INJECTION CURRENT MODULATED PARITY-TIME SYMMETRY IN COUPLED SEMICONDUCTOR LASERS

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ABSTRACT

This research investigates the characteristics of Parity Time symmetry breaking in two optically coupled, time delayed semiconductor lasers. A theoretical model is used to describe the controllable parameters in the experiment and intensity output of the coupled lasers. The PT parameters we control are the spatial separation between the two lasers, the frequency detuning, and the coupling strength. We find that the experimental data agrees with the predictions from the theoretical model confirming the intensity behaviors of the lasers, and the monotonic change in PT-threshold as a function of coupling scaled by the time delay.

CHAPTER 1. INTRODUCTION: SEMICONDUCTOR LASERS AND PARITY-TIME SYMMETRY MODEL

Parity Time (PT) symmetry is used in open quantum systems where the Hamiltonian is non-Hermitian [1], but the eigenvalues may still be real. For a system to be PT symmetric it must be invariant under the Parity (P) and Time (T) operators, where the Parity operator takes a right-handed system and turns it into a left-handed one , and the time operator reverses time.

Recently, Wilkey and co-workers [2] have demonstrated that a pair of coupled semiconductor lasers can serve as a platform for PT-symmetry. The equations that are used to model a pair of delay coupled semiconductor lasers, referred to as the modified Lang and Kobayashi (LK) rate equations are introduced below.

$$\frac{dE_1}{dt} = (1 + i\alpha)N_1(t)E_1(t) + i\Delta\omega E_1(t) + \kappa e^{-i\theta\tau}E_2(t-\tau) \tag{1.1}$$

$$\frac{dE_2}{dt} = (1 + i\alpha)N_2(t)E_2(t) - i\Delta\omega E_2(t) + \kappa e^{-i\theta\tau}E_1(t-\tau) \tag{1.2}$$

$$T\frac{dN_1}{dt} = J_1 - N_1(t) - (1 + 2N_1(t))|E_1(t)|^2 \tag{1.3}$$

$$T\frac{dN_2}{dt} = J_2 - N_2(t) - (1 + 2N_2(t))|E_2(t)|^2 \tag{1.4}$$

In these equations, $E_{1,2}$ are the intracavity electric fields of the two lasers, $N_{1,2}$ are the carrier inversions of each laser, $\Delta\omega = (\omega_1 - \omega_2)/2$ is the relative detuning between the lasers, τ is the time-delay due to physical separation between the lasers, κ is the coupling rate of light from one laser into another and $J_{1,2}$ are the pump currents to each laser. The time delay in the coupling, τ, comes from the physical distance between the two lasers. The linewidth enhancement factor is represented by α, and $J_{1,2}$ is proportional to the current divided by the threshold current, $I_{1,2}/I_{th}$. The semiconductor lasers (SCLs) operate at almost identical frequencies and the varying electric fields are defined in a symmetric frame of reference, $\theta = (\omega_1 + \omega_2)/2$ [3]. Finally, $T = \tau_s/\tau_p$ is the ratio of the carrier lifetime to the photon lifetime [4].

The parameters we control are the pump currents $J_{1,2}$, the frequency detuning $\Delta\omega$, the temporal separation between the two lasers, τ, and the feedback strength κ.

In the LK equations above, the first term in equations 1.1 and 1.2 accounts for the growth or decay of the electric field depending on *N*. The second term in these equations causes a phase shift due to α, the line width enhancement factor. The term with detuning, $\Delta\omega$, describes the change in electric field due to differences in optical frequency between the two lasers, and the final term describes the delayed coupling between the two fields [5]. It is worth noting that the electric field in the final term is evaluated at a time of $t - \tau$, which means that at time *t* SCL 1 is affected by the electric field emitted by SCL 2 at an earlier time τ.

To motivate the existence of PT-symmetry in our system, first, we will assume a steady state approximation by setting $\frac{dN_{1,2}}{dt} = 0$. This effectively eliminates equations 1.3 and 1.4. While this approximation is extreme, it does serve to simplify the system into two coupled rate equations that can be written in the form:

$$\begin{pmatrix} \frac{dE_1}{dt} \\ \frac{dE_2}{dt} \end{pmatrix} = \begin{pmatrix} (1+i\alpha)N_1(t) + i\Delta\omega & \kappa e^{-i\theta\tau} e^{-\lambda\tau} \\ \kappa e^{-i\theta\tau} e^{-\lambda\tau} & (1+i\alpha)N_2(t) - i\Delta\omega \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} \tag{1.5}$$

By setting $\tau = 0$ we produce a PT symmetric system as seen below.

