key questions at macroscopic energy and component losses level on one hand and also will answer some of the detailed microscopic level behavior of the components as seen in time series plots. Figure 2.3, shows the high level objective what we want to achieve

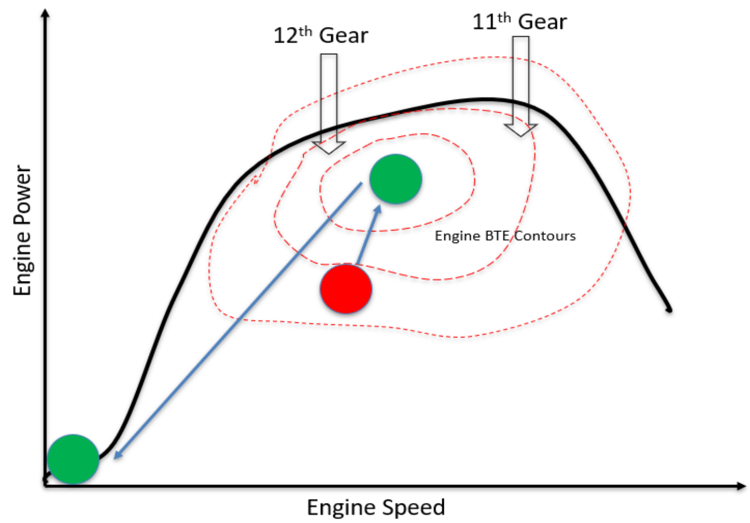


Figure 2.3. Research Objective In Fuel Map Space

in this research in fuel BTE Space. The figure shows the power curve as a function of Engine Speed. On the same plot we have the BTE contours. The 12^{th} and 11^{th} gear mark is the Isochronous speed at **65mph** vehicle speed in engine speed domain.

At this BTE contour level the objective is to move the red bubble to the green bubble zones which will increase the efficiency of the engine and save fuel. Also moving the BTE point to around 600RPM means the engine is coasting in neutral which has its own fuel benefits. The other vehicle level objectives that are analyzed in terms of fuel saving measures are the negative work reduction by the truck. Negative work involves both motoring work and well as engine and service braking. In case of the platoon system another important metrics is the aerodynamic loss reduction. As briefly described in the previous sections, this research is to study the benefits associated with running this advanced optimal targets in individual trucks following a platoon. So the objective is divided into two fold. First find the offline optimal profile for each truck given a route and vehicle configuration. Second design an online multi-agent based controller to follow the optimal target to achieve a safe operation and better fuel economy.

2.3 Offline Mode - Single Vehicle Optimality

This activity is needed whenever there is a different route introduced or the vehicle configuration changes. A full factorial based dynamic programming [62] algorithm is used to find the global optimal profiles for each control levers described above. The problem is solved in a stacked up approach where each lever is solved individually and then combined together to analyze the interaction between the different controls levers.

2.3.1 Predictive Speed Management

Figure 2.4, illustrates the speed objective which is expected to be the outcome of the optimal solution. This analogy is derived from the understanding of the cycling analogy and observing the behavior of the rider. The problem is formed as a minimization type objective

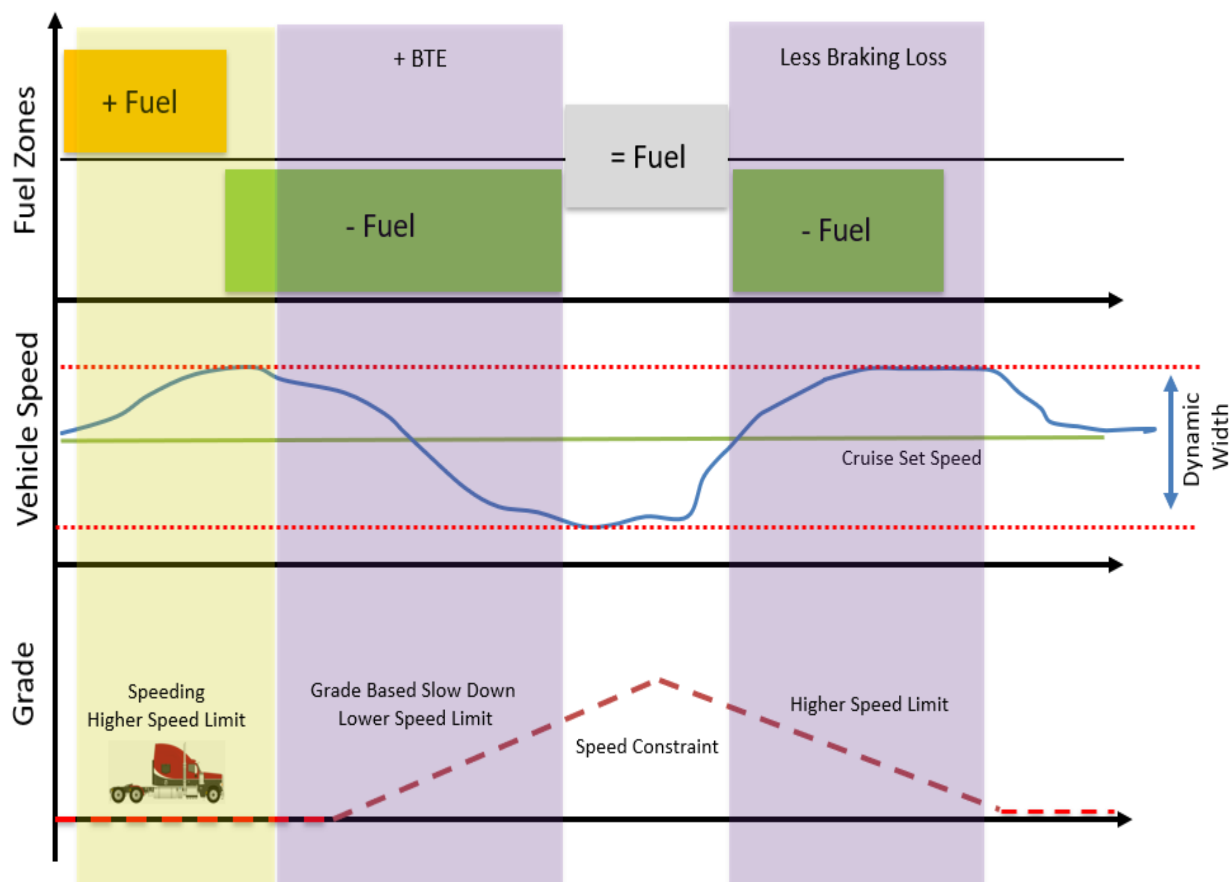


Figure 2.4. Speed Management Cartoon

with a cost function which is the weighted sum of the total fuel consumed and trip time. It is worth noting here that the trip time is included in the objective to make sure that the vehicle is not moving slow in order to achieve fuel benefits. Further, in order to include trip time which is not the free variable here, the problem is converted and solved in distance domain instead of the time domain. As a result of this the dynamics of the vehicle is converted to a distance domain.

$$\frac{\delta y}{\delta s} = \frac{\delta y}{\delta t} * \frac{\delta t}{\delta s} \tag{2.1}$$

which gives,

$$\frac{\delta y}{\delta s} = \frac{\frac{\delta y}{\delta t}}{v(s)} \tag{2.2}$$

Which essentially is dividing all rate of chance dynamics with vehicle speed in distance domain. Similarly we will also get the vehicle speed as :

$$v(s) = \sqrt{(}2 * \int \frac{\sum F}{m_{\text{eqiv}}} ds) \tag{2.3}$$

Hence, for this problem the high level objective is to let the cruise set speed dynamically vary based on look ahead road grade information. Analogically we can predict that the vehicle will speed up during the flat section before entering an uphill. Since the vehicle can now target a higher set speed on top of the baseline cruise set speed (which is decided by the tuning of the droop), it will use more fuel and try to gain kinetic energy which can be used later in the up-hill. The highlighted zone in **”Yellow”**, is the section of the route where the speed-up will occur.

The first **”Purple”** highlighted zone is the up-hill where the vehicle loose speed due to gravity and since it has started from a higher speed and it can go up to a much lower speed in this configuration there will be fuel saving by letting the engine work at a better BTE contour zone. The second **”Purple”** zone is the downhill section where the vehicle can now achieve more speed and not have to brake to target a set cruise speed. This will help with some more fuel savings by reducing losses due to braking. The speed modulation around the baseline set cruise speed is a complex design parameter to tune since it will decide on the level of fuel savings and also impact drivability.

2.3.2 Predictive Coast Management

Next concept which is extended from the rider's concept of no cadence is the neutral and engine off coast. The objective is to find out regions in the route where it is possible to coast and even turn the engine off. It is essential to select an architecture which will make it possible to disengage the engine from the drive line so that this control strategy can be implemented. Hence a P2 architecture is selected. Details of the system selection will be discussed in next chapters.

Figure 2.5, is an illustration of the concept of coast management. The ORANGE portions

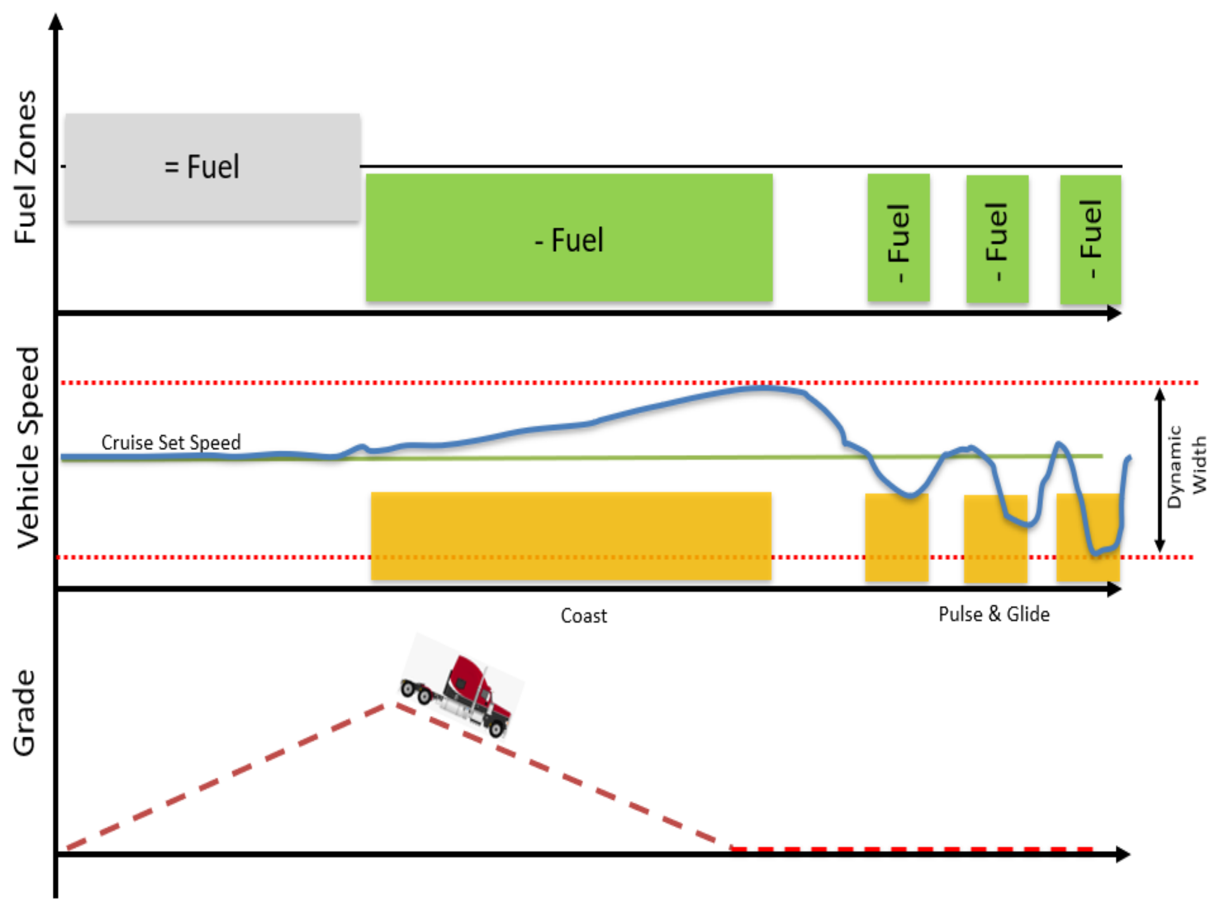


Figure 2.5. Coast Management Cartoon

are the regions where the vehicle will enter into coasting depending on operator demand power. It is expected to see a big coast event during the downhill and small frequent coast events during flat section of the route. This frequent coast events in the flat section is termed as *"Pulse and Glide"*. As in the case with speed management, we also have a dynamic speed bound between which the vehicle speed can modulate instead of targeting a fixed set speed.

2.3.3 Predictive Gear Management

The next control lever that will be studied is the modulation of gear predictively. This is expected to be more of a performance lever rather than a fuel efficient lever. As observed from the cycling analogy a downshift is needed during the uphill section in order to crest the hill. In the case of an automobile that may not be the issue but there may be speed lugs during the uphill which is a drivability issue. So, it expected to solve the drivability issue by improving on the speed lugs and also not consuming more fuel in order to do so.
Figure 2.6, is an illustration of the concept of gear management. The objective is to formulate

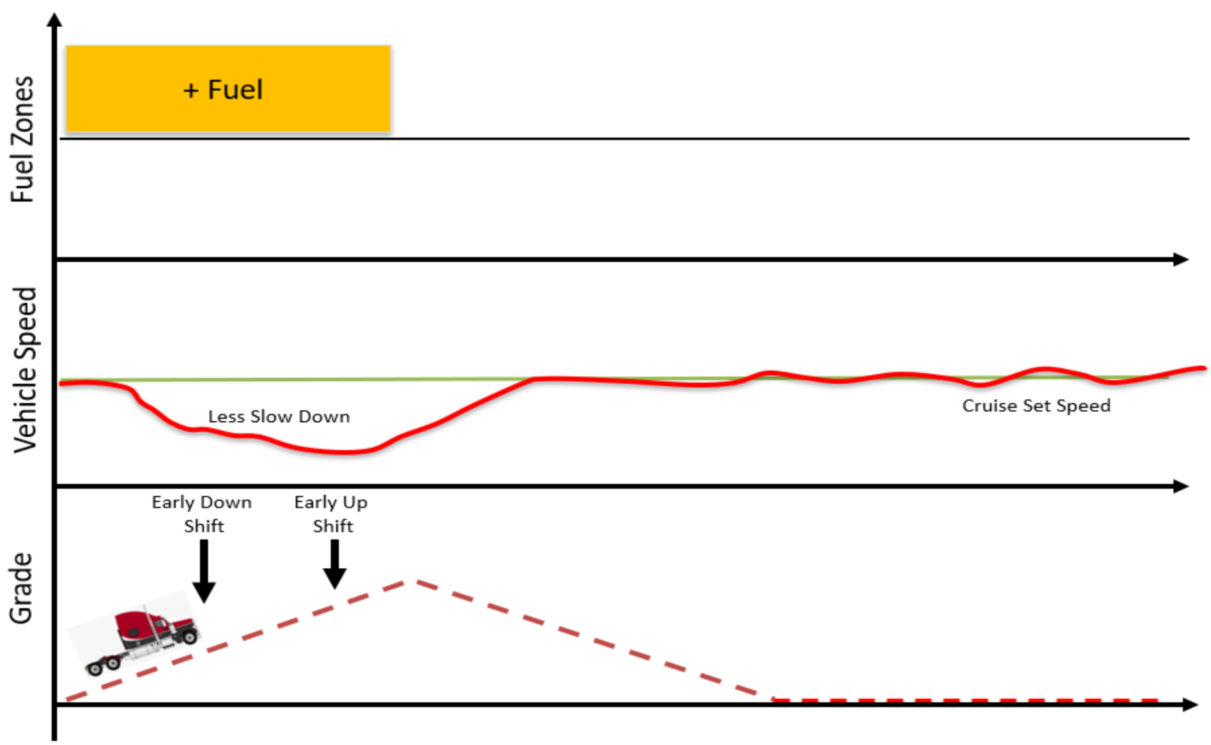


Figure 2.6. Gear Management Cartoon

a problem to see if the vehicle is predictively shifting ahead in time as compared to the reactive shifting schedules. It is also worth to note that the vehicle will also up-shift early as compared to its reactive counterpart. It is not expected to gain any fuel benefits from this objective but it is still possible to achieve a fuel benefit in case the engine efficiency based fuel tables are tuned in such a way that the engine domain isochronous speed at a lower gear (11^{th} gear) does not stretch way beyond the maximum engine BTE contours. This is discussed in detail in subsequent chapters.

2.3.4 Predictive Power Split

Lastly the objective on the single vehicle is to predictively split the power requirement between engine and hybrid electrical system. Based on the fact that this is a 48V Mild Hybrid system it is not expected to see any substantial improvement with this concept. The analogy is drawn from riding a **eBike**.

All the 4 control levers are individually solved for the single vehicle configuration given a specific route. The final optimal control trajectory is selected based on the solution of all these control levers working together. Since there is strong correlation between these levers the problem is formulated to minimize the fuel usage with all 4 controls together. A detailed problem formulation is discussed in later chapters along with recommendations to be deployed in a class 8 mild hybrid truck.

2.4 Online Mode - Multi Vehicle Optimality

The online mode is the core of the research work where the objective is to let multiple trucks operate in platoon following the target optimal trajectory as found by the offline solver. In general platooning in trucks is a very popular topic these days. The major focus in developing platooning system is more towards addressing the perception problem, connectivity algorithms, Functional Safety and Robust Sensor Technology.

Figure 2.7, shows the setup in a platooning system. The literature study in this research showed that there is insufficient study available for predictive platooning based energy domain analysis. In this work we have considered only the 1-D longitudinal dynamics of the platooning system, optimizing an energy minimization objective function. Various multi-agent methods are already established in a lot of different field with outstanding capabilities.

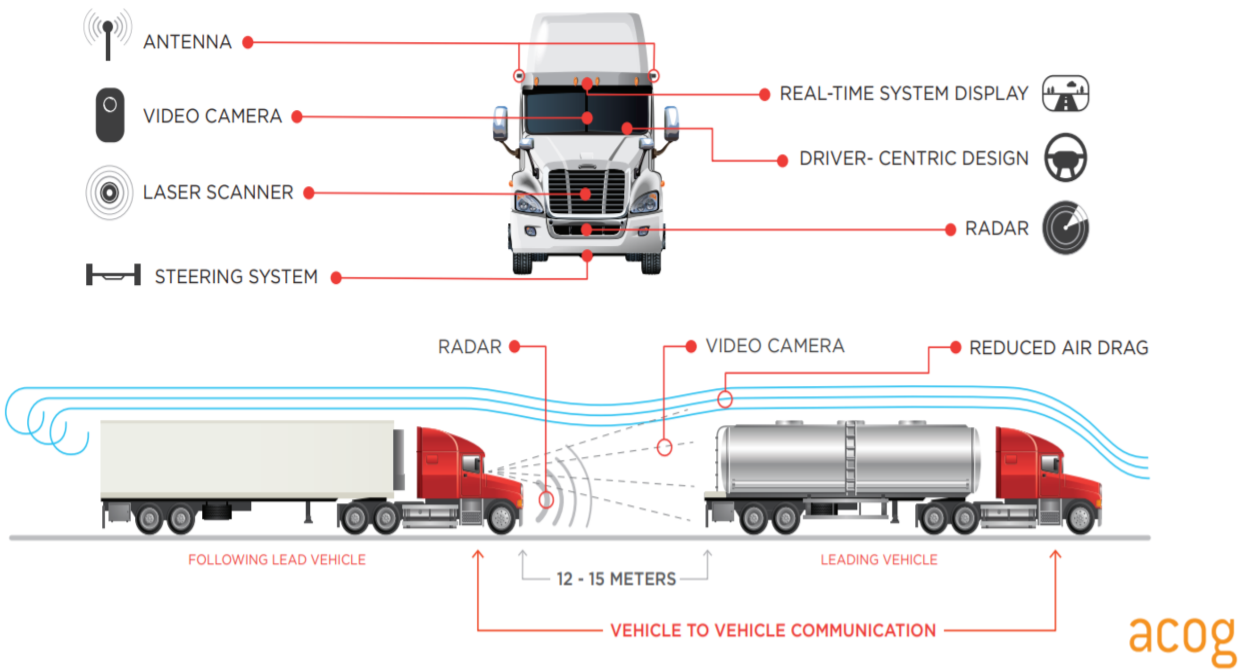


Figure 2.7. Platoon Setup Between Vehicles. Elements of Vehicle to Vehicle Communication showing a following distance of 12-15m. The key elements used to establish connectivity between two platooning trucks in close proximity.

In this work a similar approach is used using the well established metropolis algorithm based state update methods to minimize an objective function. In this research each truck share its speed profile and current optimal profile between each other. Based on the information from the different trucks in the platoon each truck runs its own local optimal algorithms to achieve safe operation (maintain distance) and achieve a better fuel efficiency and reduce emissions by their engines working at better BTE zones. The key contribution in this work is the application of the multi-agent based method to study the energy efficient predictive behavior in the platooning trucks. The questions tried to answer through this work are,

- Is the problem solvable using an multi-agent based algorithm?
- Does the algorithm show promising results?
- Is a dynamic separation distance good for platooning truck in energy domain?
- Is predictive controls required for follower trucks in platoon?

- What are the key contributors to absolute fuel economy in follower trucks in platoon?
- Predictive control challenges in grades for follower trucks?
- Interaction of multiple controls and its feasibility in application?

There are multiple ways platooning can be achieved with different objectives. In this research an energy level study is conducted to analyze and understand the overall predictive behavior in platooning hybrid trucks. The work is conducted to understand the key questions as highlighted above. The key contribution is the application of a simple multi-agent based algorithm with assumptions to maintain and achieve energy savings.

3. HYBRID VEHICLE DESIGN & COMPONENT SELECTION

This chapter is a self contained document of describing the basic principle behind designing the longitudinal dynamics of a Vehicle. To the extent feasible, the model of both a conventional diesel operated vehicle as well as a hybrid vehicle will be designed. There are an enormous number of complex models that can be designed for any vehicle. The design objectives are dependent on the use cases of the models. As an example for studying and implementing lane keeping assist systems a lateral yaw dynamics model of the vehicle is desired. Similarly for studying the vibration and shock absorber dynamics, a quad car model of the axle and wheel may be required. In the context of this research where we try to manage the energy, a simple one dimensional longitudinal force balance model is sufficient.

Figure 3.1 shows the different forces acting on a vehicle in motion. All of these forces are dynamic and are either a function of road grade or vehicle speed. Newton's 2^{nd} law of motion governs the force balance equation for the vehicle. The tractive force for the vehicle is, the

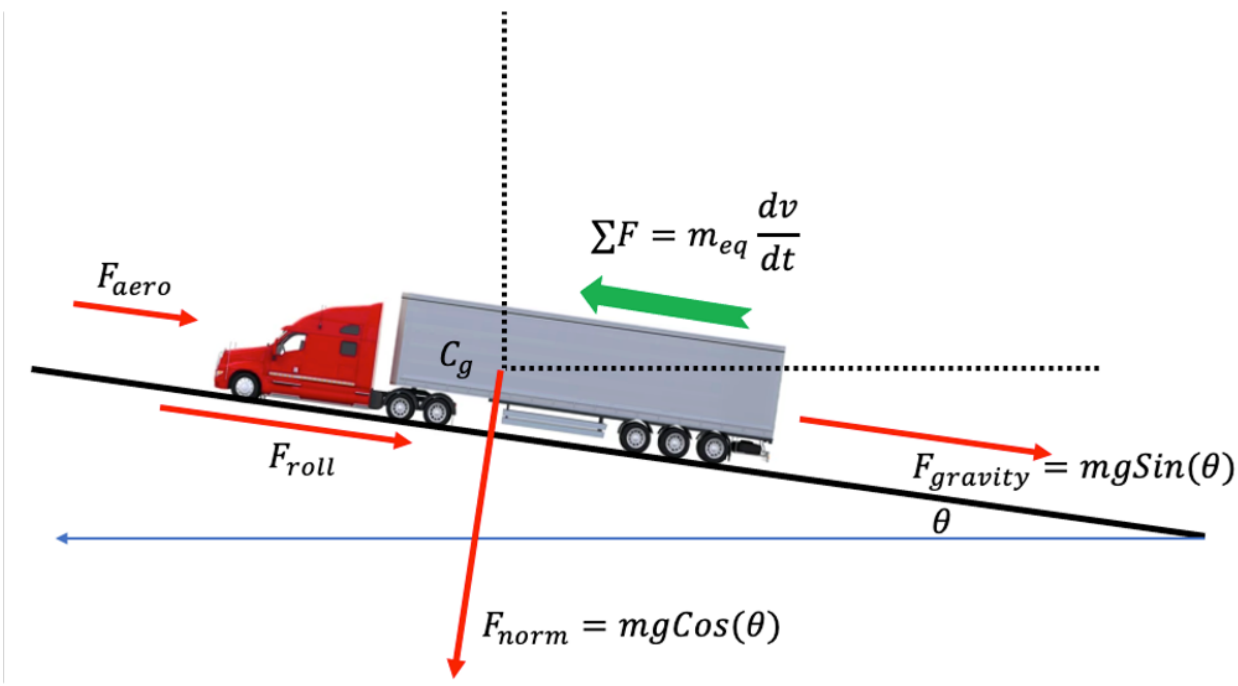


Figure 3.1. Summation of vehicle negative forces and the balancing tractive force resulting into the force balance equation providing the vehicle forward acceleration

equivalent mass of the vehicle times the forward acceleration. Hence we get our very first equation for the vehicle dynamics in 1-D longitudinal form as 3

$$F_{trac} = m_{\text{eq}} \frac{dv}{dt} \tag{3.1}$$

where, $\frac{dv}{dt}$ is the vehicle forward acceleration.

The negative forces which tries to restrict the forward motion of the vehicle are Aerodynamic drag force which is a function of how fast the vehicle is moving and its frontal area. The more streamline the vehicle frontal design is the less this drag force. The other resisting forces is the rolling resistance due to the frictional contact of the wheels with the road surface. The next force which resist the forward motion is component of gravitational pull which is a function of the Vehicle equivalent mass. In this chapter we will look at the details of the different components and how they are chosen and designed in the context of this research work. The Aerodynamic drag force is represented by equation 3

$$F_{aero} = \frac{1}{2} \rho C_d A \nu^2 \tag{3.2}$$

where, ρ is air density, C_d is aerodynamic drag coefficient which depends on Vehicle geometry, A is vehicle frontal area & ν is vehicle speed. This drag force in case of follower vehicles in a platoon changes as 3,

$$\begin{gathered} F_{aero_{follower}} = \frac{1}{2} \Phi(d) \rho C_d A \nu^2, \\ \Phi(d) = (1 - \frac{C_{D,1}}{C_{D,2} + d}) \end{gathered} \tag{3.3}$$

where, d is the separation distance between the vehicles. $C_{D,1}$ & $C_{D,2}$ are constants obtained from polynomial fit data published in open literature.

3.1 Engine

The first component in the system which is the primary power producing device is the Internal Combustion Engine. There are a number of models which can be used to design the dynamics of the engine depending on usage and need. A detailed Air Path Dynamics Model using 3 states (Boost Pressure, Exhaust Pressure and Turbine Speed) can be developed as,

$$\begin{aligned}
\frac{dP_{IM}}{dt} &= \frac{\gamma R}{V_{IM}}(\dot{m}_c T + \dot{m}_{egr} T_{egr} - \dot{m}_{eng} T_{IM}), \\
\frac{dP_{EM}}{dt} &= \frac{\gamma R}{V_{EM}}[(\dot{m}_{eng} + \dot{m}_{fuel}) T_{EM} - (\dot{m}_{egr} + \dot{m}_t) T_{EM})], \\
\frac{dN_{tc}}{dt} &= \frac{1}{J_{tc} N_{tc}}(P_t - P_c),
\end{aligned} \tag{3.4}$$

where, P_{IM} is the intake manifold pressure, V_{IM} is the intake manifold volume, T_{IM} is the intake manifold temperature, P_{EM} is the exhaust manifold pressure, V_{EM} is the exhaust manifold volume, T_{EM} is the exhaust manifold temperature.

Similarly, dividing the engine into a Block, Crank and Sump structure, we can develop a thermal model as well [63]

In this research we do not need a detailed model like described above, rather a simple map based efficiency model will suffice our purpose. The **Engine** is taken from a **Cummins Inc©15L diesel** family which has a power rating of **298-373 kW** and a torque rating of **1966-2508 N.m**. The fuel map is made up to be around **47%** efficient as shown in Figure 3.2. Its a **6 cylinder inline** configured system [64]. The solid black curve is the Engine Power Curve [kW] which shows the different power characteristics as a function of Engine Speed in RPM. The efficiency map which is made up to have an island of around 47% within a band of 1100-1250 RPM. It is worth noting here that the objective of any engine controller would be to make the engine work in this zone. This means that at top gear the engine speed should lie in this zone.

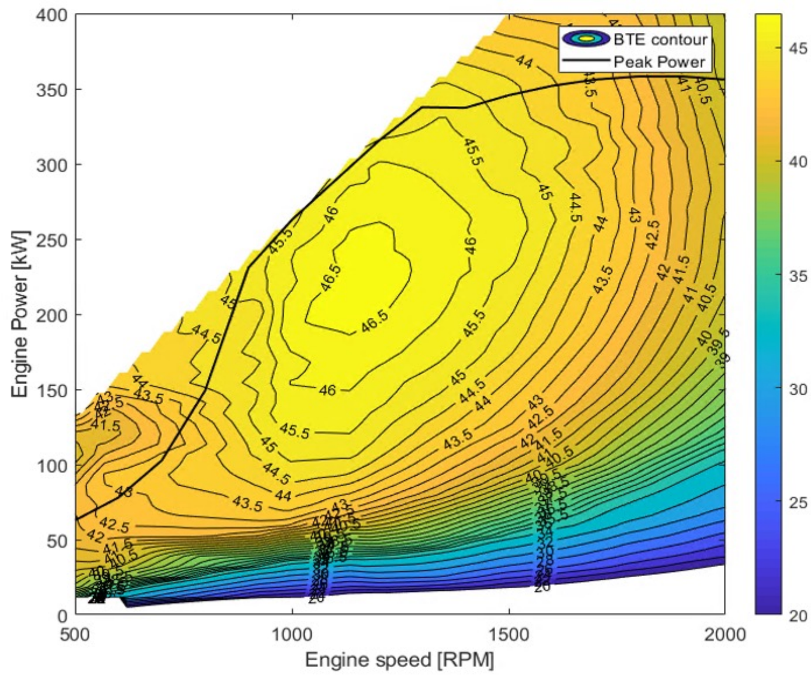


Figure 3.2. Engine Fuel Map - BTE
[64]

3.2 Clutch

Clutch is designed as a simple torque pass through device after adjusting for losses. It is assumed that clutch is a constant 97% efficient device. Equations 3.2 are the two main Clutch dynamics.

$$\begin{aligned} \omega_{eng} &= \omega_{clu}, \\ \tau_{clu} &= \tau_{eng} * \eta_{clu}, \end{aligned} \tag{3.5}$$

where, η_{clu} is the efficiency of the clutch or a measure of torque loss. It is worth to note that in the Clutch we calculate the Engine speed to be fed back upstream to the Engine system.

3.3 Transmission

The transmission system is a 12 speed overdrive EATON system. There are 12 forward ratios and 2 reverse ratios. It can support a maximum Gross Vehicle Weight (GVW) of

49895 Kg and supports a maximum torque of **2508 N.m** The gear ratios are documented

Table 3.1. Transmission Gear Ratios
[65]

Type	Gear #	Gear Ratio	% Step
Forward	1	14.43	31
	2	11.05	31
	3	8.44	31
	4	6.46	31
	5	4.95	31
	6	3.79	30
	7	2.91	31
	8	2.23	31
	9	1.70	31
	10	1.3	30
	11	1.0	31
	12	0.776	–
Reverse	1	16.92	–
	2	12.95	–

in table 3.1 and are referred from **EATON**©[65]. The shift points for the transmission is made up using vehicle speed reference. The way it is derived as a function of vehicle speed and operator throttle so that at cruising speed the transmission stays at top gear. It is also done in a way to keep the engine speed within the best operable BTE region.

3.4 Electrification - Discussion & Selection

Mild hybrid system is very popular in passenger car segment mostly due to emissions benefit and somewhat fun factor associated with it. The key aspects of a MHEV in a passenger car are,

- electrical power available (e.g. 15kW)
- voltage rating of the battery system (e.g. 48V)
- fuel consumption / CO2 reduction potential (e.g. 15%)
- functions performed by the electric machine (e.g. torque boost)

Figure 3.3 shows the market share of different types of hybrid system in the passenger car segment. where, A - Subcompact Cars, B - Compact Cars, B - Medium Cars, D - Large

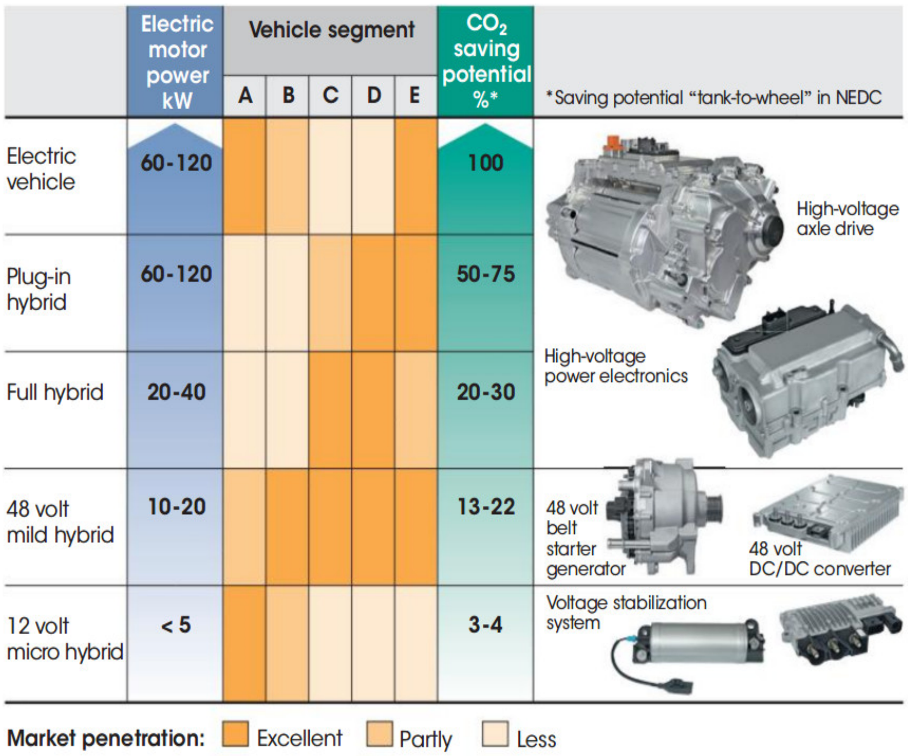


Figure 3.3. Passenger Car Market Penetration
[14]

Cars, E - Premium Cars.

Though the same range of benefits are not possible in a class-8 type application but there is scope of bringing in a number of benefits when we integrate a MHEV system in a line haul truck. There are multiple options available in terms of architecture for a **Mild Parallel Hybrid Vehicle (MHEV)**. The key objectives while looking into the different architectures are the positioning of the components (specifically the motor generator), the various control features that can be implemented and finally the advantages & disadvantages associated with fuel savings & drivability. Figure 3.4 shows the growing trend in market share that the electric and hybrid vehicles will have over a period of next 10 years. The information provides significant importance to the usage and research of electrified powertrain. Figure 3.5 categorizes the various control and systems features that can be implemented based on the selection of the choice of Hybrid Powertrain architecture. Though most of these feature is not related to our research of Class-8 application but still it provides a detailed understanding and helps with identifying the best architecture for this research. The various **Mild Hybrid Electric Vehicle** architectures are enumerated below with their description,

1. **P0** – The electric machine is connected with the internal combustion engine through a belt, on the front end accessory drive (FEAD)

2. **P1** – The electric machine is connected directly with the crankshaft of the internal combustion engine

3. **P2** – The electric machine is side-attached (through a belt) or integrated between the internal combustion engine and the transmission; the electric machine is decoupled from the ICE and it has the same speed of the ICE (or multiple of it)

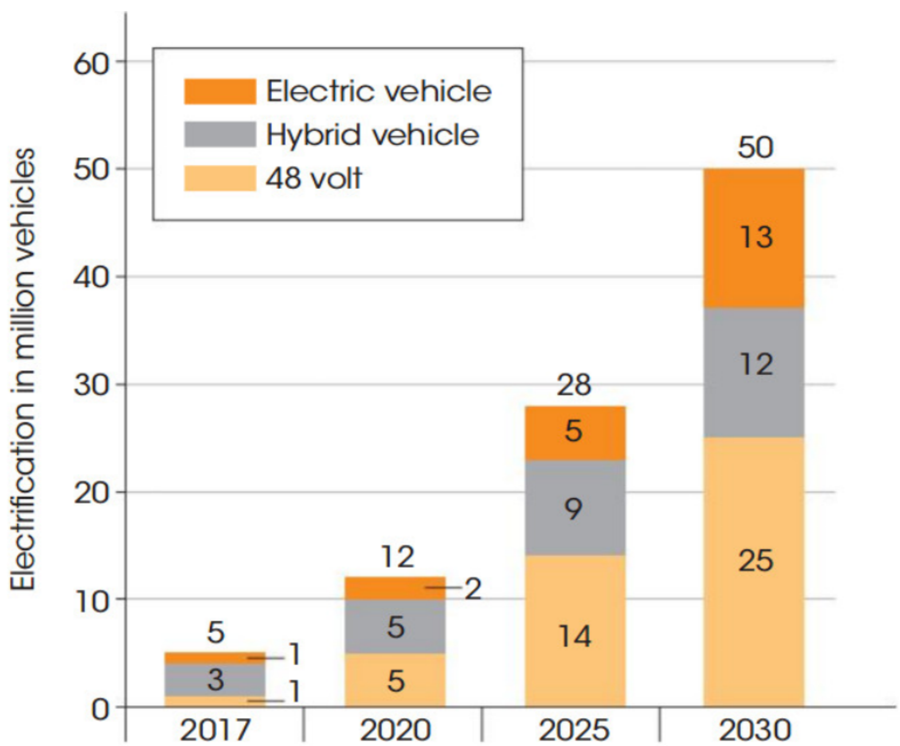


Figure 3.4. Electric Market Share
[14]

4. **P3** – The electric machine is connected through a gear mesh with the transmission; the electric machine is decoupled from the ICE and it's speed is a multiple of the wheel speed

5. **P4** – The electric machine is connected through a gear mesh on the rear axle of the vehicle; the electric machine is decoupled form the ICE and it's located in the rear axle drive or in the wheels hub

3.4.1 Drive line side electric machine MHEV architectures P(2)

Both P0 and P1 mild hybrid configurations have the electric machines on the engine side, without the possibility of mechanical disconnection. This makes torque boosting and energy recuperation not very efficient because of the torque losses. Moreover, recuperating electrical energy with the engine off, during coasting, is not possible. The P2, P3 and

	Micro Hybrid	MHEV			HEV	PHEV	EV
Topology	Regular starter	BiSG	TiMG	CiSG	Power split	Power split/Parallel	Direct Drive
Electric Power [kW]	2-4	10-15	<21	15-20	25-60	40-100	>60
Operating Voltage [V]	12	48	48	<160	150-350	<400	<450
Cold Engine Cranking	Yes	No	Yes	Yes	Yes	Yes	Yes
Idle Stop & Start	Yes	Yes	Yes	Yes	Yes	Yes	Yes
In-mission Stop & Start	Optional	Optional	Yes	Yes	Yes	Yes	Yes
Engine Load Shift	Optional	Yes	Yes	Yes	Yes	Yes	Yes
Torque Assist (fill)	No	Yes	Yes	Yes	Yes	Yes	Yes
Torque Boost	No	Yes	Yes	Yes	Yes	Yes	Yes
Sailing/Coasting	No	Optional	Yes	Yes	Yes	Yes	Yes
Energy Recuperation	Optional	Yes	Yes	Yes	Yes	Yes	Yes
Brake Regeneration	No	Optional	Yes	Yes	Yes	Yes	Yes
Electric Driving/Creep	No	No	Optional	No	Yes	Yes	Yes
External Charging	No	No	No	No	No	Yes	Yes

Figure 3.5. Different Features associated with different topology [14]

P4 mild hybrid architectures are better in terms of energy flow efficiency, mainly because of the positioning of the electric machine. In these types of configurations, the electric machine is positioned after the drive line connecting device (clutch), on the input shaft of the transmission (P2), on the output shaft of the transmission (P3) or on the rear differential (P4). In this work P2 architecture is chosen with the electric machine mounted to the input shaft of the transmission via a coupled geared assembly. This arrangement allows for all the potential hybrid benefits as well as engine to be disengaged if needed from the driveline. Figure 3.6 shows the schematic of the chosen P2 architecture with the component position in the powertrain.

