PRESERVICE TEACHERS' ENACTMENT OF MATHEMATICAL PROBES THROUGH SPEECH AND GESTURES

by

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In dedication to Peng and my parents

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PST	Preservice Teachers
CCSSM	Common Core State Standards for Mathematics

ABSTRACT

It is widely accepted that students' thinking drives teachers' teaching. There are many ways to probe students' thinking with speech or with gestures; however, the literature remains relatively distinct, not focused on both. Prior studies focus on probing students' thinking with speech (talk moves). Although Alibali and colleagues (e.g., Alibali et al., 2013; Nathan et al., 2017) did research about gestures and speech in *general teaching practices*, few studies narrow in on how gestures work together with speech in the teachers' enactment of *probing practices*. Investigating how gestures aid speech in expression of thinking is important. Further, the literature on probing assumes that what researchers consider probing is what teachers consider probing. Thus, we have seen many researchers who use a researcher's view to define and categorize teachers' ways of probing. The information about teachers' stated probes is missing. Therefore, the purpose of this study is to detail how preservice teachers (PSTs) probe students' mathematical explanations with speech and gestures and inquire into the differences between PST-identified probes and researcher-identified probes. Sources of data included videos of the preservice teachers' teaching, their identification of probes in stimulated recall interviews, and researchers' (two researchers) identification of probes. Results showed that, from the researchers' perspectives, PSTs harnessed various gestures to probe students' thinking, for example, embedding additional mathematical information (e.g., a different strategy or model) in their gestures, not in their speech partially. Also, the PSTs used more multimodal links in their probes than what the current literature reported about in-service teachers, partially because the PSTs frequently used probing gestures in every interaction with students. The PSTs' dominating identification of their probing speech highly aligned with the researchers' identification of probes, despite the PSTs' missing a majority of their gestures as probes. An influential factor that affected the PSTs' identification of their probes was the quality and quantity of students' input. The research findings provide further implications about how teacher educators teach probing practices in preservice teacher education and how future research approaches PSTs' gesture use in teaching practices.

CHAPTER 1: INTRODUCTION

1.1 Contextual Literature

With a plethora of studies on student thinking, the notion is widely accepted that students' thinking drives teachers' teaching (Carpenter et al., 1988; Jacobs et al., 2010; Van Es & Sherin, 2008). In mathematics, a good knowledge of student thinking entails a good understanding of students' strengths and weaknesses on their route to formal mathematical knowledge. Therefore, it is important for teachers to have tools that will help them understand students' thinking. Researchers have developed various constructs of noticing and supporting students' mathematical thinking. Noticing children's thinking consists of attending to their thinking, then interpreting their thinking, and deciding what actions to take based on their thinking (Jacobs et al., 2010); supporting students' thinking includes extending students' thinking (Fraivillig et al., 1999; Ozgur et al., 2015). Noticing student thinking is a "way to understand how teachers make sense of complex classrooms" (Jacobs et al., 2011, p. 98). Making sense of the various learning activities and pinpointing students' learning moments prove to be beneficial for rich classroom learning environments (Sowder, 2007). Some studies (Bobis et al., 2005; Carpenter et al., 1989; Jacobs et al., 2007) argued that students obtained better learning achievement when teachers paid attention to and built on student thinking than when teachers did not.

To uncover students' mathematical thinking, such talk moves as questioning and probing, specifically, are widely studied. Probes are follow-up discursive moves that the teacher uses to elicit student thinking after they make initial responses to mathematics problems, and from their probes, teachers can accordingly adjust their instructional responses or further probes to meet students' needs (National Council of Teachers of Mathematics [NCTM], 2000). Chapin et al, (2009) proposed that to probe students' thinking, teachers can revoice what a student says, ask others to restate what is already said, and offer prompts for more input. Adding to the list of talk moves, Ghousseini (2015), elaborated the talk moves from a more general perspective of discourse routines and included more talk moves: orienting (the teacher puts someone's idea on the spot and asks other students to comment on and contribute to that idea) and negotiating (the teacher connects different students' strategies and tries to involve students in the discussion about the similarities and differences). The teacher probes student thinking when the teacher intends to dig into student responses and discover the entailed conceptual thinking or when the teacher finds student responses confusing or ambiguous. When students explain their ideas in class, their thinking is not usually sound to all students. Teachers' probes display students' thinking in an organized and clear manner, which provides the whole class with a second chance of listening to the ideas under discussion and facilitates the whole-class understanding (Chapin et al., 2009). Also, when the teacher probes student thinking, the teacher can attract students' attention to the targeted mathematical knowledge and make that key piece prominent by purposefully selecting and sequencing student responses (Stein et al., 2008). Besides, probing can be an assessment tool to advance student thinking (Sukmadewi, 2014). Through probing, the teacher can assess what students have comprehended and then plan the next-step of instruction to advance student thinking further. Although the perspectives of studying probes vary, there seems to be a consensus that teachers *elicit* and then *probe* student thinking. Elicitation targets students' predetermined responses and probes aim at students' justification of their own responses (Sahin & Kulm, 2008).

Some factors affect mathematics teachers' decision-making as to which student thinking is good for further probes. First, teachers need to consider whether a particular student's thinking has the potential to improve the whole-class understanding of a mathematical concept, which Leatham et al., (2015) called "mathematically significant" student thinking. Second, teachers should consider whether the piece of student thinking is closely connected with their teaching goals (Leatham et al., 2015). Especially in preservice teacher education, good probes can more efficiently uncover and deepen student thinking (Sukmadewi, 2014). The probing ability is learnable if preservice teachers (PSTs) are provided with opportunities to continuously practice asking probing questions (Weiland et al., 2014).

Although teachers constantly cue students about what they want from them through pressing or probing talk moves, such speech as discursive moves do not always guarantee getting more information from students. Situations exist where students do not respond when the teacher asks for more information by repeating. Therefore, accompanying the probing speech, gestures are an essential part of the teacher's cues for more information. The discursive functions that gestures provide help reduce speakers' (in this case, teachers) discursive load when they are thinking of what to say in the moment and therefore help allocate the speakers' cognitive resources to some other essential linguistic meanings in speech (Goldin-Meadow et al., 2001).

Gestures can reveal information about whether teachers apprehend students' mathematical thinking and spot learning difficulties in the moment. Teachers may use more gestures when presenting new mathematical ideas than when connecting to previously learned knowledge (Alibali et al., 2014), and the frequency of teachers' gestures may increase when students encounter difficulty in learning (Alibali & Nathan, 2007). Because gestures accompany speech (Goldin-Meadow, 1999), the variation of gestures suggests that of speech. In the case of teachers' probing students' mathematical thinking, when teachers do not understand what students say—in which case they do not share common ground—teachers tend to use more probes to rebuild the connection between what they have understood and what they have not. With these connections, teachers tend to use more gestures to facilitate the meaning-making communication (Nathan et al., 2017). As such, the variation of gestures might align with teachers' corresponding changes of speech to meet students' learning needs.

There are many ways to probe with speech or with gestures; however, the literature remains relatively distinct. The afore-mentioned studies (Chapin et al., 2009; Ghousseini, 2015; Ozgur et al., 2015) focus on probing students' thinking with speech (talk moves). Talk moves do help PSTs execute mathematical probes. As mentioned above, talk moves can be used to break down students' explanations into meaningful and comprehensible mathematical thinking parts, based on which teachers can administrate the next-step instruction. However, talk moves do not always help PSTs get what they want from students; "the simultaneous, concurrent organization of action is equally important" (Goodwin, 2013, p. 13). Gestures have proved to be useful in facilitating meaning-making for students (Alibali & Nathan, 2012; Goldin-Meadow et al., 2009; Goldin-Meadow & Wagner, 2005; Goodwin, 2007). Few studies, particularly in mathematics education, nonetheless, have focused on gestures for requesting information. Although Alibali and her colleagues (Alibali et al., 2013a; Nathan et al., 2017) did research about gestures and speech in a general meaning-making practice, few studies narrow in on how gestures work together with speech in the teachers' enactment of probing practice (Ferrara & Sinclair, 2016). Investigating how gestures aid speech in expression of thinking is important. Examining the mismatches between gestures and speech is also equally significant because different learning opportunities can be created for student (Singer & Goldin-Meadow, 2005). Further, the literature on probing assumes that what researchers consider probing is what teachers consider probing. Thus, we have seen many researchers who use a researcher's view to define and categorize teachers' ways of probing (Fraivillig et al., 1999;

Ozgur et al., 2015; Sahin & Kulm, 2008). The information about teachers' stated probes is missing. Therefore, this study aims to fill the research gap.

1.2 Purpose of the Study

The purpose of this study is to detail how PSTs probe students' mathematical explanations with speech and gestures and inquire into the differences between PST-identified probes and researcher-identified probes. In this study, we confine probing to the speech and gestures that target a student's own mathematical ideas (Herbel-Eisenmann et al., 2013), therefore excluding situations where a student elaborates on others' ideas. Gestures contain uses of hands, head, eyes, shoulders, and other meaning-making body movements. Although prior studies focused mainly on speech-based probes (Chen et al., 2020; Moyer & Milewicz, 2002; Sahin & Kulm, 2008), the roles of speech and gestures might overlap with each other by expressing the same meaning or might each contribute something unique. There are two contrasts under investigation: 1) speech-gesture matches vs. speech-gesture mismatches; 2) researcher-identified probes vs. PST-identified probes. Speech-gesture matches and mismatches provide a panorama of PSTs' probing enactment. The researcher-identified probes provide a baseline for what is the current practice in mathematical probing investigations, especially what is identified as mathematical probes with a researcher's eyes. By contrast, PST-identified probes build on their own understanding of and their reflection on the teaching situation. Especially, most of the PST-identified probes may bear close relations with personal experience.

Therefore, it is of great significance to decompose the probing practice, focusing on speech and gestures, in fine detail. Particularly, such nuances will be explored as when the teacher's speech and gestures match in probes and when speech and gestures mismatch in probes. The main research questions to be addressed are as follows.

1. How do PSTs use speech and gestures to enact the probing practice in K-5 mathematics classrooms?
    a. In cases where PSTs use different types of probes.
    b. In cases where PSTs' gestures match or mismatch the corresponding speech in their probes.

2. In what ways do the researcher-identified and PST-identified probes align?
    a. In terms of what probes are identified.
    b. In terms of when probes are used.

1.3 Significance

Investigating PSTs' probing through the lens of speech and gestures contributes to classroom meaning-making practices, in which students understand teachers' probes and respond appropriately to make their mathematical thinking salient to teachers as well as other students. From a teacher's perspective, a good understanding of their probing can help them identify which part of student thinking attracts their attention and guides them into ensuing action.

Uses of speech alone to probe for more information often occur when common knowledge is shared (Alibali et al., 2013a; Gerwing & Bavelas, 2004). However, when new and abstract knowledge is under discussion, speech alone seems not to benefit student learning a lot (Singer & Goldin-Meadow, 2005; Valenzeno et al., 2003). A combination of speech and gestures could ground abstract and unfamiliar concepts in concrete instantiations (Alibali et al., 2014). Thus, PSTs' probes would be saliently accessible to students and elicit intended information from students as much as possible. Even when PSTs use both speech and gestures in their probes, the effects may vary because of the matches and mismatches between the meanings expressed in speech and gestures. In light of the fact that teachers' speech-gesture matches are beneficial for student learning (Wagner et al., 2004), speech-gesture mismatches have varied effect on student learning (Goldin-Meadow & Singer, 2003; Singer & Goldin-Meadow, 2005). Similarly, it will be of value to examine PSTs' multiple ways of using speech with gestures to realize their probes. Relying on PSTs' retrospective accounts of uses of probes, this study has the potential to reinforce PSTs' awareness of the combined impact of speech and gestures on the probing practice.

This study will shed light on PSTs' comprehension and articulation of their uses of speech and gestures in their probes. PSTs' prior school learning experience seems to affect, to a great extent, their uses of mathematical pedagogical knowledge (Shulman, 1987), especially in their first-year of teaching (Zeichner & Tabachnick, 1981). Novice teachers might return to their intuitive understanding and implementation of mathematics tasks based on their prior school experience. This study, through stimulated recall, may increase PSTs' awareness of their probing practice and provide an opportunity for them to articulate the probing practice, thus paving way for improving the probing practice in classrooms. With a current focus on probes through speech, this study could add to the literature on probes by an investigation of the impacts of gestures on the expression of probes. Future intentional management of speech and gestures in probes might be expected in preservice teacher education.

Investigating whether PST-identified probes align with researcher-identified probes will either lend legitimacy to the practice of current researchers or highlight needed changes in how the field interprets and investigates probing or works with preservice and in-service teachers to establish a common understanding of probing. Teacher educators can also benefit from this study on the alignment or not between PSTs and researchers. They might find a need to change their conventional ways of teaching probing techniques to PSTs. PSTs in particular might perceive a need to change their speech and gestures to enact competent probes.

CHAPTER 2: LITERATURE REVIEW

Both speech and gestures contribute to meaning making. In educational settings, in particular mathematics, speech and gestures are frequently connected with written symbols. Thus, language, gestures, and written symbols all play a communicative role in social interaction (Lemke, 1998). This study incorporates multiple sources of data (e.g., written, visual, or listening data) to make sense of a certain phenomenon “in a comprehensive and integrative manner” (Norris, 2014, p. 13) and falls under the category of multimodality research. One way to understand modality is through Goodwin’s (2013) theoretical framework, which emphasizes the dynamic and static factors in human actions. In school-based settings, Goodwin’s framework has the potential to reveal how some consistent meanings are maintained and how some new meanings are continuously added to previous topics.

2.1 The Laminated Structure of Human Actions

Goodwin's (2013) *laminated structure of human actions theory* describes the mechanism of how human actions are formed by the simultaneous application of four layers of semiotic resources, each contributing to the comprehensibility of the action in progress: a) the positioning of speakers’ bodies and broad surrounding contexts where the phenomenon under discussion occurs (the participation framework); b) the speakers’ language, including the words, tones, and pitches in speakers’ speech; c) environmentally coupled gestures, i.e., hand gestures; d) the phenomenon in the domain of scrutiny, which refers to the specific problem or topic discussed in the moment. Take an instance in Alibali and Nathan (2012) as an example. A student sitting face-to-face with a teacher worked on the y-intercepts and the slopes of three lines and aimed to reason why they were different or similar. Focusing on the red line, the teacher asked, “How much money did we start with though? Where does this red line cross this y-axis?” (Alibali & Nathan, 2012, p. 285). The accompanying hand gestures were that the teacher used her right hand to trace the red line from the top down to a point beyond the origin. According to Goodwin’s (2013) theory, to effectively communicate the questioning action to the student, the teacher positioned herself towards the student (i.e., bodily orientation) and used the hand gestures (i.e., environmentally coupled gestures) to fix her question on the red line (i.e., phenomenon under scrutiny). Together with the inquiring speech about the red line (i.e., language), the teacher made her questioning intention clear and explicit to the student. In my study, I use the *laminated structure of human actions theory* as an overarching framework that helps me weave four different layers of resources together to interpret how PSTs form and actualize their probes in mathematics discussion lessons.

The fact that the current (either discursive or motional) actions build on already existing actions (especially the preceding ones) is significant to how participants comprehend the on-going talks (Goodwin, 2013). Goodwin (2013) called the already existing actions that one speaker acted upon as the substrate. Consider an example that he provided in his article:

Tony: Why don't you get out [of] my yard.
Chopper: Why don't you *make me* get out [of] the yard.
(Goodwin, 2013, p. 9)

Chopper's talk built on what Tony said. Tony's talk acted as a substrate that Chopper could work on. Chopper kept Tony's structure and meanwhile modified it by adding some new meanings. "The process of simultaneously 1) preserving structure provided by the activities of earlier actors while 2) systematically modifying that structure to build something new, is a central, distinctive feature of human action" (Goodwin, 2013, p. 9).

Goodwin's (2013) *laminated structure of human actions theory* describes how speakers make use of a set of resources (e.g., visual, audio, written, linguistic, motional resources, etc.) available in the environment to fulfil their communicative purposes. Thus, not all resources will be used every time speakers speak. Similarly, the four layers of semiotic resources are not always present in talk but may overlap with each other in meaning-making. As for how many resources are within use, it depends on the speakers who handle accessible resources to make their meaning across to listeners.

2.2 Mathematical Probes

2.2.1 Defining Mathematical Probes

Probing questions directly follow students' responses so that teachers can better understand what their students mean rather than leading them toward a particular understanding (Sukmadewi, 2014; Teuscher et al., 2016). Although probing could be used to determine if students can articulate more advanced thinking or deeper thinking with the teacher's support (see Fraivillig et al., 1999; Hähkiöniemi, 2017; Sahin & Kulm, 2008), I define mathematical probes with a localized meaning, i.e., following up what students have said about their initial solutions. Such a definition aligns with the revoicing and prompting in Chapin et al.'s (2009) talk moves, the pressing in Ghousseini's (2015) talk moves, and follow-up (competent) questions in Moyer and Milewicz's (2002) classification of PST questions.

