# MAKING MATH REAL: EARLY CHILDHOOD TEACHERS EXPERIENCES LEARNING AND TEACHING MATHEMATICS 

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This is dedicated to Josie and Patsy, Anne, and all the dedicated early childhood teachers who pour all they have into the children in their care and classrooms. This is dedicated to Laura for pouring all she had into helping me finish.

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#### Abstract

Early childhood teachers pursuing associate degrees often repeated the college algebra course, demanding, "Why do we have to take this? We don't teach algebra!" Expectations for their training were not well-aligned with their mathematics preparation or teaching work. I have taught the mathematics courses and young children and have worked for an early childhood practice, policy, and research agency. I wanted to learn about these teachers' experiences as mathematics learners and teachers, with a goal to share the complex nature of their work with teacher educators and other stakeholders to identify better avenues for their mathematics training. I explored the questions: (1) What role, if any, do mathematical learning experiences play in early childhood teachers' mathematics teaching practice? (2) In what ways do their voices contribute to the professional dialogue regarding teaching mathematics with young children?

Dewey's (1938/1998) experience construct provided lenses to examine early childhood teachers' experiences learning and teaching mathematics. Continuity, interaction, social control, freedom, purpose, and subject matter provided insights and situated teachers' experiences within a disparate patchwork of settings and policies. Two family childcare providers participated in this narrative inquiry (Clandinin \& Connelly) through an interview on their experiences learning and teaching mathematics and three classroom observations. After analyzing data for Dewey's (1938/1998) experience constructs, I used narrative analysis (Polkinghorne, 1995) and teaching images (Clandinin, 1985) to write an emplotted narrative for each teacher, Josie and Patsy.

Josie told a turning point story (Drake, 2006) of making mathematics "real," influencing her mathematics teaching practice as she integrated "real" mathematics into everyday activities. Patsy's appreciation for mathematics and building was seen in her story of a child explaining he used the wide blocks for his base, elaborating, "He's telling me HOW he's building." While Josie and Patsy had few opportunities to learn about teaching mathematics with young children, they were eager to learn. I propose a training for early childhood teachers, iteratively working as a group to investigate a personal mathematics teaching puzzle or celebration, building on their mathematical personal practical knowledge. Adding my own story to those of the teachers, like Josie's and Patsy's, of our work together, will add to my understanding and development of my practice as a curriculum maker (Clandinin \& Connelly, 1992), as early childhood teachers' voices contribute to the professional dialogue about teaching mathematics with young children.


## CHAPTER 1 SITUATING THE RESEARCH PROBLEM IN THE EARLY CHILDHOOD EDUCATION CONTEXT

My Story - Why Is This Problem Important?

In my work with a state-funded scholarship program for early childhood teachers pursuing associate degrees, I found that teachers often repeated the required college algebra course at least once and questioned the requirement as the course content seemed disconnected to their work teaching young children. Teachers who were failing the course would demand, "Why do we have to take this? We don't teach algebra!" My co-workers were overheard responding, "It's okay, we early childhood people just aren't good at math." The required mathematics courses felt like "gatekeepers" (Moses \& Cobb, 2001) to the teachers, rather than content that was useful in their work with young children. Teachers' experiences learning mathematics left them feeling that mathematics is hard, not fun, and not useful in their work (Carroll, 1994; Richardson \& Bofferding, 2015). By asking why they had to take college algebra courses, in which they struggle and whose content was not related to the work they did with young children, the early childhood teachers implied that expectations for their education, training, and professional development were not well-aligned with their mathematics preparation or mathematics teaching work.

Because I had been both an early childhood teacher and an instructor of the required mathematics courses, I was curious about these statements. I wondered what experiences early childhood teachers have learning and teaching mathematics and what their experiences mean for their teaching mathematics with young children. Further, working for a state-affiliate of the National Association for the Education of Young Children (NAEYC) provided opportunities to interact with a variety of early childhood advocates and stakeholders, including lobbyists dedicated to early childhood education issues, board members of the state-affiliate, early childhood education teacher educators, representatives from the state Families and Social Services Administration (FSSA), and T.E.A.C.H. Early Childhood $\circledR^{\circledR}$ Scholarship developers and implementers (T.E.A.C.H. Early Childhood © National Center, 2016). With this broadened view of the early childhood education field and associated stakeholders, I wondered if there was a way for those who determined the coursework to understand the teachers' perspective and therefore plan the associate degree requirements to better align with teachers' learning experiences and their work with young
children and mathematics. I wanted to learn more about these teachers' lived experiences as learners and teachers of mathematics, with the goal of identifying better avenues for working with these teachers and sharing their stories and the complex nature of their work with early childhood teacher educators, policy makers, and other stakeholders.

## Early Childhood Context

## What Is Early Childhood Education?

Early childhood is a unique educational context with particular needs and a disparate patchwork of settings and goals (Ayers, 1989; Parks \& Wager, 2015). The National Association for the Education of Young Children (NEAYC) defines the early childhood period as birth through age 8 years (Copple \& Bredekamp, 2009), which is the largest age range for an educational grade band. An early childhood teacher might teach infants and toddlers who are zero to 36 months old; preschool children who are three to five years old; kindergarten children who are five to six years old; or children in early primary grades (first, second, and third) who are six to eight years old. These teachers work in a variety of public and private, as well as paid and unpaid, settings, including state-licensed childcare centers and family childcare homes, unlicensed but registered childcare ministries and family childcare homes, Head Start centers, and in settings labeled as preK, prekindergarten, or preschool (National Research Council [NRC], 2009; National Survey of Early Childhood Education [NSECE], 2013). Outside the regulations that each setting observes, there is great variability across the types of settings, as well as across individual centers, ministries, preschools, and homes within each category.

## Who Are Early Childhood Teachers and How Are They Trained?

The National Survey of Early Care and Education (NSECE, 2013), the most comprehensive and recent data on the early childhood workforce, highlights some of the differences between early childhood and K-12 teachers. There were approximately 1 million center-based teachers and caregivers serving children in 130,000 programs in 2012. There were also about 3.8 million home-based teachers and caregivers, with 2.7 unpaid friends and relatives who are often outside regulatory and training systems. While children from birth through age five were taught and cared for by 4.8 million teachers and caregivers, students from age six through
adulthood were taught by 5.6 million teachers (Bureau of Labor Statistics [BLS], 2016a). In addition, $96 \%$ of the childcare workforce is female, while only $68 \%$ of elementary through postsecondary teachers are female. Mean annual wages for preschool teachers are $\$ 26,870$ and are $\$ 22,930$ for childcare workers. Mean annual wages, however, for $\mathrm{K}-12$ to post-secondary teachers range from $\$ 59,270$ to $\$ 81,880$ (BLS, 2016b). These differences reflect the nature of early childhood education as "women's work" and the field's perceived lack of professionalism (Kim \& Reifel, 2010).

Training for early childhood teachers is different in several ways from that of $\mathrm{K}-12$ teachers. For example, minimum training requirements for K-12 teachers is rather consistent across the Unites States, while minimum training requirements for early childhood teachers are often inconsistent across states (Gomez et al., 2015; Pianta et al., 2009). Some states require that lead teachers possess only a four-course professional credential, although various federally- and statefunded programs, such as Head Start and the Child Care and Development Fund (CCDF), often require more training, including two-year associate and four-year bachelor's degrees. Regardless, $53 \%$ of childcare center teachers had some level of college degree, with $26 \%$ having a bachelor's degree, and $9 \%$ having a graduate degree (NSECE, 2013). Training is often started and completed after a teacher is hired to work in an early childhood education setting; very little training is preservice. In the past, the content of credential and degree programs focused mostly on child development and social-emotional development, with little to no content focused on subject matter, such as literacy or mathematics. However, four-year bachelor's degree programs now often contain some methods and may also contain some content training related to the teaching and learning of mathematics for young children. Early childhood teachers also receive a large amount of training through professional development workshops in their workplace; local agencies such as resource and referral agencies and the United Way; and local, state, and national conferences.

## What Is Early Childhood Mathematics Professional Development?

Although early mathematics ability is a good predictor of later school achievement, early childhood teachers' training in mathematics is insufficient in duration, depth, and breadth (Duncan et al., 2007; Early et al., 2007; Parks \& Wager, 2015). However, foundational mathematics, the mathematical thinking and learning preceding written symbols, is clearly not addressed by higher-
level mathematics courses such as college algebra but should be part of coursework focused on the numerical and spatial abstraction that precedes written symbols (McCray \& Chen, 2010). Goals put forth in professional preparation and early mathematics content standards also may not reach preservice and inservice teachers through training, much less through their practices (Buettner et al., 2016; NAEYC, 2009; NAEYC \& NCTM, 2002; Parks \& Wager, 2015; Simpson \& Linder, 2014). Because preservice and inservice teachers' previous negative mathematics experiences may cause fear and impede their access to postsecondary education, mathematics professional development might strive to begin with teachers' current understanding, rather than identifying professional development that attempts to give teachers the skills they "should" have (Bates et al., 2013; NAEYC, 2009; Pianta et al., 2009). To that end, Sarama and DiBiase (2004) developed a list of characteristics of effective mathematics professional development for early childhood teachers.

## Who Cares about Early Childhood Education Research and Why?

With early childhood education at the intersection of child development, education, policy, and prevention science, the public, business leaders, and policymakers have become increasingly interested in the field in recent years due to benefits these stakeholders have seen emerge (Duncan et al., 2007; Pianta et al., 2009). Ongoing research in neurology, psychology, biology, cognitive science, human development, education, and economics continues to find that high-quality early childhood education programs return more in benefits than invested to the children and families who participated, as well as to society in general (Belfield et al., 2006; Heckman, 2008; 2011; Grunewald \& Rolnick, 2010; Shonkoff \& Phillips, 2000). Even military leaders call investments in early childhood education a matter of national security since $75 \%$ of 17 - to 24 -year-olds are not eligible for military service due to issues often prevented by high-quality early childhood education (Christeson et al., 2013). As policymakers embrace ameliorative effects of early childhood education in new policies and programs, Darling-Hammond (1990) cautions them to consider, from the teachers' perspective, how policies will be transformed into teacher actions.

Because quality teaching requires well-trained teachers (Grunewald \& Rolnick, 2010), several parties have a stake in understanding factors that affect early childhood teacher training. Results of research on training for early childhood teachers is mixed, although progress continues.

For example, while Early et al. (2007) found no significant association between early childhood teachers' education and major with classroom quality or child outcomes, Buettner et al. (2016) found that two- and four-year degree programs have similar course coverage, which could be a factor in Early et al.'s results. Therefore, since many teachers receive training after being hired, program administrators and policymakers might work to understand factors in workplace environments, opportunities, practices, and policies that support teachers' on-going and on-the-job learning (Whitebook \& Ryan, 2011). Early childhood teacher educators might work to understand which aspects of prospective and inservice teachers' mathematics-related attitudes, knowledge, behavior, and pedagogical practices are affected by various professional development opportunities and activities (Parks \& Wager, 2015; Pianta et al., 2009). And the general public should be educated about the importance of foundational, preschool mathematics in order to encourage policymakers and funders to support related mathematics teaching and learning initiatives (McCray \& Chen, 2010). Each of these factors could be better understood by listening to early childhood teachers' voices.

Shonkoff and Phillips (2000) described two complementary social agendas that can drive early childhood research: economic, political, and social interests; and ethical and moral values, while policy and funding hail from all levels of government and not-for-profit agencies . Social viewpoints range from understanding early childhood education as babysitting to a fix for all manner of educational and social ills (Ginsburg, Lee, \& Boyd, 2008; Shonkoff \& Phillips, 2000). My study will take place in these policy, funding, and social viewpoints quagmires, with the hope that better understanding how these two early childhood teachers take up training and use it in their practice will shine a small light on how to bridge the gap between the macro-level policy and standards landscape and the micro-view of early childhood teachers' lived experiences within that macro-level environment.

## Problem: Missing Teacher Voices

Because teachers do not always implement the training they receive, their professional development activities do not always improve practice (Lieber, Butera, Hanson, Palmer, Horn, Czaja, Diamond, et al., 2009). In response, Dewey (1938/1998) noted that "the powers and purposes of those taught" (p.44) should be considered. Therefore, researchers called for building
on teachers' prior knowledge, experiences, skills, motivations, and dispositions to respond to their needs when planning professional development programs, preferably incorporating practice-based learning activities (Bowman, Donovan, \& Burn, 2001; Cataldo, 2013; Chen \& McCray, 2012; Huss-Keeler \& Brown, 2007; Perry \& MacDonald, 2015; Sarama \& DiBiase, 2004; Whitebook \& Ryan, 2011). Researchers should also investigate how early childhood teachers perceive mathematics in their classrooms, interpret children's activity, think critically about their practice in light of children's learning, and understand curriculum and its underlying mathematics (Ginsburg, Lee, \& Boyd, 2008; Herron, 2010). They also suggest exploring sources teachers draw on when making decisions about their practice (Brown, 2005) and memories of early learning experiences (Bowman, Donovan, \& Burn, 2001). In summary, merely increasing the quantity of early childhood teachers' training will not automatically improve teacher practice or children's outcomes, so researchers call for detailed studies that examine the ways early childhood teachers make sense of their training and use it in their work with young children, both in the broad multifaceted early care and education system, and in their individual mathematics practice (Early et al., 2007; Gomez, Kagan, \& Fox, 2015; Simpson \& Linder, 2014).

## Purposes

I have always been fascinated by stories from my elders. My father's stories of growing up in Indonesia in the 1920s and surviving the Japanese prison camps were exotic, terrifying, and heartbreaking. My mother's relatives, who lived nearby, were born in the late 1890s, when stories were the way that family history was held. The women did a lot of manual work together, such as quilting and food preservation, during which many stories, and theories, were shared. Stories from these previous generations, while not exotic in the ways my father's were, gave my sister and me insight into living through the Great Depression, in less technologically-advanced times, as well as into a culture that was more homogenized in ways and yet included my great-grandparents from Germany. This was my upbringing, these are my roots, which shed light on my inclination to "story" life, particularly my puzzles and questions. These stories of particular experiences resurface often, providing context and insight in the present, to all manner of personal and family situations.

Similarly, in narrative inquiry, the participant and researcher make meaning through storying experiences (Clandinin \& Connelly, 2000). Through the storying process, experience is examined and assimilated and accommodated into the thinking/mental structures of researcher and participant, with each party only owning their own story, no one else's. This methodology is grounded in Dewey's (1938/1998) ideas on experience as the origin of knowledge, with story as the generator of meaning and knowledge (Caine, Estefan, \& Clandinin, 2013; Clandinin \& Connelly, 2000). Researchers live alongside participants, living, telling stories of experiences, and growing and changing as stories are retold and relived, generating new understanding and knowledge.

In my own review of the literature on early childhood mathematics professional development, a majority of studies described researchers' objectives rather than teachers' descriptions of their learning or teaching experiences. Parks and Wager (2015) theorize that this is due to funding that privileges large-scale clinical trials of child outcomes over the content and pedagogy needed by teachers. Studying teachers holistically allows a broader understanding of teaching and learning that also enriches the collective understanding (Ayers, 1989).

In this work, I delve deeper into teachers' lived experiences with mathematics to understand nuances of their experiences (Pinnegar \& Daynes, 2007) by exploring two questions: (1) What role, if any, do mathematical learning experiences play in early childhood teachers' mathematics teaching practice? (2) In what ways do their voices contribute to the professional dialogue regarding teaching mathematics with young children? This is a multi-faceted puzzle that challenges the notion, or grand narrative, that, "we early childhood people just aren't good at math." We can understand the teachers through stories, (i.e., narrative inquiry), using Dewey's (1938/1998) experience constructs.

## CHAPTER 2 DEWEY'S CONCEPTUALIZATION OF EXPERIENCE AS A LENS FOR INTERPRETING TEACHER WORK

## Theoretical Framework

Experience is the theoretical framework guiding this study. So much of my understanding goes back to my early learning about experience from von Glasersfeld (1995) and Steffe's (1996) writing about radical constructivism. Their work gave me an appreciation of experience as the foundation of learning, as well as a desire to learn about the experiences of those learning mathematics and led to the idea of making models of teachers' thinking before planning professional development for them, similar to Steffe's method of working with children. My interest in working this way has its roots in my work teaching the algebra review course for students not ready for college algebra. These students often shared their "mathematics baggage stories" with me, and I felt my heart ache for them. I wanted to erase their negative experiences so they could move forward in their mathematics learning with eagerness and confidence. When I worked with early childhood teachers, I wanted to erase their sense that "we early childhood teachers are just not good at math," and again, leave them with eagerness and confidence in their ability to learn mathematics. As I studied learning theories and inquiry methodologies throughout my coursework, I found that experience was the construct that repeatedly resonated with my prior work with young children and early childhood teachers; experience just made sense to me regardless of the topic of the course. As I became more conscious of a vague awareness that experience was a common construct in my learning theory, curriculum, and inquiry courses, I started thinking about epistemology.

Because radical constructivism explains how an individual constructs understanding from his or her experiences, meaning it is a construct of what happens in an individual's mind (von Glasersfeld, 1995), empirical evidence is difficult to gather. John Dewey (1938/1998), however, looked at experience through lenses that are useful and appropriate for analysis. The two main principles of Dewey's experience construct are continuity and interaction. Other factors that play a role in experience include social control, freedom, purpose, and subject matter. Dewey's constructs are a useful lens to analyze teachers' past experiences, including ways these might limit them, as well as how their present experiences teaching mathematics with young children build on their curiosity and dispositions regarding teaching young children and allow for growth in teachers'
own mathematical thinking. These experiences also include autonomy and freedom from objective criteria that may have limited their mathematical development.

## Continuity

Dewey's (1938/1998) first principle of experience is continuity. Continuity means that a current experience is influenced by past experiences and will influence future experiences, expanding or constraining options for future experiences. For example, "mis-educative" (p. 13) experiences can arrest or distort growth by removing opportunity to reason and act autonomously. On the other hand, experiences that support autonomy, curiosity, and initiative lead to future growth. For education grounded in experience, teachers must take care to plan experiences that have the potential to "live fruitfully and creatively" (p. 16) in later experiences. Therefore, continuity implies that growth can begin again, even after it was arrested by earlier mis-educative experiences. Every experience can be judged by what it moves toward. Because the individual is modified by each experience, a slightly different person enters each new experience. As individuals move through the experiences of their lives, emotional and intellectual attitudes form and affect basic sensitivities and ways of operating and responding to conditions encountered.

Researchers study teachers' past and present mathematics learning and teaching experiences to gain insight into "how and why teachers interpret and implement curriculum materials in particular ways" (Drake, 2006, p. 601). These studies document strong internal and external influences. Internal influences from teachers' own learning experiences make up their teaching philosophies, and external influences from policies, such as standardized testing in upper grades, can dictate elements of their teaching practices (Jung and Reifel, 2011). While external influences do play a large role in teachers' curricular and pedagogical choices, internal influences collected over their lifetimes and that inform their teaching philosophies can be seen shaping their mathematics teaching practices in subtle, yet significant, ways.

## Interaction

Interaction is also a way to interpret an experience's educational function and force (Dewey, 1938/1998). Through interaction, an experience includes an interplay of two factors, objective and internal conditions, creating a situation. In a situation, interaction happens between
an individual and his or her environment, which consists of objects and other people that "interact with personal needs, desires, purposes, and capacities to create the experience which is had" (p. 41). Even when an individual is imagining or working from memory, the individual interacts with objects in the mind.