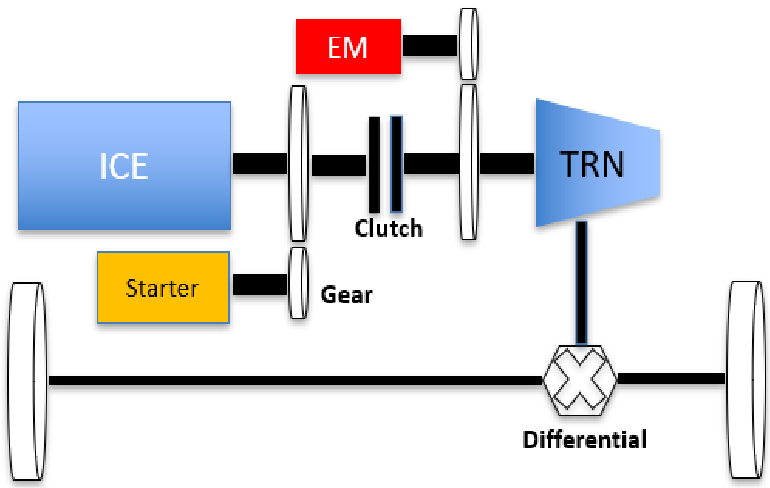


Figure 3.6. MHEV P2 Architecture – Side Mounted EM [14]

3.4.2 Motor Generator

A good choice to start with this research is to have a mild hybrid 48V system. This system provides torque boost when needed during heavy power demand as well as not make the overall vehicle very heavy. In case of a larger system the power to weight ratio will decrease and also make the load carrying capacity of the truck lower. Table 3.2 [** scalable peak torque between 50-80Nm*] [66] shows the general parameters for the selected motor

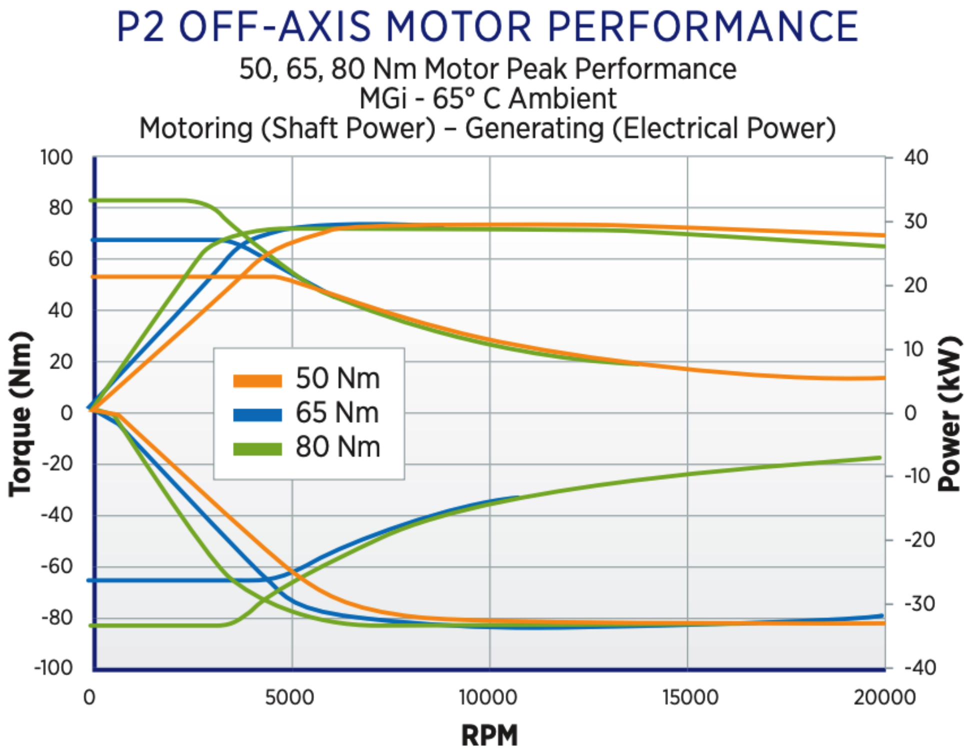


Figure 3.7. Motor Torque & Power Curves
[66]

Table 3.2. P2 OFF-AXIS 48V eMOTOR*

Feature	Performance
Continuous Power (30s)	15kW @15000 RPM
Peak Power (1s)(2x/5s)	20kW @12000 RPM
Peak Torque	50-80 Nm
Peak Torque @ Crank	150-240 Nm
Max Regeneration (30s)	$\geq$ 25 kW
Voltage	48V
Max Motor Speed Continuous	18400 RPM
Max Motor Speed	20000 RPM
Coolant	WEG, 65deg C, 6L/Min
Dimensions	ϕ 185mm,

generator. Two sets of motor specs are used from the same motor. One with a **50Nm** rating and the other with a **80Nm**.

3.4.3 Energy Storage System

There are several choices for a 48V energy storage system. In this work a simple configuration from A123 Systems is selected [67]. The battery chemistry used in this work is of Li-Ion type.

The battery is a very mild assist with a **8Ah** capacity and a nominal operating temperature of close to **25C**. At this settings it can provide continuous power of **15kW**. A more powerful custom made choice is also explored which has a rating of **60Ah** and can provide a continuous power of close to **30kW**.

A simple thermal model for the battery is designed to model the heat loss by the battery. An active cooling system is also in place to decrease the rate of heat loss by the battery. Since the battery is small and limited by power, proper heat management of the battery is necessary to utilize its full range of power capability. It is also worth mentioning that the battery is considered to always provide continuous power and the peak power switching logic is not used for this work.

Table 3.3. 48V Battery Specifications

Specification	Unit	Performance
Peak Configuration	-	14s1p
Capacity	Ah	60
Minimum Voltage	V	24
Maximum Voltage	V	54
Nominal Voltage	V	46
SOC Range	%	25 - 70
10s Discharge @ 25degC, BOL, 50% SOC	kW	28
60s Discharge @ 25degC, BOL, 50% SOC	kW	16
10s Charge @ 25degC, BOL, 50% SOC	kW	30
60s Charge @ 25degC, BOL, 50% SOC	kW	20
Usable Energy BOL @ 25degC	Wh	$\geq$180Wh
Mass	Kg	40

Table 3.3 [67] shows the different battery parameters and their chosen value as used in this work. A very simple cooling strategy is applied to maintain the battery temperature and limit the battery charge/discharge power ratings. In order to come up with the battery cooling parameters a 1D Li-Ion high fidelity electrochemical based model is used [68]. The model described by Pramanik et al., is developed for a single cell which is scaled in this work to mimic the required battery size used in this system model. This model is utilized to estimated the thermal loss of the battery pack and then calculate the coefficients needed for the cooling system.

3.5 Chassis

The vehicle assembly comprises of 2 major components in terms of torque transfer, namely the axle and the wheel. Finally the last subsystem calculates the vehicle speed from the adjusted torque in the end. A 18 wheel system is considered in this work. The gross vehicle weight is 65000lbs. A study of different condition of load is also used to get an understanding of how much weight we can put in in terms of hybrid system since with the addition of more hybrid weight we will loose hauling capacity.

3.5.1 Axle

Axle is designed as a simple torque pass through device after adjusting for losses. A relatively down sped design is used by selecting an axle ratio of close to 2.47. Equations 3.6 are the two main Axle dynamics.

$$\begin{aligned} \omega_{trn} &= \frac{\omega_{axl}}{RAR}, \\ \tau_{axl} &= \tau_{trn} * RAR, \end{aligned} \tag{3.6}$$

Axle is considered to 100% efficient in this study. It is worth to note that in the Axle we calculate the transmission speed to be fed back upstream to the transmission system. Axle torque is the forward torque transferred to the wheel for further processing in the chain.

3.5.2 Wheels

The wheel speed is given by equation 3.7

$$\omega_{whl} = \frac{\nu}{\mu_{whl}} \tag{3.7}$$

where, μ_{whl} is the wheel radius in meters The forward torque is given by equation 3.8

$$\tau_{whl} = F_{norm} + \tau_{axl} \tag{3.8}$$

3.5.3 Force Balance

Finally, the 1-D longitudinal Equation 3 can be formulated in a detailed way. The different forces at the wheel is summed up and then divided by the equivalent vehicle mass to get the acceleration. Finally the acceleration is integrated to get the velocity of the vehicle which is used to feed back to the upstream controllers for a full closed loop dynamics. The gravitational drag force as a function of the road grade is given by Equation 3.9.

$$F_{drag} = mgsin(\theta) \tag{3.9}$$

where, θ is the road grade in *radians* The aerodynamic drag as explained in the above section is a function of the vehicle speed and is reiterated by equation 3.10

$$F_{aero} = \frac{1}{2}\rho C_d A \nu^2 \tag{3.10}$$

The road normal force is also a function of road grade and is given by equation 3.11

$$F_{norm} = mgcos(\theta) \tag{3.11}$$

where, θ is the road grade in radians Finally, the forces acting on the vehicle is given by equation 3.12

$$m\alpha = F_{trac} - F_{drag} - F_{aero} - F_{norm} \tag{3.12}$$

where, α is the acceleration of the vehicle and is responsible of either speeding or slowing the vehicle. Equation 3.12 can be rearranged as Equation 3.13

$$\alpha = \frac{1}{m}[F_{trac} - F_{drag} - F_{aero} - F_{norm}] \tag{3.13}$$

Hence finally the vehicle speed is given by Equation 3.14

$$\nu = \int \frac{1}{m}[F_{trac} - F_{drag} - F_{aero} - F_{norm}]dt \tag{3.14}$$

As we have seen in earlier chapters the problem for the optimal control is solved in distance domain, we finally convert Equation 3.14 as

$$\nu = \sqrt{(}2\int \frac{1}{m\nu(s)}[F_{trac} - F_{drag} - F_{aero} - F_{norm}]ds) \tag{3.15}$$

It is worth noting here that equation 3.15 makes vehicle speed of the system as a state of the system dynamics. The assumptions made throughout this chapter while designing the system dynamics are,

- Rotational Compliance & Coupling Dynamics between components are not considered for the purpose of this research.
- Losses are considered constant instead of a function of any dependent variables.
- Map based logic is used in every calculation possible to eliminate the need of complex analytical design.

Since the research is based on energy level analysis the above considerations are justified. Finally, in a general compact state space form, the dynamics can be written as equation 3.16.

$$\boldsymbol{X} = \boldsymbol{Ax} + \boldsymbol{bu} \tag{3.16}$$

The continuous states are **Vehicle Speed**, **Vehicle Position**, **Engine Fuel Quantity**, **Battery SOC** & **Battery Temperature**. **Gear Number** & **Clutch State** are discrete integer type. Hence the problem is a mixed integer non-linear type problem. The control inputs are **Engine Throttle, Clutch Command, Brake Command, Gear Shift Request** & **Power Split Ratio**.

3.6 Corridor Information

Once the choice of vehicle component is identified and a longitudinal force balance dynamical model is designed, it is required to identify some corridor information which are exogenous inputs to the vehicle simulation. The two important inputs that is considered in this work are route grade and route speed limit. The information is used form public domain repositories such as *https://www.openstreetmap.org*. A section of the I64 route is made up which is around 82 mile long with a characteristic road grade of 1.3%.

3.7 System Performance

The baseline system dynamics are simulated in simulink for the entire duration of the route which is a generic US Route of 82 miles. The baseline simulation is done in time domain and the results are captured here. The objective here is to study the baseline system behavior and check for simulation and dynamics sanity. It is also intended to study the baseline energy behavior so that it can be compared against optimal levers. Figure 3.8 shows the simulink structure for a single vehicle which is designed as per the component specifications discussed in the above sections. Figure 3.9a shows the fuel economy numbers along with the BTE and cycle work change for a conventional vehicle of multiple loads. The first plot shows the result for 5 set of simulation. Each set has 5 bar plots (Red - Absolute Fuel Economy compared to baseline, Blue - Trip Time change compared to baseline, Green - Compensated Fuel Economy which is Absolute Fuel Economy-Trip Time Change, Yellow - Change in Cycle Work compared to baseline, Purple - Change in BTE compared to baseline). The 4 sets are baseline which is 65000lbs truck, the second one is 55000lbs truck and the rest are 70000lbs, 75000lbs and 80000lbs respectively. It is observed that the FE % change increases by over 6% with a 10000 lbs of load reduction. Similarly there is a progressive reduction in FE % change as we increase the vehicle load. This is expected since the vehicle

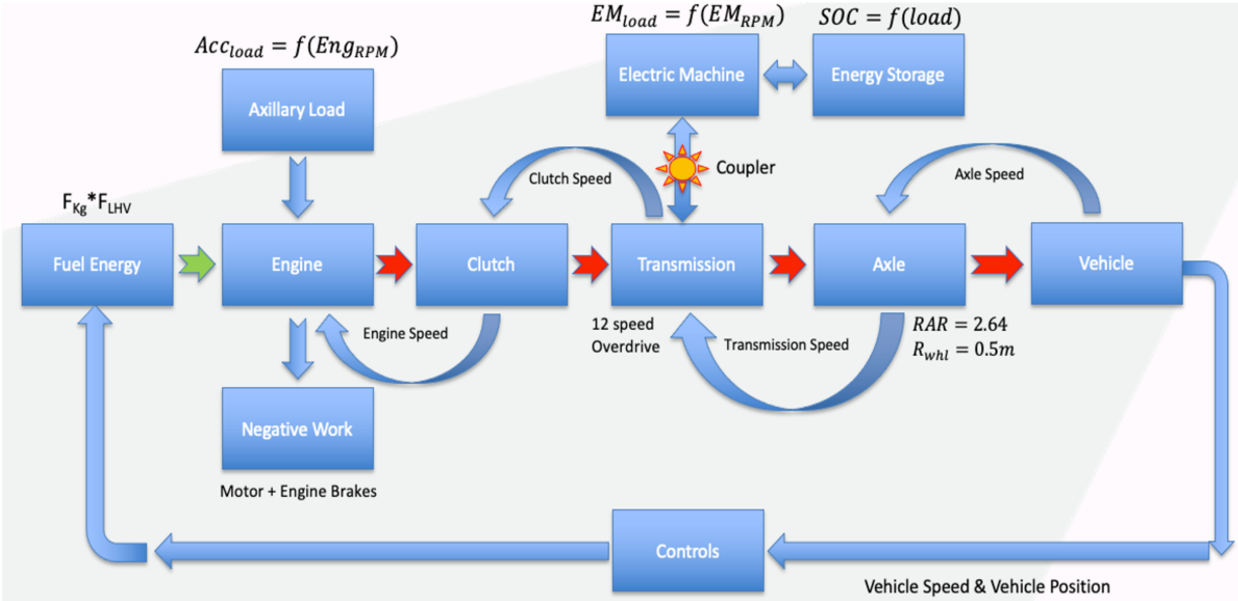
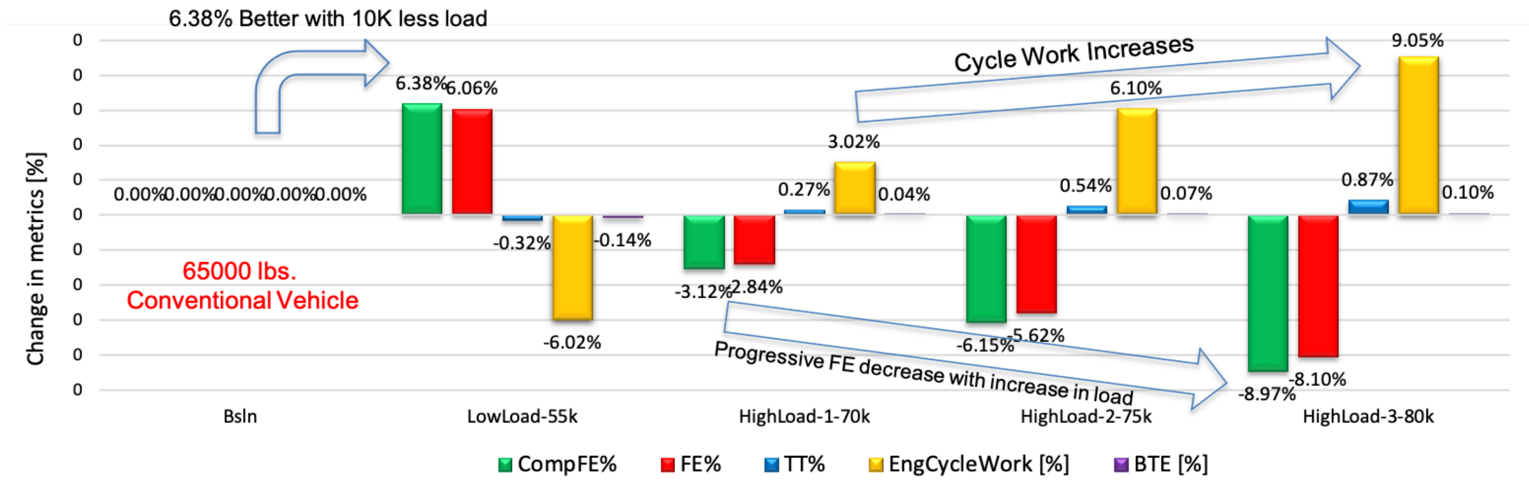
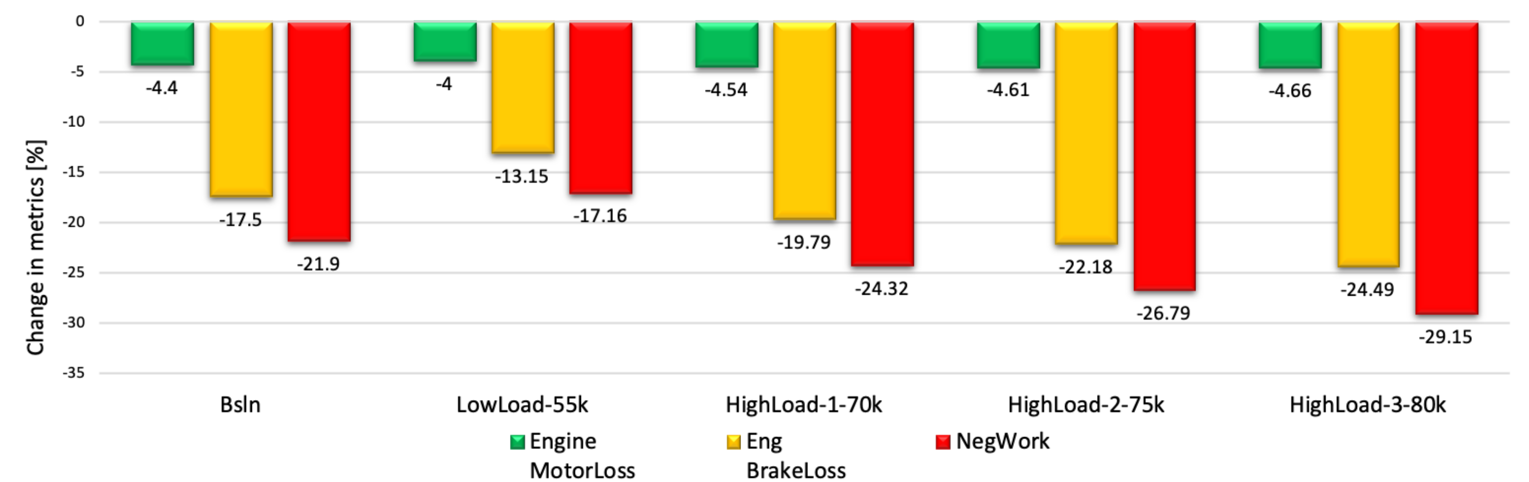


Figure 3.8. Simulink Structure of a Single Vehicle - Forward torque and backward speed feedback loop. The components used are Corridor data processing, Operator Processing [throttle, brake, gear, clutch], Power Split Sequence, Engine and Motor Generator processing, Transmission Dynamics and finally the Vehicle longitudinal model.

works more on torque curve with the increase in load. Another observation is that with the increase in load there is more cycle work being done by the vehicle which also increases proportionately with the load (the yellow bars show positive numbers with load increase which means compared to baseline its more). There is not a very significant change in BTE which signifies that the vehicle is well tuned for transmission shift points and the fuel maps are very well tuned as well. Figure 3.9b shows negative work which is the sum of the motoring work and the braking work as a percent change against baseline for a conventional vehicle of multiple loads. Similar to Figure 3.9a this plot also has 5 sets of data. The first set being the baseline simulation for the conventional truck with out hybrid system turned on and a gross vehicle weight of 65000lbs. The red bars in each set is the total negative work. The yellow bars in each set is the total engine braking work and the green bars in each set is the total motoring work. It is worth observing here that the motoring work almost remain same with load but there is a huge increase of braking work with load increase. This is anticipated since the heavier vehicle will have more kinetic energy during downhill and will have more



(a) Fuel Economy % change for the single vehicle conventional configuration with multiple payload hauling condition. Compensated Fuel Economy % is Absolute % change in Fuel Economy - Trip Time % change.



(b) Fuel Economy % change for the single vehicle conventional configuration with multiple payload hauling condition. Compensated Fuel Economy % is Absolute % change in Fuel Economy - Trip Time % change.

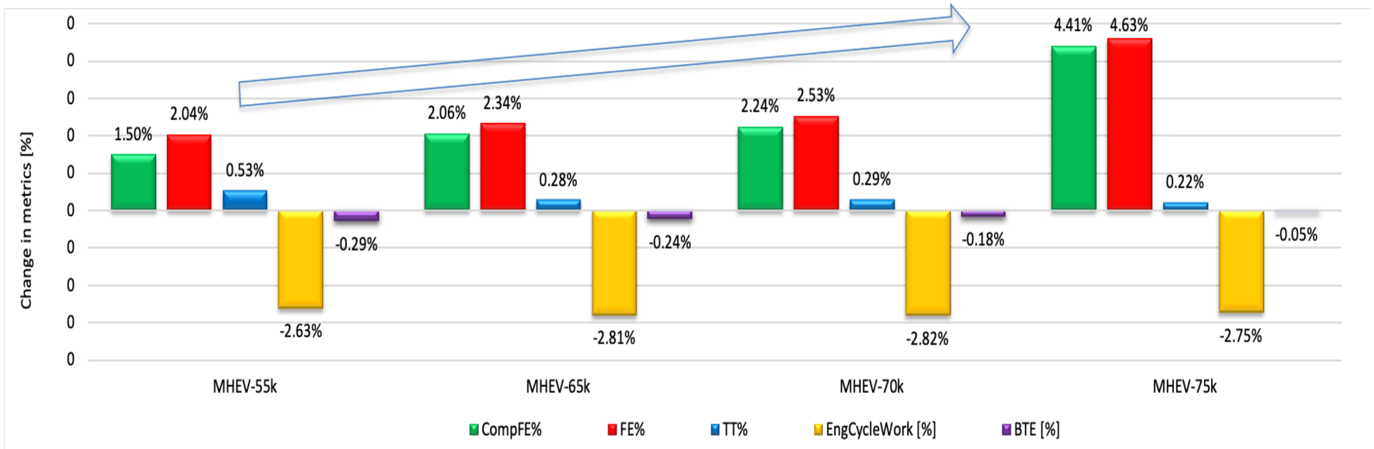
Figure 3.9. Key metrics for the conventional vehicle simulation with multiple load points.

characteristic acceleration there by going past the braking zones more than a less loaded vehicle. This fact is validated with the negative work for the less loaded vehicle where we see less braking work. These results provide a good confidence on the baseline vehicle system. Next we validate the Mild Hybrid benefits over the conventional vehicle. Figure 3.10a shows the Fuel Economy numbers with the addition of the hybrid system where the power split strategy is simple SOC target tracking within a range of 25% to 75% as specified by the battery manufacturer. The sequence of plots and nomenclature is exactly similar to what is presented in the convectional case, Figure 3.9a & Figure 3.9b. Each set here is the result of adding the hybrid system on top of the conventional counterpart. So the first set is the result of adding the hybrid system on top of the 55000lbs conventional vehicle. Similarly the second set is the result for the addition of the hybrid system on top of the conventional 65000lbs vehicle and so on. We observe substantial improvement in fueling with heavy load which is because of more regenerative braking capability associated with a heavy vehicle. We see similar cycle work all through which also shows that the electric system is compensating for the extra power demand by the additional load. Figure 3.10b shows the negative work due to motoring losses and braking for different loads. Motoring remains almost similar while braking work increases with load. Table 3.4 shows the tabulated key metrics for the

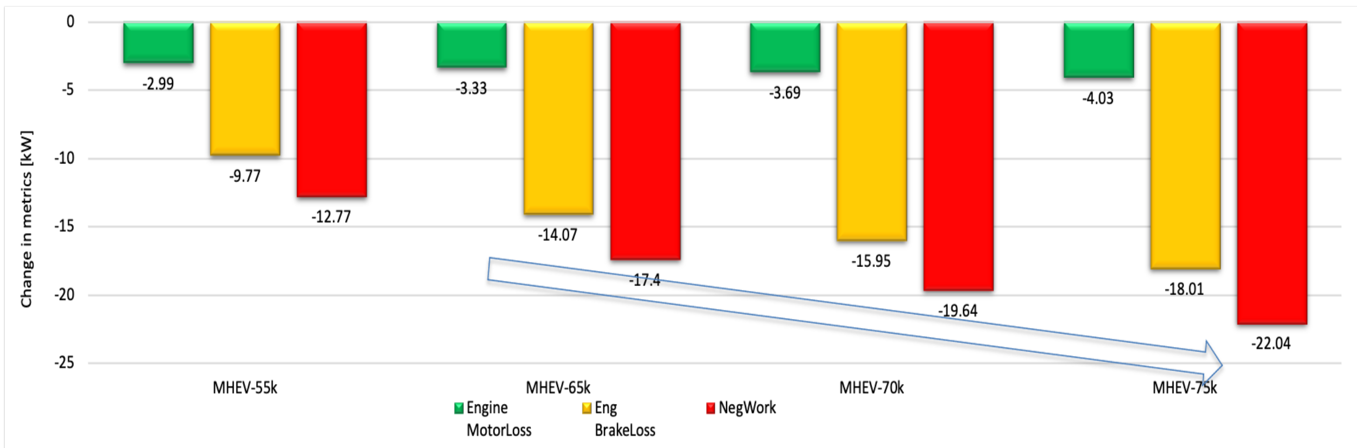
Table 3.4. Baseline simulation results with rule based control for SOC and other control levers. SOC follows a charge sustaining PI logic

Metrics	Units	Value
Distance Travelled	miles	81.92
Fuel Consumed	Kg	27.3
Fuel Economy	mpg	9.6
Trip Time	s	4600
Total Shifts	-	39
Aerodynamic Work	kWh	90.2
Cycle Work	kWh	146
BTE	%	44.8
Engine Out NOx	Kg	0.4384
Negative Work	kWh	-29.6

simulation with baseline controls. These numbers are used in the optimal controls algorithm as well as used to calculate all benefits in the subsequent sections. The baseline for this



(a) Fuel Economy % change for the single vehicle hybrid configuration with multiple payload hauling condition. Compensated Fuel Economy % is Absolute % change in Fuel Economy - Trip Time % change.



(b) Analysis of negative work for different payload condition on a hybrid vehicle configuration . The two components analysed are the motoring work and the engine braking which are added together as negative work which is lost

Figure 3.10. Key metrics for the mild hybrid vehicle simulation with multiple load points.

research work is considered as a 65000lbs truck with a mild 48V system attached to the transmission input shaft via a coupling gear. The hybrid system is managed through a simple rule based SOC target tracking proportional integral controller. The dynamics of the vehicle is modelled as described in the previous sections with simple vehicle speed feedback and torque propulsion including both engine and hybrid system. The SOS is a rule based simple target tracking control with charge sustaining algorithm, between bounds. Most literature often use such strategy for simplicity in calculation. The optimal control algorithm later described will study the effect of predictively controlling the SOC algorithm.

3.8 Conclusion

Having understood the baseline results we move on to the next part which is the problem formulation. Henceforth we will refer to the Mild Hybrid configuration running 65000 lbs load as our baseline system and all benefits will be accessed on top this system. Overall the baseline system is defined as below,

- Mild Hybrid operating a P2 architecture with the traction motor mounted to the output shaft of the transmission via a gear assembly
- The vehicle is configured to haul a load of 65000 pounds
- The vehicle is configured to run a route of nearly 82 miles with a characteristic grade of 1.3% and a road speed limit of 65mph

The optimal problem will be solved by including one lever at a time on top this newly defined baseline. All metrics associated with the control formulation will be compared against this baseline and the results produced here.

4. PROBLEM FORMULATION & APPROACH

The multi-objective minimization type problem for a system of line haul trucks in a platoon is solved in this problem. There are two objectives for this work, the first one is to understand the true global optimal behavior with multiple control levers working in conjunction with the mild hybrid system and the second one is to formulate a problem to find global optimal behavior in a platoon of trucks using multi-agent methods. The motive of the work is to understand the rules behind the control levers and how they shall operate as a function of road grade and speed limits. A two folded approach is tried here where a complex problem involving 5 states and 4 interacting controls is solved offline. In this step the behavior is analyzed for each control lever and its interaction with other controls. A set of rules that can be deployed based on predictive knowledge is recommended. The other important aspect is the need for look ahead information. It is also analyzed at this step as to what level of look ahead information is needed. The question we tried to answer here is whether we need the full route information for the true global result or a smaller section of the road ahead is good enough for the expected benefits. Once this is done the next step is to figure out the best possible way to implement the controls in the platooning trucks so that they provide true global results. It is very challenging to implement PMP or MPC based controls in the control modules that most of the trucks use today. Once we move to more GPU based processing units for the trucking industry these math heavy algorithms could be the true solution. Until then it is often needed to rely on simple rules derived from the global offline solution. A similar approach is taken here where the second step takes the offline control trajectory and tries to follow it using another simple distributed averaging based algorithm for a shorter horizon. This action is taken for each individual trucks in the 3 truck platoon in our case.

Typically for a system of this scale identification of the true objective is challenging. There are multiple factors that affect the true optimized operation of a line haul truck. Definitely the total fuel consumed is the primary objective to minimize. Though the improvement in miles per gallon number is the key element but its interaction with other components n the vehicle drives the true problem. We cannot compromise on trip time in order to save fuel.

Similarly engine out NOx numbers can also be part of the cost function in order to achieve a better emissions foot print. Carbon neutrality is another objective. Further with more advanced features like coasting while in mission can significantly lower after treatment temperatures which adversely affect conversion rates in the catalysts. Further the component sizing is another key criteria in deciding the optimal behavior of the controls. As an example engine fuel maps and rear axle ratio plays a key part in deciding optimality for predictive control of the gears.

While there are numerous ways the problem can be attacked and solved to understand some key behavioral differences in the controls, we have targeted the objective to achieve better fuel efficient, safe operation in a platoon and also do not trade off a lot on trip time and drivability. Since better fuel efficient operation also indicates a better engine operation point in the BTE contours, we also anticipate to improve on the emissions. This improvement in engine out NOx is the passive result of the better engine operation. In this research the after-treatment temperatures are not considered in the optimal problem formulation nor is the emission components part of the objective function or the constraints. This is specifically done in order to keep the problem only tied to the engine and powertrain domain. The effect of emissions is studied as a passive improvement based on the improvement in the complete powertrain. The recommendation and future work extension of this research would be to include the emissions component (both temperature and absolute NOx values) in the problem formulation to actively include those as part of the optimal behavior. There could be potential option for including externally heated after treatment models as well which may provide a different optimality. These all are viable problems for optimizing the complete powertrain.

Figure 4.1 shows the overall schematic of how the problem is laid out. It shows the two independent phases which are used to solve the problem and analyze the global optimal behavior. As shown in the figure the exogenous corridor information used in this work as predictive look ahead knowledge is the road grade and the speed limit. The offline part can be solved individually for any vehicle configuration, any load condition, any route condition with traffic or without traffic in consideration. Since this type of problem is very computational heavy it is often needed to run in offline mode either in cloud or using a high efficient

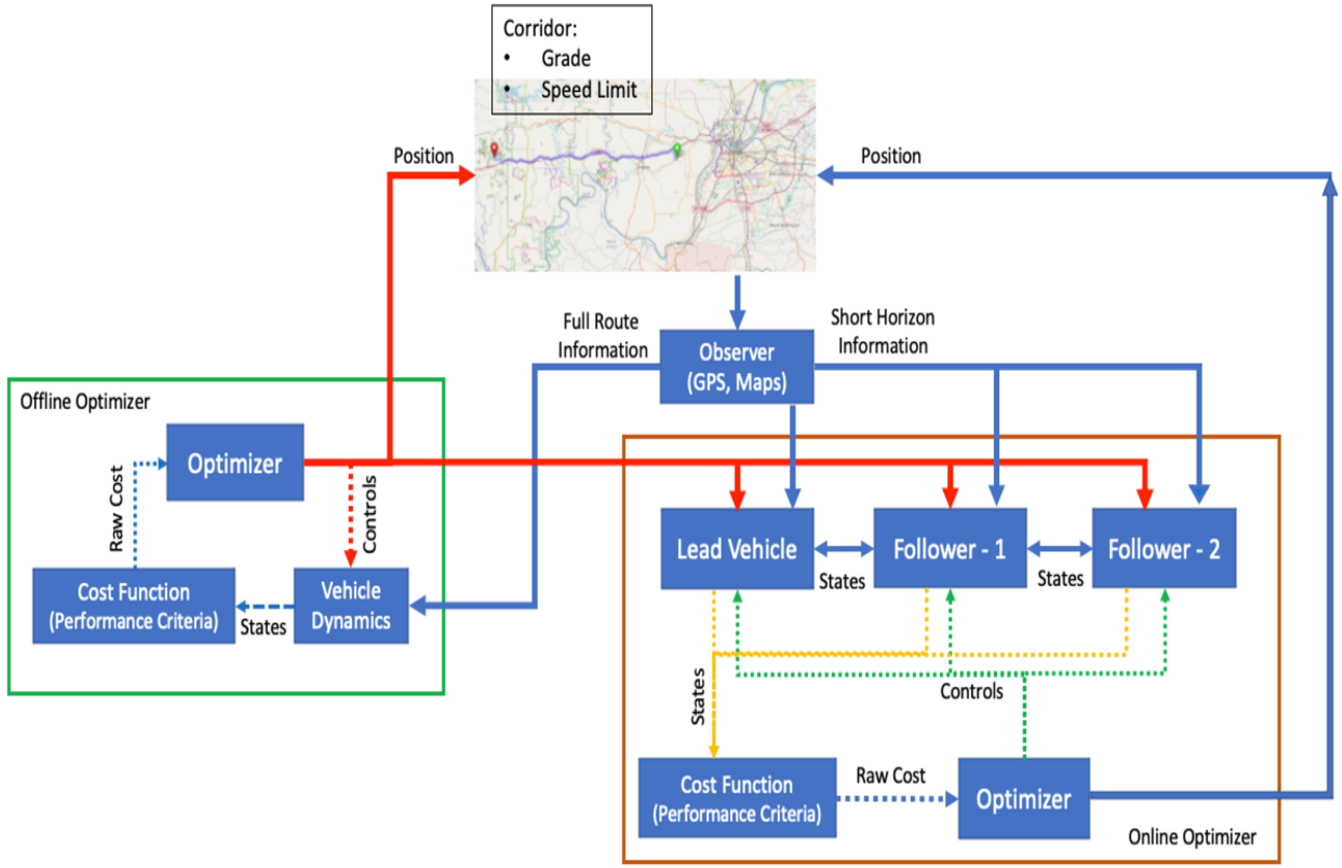


Figure 4.1. High level design architecture showing the overall problem formulation. The first major component is the offline computationally challenging dynamic programming based multi-objective optimization for a single class 8 truck. The second real time component is the multi-agent based online optimization for a 3 class 8 truck platoon. It uses a distributed averaging based gossip protocol to optimize the cost objectives for a short look ahead window.

computer. Although they are only used to understand the global true optimal behavior and not used in production environment. In this work a 4 controls, 5 states(including vehicle position) problem is solved which falls under a very challenging multi-control optimization class. The complexity of the true optimal behavior depend on a number of use cases - interacting control action, state trajectory interaction, range of absolute look needed, vehicle configuration type and so on. This problem help us realize and understand a lot of functional dependence of optimality to these control levers along with component sizing.

The vehicle system is designed and analyzed in the previous chapter and we have see the dependence of load variation with fuel economy and other key metrics. The vehicle configuration is fixed for all the 3 platooning trucks in this research and the load is set to 65000lbs for each truck. The route is also fixed to be a made up section of I64 with a hilly portion in between two relatively flat section. Further a demo route for microscopic analysis is also used which is a 3% uphill followed by a 3% downhill type of trapezoidal route. This route is primarily used to code up the problem and tune initial weights before running the final tuning on the actual route. Details of the simulation results are discussed in the next chapters. Once we have the vehicle configuration and corridor information fixed we define the objective function as Equation 4.1, It is a weighted sum of total fuel used in the route and the trip time. Since the scale is widely different for time and fuel we have normalized it with weights.

$$\min_{\forall u^* \in \mathcal{U}} [\frac{\alpha}{\mathcal{W}_{FC}}(\frac{\dot{m}_f(u,s)}{\mathcal{V}_s(u,s)}) + \frac{1-\alpha}{\mathcal{W}_{TT}}(\frac{1}{\mathcal{V}_s(u,s)})].ds \tag{4.1}$$

The problem is solved in distance as time in this problem in independent and can change depending on vehicle speed change. Hence the cost function is integrated over distance [ds]. There are only two parameters used in the cost objective. The Engine Fuel Rate and the Trip Time. As discussed in the previous section, the dynamics in time domain is converted to distance domain by dividing the state parameters in the dynamics as well as cost function by Vehicle Speed ($v(s)$). α is the weights on the fuel and time. A value of 1 for the α means the optimal behavior will be targeted towards complete fuel savings with out maintaining the trip time balance. The vehicle will slow down excessively in this case. Similarly a value of 0 for α will mean the optimality will focus on trip time saving by trying to move fast and thereby spending more fuel. We used an automated bisection search to tune alpha for each DOE since it is a time consuming effort. W_{FC} and W_{TT} are the normalizing weights on the fuel and trip time parameters since they are in different scales. These weights are estimated based on the respective fuel and trip time numbers from their corresponding baseline simulation results. The other details of the non-linear constraints and the solution method are discussed in the subsequent sections.