Building on my definition of probing, probing should help PSTs understand what students do procedurally to solve mathematics problems and understand students' conceptualization of the mathematics problems (Kazemi & Stipek, 2009). By following up what students have said, PSTs delve into how students get the answers and what mathematical reasoning they use to support their answers. For example, Hähkiöniemi (2017) proposed that secondary PSTs asked students probing questions about problem-solving methods, reasoning, causes for mathematical claims, meanings, arguments, and extension. Based on these meanings, possible situations of PSTs' probing student thinking could include when the teacher wants to know more specific or in-depth information about student responses and when the teacher finds student responses confusing or ambiguous (including unconventional responses) to herself and the whole class. On the other hand, I do not include those situations where PSTs try to extend student thinking by offering an alternative solution to create perturbation in students; neither do I include situations where PSTs ask other students to restate what their peer says. My exclusion of those instantiations does not mean they are unimportant. Instead, my focus on probing for more information and more accuracy correlates with PSTs' intentional stance that they take in the enactment of the probing practice of a particular student.

2.2.2 Uses for Probing

Previous studies (Fraivillig et al., 1999; Herbal-Eisenmann & Breyfogle, 2005; Sahin & Kulm, 2008; Sukmadewi, 2014; Teuscher et al., 2016) contribute to the reasons why mathematics teachers probe. For example, Sahin and Kulm (2008) created a model of why inservice mathematics teachers asked probing questions: a) "ask(ing) students to explain or elaborate their thinking"; b) "ask(ing) students to use prior knowledge and apply it to a correct problem or idea"; c) "ask(ing) students to justify or prove their ideas" (p. 235). Overall, there are three general intentions for when PSTs use mathematical probes: a) for PSTs themselves, i.e., PSTs want more information from students to help themselves (PSTs) comprehend students' strategies; b) for the students, i.e., PSTs intend to help students clarify their own problem-solving procedures and mathematical thinking and deepen their conceptual thinking (e.g. through connecting to prior knowledge); c) for the whole class, PSTs find the topic worthy of being attended to by the whole class and intend to make what is being said clearly and easily accessible to the whole class (Barlow et al., 2018; Kazemi et al., 2016).

Many studies have delved into fine-grained classifications when teachers use mathematical probes. Franke et al. (2009) identified four types of questions that three elementary mathematics teachers used to make students' thinking explicit and accurate: general questions (unrelated to specifics in students' responses), specific questions, probing sequences of specific questions (a series of specific questions with multiple teacher questions and student responses included), and leading questions. Only the third question type – probing sequences of specific questions— contributed to students' complete and clear explanations when students initially could not do that. Situations happen often when teachers receive ambiguous and incomplete student responses. Varying questions instead of repetitively using the same question can be helpful. As such, when teachers probe student thinking, multiple related questions with each addressing a mathematical point in students' explanations helped improve students' explanations (Franke et al., 2009).

Although not focused on mathematical probes, a later investigation conducted by Alibali and Nathan (2007) confirmed similar features of gestures going along with mathematical probes. Alibali and Nathan (2007) argued that teachers used gestures at a higher rate in two types of situations than other situations: first, when teachers introduced new mathematical ideas and second, when teachers found students confused about some mathematical content. Alibali et al. (2013a) supported this finding and argued that teachers used more gestures in turns after trouble spots than in turns prior to trouble spots. Trouble spots implied students' lack of comprehension and were characteristic of students' incorrect responses, staggering or uncertain explanations, and asking questions for clarity (Alibali et al., 2013a). Thus, when a shared understanding did not exist between teachers and students, teachers used more gestures to probe and advance student thinking.

2.3 Gesture

To keep consistent with Goodwin's (2013) theory, I will study gestures in light of head movements, eye contact, and hand gestures. In accordance with the physical stance of speakers, I also consider bodily orientations, which include three situations: directly facing toward students, partially facing toward students, and not facing toward students.

2.3.1 Functions of Gesture

Probing is meant to help clarify students' thinking. Gestures offer another window into students' minds (Goldin-Meadow, 2007). McNeill (2017) proposed a thought-language-gesture link, also called Mead's Loop. Language and gestures do not self-subsist and need to rely on the contexts as well, i.e., what Vygotsky (1987) called "material carriers," which refers to meanings in enactments or material experiences. Language and gesture are a unit that work together in meaning-making (McNeill, 2017). Gestures bring up the imagery contents of the mind or speech. No matter whether gestures are visible, the imagery contents still exist. When hand gestures are restrained, the functions of gestures may appear in some hidden forms, for example, using head movement instead of hands (McNeill, 2017).

Many studies have confirmed the benefits of teachers' gesture use in students' learning (Church et al., 2004; Valenzeno et al., 2003) and directly shed light on teacher instruction, for example, including teachers' use of pointing to or tracing mathematical objects or diagrams (Hu et al., 2015), a high frequency of gesture use (Flevares & Perry, 2001), and inclusion of additional information in gesture (Cassell et al., 1999). Alibali and Nathan (2007) argued that teachers used gesture to scaffold students' learning by grounding their instructional language when introducing new concepts, explaining abstract or complex concepts, and encountering students' confusion. Teaching with gestures proves to be more effective in attracting students' attention to target content than teaching with color highlighting (Bem et al., 2012). Moreover, Church et al (2007) argued that speech accompanied by gestures was more likely to be recalled than speech only. Thus, in classroom settings when gestures take on informative functions, the information going along with gestures can be retained well compared to information without gestures.

2.3.2 Categorization of Gesture

McNeill (1992) categorized gestures into four types: deictic gestures, iconic gestures, metaphoric gestures, and beat gestures. *Deictic gestures,* often called pointing gestures, refer to pointing movements toward concrete objects such as pointing at numbers when PSTs talk about them in a problem. *Iconic gestures* refer to the gestures that directly describe the motion included in the semantic content of speech. Consider a subtraction problem $5 + 8 = 6 + \square$. When saying, "We will move the six from the right side of the equal sign to the left side," a teacher may move her hand or fingers from right to left. The hand gesture depicts the meaning expressed in the speech and represents the movement in a similar way, hence making an iconic gesture. However, the iconic gestures can express additional information that does not exist in speech. The teacher might move her hands very slowly when she finds students are not really understanding the procedure. The slow finger movement implies that the teacher notices students' struggles and slows down her instruction as a form of scaffolding. *Metaphoric gesture*s, similar to iconic gestures in terms of referring to imagery representation, describe abstract concepts expressed in speech. An example is a grasping gesture (see Figure 1). A student says he wants to understand all that his teacher says in class though he cannot. An accompanying gesture is to open a hand and fold it to a fist as if he can grasp some objects. The knowledge or ideas that the teacher teaches is compared to something that the student can grasp. Therefore, this gesture acts as a metaphorical base to hold the concept of knowledge or ideas (McNeill, 1992). *Beat gestures* usually do not bear semantic content but are rhythmic movements that reside in the speaker' comfortable or habitual movements, like quick flicks of fingers or rapid pats on the lap. Alibali et al. (2017) and Kita et al. (2017) excluded beat gestures and define the other three types of gestures as representational gestures, which depict location, motion, trajectories, shape, and so on.

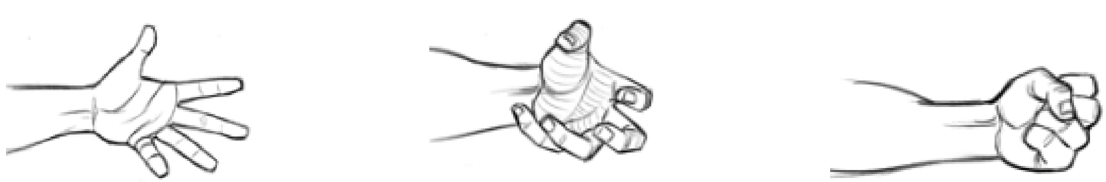

Figure 1. Grasping Gesture—An Example of Metaphorical Gesture

2.3.3 Speech and Gesture

Although beat gestures may be constrained in contributing to communicative function, deictic, iconic, and metaphorical gestures can help teachers in making their meanings across to students in classroom settings. Gestures fall on a continuum in meaning-making (McNeill, 2017). On the one hand, gestures can be communicative to a great extent, thus adding on to or reinforcing speech (Nathan et al., 2014); on the other hand, gestures can be minimal in terms of communicative function or detract from speech. Gerwing and Bavelas (2004) asked participants to describe their actions while playing with toys to those who had played with toys and those who had not. They found that when describing the action to those with playing experience, participants used less informative and less complex gestures. Therefore, when shared knowledge is accessible, gestures may be less communicative in function. When teachers probe students' thinking, shared knowledge (including the intention and the content of probing) is not always accessible to students. Teachers' use of less informative gestures can incur difficulty in comprehension for students.

2.4 Speech-Gesture Matches and Mismatches

Although not explored in most investigations of teachers' mathematical probing practices, speech and gestures jointly communicate the speaker's intended messages (Kendon, 1994; Streeck, 2002). Gestures encode information that is not always encoded in speech (Goldin-Meadow, 2005). In mathematics, speech works well to represent the logic relations among different concepts (Hord et al., 2016). Gestures enrich the logic relations with visual and spatial content (Wagner et al., 2004). Consider a teacher who is teaching children to do 5 + 3, saying, "We should add this and this together." The teacher points to 5 first with an index finger and then to 3 with a thumb and then bends these two fingers toward each other (see Figure 2). The bending gesture, an iconic gesture, reinforces the idea of adding 5 and 3 together. Along with helping students comprehend strategies, using gestures that match their speech can reduce teachers' cognitive load (Wagner et al., 2004), allowing them to focus more on other instructional decisions.

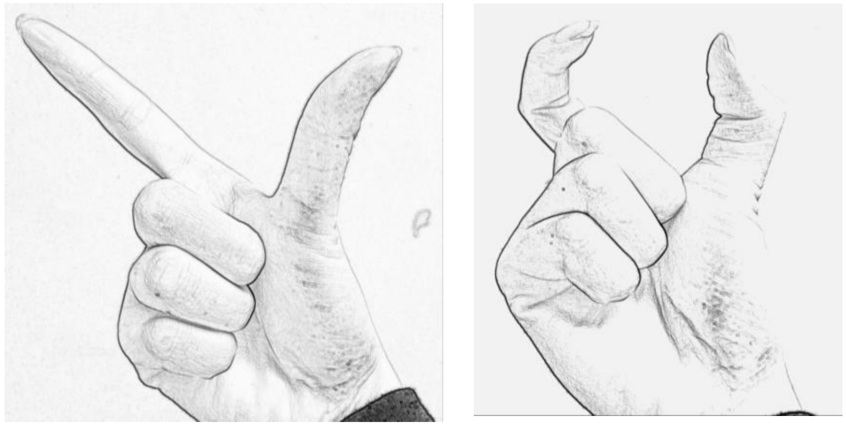

Figure 2. Hand Gestures of Adding Five and Three

Verbal instruction better facilitates students' learning when combined with gestures than without gestures (Valenzeno et al., 2003). Different learning opportunities could be created when teachers coordinate their gestures differently with their speech. When gestures stay consistent with speech, gestures reinforce students' learning. When gestures mismatch speech, students' learning outcomes may be mixed: students learn well when teachers verbally present a problem-solving strategy simultaneously accompanied by a different strategy in gesture; however, more than one strategy presented in speech with different strategies in gesture is not as effective as the previous situation (Singer & Goldin-Meadow, 2005).

Especially in lower-level elementary classrooms where student learning is primarily perceptual at the beginning (Thomas & Tabor, 2012) and depends much on visuospatial representations to conceptualize numbers (Carey, 2009), synthesizing speech and gestures is of great importance. Alibali and Nathan (2012) claimed that some mathematical knowledge is embodied in gestures when teachers and students did explanations for their mathematical ideas. Pointing gestures are most often used among all other gestures (Alibali et al., 2013b). Using hand gestures to represent the numbers is often used in K-2 classrooms, which could support students' efforts to subitize. Therefore, when PSTs enact mathematical probes, PSTs' understanding of how to use gestures to represent mathematical knowledge is embodied in their probing gestures.

Speech-gesture mismatches happen frequently when teachers teach new concepts or when students who struggle with learning are on the verge of successful comprehension (Goldin-Meadow & Singer, 2003). The information conveyed in the teacher's gestures may not explicitly match what the teacher says and often implicitly entails the connection between the contents in speech and gestures. Students can still focus on the key information, and then actually benefit from the processes (Singer & Goldin-Meadow, 2005). Some students are even able to leverage the teacher's gestures to help them make sense of the teacher's speech, Singer and Goldin-Meadow (2005) found that third and fourth graders' performances on subtraction problems were better when they were taught a strategy in speech and shown gestures that matched a different strategy than when they were shown gestures that conveyed the same strategy. The mismatching gestures were not that intrusive so that students could pick out the information from speech that they learned well.

2.5 Usefulness of Video Learning

Because gestures often happen without people thinking about them, teachers might not know what gestures they use or how students receive them. In the most recent decades, video learning (e.g., stimulated recall) has become a learning opportunity for inservice and preservice teachers to improve their professional noticing skills (van Es & Sherin, 2010; Walkoe, 2015; Walkoe et al., 2019). In teacher education, using expert teaching videos helps the PSTs notice classroom interactions in depth and encourages the PSTs to reflect on the effects of teachers' behaviors and discourse on student learning (Star & Strickland, 2008). Some teacher-researchers use their own classroom videos (i.e., self-study) to promote their teaching skills by contributing to an insider's viewpoints. When reflecting on their own teaching videos, teachers' accounts of what happened differ from observers' accounts (Roth, 2007). Conversely, teachers who go back and watch their teaching videos may partially put themselves in an observer's place and find what others could have thought of their lesson. These posteriori perspectives may promote teachers' understanding, learning, and teaching of mathematics.

However, watching teaching video clips is seldom applicable to the noticing of teachers' gesture use in instruction. Admittedly, collectively watching each other's teaching videos promotes mathematics teachers' changes in noticing students' mathematical thinking. Only a few studies mentioned the usefulness of gestures to access students' thinking (M. G. Sherin et al., 2009). Van Es and Sherin (2008) analyzed mathematics teachers' noticing of student thinking in their own teaching videos and established a learning-notice-framework to: (a) highlighting key noteworthy interactions in the lesson, (b) using teachers' knowledge of the context to explain these interactions; (c) connecting specific interactions to the broader principles in mathematics. They found that the teachers experienced noticeable changes in noticing, i.e., from noticing general interactions and actors to noticing students' mathematical thinking by exemplifying with concrete video clips. Star and Strickland's (2008) and Powell's (2005) results resonate with those findings.

Teachers' gesture use can be a noteworthy area to explore instructional meaning-making and the extent to which students receive instruction by means of teachers' verbal discourse and nonverbal movements. Videos provide an objective tool that PSTs themselves and researchers as outsiders can track back to. PSTs' descriptive accounts of their lessons may serve as a window to what the lessons look like. However, when teachers recall the interactions and events, they actually re-construct their lessons based on their perceptions of the lessons (Roth, 2007). Therefore, an outsider's perspective strengthens the objectivity of video learning.

2.6 Examples of Gesture Use in Elementary Mathematics

Elementary mathematics content lends itself to the use of gesture as much of it centers on perceptual counting and using direct modeling strategies with manipulatives or representations. The section provides some examples of gesture use when teachers deliver instruction about counting, modeling, place value, and basic operations, and makes a strong reference to the combined effects of speech and gestures on student learning.

According to the Common Core State Standards for Mathematics (CCSSM; National Governor's Association Center for Best Practices &Council of Chief State School Officers, 2010), kindergarteners begin to learn numbers and numerical operations between 1 and 10. Kindergarteners start counting with concrete objects, which is at the stage of perceptual counting (Wright et al., 2006). Perceptual counters play with concrete manipulatives to model the numerical operations. An advanced level of counting is figurative counting when students can do mental math without the presence of objects. CCSSM regulates that kindergarteners should consider subtraction as taking part or taking from. This preliminary understanding of subtraction often persists in teaching and in student learning. When teaching the concept of taking away from, K-2 teachers might draw, for example, a set of 20 circles, cover eight circles with hands, and ask students how many circles are left. This hand gesture, i.e., an iconic gesture, represents the take-away action and could facilitate students' understanding of the take-away concept.

The strategies that children use to solve basic operations in early childhood have been extensively studied (Carpenter et al., 1981; Carpenter & Moser, 1984; Roy, 2014). A consensus among these studies is that children solve basic operations by direct modeling at the very beginning and then eventually acquire such advanced strategies as using number facts, created algorithms, and number composition and decomposition. Direct modeling often involves deictic gestures when students point to objects with one-to-one correspondence. Counting without objects is also related to the direction of counting. Some students could possibly use a mental number line when they count. Thus, conscious and conscious uses of gestures may be prevalent when students solve subtraction problems. A scenario that mathematics teachers meet often is when they invite students to share their strategies, students tend to say “they counted” without further explanations of how they count. At this time, teachers usually use deictic gestures to refer to specific numbers that the students wrote on the paper or on the blackboard and ask for clarification; or teachers may use iconic gestures to imitate with fingers what students do when they work on their own to encourage students to say more.