For the youngest children, mathematical situations (Dewey, 1938/1998) emerge from their everyday experiences interacting with people and objects in their environment, such as counting toes during diapering, Cheerios when learning to eat solid food, and napkins and cups for each child during preschool lunch (Clements \& Sarama, 2014; McCray \& Chen, 2010; Parks, 2015; Vygotsky, 1986). Making the mathematics that young children encounter explicit is the teacher's role, as teachers recognize and build on children's mathematical play situations to support children in constructing mathematical ideas during play interactions (NAEYC \& NCTM, 2002; Parks \& Blom, 2013). Intentional interaction situation opportunities should also be created, especially through routines and connections to planned projects, such as conducting a sort of children in solid shirts and patterned shirts or an extended inquiry into mapping the classroom (Brownell, Chen, Ginet, Hynes-Berry, Itzkowich, Johnson, et al., 2014; Ginsburg \& Amit, 2008).

While the previous literature documents some mathematical objects that young children interact with in mathematical learning situations, little is known about what mathematics early childhood teachers identify and plan for in the activities and centers they design for children (NRC, 2009). What objectives and internal conditions interplay in their planning for mathematics instruction (Dewey, 1938/1998)? For example, Seo and Ginsburg (2004) recorded preschoolers during their play and identified the mathematical situations in which the children engaged; however, they did not identify to what extent teachers were aware of those mathematical learning opportunities or intentionally designed the environment to create situations that might support children's mathematical interactions. Steffe (1996) suggested that teachers distinguish between their own mathematical knowledge and their knowledge of the children's mathematical thinking by listening to children speak in their natural language about mathematical concepts, allowing teachers to create models of children's mathematical concepts. Without beginning with this model of children's thinking, Steffe argued that teachers are likely to attempt to teach children concepts that are independent from their knowledge, as these concepts, as objects, would not be accessible for the children to interact with in a situation. In her extensive research with children as young as one year, Kamii (e.g., Kamii \& Hausman, 2000; Kamii et al., 2007) also described the teacher’s
role in supporting children to make mental logico-mathematical relationships through interactions with their environment, which includes people, objects, and tasks. The teacher should also support children's development of autonomy, by reducing their own power and control as much as possible and by allowing children to make many decisions, which means not using rewards and punishments but instead exchanging points of view in interactions. For young children specifically, teachers should refrain from suggesting correct actions or behaviors and to allow children to decide how long to play. They suggest providing physical knowledge tasks, such as making a lever, to young children so children can determine themselves if they are correct. The goal of these tasks is for children to construct mental relationships as they interact with the physical objects. Therefore, the less interference from adults, the more opportunities children have to think (e.g., Kamii \& Hausman, 2000; Kamii et al., 2007), i.e., interact with objects in their environment and their own minds.

## Union of Continuity and Interaction

Continuity and interaction intersect and unite in the environment to provide "the measure of the educative significance and the value of an experience" (Dewey, 1938/1998, p. 43). For the teacher, this means that situations succeed one another, as the student's environment expands and contracts, allowing knowledge and skill from a previous situation to become a tool of understanding in a subsequent situation. The teacher should understand that the student enters a situation as he or she is, comes with his or her internal conditions already in place, and is responsible for understanding the students' needs and capacities. Objective conditions can provide unintended "collateral learning" (p. 49), which can form enduring attitudes and dispositions of likes and dislikes. Negative collateral learning can impair students' ability to make meaning from future experiences. Parks and Bridges-Rhoads' (2012) ethnographic project studying the mathematics teaching and learning in a rural preschool classroom of four-year-olds of color from low-income families describes collateral learning from the required use of a scripted, basic skills literacy curriculum whose instructional practices bled into the teachers' mathematics teaching. This study investigates early childhood teachers' experiences teaching mathematics by looking at possible interactions between the teachers' internal conditions, brought out through interviews, and the objective conditions of the environment, materials, and tasks related to mathematics they provide to the children. What objective conditions are seen in the physical and social environment,
how do the teachers' use these conditions to create learning experiences for the children, and how do these objective conditions reflect the internal conditions the teachers brought to the study?

Ginsburg and Amit (2008) described a four-year ethnographic case study of a preschool teacher that illustrates the unity of continuity and interaction (Dewey, 1938/1998). While the teacher previously had little training or teaching experiences with early mathematics, she spent four years with the research team as they developed and tested mathematics curriculum in her classroom. The teacher initially just observed the trial lessons, then began making comments to the research team, and eventually taught a few lessons. After making sense of what the mathematical problem entailed and how the children might think about it, the teacher carefully planned which symbols and representations she would use, how she would introduce the activity, and how she would address classroom management and individual children's identities as mathematics learners. Other studies of early childhood teachers' past and present mathematics learning and teaching experiences also shed light on ways that teachers' mathematics learning experiences were mis-educative (Dewey, 1938/1998), while also providing insights into experiences that have since supported teachers' productive mathematics learning and, subsequently, their mathematics teaching with young children (Carroll, 1994; Richardson \& Bofferding, 2015).

## Social Control

Teachers attempt to use their authority to further the interests of the classroom community rather than exhibiting authority as personal power. Using their knowledge of individuals and subject-matter, teachers select activities that encourage social organization in which everyone enjoys responsibility to contribute to the community (Dewey, 1938/1998). Ideally, causes for difficult behavior are discovered (there are no "difficult learners") so thoughtful planning can provide opportunities for all learners to explore ideas and materials individually and as a group, with the goal of continuous development (Copple \& Bredekamp). The teacher, as the "representative and agent of the interests of the group as a whole" (Dewey, 1938/1998, p. 59), is a member of the classroom community and is responsible for interactions and intercommunications that are the life of the community.

While NAEYC is the "representative and agent" (Dewey, 1938/1998, p. 59) for the interests of the early childhood education community, a multi-faceted sociopolitical context for early childhood professional preparation shapes the training teachers receive, expectations for professional development, and training topics, and may shed light on why professional development is not always well-received by early childhood teachers and does not always improve practice, and why early childhood teachers sometimes question the mathematics requirements of their professional preparation (Lieber et al., 2009; Whitebook \& Ryan, 2011).

Early childhood teachers' experiences teaching mathematics with young children are governed by a variety of federal, state, and local control, resulting in a "stunning cacophony of regulation; competing aims; blended funds; and lack of coherence in program design, curriculum, and staffing, with many programs spending precious dollars, time, and staff attention on simply managing and processing all the paperwork" (Pianta et al., 2009, p. 54). Similarly, professional development and training for early childhood education teachers is a patchwork of mixed delivery systems that is determined through federal government funding mechanisms to the states, by the states through licensing regulations and Quality Rating and Improvement Systems (QRIS), and through various educational institutions and community-based organizations (Bowman et al., 2001; Gomez et al., 2015; NAEYC, 2009). For an early childhood teacher pursuing an associate degree in a state with a QRIS, the teacher may receive scholarships, stipends, or other financial assistance by working toward a degree or professional credential, such as the Child Development Associate ${ }^{\text {TM }}$ Credential (CDA) (T.E.A.C.H. Early Childhood ® National Center, 2016). The federal government provides funding to states for these programs and other programs through multiple funding streams in multiple departments, as early childhood education policies address, for example, education and families (Federal Funds Information for States, 2014; US Department of Education, 2013; US Department of Health \& Human Services, 2014). Funding streams come with requirements and guidelines for the professional development opportunities offered in a variety of settings. These settings include two- and four- year post-secondary schools and national, state, and local organizations such as the National Association for the Education of Young Children (NAEYC) and its state and local affiliates, and Child Care Aware of America and its state and local childcare resource and referral agencies (CCAoA, 2016; NAEYC, 2009).

In addition to the national and state funding mechanisms, professional organizations and advocacy groups influence the training that early childhood teachers receive. Professional
organizations include NAEYC, who developed standards for the professional preparation of early childhood teachers (NAEYC, 2009), the Council for Professional Recognition (CPR), which confers the CDA (CPR, 2015), and the National Association for Family Child Care (NAFCC), which provides training and accreditation to family childcare providers (NAFCC, 2016). National advocacy groups, such as ZERO TO THREE whose work nurtures infants' and toddlers' wellbeing and development (Zero to Three, 2016), conduct research to ground both training they provide and policy briefs they write to influence legislation related to training the early childhood workforce. Charity organizations, such as United Way Worldwide (2016), work with private and individual funders to provide training resources for the early education workforce. This is not an exhaustive list but illustrates the range of stakeholders who influence the training of early childhood teachers, mostly through funding streams and, to a lesser degree, research.

## Professional Preparation Standards

NAEYC has created professional preparation standards for early childhood teachers (2009), as well as a blueprint for states' early childhood professional development systems (Lemoine, 2008). Professional organizations who contributed their voices to the standards include ACCESS (early childhood teacher educators in associate degree-granting institutions), the National Association of Early Childhood Teacher Educators (NAECTE, n.d.), the Division for Early Childhood of the Council for Exceptional Children (CEC/DEC), the National Board for Professional Teaching Standards (NBPTS), the National Council for Accreditation of Teacher Education (NCATE), and the NAEYC Early Childhood Associate Degree Accreditation (ECADA) (NAEYC, 2009).

Early childhood teachers may make use of social control in their mathematics teaching. Teachers may incorporate mathematics into routines that are part of their classroom management strategy, such as asking children to line up using a boy-boy-girl pattern (Richardson \& Bofferding, 2015). In a mathematics inquiry project, the teacher might consider representations that will most clearly illustrate the mathematics the children might encounter (Ginsburg \& Amit, 2008).

## Freedom

Freedom requires balance. Considered as a means of growth, teachers must give much thought to the amount and quality of free activity planned for and offered in their classrooms. However, freedom as an end in itself can impede the natural order of shared cooperative activities. Unlike external control, freedom nurtures individuals' reflection and judgment, and thereby, their own impulse control, "through a union of observation and memory, this union being the heart of reflection" (Dewey, 1938/1998, p. 74). Teachers learn about their students when students have freedom and can therefore tailor instruction to meet students' needs and purposes. Lack of freedom equates to enforced uniformity and prevents students from being their authentic selves.

Early childhood teachers want professional development in mathematics, although many post-secondary early childhood programs neglect mathematics education (Ginsburg et al., 2008; Sarama \& DiBiase, 2004; Simpson \& Linder, 2014). The early childhood mathematics professional development that is offered traditionally focuses on teachers acquiring mathematical skills and knowledge like college algebra or elementary mathematics content such as multi-digit operations rather than counting (Parks \& Wager, 2015; Richardson \& Bofferding, 2015). However, recent studies suggest that mathematics teacher educators redesign content and methods courses to include pedagogical and content knowledge of informal and foundational mathematics that occurs in play and before formal written symbols (Chen \& McCray, 2012; Parks \& Wager, 2015), better meeting the mathematics learning needs of early childhood teachers.

Some researchers determine what mathematics is and provide early childhood teachers with that information, and then work through the professional development program to help teachers take up those ideas of mathematics teaching and learning as determined by the researchers. For example, Rudd, et at., (2009) trained early childhood teachers to use more math mediated language with the children in their classrooms, while Varol, et al., (2012) assessed how teachers implemented training from the Building Blocks (Clements \& Sarama, 2008) program in their classrooms. University instructors in Gresham (2007), Herron (2010), and Huss-Keeler \& Brown's (2007) studies provided mathematics materials and curriculum to teachers, also defining mathematics. However, mathematics teacher educators can learn about early childhood teachers as mathematicians and mathematics teachers when they provide freedom (Dewey, 1938/1998) to early childhood teachers to define mathematics themselves, such as when researchers asked the
teachers in their professional development programs to consider what constitutes mathematical activity (Harkness \& Portwood, 2007) or what math is for four-year-olds (Graue et al., 2015). These teachers had the opportunity to use the ideas and knowledge they generated during the professional development as a tool of understanding as they planned lessons and tasks in their future mathematics teaching (Dewey, 1938/1998).

## Purpose

Purpose provides the motivation for learners to engage in activities that support their development (Dewey, 1938/1998). Teachers can leverage learners' purposes by being aware of their past experiences, capacities, and needs, and then collaborate with learners to develop action plans that align with learners' purposes, in a "reciprocal give-and-take" (p. 85). Researchers understand early childhood teachers as learners through child outcomes, what early childhood teachers take from professional development, and documenting early childhood teachers' experiences of professional development.

One way that researchers understand early childhood teachers as learners is by measuring children's outcomes through their achievement on mathematics assessments before and after the implementation of a professional development program. Researchers, policy-makers, and other stakeholders, looking for "what works," position teachers as a conduit for moving knowledge from outside the classroom into students (Clandinin \& Connelly, 1992) and examine many different characteristics of professional development this way (e.g., Brendefur et al., 2013; Chen \& McCray, 2012; Clements \& Sarama, 2008; Kinzie et al., 2014; Piasta et al., 2015; Yoon et al., 2007). Another way that researchers understand how early childhood teachers experience professional development is to measure or explore what teachers take from professional development. Some studies measure components from the training that are implemented in classrooms (e.g., Rudd et al., 2009; Varol et al., 2012) or teacher characteristics before and after the PD, such as math anxiety or beliefs (e.g., Gresham, 2015). Other studies document teachers' experiences of professional development (e.g., Herron, 2010; Huss-Keeler \& Brown, 2007). Researchers document stories of learning with the teachers (Clandinin \& Connelly, 1992) as they engage with the mathematics, their ideas about mathematics for young children, and even mathematical resources the children bring from home (Graue et al., 2015; Harkness \& Portwood, 2007). While these studies may
document early childhood teacher learners' past experiences, capacities, and needs, they do not document opportunities for early childhood teachers to have input in the content or the design of the learning activity or program. Therefore, we do not know what the teachers' purposes are, whether they have accepted the researchers' purposes for duration of the professional development or intervention, or they have reciprocally made it their own.

## Subject Matter

The subject matter of the classroom should arise from everyday life experience. It is governed by the continuity principle as previous experiences and development affect current learning experiences, which become the foundation for future learning. Students draw from their memories to understand their present experiences and social interactions. In addition to attaining new facts and ideas, educative experiences also include better or more organization of facts and ideas. Without organization, experiences would be chaotic. Dewey appreciates the "social and human center of the organization of experience" (1938/1998, p. 103) found in early childhood education. Subject matter moves from a social and human organization to a more objective intellectual organization as students mature. Subject matter activities should satisfy students' purposes.

According to Dewey (1938/1998), subject matter revolves around cause and effect, or the scientific method, allowing students the opportunity to understand the significance of everyday experiences. When a student does not have the opportunity to observe and test activity, activity leads nowhere intellectually. However, the scientific method requires "keeping track" (Dewey, 1938/1998, p. 110) of outcomes, where "keeping track" is a way of reflection and summarizing the meanings which will ground future experiences. Teachers should see a continuous process of reconstructing experience, with ever-expanding subject matter that progressively develops into adult form. Ideas, activities, observations, and organization are different for students of varying maturity, but all learners encounter expanding development from educative experiences.

## Subject Matter for Children

In their work to understand processes that guide cognition, Clements and Sarama (2014) reflect Dewey's (1938/1998) subject matter construct in their findings that teachers who both value
children's individual thinking and teach strategies to help children establish connections between mathematical ideas, skills, and topics are more effective than teachers who wait for children to be ready for a particular topic and those who teach by telling children how to work problems. In fact, they describe the dichotomy of explicit instruction versus a child-centered approach as false. They suggest that teachers plan for intentional instruction that uses formative assessment with learning trajectories to know where to begin and what to teach next. To maximize effectiveness, teachers should choose curricula that include a combination of skills, concepts, and problem solving, as well as address self-regulation skills through play and giving children activity choices. Teachers should also engage children in talking and writing about mathematics, including asking openended questions that encourage children to articulate their ideas and facilitating summaries of key ideas at the end of activities to support the continued organization of children's thinking.

While Clements and Sarama (2014) cautioned that developmentally appropriate practice (DAP) (Copple \& Bredekamp, 2009) or play-based (van Oers, 2010) instruction does not contain enough explicit focus on mathematics content to support children's mathematical learning, these child-centered approaches can make a valuable contribution when they are well-structured and mediated by the teacher (Parks \& Blom, 2013; Parks \& Wager, 2015). Parks (2015) adds that play is not the same as learning mathematics content. Play provides opportunities for children to gain language to articulate and abstract the big ideas from their experiences, which can be an important role of early mathematics instruction. In rich play experiences, children uncover mathematics concepts, such as composing and decomposing in number and geometry, that provide a base for teachers to build instruction. While children will not necessarily learn mathematics concepts on their own, it is important for teachers to not intervene too early. As teachers recognize and draw on children's mathematical play in their practice, they must balance providing time and productive materials for exploratory play with the formal instruction that builds on the concepts that children have already discovered. However, little research exists concerning how teachers scaffold children's play to connect their informal mathematical ideas to formal mathematics, indicating that early childhood teachers may not develop these skills through coursework or other professional development opportunities (Parks \& Wager, 2015). In fact, teachers often do not have an explicit awareness of the concepts that they are trying to build on in their practice because these foundational mathematics concepts have become invisible to them as adults (McCray \& Chen, 2010, p. 261).

## Subject Matter for Teachers

The NAEYC standards recognize that mathematics requires early childhood teachers to acquire research-based knowledge and skills (NAEYC, 2009). Teachers, however, are offered very few opportunities for professional development in mathematics, and those offered are often short, hourly sessions without follow up opportunities that would support teachers' ideas to become organized (Dewey, 1938/1998; Simpson \& Linder, 2014). PD providers rarely expect recipients of their trainings to acquire or develop their mathematical content knowledge and often work mainly with early childhood teachers in settings within public school districts, neglecting the teachers, and children in their classrooms, who teach mathematics in childcare centers, family childcare homes, or registered ministries (Simpson \& Linder, 2014). Because early childhood spans birth through age eight, the depth of coverage for each content area and age group is questionable (Early \& Winton, 2001). Sarama and DiBiase (2004) suggest that effective professional development in mathematics for early childhood teachers address and develop teachers' knowledge and beliefs about mathematics content and education, be grounded in particular curricula and materials that follow developmental learning trajectories, preferably situated in their classrooms where teachers can reflect, interact, and share their ideas and work with colleagues. They suggest that including teachers' voices and perspectives in the dialogue about their mathematics training and teaching work increases opportunities for PD to respond to individual teachers' experiences and contexts, and to focus on incremental changes guided by a consistent overarching vision.

Because children's foundational mathematics is often invisible to adults, the Early Mathematics Education Project designers used a teacher perspective to develop adult mathematics learning tasks to help preschool teachers "see" this mathematics (Brownell et al., 2014; Chen \& McCray, 2012; McCray \& Chen, 2010). The learning tasks also aimed to help teachers build confidence with and knowledge of foundational mathematics, as well as the use of math-related language that children could use to build their mathematical thinking in preparation for using written symbols in the future. To build teacher confidence, PD mathematics and coaching activities emphasized teacher strengths, such as building mathematics tasks from rich children's literature and sharing expertise with peers. Other professional development in early mathematics focused on "mathematizing" in the early childhood classroom (Clements \& Sarama, 2008; Graue et al., 2015; Harkness \& Portwood, 2007; Kinzie et al., 2014). Mathematizing is highlighting the
mathematics in everyday problems and the mathematics that grow out of children's experience and activity, rather than teaching specific skills and problem types. In PD, early childhood teachers learned how to observe and plan for children's play, for example, to elucidate rich, authentic mathematical contexts to explore with children (Graue et al., 2015). Teachers also examined their routines to find opportunities to integrate mathematics into them. In PD, teachers might work on non-standard contextual problems that honor both "school" mathematics and mathematical activity within everyday activity (Harkness \& Portwood, 2007). As a result of PD, teachers may realize that their students are capable of being problem-solving mathematicians and can do much more than memorize skills and answer questions with one right answer, becoming mathematicians themselves (Frakes \& Kline, 2000).

## Research Questions

Using Dewey's (1938/1998) experience constructs as a theoretical framework and narrative inquiry (Clandinin \& Connelly, 2000), I will address the following research questions: (1) What role, if any, do mathematical learning experiences play in early childhood teachers' mathematics teaching practice? (2) In what ways do early childhood teachers' voices contribute to the professional dialogue regarding teaching mathematics with young children?

