THE EFFECT OF SCAFFOLDED MODEL-BASED INSTRUCTION ON WORD PROBLEM-SOLVING PERFORMANCE OF ENGLISH LEARNERS WITH LEARNING DIFFICULTIES IN MATHEMATICS

by

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Dedicated to my beloved grandparents and parents

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ABSTRACT

The purpose of the present study is to examine the effectiveness of the Conceptual Model-based Problem-Solving intervention program (COMPS) along with instructional scaffolds to enhance the performance of English learners (ELs) with learning difficulties in mathematics (LDM) to solve word problems. The participants were three third-grade ELs with LDM. The participants received instruction on scaffolded COMPS strategies using culturally responsive scaffold, linguistic scaffold, visual scaffold, and interactive scaffold. Using a multiple-probe design, the study evaluated the effect of the scaffolded COMPS program on participating students' performance when solving a range of additive word problems. Through analyses of the students' problem-solving processes, teachers developed a specific profile of students' strengths and weaknesses in solving mathematics word problems and, thereby, provided appropriate and timely instructional scaffoldings for ELs with LDM. It was found that all three participants improved their performance on the researcher-developed criterion tests as well as the generalization test following the intervention. Further, this study explored the relationship between students' performances of word problem solving accuracy and process, which has the potential to pinpoint the difficulties ELs with LDM experience and, therefore, guide practitioners in teaching ELs with LDM.

CHAPTER 1. INTRODUCTION

Mathematics literacy is a necessary skill required of students in order to reason about concepts, solve problems, analyze information, and communicate both in the mathematics classroom and in their daily lives (National Mathematics Advisory Panel [NMAP], 2008). Mathematics performance has a significant influence on one's success in adulthood. The ability to understand and use mathematical concepts significantly influences students' future academic success and even their opportunities and options for future careers (National Council of Teachers of Mathematics [NCTM], 2000; Shapka, Domene, & Keating, 2006). More importantly, with the increasing involvement of technologies in the 21st century, math-related skills are in high demand (Mazzocco & Thompson, 2005).

According to the National Council of Teachers of Mathematics (NCTM, 2000), problem solving is the foundation of school mathematics. In particular, word problem solving is an area that is itself considered a challenging task because it requires an integrated set of skills, including the understanding of different scenarios, the translation of the given scenario into math equations, and the mathematics computations themselves (Shin & Bryant, 2017; Tolar et al., 2012). In particular, students who have difficulty solving word problems can be identified through an analysis of several individual skills, including reading the problems, understanding the meaning of the sentences, understanding what the problem is asking, and implementing multiple solution steps in word problems (Bryant et al., 2019).

English Learners (ELs) with disabilities represent 13.8% of that 4.6 million, constituting a group of around 635,000 students. Although the fields of English learning, or bilingual education, and special education have definitions of EL and learning disabilities (LD), this dual classification of an EL with LD poses a unique challenge to teachers because scant attention has been paid to how these two identifiers intertwine in the context of providing instruction in the classroom. According to the section on English language acquisition in Title III of the Elementary and Secondary Education Act (ESEA) reauthorized as the Every Student Succeeds Act (ESSA) in 2015, U.S. schools are accountable for the improvement of all children, including those with "disability, recently arrived ELs, and long-term ELs" (Non-Regulatory Guidance, 2016, p. 4). As such, students with limited English proficiency, or English Learners, as referred to in ESSA, must meet benchmark achievement goals and make adequate annual growth in English language proficiency and mathematics, not just English language arts.

1.1 Learning Disabilities or Difficulties in Mathematics (LDM)

According to the Individuals with Disabilities Education Act of 2004, learning disabilities can manifest in mathematical problem solving or mathematics calculations (Bryant, Bryant, & Smith, 2014). Students who have difficulty solving word problems can be observed from an analysis of several skills, including reading the problems, understanding the meaning of the sentences, understanding what the problem is asking, and solving multiple steps in word problems (Bryant, Bryant, & Smith, 2014). Mathematics disability (MD), which is also referred to as dyscalculia, is a specific type of learning disability related to a neurologically-based disorder in mathematics ability (Wadlington & Wadlington, 2008).

While the definitions of MD are meant to classify students with MD, there are various types of characteristics that students with MD generally possess. To name a few, students with MD perform worse on verbal short-term memory (Geary, Hamson, & Hoard, 2000), on phonological memory (Cirino, Carlson, Francis, & Fletcher, 2004; Hecht, Torgesen, Wagner, & Rashotte, 2001), and on math-fact retrieval skills (Geary & Hoard, 2001; Jordan et al., 2003).

Although it doesn't take long to get a sense of what MD means, a formal definition of MD can be surprisingly hard, and there is currently no consensus on its definition (Mazzocco & Thompson, 2005). Historically, LD has been defined based on IQ-achievement discrepancy (Hallahan, Pullen, & Ward, 2013). Alternatively, low math achievement scores are used as an indicator of MD, although different cutoff points are used in different kinds of literature, which may sometimes lead to different characteristics of MD (Fletcher et al., 1989; Mazzocco, 2007; Swanson, Moran, Lussier, & Fung, 2013). Educators and researchers have used the term "at risk for MD" to identify students who may be at risk for academic failure and may benefit from intervention, even though they may not have been identified as having learning disabilities in mathematics (Kong, 2017). Students whose math performance was ranked at or below the 35th percentile cutoff score on standardized measures have been identified as being at risk for learning disabilities in mathematics (LDM) (Bryant et al., 2011; Swanson, Lussier, & Orosco, 2013; Xin et al., 2017).

1.2 ELs with LDM

According to Geary (2011), around 5% to 10% of students were identified as having a mathematics disability, a group that includes many ELs. Unfortunately, ELs appear to score significantly lower in mathematics and reading in comparison to non-ELs. According to the National Assessment of Educational Progress (NAEP, 2019), the average scaled score of ELs in math in 2019 was 24 points lower than non-ELs on the fourth-grade mathematics assessment and 42 points lower on the eighth-grade mathematics assessment. Similarly, ELs scored 33 points lower than non-ELs on the fourth-grade reading assessment and 45 points lower on the eighth-grade reading assessment (NAEP, 2019).

The EL and special education fields have addressed the possible over-classification of ELs as special education due to ignoring the effects of second language acquisition on ELs' academic progress (Association for Supervision and Development & Centers for Disease Control and Prevention, 2014; Kangas, 2017) or the absence of available EL instructional services driving referrals (Kangas, 2014). Alternatively, underrepresentation of ELs in special education has also been studied, where initial identification is avoided in favor of permitting time for English mastery to take hold (Sullivan, 2011). Yet, little work to date has identified what types of instruction are furnished to dually classified ELs with LDM.

Students who are dually classified as ELs in special education fall at the crossroads of English language learning and a specific LD, making instructional service provisions challenging and often unequal, with special education provisions often taking precedence with limited consideration of students' language proficiency in English and other home or heritage languages (Collier, 2011; Kangas, 2014, 2019).

As a result, the individual EL student's distinct English proficiency level and specific special education identification do not smoothly guide what instructional practices are best suited for learning content such as mathematics and its related language or discourse. Despite federal and state requirements to meet their academic needs, around 86% of ELs do not demonstrate proficiency in mathematics (National Center for Education Statistics [NCES], 2013).

1.3 Common Practice in Teaching Word Problem Solving

An existing teaching practice that is common in many general and special education classrooms across the United States is using keywords (e.g., *more*, *less*, *together*) to refer to specific operations (Powell & Fuchs, 2019). However, as the tasks become more difficult and have more variations, the "keyword" strategy can sometimes result in incorrect answers, so it does not work for many inconsistent language problems (Xin et al., 2020; Xin, Liu, & Zheng, 2011). For example, the teacher provides a problem to students: "*Bruce had several cookies. Then Tom gave* *him 30 more cookies. Now Bruce has 68 cookies. How many cookies did Bruce have in the beginning*?" Based on the misleading keyword strategy, the keyword in this problem is *more*, and the associated operation is *addition*; however, the correct answer should be 68 minus 30, which would use subtraction as the correct operation. Therefore, the teacher should provide efficient problem-solving strategies that can be applied to solve a range of mathematical structures and more complex syntax of problems when implementing instruction and intervention.

1.4 Intervention Research in Word Problem Solving

Several reviews and meta-analyses focused on mathematical content instruction, such as word problem solving (Lei et al., 2020a; Xin & Jitendra, 1999; Zhang & Xin, 2012; Zheng, Flynn, & Swanson, 2013). Results from early meta-analysis studies (Xin & Jitendra, 1999; Zhang & Xin, 2012) indicated that teaching representation techniques (e.g., diagramming, manipulatives, linguistic training, and schema-based instruction) showed the strongest effect in improving word problem solving of students with MD. The meta-analysis conducted by Zheng et al. (2013) examined studies that focused on word problem solving and identified the sample characteristics and instructional components that influence the effects of interventions for improving the mathematics word problem-solving performance of students with MD. Results from this meta-analysis indicated that some components, such as stating instructional objectives, explaining underlying concepts, and strategy cues, provide benefits for students with MD.

Gersten et al. (2009) conducted a meta-analysis study to investigate effective instructional practices and activities (e.g., think-aloud, using real-world examples, or peer-assisted learning) to improve mathematics performance for students with LD. The results showed that heuristic intervention and explicit instruction produced large effect sizes and significant improvements in mathematics achievement for students with LD.

Cummins et al. (1988) showed that children often make mistakes in mathematical word problems due to ambiguous language in the problem statement or miscomprehension of the verbal instructions. A longitudinal study conducted by Fuchs et al. (2015) assessed the linguistic and mathematical abilities of 206 second-grade students at the beginning and end of the year. Study findings indicated that text comprehension ability is a predictor of word problem-solving ability and, thereby, concluding that word problem solving is moderated by the ability to comprehend word problem-specific language (Fuchs, Fuchs, Compton, Hamlett, & Wang, 2015). This finding was further strengthened by a more recent longitudinal study that assessed the relationships among word problem solving, language processing, and calculation abilities with 325 second graders (Fuchs, Gilbert, Fuchs, Seethaler, & Martin, 2018). In addition, Fuchs et al. (2018) also found that text comprehension was a more significant predictor of word problem-solving outcomes than calculations (i.e., addition and subtraction). Taken together, these findings support the notion that reading comprehension is critical to mathematical word problem solving. Students who struggle with language processing and reading comprehension may have more complex needs when learning to solve mathematical word problems.

1.5 Intervention Research involving ELs with LDM

ELs experience a complex process when dealing with challenging content in an academic setting that requires academic proficiency in languages (Gerena & Keiler, 2012; Kangas, 2018; Lei et al., 2018, 2020b). Although ELs may appear to be fluent in English, they may still be struggling with complex academic material or vocabulary that differs from the social language used in daily life (Gerena & Keiler, 2012; Morita-Mullaney & Stallings, 2018).