$$\begin{pmatrix} \frac{dE_1}{dt} \\ \frac{dE_2}{dt} \end{pmatrix} = \begin{pmatrix} i\Delta\omega & \kappa \\ \kappa & -i\Delta\omega \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} \tag{1.6}$$

The eigenvalues of this PT symmetric system are

$$\lambda = \pm\sqrt{\kappa^2 - \Delta\omega^2} \tag{1.7}$$

If we use solutions for the electric field that follows the form, $E \propto e^{-\lambda t}$ we expect different behaviors depending on whether the eigenvalues are real or complex. In the regions where $\kappa^2 \geq \Delta\omega^2$ we expect growth or decay of the electric field, a region of unbounded behavior. In regions where $\kappa^2 \leq \Delta\omega^2$ we expect oscillatory behavior of the electric field, a region with bounded behavior. Since the intensity is modulus squared of the electric field this is also what we expect from the intensity of the coupled SCLs.

As mentioned above, this model has discarded many important parameters. However, as will be discussed in chapter 3, this simplified model is still viable in region where the carrier densities are stable, and the expected behavior of the system will be confirmed by experiment.

CHAPTER 2. EXPERIMENTAL SETUP

Two single mode semiconductor lasers (SCL 1 and SCL 2) are optically coupled using mirrors (M1 and M2), and separated by a cavity of length τ, where τ is proportional to the distance the light travels (see Fig 2.1).

$$\tau = \frac{L}{c} \tag{2.1}$$

Here, c is the speed of light in vacuum, and L is the distance between lasers. To determine τ the physical distances from SCL1 to M1, M1 to M2, and M2 to SCL2 were measured and summed to give an $L = 0.42$m. By following the equation above and dividing by the photon lifetime of 10ps $\tau = 139$.

SCL1 and SCL2 are identical to one another except for small differences in optical frequency, and threshold pump current. The coupling is controlled by a variable neutral density filter (VND) and an independent third semiconductor laser (SCL3) is used to measure the coupling, as described by Wilkey [2]. SCL3 allows power transmission through the VND to be measured.

This is done by aligning SCL3 to pass through the same point on the VND as the coupled SCLs. Then, by measuring the power of the light from SCL3 that is transmitted through the VND, P, power in microwatts, and intensity, M, as a voltage output an equation relating the two can be written as.

$$P = \sigma M + 2.95\mu W \tag{2.2}$$

Here, $\sigma = 10.01\mu$W/Volt is a property of SCL3, and M is the intensity of SCL3 measured as a voltage output. The coupling strength, κ, is given by

$$\kappa \equiv \frac{1-r^2}{r\tau_{in}\tau_p\xi} \tag{2.3}$$

where r is the reflectivity of the laser facets, τ_{in} is the internal round-trip lifetime, τ_p represents the photon lifetime, and ξ^2 is the ratio of the power of the photons incident upon the VND, P_{in}, to the power of photons transmitted through the VND, P_{out}, which is described by equation 2.2.

$$\xi = \sqrt{\frac{P_{in}}{P_{out}}} \tag{2.4}$$

By combing the constants that form the definition of κ from equation 2.3, and using equation 2.4 we can calculate κ through the following equation.

$$\kappa = 0.0511\sqrt{\frac{P_{out}}{P_{in}}} \tag{2.5}$$

Scanning the pump current causes changes in the optical frequency. The dependency of the optical frequency with change in pump current is given by the following relation.

$$\omega = \omega_0 - k\Delta J \tag{2.6}$$

where ω_0 is the frequency of the SCL 2 at threshold, and ΔJ is the pump current with threshold pump current subtracted. The slope, k, is a property of the SCL, and was determined to be 1.90 GHz/mA. SCL 1 is held at five percent above threshold frequency while SCL 2 is swept slowly from J_2=1.3I_{th} to J_2=0.8I_{th}, where J_2 represents the pump current of SCL 2. This change in frequency is so small compared to the threshold frequency of both SCLs, we assume a linear dependence. During this sweep, the voltage output of the current controller is proportional to the pump current. The equation relating the two was experimentally determined to be

$$V = \gamma J + 1.24Volts \tag{2.7}$$

where γ = 51.569 Volts/mA where, V, represents the voltage output of the current controller, and, J, is the pump current

The scanned pump current causes changes in not only optical frequency but also intensity. Two glass slides allow for a little bit of light from the SCLs to be measured by two 1 GHz photodiodes (PD1 and PD2). A 100MHz oscilloscope allows the measurement of intensity in conjunction with the voltage output of the scanning laser. Finally, a temperature controller maintains a constant temperature in the laser cavity to ensure that the power output of the laser is only affected by changes in pump current.