Figure 4.2a shows the section of the I64 from where the road data is used. The characteristics gradient for this section is 1.3% and the entire route length is $82 miles$ with a constant cruise speed limit of $65mph$. The baseline vehicle dynamics is set to follow this constant cruise speed target with out any torque based droops. Simulation result of this baseline simulation is studied in the previous chapter.

Figure 4.2b is the dynamic grade and vehicle speed profile for the baseline simulation. It shows the how the road is graded and what is the dynamic range of vehicle speed with the any look ahead knowledge. It is observed that the engine brakes are effective to restrict vehicle speed around $71mph$ as set and the vehicle is limited to $50mph$ at heavy grades. This is the system torque map limitation of the vehicle.

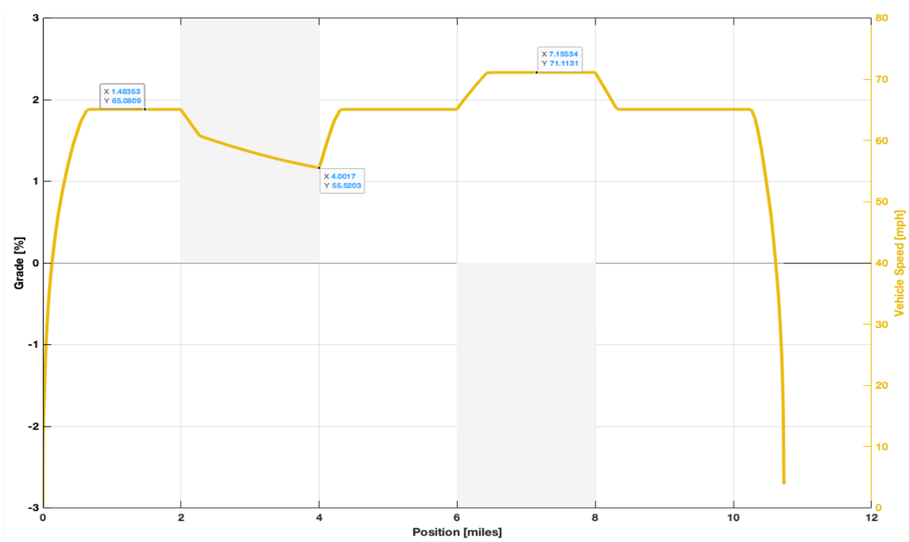
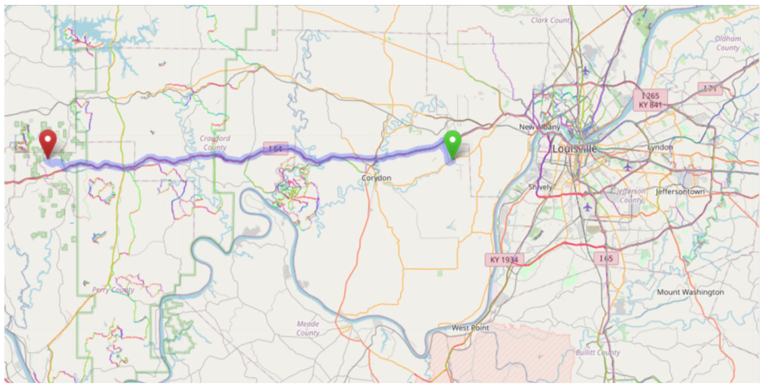
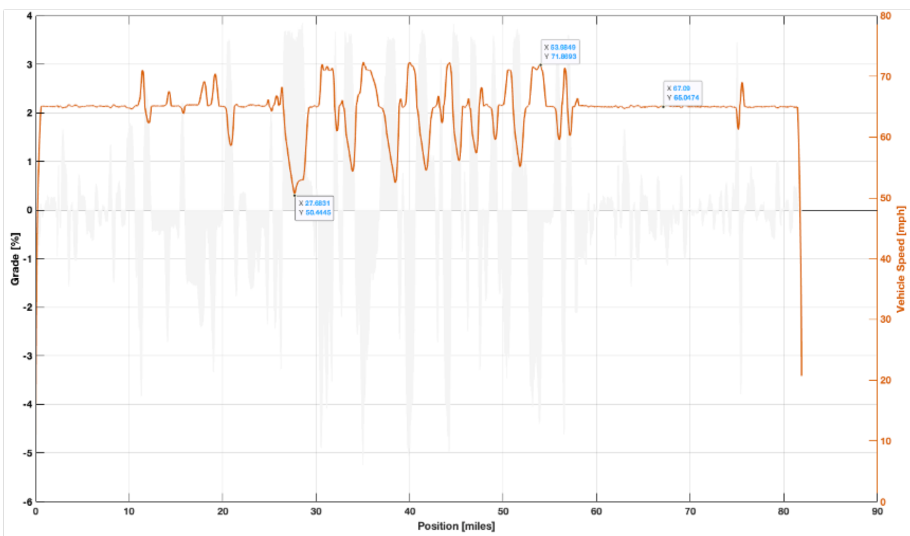


Figure 4.3. Illustration of a Demo Route for metrics analysis in terms computational correctness. This route is not meant for calculating the fuel economy numbers. This is used to computationally analyze the correctness of the algorithm implementation and also to understand the local behaviors with respect to hills and downhills.



(a) Real world drive cycle taken from a section of I64. Grade points are generated from openstreetmap.org website.



(b) Route grade and baseline vehicle speed profiles. The vehicle speed is targeting a rudimentary cruise controller set speed.

Figure 4.2. Prime Route Characteristics including real world segment and vehicle speed profile as a function of grade.

Figure 4.3 is the 10$mile$ route used as a demo short route to code, tune and analyze the initial results. It shows the typical behavior of the truck in an uphill and downhill with the speed drop, speed up and engine braking.

4.1 Dynamic Program Algorithm

Dynamic program algorithm based on the famous Hamilton-Jacobi-Bellman principle involves a complex and computationally challenging process which is described in the below steps, these steps are sequentially executed to get the optimal offline results using a single truck,

- Baseline Run :- Simulate the baseline plant to store results for comparison and gather data for dynamic program initialization

- Feasible Grid Search :- This step is the most time consuming and computation heavy process. It loops through all the combinations of feasible grid points for states & controls, then store the next iterated value for each state & cost metric parameters. This cost value is the cost to go from 1 step to another and the cost-to-go for the optimal solution

- Optimal Control Selection :- This step is heart of dynamic programming where the minimum stage cost is calculated and the optimal cost-to-go is selected from the minimum stage cost. The corresponding minimum control value for each optimal stage cost is also selected.

- Simulate Optimal Controls :- The final sequence is to apply the optimal controls generated in the previous step to see the final outcome of the dynamical plant. The optimal controls is selected based on interpolated n-D look up tables since it is a function of the number of states and the independent time vector. The figure below shows the interpolation method used to select the optimal control and the cost-to-go values,

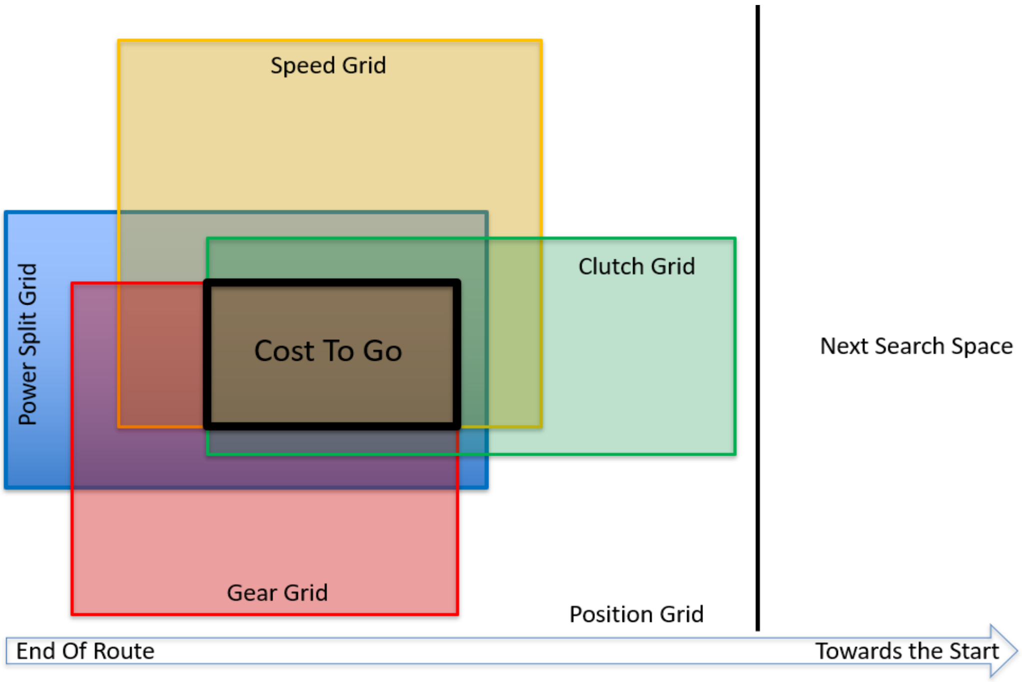


Figure 4.4. Illustration of Cost To Go Calculation. During the full factorial DOE space search starting from the end of the route to the start, the objective cost is calculated for each selected DOE space point. The cost-to-go selector then selects the minimum cost from each local cost points

Figure 4.4, shows the ***Cost-To-Go*** calculation and the selection of control variables. It depicts the admissible space among all the control levers. This admissible space is explored during the grid search stage to find the objective cost for each combination of controls and state parameters within the admissible space. This calculation is further done for each distance point for the entire route.

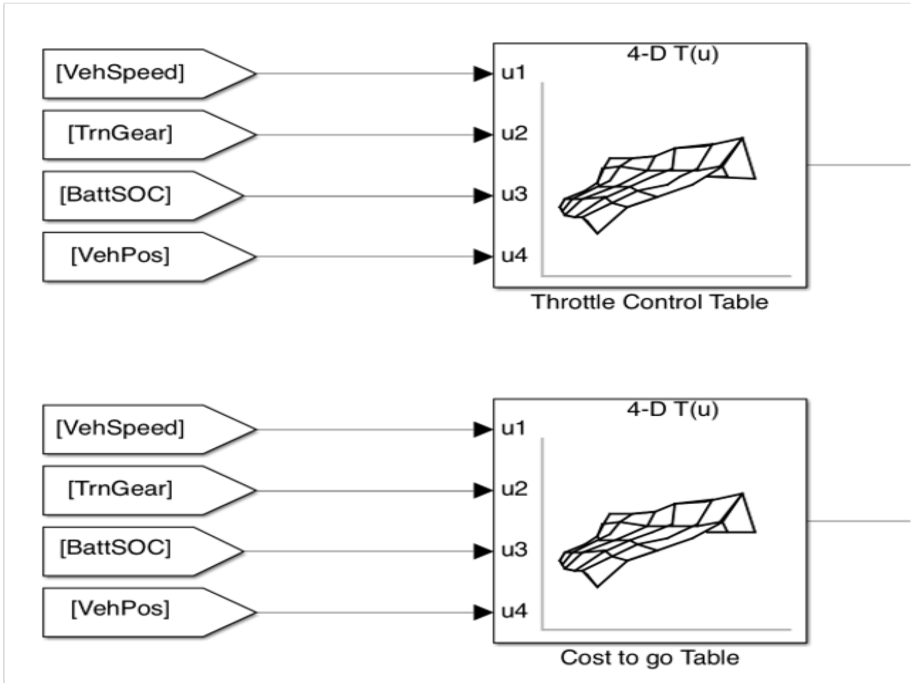


Figure 4.5. Illustration of how the optimal control is interpolated based on the state parameters and the independent vehicle position. In a similar fashion the other control levers are also chosen for the Coast, Gear and Power Split Problems

Figure 4.5 shows the simulink generic structure for the optimal control selection based on the state parameters. The n-D tables here are fed with the optimal control values as a function of each state parameter. Interpolation of the n-dimensional search space is done to select the optimal control values and the cost-to-go. It is required to do the interpolation since while going forward in simulation it is not necessary that the states coincide exactly with the grid points.

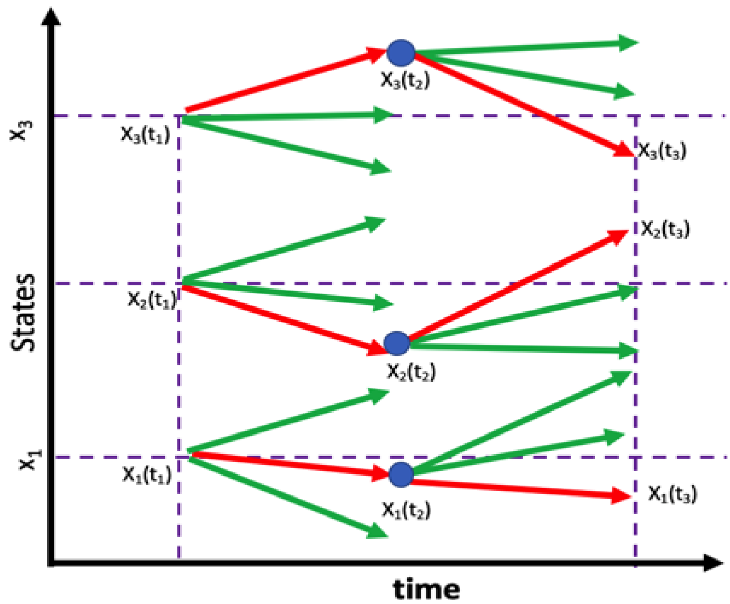


Figure 4.6. Interpolation for control action & cost-to-go

Figure 4.6 shows the propagation of state variables in every time step. The "GREEN" paths at each time instant shows the possible path that the state can take but the "RED" paths are the one which are the interpolated optimal path.

Table 4.1. Dynamic Programming Grid Setup

Parameters	States	Controls	Units	Grid Points
Vehicle Speed	Yes	No	m/s	51
Transmission Gear Number	Yes	No	$none$	4
Battery SOC.	Yes	No	$frac$	13
Clutch State	Yes	No	$none$	2
Engine Throttle	No	Yes	$frac$	11
Clutch Command	No	Yes	$none$	2
Shift Request	No	Yes	$none$	3
Power Split	No	Yes	W	13

Table 4.1 shows the discretized grid setup for the states and control variables. The grids are chosen such that the dynamics are still captured between the step size and the grid is not too large to challenge the computational cost. As can be seen from the grid setup that a total of **51*4*13*2*11*2*3*13** = **4550832** points for the states and controls. This

when multiplied with the grid size for the distance gets to a huge number which has its own computational challenge to solve. While solving for this problem the minimum cost calculated at each step is also penalized based on the violation of each associated state parameters. As an example there are soft and hard constraints on absolute vehicle speed, engine speed, gear number. Additional constraints are also on the interaction between gear shift and coast requirements. Further details on dynamic program structure and algorithm is provided in appendix section.

4.2 Online Platoon Controller

The next stage is when the platooning trucks use the optimal control profile. The problem can be solved in multiple ways. One simple way is to let the lead truck follow the optimal trajectories and the follower trucks passively follow the lead trucks ensuring that the critical distance is maintained. This is typically done using a proportional-integral feed forward controller that tracks the separation distance and adjust brake and throttle power as needed. The other gear and the coast control levers can be applied as it is ensuring dynamical requirements. This is typical to the reactive based control strategy where the lead vehicle drives the entire the platoon mostly. The follower vehicles plays in the throttle and brake space to maintain follow separation distance. while this strategy can be easy to implement it does not guarantee (without validation) whether the results are truly optimal. Later in the section we analyze whether implementing such a strategy is the best tradeoff among all the requirements.

Moving one step further the platoon problem can be solved using traditional optimal control methods such as model predictive control, Mixed integer non-linear program methods, pseudo spectral collocation methods and even Pontryagin's minimum principle. While some of them are used widely in industry for various application and also provides true optimality but it is often challenging if not impossible to implement such algorithms in real time controllers. This led to the requirement of analyzing the global optimal behavior using different methods in this research and understand the over all behavior in terms of optimal results, challenges in implementation, ability to scale up the problem and involving vehicle dynamics.

In this work a simple multi-agent based method is used where each node (trucks) in this case need to be aware of its neighbor's state. The trucks share information about their state variable and are fed with the same global optimal control signals as obtained from the offline problem. The lead and the last truck in the platoon have 1 neighbor each while the middle truck has 2 neighbors. The trucks use a shorter horizon to iterate on the state update values and minimize the cost while meeting the constraints. The state update in this case is for the vehicle speed only and is given by Equation 4.2 as studied by Boyd et al, [69][70], where each truck needs to know the separation distance between the trucks from it and then applies the formula to get is updated value.

$$x_{\mathrm{i}}(t+1) = (1 - \sum_{\mathrm{j}\in\mathcal{N}_{\mathrm{i}}} \frac{1}{(1+\max(d_{\mathrm{i}}, d_{\mathrm{j}}))})x_{\mathrm{i}}(t) + \sum_{\mathrm{j}\in\mathcal{N}_{\mathrm{i}}} \frac{1}{(1+\max(d_{\mathrm{i}}, d_{\mathrm{j}}))} x_{\mathrm{j}}(t) \tag{4.2}$$

where, N_{i} is the nodes in the network, $d's$ are the separation distance and $x's$ are the respective states. Once the state updates are available the cost function is calculated to find the minimum objective value and the penalties are applied based on the constraints. Equation 4.3 shows the objective cost function that is used in the approach. This method is iterated for the short horizon as the vehicle moves forward. More details on distributed averaging based algorithm along with short description of Model Based Control and Pontryagin's Minimum Principle are discussed in the Appendix section.

$$u^*_{1:N}(s) = \underset{u_{1:N}\in R}{\operatorname{argmin}} \sum_{n=1}^{N} \int_0^s \left\{ \frac{W_f \alpha \dot{m}_{f_n}}{v_{s_n}} + \frac{W_t(1-\alpha)v_{t_n}}{v_{s_n}} + \tau_{brake_n} \right\} ds \tag{4.3}$$

The objective of this research is to setup and understand the following key behaviors along with the major fuel efficient objective,

- Effect of mild hybridization of a line haul truck operating at high way speed
- Effect of interacting control actions based on look ahead knowledge of road grade and speed limit
- Optimality analysis for multi-objective problem as a function of predictive information
- Extension of predictive optimal solution to a 3 truck platoon system

- Analysis of predictive requirement for follower vehicles in platoon

5. SINGLE VEHICLE OFFLINE OPTIMIZATION

In this chapter the detailed problem is implemented and analyzed for each control lever at a time along with all levers working together. The addition of each control lever is done as a stacked up approach by adding 1 control lever at a time to the previous problem and analyzing the behavior.

5.1 Offline Mode - DP Based Speed Management Solver for Single Truck

Though the final problem is solved for 4 major control levers with the objective of minimizing the overall fuel usage, the method used here is by solving the problem by introducing one lever at a time.

The first lever studied here is the **Cruise Set Speed**. The objective is to understand if the cruise target speed can be predictively changed around the isochronous constant cruise speed of 65mph as function of look ahead road grade information. The general form for the cost function is modelled by Equation 5.1

$$\min_u J = \int_{t_0}^{t_f} \dot{m}_f(x, u, w) * dt \tag{5.1}$$

where, $\dot{m}_f$ is the *"fuel mass flow rate"*, (x, u, w) are states, controls and exogenous inputs. The states here in terms of solving the optimal control problem are Vehicle Speed, Gear Number & Battery SOC. The search space is discretized between a minimum and maximum set of points for all these states. Engine Speed is also a state but it is a dependant state of the vehicle speed and hence it is not needed by the solver for the control problem.

$$\omega_{eng}(t) = \frac{v(t) * w_{rad}}{\nu * RAR} \tag{5.2}$$

where, ω_{eng} is engine speed in rad/s, ν is "gear ratio", RAR is Rear Axle Ratio and w_{rad} is wheel radius. As discussed in the previous sections, since this problem is not a fixed time problem the dynamics is converted into distance domain from time domain. Total time is also included in the objective cost to make sure that total time remains within baseline limits.

So, if the single truck with out these predictive controls takes **"X" seconds** to cover the route, the optimal control should also be close to that **"X" seconds**. Hence, the final cost function is designed as discussed in previous chapter. Equation 5.3, shows the cost function with the non-linear constraints on the states and controls.

$$\min_{\forall u^* \in \mathcal{U}} [\frac{\alpha}{\mathcal{W}_{FC}}(\frac{\dot{m}_f(u)}{\mathcal{V}_s(u)}) + \frac{1-\alpha}{\mathcal{W}_{TT}}(\frac{1}{\mathcal{V}_s(u)})]\Delta s \tag{5.3}$$

$$\text{subject to,}$$

$$\dot{x}(s) = f(x(s), u(s), w(s)),$$

$$y(s) = g(x(s), u(s), w(s)),$$

$$\text{and, non-linear constraints,}$$

$$v_{min} \leq v(s) \leq v_{max},$$

$$g_{min} \leq g(s) \leq g_{max},$$

$$soc_{min} \leq SOC(s) \leq soc_{max},$$

$$\omega_{eng,min} \leq \omega_{eng}(s) \leq \omega_{eng,max},$$

$$\tau_{eng,min}(\omega_{eng}) \leq \tau_{eng}(s) \leq \tau_{eng,max}(\omega_{eng}),$$

There are three states here $x(.)$ =[Vehicle Speed, Gear Number, Battery SOC], 1 control $u(s) = Throttle$ and the primary output is $y(s) = OptimalVehicleSpeedTrajectory$.

It must be noted here that the constraints are both soft and hard. There is a hard vehicle speed limit based on absolute speed maximum and minimum thresholds. We also have a soft root mean square type error constraint based on the difference between baseline speed profile and the optimal speed profile. To make fuel benefit numbers comparable we let the base line speed increase to the upper bound for the speed before the brakes are applied. This is done in order to make a fair comparison between the optimal problem formulation and the baseline simulation environment. Figure 5.1, illustrates the different speed constraints used in the problem. The blue highlighted figure is that of the baseline simulation environment with out the optimal problem. The cruise target is set to constant **65mph**. Though it may be noted that the system is limited to torque curve to maintain this set speed at heavy grades and hence it will slowdown due to gravity on uphills. Similarly during down hills the vehicle will accelerate and speed up. In this case we have not used the brake (both engine

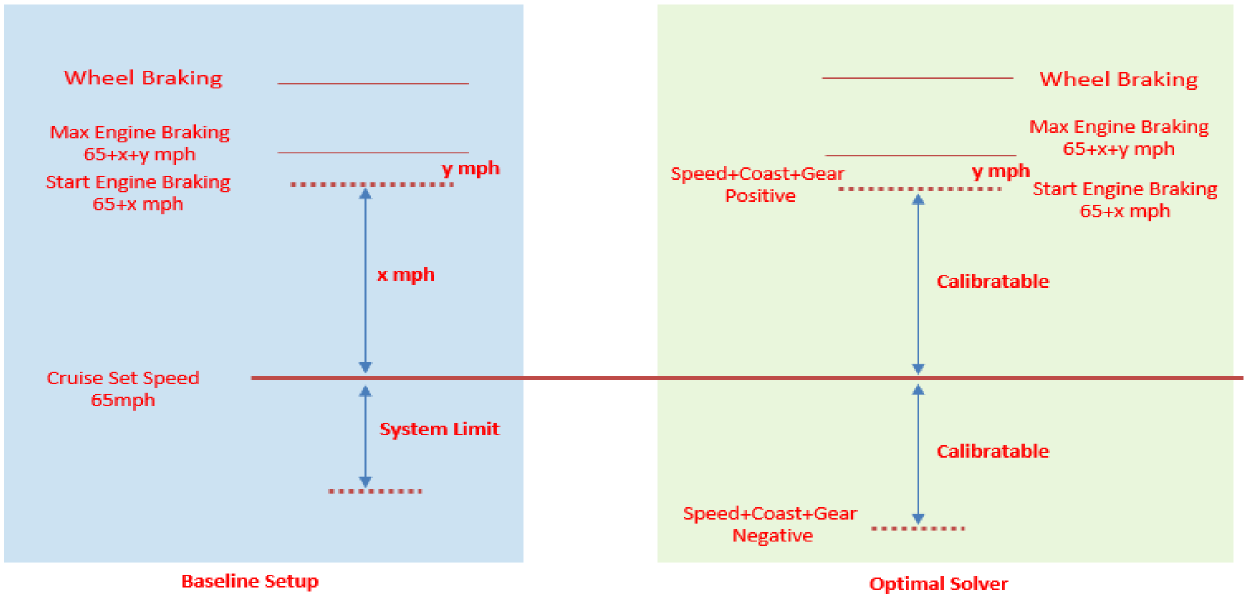


Figure 5.1. Speed Constraints for the Optimal Problem. There are

brakes and wheel brakes too prevent the vehicle from going beyond **65mph**). We let the vehicle coast up to a calibrated speed droop beyond the set speed before the engine brakes are applied and similarly another calibrated threshold before the wheel brakes are applied. This is particularly done so that it creates a fair comparison with the optima results which is formulated to dynamically move speed around the set **cruise** speed.

The sequence of code for this section is explained in Algorithm 1 The output of this solver is the optimal throttle value. This throttle control is used as input to the closed loop system to generate the optimal speed profile. The vehicle will no longer target a constant 65mph cruise set speed in this case as the optimal throttle will let the vehicle dynamically increase speed and slow down in the route based on look ahead grade information. The Vehicle Speed profile captured in this offline mode is fed as the cruise set target for the individual trucks in platoon. The multi-agent controller follows the target with other control levers in action and also include the braking system as an additional lever for maintaining safe follow through distance.

Result: Optimal Throttle, Optimal Vehicle Speed Feedback
Step 1 - Run Open Loop Forward Simulation: Initialize system parameters;
while *Vehicle Position is NOT end of Route* **do**
  Step through the route in distance;
  Accumulate fuel mass - $\dot{m}_f = f(v_s(\text{i}))$;
end
Step 2 - Generate Dynamic Programming Grid Space: Initialize system parameters;
Create full factorial tensor space for states & controls;
Simulate for 1 distance step (20 meters);
Gather states & controls output;
Step 3 - Optimal Reference Generation: Initialize system parameters;
Apply penalty to all DOE points where constraints are violated;
Select minimum throttle for each state point;
while *Vehicle Position is NOT end of Route* **do**
  Apply optimal control value interpolated for each state point;
  Integrate fuel rate for mpg;
end

Algorithm 1: Offline Global Optimality-Single Vehicle

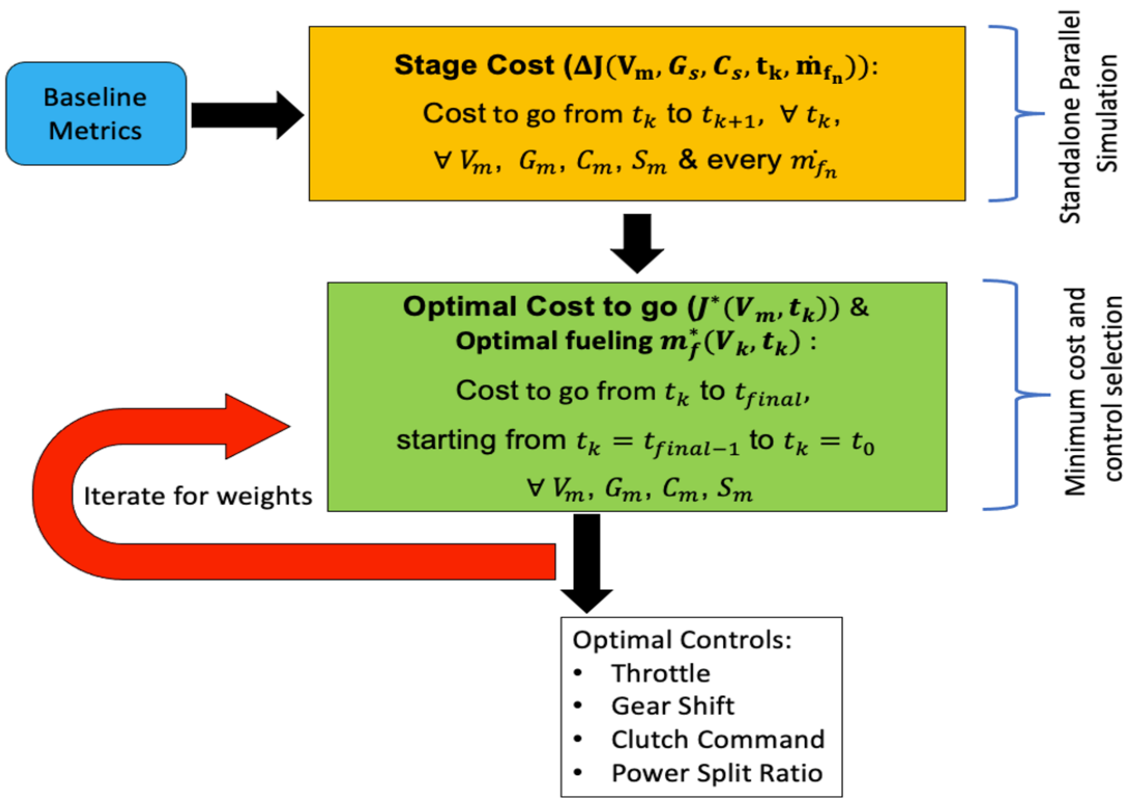


Figure 5.2. Simulation framework and flow for the single vehicle configuration. It shows the high level process and the step wise simulations that are run to generate the optimal control action.

Figure 5.2 shows the stages of the well known dynamic programming which is used in this work to solve the problem. The baseline metrics are accumulated and fed to the stage cost calculation algorithm which calculates the local stage cost for each value of control and state combinations. Finally the Optimal cost algorithm finds the minimum cost from all the state and control discretized points and selects the best control point for the minimum cost. The problem is also solved for a demo route of 10 mile with a 2 mile uphill and a 2 mile down hill section as described in Chapter 4. This is done as a sanity check for the problem and also to debug any issue. This also helps understand the local behaviors tied to individual hills. Figure 5.3 shows the optimal throttle and its contours as a function of vehicle position for a fixed gear, SOC and Clutch engaged state on the 10 mile demo route. It shows that

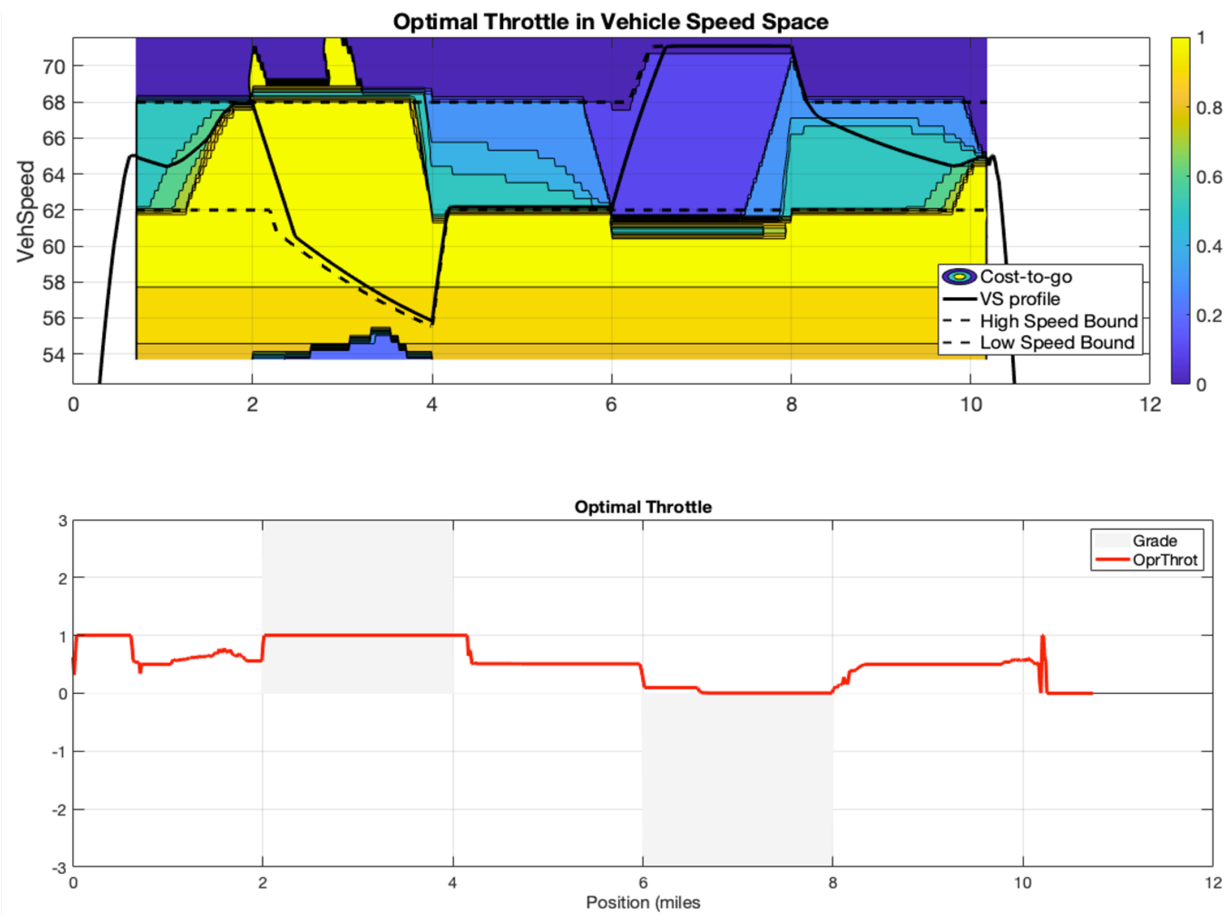


Figure 5.3. Contours of Optimal Throttle in Vehicle Speed Space for a fixed gear and SOC state point. Dotted bounds shows the constraint on Vehicle Speed which is set based on baseline simulation

the truck accelerates just before entering the hill and the throttle is maximum at that point. During the down hill the truck did not chose to apply any throttle. This contour is for a single gear state and a single discretized value of SOC which are the states in this problem. Figure 5.4 shows the cost to go calculated from the end of the route towards the start for the objective function. The Cost shown here is chosen at a particular gear and SOC point which are the state parameters in the problem.

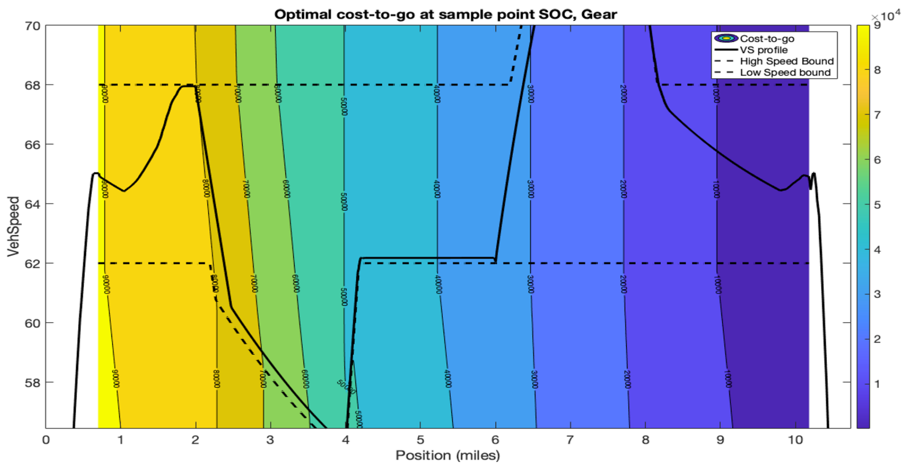


Figure 5.4. Optimal Cost-to-go contour with Vehicle Speed at a fixed gear and SOC state point.

Table 5.1 shows the key metrics for the Cruise Speed modulation problem. It shows an absolute fuel economy of 3.02% with a change of 0.07% in trip time. There is a reduction of 1% of aerodynamic work and 2.56% reduction in total cycle work. The brake thermal efficiency improved by 0.18%. Negative work reduction is mostly due to engine braking reduction.

Figure 5.3 shows the optimal throttle and its contours as a function of vehicle position for a fixed gear, SOC and Clutch engaged state.

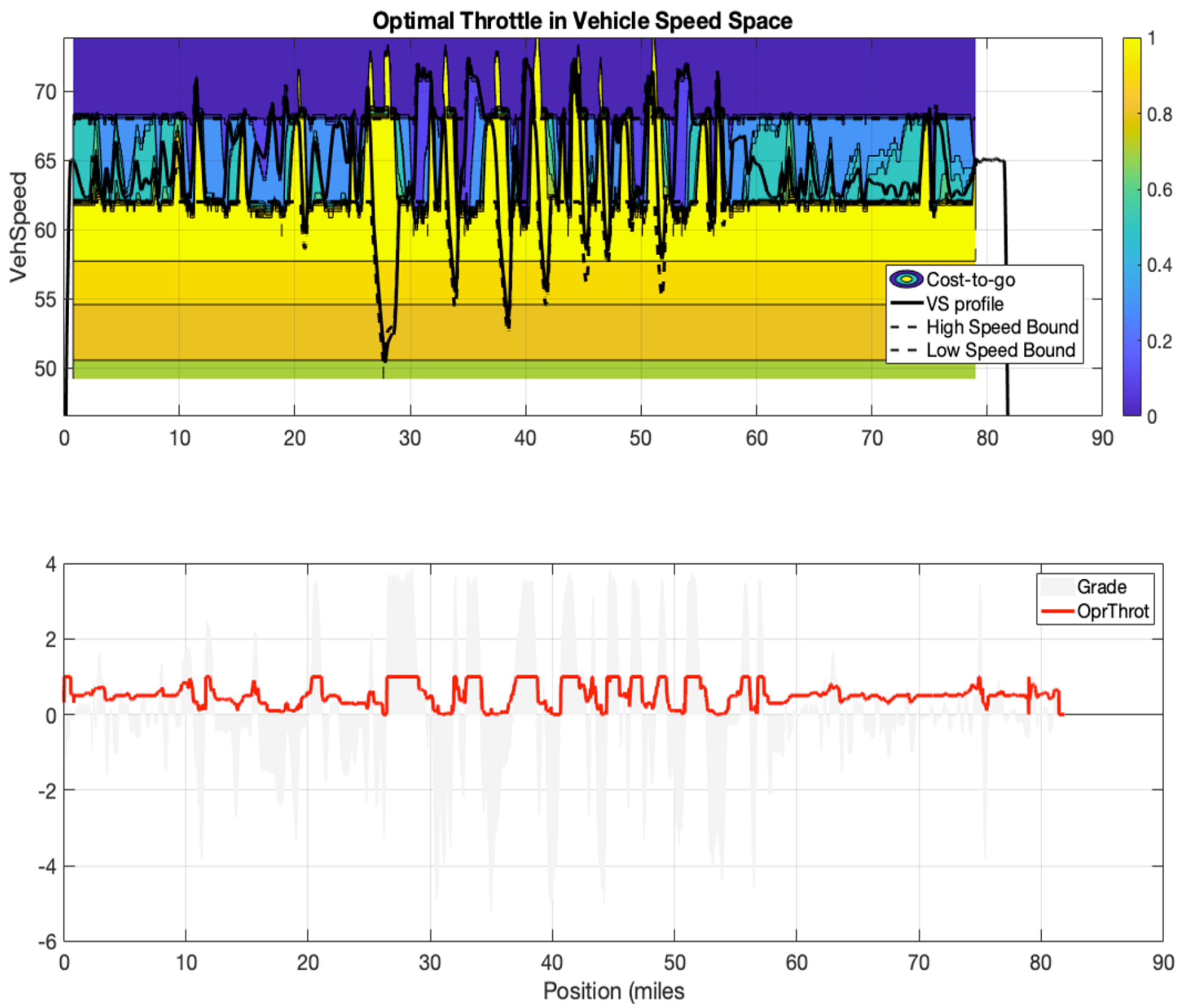


Figure 5.5. Contours of Optimal Throttle in Vehicle Speed Space for a fixed gear and SOC state point. This contour space plot is for the actual route for the entire 86miles. Dotted line bounds shows the constraint on Vehicle Speed which is set based on baseline simulation

Table 5.1. Comparison of key metrics for the first offline problem where the cruise reference speed is predictively modulated based on look ahead knowledge of the entire route

Metrics	Units	VS	Δ
Fuel Consumed	Kg	26.4984	-0.8
Fuel Economy	mpg	9.86	3.02
Trip Time	s	4602.8	0.07
Aerodynamic Work	kWh	89.26	-1.01%
Cycle Work	kW	142.34	-2.56%
BTE	%	44.95	0.18%
Negative Work	kWh	-24.1	-18.66
EONOx	Kg	0.4104	-6.41

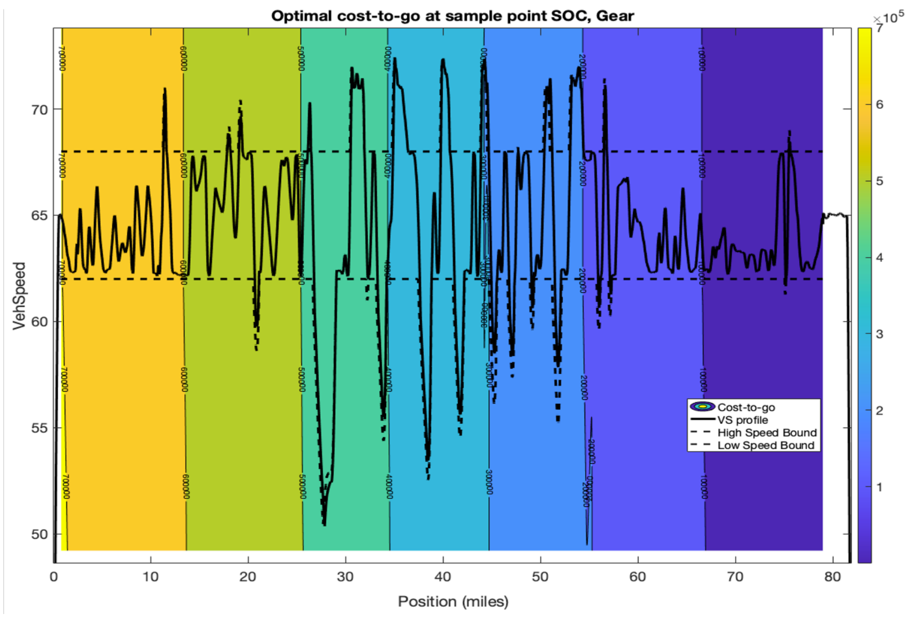


Figure 5.6. Optimal cost-to-go for the full route for one point of all the states.