A major goal in early operations instruction is to help students use such reference points as five and ten or compose and decompose numbers in their strategies (Fuson et al., 1997). For example, representations of ten-frame and base-ten blocks are accessible visually at the beginning of learning subtraction and place value and can effectively show composition and decomposition of numbers (Fuson & Briars, 1990). When solving 10 – 6, a child could see where six is on the ten frame and reach the final answer by counting the leftover dots or knowing the amount of the leftover dots by subitizing. In response to students’ counting strategies, teachers may use deictic gestures to point to the leftover dots one by one, thus implying students’ ways of counting. If students do subitizing, teachers may use metaphorical gestures like a palm up as if the leftover dots are on his/her hands (see Figure 3), thus implying seeing those leftover dots means knowing how many dots there are.

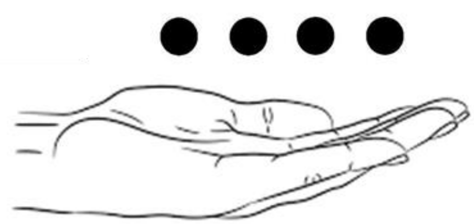

Figure 3. A Palm-up Gesture with Four Dots

When the preservice teacher education puts an increasing emphasis on PSTs' elicitation of students' mathematical thinking rather than PSTs' lecturing of content knowledge, one of the pressing questions is how PSTs elicit students' mathematical thinking. Some research suggestions include PSTs' use of discursive moves strategically (e.g., use orienting or negotiating talk moves (Ghousseini, 2015), and sequence talk moves (Franke et al., 2009). There is abundant research from the discursive perspective to study and advance PSTs' elicitation; however, little is known about how PSTs combine gesture and discursive moves in their probing enactment.

In this research, I do not attempt to study the effects of PSTs' gesture and talk move use on students' learning. I aim to investigate how PSTs encode these two modalities in their meaning-making. When PSTs probe students' mathematical thinking, PSTs try to build on students' existing responses/thinking and express their own probing intention. According to the cognitive load theory (Alibali & DiRusso, 1999; Ping & Goldin-Meadow, 2010; Pollock et al., 2002), the information conveyed through gesture saves cognitive resources and alleviates the cognitive load one processes at the same time, which means that gesture can replace some information which could have been expressed verbally and save space for other cognitive information load. Gesture can also help organize ideas by packaging several mental representations into one single gestural representation (Ping & Goldin-Meadow, 2010).

CHAPTER 3: METHODOLOGY AND METHODS

3.1 Methodology

Case study is an appropriate research method for my inquiry into PSTs' gestures. My attempt in this study was to explore PSTs' use of gestures within their regular field teaching contexts. As a researcher, I had little to no control over PSTs' teaching behaviors. My methodological orientation was "naturalistic, holistic" (Stake, 1995, p. xi), and observational (Yin, 2018). Case study methods are usually used to answer why and how research questions (Yazan, 2015). One PST was regarded a case. Regarding potential variations in gesture use, multiple cases could depict a phenomenological picture of how PSTs probed students' thinking with gestures.

I also used stimulated recall to approach PSTs' perspectives of their own probes. Stimulated recall, as a subcategory of introspective research methods, can be enacted by inviting participants to recall their simultaneous thinking in events (Lyle, 2002; Mackey & Gass, 2005). This research methods can provide a detailed account of participants' cognitive processes (Gass & Mackey, 2000). Stimulated recall is beneficial in my study to access participant's introspective views of when and why they used mathematical probes, thus minimizing researchers' subjective interpretations of PSTs' action.

3.2 Participants and Settings

The PSTs were taking an elementary mathematics methods course at a large midwestern university. The elementary mathematics methods course is part of the teacher certification program. The methods course aims to help PSTs develop elementary mathematics instruction skills based on their understanding of children's mathematical thinking. In class, PSTs were expected to learn how to elaborate mathematical concepts, how to use questioning to reveal children's mathematical thinking, how to use appropriate mathematical problems and activities to assess and enhance children's mathematical understanding, and how to organize whole-class discussions to approach the diverse mathematical thinking among children. After four-weeks of in-class learning, PSTs were placed in K-5 classrooms, observed mathematics teaching in their placement classrooms, and finally taught two mathematics lessons: a number string lesson cotaught by two PSTs and a discussion lesson taught individually; if PSTs were in a placement alone, they independently taught both lessons. PSTs' field teaching experience alternated with lectures on campus. PSTs had opportunities to discuss their field teaching experience with their instructors and their peers in class; the instructors addressed mathematics issues that were worth PSTs' attention for further improvement.

Two course readings that played a key role when PSTs investigated how to facilitate discussions around children's thinking about specific mathematical topics were Chapin et al.'s (2009) *Classroom discussions: Using math talk to help students learn* and Kazemi and Hintz's (2014) *Intentional talk: How to structure and lead productive mathematical discussions*. Chapin et al. (2009) discussed five talk moves (revoicing, asking children to restate someone else's reasoning, asking children to apply their own reasoning to someone else's reasoning, prompting children for further participation, and using wait time) and how to use these talk moves in grades K-6 classrooms. The methods course design helped PSTs gain skills in using talk moves to elicit and strengthen children's mathematical thinking. From the five discussion lesson templates Kazemi and Hintz (2014) provided in their book, the PSTs chose one for organizing their discussion lessons: a) *compare and connect* more than one strategy and discuss what makes them similar and different; b) *why? let's justify*: support students' explanations toward a mathematical generalization; c) *what's best and why*: analyze one or two strategies and decide on their effectiveness; d) *define and clarify* mathematical tools, representations, symbols, or vocabulary so that children use those tools, representations, symbols, or vocabulary with understanding; or e) *troubleshoot and revise* errors with the engagement of a whole class. Using the chosen template, PSTs did lesson planning with the instructor's and partner teacher's feedback and implemented their lesson in their placement classrooms.

During the field teaching experience, each PST taught a discussion lesson (approximately 30-minutes long). The focus of the discussion lessons was on leading productive mathematical discussions. All PSTs had interviewed one student in the class, and most PSTs had already taught a number string lesson (i.e., posing a series of related arithmetic problems) before the discussion lesson. Therefore, they had gained some familiarity with students in their placement classrooms before the discussion lesson, which will be the focus of my analyses. The PSTs wrote out their discussion lesson plans, workshopped their lesson plans in class, and submitted them for the instructor's feedback and revised their lesson plans before teaching the discussion lessons. In the discussion lessons, PSTs posed mathematics problems and asked students to solve them individually or in small groups first; then PSTs walked around and got familiar with students' strategies; finally, PSTs led whole-class discussions around the problem.

I recruited PSTs across three sections of the methods course, in which PSTs were placed in four different elementary schools. Twelve female PSTs consented to participate in the study; depending on whether they used probes or gestures in their probes (see the Sampling section for more details), seven PSTs became focal subjects of this study (see Table 1). These seven PSTs did their fielding teaching in three public elementary schools in a Midwestern city. Five PSTs were placed in a K-4 elementary school where 80.8% of the total population was economically disadvantaged and 19% was English learners; one PST—Marrisa— was placed in a K-5 elementary school where 45.9% of the total population was economically disadvantaged and 9.4% was English learners; another PST—Minnie— was placed in a K-5 elementary school where 39.9% of the total population was economically disadvantaged and 11.9% was English learners. Most PSTs' lessons were about operations with two-digit numbers. Lessons ranged from 19 to 41 minutes.

Table 1. Participants and their Discussion Lessons

Participant	Grade Level	Discussion Lesson (mins)	Topic of Each Lesson	Stimulated Recall (mins)
Clara	Grade 1	39	Two-digit number comparison	100
Minnie	Grade 1	41	Different forms of representing two-digit numbers	69
Megan	Grade 2	25	Subtraction with numbers 1-100	69
Layla	Grade 2	29	Expanded form of two-digit numbers	69
Holly	Grade 4	19	Perimeter of a shape: Adding and subtracting two-digit numbers	55
Marsha	Grade 4	34	Two-digit multiplication	77
Marrisa	Grade 5 Special Ed	41	Three-digit subtraction	79

3.3 Data Collection and Procedures

The data sources consisted of classroom observation notes, PSTs' discussion lesson plans, videotaping and audiotaping records of the discussion lessons, PSTs' and researchers' probe recording sheets, and videotaping and audiotaping records of the stimulated recall.

3.3.1 Classroom Observation Notes

I attended the methods course classes as a visitor, especially the sessions when PSTs prepared for and reflected on their discussion lessons. I sat in the corner of the classroom to minimize the influence of my presence and took notes about what instruction PSTs received in class, what mathematical ideas they came up with, and what difficulties they struggled with. My classroom observation notes provided information about the processes of PSTs' lesson planning and their reflective thinking after teaching. Additionally, regular visits to the methods course helped me gain familiarity with my potential participants. These PSTs would feel more comfortable with me in upcoming interviews, which in turn could facilitate their thought-sharing during interviews.

3.3.2 Discussion Lesson Plans

Discussion lesson plans provided information about mathematical strategies, questions, and visual representations that PSTs planned to use in their discussion lessons. I collected discussion lesson plans from the participating PSTs. The discussion lesson plans consisted of three parts: a) overall objectives, b) lesson opening and strategy sharing, and c) subject-specific content and discussions. In the overall objectives section, PSTs provided information about their lesson objectives, connection with mathematics standards, knowledge of elementary students, and students' work samples to be collected. In the lesson opening and strategy sharing section, PSTs planned on how they would open the lessons and what strategies they anticipated students would use. Planning requirements varies in the subject-specific content according to Kazemi and Hintz's (2014) book. The *compare and connect* template asks that PSTs give two examples of student strategies, argue for the mathematical connections between those strategies, anticipate what students might notice in class, and plan on responses to students' noticing. The *why? let's justify* template requires that PSTs should make appropriate responses to students' explanations and support students' reasoning toward the target ideas. The *what's best and why* template requires that PSTs emphasize the effectiveness of a particular strategy and its uses in certain problems. The *define and clarify* template requires that PSTs pinpoint the concepts related to the tool, representation, symbol, or vocabulary that they emphasize in class, anticipate students' partial understanding, and provide ways of building on students' existing understanding. The *troubleshoot and revise* template requires that PSTs determine which misunderstanding to focus on, what insight PSTs want students to learn, and what follow-up responses PSTs will use to move students out of their misunderstanding.

3.3.3 Videotaping

The field teaching took place in PSTs' placement classrooms in elementary schools. I videotaped and audiotaped PSTs when they taught their discussion lessons. The videotaping device Swivl captured PSTs' speech, gestures, and surroundings in the classrooms by following PSTs automatically. Sometimes students' responses to PSTs (verbal and gestural) helped explain the PSTs' speech and gestures. Therefore, to avoid possible limitation of focusing solely on the PSTs (Miller & Zhou, 2007), I considered students' responses, especially their gestures, in the analysis to help make sense of PSTs' use of speech and gestures.

3.3.4 Further Sampling

Twelve PSTs consented to participate in the study. I video- and audio- recorded their first discussion lessons. My goal was to select PSTs who used both speech and gestures to probe students' thinking; seven PSTs fit this group. I made the selection to the extent which the PSTs varied their speech and gesture use in their probes. Four PSTs used a high-level variance of speech and gestures in their probes; three PSTs used either few probes or few gestures or dominantly used one gesture type. Therefore, I ended up interviewing seven PSTs. Previous studies on PSTs' talk moves and use of gestures focused on one or two teachers (Alibali & Nathan, 2007; Franke et al., 2009; Ghousseini, 2015; Goodwin, 2013; Sahin & Kulm, 2008; Walkoe et al., 2019; Weiland et al., 2014). This implicates that the number of participants in this study, i.e., seven PSTs, is adequately reasonable.

3.3.5 Stimulated Recall

Within 10 days[1] after PSTs taught the discussion lessons, a stimulated recall interview was scheduled with each PST in a quiet study room. Stimulated recall "involves the use of audiotapes or videotapes of skilled behavior, which are used to aid a participant's recall of his thought process at the time of that behavior" (Calderhead, 1981, p. 212). Through stimulated recall, PSTs provided detailed accounts of their probes when their teaching videos were replayed. In this study, the foci of the stimulated recall were as follows: a) the speech and gestures PSTs used to probe; b) how they coordinated their gestures with speech to cue students for more information; c) reasons for using the specific probes, e.g., what factors in contexts PSTs took into consideration when they decided to make a probe; d) when to stop probing and why; e) a self-evaluation of whether their probes were efficient or not. The session of stimulated recall was divided into four parts.

3.3.5.1 Initial understanding interview

This interview aimed to get PSTs' initial understanding of their uses of probes in their lessons before any influence resulting from stimulated recall might occur. The interview took 5 to 10 minutes long. The interview questions built off of PSTs' lesson plans and actual lessons and targeted the probes they usually used to elicit student thinking and the responses they generally made with regards to anticipated student responses. The interview focused on five questions: a) *Why did you choose this problem to pose in your lesson?* This question would help me understand to what extent the PSTs were familiar with the problem they planned to teach (e.g., PSTs might have taught the problem before or known the target topics well). Some PSTs might also mention the relations between the problem that they chose and the broad concepts students were learning at that time. Indexing PSTs' reasons for choosing the problem in a broad context would help me determine which aspects of mathematics attracted PSTs' attention. b) *How do you probe or follow up students' thinking after getting students' answers? Give me an example when you probe or follow up students' thinking in class.* PSTs' responses provided their definitions of a probe, which paved way for their identification of probes when I replayed their teaching videos. Giving PSTs an opportunity to define probes by themselves would reduce my influence on PSTs' thoughts and

[1] As required in the methods course, PSTs will submit their lesson analyses within seven days of teaching their lessons. A 10-day limit for stimulated recall is reasonable because the lessons may still stay fresh in the PSTs' memory when they work on their lesson analyses.

lower the possibility of me imposing my thoughts on PSTs. *c) Tell me your rationale for probing or following up students' thinking like that. What else might you do as probe or follow-up and why?* This question helped me grasp PSTs' understanding of what counted as a probe (some might not interpret probes as involving gestures). d) *How might your probes or follow-ups differ in situations where students give clear responses versus ambiguous responses, and correct versus incorrect responses? (e.g., when encountering ambiguous responses, the teacher may look around the classroom and try to find an opportunity to involve other students in discussion or be ready to walk toward the blackboard and draw a picture or model on the board).* This question helped me concentrate on the situation-based differences of PSTs' planned probes and address the differential probing intent that PSTs planned to use in class. e) *How do you know it is time for you to stop a probe or follow-up?* This question provided evidence for the scope of PSTs' planned mathematical probing.

3.3.5.2 Watching discussion lesson videos

PSTs watched their lesson videos (that I recorded) with me in order to reduce the influences of external factors such as stress that results from first-time watching (Tuckwelll, 1980).

3.3.5.3 PSTs' identified probes

The stimulated recall included a reiterating watch-pause-comment process with the purpose of eliciting PSTs' thought processes about their uses of probes. I played back the teaching video to each PST, paused the video once the PST identified a probe, and filled out a recording sheet (see Appendix B) to note down the timestamps for each probe and their reflection on their uses of speech and gestures in their probes. Given the power of taking the lead, PSTs had the freedom to regard themselves as the expert on what a probe (the words "probe" and "follow-up"[2] were used at the same time in the interview) was to them. To reduce the researcher's influence, I only asked facilitation prompts to help PTSs with their think aloud, and no other training was provided for PSTs. To facilitate PSTs' thought processes, I asked prompts such as "*How do you probe or follow up students' thinking?*" and "*Tell me what math you want to get from students in your probes or*

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[2] PSTs are familiar with "follow-up" or "following-up" questions. In this study, not all follow-ups are probes. To prevent PSTs from blindly equating following up with probing and still keep the probing feature of following up questions, I used "probe" first and "follow-up" second.

follow-ups." Building on PSTs' responses, I then asked, "*How does that help you probe or follow up students' thinking?*" while avoiding such guiding questions as "Do your gestures help you do the probes or follow-ups?" I assumed that PSTs would talk more about their talk moves because verbal talk moves were discussed in the methods class; I anticipated that some PSTs might not mention gestures at all.

3.3.5.4 Initiating gestures

In this part, the stimulated recall concentrated on the integration of gestures with speech. For those who identified both speech and gestures in the previous part, I replayed two different instantiations in the videotaped lessons where PSTs used probes and asked PSTs, "*How do you think your gestures and speech helped students understand that you wanted them to tell you more?"* For those who did not initiate gestures, I would bring up two different probes (that the PSTs had identified) in which the PSTs used gestures in one probe and did not in the other probe. Prompts asked were "*Tell me something you are wondering about these two probes or follow-ups?*" "*Do you identify any differences in these probes or follow-ups?*" "*In what ways are they different?*" If the PSTs still did not identify gestures, I would take up the initiation by asking, "*Do you think your gestures help you probe or follow up students' thinking?*" and "*If you do think so, how?*" The comparison of these two different probes may focus the PSTs' attention on the use of gestures in probes. Also noticing the roles of their own movements in probing may elicit the PSTs' reasoning about the probes in a broader classroom context.

3.3.5.5 Revised understanding interview

At the end of the stimulated recall, PSTs were asked to revise their definition of a probe if they thought it necessary. Besides, I asked PTSs about their evaluation of the efficiency of their probes and asked them to provide their thoughts about when to stop probing. I was also open to something additional that PSTs themselves brought up. The duration of the stimulated recall interview with each participant varied among all seven PSTs. For those who did not probe a lot, it was faster when I replayed the teaching video to PSTs; for those who did use many probes, it took longer when we re-watched the video and paused on a frequent basis. but in general, the stimulated recall stayed within two hours. Details are presented in Table 1.