## CHAPTER 3 METHODS FOR CRAFTING NARRATIVES

This is a narrative inquiry study. While there are many research methods, methodologies, and epistemologies to choose from, those used in this study have emerged as theories from my coursework that have resonated with (Conle, 1996), and brought into clearer focus, the research puzzle that I brought into my doctoral program. This puzzle emerged when early childhood teachers recounted their experiences as mathematics learners and teachers, while the "math baggage" stories told by my Algebra Review students swirled together with my ongoing aspirations to see and document children as capable and competent humans and learners (Edwards et al., 1998). Therefore, I worked to step outside my own thinking to understand what was real and true for the children (Clemens, 1983; 1996; Paley, 1986; 1992). The learning theory radical constructivism articulates the role of experiences in learning (von Glasersfeld, 1995) and resonated with my memories of teaching with young children. With my research puzzle always in the back of my mind, this resonance led me to wonder what ideas about mathematics early childhood teachers constructed from their experiences learning and teaching mathematics. I also wondered if I could make models of early childhood teachers' thinking about mathematics and mathematics teaching to inform my teaching with them (Steffe, 1996). A professor's presentation of his narrative work showed me that I could use the stories I had a penchant for collecting to make sense of and construct knowledge and that they could be analyzed narratively (Polkinghorne, 1995) to provide insights to me, the story tellers, and others who may have lived resonating, or not resonating, stories.

I hoped that by proceeding to study how the teachers' knowledge was narratively composed, embodied in themselves, and expressed in their practices, I would be able to see ways that the early childhood teachers hold and use their knowledge to teach mathematics with young children in ways that are not commonly noticed (Clandinin \& Connelly, 2000; Elbaz, 1981), providing the finer-grained results missing in the literature (Early et al., 2007). I hoped that my narrative analysis might push back on my colleague's notion that, "We early childhood people just aren't good at math." I began by completing a pilot study to further refine my plans for data collection and analysis.

## Pilot Study

Ten early childhood teachers from a university laboratory school in the Midwest agreed to participate in the pilot study (see Table 1) (Richardson \& Bofferding, 2015).

Table 1. Participant and Classroom Descriptions

| Classroom <br> Name | Teacher | Position | Degree | Years at <br> Center | Classrooms <br> No. of <br> Children | Ages <br> (years) |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: |
| Ducks | D1 | Head | BS ECE | 17 |  |  |
|  | D2 | Associate | BS Child Dev | 11 | 20 | $3-5$ |
|  | D3 | Assistant | AS EC Dev | 6 |  |  |
| Eagles | E1 | Head | MS C\&I/BS ECE | 16 | 20 | $3-5$ |
| Squirrels | S1 | Head | BS Child Dev | 22 |  |  |
|  | S2 | Student | BS ECE | Just <br> Teacher |  | 14 |
| Higers | T1 | Head | BS Child Dev | 5 | 14 | $2-1 / 2$ |
| Koala | K1 | Head | MS Child Dev | 19 |  | $2-1 / 2-$ |
| Bears | K2 | Associate | BS Ed Studies | 1 | 14 | $4-1 / 2$ |
|  | K3 | Assistant | CDA Credential | 6 |  |  |

We (my advisor, Laura Bofferding, and I) used two data collection activities. First, I conducted semi-structured interviews with classroom teacher teams to learn about the teachers' mathematics training, classroom mathematics curriculum, who controls their mathematics curriculum, and their mathematics teaching and learning philosophies for young children. Next, I used classroom observations and modifications to photovoice (Schell et al., 2009; Wang \& Burris, 1997; Whitney, 2006), to learn about what teachers considered to be mathematics learning in their classrooms. The teachers and researchers photographed examples of mathematical activity, and the teachers discussed with us the mathematical activity in the photos and interpretations of children's mathematical learning.


Figure 1. Examples of Teacher and Researcher Photos

One outcome from conducting a pilot study was the evolution of the theoretical framework and analysis process. In my first analysis, I "chunked" transcript data by idea or activity and coded the chunks for Bourdieu's social field theory constructs (Grenfell, 1996; 2012; Grenfell \& James, 2004; Noyes, 2004), feminist theory (Brooks, 2007; Fennema \& Sherman, 1977; Hancock, 2001), teachers' experiences learning and teaching math, and mathematics content (Clements \& Sarama, 2014). Bourdieu's social field theory highlighted social factors that might have influenced the teachers' practices, including standards, regulations, administration, research, university organization, parents, and children (Richardson \& Bofferding, 2015). I created anecdotal snapshots for each classroom and a profile of each teacher that included a timeline, experiences learning mathematics and teaching it with young children, teaching and learning philosophies, and dominant themes.

Bourdieu's social field theory (Grenfell, 1996; 2012; Grenfell \& James, 2004; Noyes, 2004) provided a thorough understanding of past and present influences on the teachers' mathematics
teaching practices with young children. Missing, however, was a sense of how mathematics permeated the teachers' everyday lives and how the teachers might benefit from such an investigation. Although we, the teachers and researchers, co-constructed knowledge about what was generally mathematical in the teachers' practices during the interviews and classroom observations, these encounters did not allow for co-constructing knowledge about the teachers' strengths and needs as mathematics teachers or opportunities for them to share their photos and develop a plan to improve their lives as teachers of mathematics with young children. Stymied about my next move, I heard a professor's presentation on his narrative inquiry work and read Dewey's (1938/1998) Experience and Education and Clandinin and Connelly's (2000) Narrative Inquiry: Experience and Story in Qualitative Inquiry with my colleagues. I decided to "wrap" narrative inquiry around the photovoice project to document the project and to use Dewey's experience constructs to make sense of how the data was mathematical (see Figure 2) (Richardson, 2015). While this pilot study was not conducted with these additional methods, I tested the use of Dewey's experience constructs to analyze the data and looked for evidence of processes from narrative inquiry, such as resonance (Conle, 1996).


Figure 2. Elements of Narrative Inquiry and Photovoice. This figure illustrates discrete and overlapping elements of both methods (Richardson, 2015, p. 6).

In the Bourdieu analysis (Grenfell, 1996; 2012), I found that veteran teachers, D1 and S1, recognized the capital that novice teachers, D3 and S2, brought into classrooms. In a moment of resonance (Conle, 1996), S 1 , described her math learning experience in relation to the student teacher in her classroom, S2, who loves math: "I wish I had what she had, and I think mine has come from just self - at an older age - and I think even I grew up with the concept of I really don't like math." Bourdieu's structuring and structured structures, which are interactions between habitus, field, and capital, were seen when D2 heard a child exclaim, "Look how much I have!" as he sifted through the letters in the sensory table that she had set up as a literacy activity. In that moment, D2 was excited and surprised to see that they actually do math throughout the day, inspiring her look for more spontaneous "math moments" throughout the day, in addition to the traditional mathematics activities the teaching team enacted daily.
Dewey's (1938/1998) continuity and interaction constructs were seen in common sentiments across the group. Veteran teachers described many mis-educative experiences as K-12 and college mathematics learners, although all the teachers in the study drew on their experiences teaching mathematics with young children and more recent research-based professional development in their current practices. Teachers noted that their current practices were rooted in their curiosity about children, which they recognized increased their own understanding of mathematics (Philipp, 2007) and left them feeling newly competent and confident with mathematics. Dewey's social control and subject matter constructs surfaced when S1 explained a photo I had taken of her drumming a rhythm on the wood lid of the sensory table after she had covered it. She explained this was a routine she adapted from a conference that she used to signal that it was time to clean up (social control) and as an opportunity for the children to experience patterns and counting through listening (subject matter) (Wright et al., 2015). She marveled, "Gosh, you know what? You don't think about these things. You do it so many times, you don't think about that. I guess I've never seen a picture of that before."

While conducting my first study with teaches seemed smooth, there were some issues that we resolved in a timely manner. Scheduling interviews around the teachers' schedules was somewhat difficult, but the teachers enlisted other staff members to maintain classroom child-toteacher ratios, often during naptime. We also met during some teachers' planning time. After the first few observations, I noticed that researchers took more photos than the teachers, with some teachers not taking any photos during an observation. They were so busy teaching that it was
difficult for them to stop and take a photo. There were a few instances that a teacher asked me to take a photo, which seemed much easier for teachers. This surprised me because the teachers took photos to document children's activity and learning. At the time, these issues seemed isolated, related to being a novice researcher. However, they foreshadowed issues I would face in my dissertation study.

Although I knew more about early childhood teachers' experiences learning and teaching mathematics than I did when I started, I struggled to make a concise argument about what I had found in my pilot study. I was surprised that the veteran teachers appreciated the capital (Grenfell, 1996 ; 2012) the novice teachers brought from their recent coursework into their classrooms. I was pleased that teachers who experienced mis-educative (Dewey, 1938/1998) learning re-learned mathematics concepts in their teaching with young children. I was so excited to experience so much joy with the teachers as they uncovered mathematics teaching and learning in their practices that had before gone unnoticed. However, I still had not developed a satisfactory process to report my findings or conclusions. With my IRB for my dissertation study approved, I presented my pilot study at the Annual Graduate Study Education Research Symposium (see Figure 3) for the last time and shifted my attention to finding participants for my new study.


Figure 3. Poster Presented at the Annual Graduate Student Education Research Symposium (Richardson, 2017)

## Participants and Setting

The photovoice project required a group of participants who could meet together to discuss their photos and develop a presentation of the research to a group of stakeholders (Wang \& Burris, 1997). This made the university lab school a good fit for the pilot study. Therefore, I began my search for participants by contacting a local childcare nonprofit agency, for whom I had worked as a toddler teacher, and with multiple sites throughout the city. After unsuccessfully soliciting participants through the central office and by attempting to talk to directors directly, I contacted a past colleague from IAEYC who worked with early childhood directors and teachers through the local United Way. She arranged for me to describe my study and solicite participants during two meetings she and her team led for directors of childcare centers, registered ministries, and family childcare homes. I met a center director who introduced me to two co-teachers in the center's $O n$

My Way Pre-K program. I began data collection with the teachers, but it became increasingly difficult to contact the teachers to complete data collection, including their taking photos. I did not collect enough data to analyze.

In the meantime, my contacts at United Way introduced me to two family childcare providers who agreed to participate, bringing me full circle to my beginning in early childhood education as a family childcare provider. I emailed more information to them about the study and answered their questions. Josie and Patsy (pseudonyms) had both completed mathematics requirements for an associate or bachelor's degree in early childhood education or child development and could accommodate the interview and observation requirements of the study. I collected the following information: years taught, type of childcare setting, ages of children in classroom, position (lead teacher, assistant, etc.), gender, full-time/part-time, degrees sought and completed, and coursework completed in early childhood education and in mathematics content or methods courses (Rudd et al., 2009). I did not have a preference for any of these teacher characteristics. However, the participants did have a strong desire to participate (Elbaz, 1981) and were curious about investigating their mathematics teaching practices. Knowing when the teachers learned mathematics seemed important (Early et al., 2007; Richardson \& Bofferding, 2015); however, asking the teachers for their ages or high school graduation dates seemed too personal. I will describe each teacher and their setting in detail in their individual narrative (see Table 2).

Table 2. Participant Information

| Name | Setting \& Position | Years | No. Children \& Ages | Education |
| :--- | :--- | :--- | :--- | :--- |
| Josie | Family Childcare <br>  <br> Lead Teacher | 14 | 10 <br> 4 months to 4 years | Bachelors: Telecommunications <br> \& Psychology <br> Seeks paradigm-shifting <br> program useful to ECE |
| Patsy | Family Childcare <br>  <br> Lead Teacher | 4 | 6-10 <br> 6 months to 4 years + <br> occasional school-age | Bachelors: Business <br> Associates: Early Childhood Ed <br> Working on Masters in ECE |

## My Positionality

I am a middle-class and middle-age White woman. I have taken and taught a variety of mathematics and teacher education courses. I have attended and presented a variety of early childhood and mathematics teacher education workshop and conference sessions. I have conducted research with children and pre- and in-service teachers. I have been affiliated with a university most of my adult life. A professor in my secondary education graduate program encouraged me to consider working in early childhood education and, later, pursuing a doctoral degree.

I was an active member of the local IAEYC chapter in both communities I lived in while working in the early childhood field, helping to plan local conferences and professional development programs. As a family childcare provider, I was active in the local Family Child Care Network and initiated a program to support providers as they pursued NAFCC accreditation, which is now provided through Indiana's Paths to Quality QRIS.

As a mathematics student, I was generally successful but struggled more as the content became more abstract throughout high school. My father loved mathematics and made me feel that I may as well stop breathing if I did not pursue mathematics at every opportunity. He enjoyed posing mathematics-related riddles at the dinner table, prompting my mother to eat in another room. He suggested that I study engineering and become the chairperson of the Atomic Energy Commission. That idea did not make sense to me, so I decided I would be a good mathematics teacher because I was pretty good at mathematics but struggled at times, making me more empathetic than my teachers. Mathematics at the college level was even more challenging. I started teaching the algebra review course when I graduated. I learned a lot more about algebra working problems in front of a classroom of students. I implemented new instructional and assessment strategies from my secondary education master's program but was not allowed to use them. My job was to deliver the course material the same way the other instructors did, with no "unfair" advantages for my students.

As I began my doctoral program, my view of teaching and learning mathematics was quite "traditional," as I had little training in teaching and learning mathematics, in general, and none with young children. My role as the teacher was to show students, of any age, how to correctly work problems and to help them see and correct their mistakes. For example, when a three-yearold child in my family childcare home was drawing our guinea pig, I grabbed his second attempt
as soon as he drew four legs, as his work sample from the day. He had drawn many equidistant legs around the entire oval body for his first attempt, and I did not want his engineer mom to see that "incorrect" representation. I sensed that was probably not the best pedagogical choice, but I did not know what to do instead. Now I would be impressed with the consistent spacing of the many legs and might ask him to consider comparing his drawing to the guinea pig and invite him to tell me what he notices. Or I might look around for additional opportunities to work with the idea of "equidistant" because that can be difficult for young children. There would be no "right" answers, only invitations to look again or at something related. This memory now makes me cringe. Early in my program, I recognized that teaching mathematics with young children could probably be much more emergent and developmentally appropriate than my practice had been as an early childhood mathematics teacher, so I worked to integrate what I was learning about curriculum, learning theory, and teacher education in mathematics education with the ideas I brought into the program. For example, I created a proposed course for elementary student teachers placed in local Reggio-inspired kindergarten classrooms to help them plan for mathematics teaching when using an emergent curriculum framework (Perry, et al., 2007).

Positionality was a difficult construct to recognize while conducting my pilot study. All but one teacher was White, and they all worked in a university lab school and were, therefore, quite familiar with researchers and comfortable with data collection procedures. They had all received university-level training related to early childhood development and education. They were so similar to myself that I could not see positionality among us, except for age-related characteristics. The veteran teachers graduated from college before Fennema and Sherman (1977) found that girls' achievement in mathematics was similar to boys', going through school during a time when they were expected to not be good at mathematics nor take many mathematics courses. In contrast, the two novice teachers who were in their early 20s had positive mathematics learning experiences. My mathematics learning experiences were positioned between the veteran and novice teachers. I did not have to step outside of my own experiences before making sense of the lab school teachers' experiences.

For my dissertation study, while I can identify experiences in which I share an insider positionality (Hesse-Biber, 2007) with my participants, I can also identify more examples of outsider positionality. I am a middle-class White woman attempting to tell stories of two Black women, which is an outsider positionality. However, I share an insider positionality with my
participants as an early childcare teacher and family childcare provider, which may allow me to perceive and understand their experiences in ways that those who do not share these specific professional experiences may not (Ayers, 1989; Bowman et al., 2001; Child Care Aware of America, 2016; Gomez et al., 2015; Pianta et al., 2009; Kim \& Reifel, 2010; NAFCC, 2016; National Research Council [NRC], 2009; National Survey of Early Childhood Education [NSECE], 2013; Parks \& Wager, 2015; T.E.A.C.H. Early Childhood ${ }^{\circledR}$ National Center, 2016; United Way Worldwide, 2016). Although the three of us share a positionality as early childhood teachers and family childcare providers, our personal experiences and worldviews reveal fissures and tensions in our shared positionality. As a middle-class White woman, my view of "best" practices in early childhood education may be rooted in privilege that these Black women participants do not share. My early childhood education passions, such as for the preschools in Reggio Emilia, Italy (Edwards et al., 1998), colored my perceptions of their practices until I stepped outside of my understanding and let go of my background, so I could see them through my understanding of their experiences instead of my own.

The three of us also share a lifetime of learning. We have each spent countless hours in early childhood education workshops, obtained or are obtaining post-baccalaureate degrees, and have pursued National Association for Family Child Care Accreditation. However, I am conducting this study as the last requirement toward obtaining a doctoral degree. There is an element of using (exploiting) the participants to advance my career. In a gesture of reciprocity, I encouraged the participants to share insights from our time together, such as in program information materials for families.

I considered positionality after data analysis, not as an element of the research context or as a tool to help me understand. It was something to be addressed, almost in a perfunctory way, because the qualitative research textbook said so (Creswell, 2007; Patton, 2002). Describing my positionality has added another layer of understanding about both sets of participants and the nature of my relationship to them. For example, my positionality was a factor in gaining access to opportunities to solicit teacher participants. Also, as someone who studies early childhood mathematics teaching and learning, I overtly took on that position one time each with Josie and Patsy by giving them mathematics education language for mathematical activity they each described in their practices (structured and unstructured collections for Josie and spatial reasoning for Patsy). By not examining my positionality earlier, I failed to recognize that the easily resolved
issues from my pilot study, with teachers who had a high level of administrative support for research in a university lab school setting, might become insurmountable problems with family childcare providers who are their own administrators and whose only support comes from parttime assistants. I also failed to recognize that even when using similar language, positionality, life experiences can cause the language to mean differently, as Cohen (1990) noted in the case of Mrs. Oublier, as he compared his own perceptions of Mrs. Oublier's mathematics teaching with her own perceptions of her mathematics teaching practice.

This study received approval from the Institutional Review Board. After discussing my project and possible risks, Josie and Patsy signed consent forms to participate in the study. They also shared information about the study with the families in their programs and obtained consent for me to observe, as well as share photographs of and talk with me about the mathematical activities of the children. All names are pseudonyms.

## Data Collection Activities

As with the pilot study, I planned to engage in three data collection activities with participants, including a photovoice inquiry (Wang \& Burris, 1997), in which each participant would act as an inquirer, choosing questions and objects for photos related to the mathematical activities and materials in their classrooms. However, Josie and Patsy were even busier than the pilot study teachers, as family childcare providers who were responsible for all the jobs in their settings. They did not take any photos, so we did not complete that data collection activity. Working with my advisor, I adjusted the data collection activities and analysis plan to reflect the data I was able to collect. In the analysis process, I was able to create interim research text (Clandinin \& Connelly, 2000) images that supplement Josie's and Patsy's teacher narratives, maintaining an aspect of the visual that was lost when I could not use the photovoice methods.

To make narrative meaning of the participants' experiences learning and teaching mathematics with young children, I collected data that would provide evidence of the three dimensions of the narrative research space (personal and social interactions; continuity across the past, present, and future; and place) (Clandinin \& Connelly, 2000; Clandinin \& Rosiek, 2007; Craig, 2011; Pinnegar \& Daynes, 2007). The data I collected included an initial interview and three classroom observations with each participant.