5.2 Offline Mode - DP Based Speed & Coast Management Solver for Single Truck

This is similar to the Speed Management problem in terms of the cost function and system dynamics. Equation 5.3 is still the cost that will be used to calculate the cost to go along with the non-linear constraints on speed and gear. The only exception in this case is the addition of another state and control variable in the problem. To manage coast we need to know the current clutch state and then command the clutch to engage or disengage. Hence there is a new DP (dynamic programming required) state added to the problem which is the current clutch state. The control lever which is also added is the clutch command which dictates whether the engine is coasting or engaged.

One additional constraint here is the duration and frequency of coast events. Even though speed constraints will take care of how long we can be in coast events, there is a need for a constraint on how frequent the coast events could be. It is done by introducing a penalty on the frequency of change in clutch state. This will ensure that the system is not coasting for very less time which are practically not possible from system dynamics.

It is also needed here to handle the interaction with hybrid system. We had to answer the question of what the hybrid system will do when the vehicle is coasting. The question is more severe when we have an Engine Off Coasting. The current control action will make the hybrid system work based on SOC limits and can either regenerate or propel the vehicle. If the system is regenerating it will interact with the coast events and will reduce the speed more there by taking it out of coasting. This problem is also solved on the 10 mile short route for sanity and tuning before the problem is moved to the full route. Figure 5.7 shows the optimal behavior in speed and coast events on the 10 mile route. It is seen that the speed increases as in the speed problem alone before the hill and then slowing down before the down hill. There were 2 distinct coast events of sufficient duration, one just before entering the down hill and the other while coming out of the down hill.

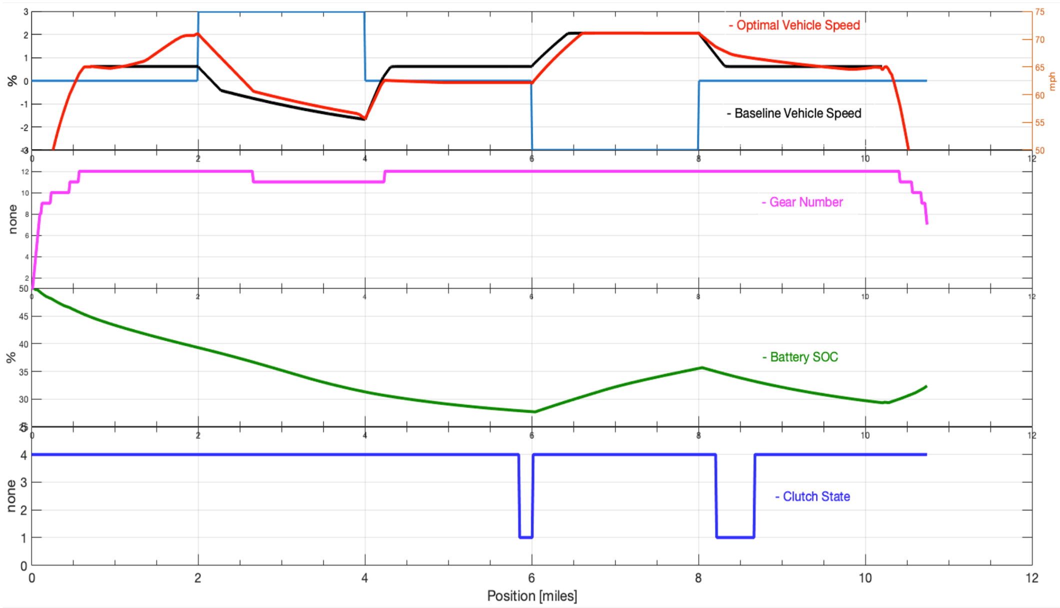


Figure 5.7. Trapezoidal 3% Route Speed Profile along with Gear and Clutch State showing Optimal Control Action

Table 5.2. Comparison of key metrics for the Coast Management problem only with Engine Idle and Engine Off Condition. The Δ% is the comparison with the baseline simulation

Metrics	Units	Case EI	ΔEI	Case EO	ΔEO
Fuel Consumed	Kg	27.03	-0.26	26.92	-0.37
Fuel Economy	mpg	9.66	0.98%	9.7	1.39%
Trip Time	s	4604.1	0.09%	4602.6	0.06%
Aerodynamic Work	kWh	89.7	-0.52%	89.1	-1.19%
Cycle Work	kW	144.97	-0.76%	144.76	-0.91%
BTE	%	44.87	0.09	44.99	0.21
Negative Work	kWh	-27.21	-8.17%	-26.1	-11.91%
EONOx	Kg	0.4304	-1.82%	0.4297	-1.98%

Table 5.3 and Table 5.2 shows the energy lost in the system in the form of negative work done. This in an important metrics to look at since the negative work done is the loss in energy which is gained at the expense of either fuel or electric energy. Since dynamic programming did not show the reason why the fuel benefits are occurring it is important to compare the reduction in negative work done which clearly indicates where the fuel economy is coming from along with the improvement in engine BTE.

Table 5.3. Comparison of key metrics for the Vehicle Speed and Coast Management problem with Engine Idle and Engine Off Condition. The Δ% is the comparison with the baseline simulation

Metrics	Units	Case EI	ΔEI	Case EO	ΔEO
Fuel Consumed	Kg	26.3345	-0.96	26.14	-1.15
Fuel Economy	mpg	9.92	3.64%	9.99	4.41%
Trip Time	s	4604.2	0.1%	4603.7	0.08%
Aerodynamic Work	kWh	87.52	-2.94%	87.53	-2.93%
Cycle Work	kW	141.92	-2.85%	140.25	-3.99%
BTE	%	45.09	0.31	44.89	0.11
Negative Work	kWh	-21.76	-26.56%	-22.12	-25.35%
EONOx	Kg	0.4021	-8.28%	0.4002	-8.71

Figure 5.9a shows the key comparison between vehicle speed only problem and vehicle speed with coast solved together. We notice that the addition of coast problem along with vehcile speed gives an added 0.6% absolute fuel economy. The problem with coast alone provided 0.88% benefits which indicates that the vehicle speed problem alone and coast problem alone benefits do not sum up completely. There is some benefit which is not realized when we solved the speed and coast problem together. Contrary to the engine idle scenario it is observed that the engine off case provides an added 1.4% benefit is achieved which is exactly the addition of vehicle speed problem and coast problem. This indicates that with engine off coasting the benefits are preserved whether we solve the problem individually or together. It is also noted that a relatively high amount of cycle work reduction took place with the engine off version of the speed and coast problem. We do not significant reduction in negative work with the coast only problem both for the engine idle as well as the engine off scenarios.

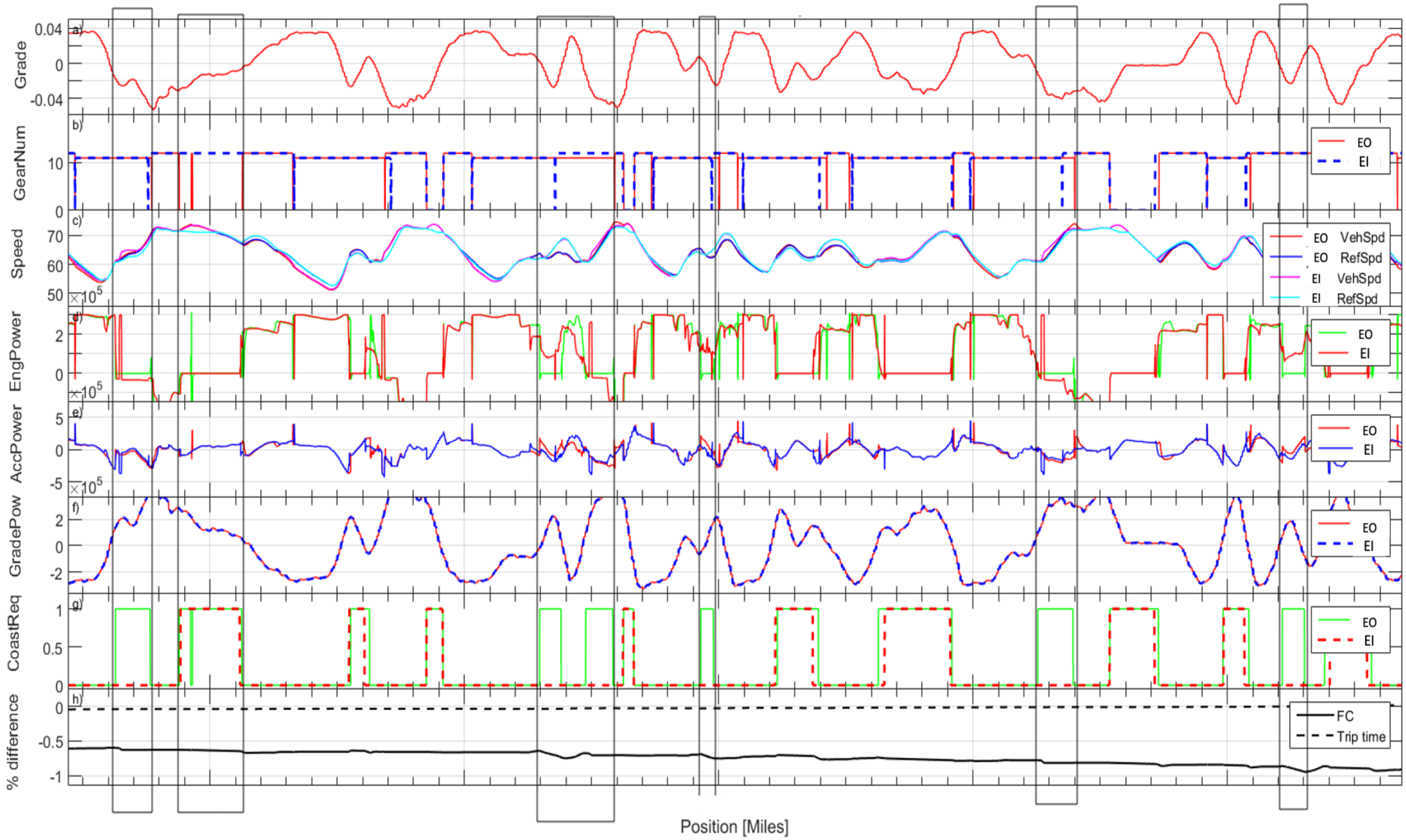
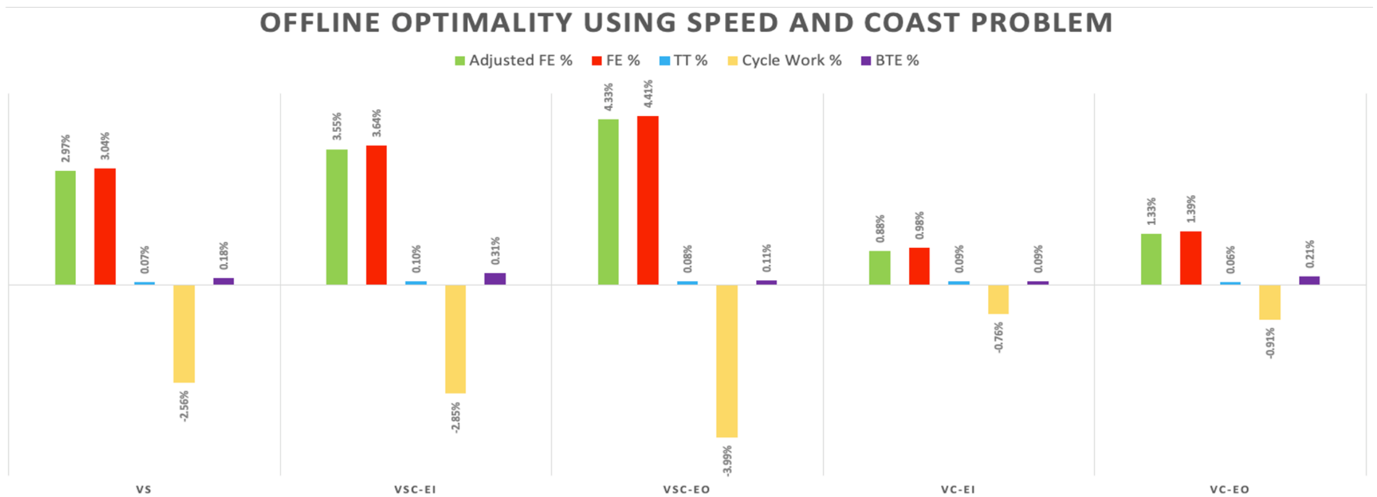
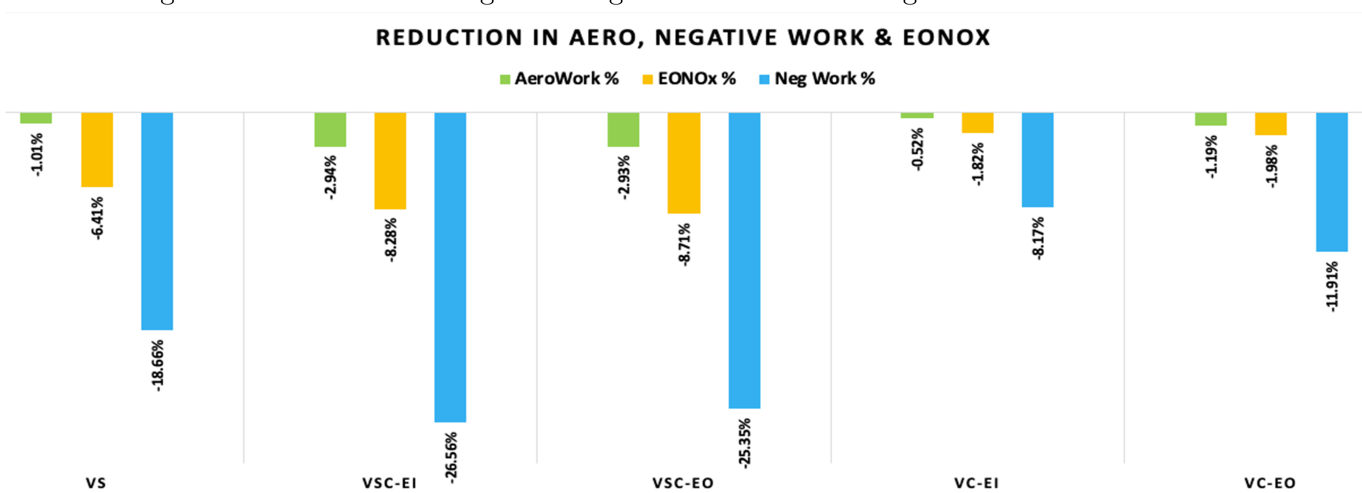


Figure 5.8. Performance Results for Optimal Solution compared to Baseline rule based control. The plot is zoomed version of stitched version of different sections in the route



(a) Key metrics showing the comparison of benefits along with Cycle work and BTE for Vehicle Speed and Coast Management Problem including both Engine Idle Coast and Engine Off Coast



(b) Reduction in Aerodynamic Work along with associated EONOx Reduction. The last bar plot shows the reduction in Negative Work which includes Engine braking, Motoring Losses and Service Braking

Figure 5.9. Key metrics for Predictive Vehicle Speed and Coast problem with Engine Idle and Off Scenario.

5.3 Offline Mode - DP Based Speed, Coast & Gear Solver for Single Truck

In this third problem we have included predictive gear control as a third lever along with speed and coast controls. The objective function remains same with the addition of an extra control input which is the gear shift command. Gear shift command command can take 3 possible states (up shift, hold gear & down shift). The objective here is to find if shifting the gear with the knowledge of road grade in the route will help achieve any fuel benefits and/or drivability improvements. As discussed in the previous chapters it is not expected to gain any fuel benefits unless the fuel maps are tuned in such a way to include high BTE zones at a lower gear. We will validate these analytical analogies with the simulation results from this problem. As per the process the initial problem is framed using a shorter made up route of 3% grade (uphill and downhill) at 2 distinct position in the route. The uphill occurs first followed by the downhill. Figure 5.10 shows the key control levers in action. We see similar expected behaviors of speed up during the pre-uphill region and slow down during the pre-downhill portion. It is worth noting that this problem has only 1 coast event just before entering the downhill. The other coast event which was coming out of the down hill is not observed here in this problem.

Table 5.4. Comparison of key metrics for the Vehicle Speed and Coast Management problem with Engine Idle and Engine Off Condition. The $\Delta\%$ is the comparison with the baseline simulation

Metrics	Units	Case EI	ΔEI	Case EO	ΔEO
Fuel Consumed	Kg	26.34	-0.95	26.1	-1.19
Fuel Economy	mpg	9.92	3.62%	10.01	4.57%
Trip Time	s	4597.3	-0.05%	4605.3	0.12%
Aerodynamic Work	kWh	90.29	0.13%	87.54	-2.92%
Cycle Work	kW	142.021	-2.78%	140.218	-4.02%
BTE	%	45.11	0.33	44.94	0.17
Negative Work	kWh	-22.3	-24.74%	-23.93	-19.24%
EONOx	Kg	0.41.	-6.48%	0.4082	-6.89

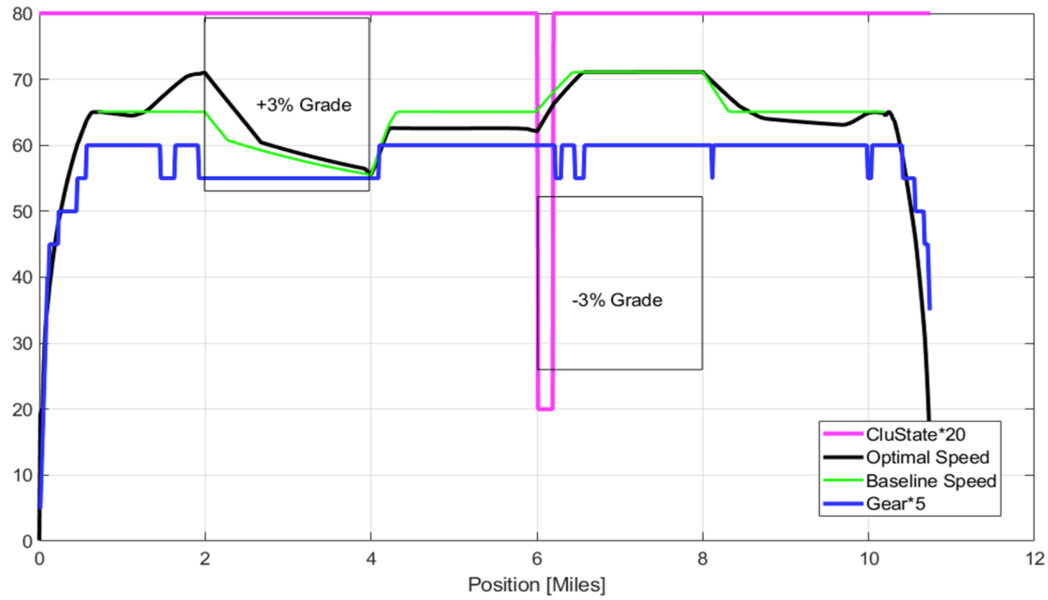


Figure 5.10. Optimal control behavior analysis using a 3% demo route. Key control levers are vehicle cruise speed, clutch command and gear shift request. Clutch State is *20 and Gear Number is *4 in the plots.

Table 5.6 shows the raw data from the vehicle speed, coast and gear management problem for both the engine idle and engine off scenarios. The Δ% are the changes compared to the baseline simulation results.

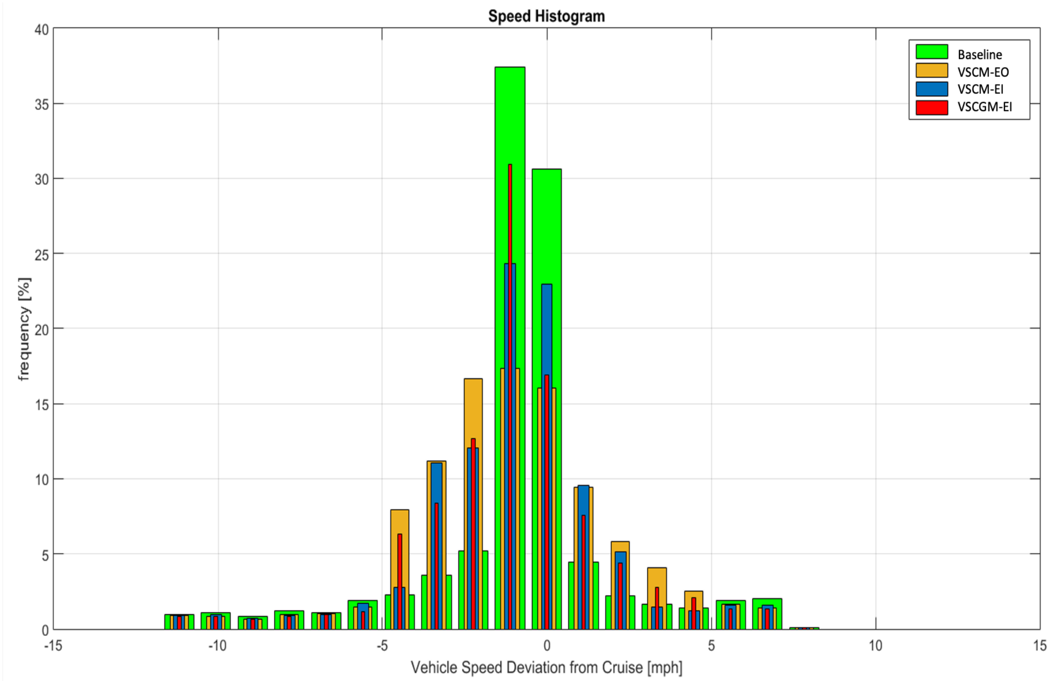
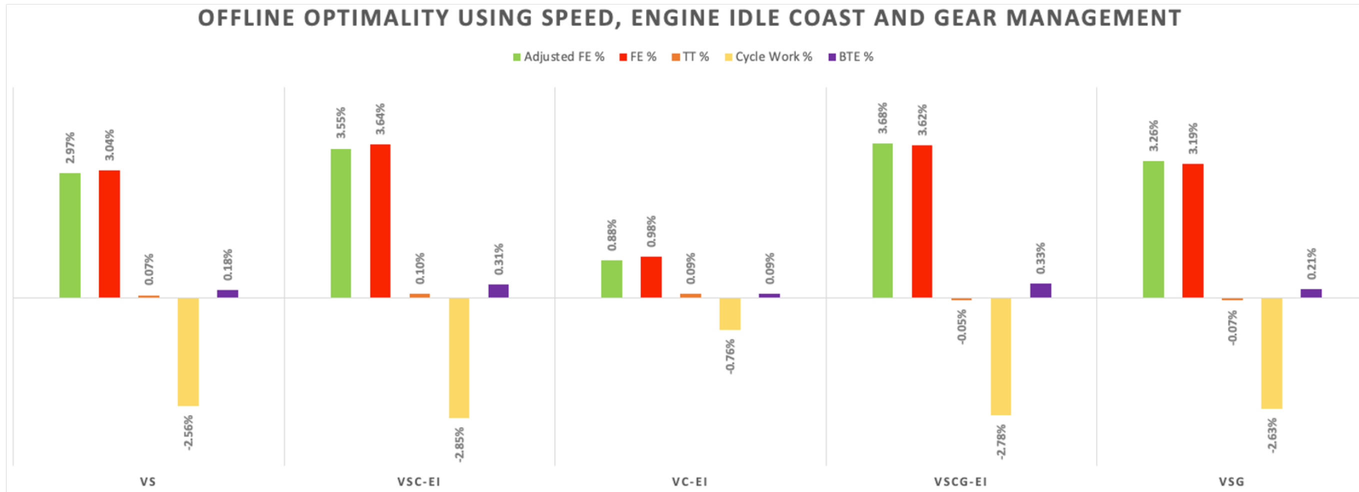
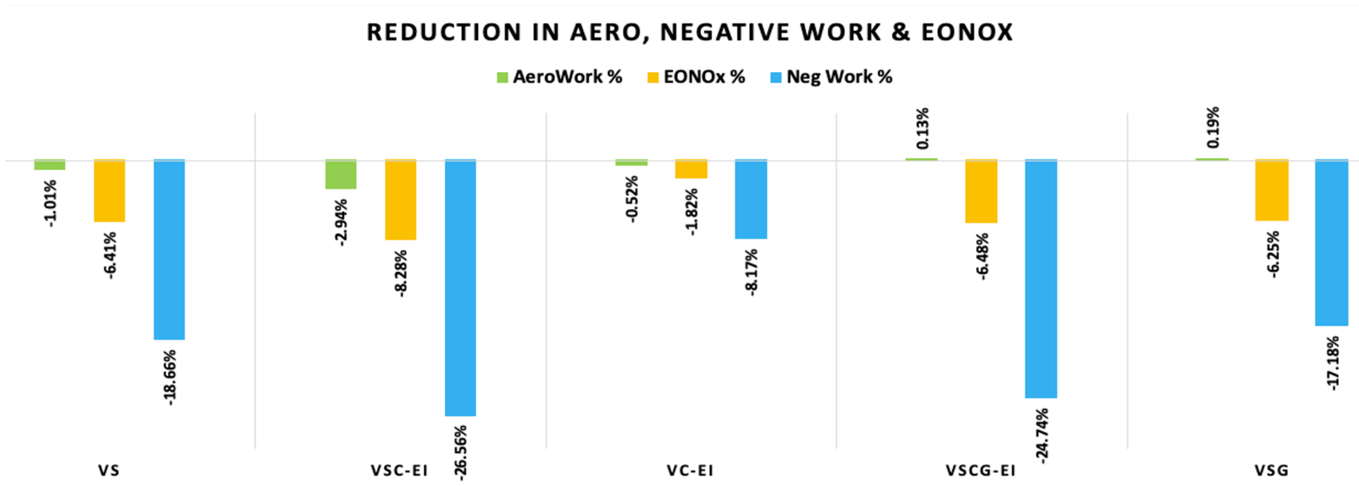


Figure 5.11. Speed histogram comparison of predictive cruise speed control with engine off coasting vs engine idle coasting and engine idle with predictive management.

Figure 5.11 shows the speed distribution between the speed, coast problem for both the engine idle and engine off case and speed, coast, gear problem together for the engine idle condition. We notice that the engine off coast problem with speed has the most standard deviation, which indicates that the vehicle is more transient in terms of speed as compared to others. It is also observed that the gear management problem reduced the speed lug-back towards the more negative speed points which are the uphill section of the route. It is also worth noting that the gear management also spends more time at the lesser negative speed points which indicates slowing down more during the flat sections of the route. Figure 5.12a shows the detailed fuel economy along with the associated key metrics. We see the progressive improvements in results and also noted that gear management is not a fuel economy but more of a performance lever as explained and studied in below sections. Most of the benefits are also associated with the BTE improvement, negative work reduction and cycle work reduction.



(a) Key metrics showing the comparison of benefits along with Cycle work and BTE for Vehicle Speed and Coast Management Problem including both Engine Idle Coast and Engine Off Coast



(b) Reduction in Aerodynamic Work along with associated EONOx Reduction. The last bar plot shows the reduction in Negative Work which includes Engine braking, Motoring Losses and Service Braking

Figure 5.12. Key metrics for Predictive Vehicle Speed and Coast problem with Engine Idle and Off Scenario along with Predictive Gear Management.

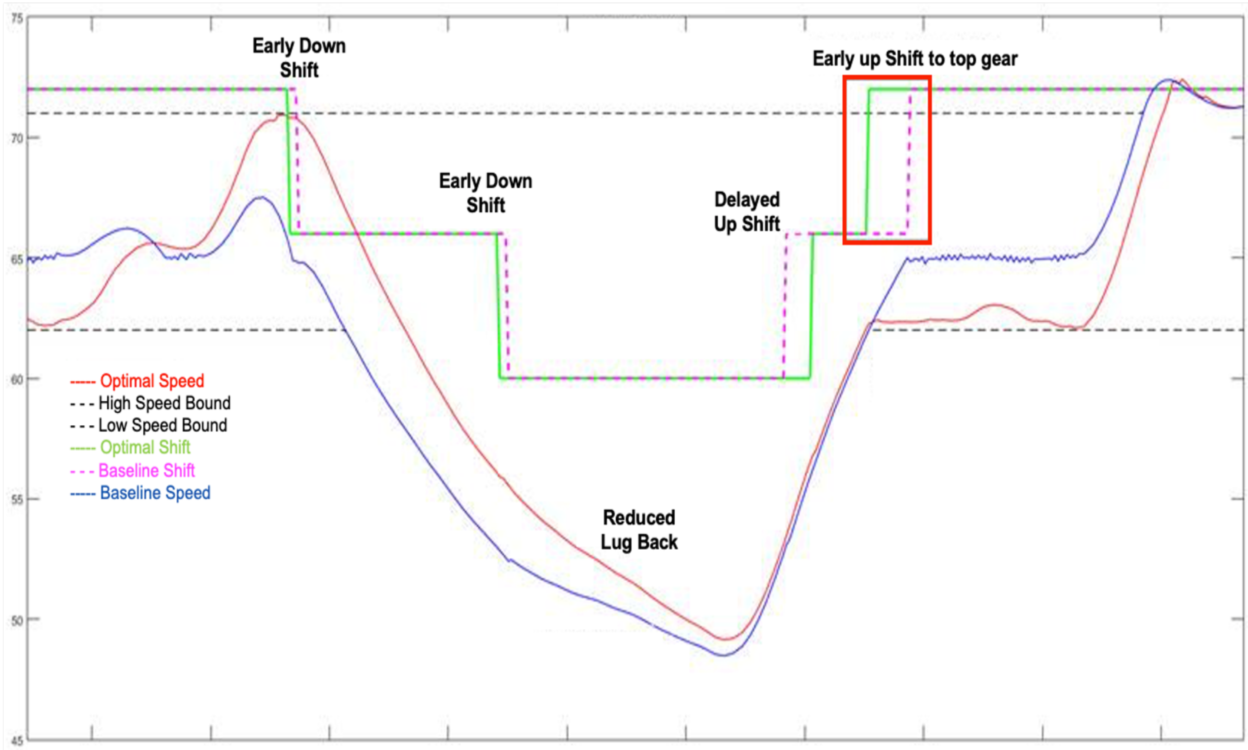


Figure 5.13. Predictive Optimality in Gear Management. The problem shows the predictive gear shift behavior in one of the up-hill section

Figure 5.13 is an exploded view of a section with details of how the gear behaves in a grade. This is for the problem with predictive gear management. The dotted black lines are the high and low vehicle speed bound which are applied as constraints for all the problems and is taken from baseline dynamics. The blue and the red plots shows the baseline and the optimal vehicle speed respectively. We notice that during the uphill section the optimal vehicle speed drops less compared to the baseline. The speed drop in this case for the optimal profile is better than the baseline by close to 2mph. Comparing the green and the dotted magenta plots which are the optimal and the baseline gear shift we notice that the gears shifts early as a result of the predictive knowledge. While this helps reducing speed lug-back in hills it will also affect fuel economy. Hence with the gear management problem we see less (top-1) gear operation and also slow down in the flat section to compensate for the fuel loss in the hills. Interestingly we also see an early upshift while coming out of the hill which is an interesting observation for fuel economy for this BTE map. The metrics results associated with behavior is discussed in the below sections.

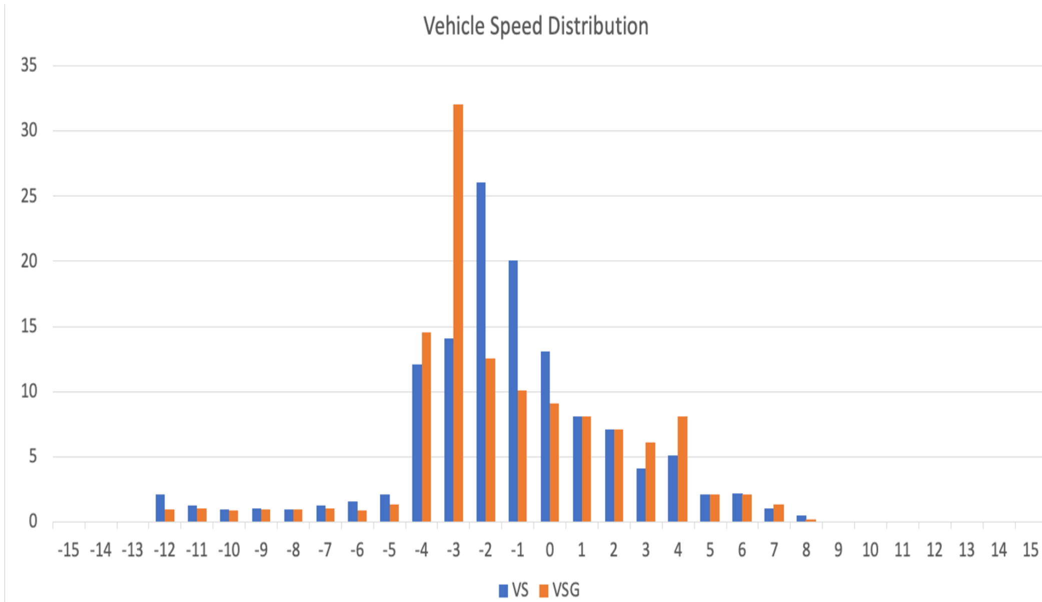


Figure 5.14. Vehicle Speed Distribution between Predictive Speed Management Problem and Predictive Speed with Gear Management Problem

Figure 5.14 shows the speed histograms for the speed and speed, gear problems. We notice less speed reduction in the heavy hills as compared to the speed only problem. The balance is done by slowing more during the flat sections. This is indicated by the taller orange bar close to 0 mph as compared to shorter orange bars closer to larger negative mph numbers.

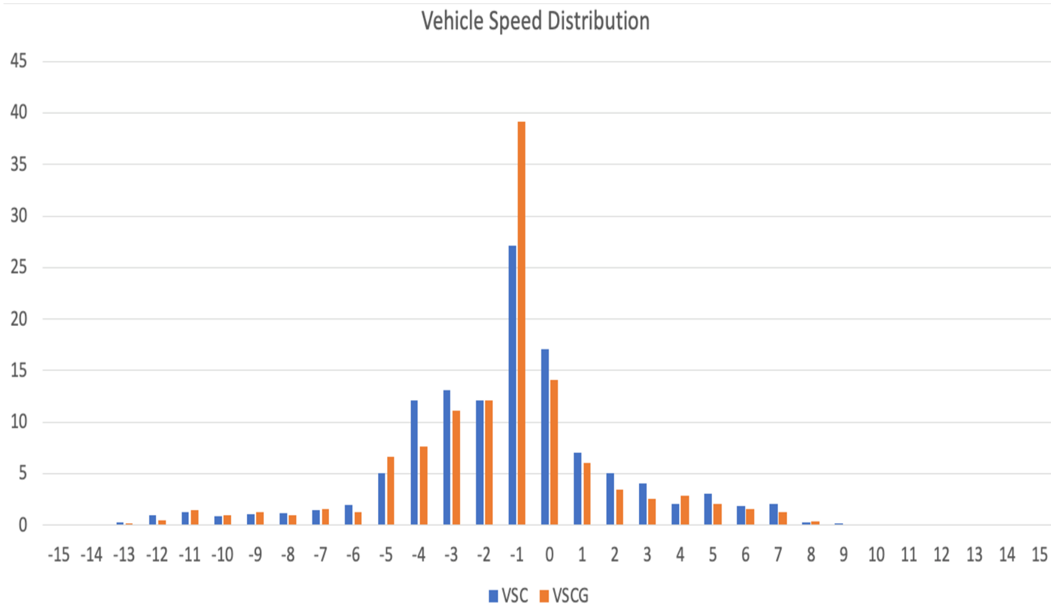


Figure 5.15. Vehicle Speed Distribution between Predictive Speed Management with Engine Idle Coast Problem and Predictive Speed with Engine Idle Coast + Gear Management Problem

Figure 5.15 compares the vehicle speed histograms for the speed, coast problem and the speed, coast, gear problem. The distribution is almost similar with a little lesser slow down which indicates a better reduction of lug-back. This is something which is shown in the below section with the time series data. It is worth noting that though the problem reduced lug-back in the heavy hills during the flat section the slow down is still more which is the optimal behavior to balance time and improve fuel. The fuel spent during the heavy hills is compensated by the slow down during the flat section and also balancing time.

Table 5.5. Comparison of key metrics for the Vehicle Speed and Gear Management problem. The $\Delta\%$ is the comparison with the baseline simulation

Metrics	Units	VSG	Δ
Fuel Consumed	Kg	26.45	-0.84
Fuel Economy	mpg	9.88	3.19%
Trip Time	s	4596.8	-0.07%
Aerodynamic Work	kWh	90.34	0.19%
Cycle Work	kW	142.24	-2.63%
BTE	%	44.99	0.21
Negative Work	kWh	-24.54	-17.18%
EONOx	Kg	0.411	-6.25%

Table 5.5 shows the raw data in tabular form for the problem speed and gear management. It is worth to note that we have tried to solve the problem for gear and speed alone to understand the behavior. It is noted that this does provide any significant improvement. % FE improvement is just 0.3% which falls under margin of error, as well it may not be realizable in actual controls. The interesting fact is that it did not eat away the benefits of speed alone problem which is important. Aerodynamic drag work increased since the truck moved a little faster than the baseline in this case.