3.3.6 Researcher-identified Probe Recording Sheets

Two researchers (including me) coded PSTs' teaching videos and identified their use of probes, serving as researcher-identified probes. I filled out a probe recording sheet (see Appendix C) and noted down the probes and the coordination of speech and gestures that I thought PSTs had utilized in teaching. My coding of probes was completed before stimulated recall to avoid the effects of PSTs' viewpoints on my coding. The second researcher[3] did his coding after all stimulated recall interviews were completed. Next, we researchers met up and discussed the probes until 100% agreement was achieved (collective researcher-identified probes).

3.4 Data Analysis

To answer the first research question: *How do PSTs use speech and gestures to enact the probing practice in K-5 mathematics classrooms?* I focused only on the speech that PSTs use to probe whereas the speech with facilitation and discipline management functions was not considered. I coded PSTs' gestures mainly from the following aspects: head movements, eye contacts, and hand gestures. For example, a PST may turn their head back and forth from the problem written on the board to a student, signaling that the student should continue their explanations about the problem; or a PST may continuously gaze at a student, which signals that the teacher is waiting for a response. The focus of hand gestures is on McNeill's (1992) deictic gesture, iconic gesture, metaphoric gesture, and beat gesture. For example, deictic gestures include any pointing gesture by uses of hands and beat gestures include small repetitive movements of hands or fingers. However, McNeill's (1992) categorization of gestures can also be applied to other body parts. An example of this is head movements. A PST may use repetitive nodding to signal that s/he is listening instead of agreeing with one another's idea.

Next, I categorized PSTs' probes and counted the number of probes that PSTs used in their discussion lessons. The categorizations of probes built on existing literature on probing or following-up questions and on grounded coding (see Table 2). Overall, I identified each probe based on what was probed, such as probing students' strategy use, reasoning about their strategy use, and clarifying potential ambiguity. As for grounded coding, I created two probes: problem-

[3] The second researcher was a mathematics-education researcher and familiar with the probing/questioning literature and PSTs' fielding teaching.

posing probe and concept probe. For example, Boaler and Humphreys (2005) excluded information-gathering and factual-knowledge-checking questions from probing questions. Together with my second coding researcher, I found that when posing problems to students, many PSTs regarded those questions as probes that they used to dig into students' prior knowledge. Therefore, I coded this type of questions as problem-posing probes. Since most PSTs covered place value in their teaching, they frequently probed students' understanding of place value and mathematical symbols. I was aware that PSTs were able to vary their use of concept probes to adjust to different students' learning needs. An addition of this concept probe would enrich PSTs' probing studies. Much caution was paid to the boundary among different probes. Situations exist when PSTs' probes shared descriptive features of more than one probe category if the probes were singled out. Hence the immediate contexts would be considered to code the probe as one probing category. For instance, a PST probed a student for why 31 was bigger than 22. Receiving such a response as, "It has three," the PST probed further by asking "Three what?" The PST was clear about 31 having three tens and there was no ambiguity or uncertainty on the PST's side from the perspective of three tens. When probing "Three what," the PST was concerned about the student's use of concise language and precise demonstration of place-value understanding. Therefore, I coded "Three what" as a concept probe rather than clarification probe.

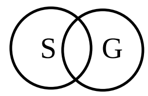
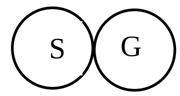
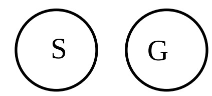
For the probes that were identified, I provided descriptive explanations for how the PSTs' speech and gestures made the probes possible. Sometimes, nonverbal probes —gestures without speech— could also act as powerfully as verbal probes. I intended to capture intricate changes of speech and gestures in my descriptions. Another instantiation of probing could be when a PST was questioning a topic, she quickly targeted the probe at a different topic before students responded to the first topic. I would not consider the incomplete questioning as a probe but would consider its contribution to the probe that the PST successfully put into effect.

Table 2. Categorizations of PSTs' Probes

Probe Type	Description	Example	Citation
Problem-posing probe	--Wants direct answer, usually wrong or right; --Rehearses known facts or procedures; --Enables students to state facts or procedures.	Have you seen these symbols? *(before posing the math comparison problem)*	Gathering information, checking for a factual method (Boaler & Humphreys, 2005)
Concept probe --place value --symbol	--Points to underlying mathematical relationships and meanings, --Makes links between mathematical ideas	Please tell me why this is bigger. *(S: It has three.)* Three what? *(S: Three ten.)*	Exploring mathematical meanings and relationships (Boaler & Humphreys, 2005)
Strategy probe	--Wants to get students' descriptions of their strategies (solutions)	How do you get 12? How do you get 20? Somebody explains how they get 12?	Elaborating problem-solving strategies (Chen et al., 2020; Franke et al., 2009)
Reasoning probe --justification --general reasoning	--Wants students to justify their problem-solving strategies	How do you know they were equal? *(S: Because the numbers, 3, 3, 7, 7.)* You said 67 is bigger than 51. Can you tell me why 67 is bigger?	Getting students to explain their thinking (Boaler & Humphreys, 2005); Uncovering students' reasoning (Franke et al., 2009) Showing why an idea/solution is true; refuting the validity of an idea; and giving mathematical defense of an idea that was challenged (Teachers Development Group, 2013, p. 41).
Clarification probe	--Asks students to clarify their explanations --Repeats students' responses to resolve uncertainty about what students said	*(S: Why are those two numbers there?)* What do you mean? Just two?	Clarifying ambiguous explanations (Franke et al., 2009); repeating students' explanations/ strategies to allow students to reconsider what they have said (Chen et al., 2020)

To answer the sub-research question: *How do speech and gestures match or mismatch each other when PSTs enact the probing practice?* I investigated the relations between the communicative meanings embedded in speech and gestures. There are four patterns of speech-gesture matches and mismatches, two of which belong to speech-gesture matches and the other two to speech-gesture mismatches (see Table 3).

Table 3. Four Patterns of Speech-Gesture Matches and Mismatches

Speech-Gesture Match		Speech-Gesture Mismatch	
Pattern 1	Pattern 2[4]	Pattern 3	Pattern 4
			

Note. S stands for speech and G stands for gestures.

In Pattern 1, speech and gestures contain the same meanings (i.e., the circle of speech and the circle of gestures are identical), which align with what is probed. In Pattern 2, speech and gestures partially share common meanings while individually bringing complementary meanings (i.e., the circle of speech intersects with the circle of gestures); both common meanings and complementary meanings align with what is probed. In Pattern 3, speech and gestures express different meanings but complement each other to make a current probe possible (i.e., the circle of speech is tangent to the circle of gestures and all covered in these two circles consists of the full meaning of a current probe). In Pattern 4, speech and gestures convey completely different meanings (i.e., the circle of speech lies outside the circle of gestures), some of which do not align with what is probed. When either speech or gestures are missing and the only existing one acts as a probe, the speech-gesture intersection or adjacency does not exist. Speech-only and gesture-only probes are not counted as speech-gesture matches or mismatches.

If speech and gestures conveyed the same or overlapping meanings, I coded it as an instance of speech-gesture match. Teachers usually use many pointing gestures in instruction. Words like “this,” “that,” “here,” or “there” convey the same meanings as the exact pointing gestures that teachers want students to pay attention to. This is an example of Pattern 1. When a teacher asks students how they solve a problem but points to a blank that awaits filling up, this is an example of a partial match (Pattern 2). Consequently, students might focus more on answers instead of justifications of their thinking processes. Speech-gesture mismatches occur when speech and gestures convey different meanings but do not necessarily contradict each other. Imagine a teacher

[4] The ways how these two circles intersect vary, e.g., one circle may lay inside of the other. The key idea lies in some common intersecting area between these two circles.

stands beside or faces a student, and asks, "Can you explain to us how you got it?" Meanwhile, the teacher goes back to the smart board to write something that suddenly occurs to her. This change of the teacher's attention, which is exemplified in gestures, is a demonstration of speech-gesture mismatch.

To answer the second research question: *In what ways do the researcher-identified and PST-identified probes align?* I compared the researchers' and PSTs' probe recording sheets and identified quantitative differences in terms of the number and times of probes and qualitative differences between their identified probes. From a quantitative perspective, PSTs may identify fewer probes than researchers. They may not intend to probe; whereas, researchers with an outsider's perspective would perceive that as a probe when taking students' responses into account. For example, a PST who was occupied with thoughts of what the next probe would be stayed silent, which in fact encouraged students to say more. From the PST's perspective, they likely deemed silence as lack of probes. However, the students' act of saying more, from the researcher's viewpoint, furnished the PST's silence as a probe. On the other hand, PSTs may identify more probes than researchers. PSTs may count a question as a probe. They might use several questions to probe the same thing but align the number of probes with that of questions.

From a qualitative perspective, I elaborated on the commonality and divergences between researchers' and PSTs' identification of probes through speech and gestures. Researchers and PSTs' probes tended to align more with each other in speech than in gestures because verbal probes are familiar to the PSTs and gestures sometimes are beyond their consciousness. Further, researchers may place a higher influence of gestures on the meaning-making of probes than PSTs themselves do. For example, a PST may habitually point to numbers under discussion, which could give hint to students and help them figure out the expected responses; however, the PST may not notice their gesture use.

CHAPTER 4: RESULTS

To answer the research questions: 1) *How do PSTs use speech and gestures to enact the probing practice in K-5 mathematics classrooms?* 2) *In what ways do the researcher-identified and PST-identified probes align?* I first provide a panoramic view of PSTs' enactment of their probes from the researchers' perspective and then unveil the details about whether their gestures and speech matched or mismatched. Next, I describe PSTs' identification of probes with speech and gestures and compare when and how their identification was different from researchers' identification.

To provide a holistic and naturalistic phenomenon of PSTs' probing practices in this research, selection of representative cases was key. I compared researcher-identified and PST-identified probes according to the coding of four patterns of speech-gesture matches and mismatches in Table 3. Situations where the probes consisted of only speech or gesture were considered. As Figure 4 shows, researchers identified all PSTs' use of *speech only* and *speech-gesture match: pattern 1*. From the researchers' perspective, Clara and Marsha were the only two PSTs who used four patterns of speech-gesture matches and mismatches; Clara is the only PST who was identified using all six probing situations shown in Figure 4. In the case of PST-identified probes, Clara was the only PST who consistently identified the use of speech only, gesture only, and *speech-gesture match: pattern 1 and pattern 2*; Minnie and Marsha identified themselves using *pattern 1* and *pattern 2* whereas Megan and Holly only identified themselves using *speech only* in their probes. Layla is the only PST who did not identify herself using any probes; whereas, Minnie, Holly, and Marrisa, to some extent, share some commonality with Clara and Marsha. As shown in Figure 5, Clara showed a high diversity of speech and gestures in her probes. Marsha showed a medium diversity and Layla showed a low diversity. This selection of PSTs does not directly associate with PSTs' effectiveness in probes. The aim thereof is to ensure different ways of probe enactment are included in this study. Therefore, I selected Clara, Marsha, and Layla for representative cases in the following analyses and discussion.

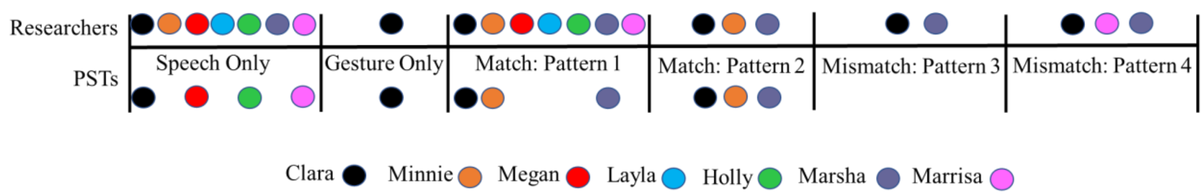


Figure 4. All PSTs: Researcher-identified and PST-identified Use of Speech and Gesture in Probes

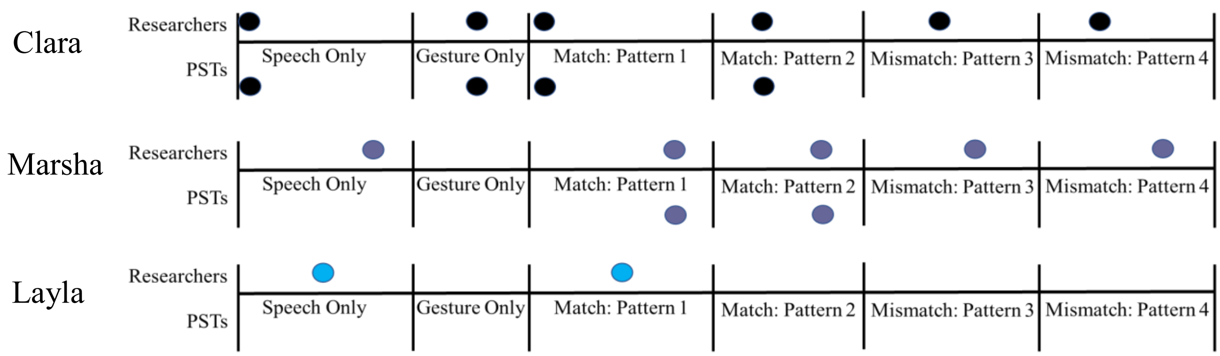


Figure 5. Three Cases: Researcher-identified and PST-identified Use of Speech and Gesture in Probes

4.1 Case One: Clara

4.1.1 Researcher-identified Probes in Clara's Class

In Clara's first-grade class, she planned to have students learn to compare (>, <, or =) two-digit numbers by using base ten blocks. Her first graders were just beginning to identify numbers represented by base ten blocks and use place value concepts in their reasoning. In the recorded lesson, Clara explained to students some rules of doing two-digit comparison, then divided students into groups, and had each group come to her table one by one (there were three other stations where students rotated around). At her table, Clara elicited students' mathematical thinking and conducted small group discussions. Finally, Clara conducted a whole-class discussion as a wrap-up.

As shown in Table 4, researchers identified that Clara probed students' conceptual understanding of place value and reasoning for their answers most often and probed students' use of strategies and clarification for their responses least, i.e., 24% of all probes together, which was smaller than either concept probes or reasoning probes. Over 50% of probes were enacted in speech and gesture. The pattern of highest frequency to lowest was similar between probes and gesture use. Thirty-four out of 49 probes (69%) were enacted with gesture. Only two out of 49 probes were enacted in gesture only.

Table 4. Researchers' Identification of Probes and Gesture in Clara's Class

Probes	Concept Probe	Strategy Probe	Reasoning Probe	Clarification Probe	Total
Overall Count	22	6*	15	6	49
Overall Percent	45%	12%	31%	12%	100%
Gesture Count	17	4	10	3	34
Gesture Percent	77%	67%	67%	50%	69%

Note. In one strategy probe, the PST's gestures were invisible and not counted in the "Gesture Count."

4.1.1.1 Concept probe

Most of Clara's probes focused on the language use (i.e., tens and ones). This was a consistent focus throughout her lesson. When probing students' conceptual understanding of place value, Clara often asked "What does this number have more of?" In a situation of students' struggle to use the tens, Clara would ask instead, "Has two more what?" by using the partial language phrase and expecting students to respond with "two more tens." Alternatively, Clara brought up the "tens" or "ones" language hints first and expected students to identify how many tens or ones by asking, "How many more tens does it have?" Clara included more hints in her concept probes if students still did not use the expected place-value language, i.e., "What does it have more of? Is it ten or one?", "You think it's just 6? Or is it sixty?", and "What place is this? The tens right?" Sometimes at the beginning of teacher-student interactions, Clara probed students' prior knowledge of the comparison signs by asking "What do you know about that?"

In this concept probe, Clara probed a student's thinking about the place value, i.e., which place is bigger between 58 and 59. She first probed, "What's bigger?" with a deictic gesture (pointed to the problem on the paper worksheet) and found out the student understood her probe as "Which is bigger, 58 or 59?" Then she revised her probe as a combination of "What does it have more of?" with the same deictic gesture. The student incorrectly answered that "It has more of ten."

Taking a step further, Clara probed by asking "What does it have more of? Is it ten or one?" with a metaphoric gesture (put right hand vertically up, then left hand vertically up, see Figure 6), thus making her probe objectively accessible to the student by restricting her probe within two choices. The metaphoric gesture with two hands vertically up abstractly represented ten and one as if the right hand stood for a side of ten and the left hand for the other side of one. Clara confined her probes by means of asking the student to choose a side to stand by; she grounded these two choices in two hands and made it accessible to the student. On the one hand, the metaphoric gesture bears a deictic aspect, referring to ten and one [gesture is imaged based (McNeill & Duncan, 2000)]; on the other hand, the metaphoric gesture represents two viewpoints, i.e., whether 59 has more of tens or more of ones than 58 (place value). Despite the student's not answering what 59 has more of, Clara herself constantly refined her speech and used increasing grounding gestures to better probe student thinking.