During data collection activities, I listened for moments of resonance with my participants and attempted to connect resonating threads and interpretations as they told stories of their lived experiences (Conle, 1996). These were moments in their stories that connected with something in my own experiences, "correspondences" (Conle, 1996, p. 304), that helped me make meaning of their experiences. For example, Josie talked about incorporating the Reggio Emilia (Edwards et al., 1998) approach into her program, which resonated with my own appreciation for the Reggio approach. In addition to being an epistemological activity for me, I also shared these moments of resonance as brief opportunities to nurture relationships with my participants (Clandinin, 2006). For example, I resonated with Patsy's story of not meeting the requirements to receive a T.E.A.C.H. ® Scholarship and expressed my appreciation for her dedication to becoming a better teacher for the children. Other times I experienced dissonance, such as Josie sharing her fondness for Saxon Math (Larson, 1994) or Patsy putting an infant in an exersaucer, which were counter to my training. These instances of dissonance, especially from the later observations, muted my memories of resonance from the interviews at the beginning of the study and hampered my initial analysis process. As I decentered from my own positionality to construct teaching images and emplotted narratives from the data, I gained more asset-based views of the teachers. Family childcare providers are very busy, and I attempted to intrude as little as possible, which restricted my ability to build robust relationships with these women. We did find opportunities outside of the study interviews and observations to get to know each other a bit more. These opportunities included meeting at the state early childhood conference and attending some sessions together, and I stayed during lunch and naptimes to help out and chat. While I made a few field notes and did audiorecord one chat, this data seemed outside my official data collection activities.

## Initial Interviews

In the first activity of the study, I interviewed each teacher about her experiences learning and teaching mathematics, which provided historical context and clues for emerging plotlines that I used in constructing their narratives (Clandinin \& Connelly, 2000). I asked the teachers to describe their training related to teaching early childhood mathematics; discuss their classroom mathematics curriculum, activities, and experiences; talk about issues related to who has control over mathematics curriculum in their school setting; and describe their philosophy. Question categories included Content, Teaching, Voice, Philosophy, and Miscellaneous. I developed the
protocol questions from the literature and to reflect the research questions (see Appendix) (Ayers, 1989; Frid \& Sparrow, 2009; Lee \& Ginsburg, 2007). The protocol was piloted with teachers at a local preschool. After listening to the initial interviews, I asked some follow-up questions during subsequent classroom observations to capture missing parts of the narrative or to clarify teachers’ stories.

## Classroom Observations

The second activity was to conduct three classroom observations with each teacher to learn about their experiences teaching mathematics with young children. Each observation lasted one to three hours and took place between breakfast and lunch, the main teaching, learning, and play time in each teacher's schedule. My observations were scheduled over a five-month period around holidays, acclimating new children and staff, and other scheduled visits and processes, such as those from evaluators for the state quality rating and improvement system or preparing for family childcare accreditation. I took field notes during the observations, noting what I saw and heard throughout the observation and paying particular attention to mathematical activities and materials.

During these visits, I attempted to live alongside (Clandinin \& Connelly, 2000) the teachers and children, providing an extra pair of hands when possible and talking with children when they approached me. I was also aware that I was somewhat of an intruder in the space and the teachers' and children's day, so I also attempted to stay out of the way as I observed and wrote notes on my clipboard. The teachers each volunteered information about their practice and the children in their care. I made field notes documenting the three-dimensional inquiry space (temporal, place or setting, and personal and social interactions) (Clandinin \& Connelly, 2000) that included time stamps, social interactions and teacher questioning, information about the classroom, and mathematics activities. My field notes and subsequent research texts are descriptive and are shaped around particular events and my own experiences, thinking, viewpoints, perspectives, identities. I began the initial analysis process and writing of research texts with the first visit to each teacher's family childcare home, thus beginning the iterative process of being in the field and writing field and research texts, in which data collection and analysis took turns driving the inquiry.

## Data Analysis

After data collection with the teachers concluded, I completed two types of analysis. My initial analysis entailed coding the data for emergent themes and Dewey's (1938/1998) experience constructs. Next, I sought to make meaning of the data through a narrative analysis process of plot construction in which I chose data elements to synthesize and configure into stories that addressed the research questions (Polkinghorne, 1995).

## Initial Analysis

Several rounds of analysis were conducted to gain a sense of the underlying relations and tensions (Clandinin, Murphy et al., 2009) that the teachers experienced in their mathematics learning and teaching with young children. Each transcript from the initial interviews was divided into chunks expressing a cohesive idea focused on the same idea or activity. Each chunk was coded for teachers' experiences learning and teaching (Early, et al., 2007; NRC, 2009), timeline (Clandinin \& Connelly, 2000), philosophy regarding teaching and learning mathematics with young children, mathematics content (Brownell et al., 2014), and any dominant themes that emerged during the analysis. Initial interview questions were analyzed from the transcript for each teacher (demographic data, societal factors, subject matter of interview questions, etc.) (Frid \& Sparrow, 2009; Lee \& Ginsburg, 2007). Next, passages from all initial and debriefing interview transcripts, as well as field notes, were coded for the main principles of Dewey's (1938/1998) experience construct, continuity and interaction. Additional components of Dewey's experience construct, purpose, freedom, social control, and subject matter were marked in each transcript. Anecdotal snapshots for each teacher were developed, and short narratives of each teacher were created that included a timeline and dominant themes found in their individual transcripts. From these pieces, I developed an "interim text" (Clandinin \& Connelly, 2000, p. 133) of each teacher as a mathematics learner and teacher in the three-dimensional inquiry space. I wondered, "What is the story?"

## Narrative Analysis

My initial analysis provided facts about each teacher's setting and how their experiences fit with Dewey's (1938/1998) experience constructs. To figure out the story, I turned to

Polkinghorne's (1995) narrative analysis guidelines and began to synthesize the data elements into a story of each early childhood teacher as a learner and teacher of mathematics. I also turned to Clandinin's (1985) teaching image, which is an embodied and enacted representation of past experiences that sheds light on present teaching events and provides guidance in future teaching experiences. In this analysis, my goal was to get a sense of how the teachers held and used their mathematics teaching knowledge, from how the mathematics in the teachers' classrooms came to be and from their earliest memories of learning mathematics to applying content and pedagogy acquired in professional development, to what they have learned about teaching and learning mathematics with the children in their classrooms (Elbaz, 1981). Through an iterative process of querying the data, rereading my narrative inquiry methodology sources (Clandinin, 1985; Clandinin \& Connelly, 2000; Drake, 2006; Polkinghorne, 1995), and talking with colleagues, I wrote research texts asking questions of meaning and social significance, looking for patterns, narrative threads, tensions, themes, and resonances (Conle, 1996) within and across the teachers' and my experiences, stories, and social settings.

I searched for data that suggested emotionality, culture, morality, and embodiment (Clandinin, 1985; Polkinghorne, 1995). This process included many false starts, doubt, and uncertainty (Clandinin \& Connelly, 2000). As the interim texts began to more clearly capture meanings that addressed my research questions, I began to consider possibilities for the narrative form of the research texts. One participant told a turning point story that left a significant and clear image in my mind of a mathematics professor who ignited her passion for mathematics, after past learning struggles, and now inspires her mathematics teaching (Drake, 2006). Finally, I could no longer resist creating a visual representation of her past mathematics learning and teaching experiences with my interpretation of her turning point story as the foundation. This image of her past mathematics experiences subsequently became an element of a visual representation of my teaching image (Clandinin, 1985) for her. As the other teacher described the surprising amount of mathematics that adults can encounter in their everyday lives, her example left a vivid image in my mind that became the foundation for my visual representation of her teaching image. These "interim text" (Clandinin \& Connelly, 2000, p. 133) images helped me organize and hold the field data in my mind while I crafted the final research texts, Josie's and Patsy's teacher narratives.

Using these methods and analysis activities, I have constructed an emplotted narrative (Polkinghorne, 1995) and teaching image (Clandinin, 1985) for both Josie and Patsy as the findings
of this study. "Chapter 4, Making Math Real: Josie's Narrative Findings" and "Chapter 5, You'd Be Surprised That Is Math: Patsy's Narrative Findings" address the first research question: What role, if any, do mathematical learning experiences play in early childhood teachers' mathematics teaching practice? "Chapter 6, A Discussion of My Narrative Findings" addresses the second research question: In what ways do their voices contribute to the professional dialogue regarding teaching mathematics with young children?"

## CHAPTER 4 MAKING MATH REAL: JOSIE'S NARRATIVE FINDINGS

Setting \& Professional Life: Past, Present, and Future

Josie, a black woman, has been a family childcare provider for about 13 years. About a year ago, she moved into her century-year-old home in a gentrifying, urban neighborhood. In her current setting, seven to ten children spend their day with Josie and her assistant, Martina, who is Hispanic. The children range in age from a few months old to four years. There is a core group that includes two infants under 12 months, three toddler girls between 18 and 30 months, a three-year-old, and a four-year-old. New infants will soon be joining the group, and there are some children who come after elementary school. In Josie's previous setting, the children were mostly preschool age, three to five years, with some before and after school children, causing Josie to now be in a constant state of reshaping her practice for infants and toddlers.

Josie is inspired by the preschools of Reggio Emilia, Italy, where "The structures, choice of materials, and attractive ways in which educators set them up for the children become an open invitation to explore" (Gandini, 1998, p. 162). Most of the first floor of her home is arranged for the children, from the enclosed front porch where families enter each morning, through what might have originally been the living room, dining room, and family room. The living room at the front of the house has little natural light, making it a natural location for the youngest children to nap throughout the day and to house an enclosed, tent-like reading area that Josie designed for coziness. The middle room receives abundant natural light and includes built-in shelves and cabinets in which Josie has curated an extensive collection of books and items made specifically for young children, as well as regular household items that are intriguing to young children and safe for them. Children choose mathematics and science materials neatly arranged on shelves and explore them at a wooden child-sized table and chair set. Children can also investigate the effects of light, shadow, color, and transparency by placing objects on a light table that Josie acquired through an early childhood education grant program. Art and pretend cooking and woodworking also happen here. A door from this room, protected by a gate, leads to the kitchen. The back room is not as light. Children's cubbies are in this room, as well as musical instruments, more books, and large soft items to crawl and climb over. Josie and the children prepare to go outside from this room, while diapering, toileting, and hygiene tasks take place in the adjacent room. In each room, there
are natural materials, white walls, wood floors with carpets, and lots of open wall space with decorative wall hangings and posted information for families and professional agencies, such as the state licensing and food program agents.

Outside, the children have many opportunities to explore sand and water. Some days, they paint on a triangular prism-shaped, child-size easel set up on the deck. Through a gate and down three steps is a grass-covered space for toddler legs to run and preschoolers to practice pulling wagons and kicking balls.

Before providing care for children in her home, Josie worked in the corporate world, until a health scare turned out to be her youngest child. By this time, she had two older children and did not want to choose between being a corporate player and a good parent. Given a second chance at life and parenthood, Josie left her corporate job and began taking parenting classes and gathering materials to help her be a teacher to her child. She decided to homeschool and sought training and resources. Although she found the local homeschooling community did not look like her and was "very conservative," she drew on their resources, such as The Well-Trained Mind: A Guide to Classical Education at Home (Wise Bauer \& Wise, 2016) and Saxon math (Larson, 1994). As she honed her emerging teaching skills with her daughter, other moms asked Josie to invite their children for play dates because they loved how Josie worked with her own daughter. Becoming a family childcare provider was "VERY organic" (Initial Interview, 8/24/19). Although Josie has immense respect and regard for children, and advocates on their behalf, she is not the kind of person who gushes that she just loves children. She was an only child and "wasn't feeling that."

Josie ran a childcare center with 40 children for a year before becoming a family childcare provider. She is planning to open more locations, a combination of centers and family childcare homes. She is very inspired by the preschools in Reggio Emilia, Italy, as well as the classical education curriculum she discovered when homeschooling her daughter. She is very passionate about combining the Reggio approach and classical education in early childhood learning environments in urban areas.

Josie: I need to get Reggio and classical education in urban areas. Do you understand that? That's what I HAVE to do. That's what I have to do!

Interviewer: And you're very driven about that, aren't you?
Josie: [chuckle] I am! (Initial Interview, 8/24/19).

To reach her goal, Josie is involved with early childhood education organizations at the national, state, and local level, such as the National Association for Family Child Care (NAFCC), the state affiliate of the National Association for the Education of Young Children (NAEYC), and local groups sponsored by the local resource and referral agency. These professional affiliations provide her with training, networking opportunities, and platforms to share her views on policies related to the early childhood profession. Through a local family childcare provider cohort sponsored by a United Way initiative, Josie met her business partner for her expansion plans, as well as future employees who want to work with young children but find they prefer to work for someone else rather than operating their own family childcare business. She seeks out these opportunities and collaborators. She is "networking like a mad dog" to get herself situated so she can "have quality individuals" staffing her future sites.

## Learning Experiences and Influences

Josie's learning career falls into three distinct periods: the "formal stuff," resources sought after her daughter was born, and early childhood trainings. She acquired a bachelor's degree in telecommunications with minors in urban and regional studies and anthropology. Later, she returned to school to earn a bachelor's degree in psychology, specifically health psychology and cognition, with a minor in international studies. She also attended law school for one year and has acquired a National Directors Credential. After her daughter was born, Josie embarked on a quest to be a better parent, seeking out parenting classes and later homeschooling resources, including classical curriculum and Saxon math. Her daughter attended an early Reggio Emilio-inspired preschool, so Josie has gravitated to those trainings as an early childhood teacher. Making the Most of Classroom Interactions (MMCI) is the most impactful training she's taken recently, resonating with her previous learning. The focus of the three-day training for preschool teachers is teacher interactions with children, which provide opportunities for teaching and learning, both academic and social-emotional. With her new cohort of younger children, she would like to take the same training specifically for infants and toddlers.

Josie has attended so many early childhood workshops, many more than required, and now she wants more "big picture stuff." She works long hours and wants the precious time that she devotes to her professional development to challenge her thinking: "I'm always about shifting my paradigm. I'm always open to having my paradigm shifted. I don't need to have my paradigms

REINFORCED." On her list of possibilities are the infant MMCI training and national conferences, such as those hosted by the National Association for the Education of Young Children (NAEYC), the National Association for Family Child Care (NAFCC), and Head Start, because she hears "they have a heavy family engagement."

I also want to go to Parents as Teachers and become recertified as an educator there. But what I really need to do, Sue Ellen, is get my [whispered]ass back in school! I need to have a doctorate or an MDiv [Master of Divinity], or I need to go back and finish my JD [Juris Doctor]. The National Directors Credential came close to what I want. I love research, and I love psychometrics. I need to do that. I need to do that. Yeah.
"I don't remember actually having any instruction in teaching math." Instead, Josie has attended local and state workshops where she has learned about the state early childhood mathematics standards and activities for teaching mathematics to young people and says, "they're good stuff," more isolated activities rather than a pedagogical framework for teaching mathematics with young children. For now, she organically fits these activities into her existing pedagogy and knowledge of how children learn through interactions, in which "there are the moments when you can do things [such as discussion around mathematics]."

When asked about her own mathematics learning experiences, Josie animatedly told the following story about her very BEST math teacher.

Ok. So, this was my second undergraduate experience, right? By this time, I enjoyed math, it was good. But here's what happened. So I walked into a finite math class. And this guy, Dr. French, this white guy, really hippy-looking dark hair, skinny guy, you know, like Birkenstocks and stuff. He's at the front of the room, and he's talking to us (chuckle), first day of class, and he's talking to us. And he starts telling us about a camping trip with some friends. And as he's telling us this story, he's drawing a picture of the mountains that they climbed, some of the fir trees that were on the mountains, rivers, and stuff. And he's taking up all of this, all of this board. Then he starts saying, he starts putting labels on the, "This is a sine, this is what it looks like. This is a cosine, this is what that looks like." He gave us a story, he gave an illustration, looped us in and made it real. THAT'S when I was stimulated, THAT'S when I decided that if I ever had the opportunity to teach math, I would. That was one thing. And the other thing was, I said I would take every math class this guy ever taught. So it turns out that he was a visual artist and just got a hankering, a bug, to learn math. And so he already had a four-year degree, and he was going to go for a graduate degree in mathematics, but he decided he would teach himself every course that he needed, that he could test out of, basically. And so he DID. In his office, he had shelves and shelves of black composition notebooks, that's what he had, and he had them labeled meticulously, what was in them, the dates. Marcus French. That's when I really began to
understand that it's all about perspective, it's all about teaching and engaging. And it's not about, it's not about parsing people out. The person on your left, the person on your right, they're not going to be here a year from now. You know, that whole approach to teaching sciences, just really sucks. It's really traditional, but this really was the antithesis of that, and so I LOVED that. Does that answer your question? That was a telling time for me. So I never had any instruction in teaching math. It's just something I, I don't remember actually having any instruction in teaching math.

When asked what future teachers should learn about teaching mathematics with young children, Josie responded:

I think that it would be helpful if people understood how much math we actually DO know and at least how much we utilize. I think if more people, especially women, understood that math is all over, and that we do mathematical functions all the time. I think it would create a better attitude about math, and I think we sorely need that.

## Teaching Mathematics with Young Children

"[T]he first thing I thought of was a star, spewww, kind of like that," Josie gestured in explanation when asked what she thinks of when she hears the word "mathematics."

When her daughter was in middle school, Josie found that she loved helping her daughter's friends who found math really difficult. She loved to make the mathematics real, like Dr. French had done for her, and she loved experiencing the ah-ah, the breakthrough moments, with these students. Since she now teaches younger children, more infants and toddlers than preschool children, the way she finds those moments to make mathematics real is evolving. She shared a story from earlier in the day that she loved, talking about mathematics concepts with green beans:

Plus, so you know, it's nutritious and they EAT it, whatever they cook, they eat! So, I took off a little end, and we talked about the green bean. We talked about it being nutritious, look how LONG it is. What do you think will happen if we break it? Let's see. So I broke it. Wow! Are these pieces bigger or smaller than the other ones? They're bigger Miss Josie! They're smaller Miss Josie! Ok. And you know what, these are called halves. Can we say it (Initial Interview, 8/24/18)?

However, other math learning and teaching moments are full of puzzles. Josie told the story, from the same day, of reading a farm book with four-year-old Mark. In vivid detail, she recounted Mark's process of counting several frogs drawn both on top of and beneath a lily pad:

Mark started at the bottom. He started at the bottom right. And he's counting going this way. And it's really weird because somehow usually he gets the right number. And I'm like, how did he, where did he start?! I don't know what happened!! So
he was over here, and then he was going, and then he was going around. And so, you know, teaching him the correct order to start, at the top, let's go across, you know, and back down, we're going to Z down and over. So after probably about four or five, well, maybe four attempts, he got it! And he went like [gestures]. But you know, by Monday, he won't remember it...I would think that it's better to teach a child that's three or four years old in a structured way, and as their brain develops, and additional capacity comes, then ok, let's go abstract. But I have a problem with that [unstructured image in the book].

While Josie is familiar with content related to early mathematics, she feels there is more for her to learn. Her greatest challenge teaching mathematics is knowing that she misses opportunities.
[W]ith the younger ones, maybe one-to-one [correspondence], building an appreciation and knowledge of a one-to-one relationship. This one's yours, and this one's mine, or these are the number. They love the magnetic tiles, so we'll have four square ones and we just count them out. But I don't know what they're getting, because these are the youngest children that I've had to work with. That's why I'm saying that I'd love to do the MMCI, Making the Most of Classroom Interactions, but it's not going to be content. And so I really need to learn more about teaching INFANTS math, it's not the same thing as preschoolers.

## Josie and the Children: Observations

As I embark on my first observation of Josie and the children, I do not know what to expect. I have spent a couple of hours talking with her already about her experiences teaching and learning mathematics, learning about her history as a family childcare provider, teacher, and business owner. I know that her childcare space is beautiful in a Reggio-inspired way, filled with an extraordinary collection of items chosen to pique children's curiosity (Gandini, 1998). The collection is numerous, vast; the items are arranged neatly, with intention. So as I pull up to the curb and park along the block, I am thinking of the beautiful materials in Josie's space and of how excited I am to meet the children and see Josie with the children. I realize that our conversation was about Josie's experiences as a learner and teacher of mathematics, more a conversation about her, about adults, rather than about children. So I am so excited to see her with the children.