Moschkovich (2015) argues in her Academic Literacy in Mathematics framework that there are both literacy and language components to learning math, along with some particular syntax and vocabulary that should be used in mathematical instructions and lessons for ELs. Since reading fluency can predict student performance in solving mathematical word problems (Kyttälä & Björn, 2014), word problems are generally more difficult for ELs, since they may lack the necessary reading fluency. In addition, good literacy skills, including reading, reading comprehension, and technical reading skills, play a significant role in students' ability to solve mathematics word problems efficiently (Kyttälä & Björn, 2014). Existing research indicates that it is necessary to provide linguistically appropriate support in addition to content-specific interventions to improve the academic performance of ELs with LD (August et al., 2012; de Araujo, Roberts, Willey, & Zahner, 2018; Lei et al., 2018, 2020b). In addition, previous literature has suggested that teachers should use scaffolds during instruction to create the conditions for ELs to comprehend input of both content knowledge and language (Gibbons, 2014; Gottlieb, 2016).

Scholars in the fields of English learning and bilingual education have recommended the use of instructional scaffolds to help convey meaning to students at varying levels of English proficiency. Scaffolds may be visual/graphic, linguistic, interactive, and kinesthetic (Gibbons, 2014; Gottlieb, 2016), and they can be used by students and teachers before, during, and after instruction to support content and content-specific language learning. Thus, scaffolds are important considerations in the planning of mathematics instruction for ELs with LDM (McGhee, 2011).

1.6 Conceptual Understanding and Procedural Fluency of ELs

According to Swafford and Findell (2001), conceptual understanding requires an ability to represent mathematical situations in different ways and knowing the different purposes for using those different representations. Basically, conceptual understanding is related to students' "comprehension of mathematical concepts, operations, and relations" (Swafford & Findell, 2001, p. 116). Procedure fluency, however, refers to "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (Swafford & Findell, 2001, p. 121). According to Rittle-Johnson and Schneider (2015), a procedure includes a series of steps or actions that are required to achieve a goal. Some examples of procedures include "(1) algorithms - a predetermined sequence of actions that will lead to the correct answer when executed correctly, or (2) possible actions that must be sequenced appropriately to solve a given problem (e.g., equation-solving steps)" (Swafford & Findell, 2001, p. 121). Although those two concepts focus on different skills, conceptual understanding and procedural fluency are always interwoven. According to Rittle-Johnson (2017), students can obtain procedure knowledge through learning mathematical concepts and vice versa. For those students who learned procedures without understanding, helping them to understand the reasons underlying the procedure becomes more difficult (Swafford & Findell, 2001).

Previous studies have identified that procedural fluency and conceptual knowledge have a positive correlation in a wide range of mathematics domains such as counting (Dowker, 2008), addition and subtraction (Canobi & Bethune, 2008), and equation solving (Durkin, Rittle-Johnson, & Star, 2001; Rittle-Johnson & Schneider, 2015). In addition to supporting students with conceptual understanding, it is also important to support procedural fluency on problem solving.

Bridging Errors. Previous studies have indicated that misconceptions about the meaning of subtraction, compensation, and place value principles may result in bridging errors (Vermeulen et al., 2020). One of the most commonly observed bridging errors is called the "smaller-from-larger" error (Brown & Vanlehn, 1980; National Research Council, 2002). For example, when solving 43 - 18 =, students who make the smaller-from-larger error usually subtract the smaller number from the larger one digit by digit, ignoring the place value with ones or tens in the minuend and the subtrahend, which means that they subtract the '3' digit of the minuend from the '8' of the subtrahend (i.e., 40-10 = 30; 8-3 = 5; 30+5 = 35). Such errors are consistently found across the U.S. educational contexts when solving multi-digit subraction problems (Narciss & Huth, 2006; Vermeulen et al., 2020).

As suggested by Moschkovich (2013), providing mathematics instruction to English learners should align with the Common Core State Standards (CCSS) in terms of balancing and connecting both conceptual and procedural knowledge. This study aims to address both skills through analyzing problem-solving processes during the conceptual model-based problem-solving intervention.

1.7 Conceptual Model-Based Problem Solving

Conceptual model-based problem solving (COMPS) (Xin, 2012) is an evidence-based intervention program for students with LDM (Xin et al., 2011, 2017). The COMPS approach (Xin, 2012) emphasizes students' understanding of mathematical relations in word problems by requiring students to represent such relations in mathematical model equations for solutions. COMPS makes the reasoning behind mathematics explicit to the students so that they are able to make sense of what they are doing with mathematical models and abstract symbols. With the COMPS approach, students are not guessing whether to add or subtract to solve the problem; rather, they represent variously situated word problems in one cohesive mathematical model equation (for additive word problems, for instance) to determine the solution (Xin, 2012; Xin et al., 2020).

Existing literature on problem-solving shows that fourth- and fifth-grade ELs use visual representations such as diagrams and symbols to help them express mathematical ideas when struggling to communicate their mathematical thought process (Turner, Dominguez, Maldonado, & Empson, 2006). Research has shown that COMPS (Xin, 2012) can be adapted to support ELs with LDM (Xin et al., 2020). In addition to providing linguistic scaffolding through *word problem* *story grammar* (Xin et al, 2008, 2020) to ELs on mathematical word-problem instruction, the mathematical model-based visual instruction of the COMPS program focuses more on nonlinguistic or nontextual modes of representation. According to existing literature (Barton & Neville-Barton, 2003a, 2003b), undergrad ELs benefited more from nontextual models than textual models of representation to make sense of mathematical problem solving when they had difficulty understanding the English text. Given that there is no existing research that applies both evidence-based instructional scaffolding from EL literature and mathematical model-based problem solving from mathematical problem-solving intervention research involving LDM, there is a need to explore the effects of a scaffolded COMPS intervention on the word problem-solving performance of ELs with LDM.

1.8 Purpose of the Study

The purpose of this study is to analyze the data from a single subject design study conducted as part of the National Science Foundation-funded *Conceptual Model-based Problem Solving: A Response to Intervention Program in Mathematics Problem Solving* project* (COMPS-RtI) (Xin, Kastberg, & Chen, 2015) during the 2017 to 2018 school year. The single-subject design study was conducted to address the need of those students who did not respond well to a computer program. Specifically, the single-subject study was carried out by a human teacher (i.e., the researcher) to evaluate the effectiveness of scaffolded COMPS on mathematics word problem-solving performance involving three ELs with LDM to solve addition and subtraction word problems.

This study aims to answer the following questions:

1. Was there a functional relation between the scaffolded COMPS intervention and the performance of ELs with LDM on solving addition and subtraction mathematics word problems?
2. Did the scaffolded COMPS intervention improve the mathematical word problem-solving performance on the post-test and generalization tests?
3. How did students' problem-solving processes related to word problem solving accuracy change before and after receiving scaffolded COMPS intervention?

CHAPTER 2. LITERATURE REVIEW

2.1 Mathematical Word Problem Solving

Problem solving is a process that includes comprehension of word problem structures and translation of these structures into mathematics equations to solve the problems (Bryant et al., 2019). Problem solving, which is an essential component of school mathematics programs (NCTM, 2000), involves complex processes that require multiple types of knowledge and skills. Specifically, Mayer (1998) indicated that, in order to solve word problems, students need to have five types of knowledge: (a) linguistic, which refers to understand English language and syntax, (b) semantic, which represents understanding the meanings of words, (c) schematic, which denotes the specific knowledge of word problem types and disregarding of irrelevant information, (d) strategic, which signifies the ability to plan and monitor solution strategies, and (e) procedural, which indicates the ability to perform a sequence of operations (Bryant et al., 2019). According to Szabo et al. (2020), problem solving is not a linear process, which requires abilities to "think forward and backward, between and across, to simplify and generalize" (p. 20). In addition, good problem solvers are goal-oriented and can adapt problem-solving skills quickly to every situation (Szabo et al., 2020), accurately identify the mathematical structure, and retain memory of the problem's structure for a long time (Krutetskii, 1976).

Good or successful problem solvers use a variety of cognitive strategies to solve word problems, such as rereading, drawing pictures, identifying important information, and ignoring irrelevant information (Montague, Enders, & Dietz, 2011). These cognitive strategies help students focus their attention on the linguistic and semantic information of the word problem structure. Therefore, using these strategies will increase students' ability to understand the meaning of a word problem and, thereby, their ability to solve the problem (Swanson & Jerman, 2006).

2.2 Instructional Scaffolding

2.2.1 Vygotsky's Zone of Proximal Development

Vygotsky formulated a theory of cognitive development to emphasize a child's ability when they are learning socially relevant tools (e.g., hands, computers) and culturally relevant signs (e.g., language, number systems) (Doolittle, 1995). Central to this cognitive development is his theoretical construct of the zone of proximal development (ZPD). Vygotsky (1978) proposed the ZPD thus:

> It is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers. (p. 86)

Vygotsky's (1978) theory of social constructivism emphasized the importance of students' interaction with more capable and knowledgeable peers or adults when provided with assistance through ZPD. Instructional scaffolding has its roots in Vygotsky's ZPD, where students achieve the goals by external factors such as instruction by a teacher or advanced peers to help them make connections between concepts.

2.2.2 Bruner's Instructional Scaffolding

Inspired by Vygotsky's social constructivism, Bruner et al. (1960) developed the vital concept of instructional scaffolding, which theorized that students could learn more than what people traditionally expected if they received appropriate support and suggested that "any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (p. 33). This support was meant to amplify students' current abilities and help fill in gaps in their knowledge such that students could then complete the task independently (Belland, 2017). Instructional scaffolding differs from other instructional support such as accommodation (e.g., extended time, use of calculator). According to Wood, Bruner, and Ross (1976), scaffolding needs to provide support built upon students' current performance, but also enable them to perform the target skills independently in the future.

2.3 Instructional Scaffoldings for ELs

Scaffolds have two key features that educators might use during their instructions to students. The first key feature of scaffolds is "strategic scaffolding," necessary for the teacher or adult to teach a strategy as an intermediate step that children can use to solve a problem (Beed, Hawkins, & Roller, 1991). The second key feature of scaffolds focuses more on a dynamic teaching situation (Yelland & Masters, 2005). Teachers provide appropriate scaffolds as a temporary tool for students during lessons as needed, and these are reduced and fade away "as the learner understands the concepts presented" (Collins, Brown, & Newman, 1989). Such instruction can also be measured by dynamic assessment (DA) in terms of students' different learning behaviors as the instruction changes from unassisted to fully assisted performance as students' experience increasingly challenging tasks (Orosco, 2014a; Vygotsky, 1978).