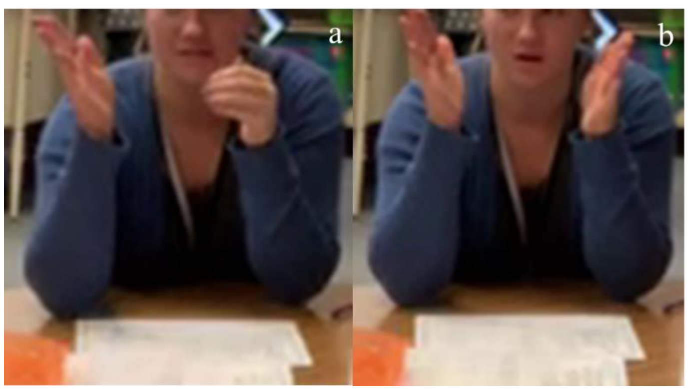


Figure 6. Concept Probe with a Metaphoric Gesture Representing Ten and One (speech-gesture match: pattern 1)

In addition, Clara's gestures could hold students responsible for continuous contributions to conversations, i.e., using environmentally couple gestures to focus on the phenomenon under discussion (Goodwin, 2013). For example, Clara probed a student about how she knew 67 is bigger than 51 and got a vague response, i.e., "This has 5, this one has 6" (see Table 5). While the student began working on another problem, Clara continued her deictic gesture and probed further the meanings of 5 and 6. The continuous deictic gesture attracted the student's attention to Clara's further probe, i.e., concept probe, which signaled that the conversation was not over yet. The student's following confusion prompted Clara to clarify her probe with another concept probe, i.e., "What are there?" accompanied with deictic gesture.

Table 5. Clara's Instance of Using an Attention-getting Gesture in Concept Probes (speech-gesture match: pattern 1)

Speech	Gesture Descriptions
Clara: *How do you know that?*	[pointed to the problem]
Student: This has 5, this one has 6.	[The student turned to another problem]
Clara: Good!	
Clara: *This 5 or 6 of what?*	[pointed to the five-ten-block number in the problem and with elbow on the table, put up left hand with palm slightly up]
Student: Which one?	[kept pointing to the number]
Clara: 5. *What are these?*	
Student: Ten six.	

4.1.1.2 Strategy probe

Clara's strategy probes targeted students' strategies in two-digit number comparison problems. Before probing students' reasoning for deciding which number was bigger, she would often ask students their ways of knowing how many the ten blocks represented with a corresponding deictic gesture, e.g., "How do you get 20?" When this strategy probe became Clara's routine in teacher-student interactions, both Clara and her students seemed to know her probing intent even without probing speech. A case in point is when Clara had continuously probed problem-solving strategies in three comparison problems, which would possibly send students a message that Clara was interested in knowing their use of strategies. At this time, a student pulled Clara's hand for attention and wanted Clara to see whether she solved the problem correctly (see Table 6). Clara did not ask any probing questions but only pointed to the ten-block visuals. Receiving Clara's probing signal, the student explained how she knew the blocks represented 41 and 73.

Table 6. Clara’s Instance of Using Only Gesture in Strategy Probes (speech-gesture mismatch)

Speech	Gesture Descriptions
	[A girl pulled Clara’s hand for attention.]
Clara: Yeah, OK.	[pointed to one ten-block visual in the problem] *(strategy probe)*
Student: 20, 30, 40, 41.	
Clara: That’s true.	[pointed to the other ten-block visual in the problem] *(strategy probe)*
Student: 10, 20, 30, 40, 50, 60, 70, 71, 72, 73.	
Clara: 73.	

4.1.1.3 Reasoning probe

Clara’s reasoning probes were used to probe students’ reasoning for their ways of knowing (e.g., “How do you know that?”) and rationale for their strategies or solutions (e.g., “Why is 91 bigger than 67?”). Her reasoning probes often went with deictic gestures to locate appropriate references in the environment (e.g., numbers or problems). Also, there were instances of holding students accountable for ongoing conversations. While asking, “How do you know that?” as a reasoning probe, Clara kept looking at the student. This continuous eyesight engaged the student’s attention to the reasoning probe. Similarly, when probing a student’s reasoning for his solution that 67 is bigger than 51, Clara put her hand under her jaw, signaling her wait time for the student’s reasoning. These verbal questions and accompanying gestures conveyed different meanings, however, they worked together as constituents of a complete probe (i.e., speech-gesture mismatch: pattern 3).

Some speech-gesture mismatches could implicitly convey the teacher’s thinking of mathematics problems and problem solving, particularly when they encountered students’ incorrect answers. In this whole-class discussion session, Clara asked students which one (20 or 12) the alligator ate. One student shouted 20 and Clara placed the alligator mouth towards 20; immediately another student answered 12, which attracted Clara’s attention for probing. As Clara said in the stimulated recall, “When somebody says an incorrect answer that is what made me probe more questions.” Clara revoiced this student’s response while slightly moving the alligator mouth between 20 and 12 without changing its direction (Figure 7a). As for the reasoning probe, Clara probed the student: “How can you think it’s gonna be the 12?” while touching those two ten sticks (i.e., 20, Figure 7b). It is only when the student explained how he got 12 that Clara pointed to the 12 blocks (Figure 7c). Herein exists a speech-gesture mismatch, i.e., Clara pointed to the comparison sign and 20 when probing 12. Grounded in such a context, the later deictic gesture (i.e., pointed to 12) acted as an interactive response to the student's explanations; whereas, the previous deictic gestures (i.e., pointed to the comparison sign and 20) supplemented Clara's probe by subtly drawing the student's attention to the previous student's response (i.e., >) and the ten blocks representing 20. Her delayed matching gesture accompanied by the student's explanation could further highlight the difference between those blocks representing 20 and 12.

Figure 7. Reasoning Probe with a Deictic Gesture (speech-gesture mismatch: pattern 4)

4.1.1.4 Clarification probe

Clara's clarification probes functioned with the purpose of prompting or pressing students to say more (Chapin et al., 2009; Ghousseini, 2015). Clara either repeated previous talk moves or directly asked students to clarify their thoughts. When using repeating, for example, Clara revoiced part of students' responses such as, "You think that one is?" or "Just two?" There were also times when Clara repeated her own questions as clarification probes to perpetuate students' efforts in working on the math problems. In one such clarification probe (see Figure 8), Clara initiated a talk turn with a student by asking "Which one is bigger?" (problem: 60 vs. 88) without using any gesture. The student responded with 100. Clara reflected in her stimulated recall, "I think to me an accomplishment was him just getting it right and actually doing the problem. So I think that's why it was different with him when I didn't say, 'How did you get it?' 'Which one is bigger?' " Then she enriched her probes by use of multiple gestures and speech. First, Clara probed by asking, "Which one, 60 or 88? Which one is bigger?" accompanied by her hands moving up and down (Figure 8a: lifting her right hand up while her left hand was ready to go up; Figure 8b: leveling up the palm of her right hand, which was ready to go down while left hand was going up; Figure 8c: both palms leveled up and were almost at the same level but in an opposite tendency of movement).

Clara moved her both hands up and down three times. She used the up-and-down gesture to denote her conception of a big number and a small number, which may originate from her learning experience of a balance scale or a vertical number line. Thus, Clara packaged the spatio-motoric information in her up-and-down gesture (Kita et al., 2017). She transferred her probe from a conceptual domain (which is bigger?) to a physical space (which is higher?), which McNeill (1992) would categorize as metaphoric gesture. Further, she did not reveal the answer by raising a hand along with the utterance of 88; instead, she signaled her questioning and offered more wait time in the motion of moving up and down three times. The series of gesture and speech well-grounded the teacher's probe tightly in the physical contexts.

Figure 8. Clarification Probe with a Metaphoric Gesture and an Iconic Gesture (speech-gesture match: pattern 2)

Since this probe was used to clarify the previous question and make Clara's probe more comprehensible, we call this a clarification probe. Clara, however, did not stop her clarification probe after using the metaphoric gesture but alternated her way of probing, "Which one is the alligator [made an alligator mouth with hands] (see Figure 2d) going to eat?" This part of her clarification probe was based on the contextual instruction that comparison signs (> & <) were like an alligator mouth and that the alligator only ate big numbers. Connecting with the alligator mouth, which the student had already been familiar with, Clara clarified her probe further with an iconic gesture (two hands forming an acute angle, a metaphoric shape of an alligator mouth).

Apart from asking the same student to clarify their mathematical thinking, Clara also asked other students to clarify their peer's thinking. When a student expressed his thoughts for using an equal sign, Clara did not really hear his explanation and turned to two other students, asking "What did you two say?" expecting them to restate what the boy said earlier.

4.1.2 Speech-Gesture Matches and Mismatches

Four different patterns of speech-gesture matches and mismatches from Table 3 were present in Clara's case. In *speech-gesture match: pattern 1*, Clara used deictic gestures to locate their speech in concrete contexts. In *speech-gesture match: pattern 2,* Clara used iconic and metaphoric gestures along with corresponding speech to probe student thinking (e.g., Figure 6 & Figure 8). When additional information was added to the gesture, Clara's probes conveyed some extra information that was not encoded in speech, for example, the metaphoric gesture in Figure 8 transmitting either the concept of a balance scale or the concept of higher and lower numbers on the number line. Interestingly, in *speech-gesture mismatch*, Clara's strategy probe only consisted of gesture while the missing speech was already understood as a routine by the teacher and students. In *speech-gesture mismatch: pattern 4*, Clara probed the student's ways of knowing 12 while pointing to a different number 20. This delayed mismatching information expressed in gesture could potentially draw the students' attention to the differences between 12 and 20.

4.1.3 Preservice Teachers' Identification of Probes

To answer the second research question: *In what ways do the researcher-identified and PST-identified probes align?* I first presented the number of probes and the number of gestures that PSTs themselves identified. Then I compared the differences between researcher-identified and PST-identified probes.

As Table 7 shows, in the stimulated recall, Clara identified that she used concept probes and reasoning probes most often, which aligned with her teaching goals about students' understanding of place value. Clara reported that all of her probes were enacted in speech, and only three (7%) were accompanied with gesture.

Table 7. PST-identification of Probes and Gesture in Clara's Class

Probes	Problem-posing Probe	Concept Probe	Strategy Probe	Reasoning Probe	Clarification Probe	Total
Overall Count	1	20	5	12	4	42
Overall Percent	2%	48%	12%	28%	10%	100%
Gesture Count	1	---	2	---	1	3
Gesture Percent	100%	---	40%	---	25%	7%

Note. Percentages may not add up to 100% due to rounding.

4.1.4 Commonality and Differences between Researcher-identified and PST-identified Probes

Unsurprisingly, the researchers identified more probes than the PST did, but both identified that concept probes and reasoning probes were used most often. Clara regarded her questioning at the problem-posing phase as probing; however, the researchers interpreted those questions as for collecting non-specific information without digging further into students' thinking, thus did not code them as a probe. Clara noticed a few gesture-related probes, which seemed not to attract her attention. The discursive rather than gestural focus may align with the instruction she received in the methods course.

When probing mathematics concepts (i.e., place value and comparison signs), Clara was consistent with her language use throughout her lesson, which contributed to her high alignment with researchers' coding of concept probes (19 vs. 22). Clara's strategy probes dug into students' strategies of knowing the cardinality of ten-block visuals. There was a high rate of consistency in the coding of speech use between researcher-identified probes and PST-identified probes (6 vs. 5). There was only one instance when Clara's strategy probe coding did not agree with researchers' coding. Clara did not identify gesture use in her strategy probes except the instance of using only gesture in a strategy probe (see Table 6). Clara's identification of the speech parts in reasoning probes overall aligned with researchers' identification (12 vs. 15) despite her not mentioning gesture use at all. With respect to the language use, Clara identified four out of six clarification probes that researchers did. Clara identified the gesture use in one probe since her other three clarification probes did not include gesture use at all.

4.1.4.1 Two consecutive probes

Clara sometimes did not notice a research-identified probe if the probe followed tightly behind another probe, regardless of whether those probed were the same students or not. Clara did not identify either the previous probe or the later probe. For example, when probing students' reasoning for why 59 is bigger than 58, Clara used a concept probe ("What's bigger?"); upon receiving an unexpected response from the student, Clara immediately reworded her concept probe as "What does it have more of?" In the stimulated recall, Clara only identified her later probe.

4.1.4.2 Probes without much students' input

Clara tended not to identify the probes followed by students' brief input or silence despite the wait time Clara sometimes used in those probes. In a case of probing students' reasoning for a comparison problem, Clara asked, "How do you know that?", which was followed by waiting gestures (i.e., steady eye contact and her jaw on the arms). The student stayed silent and began working on the worksheet; then Clara turned her head away. In the stimulated recall, Clara did not notice this case as a probe.

4.1.4.3 Probes with checklisting intent

Clara was less likely to identify her questions as probing if she was checklisting students' responses. However, some of her questions could theoretically serve as a probe. In the following scenario, Clara already asked two checklisting questions and knew that her students had noted down the numerals for the ten-block visuals. She further asked, "What do we do?" and a girl responded, "We'll put none because both of them are same." Based on previous interactions with the girl, Clara seemed to know that the girl had a good understanding of place value and therefore she might have intended to see whether the girl saw the relations between those two numbers. Clara's not identifying this research-identified probe could also be due to the influence of previous checklisting questions (e.g., "What number is this?"). The researchers coded this instance as a strategy probe because this questioning created a space for the student to explain her strategy of comparing those two numerals.

4.1.4.4 Probes with repetition of students' responses

Another obvious difference between PST-identified probes and researcher-identified probes lies in the questions that repeated students' responses. Among all the probes that Clara identified, she did not include probes with repetition of students' responses. But the researchers coded repeating questions (i.e., "Just two?" and "You think that one is?") as clarification probes that pressed students for more information by repeating. From the researchers' perspective, the repeated information prompted students to elaborate on that information picked out by the teacher. In this clarification probe (see Table 8), Clara repeated the student's response "2," which led to the student showing his ways of counting.

Table 8. Clara's Clarification Probe (speech-gesture match: pattern 1)

Speech	Gesture Descriptions
Clara: First, what number is this?	
Student: 2.	
Clara: *Just two?*	[pointed to the ten-block visual representing 22.]
Student: 20, 1, 2.	
Clara: It's 22, 22.	[pointed to the ten-block visual representing 22. Then tapped on the desk once again.]

4.1.4.5 General versus mathematical probes

The researchers sometimes did not pay attention to a PST-identified probe if the probe did not focus on mathematical concept or reasoning. However, the PST — Clara — coded as a probe the questioning of where or when students learned about mathematical symbols. For example, at the beginning of each group interaction, Clara inquired into students' experience of getting to know comparison signs (i.e., "How do you know that?") and one student responded with his childhood stories. Clara claimed this instance as probing students' prior knowledge and researchers claimed as non-mathematics specific.

Overall, Clara's enactment of probes varied through different gesture types and speech. The differences between researcher-identified and PST-identified probes were related to the PST's adjacent probes and intent, and students' responses. The following case of Marsha who used a different dominating pattern of probes would show how the same gesture was embedded in different probes.

4.2 Case Two: Marsha

4.2.1 Researcher-identified Probes in Marsha's Class

In Marsha's fourth-grade class, she used a two-digit multiplication word problem (i.e., "Sunny School has 12 classrooms with 15 students in each class. Bluewater School has a total of three times as many students. How many students does Bluewater School have?") to help students recognize that different problem-solving strategies could be used to solve the same problem. During her class, she had students work in pairs, walked around the classroom, talked with individuals to access their mathematical thinking, selected three students who used different strategies, asked them to explain their strategies in front, and finally had whole-class discussions about those different strategies. Her students used the algorithm, repeated addition, and the box method (i.e., drawing arrays) to solve the word problem.

As shown in Table 9, researchers identified that Marsha used strategy probes most often and reasoning probes least often and the other probes were used almost on an equal rate. A majority of Marsha's probes (i.e., 78%) came with gestures and speech. About one fifth of all probes were enacted in speech only.

Table 9. Researchers' Identification of Probes and Gesture in Marsha's Class

Probes	Concept Probe	Strategy Probe	Reasoning Probe	Clarification Probe	Total
Overall Count	6	12*	1	8	27
Overall Percent	22%	44%	4%	30%	100%
Gesture Count	5	9	1	6	21
Gesture Percent	83%	75%	100%	75%	78%

Note. In one strategy probe, the PST's gestures were invisible and not counted in the "Gesture Count."

4.2.1.1 Concept probe

Marsha's concept probes functioned to elicit students' conceptual understanding of the relations between mathematical ideas. Five out of six concept probes focused on the connections among mathematical strategies, and one concept probe targeted numeral relations (e.g., "What number was this representing? 10 and 2, what number?"). In Figure 9, Marsha was probing students' conceptual understanding of the links among three different strategies by asking, "What are something that we saw that were similar about their strategies?" Before asking this question, Marsha's hands stayed close to her legs. While uttering the word "something," Marsha lifted her hands up to her front (Figure 9a) and was ready to move her hands outward. When her hands moved apart, Marsha turned her left hand with an open palm in a counterclockwise direction (her right hand did not have her palm open due to a marker in her hand) (Figure 9b). In Figure 9c, Marsha's hands were far apart, and she slightly moved her hands inward during a short pause after uttering "something." These gestures were metaphoric, denoting the abstract idea of commonality of the strategies as some concrete objects that Marsha could hold.