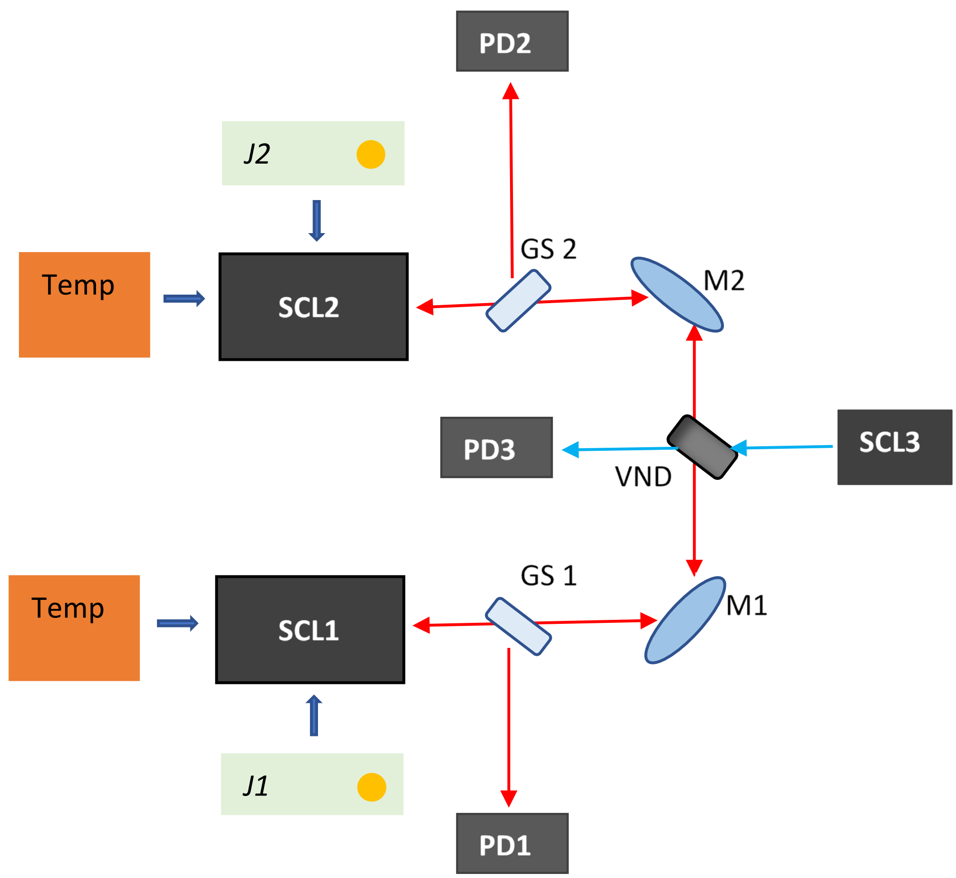

Figure 2.2: experimental setup. SCL: semiconductor laser, GS: glass slide, M: mirror, PD: photodiode, VND: variable neutral density filter. Temp: laser cavity temperature controller, J: pump current controller.

CHAPTER 3. RESULTS AND DISCUSSION

As discussed, a prediction of the PT model is a transition from bounded to unbounded behavior when the detuning frequency is equal to the coupling strength, $|\Delta\omega| = \kappa$. To experimentally determine this behavior the procedure from the previous chapter was carried out in which the detuning frequency is changed through current modulation, and all other parameters held constant. The results are shown in Fig. 3.1 where each plot show the intensities of the two lasers as a function of detuning. The intensities in this figure are in arbitrary units and the detuning is shown after scaling to the photon lifetime of 10ps. The vertical line in each figure shows the PT-threshold, i.e., where coupling equals detuning and where the transition from oscillatory behavior to growth/ decay behavior in the intensity is expected in the absence of a time delay. However, there is a gap between the PT transition and the coupling. This gap arises from time delay that is inherent to our system.