Since the term "scaffolding" was introduced, it has been used in remedial and special education for students who are provided temporary supports by teachers when they are learning new knowledge and skills. According to Dixon, Carnine, and Kameenui (1993), the effective scaffolds must be gradually dismantled in order to keep them effective. Fisher and Frey (2010) suggested that scaffolds should not be dismantled too quickly, otherwise "learning does not occur and the learner becomes frustrated in the process" (p. 2). Therefore, teacher determination to introduce and dismantle scaffolds is vital.

In the teaching-learning framework involving EL students, scaffolding is an essential support to "enable children [ELs] to perform tasks independently that previously they could perform only with the assistance or guidance of the teacher" (Gibbons, 2002, p. vii). Gibbons (2002, 2014) suggested that scaffolding can also be used for English language teaching to ELs within general education classrooms, where they spend the majority of their school day. The use of scaffolds has not been widely studied within special education, and this is also a relatively new approach within English learning (Gibbons, 2002, 2014).

Gottlieb (2016) described four types of instructional scaffolds that teachers can use and students engage in to foster understanding of target content and related language: visual, linguistic, interactive, and kinesthetic (Gottlieb, 2013).

Visual scaffolding. Visual scaffolding involves the use of drawings or photographs to connect English words, phrases, and sentences to visual images, and assists ELs in learning the target content (Gottlieb, 2013). This approach makes complex ideas feel more accessible to students and makes the language more memorable, while providing comprehensible input of the target content (McCloskey, 2005). A variety of visual supports can be used to build students' visual experience in the classroom, including manipulatives, real-life objects, and multimedia material (Carrasquillo & Rodrigues, 2002; Gottlieb, 2012).

Linguistic scaffolding. Linguistic scaffolding provides effective and responsive support for students' language output performance, which requires teachers to use language that is comprehensible to students when providing them with new and more sophisticated knowledge, including the use of a slower rate of speech, simplified vocabulary, or cycling speech with consistent reinforcement of a target set of words (Bradley & Reinking, 2011; Gibbons, 2003).

Interactive scaffolding. Interactive scaffolding involves a strategic back-and-forth between teachers and students or among students to facilitate comprehension of content and related language use. Goffman (1967) proposed the idea of "interactionism," which relates "only to those aspects of ‘context’ that are directly observable and to such immediate links between individuals as their ‘roles,’ ‘obligations,’ ‘face-to-face encounters’” (pp. 31-49). An example of instructional support for interaction involves both students and teachers taking on active roles in pair work and small-group work (Gibbons, 2008).

Kinesthetic scaffolding. Asher (1969) first introduced a strategy called Total Physical Response, which directly relates to kinesthetic scaffolding. This approach requires students to listen to a language command that may or may not be stated in their heritage language, and follow it using a physical action immediately with no expectation of speech production (Asher, 1969). This process lowers their anxiety, allowing them to produce content knowledge nonverbally, but with a related object or physical movement. Brand, Favazza, and Dalton (2012) suggested that students who use kinesthetic scaffolding benefit from “sign language, translation into another language, gestures” during sessions (p. 139), while not being restricted from participating in classroom activities due to their lower levels of English proficiency.

In order to understand how ELs construct knowledge, we need to consider their culturally diverse backgrounds. This idea is grounded in constructivist views of learning (National Research Council, 2000). The main idea behind this concept is to encourage teachers who are working with ELs to take advantage of students’ prior knowledge to help them comprehend new concepts and experiences in school settings (Villegas & Lucas, 2007). In other words, EL teachers provide supports or scaffolds for ELs to transfer what they already know (current knowledge) to what they need to learn about it (new knowledge) (Villegas & Lucas, 2007). The ELs bring their own experiences and knowledge schema from their diverse cultures and families, while the teachers deliver examples that use “a known strategy and information that was taught previously to come up with the correct answer” (Carnine et al., 2010, p. 28). Therefore, students’ retention and comprehension skills can be improved by putting the students into “linguistically familiar or culturally relevant problem situations” as they are taught mathematical problem solving (Kim et al., p. 258).

Culturally Responsive Scaffolding. One scaffolding intervention that may be used for ELs with LDM is culturally responsive scaffolding (CRS), which considers students’ diversity and culturally relevant topics in the content instruction (Lei & Xin, 2019; Lei et al., 2020a). The Madison Metropolitan School District (2015) demonstrated how culturally and linguistically responsive strategies for diverse students might look in a school: a) Plan for daily expectations and supports with grade levels and culturally and linguistically responsive text; b) Communicate classroom rules and routines clearly and fairly to all students and acknowledge all students; c) Communicate openly about their personal life experiences and family background and make links between content and students’ experiences; d) Implement cultural and linguistic behaviors into instructional activities, etc.

2.4 Classroom Discourse and EL Learning

Vygotsky’s theories (1962, 1978) emphasize the fundamental role of social interaction in the process of cognitive development and point out the powerful role of language or linguistic interaction in children’s cognitive development, where more opportunities are established for them to learn from the teacher and more skilled peers. Discourse as scaffold is a part of the delivery of content and language in classroom teaching. How teachers use their language to teach and check for understanding should be decided before and during classroom instruction. Moreover, what is perhaps more intriguing and important is “the quantity and quality of challenge and support that we provide, and the way these two dimensions interact with each other” (Mariani, 1997, p. 4). Mariani (1997) provided a teaching-learning diagram (Figure 1) that was adapted by Gibbons

(2015), which related scaffolding to the degree of intellectual challenge that students face in a task as well as the supports that are provided by teachers. The vertical axis represents the extent of "challenge" that the students are encountering in the classroom, and the horizontal axis represents the level of "support" that teachers provide in the instruction (Gibbons, 2015). There are four kinds of classroom conditions in the four quadrants, including (1) high challenge, low support; (2) high challenge, high support; (3) low challenge, high support; and (4) low challenge, low support. According to Thomas and Collier's research (1999), if the teacher's expectations of their students were high, ELs' achievement was also high. Other researchers (Gibbons, 2008; Walqui, 2007) have also found that in a high-challenge and high-support curriculum, all students' achievements will be at higher levels regardless of background and equity gaps will be diminished.

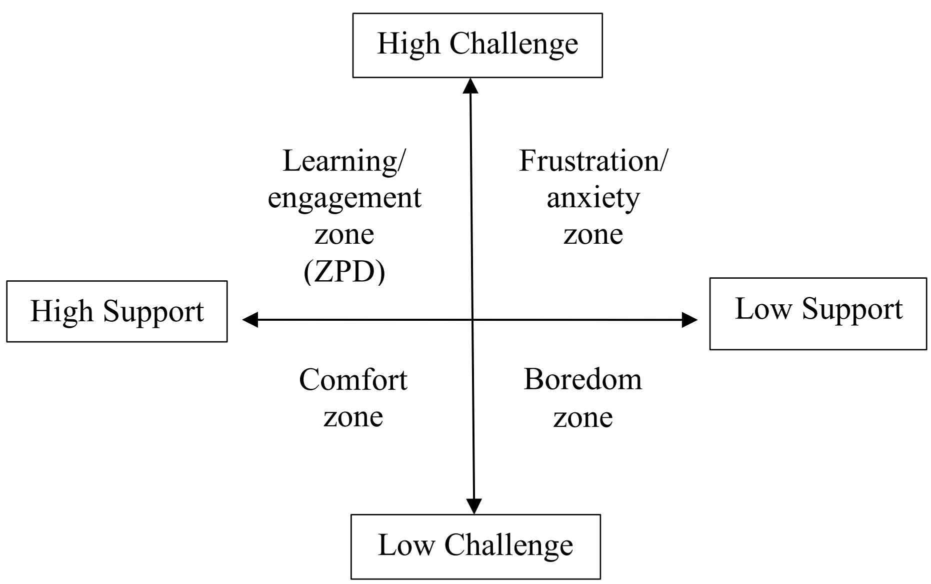

Figure 1. Four Zones of Teaching and Learning (Gibbons, 2015; Mariani, 1997)

Regarding discourse moves, Bishop and Whitacre (2010) defined the teacher's and the student's discourse moves as "give moves" when providing information and "demand moves" when requesting information during the instruction. Xin, Liu, Jones, Tzur, and Si (2016) used a similar structure to make distinctions between three levels of intellectual work when describing the teacher's and the student's discourse moves as "low," "potentially high," and "high."

In consideration of instructional scaffolds and related discourse moves, the conceptual framework that guides this study is shown in Figure 2. As illustrated, instructional scaffolds and mathematics content occur in tandem, undergirded by thoughtful preparation of content and related scaffolds.

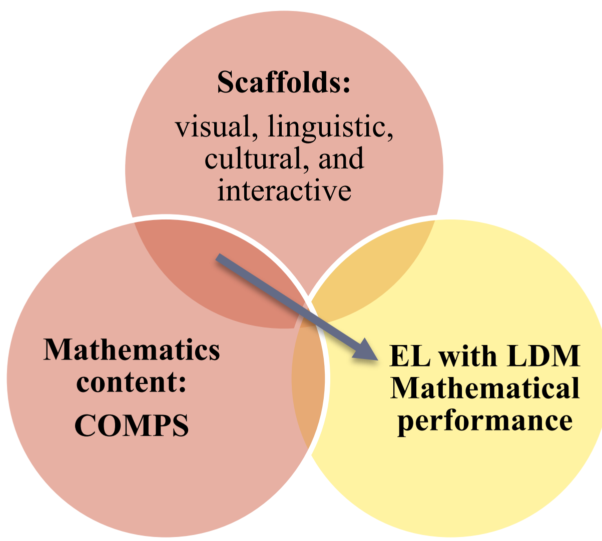

Figure 2. Teaching Framework in Mathematical Instruction for ELs with LDM (adapted from Lei et al., 2020b)

2.5 Recently Emerged Intervention for ELs with LDM

2.5.1 Interventions Targeting Reading Comprehension in Word Problem Solving

Previous researchers have suggested that students' reading proficiency strongly predicts their mathematics achievement (Jordan, 2007), and difficulties in reading have a negative effect on the development of a student's mathematics achievement (Jordan, Kaplan, & Hanich, 2003). Some cases also showed that students with limited reading abilities may have difficulty solving word problems due to their inability to read the words (Bryant et al., 2014). In addition, Krawec (2014) found that students with LDM experienced difficulties in paraphrasing what the word problems were asking. Such abilities are even more difficult for ELs at risk for MD because of their lack of academic language, mathematics difficulties, and linguistic and cultural adaptation (Kong, 2017).