On this day in August, Josie does not have an assistant. It is the time of year when there is more transition of children and staff in childcare settings. There are new children in Josie's care, as older preschoolers have moved on to kindergarten, making way for new infants and toddlers. When I arrive, it is time to go outside. Josie allows me to assist her and the children as they move
to the deck. While Josie finishes applying sunscreen and bug spray to the toddlers, I hold 4-monthold Seth while four-year-old Mark engages me in his game in which a belt renders the wearer invisible. When everyone is settled, Josie tells the children that we will stay on the deck for 10 minutes and then go to the grass. She shows the children how she is setting a timer on her phone.

Before our next observation, Josie successfully completes the accreditation process for the National Association for Family Child Care (NAFCC). She has hired and trained a new assistant, Martina, who will soon finish her associate degree in early childhood education at the local community college, which she plans to articulate into the early childhood education bachelor's degree program at the local state university campus. The fall is busy. The next time I visit, Josie is "breathing again," and planning a celebration for her staff for their next meeting and training. She has received her $\$ 500$ bonus from the state for completing the accreditation process, which is the final step to acquiring the state's highest rating for its Quality Rating and Improvement System (QRIS) for early care and education programs (Family and Social Services Administration [FSSA], 2019). In addition to the accreditation celebration, Josie would like to change the environment to better serve the younger children, including the two new infants. Top on her list is an inside climber, if she can find something that fits her space.

This January day, it is warm enough to go outside when I arrive, so I go outside with the three toddler girls and Josie. As she and the girls navigate the steps, Josie says, "Coming down steps" and counts each stair as they come down. Josie plays music from her phone. The girls are dancing (and Josie, too). Josie asks me if I know the Christian rock music of TobyMac or Ryan Stevenson. I do not, but the music has a lively beat that gets everyone moving, squealing, and giggling. Josie and the girls seem to really enjoy listening and dancing to this music!

There are two wagons, a small one that could hold, or easily be pulled by, a toddler and a large one that will hold all three girls. The girls climb in and out of the wagons, sometimes pulling each other, sometimes Josie pulling all of them in the big wagon, with Josie providing a running commentary of the girls' activity using positional words, as well as encouragement, manners, and social reminders. Josie says they will go around the yard three times. The girls and Josie count each time around the yard. Josie encourages the girls to get out by themselves and says, "Uh-oh, get back up. I see you fell."

Josie pulls Francie in "a small circle," and asks Francie to get out so her friends can have a turn.
"We did it! Thank you, that was fun!"
"Francie, I see you're getting out of the wagon."
"You're out of the wagon, out of the BIG wagon."
"Pearl, you're standing in the wagon."
"Uh-oh! You got back up. You're so strong, Francie!"
Lizette rolls a ball with Josie, who says, "Score! Now it's your turn!"
"I hear you saying, 'Help,' Pearl. That's a new word for you." (Field Notes, 1/14/19)
When I visit again in February, Josie has begun to settle into a rhythm and routine with her assistant and this group of younger children. Children can now supply the answer to "What comes next?" I arrive as the timer signals it is time for circle time to begin. In the bright middle room, Mark rushes over, but Josie asks him to wait. Jade will turn off the timer, Mark will hit the gong three times and then lead the pledge with Jade, age three, and the three toddlers, Pearl, Francie, and Lizette. Martina is with the two infants in the quiet, more dim front room.
Josie announces that she will be the song leader for four songs, and Jade will be the exercise leader. Tooty Ta is first. Josie notices that Jade is not doing the motions and tells her it is okay. She says they have sung one song and have three more to go, putting up her fingers to match the number words. Head, Shoulders, Knees, and Toes is next, and they sing it three times, slow, medium, and REALLY fast, with Josie stating the tempo each time. Showing her fingers again, Josie explains they have now sung two songs and have two more to go. For the third song, Jade suggests The Itsy Bitsy Spider. Josie agrees, with the condition that Jade must do the motions, in addition to singing. For the final song, Josie asks the children to find a friend and hold hands, hinting, but not telling, her choice for the final song. Mark guesses correctly that it is Ring around the Rosie. They sing two verses, with lots of giggling as all the children fall down for the first verse and jump up for the second.

After all the activity involved in the singing, it is time to sit on the floor. Josie asks the five children what is next, and they exclaim, "Prayer!" in unison. Josie reminds them that each child may say a prayer if they choose. Jade says a prayer, Lizette says "Amen" for her prayer. Mark chooses not to pray. Looking to Josie, Pearl points to her socks for her prayer, and Josie verbalizes, "Pearl's
prayer is for her socks." Josie says her prayer. Finally, everyone joins in singing the chorus of the gospel song, Amen.

Reading Polar Bear, Polar Bear, What Do You Hear? (Martin \& Carle, 1991), Josie asks questions while she reads, such as is the boa constrictor big or little, long or short; and what would the elephant make that noise with? Jade wonders where peacocks live, and Mark wonders where they hide from people, so Josie suggests, "Why don't we find out about where they live? Now we have a project!"

Before the group begins their peacock investigation, one of the toddlers quietly asks Martina for a baby doll, and soon all four girls want one. Although Josie calls this a "baby disruption" and indicates she will work to prevent it from happening again, she calmly gives language to the girls' actions and emotions. She brings out a new book about shapes and takes a picture of Leroy, age five months, who is being "super-attentive at circle time." Counting the sides of a triangle, Josie asks Jade for her help on what shape it is. Jade says, "square." Mark says a waffle is a square, it has butter in the middle. Josie notes, "Leroy is talking, wonderful!" When Seth, age 10 months and almost walking, falls and cries, Josie, always aware of individuals' needs and possible responses provided through her vast array of training and research, suggests to Martina to let him calm down and see what his cues are before feeding him or putting him down for a nap.

Although the structured circle time is over, Josie transcribes a list on chart paper of what the children want to know about peacocks, including where they live, if they like people, what their feathers are like, and if they can fly. Mark earlier mentioned making a book to take home, so Josie gets out a pad of colored paper and asks him what colors he needs for his peacock book (blue, black, and white). Jade counts one, two, three, four on her fingers when Josie asks how many pages she will need for her peacock book. As the toddlers play, Francie climbs a highchair, so all the highchairs go over the gate, out of reach. Josie tells Mark that he has nice manners, as his prayer ended in being kind. All the children are playing. Josie is sitting on the floor with Leroy on her lap. She asks Jade, "Did you go to the store already, Jade? What are you going to buy? (Jade says books.) Books! How many books?" Jade counts aloud while raising her fingers, 1, 2, 3 , 4. Josie counts $1-4$ starting with her thumb. Jade says no and holds up 10 fingers. Josie shows her four fingers, but Jade has changed her mind and now wants 10 .

Lizette, Pearl, and Mark play around a child-size workbench. When Lizette brings screws from the workbench, Josie gives her the word "screws." Lizette struggles to say screws, and Josie says, "I know that's hard for you." Mark tries to lift his toy shopping cart onto the workbench. Pearl is there, too. Josie tells Mark that she can see that he has an idea, but he needs to make a new idea. Mark's idea is to move the workbench away from Lizette and Pearl, so Josie tells him he has to share here.

## Teaching Images

Josie's mathematics teaching image connects her past experiences and turning point story as a mathematics learner (Dewey, 1938/1998; Drake, 2006) to her subsequent personal experiences tutoring her daughter's friends, her current professional experiences teaching mathematics with young children, and her future desire to learn how to make mathematics real for infants. This image provides a window into the hidden part of her pedagogy, the part that cannot be measured by tests or evaluative checklists and affords a view into how Josie's experiences come together in the moments of acting and deciding in her practice (Clandinin, 1985).


Figure 4. Illustration of How Josie Makes Math Real

As Josie continuously scans the learning environment for mathematics teaching opportunities during play and interactions, the teaching image of "conductor" comes to mind. Bounded by her learning, business, and social experiences, and with children and subject disciplines as instruments that Josie directs in interactions, solo notes from mathematics are occasionally heard but usually intermingle with other subjects. Like a piece of music, there is an over-arching structure to the day, with predictable rhythms and tempo changes that Josie orchestrates. As the conductor, Josie is alongside and with the children, as well as in charge. Outside the stage, like a music conductor, she is responsible for organizing the group's scheduling, the physical environment, communicating with collaborators and stakeholders (families, childcare agencies, the public), and the professional growth of herself and her employees.

Josie conducts a harmonious melody of interactions from a collection of potentially cacophonous children's purposes (Dewey, 1938/1998). The melody includes themes from Josie's influences and goals, while also allowing room to follow children's improvisations. The image of conductor feels appropriate because Josie experiences music with embodiment and emotionality (Clandinin, 1985). She sheds her intentionality and experiences the music she shares with the children, especially the music related to her own spiritual life, such as singing Amen and dancing to Christian rock with the children.


Figure 5. Josie's Teaching Image as Conductor

## CHAPTER 5 YOU'D BE SURPRISED THAT IS MATH: PATSY'S NARRATIVE FINDINGS

## Setting \& Professional Life: Past, Present, and Future

Patsy was downsized from a large pharmaceutical company after working there for many years. Beginning in production, she worked in a variety of positions and eventually moved into the corporate office, where she "did a little of everything, ordered, be mom, dad to my engineers." She was a "fixer-upper," helping those who worked overseas or out-of-state to correctly post their reports.

Looking for a new career, she answered an advertisement for a substitute teacher for a large, local childcare non-profit organization, working at different locations. After the long hours at the company and missing so much of her two sons' childhoods, she had the opportunity to, "be a kid myself!" Two years later, she left to work at a registered ministry, first as the secretary assistant and eventually working in all the jobs at the site, which was chronically short-staffed, leading to exasperation.
"You need to do your job for the kids. She [the director of the registered ministry] didn't show up till 10 or 11 o'clock in the morning. I got there, sometimes, at 7 , welcoming parents... and she leaves early, and here I was there early in the morning and staying until the last parent leaves just about."

Eventually, after working several years and in a variety of settings, Patsy concluded that she knew what it would take to open a childcare facility of her own. She had bought a house that suited her needs and opened her family childcare home in 2014, in a 1960s tri-level in a subdivision on the edge of the urban center of a Midwestern city.

And so my setup, I have a lot of people who say, wow, this is set up like a center. I wish it was bigger! But of course, it's out of my home. And so, I have my section of, my dramatic play over there, sensory is next to that, over there, there is my science area, and the library area is there. On my art wall is there, along with my bulletin [board], my kids. My infants' Pack-N-Plays are set there. I have a changing table, a sink. I think I started dreaming about doing this even before I started. Because when I moved in this house, I thought, you know, I really don't need this space, what can I use it for (Initial interview)?

When families enter the house, they take their children to Patsy's childcare space in the finished basement, passing photos of past and present children, current artwork, and required
information, such as Patsy's license and emergency exit plan, as they descend the stairs. Her license allows her to care for up to 10 children, and she usually has a combination of infants (up to 12 months), toddlers (12-36 months), preschoolers (3-5 years), and occasionally school-age children. The stairs end in a large open room where Patsy and the children spend most of their day together. Patsy has installed a variety of shelves, cabinets, and tables to separate the large room into areas for defined activities. Each area has a sign listing the early childhood standards and what children might learn from participating in activities and playing with the materials in that space. Patsy posted the standards not only for herself, but also for parents. She includes a variety of items in the space, some that the children might not know yet, because they are "absorbing...all the time." There is an open space in the middle of the room for Play Time, where the children build with blocks, dance, and play with dolls. She has exersaucers and Boppy chairs for the infants and low chairs for the toddlers.

Pack-N-Plays for infant and toddler naps line the wall adjacent to the stairs. A back corner originally housed a kitchenette and retains the sink and counter, which makes a convenient diapering area. Patsy's desk and computer are sandwiched between the diapering area and a door that leads to a bathroom and what was originally a bedroom that now houses a large, neatly organized collection of educational materials, toys, nap mats, supplies, and childcare records. Large storage boxes contain dolls, sets of blocks, and zoo animals. She rotates materials depending on the season, children's interests, and her purposes (Dewey, 1938/1998). Before her family engagement trip to the zoo, she introduced the children to her zoo animals, and the children continued playing with them long afterward. A low shelf separates the adult work areas from the children's play space and contains clear and translucent window blocks, part of Patsy's extensive collection of blocks.

Another corner houses the Circle Time and dramatic play area, which is bordered by low shelves and cabinets that contain puzzles and mathematics manipulatives. Patsy has an adult-size chair in front of the wall covered by the alphabet, shapes, a number chart that goes up to 120, and a calendar pocket chart. A child-sized kitchen and housekeeping set wraps the back of the space. During Circle Time, the children sit on the carpet while one of them or Patsy leads the activities.

As we move past the Circle Time area, we pass the door to the outside patio play space and come to the science, sensory, and social studies corner. There are images of George Washington and Abraham Lincoln, the moon and the Grand Canyon. Patsy found a hornet's nest "in the dead
of winter," bagged it up, brought it in the house, and eventually "heard a buzzing, they had warmed up!" Remnants of the vinegar and baking soda volcano they made remain on a shelf, along with a piece of bark, magnifying glasses, and a set of realistic plastic insects. A child-height table used for art, puzzles, home-made folder games, math manipulatives such as counting bears and tangrams, and fine motor skills activities such as hole punching and grabbing with pincers marks the boundary of this space.

Turning the corner, we are in the Library, bordered by shelves full of books, musical instruments, and handmade props to enhance stories. There is a rocking chair where Patsy reads, often with an infant on her lap. A three-year-old often directs the other children to, "Sit down, sit down!" when it is time for reading.

Everyone moves upstairs to the eat-in galley kitchen for breakfast, lunch, and afternoon snack. A row of three highchairs lines the back wall, while a low table and chairs sits in front of a window. Patsy sometimes has an assistant who prepares food and cleans the kitchen. The children enter the kitchen and get a boost from an adult to wash hands. Each child has a small container with their name that holds a small towel for drying hands. The preschoolers each find their towel, dry their hands, and take their place at the table. As the adults serve milk and food, the children chat, telling stories about their families, one day talking about the men working on Patsy's gutters outside the window. On a day when Patsy's assistant is gone, she talks about her future plans.

While Patsy and the children prepare for lunch, she says that her goal is to be a coach for other providers. She has been through the ups and downs and knows what they need. Children in pre-K need structure. Babies need to learn. Parents don't teach enough. Rashida's mom is busy. Patsy tells mom regularly to read to her daughter, but she's too busy (I hear disapproval in her voice). Carla's family reads to her EVERY night.

Because she missed so much with her young sons, every year Patsy creates a CD of pictures and videos for families, documenting their children's time with her. She pursued a grant for family engagement activities, providing children and their families opportunities to experience the local zoo, an amusement park, and a local dinner theater, all expenses paid.

## Learning Experiences and Reflections

## Experiences Learning about Early Childhood Education and Mathematics

Patsy's learning experiences in the field of early childhood education tell her story as a life-long learner and also illustrate the patchwork and cacophony of early childhood professional development and compensation systems (Ayers, 1989; Bowman et al., 2001; Gomez et al., 2015; NAEYC, 2009; Parks \& Wager, 2015).

Well, one thing about me, when I do a job, I'm going to do the, above and beyond. Only because, when I start substituting, you know what? I'd like to know more about what this entails, what it takes. So, I started going to school. Everything was out of pocket (Initial Interview, when asked if she felt she would stay in the early childhood field when she started her first job in early childhood education).

Because Patsy started going to school while working as a substitute teacher, going from site to site in the organization as needed, and then later worked for the registered ministry, she could not secure the sponsorship from a childcare facility director required to obtain a state-funded T.E.A.C.H. ${ }^{\circledR}$ Scholarship that would have paid for most of her school expenses (Indiana Association for the Education of Young Children [IAEYC], 2019). That changed when she became a family child provider and could sponsor herself, receiving funding for her last two courses. By this time, Patsy was in her practicum, which created another set of challenges: how to satisfy the requirement of working in another classroom after enrolling the first child in her program. She made arrangements with a veteran provider she knew from trainings and the child's family for the child to stay with the provider during the hours Patsy spent at her practicum site. Patsy, however, found an opportunity in this challenge:

So what I learned from there, she was a great mentor, also. She said, you know what I notice when [the child] plays, he plays alongside. Really? Because he was the only child I had at the time, so I didn't know. She said, yeah, the other kids will be playing, and he'll go over there, and he'll play right next to them, he'll just play, like they say here, they do that, what do they call it, solitary play, social play, no parallel play, parallel play, yeah. I'm like, yeah, ok.

Patsy is currently completing a master's degree program in early childhood education, which she is also paying for herself, as the T.E.A.C.H. ® Scholarships only fund a credential, associate degree, and bachelor's degree (IAEYC, 2019). She is taking a literacy course and shared her assignment for her dream literacy space:


Figure 6. Patsy's Dream Literacy Space

She also takes many trainings through the local resource and referral agency, as well as webinars provided by the National Association for the Education of Young Children. She enjoys school and learning.

Patsy loves mathematics and takes advantage of opportunities to learn more and to revisit topics. When she worked at a pharmaceutical company, she always took the refresher mathematics classes offered by the company, which included everything from percentages in basic classes to geometry and trigonometry. However, she did not like the story problems. Like the early childhood education learning opportunities she seeks now, she felt the company's learning opportunities kept her "fresh." She was required to take several mathematics courses for her bachelor's degree in business administration.

When she left the pharmaceutical company and found herself in the early childhood education field and acquiring an associate degree, the mathematics she had taken in her bachelor's degree program fulfilled the abstract reasoning requirement of her program. In her early childhood education associate and master's degree programs, Patsy has had very little training related to learning to teach mathematics. She was required to take sociology and biology courses in her associate degree program. While she was taking a course on early literacy development when we
first met, her master's program did not include a similar course for early mathematics development. In both programs, "they give you little doses of it [mathematics], and you really don't realize it," such as in general early childhood curriculum courses.

## Reflections on Learning to Teach Mathematics

Yeah, cause for me, in order to be good at something, especially when you have
so many lives in your hands, you really need to be on top of it. You don't want to
miss out on anything. Even though I raised two boys and my nieces and nephews,
I raised them up, I was really surprised that I didn't know this when I raised my
kids, I wish I did! And a lot of the trainings that [the resource and referral agency]
provides, when they do face-to-face, I can ask questions, what is this? If there's 15
people in there, a lot of people can give their technique that they use. You'd be
surprised at what you can learn from other people (emphasis in original, Initial
Interview).
Patsy's desires for what she would have liked to learn about teaching mathematics with young children is shaped by the responsibility she feels toward the children in her care and how important she feels mathematics is in daily life and work. Therefore, she would have liked to learn more about how to integrate mathematics throughout the day and how to highlight it in children's play. She said, "you're introducing math every day, in their blocks, when you're counting, they're solving problems, they're thinking, even when you teach them how to share." She recognizes some opportunities to integrate mathematics, such as counting the steps while going upstairs for lunch, but she yearns to learn more, especially for infants and toddlers. This is why she takes the opportunity to engage in the NAEYC webinars and the local resource and referral trainings, where she keenly listens for ideas from other providers. While these trainings often are not specifically about mathematics, they help Patsy recognize what could be mathematical about a toy or experience, "maybe I already know, but I don't know it." Patsy and I laughed together about trying to soak up everything at these trainings, that there is just so much to learn, it is overwhelming.

But when you look at it as adults, we're doing math but a lot of us don't realize. I hear a lot of young people say, "I don't need these math classes!" I say, "You don't?" When you use your GPS, what do you think all that is? Look at these highways, how they're put together. These are engineers. These roads look like they are snakes on top of each other. If you had an aerial view, you could see the structure, and how the mastermind put that together. That's all math. Everything you touch is math! I don't want to get off the topic. But if you guys give a training, I'll be the first one to sign up.