Figure 9. Concept Probe with Metaphoric Gestures (speech-gesture match: pattern 1)

The above concept probe with metaphoric gestures reinforced Marsha's concept probe of mathematical similarities. However, Marsha sometimes used re-probes to make her probing more explicit rather than open-ended. To some extent, the re-probes enacted in such a manner likely reduced the power of her initial concept probe. When probing students' conceptual understanding of students' different strategies, Marsha asked "What did you notice about them? Did they all do the exact the same way?" with a lasting deictic gesture (i.e., pointed to the posters with a spreading palm) throughout the probe. The question, "What did you notice about them?" opened up opportunities for students to express anything that they could notice; however, Marsha immediately followed with the second question, "Did they all do the exact same way?" This follow-up restricted her concept probe to a yes-or-no answer and the power of probing students' conceptual understanding was diminished.

4.2.1.2 Strategy probe

From the researchers' perspectives, Marsha used strategy probes to delve into students' problem-solving processes and strategies. Her strategy probe could be specific about a mathematical step (e.g., "How did you get from 24 to 48?") or general about strategy use (e.g., "What was your thought process in the first one?"). All Marsha's strategy probes delved into students' existing thinking processes except two instances in which the strategy probes were used to extend students' thinking (e.g., "Do you know another way you could think about it?"). When probing students' strategy use, Marsha sometimes used deictic gestures to locate the numbers on the paper or the board and sometimes used mismatching gestures to prompt students to think about different strategies. For example, in the strategy probe shown in Table 10, Marsha began gesturing immediately when she began talking. When asking, "How could we think about this problem…," she opened her arms and palms to signal that she was embracing a different strategy (Figure 10a). To emphasize her meaning of different strategies, she lifted her palms up (see the dotted hand in Figure 10a) while uttering the words "another way." Further, Marsha cued students with a hint to use "a picture or something" else. Her arms moved even higher than the previous upward movement (see the dotted hand in Figure 10b), which to some extent attracted students' attention to the gesture and the accompanying speech: "picture or something." Marsha deemed this explicit guidance as necessary, explaining, "So I kind of like guided them to think about how they could do with pictures" (stimulated recall). When probing who might have an idea about drawing a picture, Marsha changed her gestures from open arms (Figure 10c) to closed arms (Figure 10d), signaling her desire to wait for students' contributions.

Figure 10. Strategy Probe with Metaphoric Gestures (speech-gesture mismatch: pattern 3)

Table 10. Marsha's Strategy Probe (speech-gesture mismatch: pattern 3)

Speech	**Gesture Descriptions**
Marsha: *How could we think about this problem*	[spread out arms with palms up]
in another way?	[moved palms up and down a bit as if emphasizing]
Maybe with a picture or something.	[lifted arms and palms even higher and put them down]
Does anybody have any idea we could draw a picture with this problem?	[two hands held together, and played around with the marker]

4.2.1.3 Reasoning probe

Marsha used the reasoning probe to probe students' ways of knowing. Throughout her lesson, she used only one reasoning probe (i.e., "Why did you pick up those numbers?") with deictic gestures when she asked a student for her reasoning for choosing 10, 2, and 5.

4.2.1.4 Clarification probe

Marsha, according to researchers, used clarification probes to clarify students' strategy use or mathematical thinking. Sometimes the clarification probes invited students to clarify themselves (e.g., "Added them up? OK. Can you show us?") and sometimes asked students to confirm Marsha's interpretations (e.g., "So you were saying you did 24 plus 24? OK. So that would look like this?"). Figure 11 shows a scenario where Marsha identified an error in a student's problem-solving processes and tried to "help her identify where she went wrong" (stimulated recall) by using a clarification probe. Marsha asked, "For this one, did you try to solve it by adding 48 and 48 or were you thinking by multiplying?" While saying "adding 48 and 48," she pointed to the numbers on the poster, took her right hand back, and put it under her jaw (see Figure 11a, dotted circle) as if thinking of whether she got what the student meant. When saying "multiplying," Marsha lifted her left hand up (see Figure 11b); when her left hand was up in front of her face, she moved her right hand away from her jaw and put it almost on the same height with her left hand, which was ready to move downward (see Figure 11c & 11d). Marsha's hand gestures metaphorically represented two interpretations that she made about the student's strategy use: right hand referred to "adding 48 and 48" and left hand to "multiplying." Her left hand moving up and her right hand moving down, as if on a scale with one side going up and the other side going down, implied her uncertainty about the student's strategy use. Thus, the gestures in Figure 11 grounded Marsha's two interpretations.

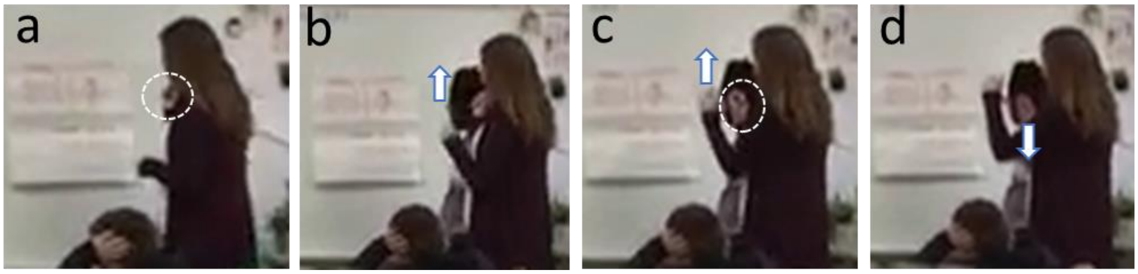


Figure 11. Clarification Probe with Metaphoric Gestures (speech-gesture match: pattern 2)

4.2.2 Speech-Gesture Matches and Mismatches

In Marsha's class, many gesture uses directly aligned with the speech, i.e., *speech-gesture match: pattern 1*. Marsha often used deictic gestures to locate numbers referred in speech, e.g., pointing to the poster when asking, "What was your thought process in the first one?" The meanings expressed in gesture sometimes made abstract concepts concrete as exemplified in Figure 9, i.e., transitioning an abstract word "something" to "something to be held" on a visuospatial platform. There were also a few scenarios when Marsha's gesture use did not fully match the meanings expressed in speech. *Speech-gesture: pattern 2* is shown in Figure 11. In this case, Marsha's right hand, representing an option for addition, and left hand, representing the other option for multiplication, indicated the similar struggle that Marsha experienced in her speech. Further, her left-hand movement, which was more visuospatially obvious than her right-hand movement, from the researchers' perspective, may suggest her personal thought about the student's use of multiplication than addition. This addition meaning contributed to the clarification probe under discussion.

Speech-gesture mismatches: pattern 3 and *pattern 4* occurred when Marsha used gestures to express different meanings from those in speech. As shown in Figure 10, Marsha verbally probed another way of solving the problem; whereas, her gestures i.e., moving hands up and down, signaled a meaning of choice or balance. Marsha used these same metaphoric gestures to probe students' noticing different strategy use, i.e., "Was your strategy different?" These same gestures worked together with different discursive moves to probe either students' strategy use or conceptual understanding of differences in strategy use. The phenomenon is called *speech-gesture* *mismatch: pattern 3,* in which speech and gesture expressed different meanings but collaborate with each other to make a complete probe. Only one *speech-gesture mismatch: pattern 4* existed in Marsha's class when Marsha used beat gesture (i.e., repetitively clicked the marker on the palm) to carry out a strategy probe, i.e., "How did you get from 24 to 48?" Beat gesture did not bring specific meanings but might represent the PST's use of wait time.

4.2.3 Preservice Teachers' Identification of Probes

Different from Clara who used concept probes and reasoning probes most often, Marsha used strategy probes and clarification probes most of the time (76%). She identified two instances of gesture use (i.e., writing gestures—writing down the student's strategy on the whiteboard) when she clarified a student's strategy. By contrast, from the PST-identified probes, the researchers identified 24 gestures; over two thirds of the probes were accompanied with gesture. Similar to the pattern of probes, according to the researchers, gesture use appeared most often in strategy probes and clarification probes.

Table 11. PST-identification of Probes and Gesture in Marsha's Class

Probes	Problem-posing Probe	Concept Probe	Strategy Probe	Reasoning Probe	Clarification Probe	Total
Overall Count	1	6	12	1	12	32
Overall Percent	3%	19%	38%	3%	38%	100%
Gesture Count	---	---	---	---	2	2
Gesture Percent	---	---	---	---	17%	6%

Note. Percentages may not add up to 100% due to rounding.

4.2.4 Commonality and Differences between Researcher-identified and PST-identified Probes

Both researchers and the PST identified the dominating use of strategy probes and clarification probes. Researchers identified fewer probes and more gestures than the PST. Researchers and the PST kept a high consistency in identifying the speech part of concept probes and strategy probes, although there were one to two cases where either researchers or the PST did not identify mathematical probes. Both parties identified the single reasoning probe. The biggest difference between researcher-identified and PST-identified probes lied in whether most probes were accompanied by gestures. The PST only noticed her gesture use when using visuals to represent a student's strategy. From the researchers' perspective, most PST-identified probes were accompanied with gestures (75%). Besides, an inconsistence appeared in terms of whether restating students' strategies were coded as clarification probes or not. The PST, not researchers, identified restatement as clarification probes. Overall, the differences between these two parties' probe identification were manifest from the following aspects.

4.2.4.1 Probes without students' input

The researchers did not identify questions that were used before students contributed to the conversations as probes while the PST did. Two instances of this type occurred when Marsha found that students had no idea how to solve the problem and suggested students use a picture without any gesture use, i.e., "Would you try a picture maybe?" and "Do you wanna try, maybe draw a picture?" Marsha explained, "I kind of just wanted him to like get us somewhere to start." The PST also tended to identify probes when she facilitated small-group discussions, e.g., "Looks like you did some adding" and "Do we get a different answer in this group?"

4.2.4.2 Probes with repetition of students' responses

Marsha, like Clara, purposefully used repetition of students' responses as probes. Although Clara's repetitions were short and accompanied with some wait time, Marsha commonly repeated all the processes in a student's strategy and provided a second chance for the whole class to listen to the strategy once again. Researchers regarded these repetitions as pedagogical summaries and not mathematical probes because no students' input followed. However, the PST identified four cases of repeating students' responses as probes. For example, in Table 12, a student came up with a strategy of adding three groups of 180, and Marsha repeated the strategy by providing a visual of three boxes. The PST thought she was probing the student's thinking by providing a visual to confirm with what the student said.

Table 12. Marsha's Clarification Probes (speech-gesture match: pattern 1)

Speech	Gesture Descriptions
Student: We could do the problem like we did. Instead of using the box method again, we could do group or something. We could do one group of- we could do three groups of 180 and then we count them all up.	
Marsha: Yeah we can add them up.	[spread hands out and put them together]
I think that's an awesome idea.	
What she is saying is we have 180. Here is a group of 180, right? We know we have three times as many. That's one, two, three. So 180,	[drew three boxes in a row to show "three times as many" and put "180" in each box]
180, 180, and her next step would be to add instead of multiply, right?	[wrote three 180s vertically on the board]
She is saying 180, 180, 180. Add those up.	[wrote down the numbers in the algorithm step by step]

Another instance of repeating a student's strategy was when the PST intended to help the student identify her error in calculation by repeating the student's and her peer's strategies, i.e., "You were thinking 94. You and Joshua over here both did 48 times 2, 48 times 2. So your method was the same right? Just ended up with a different number."

4.2.4.3 General versus mathematical probes

An obvious difference between researchers and the PST is that the PST identified some problem-posing questions as an instance of probing student thinking. In this scenario, Marsha noticed that a student was stuck in the problem and was trying to get him to start. Marsha asked by imbedding the answer in the question, "From four exhibits, how many exhibits should I try?" Seeing the student staying silent and still struggling to answer her question, Marsh thought the student was confused about the word "exhibit" and then asked, "How many cages? Yeah, exhibits. How many did we see?" which led to the student's saying four. Marsha coded this instance as a probe; whereas, the researchers did not for the reason that the PST used leading questions to restate the information in the problem instead of following up the student thinking.

These two cases above have shown that apart from deictic gestures, three other gestures could work together with speech to (successfully or not) probe students' mathematical thinking. Despite PSTs' less likeliness to identify their gestures, they were able to identify a majority of their probing speech, which researchers also identified. The following case presents a contrast between PST-identified and researcher-identied probes, offering a different perspective of PSTs' probe enactment.

4.3 Case Three: Layla

4.3.1 Researcher-identified Probes in Layla's Class

In Layla's second-grade class, she discussed the expanded form of two-digit numbers and focused on students' conceptual understanding of place value. Her second graders had worked on the expanded forms of two-digit numbers and had yet to learn the connections between the expanded form and place value. During the launching of the lesson, Layla talked about the standard forms and expanded forms of 54 and 42 with the whole class. Next, she gave each students a bag of 50 beans, asked them to show the expanded form of 21 with 10 beans in a group, and had a whole-class discussion about students' strategies of finding the expanded form with the beans. At the end of class, Layla asked students to work on 16 with the bean manipulative and show 16's expanded form with 10 beans in a bundle.

As Table 13 shows, researchers identified that Layla probed students' conceptual understanding of place value most often, twice as much as the total frequency of the other three types of probes (34%). Among all probes, gesture use occurred mainly with concept probes and strategy probes. Compared with Clara and Marsha, Layla used fewer than 50% gestures among all probes.

Table 13. Researchers' Identification of Probes and Gesture in Layla's Class

Probes	Concept Probe	Strategy Probe	Reasoning Probe	Clarification Probe	Total
Overall Count	10	3	1	1	15
Overall Percent	66%	20%	7%	7%	100%
Gesture Count	5	2	0	0	7
Gesture Percent	50%	67%	0%	0%	47%

4.3.1.1 Concept probe

Layla used concept probes to elicit students' concept understanding of the place value in the two-digit numbers. Her probes were of high consistence and took three forms in speech: "How many tens or ones?", "How many bundles of 10?", or "What's in your tens place?" Half of the concept probes were accompanied with deictic gestures when Layla located the number with her hands (*speech-gesture: pattern 1*). When asking "Do you know how many tens you have here?" as a concept probe, Layla pointed to the number 60 on the number card with "60 + 3" on it. Except for the direct location gesture, Layla sometimes used gesture as a hint to attract students' attention to specific numbers. In Table 14, the student knew "2" is in the tens place of 21, and Layla probed further to see whether the student could make connections between the "2" and "bundles of 10" asking, "How many bundles of 10 are you gonna make? without any gesture. The student returned a wrong answer, so Layla reiterated there is one in the ones place by underlining the "1" (see Figure 12a) and re-probed the student using a small change of vocabulary, "How many *groups* of 10 are we gonna make?" [underlining the "2" in 21] (see Figure 12b). The gesture of underlining the numeral "2" may be purposefully made by Layla, hinting the student that there are two groups of 10 in 21. Despite the student's imperception of the hint, Layla used a deictic gesture to provide the student with the expected response.

Table 14. Layla's Concept Probes (speech-gesture match: pattern 1)

Speech	Gesture Descriptions
Layla: What's in the tens place?	
Kathy, what's in the tens place?	[fingers on lips, as if thinking]
Student: Two.	
Layla: *How many bundles of 10 are you gonna make?*	
Student: Three.	
Layla: If we're making tens, we have one one.	[turned back to the board and underlined the 1 in 21.]
How many groups of 10 are we gonna make?	[underlined 2 in 21.]
Student: (silent.)	

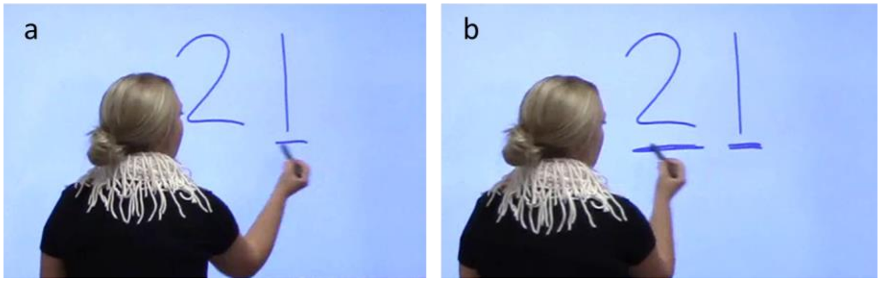


Figure 12. Concept Probes with Deictic Gestures (speech-gesture match: pattern 1)

4.3.1.2 Strategy probe

Layla used strategy probes to understand her students' ways of grouping the manipulatives into tens and ones. After Layla walked around the classroom, talking with students in a one-to-one mode and collecting information about individual student's strategy use, she selected students using different strategies to share with the whole class. She called upon a student and probed with only speech the student's way of grouping 21 beans, "Can you tell us how you split up your bundles? What did you put your bundles into?" The student responded with "circle". Layla, already knowing what the student did, finished the rest of the strategy explanations instead of probing further. In a following case, Layla probed the student's strategy use by pointing to the student, "Who else did a different strategy? Brian, I know you did something else. What did you do?" The student responded, "I did it in lines." and Layla finished the explanations by herself again.

4.3.1.3 Reasoning probe and clarification probe

Throughout this lesson, Layla use one reasoning probe and one clarification probe. When probing a student's reasoning about her previous response, Layla probed with speech only, "The five, how do you know that?" When eliciting a student's clarification, Layla probed with speech only, "Is ten here?"