An abundance of empirical literature has explored effective interventions in mathematics word problem solving for ELs at risk for or with LDM. These interventions include scaffolding with paraphrasing and visualizing word problem-solving intervention (Kong & Swanson, 2019), Dynamic Strategic Math (DSM) (Orosco et al., 2013), and culturally and linguistically responsive schema intervention (Drive & Powell, 2017), to name a few.

Paraphrasing and Visualizing. Kingsdorf and Krawec (2016) used a single-subject design to evaluate the effect of the multi-component interventions on mathematical word problem solving involving 10 third-grade ELs with or at risk for LD and/or English learners. The authors used the researcher-created word problems worksheet as an assessment to track students' paraphrasing accuracy, visualizing accuracy, and computation accuracy. The researchers used teacher-mediated explicit instruction with multiple exemplars, visual representations, and incorporation of self-strategies (e.g., a self-monitoring checklist and a paraphrasing prompt) for the intervention. The intervention targeted paraphrasing accuracy and visualizing accuracy. During the first paraphrasing intervention, the teacher presented an initial explicit instruction lesson on paraphrasing for the whole class, and students practiced with support after modeling. During the second paraphrasing intervention phase, the teacher provided direct modeling for paraphrasing a novel word problem. Afterward, students were permitted to use the paraphrasing self-monitoring checklist, which included four steps for paraphrasing guidance along with a paraphrasing prompt for each of the steps. The authors used a researcher-created word problem-solving test as the dependent measure to track participants' paraphrasing accuracy, visualizing accuracy, and computation accuracy. Based on the assessment, efficient paraphrasing practices that the teacher modeled during instruction included using abbreviations of the important information rather than complete sentences, and encouraging students to do it the same way. Afterward, at least eight of the students met the paraphrasing criterion. The students then moved on to the visualizing intervention (i.e., labels, boxes, or circles to represent the information components, a number line, or a diagram) (Kingsdorf & Krawec, 2016). The results showed that multi-component intervention (i.e., explicit instruction, a self-monitoring checklist, multiple exemplars, and assessment practices) provided a cumulative effect on computation accuracy of word problem solving for a total of 10 third-graders with or at risk for LD and/or English learners. Specifically, participants improved their performance from an average of 45% correct during the baseline (range from 14% to 86%) to an average of 68% correct (range from 38% to 94%) after the intervention. During the intervention, PND scores were all above 56% for using paraphrasing, and above 38% for using visualizing. The intervention also demonstrated skill maintenance (Kingsdorf & Krawec, 2016).

Moran et al. (2014) used a randomized control group with a pretest-posttest experimental design to evaluate the effectiveness of paraphrasing interventions on mathematical word problem-solving accuracy in 72 third-grade students at risk for mathematics disabilities. One of four intervention conditions was randomly assigned to each student: Restate (paraphrase question propositions, *n* = 16); Relevant (paraphrase relevant propositions, *n* = 19); Complete (paraphrase all propositions, *n* = 24); and an untreated Control group (*n* = 13) (Moran et al., 2014, pp. 97-100). The Restate proposition condition asked students to rewrite the question sentence from the original word problem in their own words. The Relevant proposition condition required students to rewrite all propositions including the question and needed numbers, except for any irrelevant propositions. The Complete proposition condition asked students to rewrite the questions, including any needed numbers as well as irrelevant numbers. For the first and second problems, the tutors read the problem, using explicit questioning about the target proposition and modeling restating or paraphrasing the target proposition, then providing corrective feedback for the students. The students had to finish the last three problems independently. Both the control group and the intervention group received instruction in the general education classroom with their peers. In addition, the intervention group received supplemental intervention from tutors two times a week over 10 weeks with each session lasting 25-30 minutes. Students were assessed on a curriculum-based Word Problem Solving test (WPS), as well as three standardized tests: KeyMath Revised Problem Solving, Comprehensive Mathematical Abilities Test-Problem Solving (CMAT), and Test of Mathematical Abilities (TOMA). The results showed that, compared to the Control condition, the Complete and Relevant condition had large effect sizes (0.95 and 0.93, respectively) on WPS, while the Restate condition had a moderate effect size (0.52). Also, students who received the Complete condition had a large effect size (1.04) on the TOMA, while the Complete condition and Relevant condition had similar effect sizes (from 0.21 to 0.65) on KeyMath and CMAT (Moran et al., 2014).

Dynamic Strategic Math. Dynamic assessment (DA) is a method to measure students' learning behavior from their unassisted performance to fully assisted performance on challenging tasks (Orosco, 2014a; Vygotsky, 1978). Orosco and his team conducted a series of studies (2013, 2013, 2014a, 2014b) using Dynamic Strategic Math (DSM) interventions to provide teacher-directed instructions for three levels of systematic scaffolding processes to improve the mathematical word problem-solving skills of ELLs (English language learners) (Orosco et al., 2013). Based on a scaffolding ladder, Orosco and colleagues classified the language of mathematics vocabulary into four levels, with each level providing a different linguistic modification: basic, intermediate, advanced, and technical. For each level of vocabulary, teachers provided descriptions and examples to interpret them, while also providing appropriate scaffolds to support students' development to the next higher level of word problem-solving skills (Orosco, 2013). The researchers used the Woodcock-Johnson NU Tests of Achievement 3rd Edition, Achievement Test 10: Applied Problems (WJ NU III-ACH Test 10) to measure students' mathematical skills. For the baseline phase, the participants solved mathematical word problems in the researcher-developed DSM Assessment Probe (DSMAP), which determined their starting instructional level. Students' entry levels for the word problems were determined by the number of word problems they solved correctly. The intervention included three phases. The first phase was pre-teaching the mathematical ideas, concepts, and mathematical vocabulary. The second phase included five strategies: "find the questions, find the key vocabulary, set it up, solve it, and check it" (Orosco, 2013, p. 98). The third level allowed students to practice the new method and take the lead in solving word problems on their own. DSM intervention also accesses ELLs' previous learning and provides them with learning opportunities that are consistent with their background knowledge, verbal language development, vocabulary, and their needs related to problem-solving (Orosco, 2014b). Moreover, when choosing topics for word-problem story contexts, the researchers also considered the ELLs' background knowledge (Orosco, 2014a).

Cultural and Linguistically Responsive (CLR). Driver and Powell (2017) used a quasi-experimental design to explore word-problem instruction for nine third-grade ELLs at risk of MD. In their study, they investigated the influence of the combination of culturally and linguistically responsive practices with the schema instruction (CLR-SI) on word problem solving as measured by the Pennies Test (Jordan & Hanich, 2000) (which includes total/combine, change, difference/compare, and equalize problem types), Addition Fluency (which measures computational fluency), and Subtraction Fluency (which also measures computational fluency). The CLR-SI intervention included culturally responsive teaching (CRT), linguistically responsive teaching (LRT), and schema. CLR-SI utilized problem types as schema to teach ELLs to solve mathematical word problems. Each CLR-SI lesson included a word problem related to students' personal lives, experiences, interests, and pop culture. The students were also allowed to use their native language (Spanish). All nine participants were included in the CLR-SI intervention without a business-as-usual condition, while the control group included a representative sample of third-grade students (n = 605).

The intervention tutoring sessions were conducted for 10 weeks, with three 20-30 minute sessions provided each week. Each student received two phases in the intervention stage: Basic Strategy and CLR-SI intervention. Tutors prompted students with Basic Strategy Instruction, a set of directives that included reading the problem, drawing a visual representation to illustrate the problem, solving for the unknown quantity, and explaining what they were solving for (Driver & Powell, 2017). The CLR-SI phase included several culturally and linguistically responsive mathematical instruction elements. For example, students were encouraged to explicitly state the lesson objectives, participate in oral discussions with peers, use their native language and their own experience, and use graphic organizers and manipulatives to help explain and compute the problems (Driver & Powell, 2017). In addition, the instructor utilized instructional examples from students' daily lives and pop culture (Driver & Powell, 2017). The results of this study indicated a statistically significant large effect (ES = 0.79) on the Pennies post-test due to the tutoring program. In addition, the results showed that students' English language proficiency (World-Class Instructional Design and Assessment, WIDA level) was not significantly related to their Pennies performance. Further, students' pretest scores on the Addition Fluency test (but not Subtraction Fluency scores) showed a significant interaction with their Pennies performance. These results indicated that students' word-problem performance improved after their participation in the Basic Strategy Instruction and CLR-SI intervention (Driver & Powell, 2017).

Kim, Wang, and Michaels (2015) investigated the effect of CRA on two types of fraction word problem solving for three low-performing (below 25th percentile on subtests) Asian immigrant English language learners (ELLs1) who used a different language (Chinese or Korean) at home. The CRA instruction examined in this study contained three discrete levels of representations: manipulatives, pictorial representation, and abstract symbolic representation. The authors developed an explicit CRA instruction in combination with culturally relevant teaching examples that used participants' parents and representative cultural informants to model solving both types of fraction word problems. The researchers implemented explicit instructional procedures, such as advanced organization, teacher modeling, guided practice, and independent practice, in each intervention session (Kim et al., 2015). The participants were measured by four dependent measures: Type I fraction word problems (i.e., subtract multiple parts from a whole), Type II fraction word problems (i.e., understanding part-whole relations), maintenance probes (i.e., each of the two types of fractions), and near transfer probes (i.e., both problem-solving skills) (Kim et al., 2015).

Almost all participants had 80% accuracy or higher during the intervention at each of the concrete, representational, and abstract levels (percentage of non-overlapping data = 100%) for both Type I and Type II fraction word problems. The participants' performance also demonstrated maintenance two and four weeks after the intervention (50% accuracy or higher), and all participants reached 60% for near transfer-type word problems after receiving instruction on solving Type II fraction word problems (Kim, Wang, & Michaels, 2015).

2.5.2 Interventions Targeting Mathematics Modeling in Word Problem Solving

Mathematics Modeling for ELs with LDM. A key component of teaching mathematical problem solving is the use of representation (e.g., models), which helps students process and understand the different structures of word problems rather than the surface features of problems; therefore, problem solvers can perceive and develop better plans for solving word problems (Lesh et al., 2003).