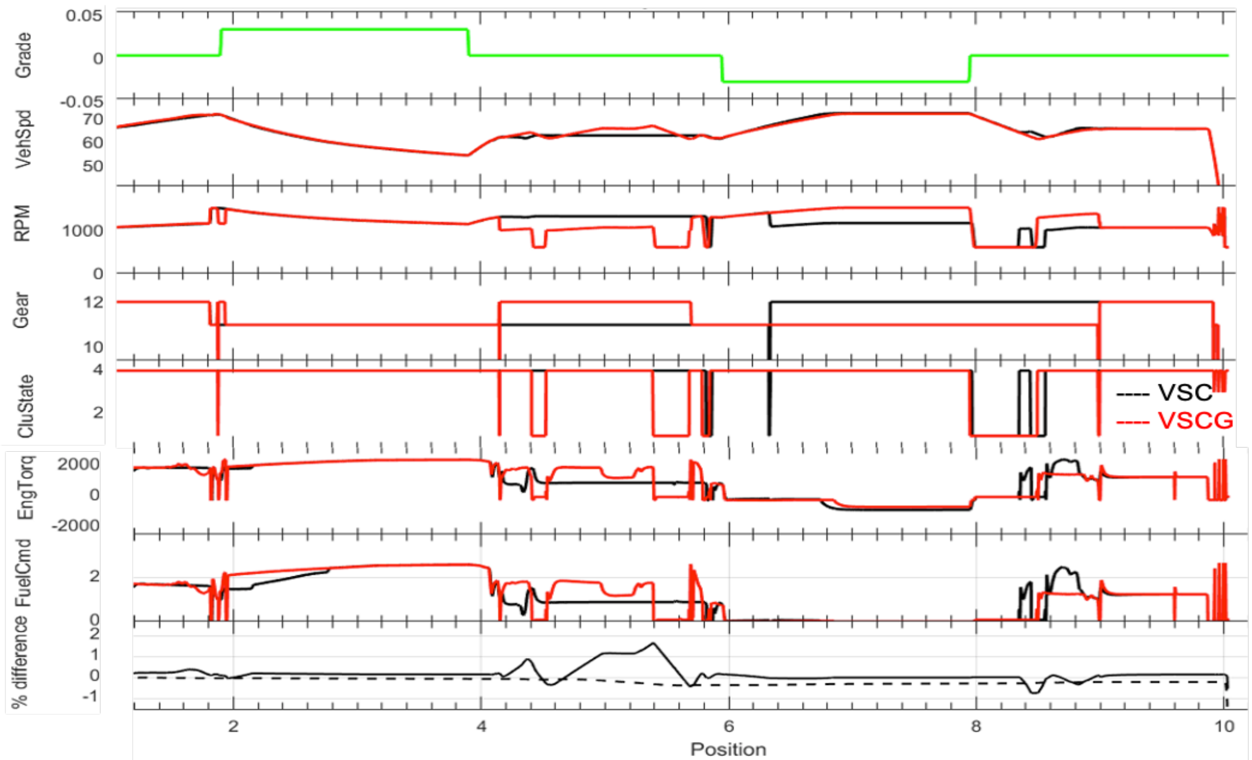


Figure 5.16. Comparison between key parameters for Speed + Coast Problem and Speed + Coast + Gear Problem

Figure 5.16 demonstrates the behavior change due to the addition of the gear problem on top of the speed and coast problem. The route is a simple 10 mile route with a 3% uphill followed by a 3% downhill. The results are not very conclusive in these plots but the overall trend is in the similar lines. The result can be tuned better with more weight adjustment which is not done in this case since this is not the prime route. In general as with all other problems, we solved the problem on the demo route and then moved to the prime route as shown in the Figure 5.17.

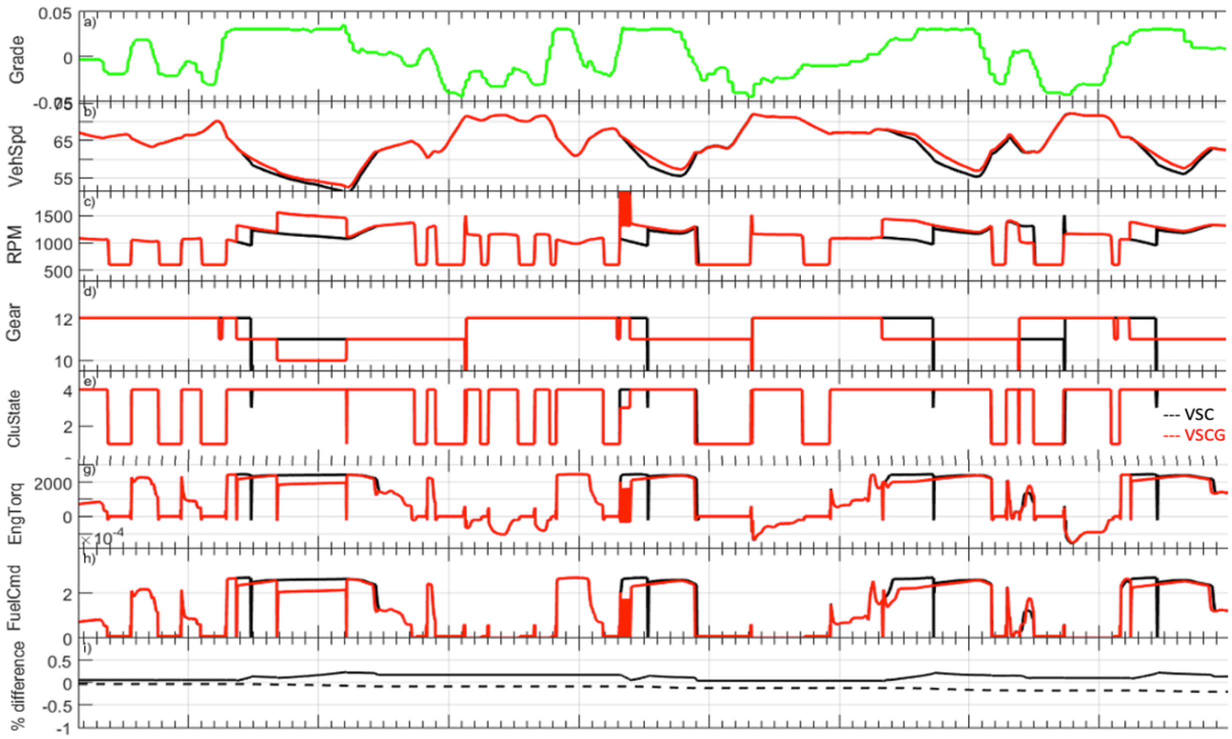


Figure 5.17. Vehicle Speed Distribution between Predictive Speed Management Problem and Predictive Speed with Gear Management Problem

Figure 5.17 shows the detailed time series plots of key signals for this problem plotted against vehicle position. The plot is stitched by taking some zoomed portion of the hilly portion. Subplot 1 shows the road grade. The subsequent subplots shows the keys signals with the red being the speed, coast and gear problem and the black being the coast and speed problem only. The reason for this comparison is to understand the microscopic differences in key signals due to the addition of the predictive gear problem. Subplot 2 shows the reduction in vehicle speed lug back at heavy hills. It is clearly seen that the red plot is not dropping as much as the black plot. On an average a $2 - 3mph$ less speed drop in the heavy grade sections are noted. This will impact fuel economy negatively but will give a better drivability performance. We tried to tune the cost weights in such a way so that we do not penalize fuel economy for this problem and hence we saw almost negligible fuel economy improvement with this problem. The subplot 3 shows the engine speed comparison between the two simulation result. Subplot 4 shows the actual gear number and difference between the two. We clearly see that there are some early downshift during pre-uphill which is typical to our predictive theoretical understanding from the introduction section. Though early up-shift is also possible but we do not have any in this portion of the route. Subplot 5 shows the coast events where the clutch state goes from 4 (locked) to open (1). We do not need any significant differences in the coast events due to the inclusion of the gear problem. There are some differences but its not huge. Subplot 6 and subplot 7 shows the variation of engine torque and fuel command during the portion of the route. The fuel command either will go top idle low speed governor fuel value or 0 depending on whether engine idle or engine off problem is solved. The last subplot 8 is the cumulative % time and fuel economy improvement. It shows the improvements as a function of dynamic signal change.

5.4 Offline Mode - DP Based Speed, Coast, Gear & Power Split Solver for Single Truck

This is an additional lever that we want to include in this research work with minimum priority. It is not expected to see a substantial improvement in results with the additional of this lever. The reasoning here for this analogy is that the hybrid system is quite small to provide substantial electric boost mode operation. A major improvement could be in the duration of coasting with the addition of hybrid boost.

In this problem there are no additional states involved but there is an extra control input for the dynamic solver. This new control input is power split ratio. The way this ratio is defined in the problem is by discretizing the entire hybrid power range including the charge and discharge limits. So if the hybrid power can range between -20kW to +20kW, then the grid is setup by discretizing this complete range of 40kW between **-ve** and **+ve** range. The resolution of the grid size matters since they impact the results based on how dynamic and responsive the particular control input is. Hence a SOC needs a more higher resolution than the power split ratio. It is also worth noting that the more the grid size the more challenging is to solve it using Dynamic Programming due to computational limits.

Table 5.6. Comparison of key metrics for the Vehicle Speed and Coast Management problem with Engine Idle and Engine Off Condition. The Δ% is the comparison with the baseline simulation

Metrics	Units	Case EI	ΔEI	Case EO	ΔEO
Fuel Consumed	Kg	26.34	-0.95	25.995	-1.30
Fuel Economy	mpg	9.92	3.63%	10.05	5.00%
Trip Time	s	4597.2	-0.06%	4601.5	0.04%
Aerodynamic Work	kWh	90.298	0.14%	89.21	-1.07%
Cycle Work	kW	141.89	-2.87%	141.67	-3.02%
BTE	%	45.07	0.29	45.59	0.82
Negative Work	kWh	-22.78	-23.12%	-23.97	-19.1%
EONOx	Kg	0.4	-8.76%	0.4023	-8.23

Table 5.6 shows the raw data for the full problem with speed, coast, gear and power split management for both the engine idle and engine off scenario. The Δ% is the comparison with the baseline simulation.

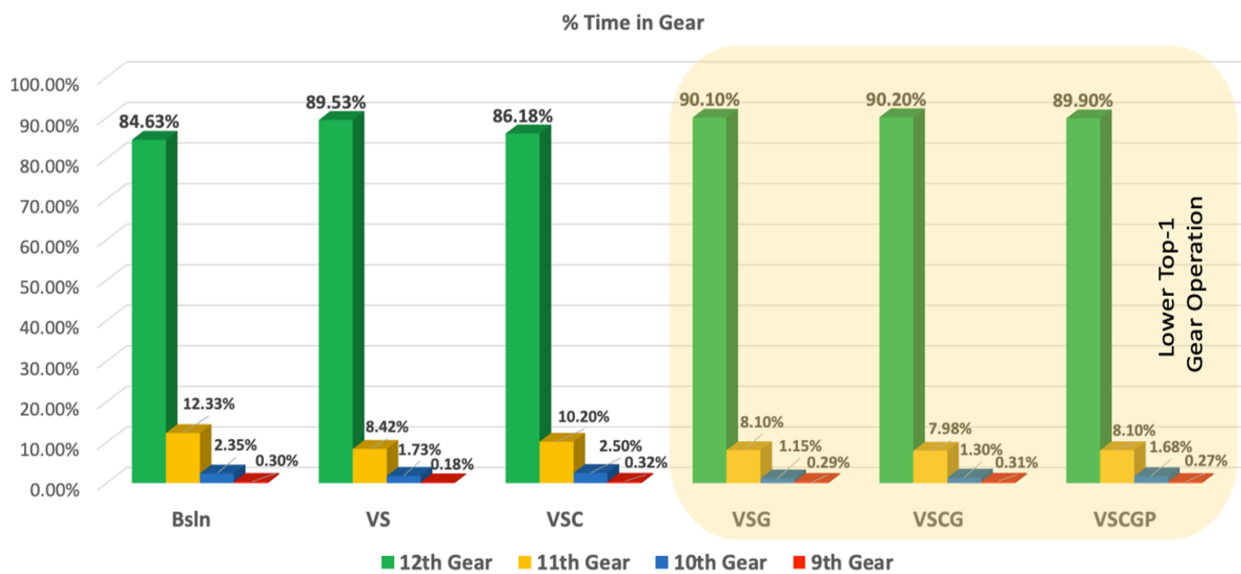
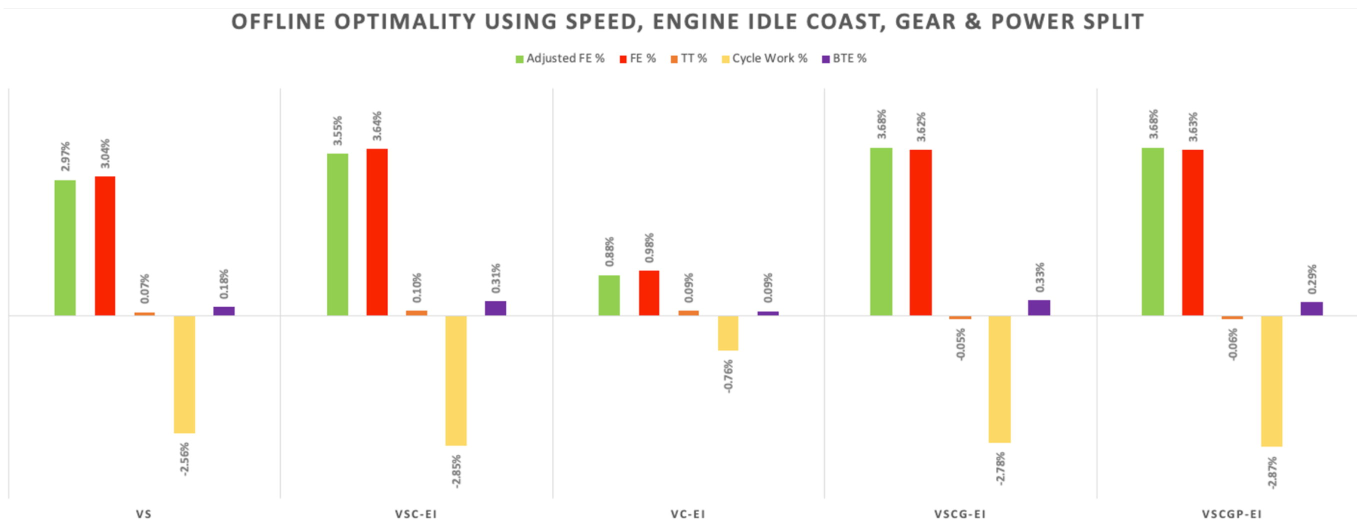


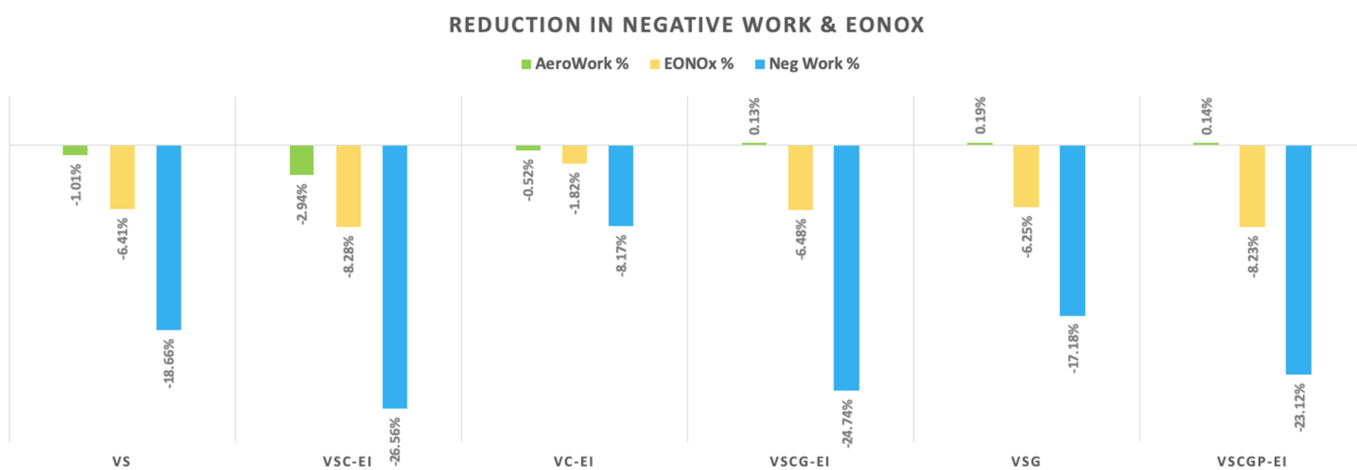
Figure 5.18. % Time in top 4 Gear for each DOE. The comparison has to be between the top 2 gears. Predictive gear tries to operate more at a lower gear while Fuel Economy tends to operate at a higher gear.

Figure 5.18 shows the total time spent in each gear for the individual problems. This metrics provides an understanding of which gear is predominantly being exercised by each problem. Since downshifting to a lower gear from top gear will take the operation outside of the maximum BTE zone it is not expected that the gear problem will try to shift down for a better fuel efficient solution. Hence for this kind of BTE map a more fuel efficient solution is practically not possible. Gear problem can expected to provide a better drivability by helping to reduce lug backs in heavy hill. It is seen in Figure 5.18 that all the problem types are trying to increase top gear operation since fuel saving will be more due to the BTE contour positioning. It is also interesting to observe that the problems with gear management is reducing the time in (top-1) gear. The coast management problem alone is the only problem which is not able to increase top gear operation much as compared to the other problems. This is due to the fact that with coast management problem since the vehicle is not predictively modulating speed and gear the speed drops are more which causes the gear to shift down more. A detailed time series of gear behavior for a portion of the route with grades is analyzed in the previous section. Figure. 5.19a shows the key metrics for the full problem with 4 predictive control levers on a single vehicle. In this problem the coast method used is with Engine idle condition. The set of bar plot as with the previous cases shows the trip time adjusted fuel economy along with the absolute fuel economy and total trip time for each case. The plots also shows the reduction in cycle work and the improvement in Brake Thermal Efficiency in each case. Figure. 5.19b shows the reduction in aerodynamic drag and the reduction in Engine Out NOx numbers. The bar chart also shows the reduction in negative work which in the single vehicle offline case includes only engine braking and motoring losses. We see a progressive improvement in economy along with an associated degree of NOx reduction. With the full set of problem a 3.7% fuel economy is achieved with a NOx reduction of around 8.3%. The corresponding BTE improvement in this case is much lower and is close to 0.3%. While the BTE improvement is on the lower side which definitely impacts the fuel economy and the NOx reduction we see substantial reduction in cycle work and negative work. in fact negative work reduction is more than the engine off counterpart.

Figure 5.20a and Figure 5.20b captures the detailed metrics for all the problems stacked up for the engine off case. It is observed that each additional lever brings in some added

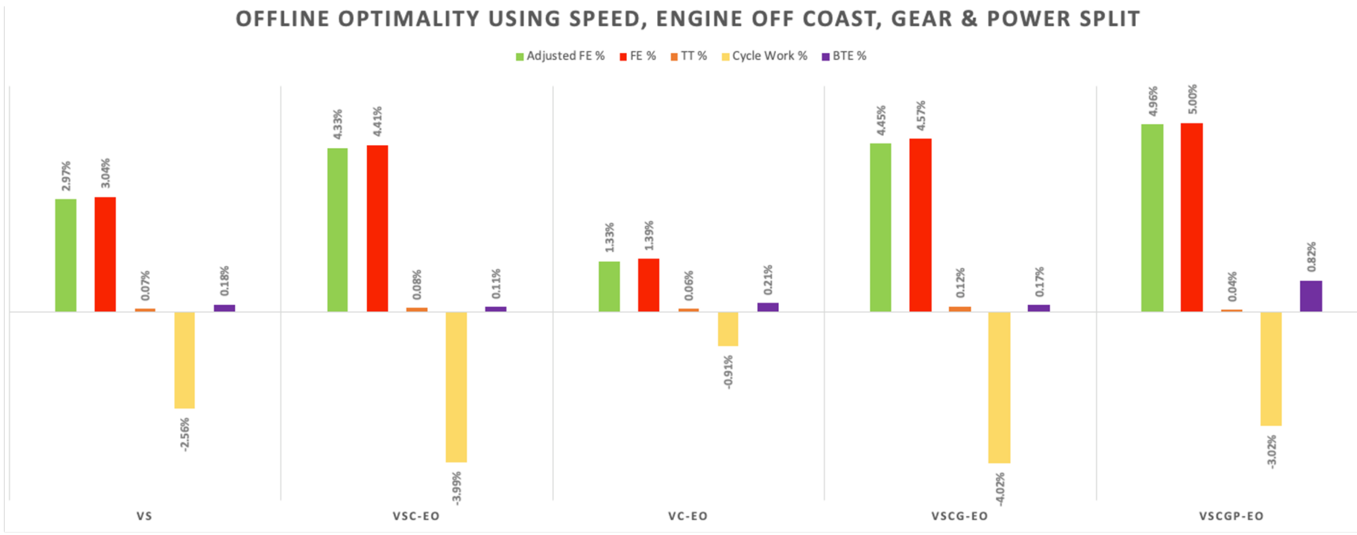


(a) Key metrics showing the comparison of benefits along with Cycle work and BTE for the complete set of problem including Speed, Coast, Gear and Power Split Management. This scenario is with with Engine Idle Coast

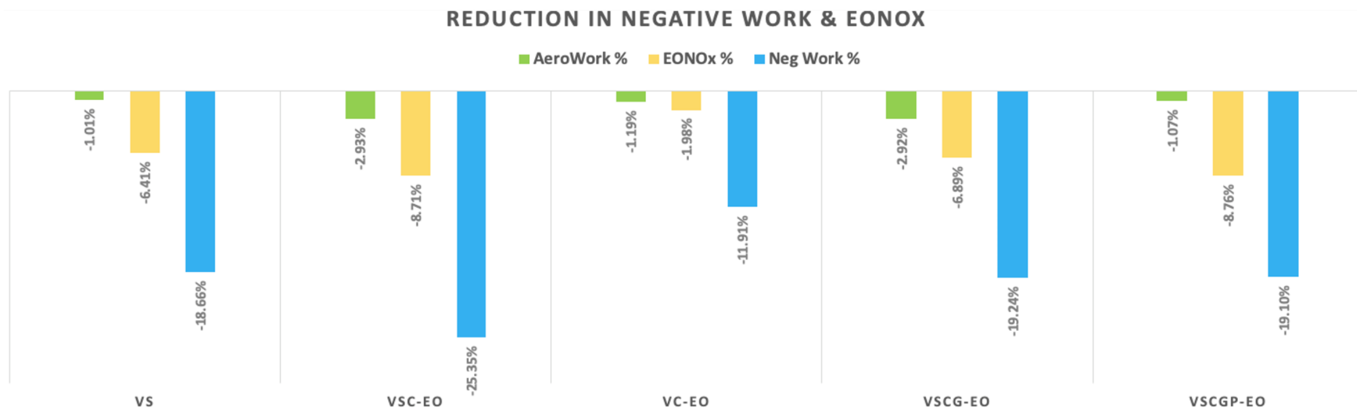


(b) Reduction in Aerodynamic Work along with associated EONOx Reduction. The last bar plot shows the reduction in Negative Work which includes Engine braking, Motoring Losses and Service Braking

Figure 5.19. Key metrics for full Predictive control problem for engine idle coast scenario only.



(a) Key metrics showing the comparison of benefits along with Cycle work and BTE for the complete set of problem including Speed, Coast, Gear and Power Split Management. This scenario is with with Engine Off Coast



(b) Reduction in Aerodynamic Work along with associated EONOx Reduction. The last bar plot shows the reduction in Negative Work which includes Engine braking, Motoring Losses and Service Braking

Figure 5.20. Key metrics for full Predictive control problem for engine off coast scenario only.

benefit though the addition of gear and power split is not significant and may not be realized in production code. This is because actual implemented production code may not get the full benefits of theoretical optimality and considering only a percentage of what we see here will actually be realized it is very hard to predict if an improvement of the order of 0.5% can truly be realized. Overall an impressive 5% fuel economy is seen with the predictive features working together with engine off coasting condition. This benefit is mostly contributed by around 0.8% improvement in BTE, 3% reduction in cycle work and 19% reduction in negative work. There is also an associated NOx reduction with each control levers. We observed a reduction of close to 8.75% of NOx when compared to baseline results. This is completely due to the fact that engine BTE has improved and other losses are reduced. Figure 5.21 shows the coast metrics for all the problems individually. The bars shows the percentage of time in coast by each problem and the line plot shows the number of coast events in each problem. We see near similar behavior with engine idle case for all the problems as well as similar behavior with the engine off case for all the problems. Interestingly the coast alone problems has some good amount of coasting events but could not provide a lot of benefit simply because of the fact that the net fuel economy is not related to coast events alone but is a combined factor of multiple scenarios including cycle work reduction, negative work reduction, BTE improvements and aerodynamic drag reduction. Further a couple of very large coast events were also observed which may not be feasible in real environment due to lube and cooling in the engine and aftertreatment restrictions. Nevertheless, the metrics gives an overview of the coast event distribution across various problem set.

Figure 5.22 shows the overall vehicle histogram for the entire set of problem. It shows a wider standard deviation in vehicle speed for the optimal problem. This is because of the fact that the vehicle speed is modulated a lot as compared to the fixed cruise speed target. In our work the speed droops used are $+6mph$ and $-3mph$ which means vehicle speed can swing between $62mph$ and $71mph$. The BTE contours shows the fuel consumed as the red bubbles. Bigger bubbles means more fuel consumed in that BTE zone. It is noted that a substantial amount of fuel spent in the idle section for the engine idle condition which directly maps to the percentage of time spent in coast. Figure 5.23a shows the % change in aerodynamic work as a function of % improvement in fuel economy. There is no concise

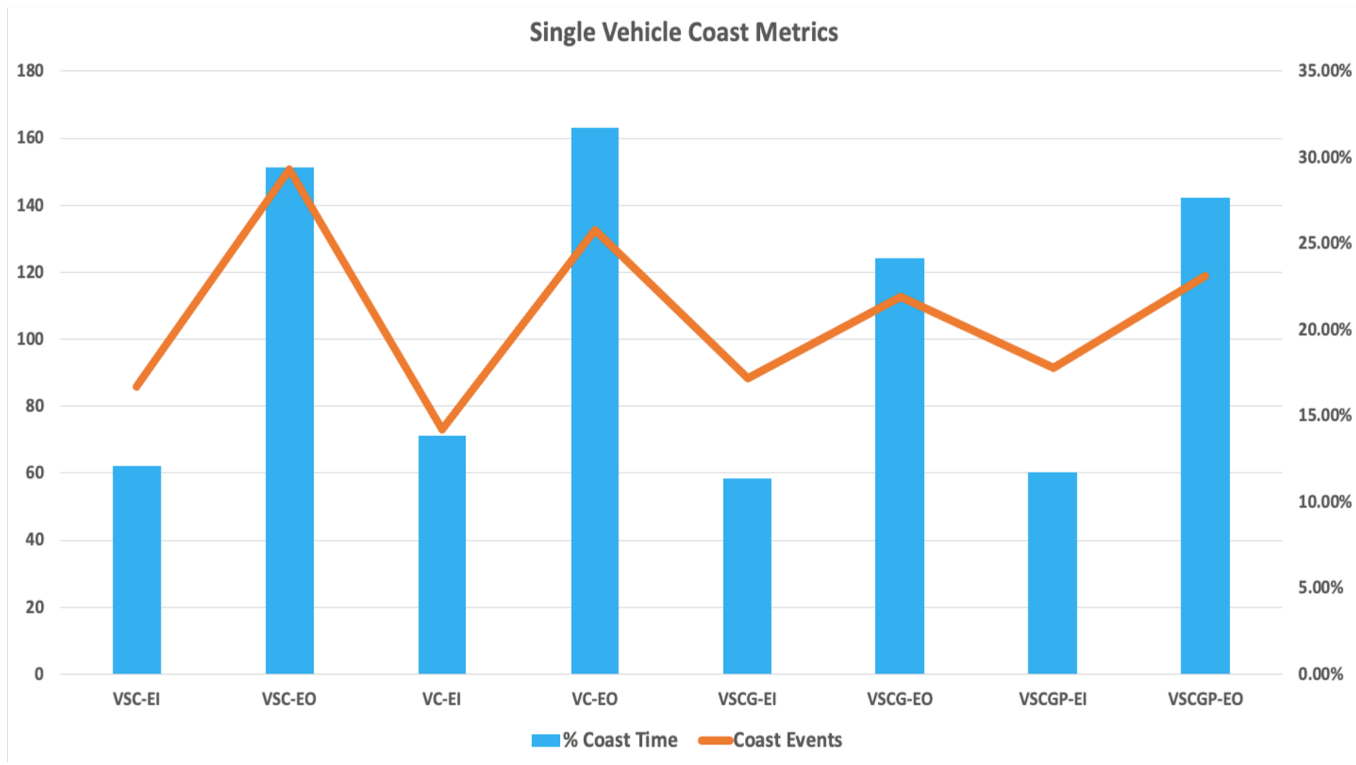


Figure 5.21. Coast Metrics for all combination of problems with Coast formulation. The bars show the % time in Coast for each problem and the plot shows the number of coast events

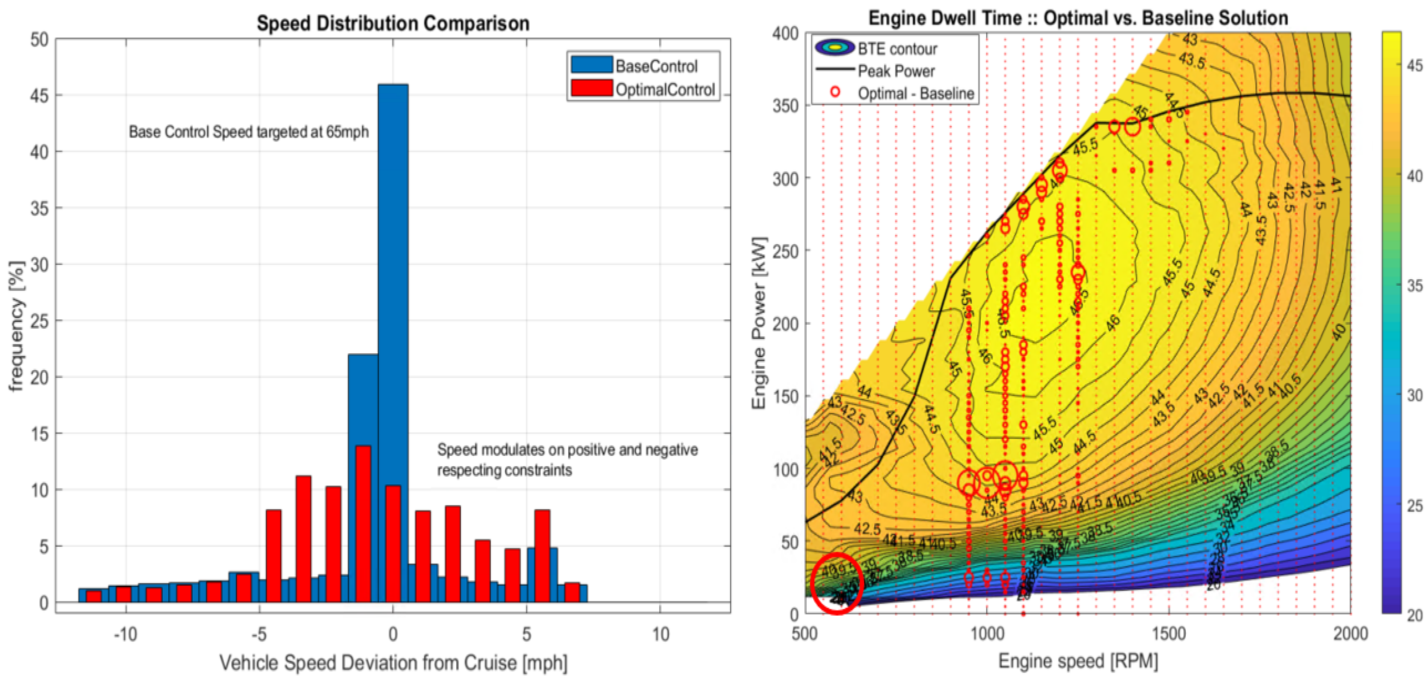
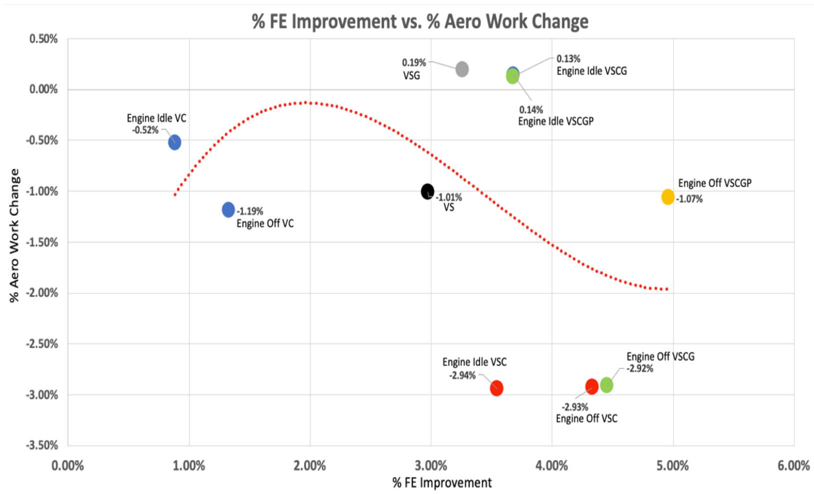
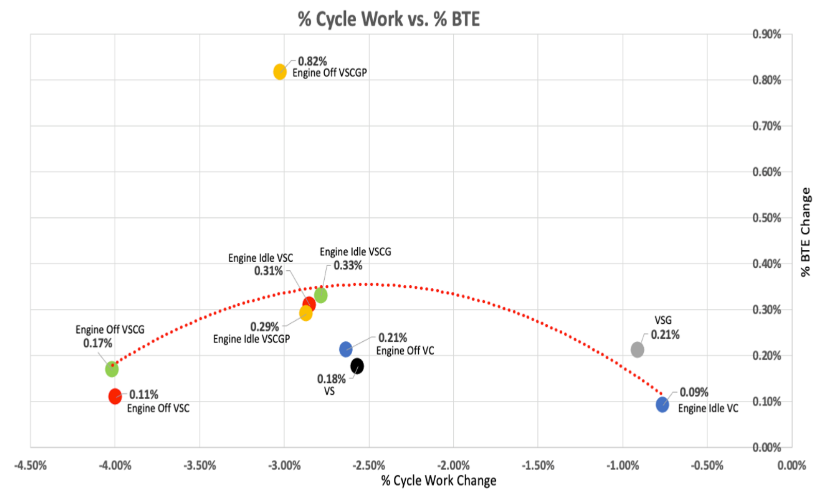


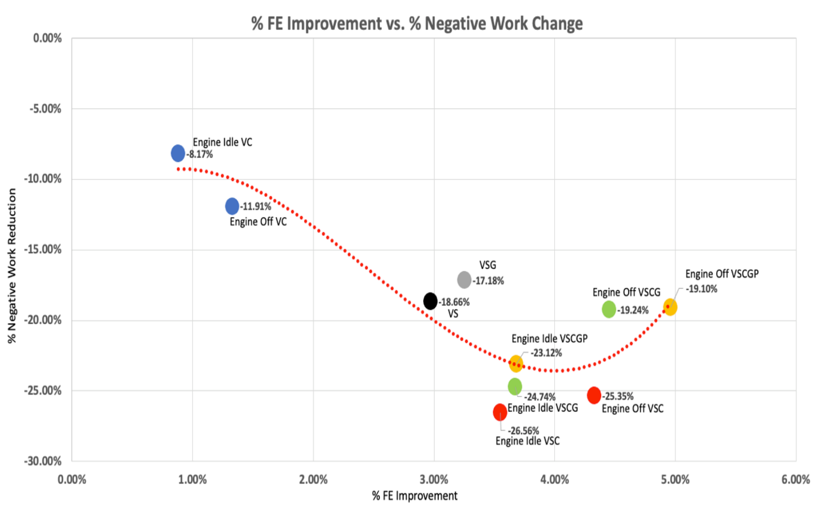
Figure 5.22. Subplot 1 is the histogram of the vehicle speed between the baseline control and the optimal control. Baseline control is less spread as it is referencing a fixed cruise speed target whereas the optimal speed has more standard deviation due to the dynamic cruise speed modulation. Subplot 2 is the Engine Operating Points in terms of fuel consumed which are shown as red bubbles. A big red bubble at the lower left corner indicates the fuel spent at coasting or sailing when the engine was not producing any positive power and is either idling or turned off



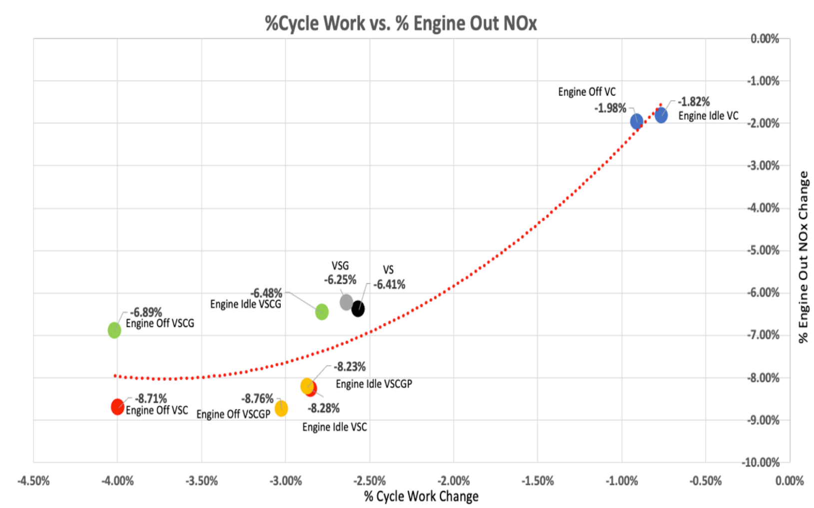
(a) % Change in aerodynamic work as a function of % fuel economy improvement for various optimal problem setups.



(b) % Change in Brake Thermal Efficiency as a function of % change in Cycle work for different combination of optimal problems.



(c) % Reduction in negative work as a of function of % fuel economy improvement for various optimal problem setups.



(d) % Reduction in cycle work as a function of % improvement in fuel economy for various optimal problem setups.