4.3.2 Speech-Gesture Matches and Mismatches

In Layla's class, researchers found that her probes only manifested *speech-gesture match: pattern 1* and no speech-gesture mismatches were identified. When gestures accompanied speech in the probes, Layla's gestures were mainly deictic gestures to locate the numbers involved and the students involved in her probes. Another frequent phenomenon was Layla probing with speech only (53% as showed in Table 13).

4.3.3 Preservice Teachers’ Identification of Probes

Layla identified herself using zero probes in her lesson because she thought herself using checklisting and leading questions a lot, not targeting at students’ conceptual understanding. She reflected,

> I feel like I prompted them maybe too much because they just do what I told him and then I like asked him how many was in the bundle, they kind of knew already, like 10. So I feel like a lot of my questions maybe aren't technically probing because they, they're really just like a checking for the right answer, but not like, maybe how they got to it (Layla’s stimulated recall).

She identified her use of “The five. How do you know that?” as an instance of probe at the beginning of the lesson. Considering the student’s response “Because it’s bigger than the four,” Layla took her words back and did not think she was probing the student’s conceptual understanding. Asking that question, she had anticipated the student’s responses such as “five is 50” or “five is fifty ones” (cited from the simulated recall). After re-watching her own lesson, Layla began to consider what kind of probing questions she would ask if she would teach the lesson again. She said that she would provide students more time to figure out their own strategies instead of her offering much guidance and would ask such probing questions as “How do you know there was three bundles of ten?” and “How did you know to group them into bundles of 10? Why not 12?” This shows that Layla was considering using more reasoning probes to elicit students’ thinking.

4.3.4 Commonality and Differences between Researcher-identified and PST-identified Probes

Obvious differences exist between researcher-identified and PST-identified probes. Researchers identified situations when Layla was aware that her students could identify the tens place and the ones place but could not connect with the concepts that one in the tens place refers to one ten or ten ones, she probed students’ conceptual understanding of the place value by consistently asking “How many tens or tens…?” Researchers disregarded students’ responses when coding probes whereas Layla considered students' responses as a criterion for her using probes or not.

CHAPTER 5: DISCUSSION AND IMPLICATIONS

5.1 PSTs' Enactment of Probes with Speech and Gesture: A Researchers' Perspective

5.1.1 Multimodal Links in Probe Enactment

The results across the cases reveal a continuum in terms of the variety of their probe use in speech and gesture. From the researchers' perspective, Clara's class showed the most diverse probe use, including probes with speech only, gesture only, and four speech-gesture patterns. Marsha's class manifested medium to high diversity in her probe use, including probes with speech only and four speech-gesture patterns. Layla's class showed a low diversity in her probe use, including probes with speech only and one speech-gesture pattern.

The PSTs enacted more than 50% of their probes with gestures, dominantly deictic gestures, to index the on-going topics in specific contexts (Goodwin, 2013). The finding that the PSTs probed students' thinking multimodally (i.e., speech, gestures, visuals) in instruction is consistent with Alibali et al. (2014) and Flevares and Perry (2001), in which they investigated elementary and middle-school in-service teachers' gesture multimodality. My dissertation study expands current literature to focus on PSTs' teaching practices and especially on their probing enactment.

Concerning the probes implemented with speech and gesture, the PSTs used multimodal links to enact their probes, which is consistent with previous studies (e.g., Alibali et al., 2014), but only decreased their frequency of gesture use when they continued probing the same student. Alibali et al. (2014) reported that teachers made more multimodal links (i.e., connecting different representations such as written, graphic, and discursive forms) in instruction when they introduced mathematical information for the first time than when they reviewed previous mathematical information. In my dissertation, the trend is different: the PSTs were found using multimodal links frequently, regardless of whether the target information was mentioned or not. Gesture use in probe enactment depended more on whether the PSTs probed the same student or a different student. For example, in Clara's concept probes (e.g., what does it have more of?), strategy probes (e.g., how do you get it?) and reasoning probes (e.g., how do you know that?), Clara made multimodal links when she interacted with a different student or talked about a different problem with the same student. Marsha also located her probes with multimodal materials (e.g., board writing, posters) and kept referring to mathematics representations in her probes when interacting with different students. Layla was the one who used multimodal links least among all three PSTs. When probing students' thinking of how many ones and tens there were in her exemplar task, she frequently used deictic gestures to implement her probes. Clara and Marsha sporadically used speech-only probes during continuous interaction with the same individuals, which were times when fewer multimodal links were used.

Regarding probing enactment among three PSTs, some factors may contribute to their dominant use of certain probes compared to other probes. Alibali and her colleagues studied the same teacher's varied use of gesture in classroom sessions. Their findings partially hold in my dissertation when different teachers' gesture uses were examined across different mathematics topics. Clara and Layla aimed to reinforce students' understanding of the place value concept, which their students were still struggling to learn; a prevailing use of concept probes was evident in their teaching. Marsha built on students' prior knowledge of multiplication problem solving and focused on students' conceptual understanding of the connections among different strategies; a majority of her probes were strategy probes. From the researchers' perspective, Clara's and Marsha's gesture use in their concept probes is consistent with prior studies (Alibali et al., 2013a; Alibali & Nathan, 2007). Alibali and Nathan (2007) and Alibali et al. (2013a) found that mathematics teachers increased their frequency of gesture use in instruction when students struggled in comprehension. Similarly, in the domain of probing practices, Clara and Marsha maintained a higher frequency of gesture use in their concept probes (often trouble spots where students needed more scaffolding) in contrast to other probes. For example, in Table 14, Layla made a speech-only probe, "How many bundles of 10 are you gonna make?" Receiving an unexpected answer from the student, Layla made another probe (i.e., How many groups of 10 are we gonna make?) with multimodal links to the board, the number 21, the ones place "1," and the tens place "2." The underlying gesture gave the student a cue of the expected answer. In Layla's class, however, only 50% of concept probes (i.e., lowest frequency among three PSTs) were accompanied with gestures. In the case of Layla, her overall lower gesture use (i.e., 47%) may result from some external (e.g., comfort with the teaching environment) or internal factors (e.g., her probing style), an area that future research could investigate.

In a word, the PSTs in my study tended to use multimodal links when they probed individual student, regardless of whether they already used the same probes with other students or with the same students earlier. Different from the prevailing claim in existing literature that in-service teachers used fewer multimodal links in discussion of already mentioned materials, this finding extends the existing literature by supplementing PSTs' use of multimodal links in their probing practices. The following topic—re-probing—is another contribution to the current literature.

5.1.2 Re-probing

The PSTs re-probed students' thinking (i.e., using multiple discursive and gestural representations) in one single talk turn when encountering students' struggles in understanding. If the PSTs used gestures in their initial probe, they were likely to strengthen the probing power by using different gestures that brought additional information in their re-probes. For example, Clara used metaphoric and iconic gestures when re-probing the student's clarification for which number was bigger (see Figure 8). Marsha did her re-probe by suggesting the use of "a picture or something" else in Figure 9. If the PSTs did not make noticeable changes in speech or gestures in their re-probes, they were likely to keep or reduce the probing power. For example, Layla did her re-probe about a student's way of splitting the bundles, albeit seemingly more accessible to the student, reducing the power of her initial probe about the student's reasoning (see section 4.3.1.2). The temporal gaps between the PSTs' initial probe and the re-probe were usually very short, sometimes as an immediate follow-up, when students stayed silent or were hesitant in their responses (e.g., uhm…). Overall, the re-probes with speech or gestures were more explicit and accessible than the initial probes.

On the speech level, the PSTs' probes became more explicit in their re-probes, shifting away from the implicit questions in their initial probes. When probing a student's way of knowing that two pictures of hundred blocks represented the same number, Clara made a speech-only probe, "What do you think about that?" to elicit the student's reasoning for the equality. Noticing the student did not pay attention to her initial probe, Clara re-probed, "How do you know they were equal?" [spread out two palms, almost on the same level]. Compared to the initial implicit and open probe that could encourage the student to say anything, this re-probe made it explicit to the student that her reasoning for equality was questioned, i.e., an explicit open question according to Parks (2009). Thus, Clara's re-probe enacted with an iconic gesture and explicit open question strengthened her probing power. Similarly, Marsha started her probe with an implicit open question, "How could we think about this problem in another way?" with metaphoric gestures (see

Table 10 and Figure 10). Immediately following up, she re-probed with an explicit open question, “Does anybody have any idea we could draw a picture with this problem?” which offered students with a clear direction to work toward but did not restrict the re-probe to a fact-eliciting or yes-no question. This speech changes in re-probes further confirmed with Parks' (2009, 2010) findings that explicit open questions could still promote students’ reasoning and clarification and meanwhile provide clarity and openness for students to know how to participate in classroom discussions. In addition, there were situations when speech-only re-probes did not provide additional explicitness and flexibility for students’ engagement in the conversations, thus restraining the power of their initial probes. Layla knew a student’s strategy when walking around the classroom during the partner work, and later called her up to explain her strategy to the whole class. Layla probed with speech only, “Can you tell us how you split up your bundles? *[initial probe]* What did you put your bundles into? *[re-probe]*” Both questions were explicit open questions, inviting the student to talk about her way of splitting 21 beans into bundles of tens. However, the immediate re-probe was narrowly interpreted by the student who provided a one-word response, “Circle.” It could also result from the misleading what-question without any gesture that restrained the probing power.

On the gesture level, the PSTs’ re-probes either added more information to or repeated the sense-making in the speech. As shown in Figures 6 and 8, Clara’s metaphoric gestures enriched her re-probes with visuospatial information (i.e., two concrete sides, a vertical number line or a scale) (Kita et al., 2017). Deictic gestures were often used to synchronize the meanings in speech. For example, Clara circled two students’ answers in her initial probe and re-probe.

5.1.3 Speech-Gesture Matches and Mismatches

Among these three cases, at least half of probes were enacted with gestures, which provided a space for discussing gesture-speech matches and mismatches. There was no correlation between gestures, probes, and speech-gesture matches and mismatches. The PSTs used deictic, metaphoric, and iconic gestures in all types of probes, regardless of whether the speech and gesture matched or not. Only Marsha used beat gestures, which did not carry semantic meanings and therefore did not match the meanings in speech.

5.1.3.1 Speech-gesture matches

Speech-gesture: pattern 1 often came with deictic gestures when the PSTs connected with the concrete contexts by pointing to the written mathematics symbols or learning manipulatives. Similar to the frequent use of deictic gestures in instructional explanations (Alibali et al., 2013b), this study provided evidence that deictic gestures were also used on a frequent basis in teachers' questioning. *The speech-gesture: pattern 1* could entail different information in instruction, therefore possibly producing different effects on students' learning. First, using a matching deictic gesture (e.g., pointing to the writing on the board or the mathematics problems on the paper) in probes, the PSTs could hold students' attention to the topics in discussion (see Table 5), which was consistent with Farsani et al. (2020). Second, using matching metaphoric and iconic gestures in probes, the PSTs could shift abstract mathematical concepts into visuospatial information that provided easy access to students (see Figures 5 & 8). Third, using matching deictic gestures, the PSTs could provide additional helpful hints for students' understanding, although this could be context dependent. For instance, in Figure 12, Layla built on the student's knowledge that "2" was in the tens place and probed how many groups of 10 there were in 21 while underlining the numeral "2" in 21. This deictic gesture could attract the student's attention to the underlined numeral and explicitly pass the message that the answer was the numeral "2" as well.

Similar to Alibali et al.'s (2011) and Goldin-Meadow et al.'s (2009) findings that adults and elementary students benefited from their gesture use in meaning making, the gestures used in *speech-gesture: pattern 1* in this study helped the PSTs enact effective and clear probes, although these probes elicited students' correct or incorrect responses. Future studies could investigate to what extent students notice and respond to the meanings embedded in speech and gestures in teachers' probes and whether there are differences in terms of speech-type meanings and gesture-type meanings.

Gestures could also be evidence for the thinking processes; in other words, knowledge expressions are embodied in gestures (Gibbs, 2005; Hostetter & Alibali, 2008). In *speech-gesture match: pattern 2,* gestures contained additional information that supplemented the information provided in the speech, thus making a complete probe (see Table 3). Singer and Goldin-Meadow (2005) reported that third and fourth graders benefited more from their teachers talking about one strategy in speech and a different strategy in gestures than from their teachers talking about two strategies in speech with mismatching gestures. Consistent with and meanwhile broadening their finding, this study found that the PSTs' expressions of gestures were not necessarily different from those in speech; instead, extended the meanings in speech (e.g., expanding a concept to a physical space in Figure 8). This gestural embodiment was beneficial to students' understanding in classroom interaction, which was evidenced by the student's success in finding the bigger number in section 4.1.1.4.

The PSTs used gestures frequently when speaking and teaching in classrooms (McNeill, 1992). Gestures sometimes were used for a communicative purpose in instruction (Goldin-Meadow, 1999) and sometimes for personal expressions of thoughts (Kita et al., 2017; Kita & Davies, 2009), similar to students' using gestures in explanations (Goldin-Meadow & Wagner, 2005). In some situations of *speech-gesture match: pattern 2*, the PSTs' gestures might not be explicitly visible to students and acted as a modality of cognition by accompany speech (Edwards, 2009). As shown in Figure 11, Marsha embodied in metaphoric gestures two interpretations of the student's strategy use. The student stood facing the white board and Marsha stood behind her. Marsha's gestures obviously served herself in communication of her probe, not for the student to perceive her probe through gestures . Besides, Marsha moved her left hand (corresponding with "multiplying" in speech) higher than her right hand (corresponding with "adding 48 and 48" in speech). This could be a hint that Marsha might be inclined toward the multiplication strategy. However, to avoid overinterpretation of PSTs' gesture use, future research could focus on PSTs' explanations of their gesture use and study whether they intended their gesture use for certain instructional or non-instructional purposes. Alibali et al. (2011) reported that gestures affected college students' choices of problem-solving strategies when their hands were restrained versus free to move around. Considering the roles that gestures played in the PSTs' probe use, future studies could investigate whether the PSTs or in-service teachers will use different probes when they are restrained versus free to use their hands and other body parts.

5.1.3.2 Speech-gesture mismatch

In this study, I also find cases where the PSTs used gestures to describe the content visually or metaphorically in speech, e.g., in *speech-gesture: pattern 3*. The iconic and metaphoric gestures could be simulation of action (Streeck, 2002), e.g., drawing a triangle in the air based on the action of drawing one on paper, and grasping a concept based on the action of holding an object. Lakoff and Johnson (1980) claimed that metaphoric gestures reflect the conceptual system and come from people's familiar ways of expressions. For example, when Marsha probed a different strategy that students could use, her metaphoric gestures, especially her hands moving up and down, seemed to originate from her way of weighing two objects (see Figure 10 & Table 10). This connection with the PST's lived experience were also evident when Marsha used the same metaphoric gestures to probe a student's thinking of two different strategies

Interestingly, among the PSTs existed a speech-gesture mismatch in which Clara only used deictic gestures to probe a student's strategy use (see Table 6). Deictic gestures are often used to index concrete and present objects. Although Clara indexed her pointing gestures to the comparison problems on the worksheet, she was probing something abstract, i.e., the problem-solving strategy. According to McNeill (1992) and Alibali and Nathan (2012), deictic gestures could be referred to nonpresent objects as well, e.g., referring to a visual by pointing to the place where it was drawn on the board earlier and to a person by pointing to a seat where they had sit before. Therefore, deictic gestures connect present objects with the nonpresent but associated objects. In Clara's case, her deictic gestures were associated with her usual way of probing students' strategies by pointing to the problems on the worksheet. When both the teacher and students establish a common communicative background, gestures only can be effective probes.

Speech-gesture mismatches could potentially reveal the teacher's attitude towards the topics in discussion or indicate information related to the teaching content (Alibali & Nathan, 2012; Goldin-Meadow et al., 1999). In Figure 7, Clara used mismatching deictic gestures to refer to the 20 number blocks and the comparison sign while probing the student's reasoning for 12 being bigger than 20. Through the mismatching gestures, Clara might convey a message that 12 was not the bigger number. Goldin-Meadow et al. (1999) reported that elementary and secondary teachers tended to use gestures with speech rather than use speech only when addressing incorrect information in students' strategies. In my study, speech-gesture mismatches pertaining to probes were used at a very low frequency. All the PSTs were likely to address students' incorrect strategies in instructional statements rather than in probes. Future research could investigate whether teachers use speech-gesture mismatches frequently when teaching mathematically struggling students (e.g., English language learners or students with learning disabilities), and further explore how teachers' mismatching gestures reveal correct mathematics-relevant information.

5.2 Comparison of Researchers' and Preservice Teachers' Perspectives

5.2.1 Preservice Teacher-identified Probes

PST-identified probes were characteristic of mainly using speech. This is consistent with a continuous emphasis on verbal questioning in teacher preparation program, i.e., PSTs are educated to use various questions to elicit students' thinking, based on which they will further promote students' thinking and adjust their instruction to students' learning needs (e.g., Chapin et al., 2009; M. Franke et al., 2011; Ghousseini, 2015; Moyer & Milewicz, 2002). For example, in order to attract whole-class attention to a strategy, the PSTs in my study usually revoiced the strategy or compared with a different strategy. The verbal revoicing (Chapin et al., 2019) and comparisons (Ghousseini, 2015) are discursive moves to probe students' thinking.