Sharp and Dennis (2017) utilized the Model Drawing Strategy (MDS) intervention package with three Hispanic students with LD in fourth grade with a multiple probe design that focused on word problem solving pertinent to fraction comparison and ordering as measured by curriculum-based assessment (CBA) probes and a research-created generalization test (i.e., students were able to order the fractions from the smallest to the largest). Model Drawing Strategy explicitly taught students to draw a bar diagram to represent the qualitative relations presented in the word problem or to represent components in a problem scenario (Sharp & Dennis, 2017). The MDS intervention package designed by the researchers included two parts: training lessons and MDS lessons. In the training lessons, students learned some prerequisite skills such as basic fraction concepts, naming the numerators, and drawing a rectangular bar to represent the whole. After the training lesson, students received six 30-minute MDS lessons. MDS lessons required students to read the problem, list relevant information, and draw a rectangular bar to represent relationships between the whole and part of the fraction (Sharp & Dennis, 2017). Overall Tau-U effect size for the intervention package was 1.0, which means that this intervention package had a large effect on improving the fraction word problem solving of students with LD. The study also demonstrated maintenance effects both two weeks and four weeks after the intervention. Also, all participants' performance on the generalization tests improved from 0% to at least 50% on generalization tests (Sharp & Dennis, 2017).

Xin et al. (2017) conducted a study using a Computer-Assisted Instruction (CAI) called Please Go Bring Me-Conceptual Model-Based Problem Solving (PGBM-COMPS), an intelligent tutor program designed to promote the multiplicative reasoning and problem-solving abilities of students with learning disabilities or difficulties in mathematics (including four ELs). The study employed a pretest-posttest comparison group design with random assignment of participants to groups. The study involved third-grade and fourth-grade students with LDM. The PGBM-COMPS intelligent tutor program contained two parts. The first part consisted of "Please Go Bring Me …" (PGBM) turn-taking games to build a learner's construction of "fundamental ideas in multiplicative reasoning" (Xin et al., 2017, p. 5). The second part consisted of COMPS, which emphasized "understanding and representation of word problem structures in mathematical model equations" (p. 5). The dependent measures include (a) a researcher-developed, 11-item multiplicative reasoning and problem-solving criterion test, and (b) a Mathematics Problem Solving subtest of Stanford Achievement Test (SAT)-10. The SAT-10 was used as a far transfer measure to assess the generalization effect of the interventions. Participants who were in the

PGBM-COMPS group worked with the web-based PGBM game one-on-one through all four modules (A, B, C, and D) in sequence. Students who were in the Teacher Delivered Instruction (TDI) group worked with a school teacher (Business as Usual Condition, BAU). Both groups received 25-minute sessions four times per week over nine weeks for a total of 36 sessions. The results showed that students in the PGBM-COMPS groups outperformed the BAU group on a researcher-developed multiplicative word problem-solving criterion test (effect size = 1.99). More importantly, only the PGBM-COMPS group significantly improved their performance on the far transfer measure, a norm-referenced standardized test (Problem Solving subtest of the SAT-10), with a large effect size of 1.23.

Another study conducted by Xin and colleagues (2020) used a single-subject design to evaluate the effect of the Conceptual Model-based Problem Solving (COMPS) computer tutor on additive word problem-solving performance of four third-grade students with LDM (including three ELs). The COMPS intervention involved two components: (a) "content specific visual scaffolding including mathematical model-based representation," and (b) "the linguistic scaffolding involving a series of word problem story grammar prompting questions" (p. 110). Specifically, mathematical model-based problem representation included three modules. Module A focused on using virtual manipulatives such as unifix cubes to cultivate students' fundamental mathematical ideas. After building on the fundamental ideas, Modules B and C were then introduced to students. Module B used the "cohesive mathematical model equation (P + P = W)" to teach students to represent and solve a range of *combine* and *change* problem types, while Module C engaged students in representing and solving various additive comparison problems using a similar mathematical model equation ($P_{\text{smaller quantity}} + P_{\text{difference quantity}} = W_{\text{bigger quantity}}$). The linguistic scaffolding consisted of a set of "word problem [WP] story grammar" prompting questions (p. 112). For instance, when presented with an additive comparison problem, students first needed to find and underline the comparison sentence in the story. Then they were prompted to ask themselves a set of *WP story grammar* questions such as "*who is compared to whom,*" "*who has more and who has less*," "*which quantity is the bigger one,*" and "*which quantity is the smaller one*" (p. 112). After comprehending the word problem story situation and the mathematical relations involved, students were then asked to represent the information in the diagram equation (i.e., $P_{\text{smaller quantity}} + P_{\text{difference quantity}} = W_{\text{bigger quantity}}$). Through this linguistic scaffolding, students were able to understand three elements (Part, Part, and Whole) of word problems and were able to use the mathematical model equation to represent and solve a variety of problem situations. Results of this study showed that the intervention improved students' performance on researcher-developed tests and a generalization test. Overall Tau-U effect size for the COMPS computer tutor was 0.96, which indicates that the intervention had a strong effect on improving the performance of additive word problem solving of ELs with LDM. Furthermore, the study also demonstrated that two of the four students showed improvement in a far transfer measure Mathematics Problem Solving subtest of the SAT-10.

2.6 Summary

Instructional scaffoldings play a significant role in mathematics instruction and learning. Especially for ELs with LDM, instructional scaffoldings make contributions to both reading and problem solving during mathematics interventions. The previous studies have not explored effective mathematical instruction and interventions to support dually classified ELs with LDM through a multidisciplinary education approach. Xin et al. (2020) applied word problem story grammar that is relevant to the elements in word problem structure but did not apply other scaffoldings such as culturally responsive scaffolding and scaffoldings for English comprehension including vocabularies.

This study integrated instructional scaffolding theories and culturally responsive pedagogy from the field of English language education, and evidence-based practices from special education, to develop mathematical interventions for students who are ELs with LDM. It systematically applied scaffolding theory from EL literature combined with content-specific scaffolding from mathematical problem-solving intervention research involving students with LDM. The ultimate goal of this study was to provide scaffolded mathematics interventions for ELs with LDM as well as to address critical research gaps on guiding teachers' decisions on how to introduce and dismantle scaffolds appropriately based on students' needs.

CHAPTER 3. RESEARCH METHODOLOGY

3.1 Participants and Setting

This study took place in a Midwest suburban public elementary school in the United States. The participants were three students with learning difficulties in mathematics (Table 1). The participants were characterized by the following criteria: (a) they spoke English as a second language or had a language other than English in their background; (b) they were identified by the school as having learning disabilities in mathematics; and (c) they scored lower than the 30th percentile (on the *Mathematics Problem Solving* subset) of the Stanford Achievement Test (SAT-10, Pearson Inc., 2004). Additionally, the students' pre-intervention performance on the research-developed criterion test was below 60%. All sessions were implemented in a vacant conference room. All sessions were videotaped for data analysis.

Table 1. Students' Demographic Characteristics

Variable	Laura	Cam	Sara
Gender	Female	Male	Female
Ethnicity	Latina	Latino	Latina
Age (year-month)	9-7	8-7	8-2
Grade	3	3	3
Social Economic Status	Low	Low	Low
Reduced/Free lunch	Free	Reduced	Free
Duration of RtI support (min.)	Tier I-60 Tier II-30	Tier I-60 Tier II-30	Tier I-60 Tier II-30
% in General Education class	100%	100%	90%
Otis Lennon school ability test score			
Full Scale	89	72	74
Verbal	95	79	78
Performance	83	68	74
SAT mathematics problem solving (in percentile rank)	13	9	11

Note. RtI: response-to-intervention.

3.2 Dependent Measures

The types of word problems found in elementary curricula are typically *join*, *part-part-whole* (PPW), *separate*, and *compare* for addition and subtraction (Van de Walle et al., 2012). Table 2 provides sample problem situations for PPW and additive compare word problems (adapted from Xin et al., 2008). Types of PPW word problems include *combine*, *change-join*, and *change-separate* problems, while additive compare (AC) problems include *compare-more* (where one quantity is "more") and *compare-less* (where one quantity is "less" than another quantity) (Xin, 2012). This study included all 14 types of word problems in the generalization test, and eight types of PPW word problems in the criterion tests.

Table 2. Part-Part-Whole and Additive Compare Problem Types (adapted from Xin et al., 2008)

Problem Type	Sample Problem Situations
Part-Part-Whole	***Combine***
Part (or smaller group) unknown	1. Rina and Bill have 80 stickers altogether. Rina says that he has 33 stickers. How many stickers does Bill have?
Whole (or larger group) unknown	2. Mike made 19 hats in his class. His brother Taylor made 69 hats. How many hats do they make altogether?
	Change-Join
Part (or smaller group) unknown	1. Alex had 80 rubber bands. Then his sister Ivy gave him some more rubber bands. Now he has 99 rubber bands. How many rubber bands did Ivy give Alex? 2. Emily had several crayons. Then her friend Paul gave her 60 more crayons. Now Emily has 76 crayons. How many crayons did Emily have in the beginning?
Whole (or larger group) unknown	3. James has 36 candy bars. His brother gives him 26 more candy bars. How many candy bars in total does James have now?
	Change-Separate
Part (or smaller group) unknown	1. Jessica had 55 fish in her fish tank. Then, one day she lost 16 of them. How many fish does Jessica have now? 2. Leslie had 40 tickets. She sold some tickets on the first day of her trip. On the second day, she had only 19 tickets left. How many tickets did Leslie sell on the first day?
Whole (or larger group) unknown	3. Tony had many stamps. Then he gave away 45 of his stamps to his brother Denzel. Now Tony has 18 stamps. How many stamps did Tony have in the beginning?
Additive Compare	***Compare-more***
Larger quantity unknown	1. Nancy bought 67 goldfishes. On Friday, she found out that her sister Lucy has 28 more goldfishes than what she bought. How many goldfishes does Lucy have?
Smaller quantity unknown	2. Davis collects rocks. As of today, he has 53 rocks in his collection. Davis has 18 more rocks than Lucas. How many rocks does Lucas have?
Difference unknown	3. Helen has 62 baseballs in a box to practice hitting. Stephanie has 45 baseballs. How many more baseballs does Helen have than Stephanie?
	Compare-less
Larger quantity unknown	1. Richard blew up 82 balloons for a party. Richard blew up 25 fewer balloons than his friend Amy. How many balloons did Amy blow up?
Smaller quantity unknown	2. Katelyn ran 32 miles in one month. If Kevin ran 11 fewer miles than Katelyn, how many miles did Kevin run?
Difference unknown	3. If Deanna has 62 tiny fish in her aquarium and Gerald has 71 tiny fish in his aquarium, how many fewer tiny fish does Deanna have than Gerald?

3.2.1 Word Problem-Solving Performance

Part-Part-Whole (PPW) Criterion Test. The primary dependent measure used alternate forms of a researcher-developed criterion test (Xin, 2012) in the baseline, intervention, and post-test phases. The criterion test was comprised of eight *Part-Part-Whole* (PPW) additive mathematical word problems with either the part or the whole as the unknown quantity. There are three types of story situations for the PPW problems: *combine*, *change-join*, and *change-separate*. See samples problems in Table 2.