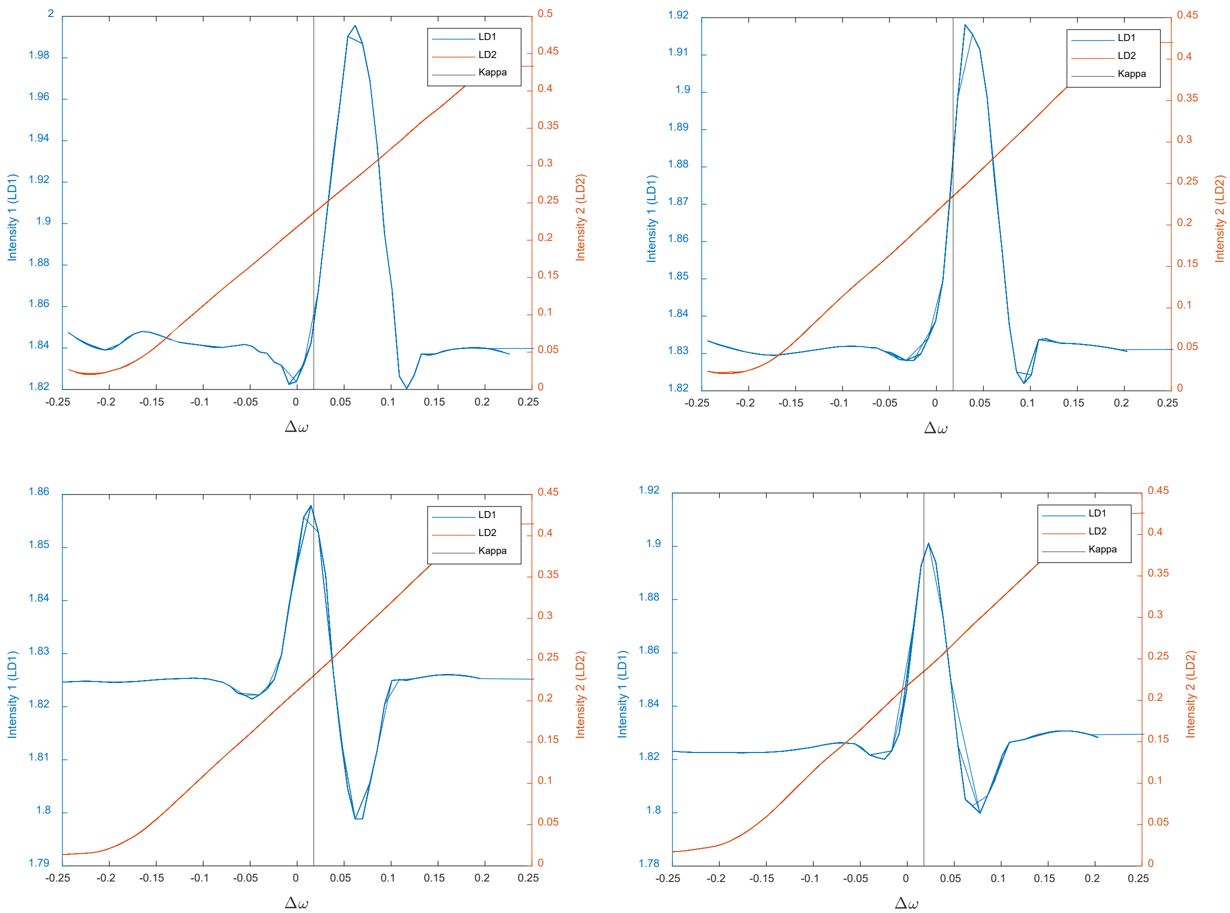

Figure 3.3: Experimental data confirming the presence of a PT transition. Data showing SCL intensity (arbitrary units) vs. $\Delta\omega$. All data was taken at τ=139 (scaled to photon lifetime of 10ps). Here the SCL 2 (orange line) is being decreased from $1.3I_{th}$ to $0.8I_{th}$. Top left was taken at κ=0.0274. Top right was taken at κ=0.0231. Bottom left was taken at κ=0.0195. Bottom right was taken at κ=0.0244.

SCL1 is kept at a constant current and temperature. As SCL2's pump current is decreased, the frequency of SCL2 is changed. Thus, the sweep of SCL2's pump current causes a change in the detuning. This change in pump current also causes a decrease in intensity for SCL2. The intensity of SCL1 shows an abrupt increase near the PT threshold as predicted by the theoretical model. For all plots in Figure 3.1 we see that beginning of the growth does not happen at black line representing where the PT transition as would be expected. As discussed above this is anticipated and is caused by the time delay in our experiment.