Figure 5.23. Energy metrics analysis for full predictive control problem.

co-relation between the the two in terms of the different problems. This is because the trip time is balanced with baseline trip time. Hence the overall increase in speed and decrease in speed tends to balance each other. The vehicle speed and coast only problems show typically more aerodynamic work reduction. This is due to the fact that they are only slowing down the vehicles whenever possible by going to coasting along with speed modulation. There is no precise rule which can be extracted by these behaviors. Figure 5.23c shows similar trends as compared to the cycle work reduction where we see limited reduction with the coast alone problems due to no speed modulation. Since negative work is due to the speed band operating at regions beyond the engine braking limits, with the coast alone problem vehicle speed is not intentionally modulated to a higher or lower value at the expense of the fuel hence the reduction is less as compared to the baseline results. In this case the speed modulation typically follows the baseline numbers. The other problems have a lesser spread with the engine idle problem as compared to the engine off problem. It is noted that there is a linear trend in fuel economy and negative work reduction for al problems except the problems with the addition of the gear modulation. Figure 5.23d shows the reduction in total cycle work of the engine as a result of the predictive knowledge of the road grade. The bubbles shows the reduction in Engine Out NOx as a function of the reduction in % Cycle Work by the engine. Though it can also be seen that the reduction is more in case of Engine off case which is due to the fact that the engine idle work is taken away in this case. In case of the Engine idle scenario the reduction for all the problems are around -2.75% while the problem with engine idle coast only is around -0.76% while with the Engine off scenario the problems with vehicle speed along with coast, gear and power split provides added reduction as compared to vehicle speed alone problem only. This clearly demonstrate the fact that the problem with engine idle and engine off case are completely different in behavior and cannot be determined by interpreting zero fuel consumption by engine idle problem during the idle sections. This is an important observation. Similar trends are also observed with the negative work reduction for both the engine off and engine idle coast cases. Negative work in this case is comprised of engine braking work and motoring work. Similarly Figure 5.23b shows the variation of Cycle work reduction to Brake Thermal Efficiency improvement. There is no strong co-relation between any problem and the general behavior. Extraction of rules based off only BTE behavior is not very feasible.

Another analysis done while working on the single truck optimal problem is to run the same optimal formulation for a shorter section of the route to validate the optimal behavior relation to look ahead distance requirement. The complete route is divided into two sections of 40 mile each, one for the first half and the next for the second half. Table 5.7 shows the % Fuel Economy numbers for the two sections of the route. The results from Table 5.7

Table 5.7. Predictive fuel economy numbers for different section of the route

Route Section	FE	Trip Time	Full Route FE	Full Route TT	Coast Events
1^{st} 40 miles	2.41	-0.05	5.00	0.04	Decreased
2^{nd} 40 miles	2.39	0.02			Increased
Hilly 10 mile	0.053	-0.86			None
Flat 10 mile	1.03	0.27			Regular

shows that the over all behavior and fuel economy numbers stays near similar if we shorten the route to half. Since the route is not exactly symmetrical the numbers are not equally divided. The coast events also reduced a little for the first half of the route and increased marginally for the second section. This is solely due to the fact that the grade profile is not similar. It is also noted that the optimal control shows similar physical behavior during the very short hilly section where there was no coast events observed and the vehicle speed modulation was also not effective. The predictive gear played a role by reducing the lug back. It is noted that the Fuel Economy is not at all achieved in this section. While in the flat section there is usual behavior of coast events and the problem was able to achieve around 1% benefit. There is also more slow down of the vehicle due to the fact that there was coast events which slowed the speed down. Overall if these results are compared with the full route solution it is not observed that the benefits are hugely sacrificed. Specifically for the 40 miles route it is noted that the benefits are almost equally divided between the two segments and adds up to get close to the full route benefits.

5.5 Conclusion

In this Section a true global predictive optimal problem is designed for a class 8 mild hybrid truck involving 4 interactive controls. This type of problem is never solved in energy domain to the best of the author's knowledge. The interaction analysis including the 4 independent controls for a class 8 truck is not studied well in literature. There is no literature available prescribing what shall be the different control levers that could be applied to a class 8 truck as a function of look ahead road grade. This analysis is crucial in understanding the optimality in interacting controls and will help design fuel efficient algorithms for class 8 mild hybrid trucks. This analysis is also precursor to predictive platooning systems. The usage of this formulation in platooning system is discussed in the next chapter. Based on the analysis done in this chapter the following algorithm suggestions are made which are novel to the best of the author's knowledge in a class 8 mild hybrid truck application,

- Predictive road grade knowledge can help design control algorithms that will enable fuel savings depending on road grade profile,
- Vehicle cruise speed shall be increased within acceptable bounds (calibrated for drivability) before entering an uphill,
- Vehicle cruise speed shall be reduced within calibratible bounds before entering a downhill,
- Down shift gear to a lower value predictively before hitting speed lug back in up-hill,
- Up Shift gear predictively while still on uphill and before completely coming out of the hill,
- Engine shall be disengaged and turned off in mild down grade,
- Engine shall be disengaged for short duration during flat section of route with predictive speed modulation (increase speed then disengage),

6. MULTI-VEHICLE REAL TIME CONTROL

6.1 Online Mode - Multi-Agent Control

This step is the second phase of the problem where the offline optimal control is fed to the 3 trucks in platoon. The objective of the 3 trucks is to follow the optimal control as much as possible to come to a consensus based on the objective cost and the non-linear constraints. Each vehicle in the platoon is cognizant about the optimal control trajectory as obtained by the offline optimizer. This optimal trajectory is same for all the trucks in platoon since they are configured to have the same architecture. The task of the online multi-agent based optimizer is to use the global offline optimal trajectory and optimize the 3 trucks in the platoon following a different constrained optimization algorithm for a short look ahead horizon. The decision of using a short horizon is based on the analysis and understanding of the global optimal behavior of a single truck optimized over the entire route. As noted with the single vehicle optimization, the major benefits associated with the predictive information is tied to local optimal points and is not related to the entire route. We understood that the major components in achieving the fuel economy are the predictive cruise control and predictive coasting (both engine idle and engine off). We also found that the predictive gear shift is more of a performance criteria and does not provide any fuel economy for this type of powertrain and engine BTE maps. Similarly the predictive power split management also does not provide any fuel economy and it is not surprising since the hybrid system is a mild 48V system with limited capacity. It is never designed to propel the truck on electric power alone. It is worth noting here that when we say the predictive hybrid system is not providing any fuel economy we are referring to the fuel economy on top of the rule based SOC management strategy which also has the hybrid system. So essentially the predictive power split control is not bringing in any benefit as compared to the baseline.

It is also understood from the offline results that the region of operation for the predictive cruise control and coasting are mutually exclusive and does not eat away each others benefit. While the predictive cruise control is more prevalent during the pre-hill sections, coasting is more effective during long flat sections of the route. This is a good key learning with respect to implementation details as it will help design rule based controllers which can work much more efficiently with out having to interact with each other often. Figure 6.1 shows the high level architecture of the platoon system. The offline problem is either solved in cloud or in a high performance computer. This step is super challenging in terms of computational power. This type of predictive global optimal control can never be run in a real time controller. Hence it is always desirable to run these simulations in offline and extend the learning to real time controllers. Full route grade information along with speed

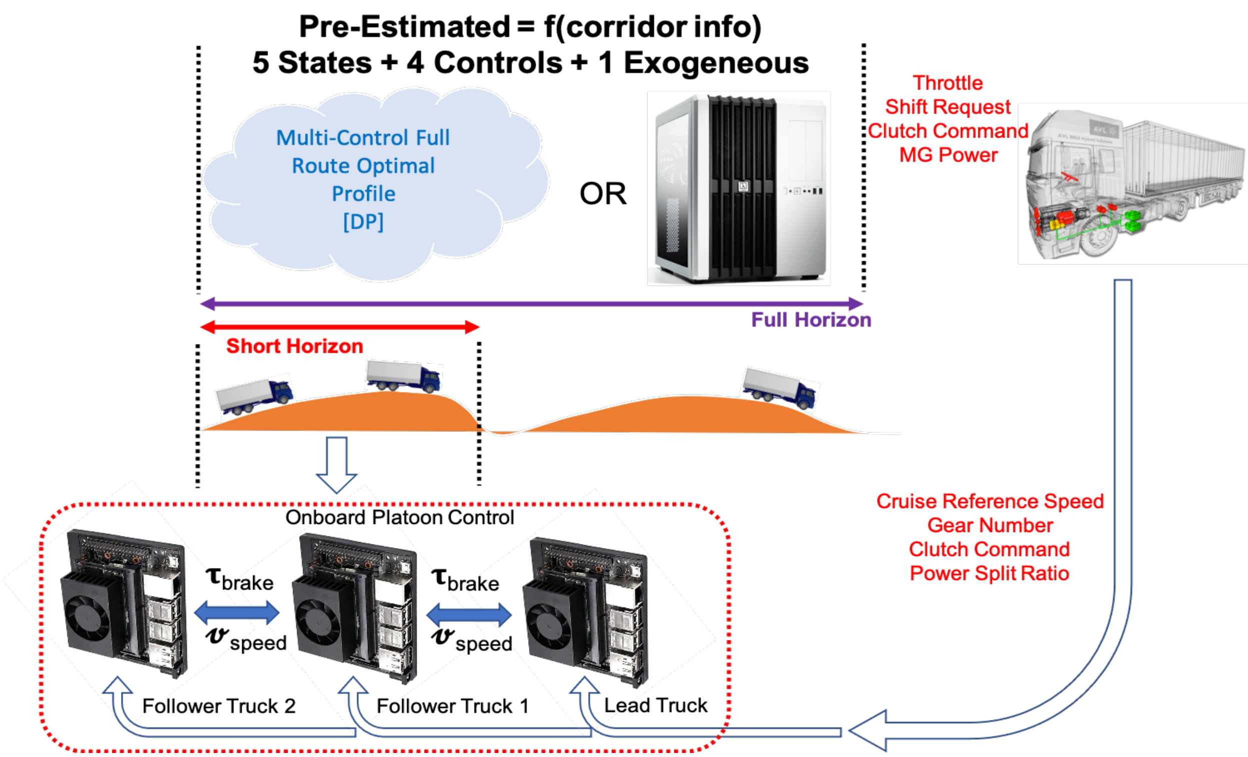


Figure 6.1. High level overview of the full control formulation and hierarchy of the process. The full horizon is used to conclude the optimality for the single vehicle. A short horizon is used to achieve cooperative consensus among the platooning trucks.

limit is used in this problem as predictive information. The result obtained is the global behavior of the control levers. These optimal levers are provided to the 3 trucks in platoon and each one run its own iterative process to minimize the modified cost function with the non-linear constraints applied. This problem is not easy to solve as it involves Mixed Integer Non-linear variables. As an example the gear number and the clutch state are integer type state variables. One way to solve this type of problem is to relax the integer type state and then solve for the gradient following which the state variables are restored from the relaxed state. One way to form the problem is to use the Hamiltonian of the cost and constraints and then solve the problem using Pontryagin's minimum principle. A general quick note on the Hamilton-Jacobi-Bellman PDEs and Pontryagin's Minimum Principle,

- Often, we only care about the optimal trajectory for a specific initial condition x_0. Exploiting that we need less information, we can arrive at simpler conditions for optimality – Pontryagin's Minimum Principle

- The PMP **does not apply to infinite horizon problems**, so one has to use the HJB equations in that case

- The HJB PDE is a sufficient condition for optimality (it is possible that the optimal solution does not satisfy it but a solution that satisfies it is guaranteed to be optimal)

- The PMP is a necessary condition for optimality (it is possible that non-optimal trajectories satisfy it) so further analysis is necessary to determine if the candidate PMP policy is optimal

- The PMP requires solving an ODE with split boundary conditions (not easy but easier than the nonlinear HJB PDE!)

The Hamiltonian can be formed as,

$$
\begin{aligned}
H(v_s, \eta_n, C_s, B_soc, \lambda_1, \lambda_2, \lambda_3, \lambda_4, s) = & \sum_{n=1}^{N} \int_0^s \left\{ \frac{W_f \alpha \dot{m}_{f_n}}{v_{s_n}} + \frac{W_t(1-\alpha) v_{t_n}}{v_{s_n}} + \tau_{brake_n} \right\} + \\
& \sum_{n=1}^{N} \lambda_{1_n} \frac{\partial v_{s_n}}{\partial s_n} + \sum_{n=1}^{N} \lambda_{2_n} \frac{\partial \eta_n}{\partial s_n} + \\
& \sum_{n=1}^{N} \lambda_{3_n} \frac{\partial C_{s_n}}{\partial s_n} + \sum_{n=1}^{N} \lambda_{4_n} \frac{\partial B_{soc_n}}{\partial s_n}
\end{aligned} \tag{6.1}
$$

where, v_{s_n} is the vehicle speed for each truck, η_n is the gear number, C_{s_n} is the clutch state and B_{soc_n} is the battery state of charge.

This problem is not easy to solve analytically and definitely not possible to solve real time even for a very short horizon. Hence we do not explore this option beyond. Details on

Pontryagin's Minimum Principle is highlighted in Appendix section including some basic proofs.

Another implementable but challenging solution is to use Model Predictive Control approach. We have tried to solve the problem using model predictive control. Liu et al., used a similar offline simulation and then used the optimal cruise control speed target in a 2 truck platoon [71]. They used a moving window based simple model predictive control approach to solve the objective cost as shown in Equation 6.2

$$\begin{aligned} \min_{u} J = \int_{t}^{t+\Delta t} & (w_{\dot{m}_f} * \dot{m}_f(x,u,w).w_v(v(t) - v_{ref}(t))^2) \\ & + w_{\delta T_{eng}} \delta T_{eng}(t) + w_{\delta T_{brake}}.\delta_{T_{brake}})dt \end{aligned} \tag{6.2}$$

This cost function is discretized over the prediction horizon Δt and is minimized subject to the vehicle and powertrain dynamics and constraints. In the cost function, v_{ref} is the optimal speed profile computed by offline algorithms, δT_{eng} and δT_{brake} are the change in engine and brake torque inputs, and $w_{\dot{m}_f}$, w_v, $w_{\delta T_{eng}}$ and $w_{\delta T_{brake}}$ are the weight factors to tune the MPC performance. This problem is solved in real-time using a nonlinear programming optimizer based on interior-point methods as in [72] and is applied in real-time in the framework of MPC. The application of this controller in car following operation mode is a fuel efficient Adaptive Cruise Controller (ACC) in which MPC controller has the flexibility to optimize the distance to the front vehicle within its limits with respect to the integrated vehicle and powertrain dynamics, constraints and the fuel consumption cost function. While this is a very elegant technique but it has its own challenge of implementing specifically for multi-objective problems with 5 states 4 controls. MPC is evaluated theoretically and not pursued due to the challenges associated with large number of states and controls.

Another approach which is tried in this work as a comparison is the pseudo-spectral collocation method using the SNOPT (*Sparse Nonlinear OPTimizer*, is a software package for solving large-scale nonlinear optimization problems written by Philip Gill, Walter Murray and Michael Saunders[73]) library and matlab to solve the non-linear mixed integer problem by relaxing the integer type gear state variable and the clutch state. This is a very interesting solver used to find global optimality in trajectory optimization. though it is nice way to solve a complex mixed integer non-linear problem but the implementation of such an algorithm was again very challenging in terms of formulating the problem for a large number of states and controls, specially for mixed integer type.

Both the above methods showed average benefits in fuel economy close to 10% and 8.4% respectively. While this numbers are not analyzed in details in this work but they provide a relevant ball park estimate of what is achievable. Based on the analysis of the different available methods a novel method is used in this work. A modified version of Metropolis Algorithm is used which is very common in multiple flying robot formation controls. Having discussed this background we shift to the multi-agent based approach and analyze the results we obtain.

6.1.1 Distributed averaging based consensus

Multi-agent systems (MASs) have gained wide attention in recent years due to its multi-faceted practical applications, especially in wireless sensor networks, formation control in robots, transportation network optimization, vehicle ecosystem development, etc. One of the most important and intensively investigated issues in MASs (and their applications) is the consensus problem due to its attraction in both theoretical and applied aspects (Olfati-Saber and Murray, [74]; Olfati-Saber et al.,[75]; Ren[76]). Recently there is a lot of traction in more advanced problems of constrained consensus analysis and design using MAS's. In networks of agents (or dynamic systems), "consensus" means to reach an agreement regarding a certain quantity of interest that depends on the state of all agents. A "consensus algorithm" (or protocol) is an interaction rule that specifies the information exchange between an agent and all of its neighbors on the network. Distributed computation over networks has a tradition in systems and control theory. [77],[78] also discusses the distributed computing structure using multi agent modes. The distributed averaging problem is a consensus problem whose objective is to devise a protocol which will enable all the members of a group of autonomous agents to compute the average of the initial values of their individual consensus variables in a distributed manner. Gossip algorithms can provide information exchange and computation for autonomous vehicles in a group, where each vehicle must make estimates and decisions, while ensuring consensus at the group level. Periodic gossiping is a deterministic method for solving the distributed averaging problem by stipulating that each pair of agents which are allowed to gossip, do so repeatedly in accordance with a pre-specified periodic schedule. Agent pairs which are allowed to gossip correspond to edges on a given connected, unidirected graph. In general, the rate at which the agents' consensus variables converge to the desired average value depends on the order in which the gossips occur over a period. A more detailed description of distributed averaging is provided in the appendix section.

6.1.2 Problem Formulation

The objective for the online controller running distributed mode calculations is formulated in Equation 6.3

$$u_{1:N}^{*}(s) = \underset{u_{1:N}\in R}{\operatorname{argmin}} \sum_{n=1}^{N} \int_{0}^{s} \left\{ \frac{W_f \alpha \dot{m}_{f_n}}{v_{s_n}} + \frac{W_t(1-\alpha) v_{t_n}}{v_{s_n}} + \tau_{brake_n} \right\} ds \tag{6.3}$$

subject to,

$$\dot{x}(s) = f(x(s), u(s), w(s)) \tag{6.4}$$

$$y(s) = g(x(s), u(s), w(s)) \tag{6.5}$$

$$\dot{d}_n(s) = v_n(s) - v_{n-1}(s) \tag{6.6}$$

and, non-linear constraints

$$v_{min} \leq v(s) \leq v_{max} \tag{6.7}$$

$$\tau_{brk,min} \leq \tau_{brk}(s) \leq \tau_{brk,max} \tag{6.8}$$

$$d_{min} \leq d(s) \leq d_{max} \tag{6.9}$$

The cost objective is a summation of fuel consumed and total trip time. Trip time is added to compensate for the excessive slow down of the vehicle in order to save fuel. The other component in the cost function is the braking work. The vehicle will try to brake in order to maintain the safe distance between the trucks. The addition of braking work will make sure that the trucks are not utilizing excessive braking.

The problem is solved by considering each truck as an agent with the other trucks being it's neighbor. Hence the lead truck and the last truck in the platoon has 1 neighbor each while the middle truck has two neighbors. Hence the middle truck has two edges. Only the vehicle speed state in this case is update using a generalized metropolis algorithm and the other control levers are applied as its from the offline optimal results as it is. If the optimal control violates the constraints then the constraints gets the priority and the truck comes out of the optimal profile. As, an example if the truck cannot be in coast mode in the platoon due to a constraint violation then it comes of coast mode and runs normal operation. In general a consensus process recursively evolves with respect to a discrete time scale. In general for a consensus algorithm, agent i sets the value of its own agreement variable at time $t+1$ based on the average of its current value and the neighbor's value,

$$x_{\mathrm{i}}(t+1) = \frac{1}{(1+d_{\mathrm{i}})}(x_{\mathrm{i}}(t) + \sum_{\mathrm{j}\in\mathcal{N}_{\mathrm{i}}} x_{\mathrm{j}}(t)) \tag{6.10}$$

where, N_{i} is the set of indices of agents of $\mathrm{i}'s$ neighbors and d_{i} is the number of indices in N_{i}. Boyd et al, [69][70][79] provided a better algorithm called **Metropolis Algorithm** where each agent needs to know the number of neighbors of each of its neighbors, The state update is given as,

$$x_{\mathrm{i}}(t+1) = (1 - \sum_{\mathrm{j}\in\mathcal{N}_{\mathrm{i}}} \frac{1}{(1+\max(d_{\mathrm{i}}, d_{\mathrm{j}}))})x_{\mathrm{i}}(t) + \sum_{\mathrm{j}\in\mathcal{N}_{\mathrm{i}}} \frac{1}{(1+\max(d_{\mathrm{i}}, d_{\mathrm{j}}))}x_{\mathrm{j}}(t) \tag{6.11}$$

We used this algorithm with the vehicle as the state variable which is updated at each time step for the short horizon in real time following the minimum objective cost and constraints. d for the lead and the last truck is 1 and the middle truck is 2 depending on number of edges. The other control levers are not updated based on this algorithm as it will make the problem challenging and it is not expected to get much benefit by doing so.

The sequence of code for this section is explained in Algorithm 2 The optimal speed trajectory

Result: Platoon Total Fuel Benefit in % mpg & Inter-vehicular Separation
Initialize Vehicle Parameters;
Initialize Inter-vehicular Separation;
while *Vehicle Position is NOT end of Route* **do**
    Set Cruise Reference Speed bucket;
    Set Braking Bucket;
    Calculate Fuel Consumption for the Platoon;
    $\dot{m}_f(t) = f(v(t))$;
    Calculate Inter-vehicular Distance;
    $s(t) = f(v_1(t), v_2(t), v_3(t))$;
    Update State parameter;
    if *Fuel Consumption is Minimum & Inter-vehicular Separation within Limit* **then**
        Select $v(t+1) = v^*(t)$;
    else
        Run next set of speed reference;
    end
end

Algorithm 2: Online Local Optimality - Multiple Vehicle

for the single vehicle in this route is captured from global offline optimal solution and is fed as the cruise target speed for the individual vehicles in the platoon. The job of the online multi-agent controller [77][78] is to coordinate with each vehicle in the platoon to follow the set reference speed and maintain a safe inter-vehicular separation using the proposed Metropolis Algorithm [70][79]. This control is needed because simply feeding the speed target will make the trailing vehicles run faster than the lead vehicles and collide with each other since the trailing vehicles will have less aerodynamic drag and will speed up more. Figure 6.2 shows the relative change in drag coefficient as a function of vehicle separation [80][81]

The reduction in Aerodynamic drag coefficient is given in Equation 6.12 which is the fraction by which the aerodynamic drag coefficient will change based on the separation distance. The constants $C_{D,1}$ and $C_{D,2}$ are adjusted based on polynomial fit from open literature data.

$$\Phi(d_{\mathrm{i}}) = (1 - \frac{C_{D,1}}{C_{D,2} + d_{\mathrm{i}}}) \tag{6.12}$$

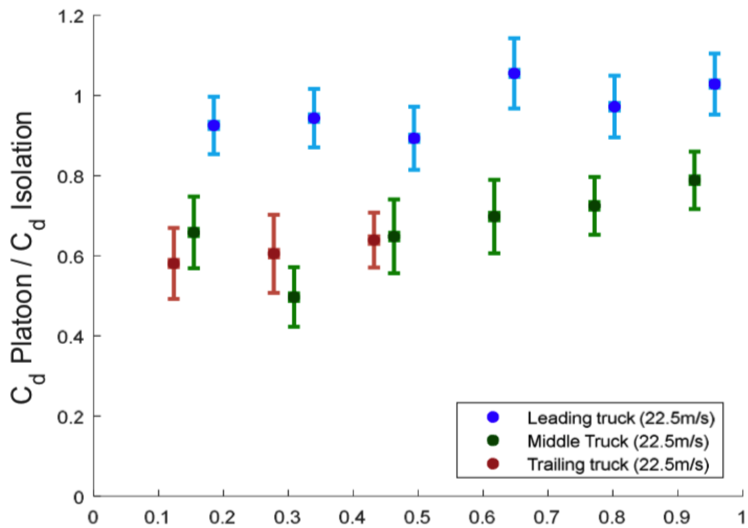


Figure 6.2. Drag Coefficient as a function of inter vehicular separation

It is worth noting here that the multi-agent controller will run discrete control optimizer in each truck knowing the grade information and complete optimal optimal profile for each truck in the platoon. Each individual agent will try to solve the cost for its own which there by in conjunction with the global optimal input target will achieve best fuel economy for the entire fleet. Additional control levers in this case is the braking effort and the inter vehicular dynamics are also included.

6.1.3 Optimal Behavior Analysis

The problem as described above is solved for the 3 trucks in platoon. The separation distance between the trucks are dynamically modified with the intention of spending the least amount of energy as well as maintaining the separation distance. The braking effort is part of the objective function to make sure that the system will not have to brake too often to loose kinetic energy which is a loss at the expense of the fuel energy. In this section the benefits are analyzed and studied. The first Figure 6.3 is the radar plot of the adjusted Fuel economy of the 3 trucks. There are 2 sets of data in the plot. One is for the coast events when engine is idle and the other for the case with engine off. On an average the 3 truck platoon achieved 9.42% better fuel economy over baseline simulation results. This result is

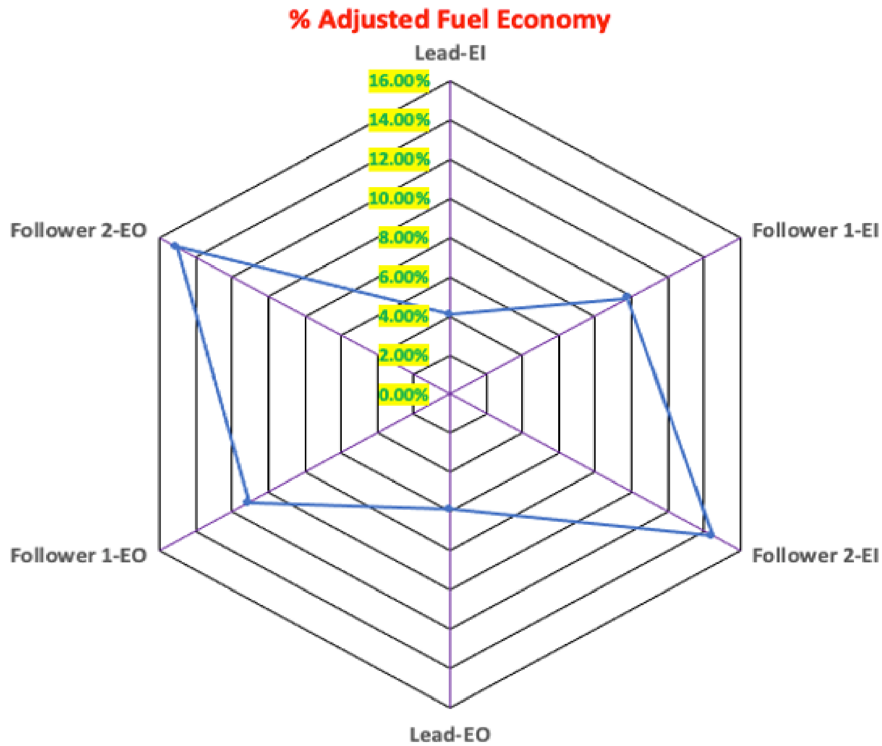
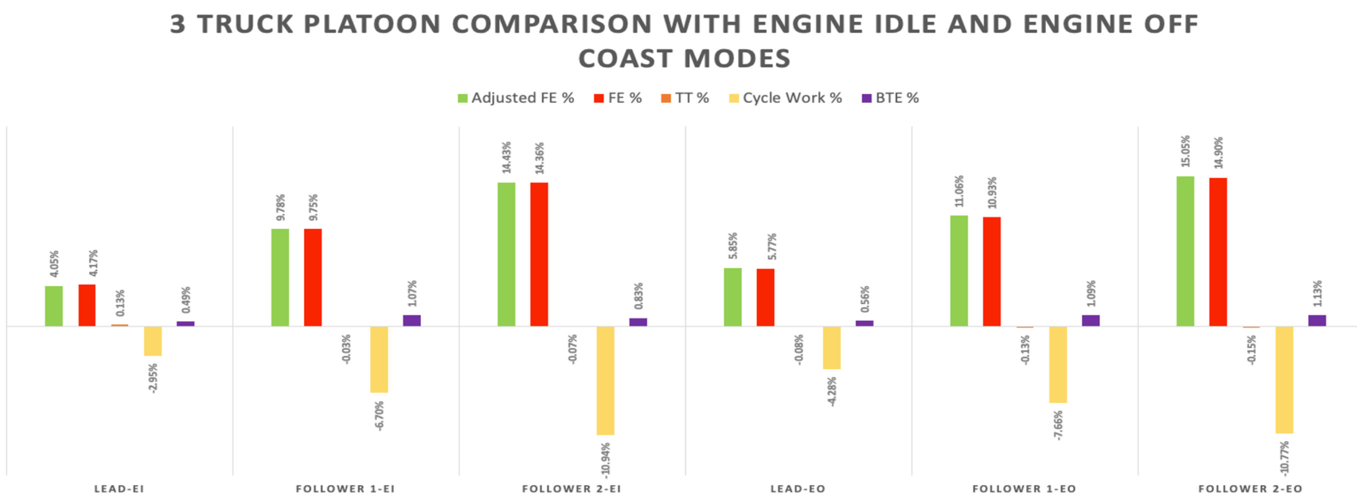
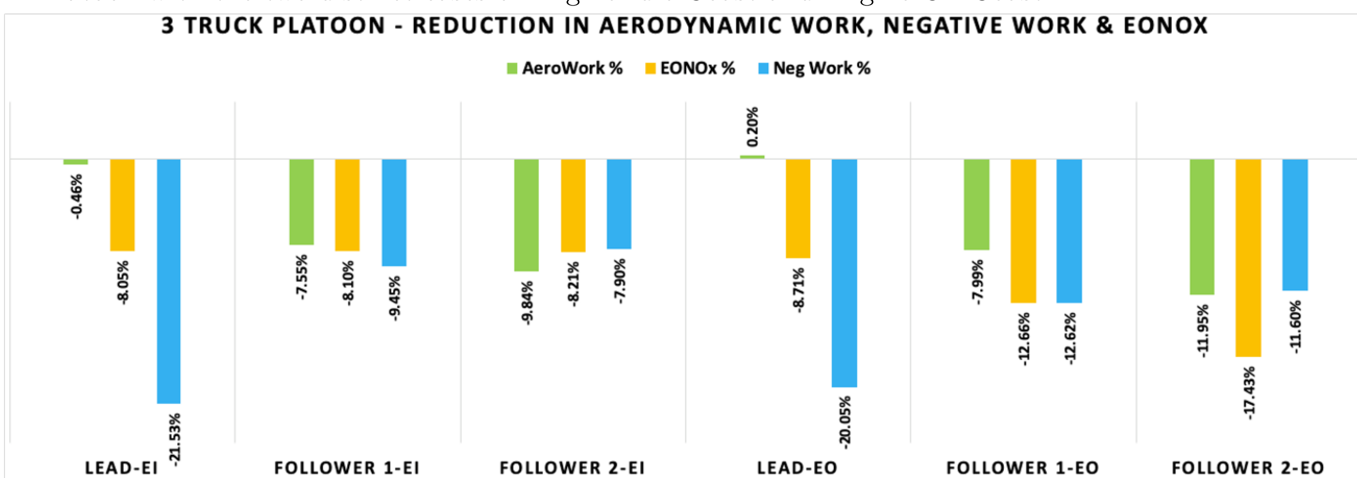


Figure 6.3. % Fuel Economy radar for the 3 platooning trucks - Lead, Follower 1 and Follower 2. The fuel economy radar shows the numbers for both engine coast condition as well as engine off coast conditions

for the engine idle coast event. Similarly for the engine off coast scenario the average went up to 10.65%. The trend in improvement is similar though for both the scenario. The engine off scenario made the lead truck do more better in terms of fuel economy. Figure 6.4a shows the key metrics related to fuel consumption and the associated parameters affecting it. It is observed that an average of 9.5% fuel benefit is achieved in the engine idle scenario for the platoon. The engine off case shows an average of 10.7% for the platoon. The plot show 6 sets of bar plots. Each set comprises of 5 key metrics (Green - Adjusted Fuel Economy % Change, Red - Absolute Fuel Economy % Change, Orange - Absolute Trip Time % Change, Yellow - Engine Cycle Work % Reduction, Purple - BTE % Change). The 3 sets of bar plots are for engine idle case while the last 3 sets are for the engine off case. The lead vehicles in both the cases shows almost similar behavior to the single vehicle optimality. The associated benefit is a result of cycle work reduction and aerodynamic work reduction. There is almost



(a) Key metrics showing the comparison of benefits along with Cycle work and BTE for the 3 class 8 truck in Platoon with the two distinct cases of Engine Idle Coast and Engine Off Coast



(b) Reduction in Aerodynamic Work along with associated EONOx Reduction. The last bar plot shows the reduction in Negative Work which includes Engine braking, Motoring Losses and Service Braking

Figure 6.4. Key metrics for the 3 truck predictive platoon system

near similar improvement in brake thermal efficiency in all the cases. Figure 6.4b shows the comparison of different negative work reduction metrics. The bar plots in green shows the reduction in aerodynamic drag work. The reduction of the lead trucks in both the Engine Idle as well as the Engine Off case is negligible compared to baseline results. This is considering the fact that the lead trucks follow the optimal speed profile almost perfectly. The follower trucks shows more reduction due to the reduction in aerodynamic drag in the following trucks. The reduction is more in the second following truck than the first as expected due to more reduction in aerodynamic drag in the second follower than the first. Engine off case shows a bit more reduction in aerodynamic work loss. Blue bars show the reduction in negative work which includes motoring loss, engine braking along with service braking. The follower trucks in both engine idle as well engine off scenario shows less reduction due to the application of more service brakes in order to maintain safe operable distance between the trucks. Engine idle scenario shows less reduction in negative work than the engine off case. Figure 6.5 shows the detailed time series plots of the 3 trucks in platoon as a function of vehicle position in x-axis. The trucks show dynamically varying separation distance with the trailing truck almost going 120m during heavy hills. This can pose challenge with cut-ins. This was because of a coast event before a hill. This large separation distance also reduce the benefits associated with aerodynamic drag reduction. This is an anomaly observed in the solution space. This can be better tuned by making the separation constraint more stringent. The battery SOC is pretty much dependent on the reactive grade profile. It is also observed that the wheel braking increased a lot. This is also shown in Figure 6.4b. The blue bar plots in this figure shows significant less reduction in the negative work which is due to the fact that wheel braking has increased. Figure 6.6 shows the data for the coast events and % time in coast for the system of trucks in platoon. The plot depicts 2 sets of data one with the engine idle coast scenario and the other for the engine off coast. It is noted that for the following trucks the total number as well as the total time in coast is significantly lower than the lead truck. This behavior is similar to both the engine idle and engine off coast case. This is analytically because of the speed modulation in the follower trucks which made the trucks go out of coast in most of the cases or not get into coast at all. Figure 6.7 shows the key signals for total fuel consumed and the Engine Out NOx values. It shows the progressive

Figure 6.5. Subplot 1 is the Vehicle Speed Trajectory of two trucks in platoon. Subplot 2 is the following distance of the second truck in the platoon. Subplot 3 is the engine out NOx for the lead as well as the follower truck which shows no improvement in NOx reduction by the follower truck.

trend in decreasing fuel consumption and engine out NOx numbers. Figure 6.8a and Figure 6.8b shows the relation of % change in fuel economy as function of change in Brake Thermal Efficiency. In both the case it is observed that the BTE improves progressively with increase in Fuel Economy. In the Engine Idle case the BTE change reduced a bit for the last truck in the platoon but still it shows better fuel economy. The benefits associated here is more contributed by the reduction in aerodynamic drag reduction. The BTE did not improve a lot because of more gear shifts with the predictive knowledge as well as to maintain separation distance.