Generalization Tests. The additive word problem solving test (AWPS, Xin et al., 2020) included eight PPW problems as well as six AC problems (including situations such as "… more than …" or "… less than …") (Xin, 2012). In addition to the eight types of PPW problems that are similar to the items on the criterion test, the Generalization test also included *compare-more* and *compare-less* types of story situations (Xin, 2012). See sample problems in Table 2.

Scoring. The researcher scored all tests, including the criterion tests and generalization tests, using the answer sheets. All tests were analyzed based on the accuracy of problem solving. Each correct answer to a word problem was scored one point. An incorrect calculation or answer paired with correct problem representations was scored half a point. The accuracy was calculated by dividing the total points earned by the total possible points and multiplying by 100.

3.2.2 Word Problem-Solving Process

Video footage of the students' word problem-solving process was coded based on a coding scheme developed by the researcher. Table 3 presents a description of each element in the problem-solving process and the corresponding coding scheme.

Table 3. Examples of Problem-Solving Process Coding

Word problem-solving process	**Description**	**Examples**
Reading	Read the problem	Student reads the problem with teacher.
Visualizing	Highlight and/or label part/whole sentences	Student highlights three components of the problem and/or writes the corresponding letters "W" or "P".
Explaining	Explain the unknown quantity "*a*", part, and whole	Student explains the unknown quantity "*a*" and answers what we are asked to solve for.
Representing/Story mapping	Map all numbers onto the diagram	Student maps all numbers onto the diagram corresponding to the PPW equation.
Solution Planning	Write the equation to solve the unknown quantity "*a*"	Student writes the equation for solving the unknown quantity "*a*" on paper.
Solving	Solve the unknown quantity "*a*"	Student chooses a strategy to solve the unknown quantity "*a*" (e.g., mental computation, counting down/up, standard algorithm, or a calculator).

Scoring. Table 4 shows the scoring of the elements used by the student during the problem-solving process. The frequency of the occurrence of each element used by the student in the problem-solving process during each session was recorded. The percentage of student use of each element was calculated by dividing the actual frequency counts of each element by the total frequency counts across all elements in each session and multiplying by 100.

Table 4. Examples of Scoring Problem-Solving Processes as 1, 0.5, or 0 Points

Word problem-solving process	**Scoring**
Reading	Read the entire problem independently: 1 point
	Read part of the problem independently or with teacher: 0.5 point
	Did not read: 0 points
Visualizing	Highlighted and/or labeled all sentences as “W” or “P”: 1 point
	Highlighted and/or labeled one sentence: 0.5 point
	Did not highlight or label any sentences: 0 points
Explaining	Explained both parts, whole, and unknown quantity “*a*”: 1 point
	Explained each element only partially: 0.5 point
	Did not explain any elements: 0 points
Representing/Story mapping	Correctly mapped all numbers/variables onto PPW equation: 1 point
	Put wrong numbers/variable or did not put numbers into the PPW equation: 0 points
Solution Planning	Wrote a correct equation for solving the unknown quantity “*a*”: 1 point
	Wrote an incorrect equation for solving the unknown quantity “*a*”: 0 points
Solving	Solved the unknown quantity “*a*” correctly: 1 point
	Did not get the correct solution: 0 points

3.3 Design

This study used a multiple-probe-design (Horner & Baer, 1978) across participants (Horner et al., 2005) to measure the potential functional relation between the scaffolded COMPS intervention and participants’ mathematical word problem-solving performance and problem-solving processes. The multiple-probe design demonstrates the intervention to different baselines or participants at different time points. “If each baseline changed when the intervention is introduced, the effects can be attributed to the intervention rather than to extraneous events” (Kazdin, 1982, p. 126).

The rationale for choosing a single-subject research design was that the design provides a well-suited methodological approach to conduct single case and/or group comparison studies. The intervention's effects are demonstrated "when a change in each person's performance is obtained at the point when the intervention is introduced and not before" (Kazdin, 1982, p. 132). The design included a baseline (criterion tests and generalization tests), an intervention (repeatedly probed on criterion tests), post-intervention (criterion and generalization tests) phases.

3.4 Procedures

The researcher, a doctoral student in special education, went to the school to work with the students each afternoon for four days per week. Each participant received the scaffolded COMPS intervention in a one-on-one setting of approximately 20-30 minutes each day.

3.4.1 Baseline Assessment

All participants began the experiment on the same day by taking the PPW criterion test. Students could ask to complete the problems on separate days, but they were not offered help during the baseline sessions. After all three participants completed a criterion test in the first session, the researcher randomly selected one student to take two more alternate forms of the criterion test. Once the first student (Cam) showed a stable trend, the scaffolded COMPS intervention was introduced to him. In the intervention phase, when Cam's WPS accuracy score showed an accelerating trend, the intervention was introduced to the second participant (Laura) after she took three additional alternative criterion tests. The third participant (Sara) followed the same sequence and took four more additional alternative criterion tests. Each participant took at least two additional criterion tests before entering the intervention condition. All participants took the generalization test during the last session of their baseline.

3.4.2 Intervention Sessions

The scaffolded COMPS intervention was developed based on Xin's (2012) COMPS approach, which integrated the following components: (a) Culturally and Linguistically Responsive Scaffolding; and (b) Mathematical model-based (COMPS, Xin, 2012) visual scaffolding (Figure 3) (see Appendix A for a detailed teaching script for the scaffolded COMPS intervention).

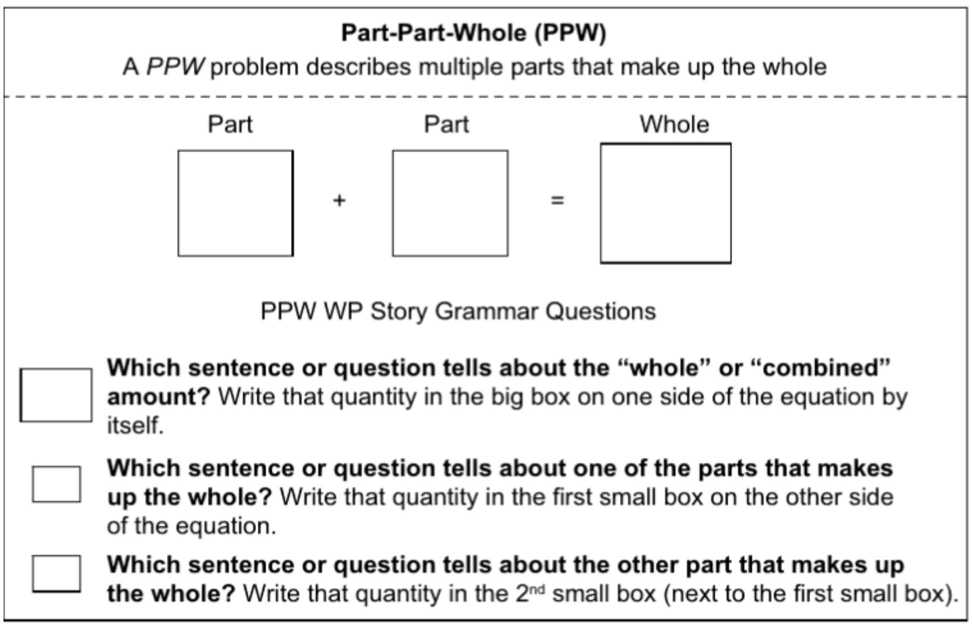

Figure 3**.** A Part-Part-Whole Word Problem Description (Xin, 2012, p. 47)

Phase 1: Culturally and Linguistically Responsive (CLR) scaffolding (Figure 4). The CLR scaffolding phase included communication and teaching mathematical vocabulary instruction. At the beginning of the intervention, the teacher held a three-minute conversation with the students to talk about some stories from the students' countries of origin. The teacher initiated dialogue journals between the teacher and the student during this conversation. During this process, students were asked about interesting stories, and the teacher also encouraged students to revise mathematical word problems if they so wished. The content of the problems related to their personal lives, native language, traditions, experiences, interests, and culture. This process provided motivational intervention for students to help familiarize themselves with the problem. This step also included mathematics vocabulary/phrases and sophisticated language teaching and explanation during mathematics instruction (Moschkovich, 2013) in order to help students comprehend mathematical relations in the problems. Some mathematics vocabulary examples are *altogether*, *addition*, *subtraction*, *combine*, *together*, *part*, *total*, *whole*, *in all*, *take away*, etc. In each session, students learned two types of *Part-Part-Whole* word problems. In total, the entire intervention contained six *Part-Part-Whole* word problem types (Xin, 2012).

For example, the student was told, "We are going to talk about some interesting stories together. You can tell me any interesting things you want to let me know about your daily life. There are several colors of pens and blank paper in front of you on your desk. You can draw on the paper if you want." The student was allowed to modify the story to fit their cultural background. A sample given problem was: "*Bobby had 87 cards in a box. He gave some cards to his brother, Jeff. Then Bobby had 62 cards left in his box. How many cards did he give to his brother Jeff*?" In this way, the students were encouraged to take advantage of their prior knowledge to create and comprehend new concepts (Villegas & Lucas, 2007) in solving word problems. Moreover, culturally relevant problem situations can improve students' retention and comprehension skills (Kim et al., 2015) and motivate them to solve mathematical word problems.

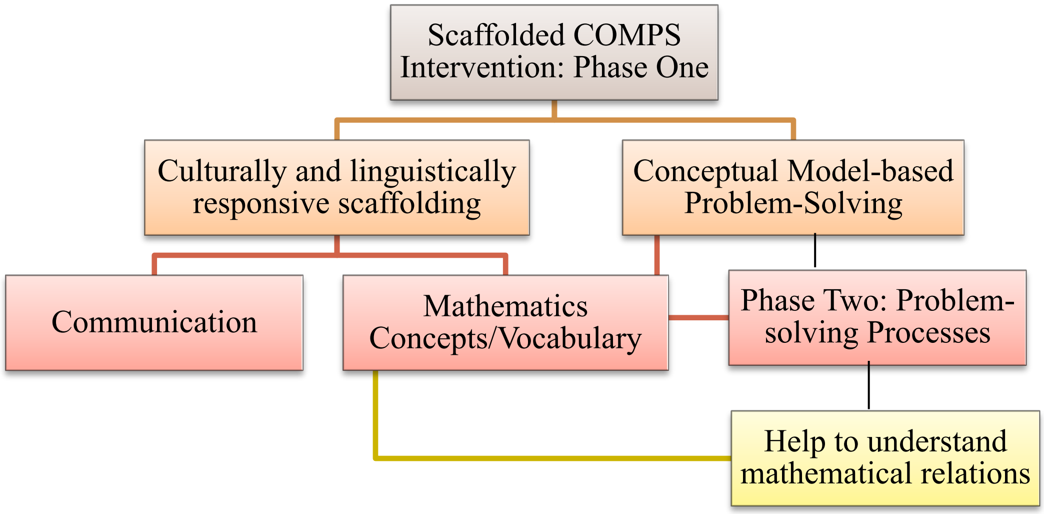

Figure 4. Scaffolded COMPS Intervention Phase One

After providing CLR scaffolding to students, the teacher used six steps to teach students how to solve *Part-Part-Whole* word problems using mathematical model-based visual scaffolding. Table 5 presents an example of the six general steps that guide the scaffolded COMPS intervention.