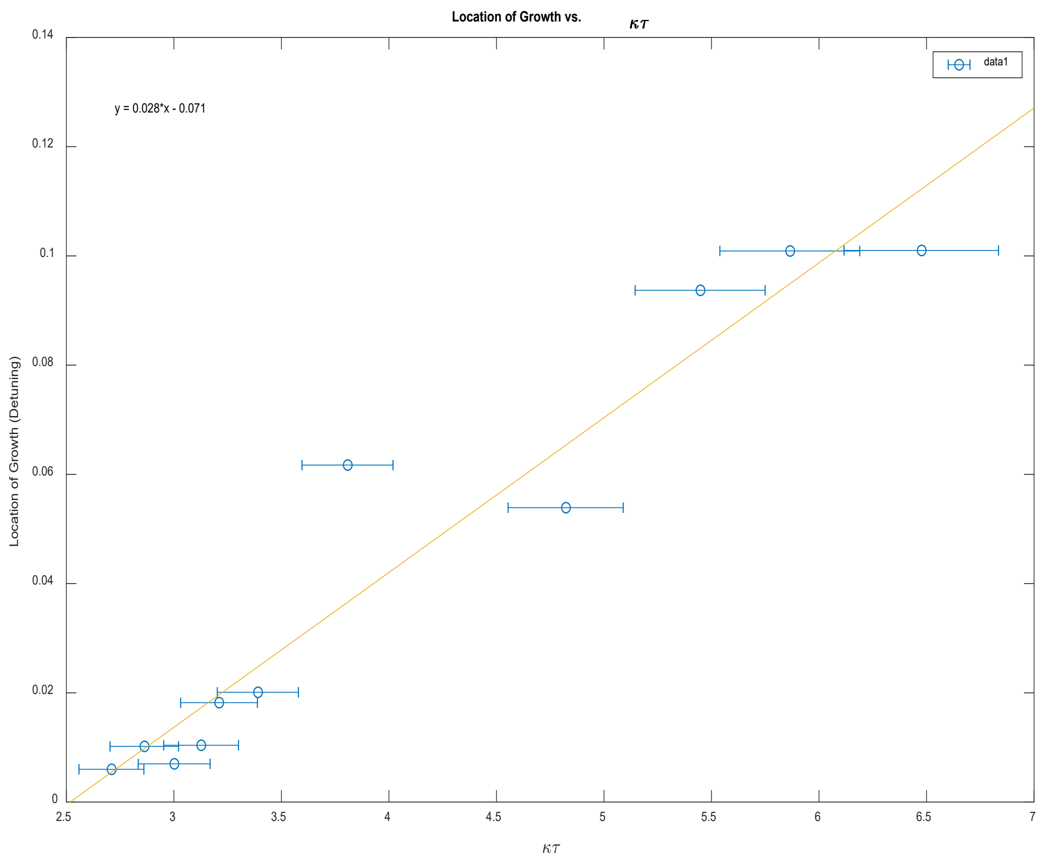

Figure 3.4: Experimental plot of the detuning location where the growth begins for increasing values of κ. The blue dots represent varying values of $\kappa\tau$ plotted verses detuning. The slope of the best fit line, illustrated in yellow is 0.028. Error in $\kappa\tau$ was determined using 95% confidence bands in measurements of the output voltage see equations 2.2-2.5. Errors in detuning where not considered as the value for each was taken directly from each plot see Figure 3.1

Finally, another prediction of the simplified model is a monotonic increase in the location of the PT transition as coupling strength increases. Since, the PT transition occurs where $\Delta\omega = \kappa$, in the absence of a time delay if we were to plot $\Delta\omega$ versus κ we would expect this linear trend to have a slope of one. Thus, as κ is increased, the PT transition will increase to higher values of detuning.

In figure 3.2 we show that this monotonic increase is still present even with the time delay in system. Here we have plotted the location of the PT transition in detuning versus the coupling strength scaled by the time delay. Where the start of the PT transition was identified by selecting the value of detuning at which the intensity is at a minimum preceding the expected PT transition. Due to the time delay, the slope is no longer equal to one. This plot, though not a perfect monotonic increase shows the linear trend that is expected from our simplified model, until it breaks down for higher values of $\kappa\tau$. The departure from linearity is primarily due to the back scattering of reflected photons off the laser facet causing those photons to be reinjected into the SCL that original emitted them. It is also worth noting that as κ becomes very small the experimental determination of the PT transition becomes impossible due the decrease in the intensity changes no longer being detectable. Thus, the exact behavior in the region of κ less than 0.0195 was not determined.

CHAPTER 4 SUMMARY

In conclusion we found that our experiment verified the expected intensity behaviors of delay coupled SCLs that our simplified PT model predicted. While the smoothing of the data lead to a loss of some expected behaviors, specifically in the region where oscillatory behavior was expected, the growth in intensity marking the PT transition was obvious. Furthermore, experiments confirmed predictions of the model showing a monotonic increase in the detuning at which the PT transition occurs as coupling strength increased. While higher values of κ stray from linearity, and values near zero remain immeasurable, the overall trend is linear and remains as a testament to our model.

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