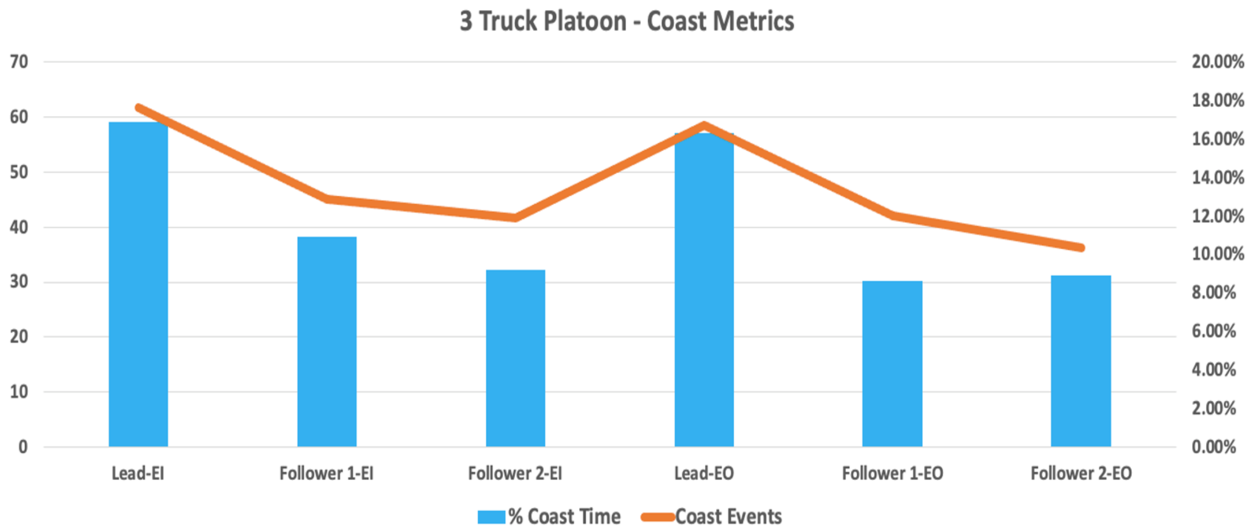


Figure 6.6. Coast metrics for the 3 Trucks in Platoon - The Engine Idle and Engine Off metrics shows clear difference in optimal behavior

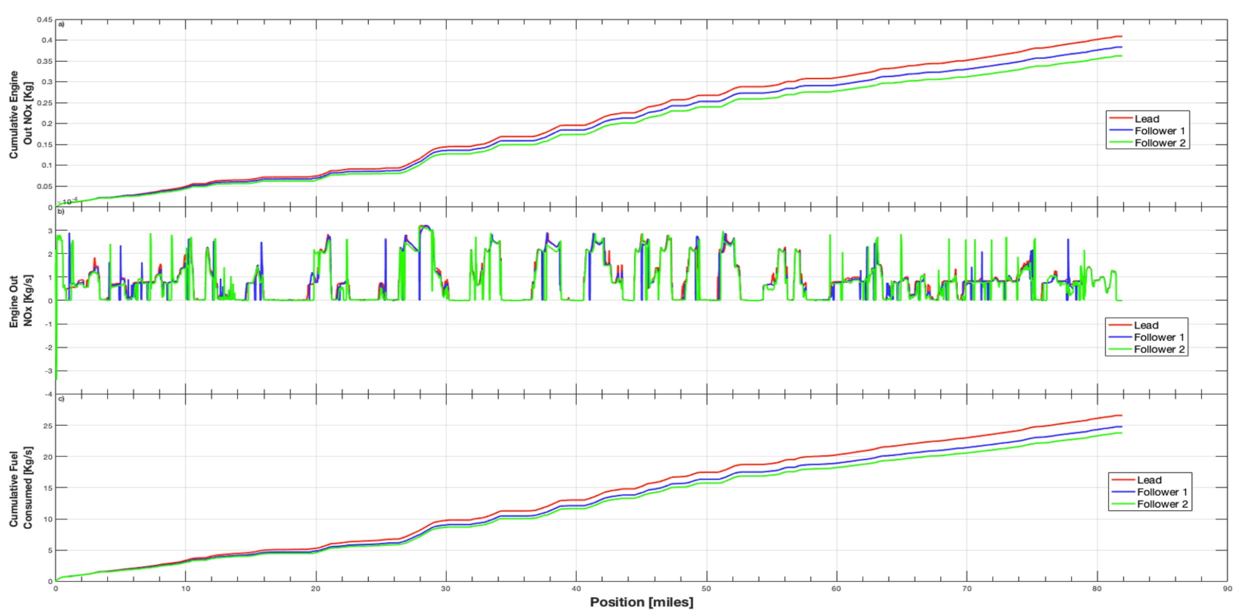
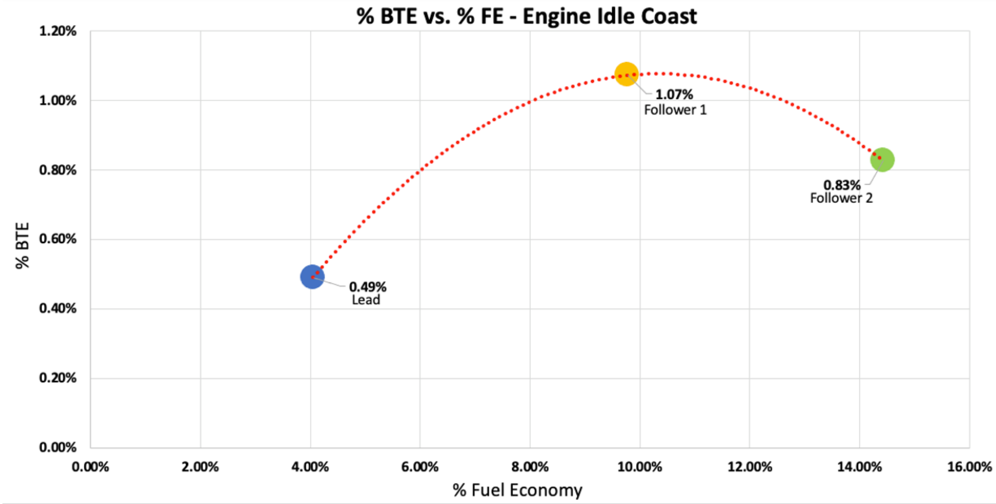


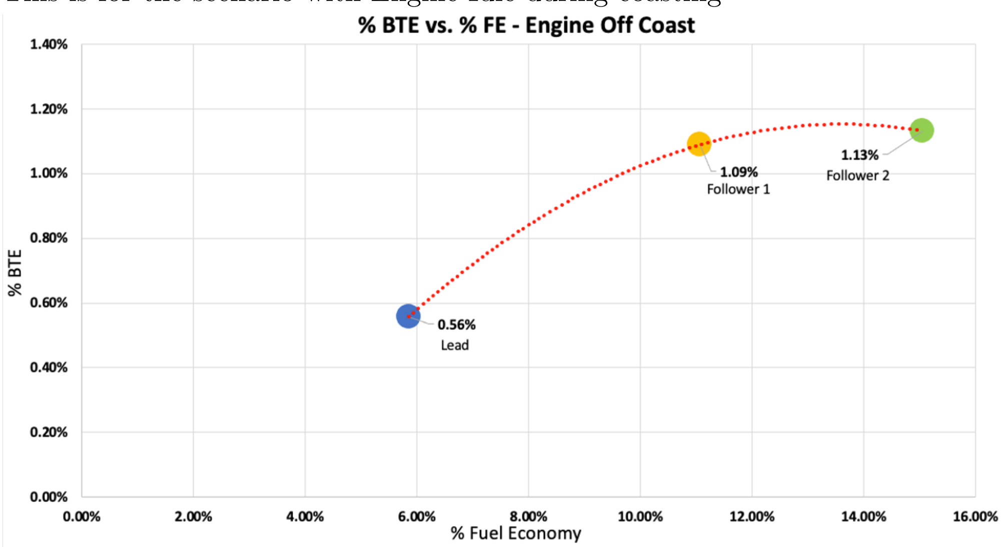
Figure 6.7. Engine Out NOx and Fuel Consumed plots. The first subplot is the cumulative Engine Out NOx and the second one is the instantaneous Engine out NOx value

6.2 Conclusion

In this research the general metropolis algorithm for multi agent based distributed averaging problem is used to study an implementable algorithm for the 3 truck platooning



(a) Brake Thermal Efficiency improvement as function of % Fuel Economy. This is for the scenario with Engine Idle during coasting



(b) Brake Thermal Efficiency improvement as function of % Fuel Economy. This is for the scenario with Engine Off during coasting

Figure 6.8. Brake Thermal Efficiency as compared to Fuel Economy for Engine Off and Engine Idle Coast scenarios.

system with predictive optimal control. The intent of the work is to understand the global optimality and the general behavior more than trying to design a control algorithm. In most of the real time controller rule based algorithms are suitable so it becomes essential to understand these underlying behaviors which gives true optimal results. It is also compared to a simple model predictive control based algorithm from literature where the predictive cruise control is studied alone. It shows near similar results and behavior. Obviously the results will not match exactly since the it depends on vehicle configuration and architecture. Figure

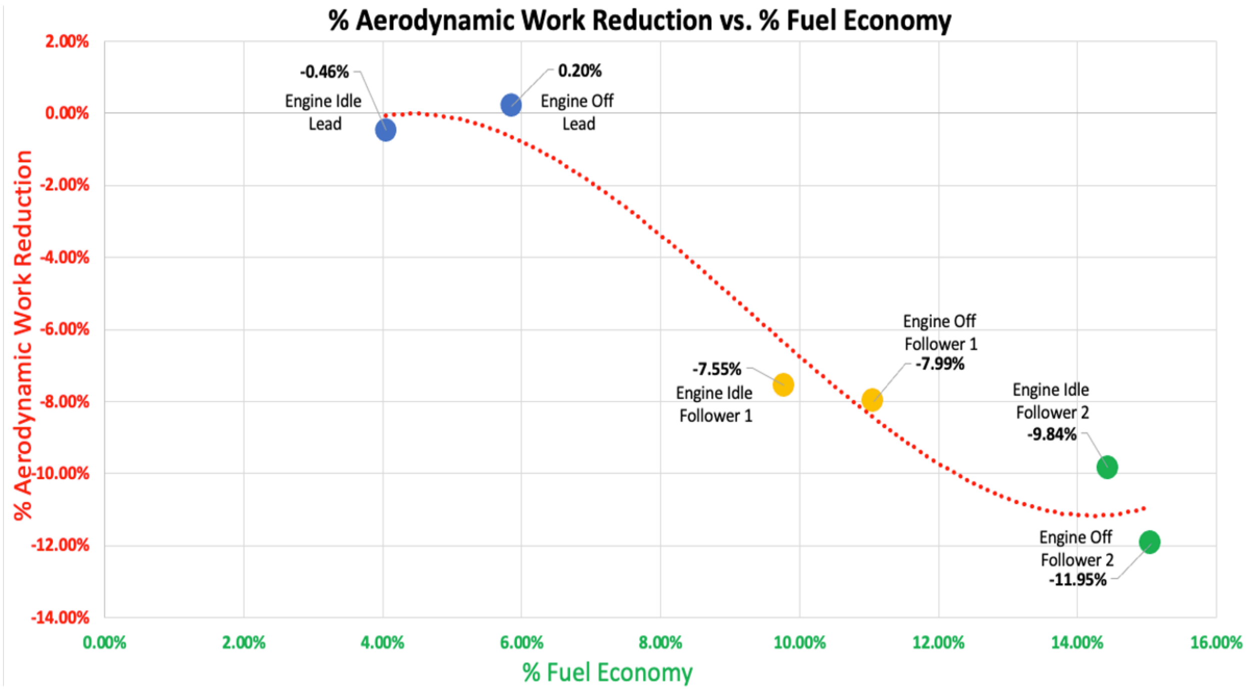
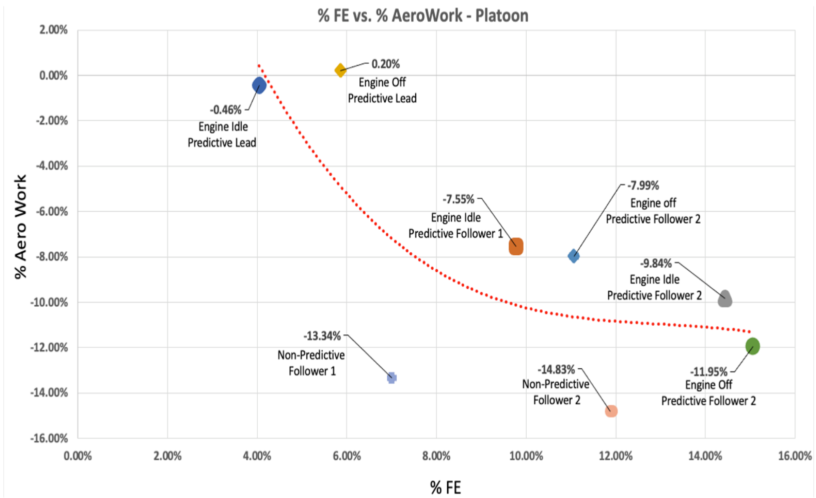


Figure 6.9. % Aerodynamic drag work reduction as function of % Fuel Economy. The % reduction in aerodynamic drag work is calculated based on baseline simulation results

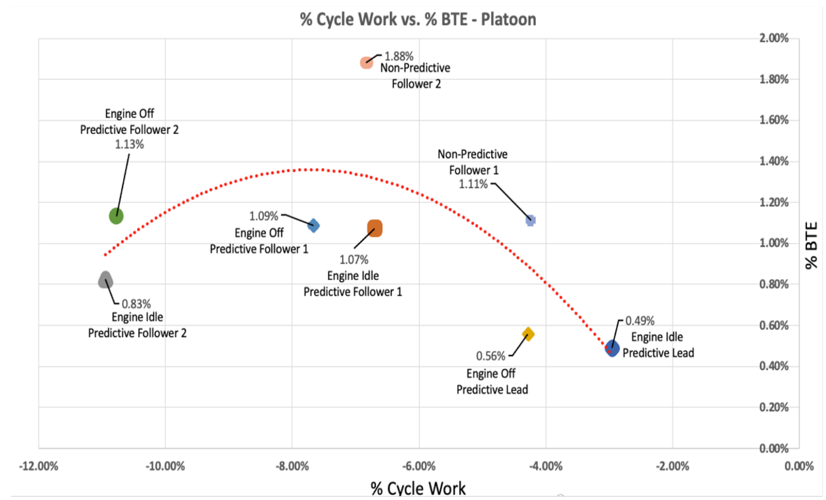
6.9 is another nice metrics to analyze and look at. This indicates the fuel benefits associated with overall aerodynamic drag work reduction. The major fuel benefits are definitely due to the reduction in drag coefficients in the following trucks. The work reduction is definitely affecting fuel economy but there are other contributors as well in the benefit such as negative work reduction, and BTE improvement as seen in Figure 6.8a and Figure 6.8b. These figures shows the relation of BTE to fuel economy.

Figure 6.10a shows the bubbles for % change in Fuel Economy as a function of % change in Aerodynamic work reduction. The bubbles are represented for both engine idle and engine off coast cases. The "Blue" bubbles are for the lead truck. The "orange" bubbles are for the follower 1 and the "green" bubbles are for the follower 2 truck. It is clear that the reduction in aerodynamic drag and fuel economy is proportional to each other and the progressively compliment each other. The "gray" bubble is for the follower 1 truck with no predictive control and the ”yellow” bubble is for the follower 2 truck with no predictive controls. Though the reduction in aerodynamic drag is much more in the non-predictive case because of the trucks following constant separation distance but still the fuel economy benefits are less than the predictive case. This shows that the look ahead knowledge and predictive control can add more fuel efficient behavior. Similarly, Figure 6.10b shows the respective cycle work reduction as function of % improvement in Brake Thermal Efficiency. There is minor BTE improvement in the follower trucks but Cycle work reduction is significant in the follower trucks. This contributes to the fuel economy improvement in the follower trucks. It is also important to note that the follower trucks in case of non-predictive control did not show much improvement in cycle work reduction. Though the BTE is improved for the follower truck 2 a lot. This shows that fuel economy is a combination of multiple factors including aero dynamic drag reduction, BTE improvement, Cycle work reduction as well as reduction in negative work. Figure 6.10c shows the % change in Fuel Economy as a function of % in Negative work which includes wheel braking, Engine Braking and Motoring. The follower trucks in the predictive control shows increase in negative work due to the application of more wheel brakes in order to adjust for dynamic speed modulation and separation distance. The non-predictive controls has to do more wheel braking in order to maintain a fixed separation distance and hence the reduction is much less which also contributes to less fuel economy. Lastly, Figure 6.10d shows the relation of % change in Engine Out NOx as function of % reduction in Cycle Work. The NOx reduction is passive in this case as a result of better engine operation. The NOx reduction is not part of the optimal control formulation. The engine off coast scenario for the follower 2 truck shows the most reduction in % NOx. This is also due to the fact that the aerodynamic drag is the least as well as the cycle work is very low and also there is some improvement in BTE operation. Table6.1 shows the improvement in Fuel Economy when predictive control is used as compared to non-predictive controls. It shows that on an average for the 3 truck platoon there is an overall net fuel economy improvement of 2.94% for the predictive controls with Engine Idle scenario and 3.99% for the Engine off scenario. Table 6.2 captures the detailed metrics of the multi-agent based optimal result for the 3 truck in platoon. The results are from the problem with Engine Idle Coast condition. The detailed metrics show the absolute numbers and how they change with different scenarios for the lead and follower trucks. Table 6.3 captures the detailed metrics of the multi-agent based optimal result for the 3 truck in platoon. The results are from the Engine Off Coast condition. Finally, based on the optimal behavior using predictive look ahead knowledge the platooning system can have the following recommended control actions,

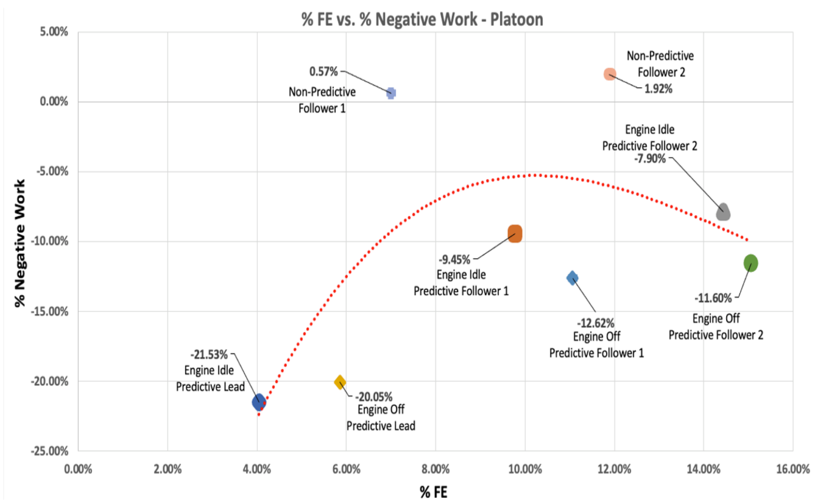
- Adaptive speed modulation can provide fuel benefits in platooning trucks,
- Disengage engine and use idle conditions in flat section of the route,
- Follower trucks shall not need predictive gear shifts,
- Follower trucks shall save electric energy during heavy grade and use it to supplement longer coast events in the flat section,
- Dynamic separation shall be limited to 20 to 120 meters,



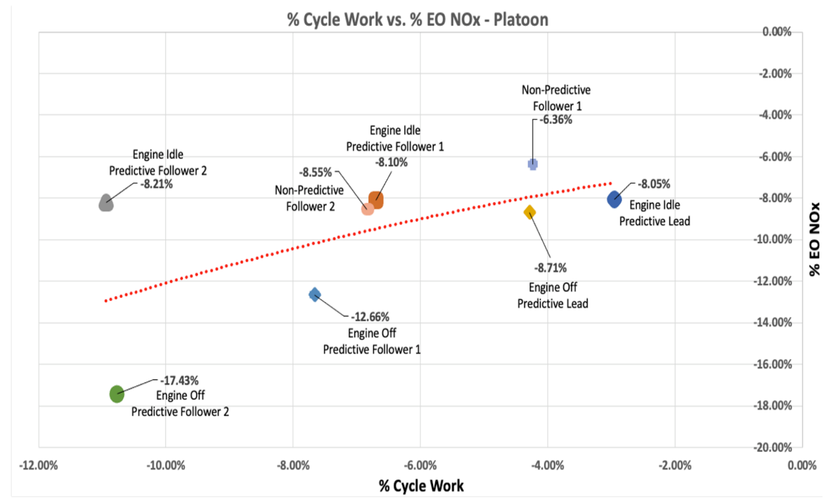
(a) Bubbles showing % FE improvement over % Aerodynamic Drag reduction



(b) Engine Cycle work change as a function of Brake Thermal Efficiency Improvement



(c) % reduction in negative work which includes motoring, engine braking and service braking vs. respective improvement in Fuel Economy.



(d) % reduction in Cycle Work and subsequent reduction in % Engine Out NOx

Figure 6.10. Predictive energy metrics bubble plots for key parameters.

Table 6.1. Comparison of key metrics between predictive look ahead based optimal control vs. non-predictive controls in the 3 truck platoon system

	Engine Idle			Engine Off			Non-Predictive	
Metrics	Lead	Follower 1	Follower 2	Lead	Follower 1	Follower 2	Follower 1	Follower 2
Fuel Economy	4.05%	9.78%	14.43%	5.85%	11.06%	15.05%	7.01%	11.88%
Aerodynamic Work	-0.46%	-7.55%	-9.84%	0.20%	-7.99%	-11.95%	-13.34%	-14.83%
Cycle Work	-2.95%	-6.70%	-10.94	-4.28%	-7.66%	-10.77%	-4.24%	-6.84%
BTE	0.49%	1.07%	0.83%	0.56%	1.09%	1.13%	1.11%	1.88%
Negative Work	-21.53%	-9.45%%	-7.90%	-20.05%	-12.62%	-11.60%	0.57%	1.92%
EONOx	-8.05%	-8.10%	-8.21%	-8.71%	-12.66%	-17.43%	-6.36%	-8.55%

Table 6.2. 3 Truck Platoon Metrics Running Optimal Control. All the vehicles have knowledge of the offline optimal control trajectory. The individual trucks are running consensus agent based algorithm to calculate the final optimal path. The metrics shown are with Engine Idle Coast scenario.

		Lead Truck		**Follower Truck 1**		**Follower Truck 2**	
Metrics	**Units**	**Absolute value**	**% change**	**Absolute value**	**% change**	**Absolute value**	**% change**
Fuel Economy	mpg	9.97	4.17	10.5	9.75	10.95	14.36
Trip Time	s	4605.6	0.13	4598.3	-0.03	4596.4	-0.07
Number of Shifts	-	37	-	35	-	36.	-
Cycle Work	kWh	141.77	-2.95	136.3	-6.7	130.1	-10.94
BTE	%	45.27	0.49	45.85	1.07	45.60	0.83
Negative Work.	kWh	-23.25	-21.53	-26.83	-9.45	-27.29	-7.9
Aerodynamic Work	kWh	89.76	-0.46	83.364	-7.55	81.298	-9.84

% change is compared to baseline simulation running rule based SOC control without any predictive knowledge

Table 6.3. 3 Truck Platoon Metrics Running Optimal Control. All the vehicles have knowledge of the offline optimal control trajectory. The individual trucks are running consensus agent based algorithm to calculate the final optimal path. The metrics shown are with Engine Idle Coast scenario.

		Lead Truck		**Follower Truck 1**		**Follower Truck 2**	
Metrics	**Units**	**Absolute value**	**% change**	**Absolute value**	**% change**	**Absolute value**	**% change**
Fuel Economy	mpg	10.12	5.77	10.62	10.93	11	14.90
Trip Time	s	4596.1	-0.08	4593.8	-0.13	4592.8	-0.15%
Number of Shifts	-	37	-	41	-	40	-
Cycle Work	kWh	139.83	-4.28	134.89	-7.66	130.354	-10.77
BTE	%	45.33	0.56	45.87	1.09	45.91	1.13
Negative Work.	kWh	-23.69	-20.05	-25.89	-12.62	-26.192	-11.6
Aerodynamic Work	kWh	90.356	0.20	82.97	-7.99	79.393	-11.95

% change is compared to baseline simulation running rule based SOC control without any predictive knowledge

7. CONCLUSIONS

48V mild hybrid driven P2 powertrain architecture for a class 8 type line haul application is a complex system in itself. There are a number of design options available in terms of systems engineering. Further there are a different variety of control algorithms that can be implemented for achieving a better overall powertrain efficiency. The problem becomes more interesting and challenging when we include a platoon of such trucks driving at highway speed following a autonomous path planning algorithm. Though in this work the autonomy is studied but an attempt to understand the overall energy efficient behavior of each vehicle in a platoon is made. The problem in this work is setup in an incremental stacked up fashion. While the first few chapters described the overall motivation for the work along with the background, chapter 4 discussed the hybrid architecture that is used in this work. A few assumptions are made while conducting this system design which are as follows,

- The platoon consists of three trucks
- All three vehicles have the same architecture running the similar duty cycle
- Corridor information consists of only road grade and route speed limit. Traffic information is not considered in this work
- Vehicle load is kept constant throughout this analysis

The system runs a proportional integral based feed forward control for route speed limit tracking and braking. The hybrid system is a simple SOC trajectory tracking based rule based control which operates in charge sustaining mode between 25% and 75% SOC levels. This system is defined throughout as baseline and all metrics are compared against such a system. The follower trucks in the baseline system are exactly similar to the lead truck except for the fact that they experience a less aerodynamic drag. The braking system also takes into effect the separation distance from the lead truck.

Once the baseline system is defined, chapter 5 outlines the problem formulation methodology along with the different objectives. It talks about the high level objective of actively using predictive information to study the optimal behavior of platooning class 8 trucks. It is also discussed here about the multi-objective optimization challenge including multiple control levers together. It is of prime importance to understand the co-relation of these control levers with each other. This is needed in order to design specific embeddable controllers which can operate in their own zones without interfering with other controllers. The other key question is to understand the benefits of such predictive controllers in platooning vehicles. The big question here is *"Is it necessary for follower trucks to know the predictive information or they can passively follow the lead truck running predictive controls?"*

Chapter 6, defines the individual predictive control problems one at a time for the single, lead vehicle only. The idea presented in this work is to apply a global optimizer like dynamic programming to find the optimal trajectories for the 4 control levers. This is done as an offline optimal problem and is highly computation heavy. This kind of problems either need a very efficient high end processor or a cloud structure. Hence these are often used to understand and analyze the optimal behavior and then a better controller efficient rule based algorithm can be designed. In this work dynamic programming is used using a high efficient tower workstation to solve the multi-objective problem in the following stacked up sequence,

- ***Predictive Cruise Speed -*** In this problem formulation the look ahead predictive information used is still the road grade for the entire route and the speed limit. The cost function used is to minimize fuel along with no compromise on the total trip time. The time is considered as a part of the cost function so that the truck does not slow down in order to save fuel. The trip time for the entire optimal solution has to be close to the baseline trip time. The objective is to find the adaptive cruise speed target based on the predictive road grade knowledge and the optimal u^* which in this case is the engine throttle. The cost function is,

$$u^*(s) = \operatorname*{argmin}_{u \in [0,1]} \int_0^s \left\{ \frac{W_f \alpha \dot{m}_f}{v_s} + \frac{W_t(1-\alpha)v_t}{v_s} \right\} ds \tag{7.1}$$

  where, W_f & W_t are the normalizing weights for the trip time and the fuel consumed, α is the tuning weight for the fuel and trip time. $\alpha = 1$ means a complete optimality for fuel saving with out considering trip time and vice versa. The non-linear constraints are applied on vehicle speed, engine speed, gear number. This optimal problem formulation shows a 2% fuel savings for the single truck with out compromising on the total trip time. The characteristic grade for the entire route is close to 1.3%. Some key observations from this problem is that the predictive cruise control modulates speed around hills and downhills. Specifically it increases speed before entering a hill and decreases the speed before entering a down hill. This is also in line to our understanding from the behavior seen in the cyclist's situation. In energy domain it is similar to gaining energy when it is easy to do in the flat section and then utilize the kinetic energy gained in the hill section to overcome the grade drag. Similarly during the downhill it is efficient to slow down a bit before entering the downhill to save energy (fuel) since it is expected to increase speed during the downhill and will have to brake thereby wasting energy which is gained at the expense of fuel. The main objective here is to reduce the negative works in the form of reduced engine braking. Speed modulation during the flat sections are not very common. During the flat section the truck follows the usual route speed limit which is $65mph$ or $29m/s$ in this work. This behavior is also validated using the dummy $10mile$ route with a trapezoidal 3% uphill and downhill section. The emissions are also improved as a passive component due to the engine operating point change. Now that the engine operates at a more better BTE zone consuming less fuel, we observed a better NOx numbers. The Normalized NOx reduction from baseline simulation is around 5% in this optimal problem formulation.

- ***Predictive Cruise Speed & Coasting*** **-**This problem is exactly similar to the previous problem but an added control variable is included. The new control lever is the clutch command which can disengage the master clutch and let the engine idle. The extra fuel saving associated with this approach is when the engine goes to idle and consume idle fuel instead of torque curve fuel. It is also worth noting here that the coasting events shall not replace motoring event where there is no fuel consumption at all. There is another class of problem solved here which is similar to coasting but instead of the engine idling it shuts the engine there by consuming no fuel at all. This method can replace motoring events but will lack the ability of engine braking. So with the coasting problem the speed constraints are critical in order to assure that when engine braking is needed the system gets out of coasting to let the master clutch engage and help with engine braking. In this case for the given route of $86miles$ with a characteristic grade of 1.3% we observed a fuel saving of close 0.8% when the engine is let to idle during coast events. In case of coasting with engine off an additional 1% totaling to a 1.8% fuel economy is achieved. It is also observed that the coasting events are mostly active during the flat section of the route as compared to the hilly section where the predictive cruise control was mainly active. This is indeed good in order to develop controls which are not encroaching each others space. It is also observed that coasting with engine off has a different set of operating region as a function of grade. It is mostly active during the down grade portions of the route specifically coming out of the down hill. It is also noticed that coasting with engine off has less but longer coasting events as compared to coasting with engine idling. Engine idle coast events are similar to pulse and glide type behavior during the flat section of the route. The coasting problem is also solved as an independent problem with out the cruise speed and it is observed that the benefits are little less than what is achieved with the cruise speed and coast problem together. The decrease in benefit is around 10% for the coasting with engine idle problem alone and $15\%-20\%$ for the coasting with engine off problem alone. The behavior and operating zone is still the same but the duration of events reduced. This is also analytically justified since with the speed modulation the truck gets more opportunity to stay in coast without violating speed constraints. The improvement in passive NOx reduction with this problem formulation was around 7.4% with engine idle coast events and around 8% with engine off coast events. The improvement with engine off coasting case is not substantial since the NOx production is a little more during engine crank and start cycles. It is also worth pointing out that shutting out the engine will lower the after treatment temperatures and hence it will impact the conversion efficiency of the active selective catalytic reduction components. This will pose a challenge in using engine off coasting too frequently unless there is an active heating element integrated in the system to maintain after treatment temperatures.

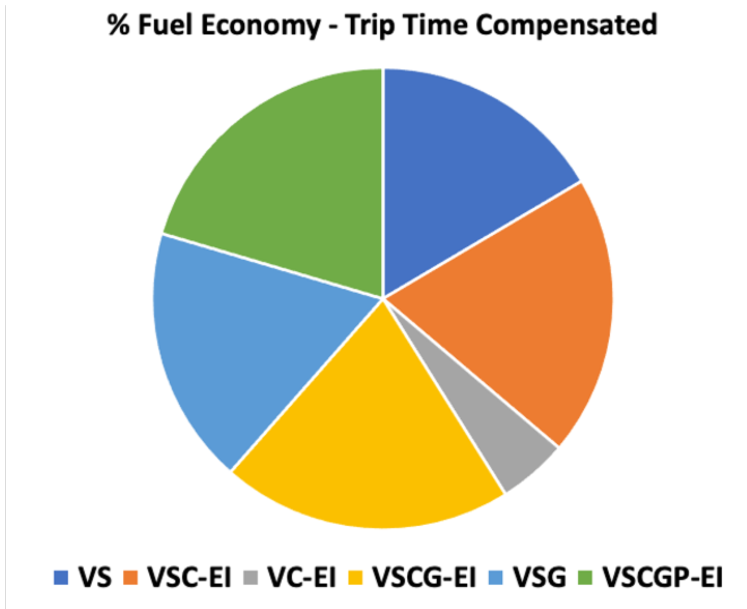
This is an altogether different problem to solve but indeed a potential candidate for future multi-disciplinary optimization techniques.

- ***Predictive Cruise Speed, Coasting & Gear Management -*** In this problem a third control lever is added in the multi-objective optimization. Though gear is more of a performance lever it is not expected to gain a lot of fuel economy by predictively modulating gear but it is solved to understand the behavior and study its co-relation with the other levers together. It is also worth pointing out here that the engine efficiency maps used for this work has its peak BTE zone around a range of engine speed which corresponds to top gear ratio of the transmission system. This means that if we get down to a lower gear the system will compromise on fuel savings. Hence to achieve the minimum cost of fuel savings the system will not down shift. This is also seen with the optimal solution. This problem did not provide any fuel benefits but neither did penalize fuel savings. It was a hard problem to tune for achieving at least the same fuel economy as with the previous problem. The problem is tuned to provide the same benefits as the previous problem and with this settings it is observed that the system down shifts a little early while in the grade, and stays at a lower gear a little more after coming out of a grade. The way this problem is setup is that when a coasting event is on the gear cannot shift and if there is a shift request the system cannot go to a coasting event. This is implemented as a penalty in the cost function in order to make sure that system dynamics are not affected and the vehicle can shift and do coasting when needed. With this in mind, it is observed that going out of a down hill when the vehicle is coasting the gear is held usually at the top gear and then after the coast event the gear is shifted if there is a request. On the performance side it is observed that with predictive gear shifts the truck was able to maintain a higher speed in the uphill sections. On an average a $2mph$ less speed reduction is achieved in all the uphills. The impact of NOx improvement is not at all substantial to justify that addition of predictive knowledge for gear management can improve NOx production in the system. In fact for some tuning cases it increased the NOx production a bit due to the gear operation at a lower gear. This is analytically justified as well since a lower gear operation means better performance rather than a better BTE zone operation. The good observation is that even with the fuel efficient tuning for the optimal parameters it did not penalized NOx improvement drastically.

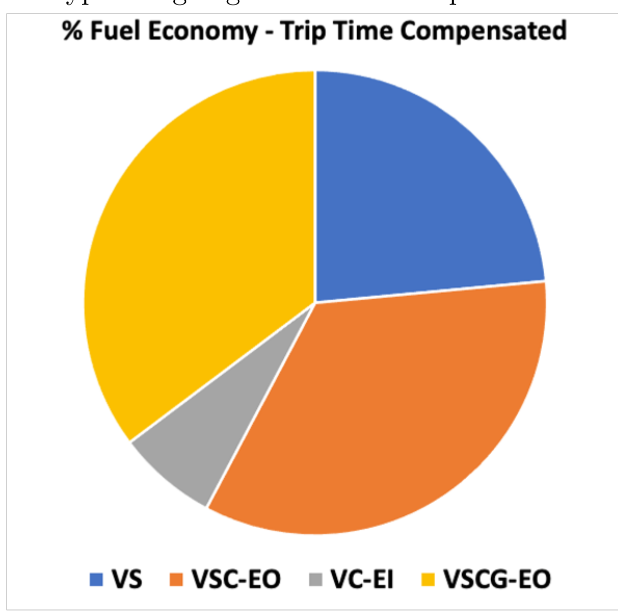
- ***Predictive Cruise Speed, Coasting, Gear & Power Split Management -*** The last problem solved is including the power split strategy based on the look ahead information. Though the SOC profile changed a bit but it did not provide any substantial benefit in fuel savings. The size of the hybrid system limited the power split strategy to substantially do any thing beyond the baseline rule based charge/discharge strategy. It is still observed that the hybrid system is discharged at its full capacity in the uphills but then charged quickly during the downhills. Any other strategy to charge the battery at the expense of fuel is not considered as beneficial in the optimal solution. During heavy grades it is seen that the gear shifts early and the hybrid system is exhausted much earlier than the baseline. Potential improvements can be made by increasing the size of the hybrid system but it will come at a cost of increased weight and thereby compromising on the total freight carrying capacity. The constraints added in this problem are on vehicle speed minimum and maximum deviation from the route speed limit, gear numbers, engine speed, coasting penalty during shift requests and vice versa and finally on SOC minimum/maximum limits. The NOx improvement in this problem as well did not show a significant positive trend. The same analogy can be derived as presented in the previous problem since gear management is as well a part of this problem.

The key outcome of the offline multi-objective optimal control theory results are that there are separate zone of operation for the individual control levers and that the levers do not fight with each other for objective minimization, that is more fuel economy. Fuel efficient control levers are cruise speed as a function of look ahead road grade and coast management. Predictive gear management does not provide fuel benefit but is a performance lever. Similarly due to the size of the mild hybrid system there is no significant energy efficient behavior with predictive look ahead knowledge.

Figure 7.1a shows the high level absolute fuel benefits compared to baseline rule based



(a) Pie distribution of absolute fuel economy for the various problem type using engine idle coast sequence



(b) Pie distribution of absolute fuel economy for the various problem type using engine off coast sequence

control simulation results. The pie chart shows the different DOE scenarios with engine idle condition only. We see the trend that predictive speed modulation with coast provides the maximum fuel benefit while gear and power split are not significant players in achieving fuel efficient solution for this vehicle configuration and route type. Similarly Figure 7.1b shows the same DOE's with engine off coast sequences. Though the overall fuel economy improved in this case as compared to its engine idle counterpart but the general trend still remains the same. We see benefits exclusively from speed modulation and coast whereas gear management is more of a performance lever and power split kind of limited to it small range and could not provide any significant benefits.

The next approach to this research was to take the optimal behavior to a platoon of 3 trucks. The lead truck is the prime mover here with the follower trucks limited to dynamically changing the control levers obeying the separation distance. The separation distance was not set to a fixed value but rather considered as a dynamic range between an upper and lower bound. We used model predictive control as an option to compare the results with the proposed multi-agent based distributed averaging method. While MPC is the analytical optimal solver, it is always not feasible to implement MPC in real time controllers. Hence the distributed averaging based algorithm is chosen as an alternative. It is observed that distributed averaging method is able to get close to MPC results.

The objective function for the platooning trucks, is,

$$u^*_{1:N}(s) = \underset{u_{1:N}\in R}{\operatorname{argmin}} \sum_{n=1}^{N} \int_0^s \left\{ \frac{W_f \alpha \dot{m}_{f_n}}{v_{s_n}} + \frac{W_t(1-\alpha)v_{t_n}}{v_{s_n}} + \tau_{brake_n} \right\} ds \tag{7.2}$$

where, W_f & W_t are the normalizing weights for the trip time and the fuel consumed for individual trucks, α is the tuning weight for the fuel and trip time. $\alpha = 1$ means a complete optimality for fuel saving with out considering trip time and vice versa. N = 3 is the total number of trucks in the platoon and τ_{brake_n} is the baring work for each truck. The other constraints applied are on the inter-vehicular separation which is not fixed but can vary between a lower and upper bound. Another penalty was added as a soft constraint on the deviation of the control signals of each truck from the optimal control signal estimated by the offline controller. Overall the platoon average fuel economy increased by 9%. This benefit is compensated against baseline trip time, which is considering that the trucks shall not take more time than what the baseline simulation provided. This is to ensure that the fuel benefits are not achieved by penalizing trip time. There was slight reduction in fuel benefits on the lead truck which indicates that the truck now in the platoon is not exactly following the optimal offline control trajectories. This is based on the fact that all the three trucks are now trying to come to a consensus in the platoon. The truck right behind the lead truck achieved around 9.5% fuel benefit and the last platooning truck achieved a combined benefit of 12%. The follower trucks showed significant reduction on the coast events and the predictive gear shift events. Coast events gets penalized due to the variation in inter-vehicular separation. Follower vehicles chose not to do any coasting during the hilly portion of the route but did do a couple of events during the flat section. The events were also reduced in time, which is why there was no significant difference in the solution between engine off verses the engine idle coast formulation.

Due to the reduction in aerodynamic drag in the follower vehicles the engine power was sufficient to keep the vehicle speed well within limit in the hills. Hence we did not notice a lot of predictive gear operation. Even from the performance factor an early downshift was not needed by the follower trucks. Predictive hybrid management as well did not provide any significant difference in behavior. The follower trucks showed a much less transient battery throughput. Since the net drag in the system is reduced the battery charge/discharge cycles were not that transient.

Overall there was 7% increase in braking work by the by the follower truck 1 and around 10% increase in braking work for the follower truck 2. Engine braking work in the follower trucks reduced and was replaced by mostly braking work increase. Follower truck 1 had around 5.5% reduction and follower truck 2 around 9%. Motoring work increased by around 1% for follower truck 1 and 0.6% for follower truck 2. In general the overall negative work increased for the follower trucks which is counter intuitive to fuel savings. This analytically provides sufficient understanding that predictive features may not provide a lot of fuel savings for follower trucks in a platoon. The benefit in fuel savings in a platoon is mostly by the reduction of aerodynamic drag. It is also observed that the drivability is impacted by the follower trucks in order to keep the separation distance within bounds but dynamically changing it. It could be a better solution to maintain a fixed separation distance between two trucks just by following the lead truck dynamics. It is also noted that there is no significant improvement in NOx, numbers by predictively controlling the follower trucks. In fact NOx numbers are around 2% better if a fixed separation distance is maintained.

8. FUTURE WORK

The problem analyzed in this work is very complex with multiple levers and vehicle configuration involved. It is highlighted throughout the paper that different assumptions are made. There is a huge potential in including other key control levers or bringing in more objectives in the problem. The current problem helped us understand the impact and interaction of 4 control levers namely adaptive cruise speed, gear selection, coasting (both engine idle and engine off) and hybrid power split. We have seen the effectiveness of these levers independently as well as when interacting together. It is also noted that levers have unique operating regions when compared to different drive cycles with a varying road grade. There is potential to include other control levers to the problem set. This will give more holistic optimal behavior for the complete system. The objective can also be changed to include emissions and tailpipe temperature for a direct optimal control involving active emissions strategy. While doing the fleet control optimization the lateral dynamics are not studied which can also be another degree of freedom for the study. The below key improvements and research are either being conducted by the author or are potential candidate for future research.

Total Cost of Ownership optimization is a key driver for a lot or industrial conglomerate these days. This is kind of a travelling sales man problem where any genetic algorithm based static optimization can be run in order to figure out the best possible route in terms of trip time, energy savings, shortest route as well as meeting the delivery schedule for a stop and go kind of application. This kind of optimization will also provide the best powertrain architecture that is suitable for this application along with the component sizing. It will also help understand the charging station installation need over the entire route. This is a real life challenge and interesting problem to solve. Another key objective is sizing the right components for a particular class of vehicle hauling a fixed load. In conjunction with predictive levers the right component size will make the system the best fuel efficient as well as attain other objective of load carrying capacity and vehicle gross weight reduction.

The next problem is to understand the impact of bringing in the after treatment components in the optimization method. It will ensure the proper operation of the after treatment in terms of maintaining temperatures as well as conversion efficiency down stream of the engine. This will help study the need to have externally heated systems for cold conditions. Another situation is to use more control levers together such as dynamic torque control, load based predictive control, predictive braking, etc. All these interacting control will improve the over all performance and help understand the global optimality in a much diverse space. At this point with various multiple levers it is almost impossible to frame any analytical minimization problem. So there is a need to distributively frame the problem in such a way so as to understand the interacting lever in a much better way is possible.

In platoon system there is a lot that can be achieved by introducing more predictive and smart algorithms. Traffic based distributed platoon algorithms can be realized which can optimize their controls based on the future need to come to a platoon. So multiple trucks operating at nearby regions can optimize their trajectory based on the predictive information of road grade, weather, traffic and plan their route accordingly so that they can get to a platoon when needed in the best optimal way.

The current setup is just a way to have the basic system ready and can be scaled easily to any problem as needed. Even different fuel source can be modelled easily to study the effects. This field has huge potential and with increasing norms around the world and an urge to reduce carbon footprint for a better tomorrow this will pave the way for a lot more research to come in near future.