## Teaching Mathematics with Young Children

With two infants, two toddlers (one who has special needs), two three-year-olds, one four-year-old, and four occasional school-agers (ages eight and nine), Patsy works to introduce all the children to many different mathematics topics. She introduces "a little dose at a time" and uses lots of repetition, as well as routine. The children know they will work on the alphabet and numbers during Circle Time after they finish their morning snack, followed by reading a book, then play time. While the children feel Patsy's organized schedule, she also interjects learning opportunities throughout the day that grow out of the children's actions. Reading is one of her favorite times to integrate mathematics. When asked if her current literacy course is helpful to her for teaching mathematics, Patsy responded as follows:

Patsy: Well, they always say math and literacy kinda go hand-in-hand. I mean, but ok, exactly what is literacy? Literacy could be a lot of things. You're reading to your child. You're giving them understanding. And in all the same, you'd be surprised, in all that communication between that child, math is coming through. Because everything we do is math. When they're writing, they're focusing on, they're thinking, they're solving problems. When I see them doing things. One little boy takes Thomas the Train, he hooks up the magnet, he rode Thomas around the edge of the thing. Isn't that what an engineer does, he has to figure out how much space he needs to have, to find the edge.

Interviewer: He's using his spatial reasoning, that's our word.
Patsy: Oh, spatial reasoning, I like that!
Interview: He's doing his spatial reasoning work right there. And ALL of this, like with the blocks, and when the little ones are trying to figure out how to fit the pieces into these holes, that's spatial reasoning.

Patsy: Ok, thank you! I can use that with them, I'll go, spatial reasoning, that's what we're doing. If you can make it sound exciting, they'll go, wow!

Interviewer: Any time they are trying fit things together, or block related. It's also your body space. There's a lot. It's very interesting. I love that field. Oh, when you were talking about those bristle bricks. We had some of those, they were really old. All of those, when you put together those big blocks, they do a lot of spatial reasoning with that, too.

Patsy: Oh, yeah, my kids love to build. One boy liked to build tall. I tell them don't go higher than you can reach. I asked, "What do you have right here?" He's 4. He said, "I used the yellow ones to put down because they're wider." I said, ok, ok! He's telling me HOW he's building.

Interviewer: It's his spatial reasoning again! He's thinking!
Patsy: Yes! Now that you have given me a word, that connects!
Interviewer: So he's putting the yellow ones down first because they're wider, they're a better base.

Patsy: Yes, single one versus, they're double, so you can put these down for the foundation, and it's sturdier, so when the babies come by, they can't knock them over. They can build up high. But I like the fact, when they are using these right here, they say, "Look, I'm building a house!" They're thinking of what they are making. They're pulling apart and redoing again.

Interviewer: Do they seem to like the texture?
Patsy: They must, because when they sit at the table, they mash them. But the fact that they are building something, I like that.

To help her and families know what children are learning during free play, such as puzzles, sorting, and blocks, she has posted sets of bullet points related to the standards that she received in a training. Patsy noted that children are thinking mathematically when they sort and look at the shapes of puzzle pieces to put them together. They do a lot of sorting, using a variety of attributes, such as color and shape, to classify. Guided by the value she places on mathematics, building, imagination, and perhaps a bit of feminism, Patsy also intentionally places books with building sets:

Because these are ENGINEERS, engineers need to have a LOT of different math.
And my girls, I have books over here, so they can look through here, hey, here's a bulldozer, what does a bulldozer have to do with blocks?

Interviewer: So you intentionally put this in here as provocations?
Patsy: Yes! Because they look at this, and they're building. They have a lot of free time. My little girl is working things out, solving problems, with the blocks. She'll get one block, put it over here on the floor and go back to get another one. She'll make 50 trips back and forth with the blocks until she has a long train. I'll go what have you created here. She'll say it's a, a, a, a tunnel! Oh, wow, I like your tunnel! Then she said, I'm going to put this up here, and this is going to be a, a, a. Well what do you think it might be? Well, it's a lid!... Look, I'm building a house! An older boy said, "It looks like China!" It does, looks like China.

Interviewer: Oh, with the roof.
Patsy: Yes, it's their imaginations.

## Patsy and the Children: Observations

After interviewing Patsy in her empty space at night, I am eager to see it in the daylight with the children. I arrive at snack time, and her galley kitchen is full, with infants and a toddler in highchairs and children sitting around a low table at the end of the space under a window. Patsy had mentioned that she would have three school-age girls the first day I visited. Patsy introduces me to the children and shows me where to find a chair so I can observe and not be in the way. The older girls introduce themselves and the younger children. The children are happy to have a new audience and chat with me. On subsequent visits, I feel the flow of Patsy's schedule.

## Circle Time

After morning snack, Patsy and the children transition from the kitchen back downstairs to the Circle Time space. The infants sit in exersaucers with favorite toys, while the toddlers and preschoolers sit on the floor facing Patsy and the wall that houses teacher supply store materials such as the alphabet, calendar, and shapes. During Circle Time, which lasts about 20 minutes, Patsy and the children explore social knowledge (Kamii \& Housman, 2000) topics, such as letters of the alphabet, counting numbers from one to 30 , shapes, days of the week, holidays, and manners.

We do our ABCs and we count, our numbers. We go through the days of the week, and we say what day is it. We say, "It's FRIDAY!" Then my kids also, this is my math cabinet for them.

Patsy usually solicits the children for a starting activity. One day it is shapes, another day it is the alphabet. One day is a school holiday, and one of the three school-age girls leads the circle time. Patsy interjects questions, such as where they see an octagon and asks Annie to get their stop sign so they can count the sides, reminding the children that there are eight sides. She shows the children three-dimensional block shapes that correspond to the two-dimensional cardstock shapes on the wall. Another school-age girl leads the number counting, using a chart that goes to 120. Patsy asks what the numbers in the first column have in common. She says she wishes someone had taught her that! After discussing ways to see patterns in the chart, the older girls find several. Patsy points out in the 5 column that the numbers go up by 10. Patsy talked about how they did not get to look at those patterns when she was growing up learning math. While the children count, she encourages individuals to "pronounce the numbers clearly" and to speak up because "we want to hear you!"

Another day, each of the young children has a turn pointing to the shapes while the others identify them. Patsy says they are "little teachers." Patsy puts " 26 " on the calendar and asks what's coming at the end of October. Halloween, she exclaims, and then she and the children count the days on the calendar, one to 26 . She taps out the syllables in a child's name on her thigh, and the child copies her. This day, she brings out a board with collections of one to 10 different items to count. She asks the children to name the item and count how many of each one, which they do with much excitement. They use the Spanish word, "uno," for "one."

## Reading Carpet/Literacy

After Circle Time, it is Story Time, for about 15 minutes. The children gather on the reading carpet, and Patsy gets situated in her reading chair. The youngest infant is ready for a nap, and Patsy holds the older one on her lap. One day, Patsy reads Five Little Monkeys, in which an alligator snatches one monkey at a time. After the first monkey is snatched, Rashida said there are four monkeys left. Patsy holds up three fingers after the next snatch, and Omar says there are three left. When there is one monkey left, Patsy says, "Hands up! One monkey is asleep" and gestures to Dennis sleeping in his pack-n-play. Another day, Rashida wants Patsy to read The BIG Eating Book. There is a baby in the story, and Patsy says Carla is a baby. The book says, "Everyone has to eat." Patsy makes the sign language sign for "eat" and asks the children to do it. The squirrels and chipmunks eat acorns. Patsy asks, "How many acorns?" and asks Rashida if she is paying attention. There is a goat with birds on its back. "How many birds?" Annie says one. Patsy and the children count five butterflies. This story time ends with Patsy reading, "Josh eats three desserts," and inviting, "Let's count them. One, two, three."

## Play Time!

Next, Patsy announces that it is Play Time! As Patsy and the children transition from the literacy area into free play, Patsy addresses diaper changes and toileting. The infants might be napping or in an exersaucer while she pulls out materials. One day, Patsy brings out bottles she has filled with water and soil. The children explore liquid and dry solid volume measurement. Patsy and the children notice that the soil compacts so the bottle with liquid looks like it has more volume. Another day, Patsy gets a puzzle out for Annie, Rashida, and Omar. Patsy shows the
girls how to put the pieces together, asking "See how it matches?" Rashida puts two pieces together by herself. Patsy tells her to turn the pieces around and suggests a place for Annie to try her pieces. After the puzzle is finished, Patsy, Annie, and Rashida count the pieces as they put it back in the box. Later, Patsy brings out six four-inch dice of different colors. She and the children discuss that the die is a cube, like a square. Rashida says it is blue. Patsy asks the girls to find the side with one dot. She shows them the side of the cube with one dot. Annie finds the side with one dot, then Rashida finds it. Omar has a die and finds one dot. While Carla moves around in a walker, the older children all find the side with four dots. Annie stacks four cubes, Rashida stacks hers on top, then they count six cubes in the tower. Patsy arranges them into a square, then into a rectangle, and finally into an L-shape.

Another day, Patsy brings out a box of window blocks (typical wooden building blocks with clear windows) and color blocks (windows are translucent colors). She asks who wants to be measured, and Javonte exclaims, "Me!" The children and Patsy stack blocks as tall as Javonte, then he knocks the blocks over. Javonte begins building a new structure, a house. Patsy points out Javonte's house, which is now a tall tower. "He's an architect," she says. As Carla crawls around the play space, she knocks over Javonte's creation, and Javonte claps and cheers for her.

## Teaching Images

Patsy's story offers clues, breadcrumbs from her past, present, and future that form threads of continuity and interaction (Dewey, 1938/1998; Clandinin \& Connelly, 2000) from her childhood experiences, through her pharmaceutical work experiences, parenting experiences, to her current work in early childhood education settings in a childcare center, registered ministry, and her family childcare home. Two recurring threads in her story are nurturing and engineering or building.

## Nurture

Merriam-Webster (n.d.) defines nurture as 1) training, upbringing; 2) something that nourishes; and 3) the sum of the environmental factors influencing the behavior and traits expressed by an organism. Beginning with her own parents and early childhood experiences speaking "gibberish" when she started school and her third-grade teacher who took an interest in
her and her siblings, Patsy's story includes elements of training and upbringing. As she raised her own sons, nieces and nephews, and foster children, the sum of these environmental factors are reflected in the best decisions she knew to make at the time, encouraging a troubled foster child to overcome his circumstances by using strategies she had used to overcome her own adversity, including no excuses. As an administrative assistant, she took pride in nourishing her colleagues in her roles of being mom and dad to the engineers in her work group, and the problem-solver, "fixer-upper." Patsy continued nurturing children, parents, and families when she began working in early childhood education. The sum of environmental factors, such as missing important moments of her sons' childhoods and overcoming her own obstacles to learning as a child, influence her practice as a family childcare provider, encouraging Patsy to provide an annual CD record to families of their child's time with her, providing family engagement activities, and learning all she can about teaching young children. She yearns to support busy parents who lack resources, like her third-grade teacher did for her. Each day with the children, Patsy nurtures them by preparing them for school through the environment and instructional activities she has carefully created. Looking to the future, after she finishes her master's degree, she would like to continue nurturing families, as well as other providers, providing them with nourishing training that would grow out of her knowledge and experiences of what children need.

Patsy nurtures Omar, almost two years old and with special needs, with training she has received from his therapists to support his development and gets very excited about his progress. She watches Omar, encourages him, suggests he try something again at his current level or something new just beyond this level. She seems to notice the "edge" of his ability, which could be a place to build from in professional development with Patsy.

## Engineering and Building

The engineering and building thread originates as Patsy recounts her professional life, which began after she received a business administration degree and went to work at a pharmaceutical company as an administrative assistant to a group of engineers. Her chronicle is laced with respect for the engineers, who she takes pride in serving, helping them with late reports. In addition to her respect for others who design and engineer, Patsy also designs and creates learning materials for the children, such as a literacy and math "clock" and a rod with rotating blocks for children to turn and tell her the number they see. She notices mathematics, engineering,
and building all around her and has implored young people around her to also appreciate the mathematics in the tangle of interstates in the city. Her appreciation for building and engineering are most manifested in the large number and variety of block sets in her space. She chuckles as she says, "And over here, I have more blocks, maybe I really love blocks" (Initial Interview).

The anecdotes Patsy shares related to engineers, building, and blocks illustrate that she sees females as capable of becoming engineers, scientists, mathematicians, and builders. She inferred that the engineers she worked with at the company were both female and male, and the mom of one of the children in her care is a scientist. With the children, she intentionally places books about bulldozers with blocks for the girls because they are "problem-solvers."


Figure 7. Patsy's Teaching Image of Engineering and Building

## CHAPTER 6 WHAT DO THEIR VOICES TELL US?

## What Do Josie's and Patsy's Stories Tell?

This section responds to the first research question: What role, if any, do mathematical learning experiences play in early childhood teachers' mathematics teaching practice?

## Early Childhood Teachers Want to Learn to Teach Mathematics with Young Children

Contrary to the grand narrative that early childhood teachers are not good at mathematics and are not interested in teaching or learning about teaching mathematics (Clandinin \& Connelly, 2000; NRC, 2009; Richardson \& Bofferding, 2015), both Josie and Patsy shared enthusiasm for learning mathematics and teaching mathematics with young children in the present. Narrative analysis (Polkinghorne, 1995), used with Dewey's (1938/1998) continuity and interaction constructs, helped me uncover a possible path for how these women evaded this grand narrative. Josie's turning point story (Drake, 2006) alludes to resolution of her prior "mis-educative" (Dewey, 1938/1998, p. 13) experiences with mathematics and identifies the specific moment in her mathematics learning when her curiosity for mathematics and interest in teaching it began. In Josie's narrative, this is the moment when mathematics became "real" to her. As she interacted with her stories in her mind and told them to me, perhaps re-storying them in her telling, she connected them to her own narrative of making mathematics real. I observed Josie find multiple authentic opportunities to seamlessly integrate counting, measuring, positional words, and problem solving into daily activities and routines, highlighting how she sees mathematics as real and part of everyday activities.

Patsy's narrative conveys a lifelong appreciation for learning and doing mathematics, as well as teaching it now, demonstrating continuity. In the moment as Patsy interacted with her thoughts on how her literacy course could be helpful to her mathematics teaching, she seemed to re-story her conception of literacy so that it includes mathematics. Reflecting on their lack of opportunities to learn about teaching mathematics with young children in the past, both women are eager to learn in the future. Josie exclaimed, "I really need to learn more about teaching INFANTS math, it's not the same thing as preschoolers," while Patsy exclaimed, "If you guys give a training, I'll be the first one to sign up." Josie and Patsy both enjoy mathematics themselves and
put great value in the children learning mathematics. Their stories highlight a lack of opportunities to learn.

## Mathematics Teaching in Josie's and Patsy's Classrooms

Josie and Patsy both gather the children after breakfast for whole group "planned teaching" (Clements \& Sarama, 2014, p. 6). Josie includes songs and prayer, while Patsy includes shapes, calendar, alphabet, manners, and reading at least one story. During this structured teaching time, traces of each woman's mathematics learning experiences can be seen by examining their purposes in their instruction and use of social control (Dewey, 1938/1998). Determined that the children in her care are prepared for kindergarten, as she was not prepared as a child and did not know to prepare her own children, Patsy explicitly teaches shapes and counting every day (Lee \& Ginsburg, 2007). Her previous time spent structuring the engineering group office is reflected now in her consistent schedule that the children know well. Within that schedule, though, after shapes and counting, as well as saying the alphabet, Patsy chooses a mathematics activity based on the interests or needs of the group (Parks \& Bridges-Rhoads, 2012).

Josie's influences from classical education for homeschooling (Saxon, 1994; Wise Bauer \& Wise, 2016) and Making the Most of Classroom Interactions are visible as she integrates counting opportunities into this planned teaching time, such as when she and the children determine the number of songs they will sing and have completed singing. These influences are also visible as Josie employs social control through rituals she embeds during this time, including her use of a timer as a signal to the children that it is time to gather and Mark hitting the gong three times to signal that their learning time has officially begun. Josie's past training tells her that the rituals, counting, and built-in waiting time draw the children's attention and support executive function development (Ginsburg \& Amit, 2008; Richardson \& Bofferding, 2015). Both teachers have structured this time so that each child contributes to and benefits from membership in the community (Dewey, 1938/1998).

During the hour or so before lunch, children in both classrooms engage in self-directed free play (van Oers, 2010). Mathematics instruction during this time is found in teachable moments (Clements \& Sarama, 2014), such as Josie counting the steps as Francie descends or Patsy noting that Javonte's tower is so tall. These are the opportunities when children try out their mathematics subject matter (Dewey, 1938/1998) on their own terms, such as identifying relations between the
shapes that they named earlier during the planned instruction and the three-dimensional blocks they are now building with (Parks, 2015). During this time, the teachers also learn about how the children think as the children experience freedom (Dewey, 1938/1998) during play activities, such as when Javonte told Patsy, "I used the yellow ones to put down because they're wider," and Patsy explained, "He's telling me HOW he's building." While the girls in Josie's classroom are playing with wagons outside, she is reflecting their freedom of movement back to them in her running dialogue of positional words and counts of the girls' movements. Her running dialogue also helps the girls learn about sharing space together in an organized, cooperative way, providing social control (Dewey, 1938/1998).

## What Role Do Teachers' Learning Experiences Play in Their Mathematics Teaching?

To answer this research question, I looked to the teachers' narratives. Josie's turning point story (Drake, 2006) marked a specific moment in her mathematics learning experiences that continued to live in her present mathematics teaching. Patsy, however, only said that she always enjoyed math, which was evident in her enthusiasm for integrating mathematics into her literacy teaching practice, open-ended play time, and planned circle time teaching. I was surprised that Josie's and Patsy's mathematics learning experiences did not play a larger role in their mathematics teaching. While my own personal practical knowledge of my Algebra Review students' "math baggage stories" of their past and present mathematics learning experiences led me to expect a similar connection between the early childhood teachers' mathematics learning experiences and their mathematics teaching practices, Clandinin (1985) reminds that "personal practical knowledge need not be clearly articulated and logically definable in order to exert a powerful influence in teachers' lives" (p. 383).

An early idea that emerged from the data was that the teachers' mathematics teaching philosophies, rather than their mathematics learning experiences, were driving their mathematics teaching and their inquiries about teaching mathematics with young children. Neither teacher shared details of her K-12 mathematics learning experiences, unlike the early childhood teachers in my pilot study (Richardson \& Bofferding, 2015). However, developing teaching images for Josie and Patsy provided a mechanism with which to examine how the knowledge they use in their mathematics teaching practices came to be and how each of them uses this knowledge in their teaching. The role that each teacher's mathematics learning experiences play in her mathematics
teaching with young children is not obvious nor causally connected, as my research question hints. Instead, Josie's and Patsy's mathematics learning experiences are single elements of many that contribute to their emotional and moral personal practical knowledge (Clandinin, 1985).

## How Can MTEs Hear Early Childhood Teachers' Voices in Planning for Professional Development?

I did not set out to investigate early childhood teachers' mathematics teaching images or even the personal practical knowledge they use in teaching mathematics with young children (Clandinin, 1985). I set out to document early childhood teachers' experiences learning and teaching mathematics so that policymakers and other stakeholders could hear early childhood teachers' expert voices on what they already know and want to learn about teaching mathematics with young children. Following my resonance with the role of experience in my mathematics education learning course (von Glasersfeld, 1995) to Dewey's (1938/1998) experience constructs led me to narrative inquiry (Clandinin \& Connelly, 2000). Polkinghorne (1995) helped me narratively analyze my data, while Clandinin's (1985) teaching image helped me understand how the knowledge teachers use in teaching moments comes to be and is used. To begin to answer how teachers' voices contribute to the professional dialogue regarding teaching mathematics with young children, I will consider what role, if any, Josie's and Patsy's teaching images might play in professional development and then describe a professional development program that encompasses my findings.