Phase 2: Mathematical model-based visual scaffolding (Figure 5). In the first step, the student was asked to read the word problem independently or with the teacher. If the student could not read the problem independently, the teacher used a slower rate of speech to read the problems. Moreover, the student was asked the simple phrases: "which sentence tells about the whole," "which sentences tell about the two parts," and "what are we asked to solve for" (Xin, 2012). These simple, structured and repeating grammar prompts provide students with the elements of a story (Rand, 1984) in order to scaffold students' comprehension and recall of the word problems (Xin, 2012). Similarly, story grammar can also be applied by asking the five W questions (i.e., who, what, where, when, and why) and an H question (i.e., how) when teaching reading comprehension to students (Gurney, Gersten, Dimino, & Carnine, 1990). When applied to mathematical word problems, Xin (2008) established the *Word Problem Story Grammar*, which provides a mathematical word problem structure across a series of word problem situations to help students represent word problems through mathematical model equations (Xin, 2012).

Table 5. Example of Scaffolded COMPS (adapted from Xin, 2012)

Description of the teaching script
Example question: Bobby had 87 cards in a box. (W) He gave some cards to his brother, Jeff. (P) Then, Bobby had 62 cards left in his box. (P) How many cards did he give to his brother Jeff? (*a*) $\square + \square = \square$ Part (P) or (*a*) Part (P) Whole (W) a = 87 - 62 ANSWER: ______________________ Teaching steps: 1. Students read the problem independently or with the teacher. Teach mathematics concepts/vocabulary. 2. Highlight "**which sentence tells about the 'whole'**" (in red) And write a letter "W" next to the sentence. Highlight "**the two sentences that talk about the two parts**—the cards he gave Jeff and the cards he had left" (in yellow) Write a letter "P" next to each of the sentences. 3.Ask students "**what are we asked to solve for**?" Tell them that is the unknown quantity that we need to solve for. Write the letter "*a*" next to that sentence. 4. Let students put all numbers into the diagram corresponding to the PPW equation. 5. Teach to rewrite the unknown quantity "*a*". 6. Teach to solve for the unknown quantity "*a*" in the equation.

With the COMPS approach, students are not guessing whether to add or subtract to solve the problem; rather, different word problems are represented in one mathematical model equation (Figure 5), and then the model equation determines the operation needed to find the solution (Xin, 2012). For example, in Step Two, students were asked to highlight the sentence that tells about the whole and write the letter "W" next to the sentence. In addition, students were asked to highlight two sentences that tell about the parts and write the letter "P" next to each of the sentences. Highlighting helps ELs with LDM follow along with the text and provides scaffolds to students as they read each word problem (Ybarra & Green, 2003). In Step Three, students were asked: "What are we asked to solve for?" The teacher taught students that the unknown quantity is what we need to solve for and to use the letter "*a*" to represent it. Students were then asked to find the sentence that includes the unknown quantity and write the letter "*a*" next to that sentence. Then, the teacher guided students to put all the numbers (including the unknown quantity) into the mathematical diagram corresponding to the PPW equation. This step, also called story mapping, assists students in identifying, organizing, interpreting, and representing mathematical relations in the PPW diagram equation, and it also increases reading comprehension skills (Sorrell, 1990; Xin et al., 2020). Previous research has identified that explicit instruction on story-grammar elements using story mapping as a visual aid has contributed to the reading comprehension skills of students with and without learning disabilities (e.g., Boulineau et al., 2004; Xin et al., 2020). These studies also explicitly teach each problem-solving step via different types of scaffolding for guided practice. Step Five is planning the solution, which required the student to rewrite the equation to solve for the unknown quantity "*a*" on the paper (e.g., a = 87 - 62). Then, the student was allowed to choose a counting strategy to solve for the unknown quantity *a* (e.g., mental computation, counting down/up, standard algorithm). If not successful, the teacher asked the student to use the calculator to solve for the unknown quantity. The teacher also monitored whether the student was using the calculator correctly. After finding the solution, the student wrote the answer on the answer line.

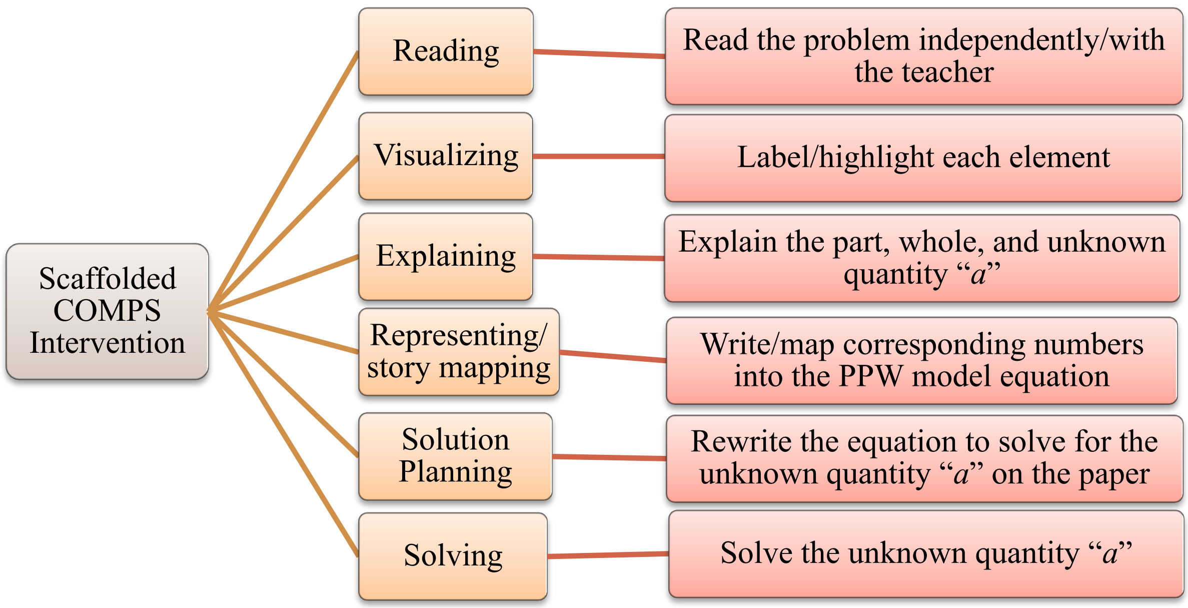

Figure 5. Scaffolded COMPS Intervention Phase Two

3.5 Fidelity of Implementation

To ensure the fidelity of implementation for all elements of the study procedure, the researcher developed a checklist that included all treatment components (see Appendix B). The treatment delivery of each element is listed in the checklist to assess the fidelity of implementation of the Intervention program. An independent observer observed 30% of the total sessions and completed the fidelity checklist for each of the observed sessions. The fidelity was calculated by the percentage of correctly implemented treatment elements and it was 97%.

In addition, video footage of the teacher's use of each type of scaffolding during scaffolded COMPS instruction was coded and analyzed. NVivo 12 (QSR International Pty Ltd., 2018) was used in this study to code the teacher's verbal and nonverbal scaffolded COMPS instruction across sessions. The teacher's scaffolded COMPS instruction was coded based on four different scaffolding categories: visual scaffold, linguistic scaffold, interactive scaffold, and culturally responsive scaffold. Table 6 illustrates the coding scheme with examples from the teacher's scaffolded COMPS instruction. The frequency counts of each type of scaffolding used by the teacher during the instruction in each session were recorded.

Table 6. Scaffolding and Coding Scheme

Scaffolding	Examples
Visual Scaffold	T: Correct, we do not know how many cookies the parents baked. Let's highlight this sentence and use the letter *a* to represent the unknown quantity.
Linguistic Scaffold	Example one: T: Let's think about what is *altogether*? S: Total... T: That's right, when we combine, we put things altogether, which is the same meaning as *total*. Example two: T: Now we will find the number of cookies Mom baked and the number of cookies Dad baked. So, which sentences tell about the two parts? S: "*Mom baked 62 cookies for the class party. Dad baked 26 cookies for the party*."
Interactive Scaffold	T: What we are asked to solve for? S: "*The number of cookies the parents baked*?" T: That's right, this is the unknown quantity that we need to solve for. So, from reading the problem, do you know which quantity is the total? S: "a" T: Great, let's write *a* in the box designated for the total in the equation.
Culturally Responsive Scaffold	T: We are going to talk about some interesting stories together. You can tell me any interesting things you want to let me know about your daily life.

3.6 Interrater Reliability

The researcher scored all tests, including the criterion tests, generalization tests (using an answer sheet), and word problem-solving process coding (using the coding protocol). In order to ensure the reliability of this study, a graduate student in the department of special education rescored 30% of the test items. The graduate student also checked 30% of the WPS process coding independently. Interrater reliability was computed by dividing the number of agreements by the number of agreements and disagreements and multiplying by 100. The interrater reliability for the test scores was 100%, and for the process coding scores was 100%.

3.7 Social Validity

To investigate the social validity of the scaffolded COMPS intervention, the researcher developed a 5-point Likert scale (Appendix C) to assess students' perspectives about the intervention. After the intervention phase, students were asked their opinions on how difficult the probe sessions were, how they liked them, and how effective and helpful the intervention sessions were. A 5-point rating indicates strong agreement and a 1-point rating indicates strong disagreement.

3.8 Data Analyses

The study used visual analysis to evaluate word problem solving accuracy and process performance collected from the study. According to Kennedy (2005), visual analysis is "the most revealing way of analyzing the data and provides the most information to the viewer" because the results from visual analysis can also probe into various aspects and patterns of the data (p. 192). In order to do a visual inspection of the data, the results were presented in line graphs. Six features were used to determine within- and between-phase data patterns, including level, trend, variability, immediacy of the effect, overlap, and consistency of data patterns across similar phases (Kennedy, 2005). Further details will be discussed in the Results section.