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APPENDIX

A.1 Dynamic Programming Background

Once the performance measure of the system or the cost function is determined the next major task is to define a control function that would minimize the performance criteria.
Two widespread methodology to accomplish this task are minimum principle of Pontryagin and the method of Dynamic Programming. Pontryagin's minimum principle is a variational approach that lead to a non-linear two point boundary value problem which is solved to get the optimal control.
Dynamic Programming (DP) is both a controls methodology and a computer programming method to numerically solve a optimization problem given a set of admissible controls and state space grid vectors. It always satisfies global optimality as it finds the minimum value of the cost/objective function from all admissible search space.
Since it has to traverse a full factorial DOE of search space for all the controls & states it is often challenged by the curse of dimensionality [82][83]

A.1.1 Principal of Optimality

In controls literature a general control law is defined as

$$u_{\mathrm{i}}^{*}(t) = f(x_{\mathrm{i}}(t), t) \tag{A.1}$$

which is a closed loop or feedback optimal control. The functional relationship **f** is called the optimal control law or optimal policy. The control law specifies how to generate the control law from the states at a given time, since this is a time varying control formulation. Dynamic programming specifically solve the controls problem applying the principle of optimality.
Bellman's original Optimality Principle, states:
An optimal policy has the property that whatever the initial state and initial decisions are, the remaining decision must constitute an optimal policy with regard to the state resulting from the first decision. In figure A.1, if J_{abe} is the minimum cost to go from a-e, then from

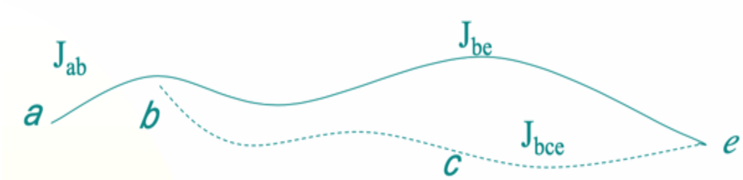

Figure A.1. Illustration of the Principle of Optimality

b-e the minimum cost has to be J_{be} and J_{bce} cannot be the optimal path.
That is $J_{bce} > J_{be}$
Dynamic programming is based on the same principle to find the optimum cost at each

time step traversing backwards and then figuring out the optimal cost to go in the forward simulation which by the principle of optimality is claimed to the global optimal result.
Figure A.2 depicts such a condition of admissible control selection. An alternative notation

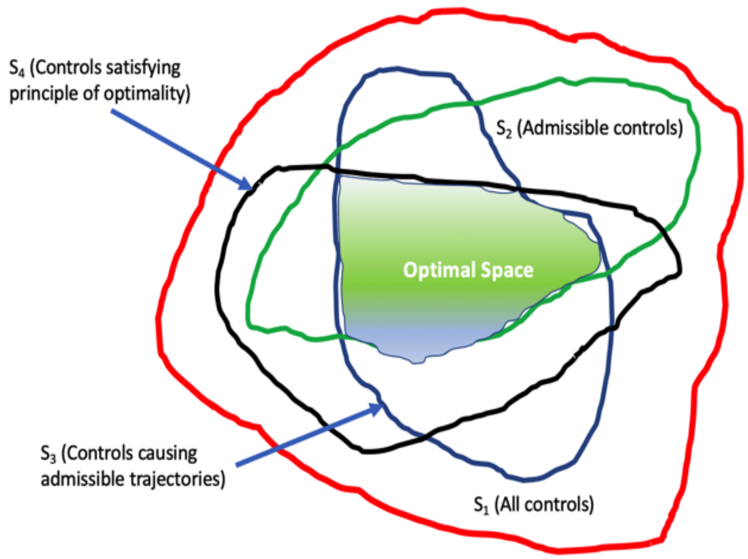

Figure A.2. Illustration of the Principle of Optimality

for the computational formulation of the dynamic program algorithm is:

$$\begin{aligned} J_K^*(x(N-K)) = \min_{u(N\text{-}K)} [&g_D(x(N-K), u(N-K)) \\ &+ J_{K-1}^*(a_D x(N-K), u(N-K))] \end{aligned} \tag{A.2}$$

with, K being each stage during the search process and J^* is calculated for each stage K, which is known as stage cost. g_D & a_D comes from the definition of the system model dynamics which can be ignored in this section.
Since a direct search is used to solve the functional recurrence equation, the solution obtained is absolute (or global) minimum. Dynamic programming makes the direct search feasible because instead of searching among the set of all admissible that cause admissible trajectories, we consider only those controls that satisfy additional necessary condition - principle of optimality.

A.2 Formation Graphs and Deviation Variables

This section describes some background on graph theory and its application to manipulation of formations of multiple vehicles. A graph is denoted by $\mathcal{G} = (V, \mathcal{E})$ where V is the set of vertices and $\mathcal{E} \subset \mathcal{V}x\mathcal{V}$ is the set of edges of the graph. We assume all the graphs are undirected with no edges $(v\mathrm{i}, v\mathrm{i})$, $\forall \mathrm{i} \in \mathcal{I}$ from a node to itself. Each edge is denoted by $\mathrm{e}_{\mathrm{ij}} = (v\mathrm{i}, v\mathrm{j}) \in \mathcal{E}$ or $\mathrm{ij} \in \mathcal{E}$ for simplicity of notation where $\mathrm{i}, \mathrm{j} \in \mathcal{E} = (1, ..., n)$. An orientation of the edges of the graph, $\mathcal{E}_{\prime} \subset \mathcal{E}$, is the set of edges of the graph which contains one and only one of the two permutations of $\mathrm{ij} \subset \mathcal{E}$ (ij or ji) for all the edges $\mathrm{ij} \subset \mathcal{E}$.
A triangulated graph is a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{F})$ with the set of faces $\mathcal{F} \in \mathcal{V}x\mathcal{V}x\mathcal{V}$ with elements $f_{\mathrm{ij}k} = (v_{\mathrm{i}}, v_{\mathrm{j}}, v_k)$ or simply $\mathrm{ij}k(\mathrm{i}, \mathrm{j}, k \in \mathcal{I})$ satisfying the consistency condition that for all faces $f_{\mathrm{ij}k} = (v_{\mathrm{i}}, v_{\mathrm{j}}, v_k) \subset \mathcal{F}$, the following holds,
$(v_{\mathrm{i}}, v_{\mathrm{j}}) \in \mathcal{E}$, $(v_{\mathrm{j}}, v_k) \in \mathcal{E}$, $(v_k, v_{\mathrm{i}}) \in \mathcal{E}$
Similarly, an orientation of the faces of a triangulated graph $\mathcal{G}$ is a set of faces $\mathcal{F}_{\prime} \subset \mathcal{F}$ that contains one out of the six permutations .of each face $\mathrm{ij}k \in \mathcal{F}$. Define the dual graph $D(\mathcal{G})$ of a triangulated graph $\mathcal{G}$ as a graph with $|\mathcal{F}_{\prime}|$ number of nodes, one corresponding to each (oriented) face of $\mathcal{G}$. There is an edge between two distinct faces f_1, $f_2 \in \mathcal{F}_{\prime}$, if and only if f_1 and f_2 share a common edge $\mathrm{e} \in \mathcal{E}$. A *triangulated formation graph* is a quintuple,
$\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{D}, \mathcal{F}, \mathcal{A})$,
with a connected dual graph $D(\mathcal{Q})$. Let $q_{\mathrm{i}} = (x_{\mathrm{i}}, y_{\mathrm{i}}) \in \mathcal{R}^2$ denote the position of the node v_{i}. Here, $D = d_{\mathrm{ij}} : \mathrm{ij} \in \mathcal{E}$ is the set of desired distances and $A = a_{\mathrm{ij}k}, \mathrm{ij}k \in \mathcal{F}$ is the set of desired areas of triangular faces. The *signed area of a triangle* is given by

$$h(q_{\mathrm{i}}, q_{\mathrm{j}}, q_k) = \det \begin{bmatrix} x_{\mathrm{i}} & y_{\mathrm{i}} & 1 \\ x_{\mathrm{j}} & y_{\mathrm{j}} & 1 \\ x_k & y_k & 1 \end{bmatrix} = (q_k - q_{\mathrm{i}})^T S (q_{\mathrm{j}} - q_{\mathrm{i}}), \tag{A.3}$$

where,

$$S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{A.4}$$

Delaunay triangulation is used as a set of points to obtain the triangulated graphs. In Figure A.3 a triangulated formation of six vehicles is shown with,
$\mathcal{V}$ =1,2,3,4,5,6,
$\mathcal{E}_{\prime}$ =12,13,23,24,25,35,36,45,56,
$\mathcal{F}_{\prime}$ =123,245,253,356,

Fix the edge and face orientations of the triangulated graph $\mathcal{G}$ such that for all the faces $a_{\mathrm{ij}k} \geq 0$, i.e. if for the face $\mathrm{ij}k \in \mathcal{F}_{\prime} \subset \mathcal{F}$, $a_{\mathrm{ij}k} \leq 0$ then replace the triplet $(v_{\mathrm{i}}, v_{\mathrm{j}}, v_k) \in \mathcal{F}_{\prime}$ by $(v_{\mathrm{j}}, v_{\mathrm{i}}, v_k)$ to change the sign of the determinant in Equation A.3. The following edge and

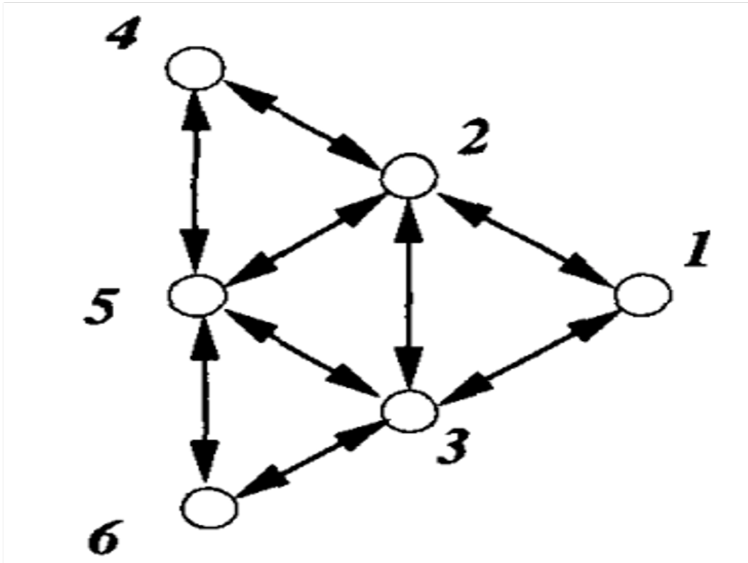

Figure A.3. Illustration of a triangulated six vehicle formation

face deviation variables (also known as shape variables [84]) associated with the edges and faces of the triangulated graph $\mathcal{G}$ are defined, respectively, as

$$\begin{aligned} \eta_{ij} &= ||q_j - q_i|| - d_{ij}, \\ \delta_{ijk} &= q_{ik} \bigotimes q_{ij} = (q_k - q_i)^T S(q_j - q_i) - a_{ijk}, \end{aligned} \tag{A.5}$$

for edges ij $\in \mathcal{E}_{\prime}$ and faces ij$k \in \mathcal{F}_{\prime}$. In addition, $q_{rs} := q_s - q_r$ and the tensor product $\bigotimes$ is defined by $\alpha \bigotimes \beta := \alpha^T S \beta$ for $\alpha, \beta \in \mathcal{R}^2$ Let $p_i = \dot{q}_i$ denote the velocity of each node $v_i \in \mathcal{V}$. Then, the *edge and face deviation rate variables* (also known as shape velocities [84]) associated with the set of edges and faces of the graph $\mathcal{G}$ are defined, respectively, as follows,

$$\begin{aligned} v_{ij} &= n_{ij}^T.(p_j - p_i), \\ \zeta_{ijk} &= (p_k - p_i)^T S(q_j - q_i) + (q_k - q_i)^T S(p_j - p_i), \end{aligned} \tag{A.6}$$

where, $v_{ij} = \dot{\eta}_{ij}$, $\zeta_{ijk} = \dot{\delta}_{ijk}$, $n_{ij} = q_{ij}/||q_{ij}||$ for $q_i \neq q_j$. Using the notation $p_{rs} = p_s - p_r$ and $\alpha^{\perp} = S\alpha$ (thus $\alpha \bigotimes \beta = \alpha^T.\beta^{\perp}$), we can simplify the expression for the shape velocities as,

$$\begin{aligned} v_{ij} &= n_{ij}^T.p_{ij}, \\ \zeta_{ijk} &= p_{ik} \bigotimes q_{ij} + q_{ik} \bigotimes p_{ij} = p_{ik}^T.q_{ij}^{\perp} - p_{ij}^T.q_{ik}^{\perp} \end{aligned} \tag{A.7}$$

A.3 Model Predictive Control

The model-based predictive control (MPC) methodology is also referred to as the moving horizon control or the receding horizon control. The idea behind this approach can be explained using an example of driving a car. The driver looks at the road ahead of him and taking into account the present state and the previous action predicts his action up to some distance ahead, which we refer to as the prediction horizon. Based on the prediction, the driver adjusts the driving direction. The MPC main idea is illustrated in Figure 1. The MPC is constructed using control and optimization tools. The objective of this write-up is to introduce the reader to the linear MPC which refers to the family of MPC schemes in which linear models of the controlled objects are used in the control law synthesis. In the MPC approach, the current control action is computed on-line rather than using a pre-computed, off-line, control law. A model predictive controller uses, at each sampling instant, the plant's current input and output measurements, the plant's current state, and the plant's model to,

- calculate, over a finite horizon, a future control sequence that optimizes a given performance index and satisfies constraints on the control action;

- use the first control in the sequence as the plant input.

The MPC strategy is illustrated in Figure 2, where Np is the prediction horizon, $u(t+k|t)$ is the predicted control action at t + k given u(t). Similarly, y(t + k|t) is the predicted output at $t+k$ given $y(t)$.

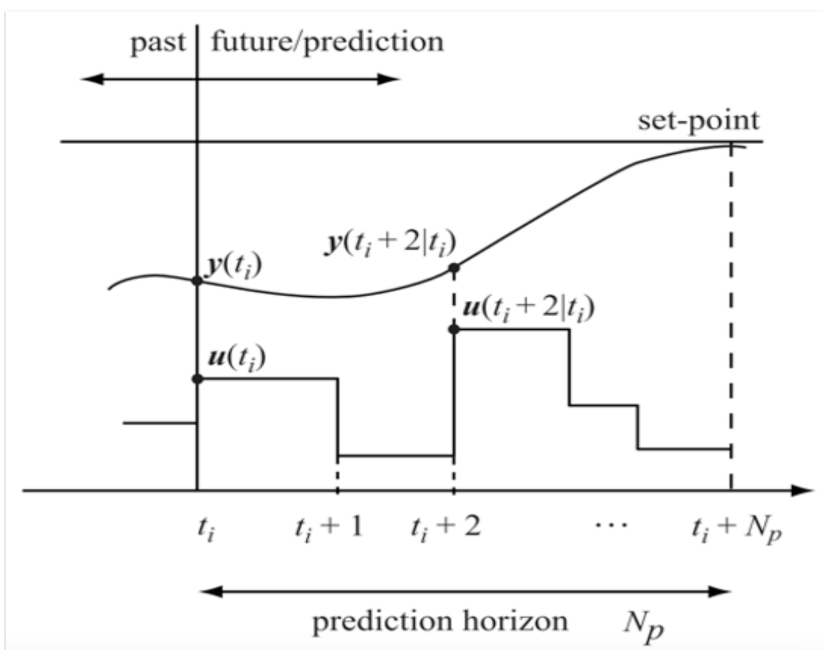

Figure A.4. Model Predictive Control illustration for the predictive horizon and the control horizon.

A.3.1 Basic Structure of MPC

In Figure A.5, we show a basic structure of an MPC-controlled plant, where we assume that the plant's state is available to us.

A.3.2 From Continuous to Discrete Models

Our objective here is to present a method for constructing linear discrete-time models from given linear continuous-time models. The obtained discrete models will be used to perform computations to generate control commands. We use a sample-and-hold device that transforms a continuous signal, f(t), into the staircase signal, $f(kh), kht < (k+1)h$, where h is the sampling period. The sample and zero-order hold (ZOH) operation is illustrated in Figure A.6

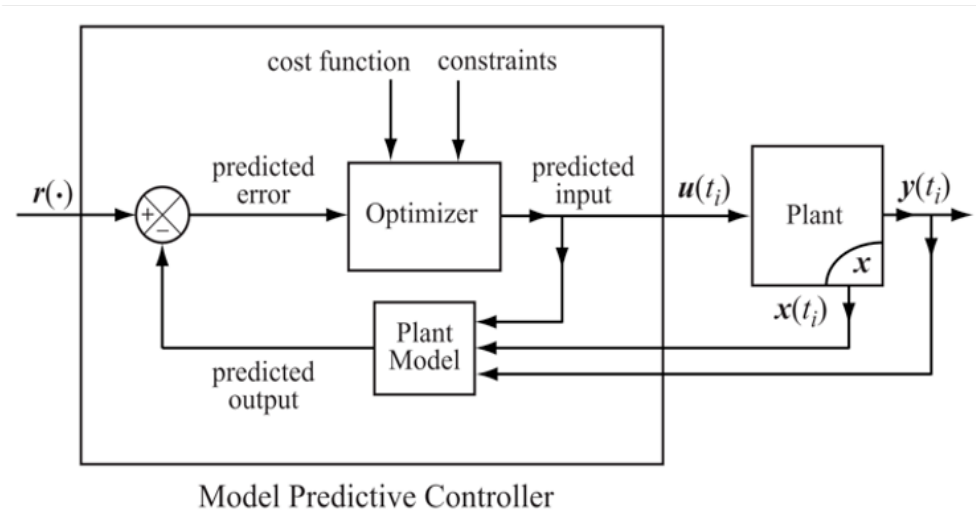

Figure A.5. State feedback model predictive controller

Suppose that we are given a continuous-time model,

$$\dot{x}(t) = Ax(t) + Bu(t), x_0 = x(t_0) \tag{A.8}$$

$$y(t) = Cx(t) \tag{A.9}$$

The solution to the state equation is

$$x(t) = \mathrm{e}^{A(tt_0)}x(t_0) + \int_t^{t_0} \mathrm{e}^{A(t-\tau)}Bu(\tau)d\tau \tag{A.10}$$

We assume that the input to the system is generated by a sample-and-hold device and has the form,
$u(t) = u(k), kht < (k+1)h$ Let $t_0 = kh$ and $t = (k+1)h$ and let us use shorthand notation,

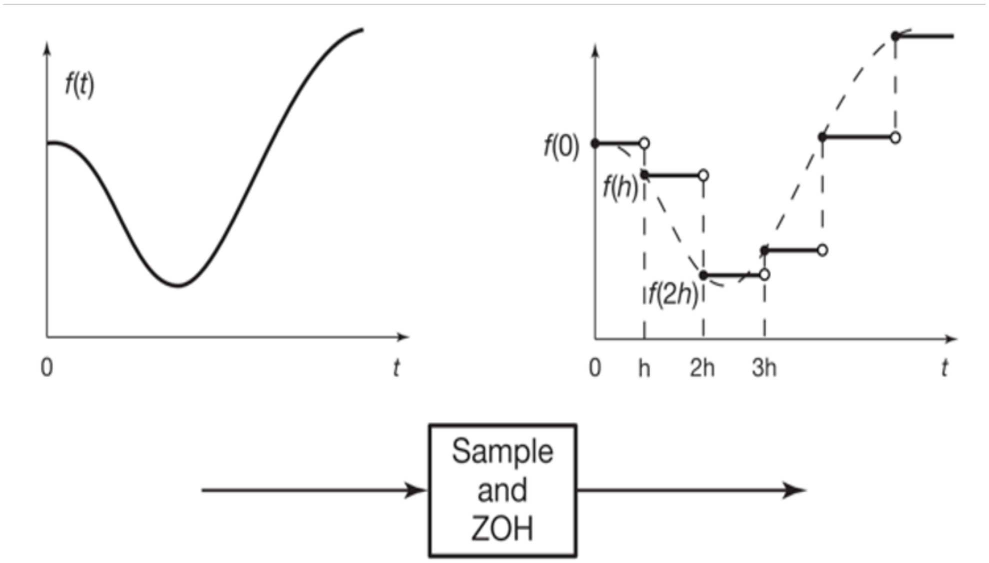

Figure A.6. Sample and zero-order hold (ZOH) element operating on a continuous function

$x(kh) = x(k)$ Then taking into account that u(k) is constant on the interval $[kh, (k+1)h)$ we represent Equation A.10 as,

$$\begin{aligned} x(k+1) &= \mathrm{e}^{\mathbf{A}h} x(k) + \int_{kh}^{(k+1)h} \mathrm{e}^{\mathbf{A}(kh+h\tau)} \mathbf{B}\mathbf{u}(k) d\tau \\ &= \mathrm{e}^{\mathbf{A}h} \mathbf{x}(k) + \int_{kh}^{(k+1)h} \mathrm{e}^{\mathbf{A}(kh+h\tau)} \mathbf{B} d\tau \mathbf{u}(k) \end{aligned} \tag{A.11}$$

Consider now the second term on the right-hand side of the above equation. Let $\eta = kh + h\tau$ Then we can represent Equation A.11 as,

$$\begin{aligned} x(k+1) &= \mathrm{e}^{\mathbf{A}h} \mathbf{x}(k) + \int_{kh}^{(k+1)h} \mathrm{e}^{\mathbf{A}(kh+h\tau)} \mathbf{B} d\tau \mathbf{u}(k) \\ &= \mathrm{e}^{\mathbf{A}h} \mathbf{x}(k) + \int_{0}^{h} \mathrm{e}^{\mathbf{A}\eta} \mathbf{B} d\eta \mathbf{u}(k) \\ &= \phi \mathbf{x}(k) + \Gamma \mathbf{u}(k), \end{aligned} \tag{A.12}$$

where,
$\mathbf{\Phi} = \mathrm{e}^{\mathbf{A}h}$ and $\mathbf{\Gamma} = \int_0^h \mathrm{e}^{\mathbf{A}\eta} \mathbf{B} d\eta$ The discrete output equation has the form,
$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k)$

A.3.3 Simple Discrete Time MPC

We consider a discretized model of a dynamic system of the form,

$$\mathbf{x}(k+1) = \mathbf{\Phi}\mathbf{x}(k) + \mathbf{\Gamma}\mathbf{u}(k) \tag{A.13}$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) \tag{A.14}$$

where, $\mathbf{\Phi} \in R^{nxn}$, $\mathbf{\Gamma} \in R^{nxm}$ and $\mathbf{C} \in R^{pxn}$
Applying the backward difference operator, $\Delta\mathbf{x}(k{+}1) = \mathbf{x}(k{+}1)\mathbf{x}(k)$, to Equation A.13 gives,

$$\Delta\mathbf{x}(k+1) = \mathbf{\Phi}\mathbf{\Delta x}(k) + \mathbf{\Gamma}\mathbf{\Delta u}(k) \tag{A.15}$$

where, $\mathbf{\Delta u}(k+1) = \mathbf{u}(k+1) - \mathbf{u}(k)$
We now apply the backward difference operator to Equation A.14 to obtain,

$$\begin{aligned}\mathbf{\Delta y}(k+1) &= \mathbf{y}(k+1) \\ &= \mathbf{C}\mathbf{x}(k+1) - \mathbf{C}\mathbf{x}(k) \\ &= \mathbf{C}\mathbf{\Delta x}(k+1)\end{aligned} \tag{A.16}$$

Substituting into the above Equation A.15 yields,

$$\mathbf{\Delta y}(k+1) = \mathbf{C\Phi\Delta x}(k) + \mathbf{C\Gamma\Delta u}(k) \tag{A.17}$$

Hence,

$$\mathbf{y}(k+1) = \mathbf{y}(k) + \mathbf{C\Phi\Delta x}(k) + \mathbf{C\Gamma\Delta u}(k) \tag{A.18}$$

We combine Equation A.16 and A.17 into one equation to obtain,

$$\begin{bmatrix}\Delta\mathbf{x}(k+1) \\ \mathbf{y}(k+1)\end{bmatrix} = \begin{bmatrix}\mathbf{\Phi} & \mathbf{O} \\ \mathbf{C\Phi} & \mathbf{I}_p\end{bmatrix}\begin{bmatrix}\mathbf{\Delta x}(k) \\ \mathbf{y}(k)\end{bmatrix} + \begin{bmatrix}\mathbf{\Gamma} \\ \mathbf{C\Gamma}\end{bmatrix}\mathbf{\Delta u}(k) \tag{A.19}$$

We represent Equation A.14,

$$\mathbf{y}(k) = \begin{bmatrix}\mathbf{O} & \mathbf{I}_p\end{bmatrix}\begin{bmatrix}\mathbf{\Delta x}(k) \\ \mathbf{y}(k)\end{bmatrix} \tag{A.20}$$

We now define the augmented state vector,

$$\mathbf{x}_a(k) = \begin{bmatrix}\mathbf{\Delta x}(k) & \mathbf{y}(k)\end{bmatrix} \tag{A.21}$$

Let, $\mathbf{\Phi}_a = \begin{bmatrix}\mathbf{\Phi} & \mathbf{O} \\ \mathbf{C\Phi} & \mathbf{I}_p\end{bmatrix}$, $\mathbf{\Gamma}_a = \begin{bmatrix}\mathbf{\Gamma} \\ \mathbf{C\Gamma}\end{bmatrix}$ and $\mathbf{C}_a = \begin{bmatrix}\mathbf{O} & \mathbf{I}_p\end{bmatrix}$

Using the above notation, we represent Equation A.17 and A.18 in a compact format as,

$$\mathbf{x}_a(k+1) = \mathbf{\Phi}_a \mathbf{x}_a(k) + \mathbf{\Gamma}_a \mathbf{\Delta u}(k) \tag{A.22}$$

$$\mathbf{y}(k) = \mathbf{C}_a \mathbf{x}_a(k) \tag{A.23}$$

where, $\mathbf{\Phi}_a \in R^{(n+p)x(n+p)}$, $\mathbf{\Gamma}_a \in R^{(n+p)xm}$, $\mathbf{C}_a \in R^{px(n+p)}$ Suppose now that the state vector $\mathbf{x}_a$ at each sampling time, k, is available to us. Our control objective is to construct a control sequence,

$$\mathbf{\Delta u}(k), \mathbf{\Delta u}(k+1), ..., \mathbf{\Delta u}(k+N_p-1) \tag{A.24}$$

where N_p is the prediction horizon, such that a given cost function and constraints are satisfied. The above control sequence will result in a predicted sequence of the state vectors,

$$\mathbf{x}_a(k+1|k), \mathbf{x}_a(k+2|k), ..., \mathbf{x}_a(k+N_p|1) \tag{A.25}$$

which can then be used to compute predicted sequence of the plant's outputs,

A.4 Pontryagin's Minimum Principle

The conditions for an optimal trajectory is provided by the well known Pontryagin's Minimum Principle which closely follows the established Hamilton-Jacobi-Bellman principle. However, the minimum principle provides necessary conditions, but not sufficient conditions, for optimality. In contrast, the Hamilton-Jacobi-Bellman equation offered sufficient conditions. Using the minimum principle alone, one is often not able to conclude that a trajectory is optimal. In some cases, however, it is quite useful for finding candidate optimal trajectories. Any trajectory that fails to satisfy the minimum principle cannot be optimal. The method of Lagrange [85], a control Hamiltonian function $\mathcal{H}$ can be constructed by appending the state equation to the integrand $\mathcal{L}$ using the Lagrange multipliers, $\lambda(t)$, as follows,

$$H(u(t), z(t), \lambda(t), t) = L(u(t), z(t), t) + \lambda^T(t) f(u(t), z(t), t), \tag{A.26}$$

where $z(t)$ is the optimal control, and $u(t)$ is the corresponding optimal state. Then the Pontryagin Minimum Principle (PMP, [85]) states that there exists a continuous function λ, known as an *adjoint function*, that is the solution of the *adjoint equation*,

$$\lambda(t) = -H_u(u(t), z(t), \lambda(t), t), \tag{A.27}$$

along with the appropriate initial (or final) condition of λ. In Equation. A.27, H_u denotes the differentiation of the Hamiltonian function with respect to the state. In particular, the adjoint function is a Lagrange multiplier that brings the information of the state equation constraint to the optimization problem. According to the PMP, the optimal control, z(t), and corresponding optimal state, $u(t)$, and adjoint, $\lambda(t)$, must minimize the Hamiltonian so that,

$$H(u(t), z(t), \lambda(t), t) \leq H(u(t), z^*(t), \lambda(t), t), \tag{A.28}$$

for all time and for all admissible (i.e., feasible) trajectory control variables $z^*(t)$, while the adjoint equation Equation. A.27 is satisfied. Admissible trajectories are defined as a set of variables that lay in the neighborhood of the minimal solution and satisfies all of the constraints. With the above considerations, we can now define the necessary and sufficient conditions for optimality. For a feasible trajectory that satisfies the minimum principle, condition Equation. A.28 implies that the Hamiltonian is minimum at the optimal control $z(t)$, such that,

$$Hz(u(t), z(t), \lambda(t), t) = 0 \tag{A.29}$$

Equation. A.29 is the *first order necessary condition* for optimality and corresponds to a special case of the Euler-Lagrange equation of the calculus of variations. The Pontryagin Minimum Principle also leads to the positive semi-definiteness of the Hamiltonian's Hessian matrix as,

$$H_z z(u(t), z(t), \lambda(t), t) \geq 0, \tag{A.30}$$

that is termed the Legendre-Clebsch condition and it is a *second-order necessary condition* for optimality. In addition to the necessary conditions derived from the Pontryagin Minimum Principle, if the strengthened Legendre-Clebsch condition,

$$H_z z(u(t), z(t), \lambda(t), t) > 0, \tag{A.31}$$

holds, it guarantees that $z(t)$ is the local minimizer of the Hessian. The condition in Equation. A.31 is known as the second-order sufficient condition for local optimality. For a purely convex objective functional with respect to the control variables, the necessary conditions are sufficient to guarantee the optimal control variable. We refer the interested readers to [1, 2] for the proof of these theorems.

In summary, the Pontryagin Minimum Principle converts the optimal control problem into a multipoint boundary value problem. That is, the optimality condition $H_z = 0$ results in control expressed as,

$$z(t) = G(u(t), \lambda(t), t), \tag{A.32}$$

and the optimal control variable and corresponding state and adjoint can be computed by solving an ODE system,

$$\begin{aligned} \dot{u}(t) &= f(u(t), G(u(t), \lambda(t), t), \\ \dot{\lambda}(t) &= -H_u(u(t), G(u(t), \lambda(t), t)), \end{aligned} \tag{A.33}$$

with appropriate initial and end time conditions, while Equation. A.31 ensures the optimal control is a minimizer.

A.4.1 Theorem - Pontryagin's Minimum Principle

Let $u^*(t) : [t_0, T] \Rightarrow \mathcal{U}$ be an optimal control trajectory
Let $x^*(t) : [t_0, T] \Rightarrow \mathcal{X}$ be the associated state trajectory from x_0,
Then, there exists a **costate trajectory** $p^*(t) : [t_0, T] \Rightarrow \mathcal{X}$ satisfying,

- Canonical equations with boundary conditions:

$$\begin{aligned} \dot{x}^*(t) &= \Delta_p H(x^*(t), u^*(t), p^*(t)), x^*(t_0) = x_0, \\ \dot{p}^*(t) &= \Delta_x H(x^*(t), u^*(t), p^*(t)), p^*(t_0) = \Delta_x g_T(x^*(T)), \end{aligned} \tag{A.34}$$

- Minimum principle with constant (holonomic) constraint:

$$\begin{aligned} u^*(t) &= arg \min_{u \in \mathcal{U}} H(x^*(t), u, p^*(t)), \forall t \in [t_0, T], \\ H(x^*(t), u^*(t), p^*(t)) &= constant, \forall t \in [t_0] \end{aligned} \tag{A.35}$$

A.4.2 Proof of Pontryagin's Minimum Principle

First Order Necessary Condition for Optimality -
Let f be a continuously differentiable function on $\mathcal{R}^m$ and $\mathcal{U} \subseteq \mathcal{R}^m$ be a convex set. If u^* is a minimizer of $min_{u \in \mathcal{U}} f(u)$, then:
$\Delta f(u^*)^T(v - u^*) \leq 0$, $\forall v \in \mathcal{U}$

Proof - Suppose $\exists\ w \in \mathcal{U}$ with $\Delta f(u^*)^T(w - u^*) \leq 0$. Consider $z(\lambda) := \lambda w + (1-\lambda)u$ for $\lambda \in [0,1]$. Since $\mathcal{U}$ is convex, $z(\lambda) \in \mathcal{U}$ and

$$\frac{d}{d\lambda} f(z(\lambda))|_{\lambda=0} = \Delta f(u^*)^T(w - u^*) \leq 0 \tag{A.36}$$

implies that $f(z(\lambda)) \leq f(u^*)$ for small λ, contradicts that u^* is optimal.

Lemma: Δ - min Exchange
Let $F(t,x,u)$ be a cont.-diffable function of $t \in \mathcal{R}$, $x \in \mathcal{R}^n$, $u \in \mathcal{R}^m$ and let $\mathcal{U} \subseteq \mathcal{R}^m$ be a convex set. Furthermore, assume $\boldsymbol{\pi}^*(t,x) = arg \min_{u \in \mathcal{U}} F(t,x,u)$ exists and is cont.-diffable. Then, for all t and x:

$$\frac{\partial(min_{u \in \mathcal{U}} F(t,x,u))}{\partial t} = \frac{\partial F(t,x,u)}{\partial t}|_{u=\boldsymbol{\pi}^*(t,x)} \Delta_x(\min_{u \in \mathcal{U}} F(t,x,u)) = \Delta_x F(t,x,u)|_{u=\boldsymbol{\pi}^*(t,x)} \tag{A.37}$$

Proof - Let $G(t,x) := \min_{u \in \mathcal{U}} F(t,x,u) = F(t,x,\boldsymbol{\pi}^*(t,x))$. Then:

$$\frac{\partial G(t,x)}{\partial t} = \frac{\partial F(t,x,u)}{\partial t}|_{u=\boldsymbol{\pi}^*(t,x)} + \underbrace{\frac{\partial F(t,x,u)}{\partial u}|_{u=\boldsymbol{\pi}^*(t,x)} \frac{\partial \boldsymbol{\pi}^*(t,x)}{\partial t}}_{since \Delta_u F(t,x,\boldsymbol{\pi}^*)(\boldsymbol{\pi}^*(t+\epsilon,x) - \boldsymbol{\pi}^*(t,x)) \geq 0} \tag{A.38}$$
$$=0$$

A similar derivation can be used for the partial derivative wrt. x.

Step 1 - Hamilton-Jacobi-Bellman Partial Differential Equation gives $J^*(t,x)$
Extra Assumption: The PMP is proved under the assumption that $J^*(t,x)$ and $\boldsymbol{\pi}^*(t,x)$ are cont.-diffable in t and x and $\mathcal{U}$ is convex. These assumptions can be avoided in a more general proof.
If the cost-to-go is cont.-diffable, the HJB PMP is also a necessary condition for optimality:

$$\begin{aligned} J^*(T,x) =& g_T(x), \forall x \in \mathcal{X} \\ =& \min_{u \in \mathcal{U}} \underbrace{(g(x,u) + \frac{\partial}{\partial t} J^*(t,x) + \Delta_x J^*(t,x)^T f(x,u))}_{:= F(t,x,u)}, \forall t \in [t_0, T], x \in \mathcal{X} \end{aligned} \tag{A.39}$$

with, $\boldsymbol{\pi}^*(t,x)$ a corresponding optimal policy.

Step 2 - Δ - min Exchange Lemma
Apply the Δ-min Exchange Lemma to the HJB PDE:

$$\begin{aligned}
0 =& \frac{\partial}{\partial t}(\min_{u\in\mathcal{U}} F(t,x,u)) = \frac{\partial^2 J^*(t,x)}{\partial t^2} + [\frac{\partial}{\partial t}\Delta_x J^*(t,x)]^T f(x,\boldsymbol{\pi}^*(t,x)) \\
0 =& \Delta_x(\min_{u\in\mathcal{U}} F(t,x,u)) \\
=& \Delta_x g(x,u^*) + \Delta_x \frac{\partial J^*(t,x)}{\partial t} + [\Delta_x^2 J^*(t,x)] f(x,u^*) + [\Delta_x f(x,u^*)]^T \Delta_x J^*(t,x)
\end{aligned} \tag{A.40}$$

where, $u^* := \boldsymbol{\pi}^*(t,x)$
Evaluating this along the trajectory $x^*(t)$ resulting from $\boldsymbol{\pi}^*(t,x^*(t))$:

$$\dot{x}^*(t) = f(x^*(t),u^*(t)) = \Delta_p H(x^*(t),u^*(t),p)^T, x^*(0) = x_0 \tag{A.41}$$

Step 3 - Evaluate along $x^*(t), u^*(t)$
Evaluating the results of **Step 2** along $x^*(t)$:

$$\begin{aligned}
0 =& \frac{\partial^2 J^*(t,x)}{\partial t^2}|_{x=x^*(t)} + [\frac{\partial}{\partial t}\Delta_x J^*(t,x)]^T \dot{x}^*(t) \\
=& \frac{d}{dt}(\underbrace{\frac{\partial J^*(t,x)}{\partial t}|_{x=x^*(t)}}_{:=\, r(t)}) = \frac{d}{dt} r(t) \Rightarrow r(t) = C.\forall t
\end{aligned} \tag{A.42}$$

and

$$\begin{aligned}
0 =& \Delta_x g(x,u^*)|_{x=x^*(t)} + \frac{d}{dt}(\underbrace{\Delta_x J^*(t,x)|_{x=x^*(t)}}_{=:\, p^*(t)}) + [\Delta_x f(x,u^*)|_{x=x^*(t)}]^T [\Delta_x J^*(t,x)|_{x=x^*(t)}] \\
=& \Delta_x g(x,u^*)|_{x=x^*(t)} + \dot{p}^*(t) + [\Delta_x f(x,u^*)|_{x=x^*(t)}]^T p^*(t) \\
=& \dot{p}^*(t) + \Delta_x H(x^*(t),u^*(t),p^*(t))
\end{aligned} \tag{A.43}$$

Step 4 -
The boundary condition $J^*(T,x) = g_T(x)$ implies that $\Delta_x J^*(T,x) = \Delta_x g_T(x)$ $\forall x \in \mathcal{X}$ and thus $p^*(T) = \Delta_x g_T(x^*(T))$,
from the HJB PDE we get,

$$-\frac{\partial J^*(t,x)}{\partial t} = \min_{u\in\mathcal{U}} H(x,u,\Delta_x J^*(t,.)) \tag{A.44}$$

which along the optimal trajectory $x^*(t), u^*(t)$ becomes,

$$-r(t) = H(x^*(t), u^*(t), p^*(t)) = C \tag{A.45}$$

where, C is a constant
Finally, a quick note,

$$\begin{aligned} u^*(t) =& arg \min_{u \in \mathcal{U}} F(t, x^*(t), u) \\ =& arg \min_{u \in \mathcal{U}} g(x^*(t), u) + [\Delta_x J^*(t, x)|_{x=x^*(t)}]^T f(x^*(t), u) \\ =& arg \min_{u \in \mathcal{U}} g(x^*(t), u) + p^*(t)^T f(x^*(t), u) \\ =& arg \min_{u \in \mathcal{U}} H(x^*(t), u, p^*(t)) \end{aligned} \tag{A.46}$$

VITA

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