What Role Can Josie's and Patsy's Teaching Images Play in Professional Development?Because early childhood teachers may not take up mathematics professional development as intended (NRC, 2009; Sarama \& DiBiase, 2004), mathematics professional development might strive to begin with teachers' current understanding, rather than identifying professional development that attempts to give teachers the skills they "should" have (Buettner et al., 2016; NAEYC, 2009; NAEYC \& NCTM, 2002; Parks \& Wager, 2015; Pianta et al., 2009; Simpson \& Linder, 2014). Clandinin (1985) explains that "teachers are expected to facilitate someone else's intentions. This failure to understand the teacher as an active holder and user of personal practical knowledge helps explain the limited success of curriculum implementation" or mathematics professional development (p. 364). I have developed teaching images for Josie and Patsy to understand how they hold and use their personal practical knowledge. Their narratives
show each teacher as a mathematics curriculum maker (Clandinin \& Connelly, 1992). However, early childhood teachers, including Josie and Patsy, do not receive sufficient training for teaching mathematics with young children (NRC, 2009), although Josie and Patsy indicate that they are eager to learn.

Josie wants to learn about teaching mathematics with infants, including ways to assess infants' mathematical development, explaining "I don't know what they're getting, because these are the youngest children that I've had to work with." As she told her puzzle of reading a book with frogs above and below lily pads with four-year-old Mark, who surprisingly could not keep track of his counts, my knowledge of early counting caused me to suggest that Mark might be struggling with this counting task because the countable items (frogs) were in an unstructured arrangement. Josie was happy to have vocabulary that explained what she saw. Patsy's narrative includes a similar story about the term spatial reasoning.

## Professional Development Proposal

I propose a series of six, two-hour evening meetings provided through the local resource and referral agency's training schedule. Each teacher would bring a mathematics teaching celebration or puzzle to share with the group, thereby beginning the program with each teacher's purpose and subject matter grounded in their own experiences (Dewey, 1938/1998). Josie's purpose might be to make the mathematics real, while Patsy's might be preparing the children for kindergarten. We will work together as a group to create next steps that address their celebrations or puzzles, which could be improving the instructions around a puzzle, or gaining new ways to deepen learning. My role would be to provide guidance from my knowledge of the discipline, to narrow the field of options. Patsy will appreciate hearing each teacher's celebration or puzzle and the group discussion, as "You'd be surprised at what you can learn from other people." Although Josie wants her "paradigms shifted" in any future trainings, she would be eager to look at her mathematics teaching puzzles and celebrations with someone more knowledgeable and to receive guidance on next steps. The teachers' homework would be to take next steps in their classrooms and report on the interactions (Dewey, 1938/1998) they have in the environment and with the children around the next steps for their mathematics teaching celebration or puzzle. My homework would be to observe them in their classrooms, begin sketching teaching images for each teacher,
and to develop an activity to explore that is related to the teachers' teaching images. The resources I would consult first are Big Ideas of Early Mathematics: What Teachers of Young Children Need to Know (Brownell et al., 2014) for mathematics content, and Exploring Mathematics through Play in the Early Childhood Classroom (Parks, 2015). Teachers might feel vulnerable in this setting, so I would have to work to build trust and relationships within the group (Clandinin \& Connelly, 2000). I envision learning together with a small group of teachers to refine the process.

In my experience, early childhood teachers asked why they have to take a college algebra course that is not related to the mathematics they teach with young children, implying that the institution who holds authority over their coursework is exhibiting its personal power rather than exhibiting its power on behalf of the group, causing this requirement to feel arbitrary rather than just and fair (Dewey, 1938/1998). The analysis completed in this study and this mathematics professional development proposal are first steps toward recognizing early childhood teachers' personal practical knowledge in mathematics and providing teachers an opportunity to build on their professional knowledge, for themselves, the children in their classrooms, and for early childhood teachers in general. Prologue: What Does My Voice Tell?

## Images

The roots of the images I made as interim research texts (Clandinin \& Connelly, 2000) for Josie's and Patsy's narratives are found in my study of the Reggio Emilia preschools and one of their fundamental principles, documentation as communication: "Teachers' commentary on the purposes of the study and the children's learning process, transcriptions of children's verbal language (i.e., words and dialogue), photographs of their activity, and representations of their thinking in many media are composed in carefully designed panels or books to present the process of learning in the schools" (Cadwell, 1997, p. 6). Documentation not only records and communicates the children's learning process, but also provides opportunities for teachers to learn about children's thinking and perceptions of the world, as well as opportunities for children's ongoing sense-making as they revisit and expand documentation with their families, teachers, and peers (Rinaldi, 1998). I saw photovoice (Wang \& Burris, 1997) as a similar tool I could use as a researcher to document early childhood teachers' experiences learning and teaching mathematics with young children, as well as for the teachers to communicate their experiences, strengths, and needs to stakeholders (Richardson, 2015).

Although Josie and Patsy did not take photos for the study, I was still documenting their experiences in the data I collected and analyzed. There came a point in Josie's analysis as I was
creating her teaching image (Clandinin, 1985), wrestling with the vivid mental image her turning point story provoked (Drake, 2006), that my ability to use words to capture my understanding of the data and analysis failed. At that point, I turned to my favorite tool for visually capturing my ideas, PowerPoint, and created my own documentation panels (Cadwell, 1997; Rinaldi, 1998). Going back and forth between words and images provided more ways to make sense (Bach, 2007). Creating these images helped me see more of Josie and Patsy's strengths than I could see initially when I was only writing. Conle (1996) noted that trigger stories that promote resonance "contained concrete imagery" (p. 319). In this respect, my images seem related to resonance. I talked with our violin teacher, Candida Wiley, about characteristics of conductors so I would get the words right. Words, images, and voices seemed to swirl together at times, as Josie's email response to my notes from our first observation reflect:

Wow, did I do that?
What an observation! Thank you for letting me have the chance to see my practice through your eyes; what a gift.

Would you mind if I used this as part of my testimonials? I don't have any, and both your observation and your email have words in them that might help others understand our practice, values, and philosophies.

Again - what a gift, Sue Ellen. Thank you.

## Remaining Questions

Words, images, and voices swirling together, as seen in Josie's note, makes me wonder about the role and affordances of this aspect of the work. I set out to do a photovoice (Wang \& Burris, 1997) study, which would have provided an opportunity for stakeholders to hear teachers’ voices as they tell their stories and share photos of learning and teaching mathematics with young children. Instead, my voice and images tell the story of my experience of Josie's and Patsy's teaching and learning experiences, which was quite valuable to Josie in that moment. I adjusted the methods to reflect the data I gathered, rather than considering the implications of substituting my voice and images for Josie's and Patsy's.

I have a lot to learn about how to recognize positionality and how to write about it. I also have to learn how and when to write about my own experience of the research, especially when I encounter "messiness." I would also like to learn more about the images I made, where this activity
might fit within the research literature, and how I could use documentation as a mathematics teacher educator. I am very curious about using teaching images (Clandinin, 1985) and models of teachers' thinking (Steffe, 1996) in my work. Clandinin and Connelly's (1992) writing on teachers as curriculum makers shows a path. In addition to stories of teachers' practices, such as Josie's and Patsy's, they propose contributing their own researcher stories of their work with teachers, thus co-constructing stories of teachers as curriculum makers with teachers and providing a mechanism for early childhood teachers' voices to contribute to the professional dialogue about teaching mathematics with young children. Their researcher stories would document their learning with the teachers. This would be a different way of working as a mathematics teacher educator, one I am eager to explore. And so my narrative continues...

## REFERENCES

Bach, H. (2007). Composing a visual narrative inquiry. In D.J. Clandinin (Ed.) Handbook of narrative inquiry: Mapping a methodology (pp. 280-307). Sage Publications. https://doi.org/10.4135/9781452226552.n11

Bates, A. B., Latham, N. I., \& Kim, J. A. (2013). Do I have to teach math? Early childhood preservice teachers' fears of teaching mathematics. Issues in the Undergraduate Mathematics Preparation of School Teachers, 5, 1-10.

Belfield, C.R., Nores, M., Barnett, S., \& Schweinhart, L. (2006). The High/Scope Perry Preschool Program: Cost-benefit analysis using data from the age-40 followup. The Journal of Human Resources, 41(1), 162-190. https://doi.org/10.3368/jhr.xli.1.162

Bowman, B.T., Donovan, M.S., \& Burns, M.S. (Eds.). (2001). Eager to learn: Educating our preschoolers. National Academies Press. https://doi.org/10.17226/9745
Brendefur, J., Strother, S., Thiede, K., Lane, C., \& Surges-Prokop, M. J. (2013). A professional development program to improve math skills among preschool children in Head Start. Early Childhood Education Journal, 41(3), 187-195. https://doi.org/10.1007/s10643-012-0543-8

Brooks, A. (2007). Feminist standpoint epistemology: Building knowledge and empowerment through womens' lived experience. In S.N. Hesse-Biber \& P.L. Leavy (Eds.), Feminist research practice (pp. 53-82). Thousand Oaks, CA: Sage Publications. https://dx.doi.org/10.4135/9781412984270.n3

Brown, E.T. (2005). The influence of teachers' efficacy and beliefs regarding mathematics instruction in the early childhood classroom. Journal of Early Childhood Teacher Education, 26(3), 239-257. https://doi.org/10.1080/10901020500369811

Brownell, J.O., Chen, J., \& Ginet, L. (2014). Big ideas of early mathematics: What teachers of young children need to know. Upper Saddle River, NJ: Pearson Education, Inc.

Buettner, C.K., Hur, E.H., Lieny, J., \& Andrews, D.W. (2016). What are we teaching the teachers? Child development curricula in US higher education. Child Youth Care Forum, 45, 155-175. https://doi.org/10.1007/s10566-015-9323-0

Bureau of Labor Statistics, United States Department of Labor (BLS). (2016a). Labor force statistics from the current population survey. Retrieved from http://www.bls.gov/cps/cpsaat11.htm.

Bureau of Labor Statistics, United States Department of Labor (BLS). (2016b). Occupational employment statistics. Retrieved from https://www.bls.gov/oes/current/naics4_624400.htm\#25-0000.

Caine, V., Estefan, A., \& Clandinin, D.J. (2013). A return to methodological commitment: Reflections on narrative inquiry. Scandinavian Journal of Educational Research, 57(6), 574-586. https://doi.org/10.1080/00313831.2013.798833

Cadwell, L.B. (1997). Bringing Reggio Emilia home: An innovative approach to early childhood education. Teachers College Press.

Cataldo, P. (2013). From classroom to coach: One teacher's journey. Teaching Children Mathematics, 20(2), pp. 110-115. https://doi.org/10.5951/teacchilmath.20.2.0110
Chen, J. \& McCray, J. (2012). A conceptual framework for teacher professional development: The whole teacher approach. NHSA Dialog, 15(1), 8-23. https://doi.org/10.1080/15240754.2011.636491

Child Care Aware of America (CCAoA). 2015. Child care providers. Arlington, VA. Retrieved from http://usa.childcareaware.org/about/child-care-providers/.

Christeson, W., Bishop-Josef, S., Taggart, A.D., \& Beakey, C. (2013). A commitment to prekindergarten IS a commitment to national security: High-quality early childhood education saves billions while strengthening our military and our nation. Mission: Readiness: Military Leaders for Kids. Retrieved from https://strongnation.s3.amazonaws.com/documents/26/77b8b81d-efb0-4f6a-a921-d3896d048db7.pdf?1469640394\&inline;\ filename=\"How\ PreK\ Helps\ National\ Security.pdf\"

Clandinin, D.J. (2006). Narrative inquiry: A methodology for studying lived experience. Research Studies in Music Education 27(1), 44-54. https://doi.org/10.1177\%2F1321103X060270010301

Clandinin, D. J. \& Connelly, F. M. (1992). Teacher as curriculum maker. In P.W. Jackson (Ed.) Handbook of research on curriculum (pp. 363-401). New York: MacMillan Publishing.

Clandinin, D.J. \& Connelly, F.M. (2000). Narrative inquiry: Experience and story in qualitative research. Jossy-Bass.

Clandinin, D. J., Murphy, M. S., Huber, J., \& Orr, A. M. (2009). Negotiating narrative inquiries: Living in a tension-filled midst. The journal of educational research, 103(2), 81-90. https://doi.org/10.1080/00220670903323404
Clandinin, D.J. \& Rosiek, J. (2007). Mapping a landscape of narrative inquiry: Borderland spaces and tensions. In D.J. Clandinin (Ed.) Handbook of narrative inquiry: Mapping a methodology (pp. 35-76). Sage Publications. https://doi.org/10.4135/9781452226552.n2
Clemens, S.G. (1983). The sun's not broken, a cloud's just in the way: On child-centered teaching. Gryphon House, Inc.

Clemens, S.G. (1996). Pay attention to the children: Lessons for teachers and parents from Sylvia Ashton-Warner. Rattle OK Publications.

Clements, D.H. \& Sarama J. (2008). Experimental evaluation of the effects of a research-based preschool mathematics curriculum. American Educational Research Journal, 45(2), 443494. https://doi.org/10.3102\%2F0002831207312908

Clements, D.H. \& Sarama, J. (2014). Learning and teaching early math: The learning trajectories approach, $2^{\text {nd }} e d$. Routledge.

Cohen, D.K. (1990). A revolution in one classroom: The case of Mrs. Oublier. Educational Evaluation and Policy Analysis, 12(3), 311-329. https://doi.org/10.2307/1164355
Conle, C. (1996). Resonance in preservice teacher inquiry. American Educational Research Journal, 33(2), 297-325. https://doi.org/10.3102\%2F00028312033002297
Copple, C., \& Bredekamp, S. (2009). Developmentally appropriate practice in early childhood programs serving children from birth through age 8. National Association for the Education of Young Children.

Council for Professional Recognition (CPR). (2015). CDA Competency Standards. Council for Professional Recognition. Retrieved from http://cdacouncil.nonprofitsoapbox.com/about/cda-credential/competency-standards.
Craig, C. (2011). Narrative inquiry in teaching and teacher education. Narrative Inquiries into Curriculum Making in Teacher Education, 13, 19-42. https://doi.org/10.1108/s14793687(2011)00000130005

Darling-Hammond, L. (1990). Instructional policy into practice: "The power of the bottom over the top." Educational Evaluation and Policy Analysis, 12(3), 339-347.
https://doi.org/10.2307/1164357
Dewey, J. (1938/1998). Experience and education: The $60^{\text {th }}$ anniversary edition. West Lafayette, IN: Kappa Delta Pi.
Drake, C. (2006). Turning points: Using teachers' mathematics life stories to understand the implementation of mathematics education reform. Journal of Mathematics Teacher Education, 9, 579-608. https://doi.org/10.1007/s10857-006-9021-9

Duncan, G.J., Dowsett, C.J., Claessens, A., Magnuson, K., Huston, A.C., Klebanov, P., et al. (2007). School readiness and later achievement. Developmental Psychology, 43(6), 14281446. https://doi.org/10.1037/0012-1649.43.6.1428.supp

Early, D.M., Maxwell, K.L., Burchinal, M., Bender, R.H., Ebanks, C., Henry, G.T., et al. (2007). Teachers' education, classroom quality, and young children's academic skills: Results from seven studies of preschool programs. Child Development, 78, 558-580. https://doi.org/10.1111/j.1467-8624.2007.01014.x

Early, D.M. \& Winton, P.J. (2001). Preparing the workforce: Early childhood teacher preparation at 2- and 4-year institutions of higher education. Early Childhood Research Quarterly, 16, 285-306. https://doi.org/10.1016/s0885-2006(01)00106-5

Edwards, C., Gandini, L., \& Forman, G. (Eds.). (1998). The hundred languages of children: The Reggio Emilia approach-advanced reflections (2 $2^{\text {nd }}$ ed.). Ablex Publishing Corp.

Elbaz, F. (1981). The teacher's "practical knowledge": Report of a case study. Curriculum Inquiry, 11(1), 43-71. https://doi.org/10.1080/03626784.1981.11075237

Family and Social Services Administration [FSSA]. (2019). Program incentives: What are the benefits of participating in Paths to Quality ${ }^{\mathrm{TM}}$ ? Indiana: FSSA. Retrieved from https://www.in.gov/fssa/pathstoquality/info-for-programs/program-incentives/.

Federal Funds Information for States (FFIS) (2014). Programs we track. FFIS. Retrieved from https://www.ffis.org/about/programs-we-track.

Fennema, E. \& Sherman, J. (1977). Sex-related differences in mathematics achievement, spatial visualization and affective factors. American Educational Research Journal, 14, 51-71.

Frakes, C. \& Kline, K. (2000). Teaching young mathematicians: The challenges and rewards. Teaching Children Mathematics, 6(6), 376-381. https://doi.org/10.3102/00028312014001051
Frid, S., \& Sparrow, L. (2009). "You Just Have to Take a Bit of a Risk Sometimes": Breaking the "Cycle of Tradition" in Primary Mathematics. Mathematics Teacher Education and Development, 1136-53.
Gandini, L. (1998). Educational and caring spaces. In C. Edwards, L. Gandini, \& G. Forman (Eds.), The hundred languages of children: The Reggio Emilia approach-advanced reflections ( $2^{\text {nd }}$ ed.). Ablex Publishing Corp.
Ginsburg, H.P. \& Amit, M. (2008). What is teaching mathematics to young children? A theoretical perspective and case study. Journal of Applied Developmental Psychology, 29, 274-285. https://doi.org/10.1016/j.appdev.2008.04.008
Ginsburg, H.P., Lee, J.S., \& Boyd, J.S. (2008). Mathematics education for young children: What it is and how to promote it. Social Policy Report: Giving Child and Youth Development Knowledge Away, 22(1), 3-23. https://doi.org/10.1037/e640072011-001
Gomez, R.E., Kagan, S.L., \& Fox, E.A. (2015). Professional development of the early childhood education teaching workforce in the United States: An overview. Professional Development in Education, 41(2), 169-186. https://doi.org/10.4324/9780203712658-1
Graue, E., Karabon, A., Delaney, K.K., Whyte, K., Kim, J., \& Wager, A. (2015). Imaging a future in PreK: How professional identity shapes notions of early mathematics. Anthropology \& Education Quarterly, 46(1), 37-54. https://doi.org/10.1111/aeq. 12086
Grenfell, M. (1996). Bourdieu and initial teacher education: A post-structuralist approach. British Educational Research Journal, 22(3), 287-303. https://doi.org/10.1080/0141192960220303
Grenfell, M. (Ed.) (2012). Pierre Bourdieu: Key concepts. Acumen Publishing.
Grenfell, M. \& James, D. (2004). Change in the field: Changing the field: Bourdieu and the methodological practice of educational research. British Journal of Sociology of Education, 25(4), 507-523. https://doi.org/10.1080/014256904200026989
Gresham, G. (2007). A study of mathematics anxiety in pre-service teachers. Early Childhood Education Journal, 35(2), 181-188. https://doi.org/10.1007/s10643-007-0174-7

Grunewald, R., \& Rolnick, A. J. (2010). An early childhood investment with a high public return. The Regional Economist, (Jul), 12-13. Retrieved from https://www.stlouisfed.org/~/media/Files/PDFs/publications/pub_assets/pdf/re/2010/c/chi ldhood_education.pdf

Hancock, S.J.C. (2001). The mathematics and mathematical thinking of four women seamstresses. In J.E. Jacobs, J.R. Becker, \& G.F. Gilmer (Eds.). Changing faces of mathematics: Perspectives on gender, (pp. 67-77). NCTM.