What range of probes did the PSTs identify in their teaching? Clara and Marsha identified various instances of themselves probing students' conceptual understanding, strategy use, reasoning, and further clarifications. This aligns with previous studies about using talk moves to probe students' conceptual understanding, problem-solving reasoning, and clarifications or justifications of their reasoning (Carpenter et al., 1999; Chen et al., 2020; Kawanaka & Stigler, 1999; Sahin & Kulm, 2008). Both Clara and Marsha identified one problem-posing probe. Clara identified herself probing students' learning (or lived) experience of when or where they learned about the comparison symbols. Admittedly, Clara's problem-posing probe could be a probe in a broad sense; however, this study examined the PSTs' probes about mathematical meanings and this instance was not coded as a probe from the researchers' perspective. Regarding Marsha's problem-posing probe, she reported that by using some task-related leading questions (e.g., How many exhibits should I try?) she intended to help the student start working on the task somewhere because the student did not show any work on her paper yet. Knowing her student struggled to solve the problem, Marsha seemed not to elicit the student's struggles but went forward to probe the student's understanding of the basic task-related information, i.e., the vocabulary involved (see section 4.2.4.3), an area where Marsha thought the student were probably struggling with (cited from the stimulated recall). Because Marsha did not probe the student's self-identified struggle, a potential challenge for Marsha could be which piece of the student's mathematical thinking to build on and probe further so that she scaffolded the student in learning. Marsha's challenge—how to effectively elicit and build on students' mathematical thinking— has been documented by many preservice and in-service teachers (Ball, 2001; Leinhardt & Steele, 2005).

The quality and quantity of student responses that are elicited through teachers' probing seems to be a touchstone of the probing effectiveness. The most noticeable finding was that Layla did not identify herself using any probes in her lesson. Layla considered students' responses as a determining factor for whether she used probes or not (see section 4.3.3), which aligns with Teuscher et al.'s (2016) view that student responses were critical in the probing practices. Despite her acknowledgement, at the beginning of the stimulated recall, that her probes would target students' problem-solving procedurally and conceptually, she reported that she was not probing the students' thinking when asking checklisting questions to get students talking about their procedures (cited from the stimulated recall). This, to some extent, reflected Layla's consistent understanding of what a probe meant to her. Layla was aware that in the probing practice, she was expected to elicit students' procedural and conceptual understanding, and in the teaching practice, she could recognize that she did not ask probing questions to achieve that eliciting purpose. This may imply that there is a gap between attending to the probing practice and applying the probing practice to real teaching. Existing research on video learning (Sherin et al., 2011; van Es & Sherin, 2010; Walkoe et al., 2019) pays much attention to the attending part. Future research could further investigate the transitions from the attending to enactment in actual teaching (e.g., How much knowledge could experience successful or failing transfer? What factors may affect the transfer?).

5.2.2 Researcher-identified versus Preservice Teacher-identified Probes

PSTs were inclined to notice their discursive use of probes if their teaching contained much variety in probes. The teaching with medium to high diversity in probes (i.e., Clara and Marsha) manifested few differences in the number of probes; nevertheless, significant differences were visible in the number of gestures. These two PSTs identified approximately 47% to 72% fewer gestures in comparison to researchers. In the low probe diversity class, the differences between the PST (i.e., Layla) and researchers were most obvious both on the speech and the gesture levels. This might indicate that the PST was less likely to notice her probes with speech and gesture when she did not use many in teaching. It is possible that Clara and Marsha could unconsciously use gestures in their probes. When zeroing in on their own teaching videos, they were more likely to notice their gesture use than those who did not use many probing gestures. Therefore, if preservice teacher educators aim to teach PSTs how to notice and apply various probing gestures in teaching, they might find that the PSTs who habitually use probing gestures in teaching could be more ready to benefit from the gesture training than the PSTs who do not.

Besides, this study found differences of probe enactment on the speech level, not to mention the gestures thereof. The researchers identified more probes with speech and gestures than the PSTs did when conceptual understanding, strategies, and reasoning were probed. As for clarification probes, interestingly Marsha identified more probes than researchers because she reported her writing and drawing gestures when revoicing her students' strategies. For example, when showing three groups of 180, she drew three circles representing the groups. In the stimulated recall, she reported that she probed the students' thinking by drawing visuals. This aligned with the use of revoicing talk move, not a probing move, to elicit students' thinking (Chapin et al., 2009). Meanwhile, Marsha's successful noticing of her hand movements was encouraging.

Some common themes arose to explain the differences between researcher-identified and PST-identified probes, which indicates the diverse criteria that PSTs used to assess whether they were probing or not. The most common theme is probes with little to no students' input. Using little to no students' follow-up input as a criterion, Clara and Layla did not identify their relevant speech and gestures as probes; whereas, Marsha's intent for probing students' thinking, even before students shared anything, acted as a criterion for Marsha to decide that she was probing. A second theme is oriented with repetitions of students' responses. Clara's repetitions were usually short and aimed to clarify the information that students provided in their explanations, but she did not identify them as probes; Marsha's repetitions resembled a summary of students' explanations in a clear and organized manner, and she did identified them as probes. It is possible that whether the PSTs classified the repetitions as probes depended on their own understanding of students' thinking. Their teacher knowledge of students' thinking may have reinforced their certainty with the probes. In cases where they struggled with students' thinking as shown through their repetitions of and multiple interpretations of students' responses, they tended not to identify those interactions as probes.

Probing is tightly associated with eliciting students' thinking. Teachers elicit students' thinking first and then probe students' thinking to further deepen their thinking (e.g., Sahin & Kulm, 2008). In this study, the PSTs seemed to treat the elicitation as probes and focused less on the necessity of building their probes on specific students' thinking; instead, they used checklisting questions and general questions to check whether students had their expected answers. This could possibly be explained by the lack of distinction between elicitation and probes in preservice teacher education. Consequentially, the PSTs were inclined to believe that they were probing when they got pieces of (mathematical or not) information from students. This calls for a need in preservice teacher education that PSTs should know how to distinguish probing students' deep thinking from eliciting students' explanations.

5.3 Cross-case Synthesis

This study sought to examine ways of how PSTs' gestures and speech, when probing, match and mismatch and how PSTs interpret probes differently from researchers. These three cases demonstrate the popular use of deictic gestures by grounding their verbal information in concrete objects or places and by transferring concrete concepts to spatio-motoric information.

Why are gestures repetitively used in probes with individual students whilst PSTs are unlikely to notice their probing gestures? Existing literature has reported a tendency of teachers' high-frequency gesture use in trouble spots in instruction and low-frequency gesture use in presenting already-mentioned information (e.g., Alibali et al., 2013a). Instructional discourse is usually led by teachers who can organize their thoughts continuously without interruption or much feedback from students. However, probing discourse emphasizes teacher-student interaction with the teacher demanding responses from students. Therefore, mutual understanding is very important in probes. This difference could partially explain why the PSTs repeated similar gestures whenever they used the probes with a particular student. Using gestural and discursive modalities, the PSTs likely made their probes as comprehensible as possible to students. Even though researchers could study PSTs' gestures with scrutiny, PSTs' identification their probing gestures was noticeably rare. The functions of gestures in their probing practices may not have attracted the PSTs' attention and in fact, gestures were not discussed in their mathematics methods course. Without further attention to their gestures, PSTs may not realize when their gestures are supporting students' learning or potentially causing confusion or distraction. Situations indeed occur when gestures help teachers get their probes across in interactions with students. Some metaphoric gestures entailing familiar concepts such as scales could help students understand associated concepts such as the numerosity of numbers; some deictic gestures entailing extra hints could help students focus on what is expected. These gestures seem to be common in mathematics teaching. Also, some gestures bring up idiosyncratic styles of one's body movements, for example, in the case of waving hands up and down to show emphasis. Teachers might use different idiosyncratic gestures to express similar meanings. When teachers keep using their idiosyncratic gestures regularly, the gestures could become part of their classroom norms.

Why is students' thinking so influential on PSTs' identification of probes? This study found that the PSTs tended not to identify their talk moves as probes when encountering little input from students. In other words, the quality of student input encouraged teachers' probing follow-ups in case of ambiguous or confusing student responses, or discouraged teachers' probing follow-ups in case of little student input. Teachers must make decisions in the moment to respond to students' mathematical thinking; upon receiving more information from students, teachers make further decisions about whether to continue probing (e.g., Sahin & Kulm, 2008). Interestingly, because of little student input, the PSTs did not notice their efforts in probing students' thinking. PSTs' lack of attention to these broader probing efforts could prevent them from delving into how they could use alternative probes to access student thinking. Explicit conversations about PSTs' probes may influence how they interpret them and eventually how they use them in teaching. Further investigations should illustrate how PSTs' probes with gestures change when they are brought to their attention.

5.4 Limitations

This study provides some interesting insights into the PSTs' gesture use in probes and extends the existing literature on multimodality research about instructional explanations by adding the multimodal perspective to instructional probes. However, this study has some limitations. First, the number of PSTs in the detailed investigation (i.e., three) is small. Albeit a frequent use of *speech-gesture match: pattern 1*, a limited number of the other three patterns were identified. It would be insightful to use larger data sets to depict a clear picture of how PSTs synchronize (or not) their gestures with their speech in probing discourse. Second, the positioning of PSTs' bodies and broad surrounding contexts, i.e., one layer of Goodwin's theory, were not equally discussed as three other layers (language, gestures, and phenomenon under discussion) were. This study focused on PSTs' speech, gestures, and the phenomenon under discussion and only a few PSTs' spatial positions in the proximal surroundings (e.g., Figure 11) were studied. The fact that Clara sat down with students most of the class time and Marsha and Layla walked around may or may not have influence their gestural frequency and variety, which awaits further investigation. It would be interesting to examine how PSTs coordinate their positioning with their hand gestures and how body positioning contributes to meaning making in teaching practices. Third, the quality of gesture images still has space for improvement. To minimize the influence on the PSTs' teaching, the videorecorders were in a back corner of the classroom, and we used a stationary camera to capture the whole-class images. Sometimes the camera was blocked by PSTs' or students' movements. This resulted in failing to capture some details of the PSTs' gestures. For example, deictic gestures included pointing with hands or markers; the place that the PSTs pointed to on the students' worksheets may also reveal additional information about their attitudes about mathematics problems. Finally, even though I tried to stay closer to the communicative meanings embedded in the PSTs' gestures, I was aware of the danger that I might overinterpret the PSTs' gesture use from a researcher's perspective. For future research, a follow-up interview with the PSTs (after the stimulated recall) could be conducted to elicit PSTs' interpretations of their gesture use that was brought up by the researchers. Thus, the triangulation of PSTs' probe enactment with gestures and speech could be evidenced in researchers' interpretations, and PSTs' self-identified, and follow-up prompted interpretations.

5.5 Implications and Future Work

For potential implications for future work on probing gestures, I raise the following question that is worthy of further attention: *How do researchers and teacher educators distinguish PSTs' intentional gestures from idiosyncratic gestures?* In this study, researchers were inclined to notice the attributions of gestures in PSTs' probes whilst PSTs rarely identified their intention of using gestures in communication. As Melinger and Levelt (2004) reported, it is difficult to decide whether gestures, especially iconic gestures, are communicatively intended. PSTs could unconsciously and habitually use some gestures with concurrent speech; upon directed reflection, they might notice why they used that gesture, especially when students suddenly understood the teacher's probes as in Clara's case of iconic gestures in Figure 8. Some probing gestures (e.g., Marsha) entail strong idiosyncratic styles. It would be insightful to help PSTs become aware of whether their gestures are communicatively helpful in interactions with students. Some PSTs used a wide variety of gestures while others used the same gesture over and over. Variety might help a wider range of learners while consistent gestures might promote understanding. Further, whether a variety of gestures or consistent gestures are more helpful might depend on the context. Future work needs to tease apart these situations in concert with the teachers and students to better understand their impact. Besides, it would be interesting to examine to what extent students receive the intended information that PSTs embedded in their probes. Since the effects of probing gestures on students' learning were not a focus in this study, future work should incorporate students' attention to teachers' probing gestures (e.g., characteristics of gestures, connections with known concepts) that assist them in understanding and learning mathematics. Such work as connected with students' learning can contribute to the establishment of gestural teaching practices in preservice teacher education.

5.6 Conclusions

Overall, this work demonstrated that from the researchers' perspective, the PSTs used various gestures to probe students' thinking with some gestures being very helpful encouraging students to contribute more, targeted information. There are still gaps between the PSTs' attention to what probes should be like and their application of probes in the teaching practice. A knowledge of one's own multimodal functions in meaning-making is likely to be applicable for students' meaning-making through their gestures. With an increasing attention on gesture use in instruction, it is worthwhile investigating the effects of teachers' pedagogical gestures on students' learning, the reception of teachers' pedagogical gestures by different cultural groups, and teachers' beliefs about their own gesture use. All these efforts will contribute to our understanding of the role of gestures in teaching and learning.

APPENDIX A. STIMULATED RECALL PROTOCOL

Thanks for participating in my dissertation study. The purpose of my study is to learn how preservice teachers probe or follow up students' mathematical explanations through talk and body movements.

Part I Before watching the video

Before we watch the video, I'd like to get your ideas on how you probe or follow-up students' mathematical explanations in general.

1. What does probing or following up students' mathematical explanations involve?
2. Why did you choose this problem (refer to their lesson plan) to pose in your lesson? (not for those who gave me their lesson plans)
3. How do you probe or follow up students' mathematical explanations after getting their initial answers? Give me an example when you probe or follow up students' mathematical explanations.
4. (alternative) What strategies do you usually use to probe or follow up students' mathematical explanations?
5. Why do you probe or follow up students' mathematical explanations like that? What else might you do as probes or follow-ups and why?
6. What might influence how and when you probe or follow-up on students' mathematical explanations?
    a. How might your probes or follow-ups differ in situations where students give clear responses versus ambiguous responses
    b. How might your probes or follow-ups differ in situations where students give correct versus incorrect responses?
7. How do you know it is time for you to stop a probe or follow-up?

Part II Watching the video

I will play your discussion lesson video. I'd like you to tell me to pause the video every time you probe or follow up students' mathematical explanations. Then at each pause, I will ask you some clarification questions.

At each pause, I ask the following prompts *(caution: no initiation of gesture if PSTs do not mention it)*:

1. Tell me about why you chose to probe or follow-up here.
2. What information were you hoping to get from students in this probe or follow-up?
3. How did you probe or follow up students' explanations.?
4. (based on PSTs' responses) How does that help you probe or follow up students' explanations?
5. Were you satisfied with the student's explanations? Why or why not?
   a. If not – Did you probe or follow-up their thinking further? Why or why not?
   b. If you had the chance, would you change the way you probed followed-up here?

For the PSTs who mention gesture in their probes or follow-ups,

1. Replay two different instantiations in the videotaped lessons where PSTs use probes differently (maybe one with gesture and the other with talk and gesture OR both with gesture but with different gesture).
2. Ask
   a. How do you think your gestures and speech helped students understand that you wanted them to tell you more?

For the PSTs who do not mention gesture at all,

1. Replay two different probes (that the PSTs have identified) in which the PSTs use gestures in one probe and do not in the other probe.
2. Ask
   a. Do you identify any differences in these probes or follow-ups? If yes, in what ways are they different?
   b. What else are you wondering about these two probes or follow-ups?
3. If the PSTs still do not identify gestures, I will take up the initiation by asking
   a. Do you think your gestures help you probe or follow up students' explanations.?
   b. If you do think so, how?

Part III After watching the video

Now that you've had a chance to reflect on your lesson, I'm wondering if you'd like to add anything to your original answers about following-up student thinking.

1. What does probing or following up students' mathematical explanations involve?
2. Why did you choose this problem (refer to their lesson plan) to pose in your lesson? (not for those who gave me their lesson plans)

3. How do you probe or follow up students' mathematical explanations after getting their initial answers? Give me an example when you probe or follow up students' mathematical explanations.
4. (alternative) What strategies do you usually use to probe or follow up students' mathematical explanations?
5. Why do you probe or follow up students' mathematical explanations like that? What else might you do as probes or follow-ups and why?
6. What might influence how and when you probe or follow-up on students' mathematical explanations?
    a. How might your probes or follow-ups differ in situations where students give clear responses versus ambiguous responses
    b. How might your probes or follow-ups differ in situations where students give correct versus incorrect responses?
7. How do you know it is time for you to stop a probe or follow-up?

APPENDIX B. PST-IDENTIFIED PROBE RECORDING SHEET

Please identify moments when you used speech and gestures to PROBE/follow-up student responses. Check the boxes when you used speech or gestures in your probes. Feel free to add more columns. An example is provided in the table.

Timestamp of Probe (minutes & seconds)	Talk	Gesture/Body Movement
5:27 – 6:03	☑	☑
	☐	☐
	☐	☐
	☐	☐
	☐	☐

APPENDIX C. RESEARCHER-IDENTIFIED PROBE RECORDING SHEET

Timestamp of Probe (minutes & seconds)	Speech	Gestures	What did PSTs probe for?	How did the PST coordinate speech and gestures in the probe?

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