Harkness, S. S. \& Portwood, L. (2007). A quilting lesson for early childhood preservice and regular classroom teachers: What constitutes mathematical activity? The Mathematics Educator, 17(1), 15-23. Retrieved from https://files.eric.ed.gov/fulltext/EJ841558.pdf

Heckman, J. J. (2008). Schools, skills, and synapses. Economic inquiry, 46(3), 289-324. https://doi.org/10.1111/j.1465-7295.2008.00163.x
Heckman, J. J. (2011). The Economics of Inequality: The Value of Early Childhood Education. American Educator, 35(1), 31-35. Retrieved from https://files.eric.ed.gov/fulltext/EJ920516.pdf
Herron, J. (2010). An Evolution of Mathematical Beliefs: A Case Study of Three Pre-K Teachers. Journal Of Early Childhood Teacher Education, 31(4), 360-372. https://doi.org/10.1080/10901027.2010.523771
Hesse-Biber, S. N. (2007). The practice of feminist in-depth interviewing. In S.N. Hesse-Biber \& P.L. Leavy (Eds.), Feminist research practice (pp. 111-148). Sage Publications. https://doi.org/10.4135/9781412984270.n5
Huss-Keeler, R. \& Brown, S. (2007). Meeting diverse learning needs: Differentiating instruction in graduate early childhood mathematics classes. Journal of Early Childhood Teacher Education, 28(1), 41-57. https://doi.org/10.1080/10901020601184390

Indiana Association for the Education of Young Children (IAEYC). (2019). T.E.A.C.H. ® Teacher education and compensation helps [Brochure]. https://inaeyc.org/wp-content/uploads/2019/04/IAEYC_TEACHBrochure_2019_Web-1.pdf
Jung, H.Y. \& Reifel, S. (2011). Promoting children's communication: A kindergarten teacher's conception and practice of effective mathematics instruction. Journal of Research in Childhood Education, 25, 194-210. https://doi.org/10.1080/02568543.2011.555496

Kamii, C. \& Housman, L.B. (2000). Young children reinvent arithmetic: Implications of Piaget's theory ( $2^{\text {nd }} E d$. .). Teachers College Press.

Kamii, C., Miyakawa, Y., \& Kato, T. (2007). Trying to make a lever work at ages 1 to 4: The development of "functions" (Logico-mathematical thinking). Early Education and Development, 18(1), 145-161. https://doi.org/10.1080/10409280701274774

Kim, M. \& Reifel, S. (2010). Child care teaching as women's work: Reflections on experiences. Journal of Research in Childhood Education, 24(3), 229-247. https://doi.org/10.1080/02568543.2010.487402

Kinzie, M.B., Whittaker, J.V., Williford, A.P., DeCoster, J., McGuire, P., Lee, Y., \& Kilday, C.R. (2014). MyTeachingPartner-Math/Science pre-kindergarten curricula and teacher supports: Associations with children's mathematics and science learning. Early Childhood Research Quarterly, 29(2014), 586-599. https://doi.org/10.1016/j.ecresq.2014.06.007

Larson, N. (1994). Math 1: An incremental development, Home study meeting book. Norman, OK: Saxon Publishers, Inc.

Lee, J.S. \& Ginsburg, H.P. (2007). Preschool teachers' beliefs about appropriate early literacy and mathematics education for low- and middle-socioeconomic status children. Early Education \& Development, 18(1), 111-143. https://doi.org/10.1080/10409280701274758

Lemoine, S. (2008). Workforce Designs: A Policy Blueprint for State Early Childhood Professional Development Systems. NAEYC Public Policy Report. Washington, DC: National Association for the Education of Young Children. Retrieved from https://www.naeyc.org/sites/default/files/wysiwyg/user-74/workforce_designs.pdf

Lieber, J., Butera, G., Hanson, M., Palmer, S., Horn, E., Czaja, C., Diamond, K., GoodmanJansen, G., Daniels, J., Gupta, S., \& Odom, S. (2009). Factors that influence the implementation of a new preschool curriculum: Implications for professional development. Early Education and Development, 20(3), 456481.https://doi.org/10.1080/10409280802506166

Martin, B., \& Carle, E. (1991). Polar Bear, Polar Bear, What Do You Hear? Julia MacRae Books.

McCray, J. S., \& Chen, J. Q. (2010). Foundational mathematics: A neglected opportunity. In B. Atweh, M. Graven, W. Secada, \& P. Valero (Eds.), Mapping equity and quality in mathematics education (pp. 253-268). Springer. https://doi.org/10.1007/978-90-481-9803-0_18

Merriam-Webster. (n.d.). Nurture. In Merriam-Webster.com dictionary. Retrieved from https://www.merriam-webster.com/dictionary/nurture.
Moses, R. P. \& Cobb, C. E., Jr. (2001). Radical equations: Math literacy and civil rights. Beacon Press.

National Association for the Education of Young Children. (2009). NAEYC standards for early childhood professional preparation: A position statement of the National Association for the Education of Young Children. Retrieved from http://www.naeyc.org/files/naeyc/file/positions/ProfPrepStandards09.pdf.
NAEYC \& NCTM. (2002). Executive summary: Early childhood mathematics: Promoting good beginnings: A joint position of the National Association for the Education of Young Children (NAEYC) and the National Council of Teachers of Mathematics (NCTM). Retrieved from http://www.naeyc.org/files/naeyc/file/positions/Mathematics_Exec.pdf.

National Association of Early Childhood Teacher Educators (NAECTE). (n.d.). Position statement on early childhood certification for teachers of children 8 years old and younger in public school settings. Retrieved from http://naecte.org/wp-content/uploads/ECE-certification-position-statement.pdf.

National Association for Family Child Care (NAFCC). (2016). Your home. Your profession. Our commitment. Retrieved from https://www.nafcc.org/.
National Research Council. (2009). Mathematics learning in early childhood: Paths toward excellence and equity. The national Academies Press. https://doi.org/10.17226/12519
National Survey of Early Care and Education Project Team (NSECE). (2013). Number and characteristics of early care and education (ECE) teachers and caregivers: Initial findings from the National Survey of Early Care and Education (NSECE). OPRE Report \#201338. Office of Planning, Research and Evaluation, Administration for Children and Families, U.S. Department of Health and Human Services. Retrieved from https://www.acf.hhs.gov/sites/default/files/documents/opre/nsece_wf brief_102913 0.pd f

Noyes, A. (2004). (Re)producing mathematics educators: A sociological perspective. Teaching Education, 15(3), 243-256. https://doi.org/10.1080/1047621042000257180

Paley, V.G. (1986). On listening to what the children say. Harvard Educational Review, 56(2), 122-131. https://doi.org/10.17763/haer.56.2.p775487x30tk69m8

Paley, V.G. (1992). You can't say you can't play. Cambridge, MA: Harvard University Press.
Parks, A.N. (2015). Exploring mathematics through play in the early childhood classroom. Teachers College Press.

Parks, A.N. \& Blom, D.C. (2013). Helping young children see math in play. Teaching Children Mathematics, 20(5), 310-317. https://doi.org/10.5951/teacchilmath.20.5.0310

Parks, A. N., \& Bridges-Rhoads, S. (2012). Overly scripted: Exploring the impact of a scripted literacy curriculum on a preschool teacher's instructional practices in mathematics. Journal of Research in Childhood Education, 26(3), 308-324. https://doi.org/10.1080/02568543.2012.684422

Parks, A.N., \& Wager, A.A. (2015). What knowledge is shaping teacher preparation in early childhood mathematics? Journal of Early Childhood Teacher Education, 36(2), 124-141. https://doi.org/10.1080/10901027.2015.1030520

Perry, B., Dockett, S., \& Harley, E. (2007). Learning stories and children's powerful mathematics. Early Childhood Research \& Practice, 9(2). Retrieved December 6, 2009, from http://ecrp.uiuc.edu/v9n2/perry.html

Perry, B. \& MacDonald, A. (2015). Educators' expectations and aspirations around young children's mathematical knowledge. Professional Development in Education, 41(2), 366381. https://doi.org/10.1080/19415257.2014.990578

Philipp, R. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), Second handbook of research on mathematics teaching and learning (pp. 257-315). IAP.

Pianta, R.C., Barnett, W.S., Burchinal, M., \& Thornburg, K.R. (2009). The effects of preschool education: What we know, how public policy is or is not aligned with the evidence base, and what we need to know. Psychological Science in the Public Interest, 10(2), 49-88. https://doi.org/10.1177/1529100610381908

Piasta, S.B., Logan, J.A., Pelatti, C.Y., Capps, J.L., \& Petrill, S.A. (2015). Professional development for early childhood educators: Efforts to improve math and science learning opportunities in early childhood classrooms. Journal of Educational Psychology, 107(2), 407-422. https://doi.org/10.1037/a0037621

Pinnegar, S. \& Daynes, J.G. (2007). Locating narrative inquiry historically: Thematics in the turn to narrative. In D.J. Clandinin (Ed.) Handbook of narrative inquiry: Mapping a methodology (pp. 3-34). Sage Publications. https://doi.org/10.4135/9781452226552.n1

Polkinghorne, D.E. (1995). Narrative configuration in qualitative analysis. Qualitative Studies in Education, 8(1), 5-23. https://doi.org/10.1080/0951839950080103
Richardson, S.E. (2015). A multi-method approach to investigate early childhood teachers, experiences learning and teaching mathematics. Unpublished manuscript (AERA proposal), Purdue University.
Richardson, S.E. (2017). Early childhood teachers' experiences learning and teaching mathematics as seen through Dewey and narrative inquiry. Unpublished poster, Annual Graduate Student Educational Research Symposium, Purdue University.

Richardson, S.E. \& Bofferding, L. (2015). Through their eyes: Early childhood teachers as learners and teachers of mathematics. In T. G. Bartell, K. N. Bieda, R. T. Putnam, K. Bradfield, \& H. Dominguez (Eds.), Proceedings of the 37th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (pp. 804-811). Michigan State University.

Rinaldi, C. (1998). Projected curriculum constructed through documentation-Progettazione: An interview with Lella Gandini. In C. Edwards, L. Gandini, \& G. Forman (Eds.), The hundred languages of children: The Reggio Emilia approach-Advanced reflections (2 ${ }^{\text {nd }}$ ed., pp. 113-125). Ablex Publishing.

Rudd, L.C., Lambert, M.C., Satterwhite, M., \& Smith, C.H. (2009). Professional development + coaching $=$ enhanced teaching: Increasing usage of math mediated language in preschool classrooms. Early Childhood Education Journal, 37, 63-69. https://doi.org/10.1007/s10643-009-0320-5

Sarama, J. \& DiBiase, A.M. (2004). The professional development challenge in preschool mathematics. In D.H. Clements, J. Sarama, \& A.M. DiBiase (Eds.), Engaging young children in mathematics: Standards for early childhood mathematics education (pp. 415446). Lawrence Erlbaum Associates.

Schell, K., Ferguson, A., Hamoline, R., Shea, J., \& Thomas-Maclean, R. (2009). Photovoice as a teaching tool: Learning by doing with visual methods. International Journal of Teaching and learning in Higher Education, 21(3), 340-352.

Shonkoff, J. P. \& Phillips, D. A. (Eds.). (2000). From Neurons to Neighborhoods: The Science of Early Childhood Development. National Academies Press. https://doi.org/10.17226/9824
Simpson, A. \& Linder, S.M. (2014). An examination of mathematics professional development opportunities in early childhood settings. Early Childhood Education Journal, 42, 335342. https://doi.org/10.1007/s 10643-013-0612-7

Steffe, L. P. (1996). Social-cultural approaches in early childhood mathematics education: A discussion. In H. Mansfield, N.A. Pateman, \& N. Bednarz (Eds.), Mathematics for tomorrow's young children (pp. 79-99). Netherlands: Springer.
T.E.A.C.H. Early Childhood ${ }^{\circledR}$ National Center. (2016). Providing the early learning workforce with access to debt-free college education, better compensation and job stability since 1990. Chapel Hill, NC: Child Care Services Association. Retrieved from http://teachecnationalcenter.org/t-e-a-c-h-early-childhood/.
US Department of Education. (2013). Race to the Top - Early Learning Challenge awards [online]. US Department of Education. Retrieved from http://www2.ed.gov/programs/racetothetop-earlylearningchallenge/awards.html.

US Department of Health \& Human Services. (2014). Early Childhood Training and Technical Assistance System. Washington, DC. Retrieved from https://childcareta.acf.hhs.gov/.
United Way Worldwide. (2016). Education is a cornerstone for success. United Way Worldwide. Retrieved from http://www.unitedway.org/our-impact/focus/education.
van Oers, B. (2010). Emergent mathematical thinking in the context of play. Educational Studies in Mathematics, 74, 23-37. https://doi.org/10.1007/s10649-009-9225-x
Varol, F., Farran, D.C., Bilbrey, C., Vorhaus, E.A., \& Hofer, K.G. (2012). Improving mathematics instruction for early childhood teachers: Professional development
components that work. NHSA Dialog, 15(1), 24-40.
https://doi.org/10.1080/15240754.2011.636488
Von Glasersfeld, E. (1995). Radical constructivism: A way of knowing and learning. RoutledgeFalmer.

Vygotsky, L.S. (1986). Thought and language (Rev.). A. Kozulin (Ed.). MIT Press.
Wang, C. \& Burris, M. A. (1997). Photovoice: concept, methodology, and use for participatory needs assessment. Health Education \& Behavior, 24(3), 369-387.
https://doi.org/10.1177/109019819702400309
Whitebook, M., \& Ryan, S. (2011). Degrees in context: Asking the right questions about preparing skilled and effective teachers of young children. National Institute of Early Education Research and Center for the Study of Child Care Employment. Retrieved from https://cscce.berkeley.edu/wp-content/uploads/2011/DegreesinContext_2011.pdf
Whitney, J.C. (2006). My education: Students with disabilities describe high school in pictures and words. TEACHING Exceptional Children Plus, 3(2), 1-17.

Wise Bauer, S \& Wise, J. (2016). The well-trained mind: A guide to classical education at home (4 ${ }^{\text {th }}$ Ed.). W.W. Norton \& Company.
Wright, R.J., Stanger, G., Stafford, A.K., \& Martland, J. (2015). Teaching number in the classroom with 4-8 year-olds, (2 $2^{\text {nd }}$. Ed.). Sage Publications.

Yoon, K.S., Duncan, T., Lee, S.W., Scarloss, B., \& Shapley, K.L. (2007). Reviewing the evidence on how teacher professional development affects student achievement (Issues \& Answers Report, REL 2007-No. 033). Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory Southwest. Retrieved from http://ies.ed.gov/ncee/edlabs
Zero to Three. (2016). Early Workforce Innovations. Washington, DC: ZERO TO THREE: National Center for Infants, Toddlers, and Families. Retrieved from https://www.zerotothree.org/resources/services/early-childhood-workforce-innovations.

## APPENDIX A. PILOT STUDY INTERVIEW PROTOCOL

## Interview

A) Number Sense
B) Computation
C) Algebra and Functions
D) Geometry
E) Measurement
F) Problem Solving

## Questions:

How old are the children in your classroom?
How long have you worked in early childhood education (and in what positions)?
What early childhood education training programs have you participated in? Degrees received?
What early childhood courses have you taken? What mathematics courses have you taken?

## Content:

When you think of your experiences learning (math topic), what stands out to you?
a. What have you learned about (math topic) in your content course(s) or professional development?
b. Do you use this content information in your teaching? If so, how? If not, why not?
c. Would you have liked something different in your content course(s) related to learning (math topic)?
d. What content is involved in the children's learning of (math topic)?
e. What would you like to learn more about regarding children's learning of (math topic)?
f. What knowledge do you think future teachers need in order to teach (math topic)?

## Teaching:

When you think of your experiences learning to teach mathematics, what stands out to you?
a. What strategies for teaching mathematics did you learn?
b. What strategies for teaching (math topic) did you learn?
c. Do you use any of the ideas from these classes or other training in your teaching of (math topic)? If yes, what and how? If no, why not and what do you do instead?
d. What does teaching (math topic) involve for you?

## Voice:

1) What was the most important part of your training? Least important part? How come?
2) What have you learned about (math topic) in working with the children?
3) Who do you feel exerts the most control over what and how you teach?
a. What would you like them to know about teaching (math topic) to young children? Why is that important for them to know?
b. What prerequisites do they think are necessary for teaching preschoolers mathematics?

Do you agree/disagree, why?

## Philosophy:

1) How do you think children learn mathematics?
2) How do you think children learn (math topic)?
3) How do you include (math topic) in the classroom?
a. What (math topic) materials do you and your students use?
b. What (math topic) activities do you use?
c. Is (math topic) incorporated with any other subject? Should it be?
4) What makes a material or an activity mathematical?
5) How much time of the day do you think the average student spends on math-related activities?

## Other questions:

a) Some teachers think that preschoolers learn (math topic) without support from teachers and some teachers think the teacher should play a central role in preschool (math topic) activities. What do you think?
b) Some teachers think they should show preschoolers the correct way to solve a (math topic) problem. What do you think?
c) Some teachers think (math topic) is an important part of the preschool curriculum and some teachers think it is better to wait until kindergarten to do (math topic) activities. What do you think?
d) Some teachers think preschoolers are not socially or emotionally ready for (math topic). What do you think?
e) Some teachers think (math topic) is confusing for preschoolers. What do you think?
f) Some teachers do not know how they could support (math topic) learning in preschool. What do you think?
g) Is there anything/anything else children need to know about (math topic) before entering kindergarten?
h) Some teachers think that preschoolers should use flashcards to learn some (math topic) concepts. What do you think?
i) Some teachers think (math topic) worksheets are not appropriate for preschoolers and others think they are appropriate. What do you think?

## Questions Adapted from Ayers, 1989, p. 8

The Reflective Practitioner and Autobiographer:
What do you like most about teaching mathematics? What are the rewards for you? When do you feel best as a mathematics teacher? What are your favorite moments?

What is most difficult about teaching mathematics? If you could, what things would you change in your work teaching mathematics?

Tell me about the role of parents in your work, especially in teaching mathematics?
Why is your space arranged the way it is? Why do you follow particular routines, especially related to teaching mathematics?

Why do you teach mathematics as you do? What criteria do you have in mind? What do you take to be valuable in your teaching? What other teachers do you admire? Why?

What is your role in the mathematical lives of children and families? What are your mathematics goals for children? How do you meet these goals?

What is your role in preparing children for the future in mathematics? Are there any conflicts between your goals and the school's mathematics goals? Society's goals? If so, do the conflicts affect the children?

Tell me how you decided to become an early childhood teacher, what interested you?
Do you remember any outstanding teachers from your years as a student, particularly in mathematics?

What has your formal teacher education been like, especially in mathematics? Did it prepare you for the realities of teaching mathematics? Is teaching mathematics with young children pretty much what you expected? Have you had any mentors or colleagues who have influenced you in your mathematics teaching? How?

Can you remember when you felt comfortable as a mathematics teacher, confident with your own philosophy and practical knowledge?

Can you think of early experiences that continue to influence what and how you teach mathematics now? Can you describe the central teaching ideas that guide your mathematics teaching, and how you came to adopt them?

How have you changed as a mathematics teacher over the years?

## The Whole Person:

What is of value to you beyond teaching? Are you involved in any social or political groups? What concerns you most about children and families today? About the state of society or the world?

Are you involved in any other projects or interests outside of teaching? What? How are they important to you?

What have you read recently that was significant to you?
What do you imagine you'll be doing in five years? In ten years?

## APPENDIX B. OBSERVATION RECORDING SHEET

| Teacher: |
| :--- |
| Children's Ages: Total Children: Girls: Boys: <br> Classroom Information:    <br> What's going on (routines, centers, circle, outside, meal, drop-off/pick-up):    <br> Mathematics Topics/Activities/Curriculum:    <br> Teacher Questioning (whole group vs. individuals, boys vs. girls, spontaneous vs.    <br> curriculum):    |

Observation Notes (with time stamps and social interactions):

Informal Interview Notes (note any feelings, conflicts, constraints, possibilities, dissonance):