To estimate the treatment effect of the intervention, the Tau-U effect size was calculated using a web-based application developed by Vannest et al. (2016). According to Parker, Vannest, Davis, and Sauber (2011), Tau-U is an effect size statistic that measures the degree of non-overlap between phases and quantitatively analyzes the treatment effect of single-subject design studies. The three levels of intervention effects of Tau-U are small effect (0 – 0.62), moderate effect (0.63-0.92), and strong effect (0.93 – 1.00). The reasons for choosing Tau-U effect sizes is because it shows evidence of being more robust and more precise than other non-overlap effect sizes, and also it provides confidence intervals (CI) (Parker et al., 2011).

CHAPTER 4. RESULTS

4.1 Baseline Analysis

The participant's performance during baseline sessions included both word problem solving (WPS) accuracy and WPS process. WPS accuracy used the PPW criterion tests to analyze the students' baseline accuracy of solving eight types of PPW word problems. The WPS process data was recorded as a percentage of steps followed during the problem-solving process.

Cam: (1) WPS accuracy: Cam's accuracy of solving PPW problems was low and stable, with a low degree of variability. The percentage correct on all three tests was 37.5% (see Table 7) with a mean of 37.50% correct, a median of 37.50% correct, and the trend direction was zero-accelerating (see Figure 10 in 4.2 Intervention Analysis). Cam made exactly the same mistakes on three parallel criterion tests. On each test, he got correct answers only on three types of PPW problems: *combine* situation with whole or larger group unknown, *change-join* situation with whole or larger group unknown, and *change-separate* with whole or larger group unknown. Basically, he could only solve problems using addition operations for all eight types of PPW problems.

(2) WPS process: Based on the video of Cam's problem-solving process, I observed that Cam's problem-solving process only included one step (i.e., Step Six: solving) across all three criterion tests. The percentage of problem-solving steps used were low and stable, with 12.5% correct for first two criterion tests, and 10.42% correct for the third criterion test (see Table 7). Cam also had difficulty representing information on the PPW diagram equation. For example, he did not know how to use letter *a* to represent an unknown quantity, or how to put three numbers into the PPW diagram equation correspondingly (see Figure 6 for an example). As shown in the

example, Cam put the two numbers and the solution directly into the PPW equation without using a letter or a variable to represent the unknown quantity.

In addition, Cam used standard algorithm and finger counting as his counting strategies to solve for the problems in the baseline phase, but he could not always get the correct answers without using his calculator. Therefore, Cam's performance indicated the need for intervention and scaffoldings on both representation and calculation skills.

Table 7. Cam's Percentage Correct on the WPS Accuracy and Percentage of WPS Process Elements During the Baseline Condition

	Session 1	Session 2	Session 3
WPS accuracy	37.5%	37.5%	37.5%
WPS processes	12.5%	12.5%	10.42%

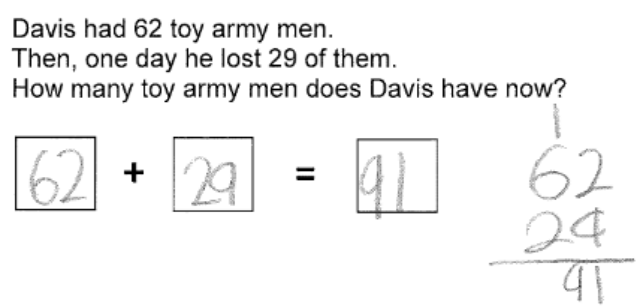

Figure 6. An Example of Cam's WPS Processes

Laura: (1) WPS accuracy: Laura's data showed a low degree of variability, with a mean of 31.25% correct, a median of 37.5% correct, and a slightly upward trend (see Figure 11 in 4.2 Intervention Analysis). Similar to Cam, Laura only got correct answers for problems that needed addition operations during the four baseline sessions. The problem types she answered correctly in Session Two to Session Four were *combine* situation with whole or larger group unknown, *change-join* situation with whole or large group unknown, and *change-separate* with whole or larger group unknown. However, Laura did not know how to represent problem information in the PPW diagram equation, so she appeared to use a "keyword" strategy as her counting strategy. For example, when she encountered the word "more," she opted to use addition operation to solve the problem and ignored the relationship expressed in the word problems (see Figure 7).

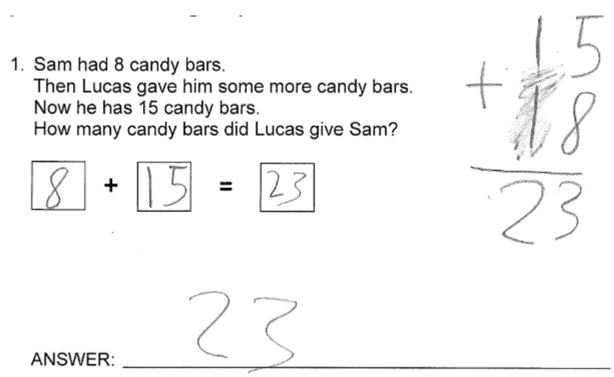

Figure 7. An Example of Laura's Counting Strategy

(2) WPS process: The elements of Laura's problem-solving process also only included Step Six (solving). The percentage of the problem-solving steps used was very low, with a mean of 11.98% correct, a median of 12.50% correct (see Table 8), and a slightly downward trend (see Figure 11 in 4.2 Intervention Analysis). She had difficulty using the PPW diagram equation to represent word problems (see Figure 8). As shown in Figure 8, Laura only put the solution into the PPW equation without putting numbers into the two boxes for parts. Laura also used standard algorithm as her main counting strategy to solve for the problems, whereas she always got wrong solutions without using her calculator. Moreover, Laura frequently made mistakes in calculation, such as switching the minuend and subtrahend, and misconceptions of subtraction, regrouping and place value principles (e.g., 17-21 = 16; see Figure 8). Therefore, she needed the intervention

beginning with comprehending the semantics of word problems, representing the unknown quantity using the PPW equation, as well as counting strategies.

Table 8**.** Laura’s Percentage Correct on the WPS Accuracy and Percentage of WPS Process Elements During the Baseline Condition

	Session 1	Session 2	Session 3	Session 4
WPS accuracy	12.5%	37.5%	37.5%	37.5%
WPS process	8.33%	14.58%	12.5%	12.5%

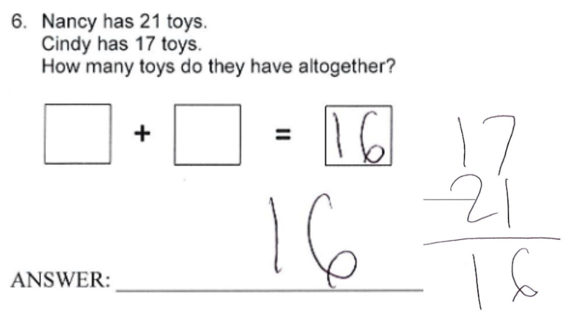

Figure 8**.** An Example of Laura’s Problem-Solving Processes

Sara: (1) WPS accuracy: Sara’s data showed a low degree of variability, with a mean of 25% correct, a median of 37.50% correct, and a moderate downward trend (see Figure 17 in 4.2 Intervention Analysis). Similar to Cam and Laura’s solving strategy, Sara also added all three numbers together for eight types of problems during the five sessions. Sara had difficulty solving part (or smaller group) unknown problems in *combine*, *change-join*, and *change-separate* situations. Moreover, Sara’s arithmetic calculation abilities were low. For example, Sara got 0% correct in Session One because she made mistakes on all operations although she used a correct algorithm to solve for the final solution. It seemed that Sara had difficulty accurately solving a problem for both addition and subtraction operations.

(2) WPS processes: Sara's average performance of WPS processes was 8.33% correct, with moderate degree of variability, and a moderate downward trend (see Figure 17 in 4.2 Intervention Analysis). Sara did not read any problems before solving them. In addition, Sara did not know how to use the PPW diagram equation to represent the relationship of word problems. Instead, she put two numbers in the boxes designated for the two parts (see Figure 9). However, without using a calculator, she got a wrong answer of 38. Therefore, Sara needed intervention to help her representation skills using PPW equation and algorithm-solving strategies to solve change-unknown and beginning-unknown problems, and scaffolding with reading and comprehension skills. Similar to Cam and Laura, Sara's problem-solving elements only included Step Six (solving the problem).

Table 9. Sara's Percentage Correct on the Criterion Test and Percentage of Problem-Solving Process During the Baseline Condition

	Session 1	Session 2	Session 3	Session 4	Session 5
WPS performance	0%	37.5%	37.5%	37.5%	12.50%
WPS processes	0%	12.5%	12.5%	12.5%	4.17%

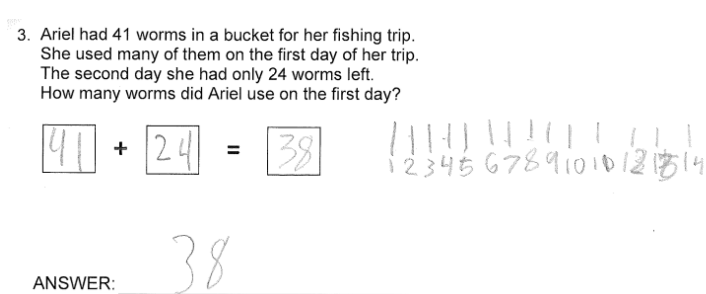

Figure 9. An Example of Sara's Counting Strategy

4.2 Intervention Analysis

Cam: (1) WPS accuracy: Cam's problem-solving performance from his baseline to his intervention phase (increased from 37.5% correct to 43.75% correct), which demonstrated an immediate change in level between conditions (Gast & Ledford, 2014; refer to Figure 10). His WPS accuracy performance was above 50% correct in the following sessions, and ranged from 43.75% correct to 87.50% correct, with a median score of 62.5% correct. The average of Cam's WPS accuracy increased from 37.50% correct in the baseline phase to 62.5% correct in the intervention phase. The trend direction of his data was accelerating. The Tau-U effect size of Cam's improvement in WPS accuracy was 1.0 (90% CI = [0.292, 1.0]).

Table 10**.** Cam's Percentage Correct on the WPS Accuracy and Percentage of WPS Process Elements During the Intervention Condition

	Session 1	Session 2	Session 3	Session 4	Session 5	Session 6
WPS accuracy	43.75%	50%	62.50%	62.50%	68.75%	87.50%
WPS process	37.50%	52.08%	58.33%	62.50%	64.58%	77.08%

(2) WPS processes: Cam's WPS process increased from 10.42% correct to 37.50% correct at the first session of the intervention condition, demonstrating a change in level between conditions (Gast & Ledford, 2014). Cam's WPS process scores ranged from 37.50% correct to 77.08% correct, with the mean of 58.68% correct and median of 60.42% correct. Figure 10 shows Cam's WPS accuracy and process performance in the baseline and intervention phases, respectively. Cam's performance of WPS process changed from a decreasing trend to an increasing trend from the baseline phase to the intervention phase.

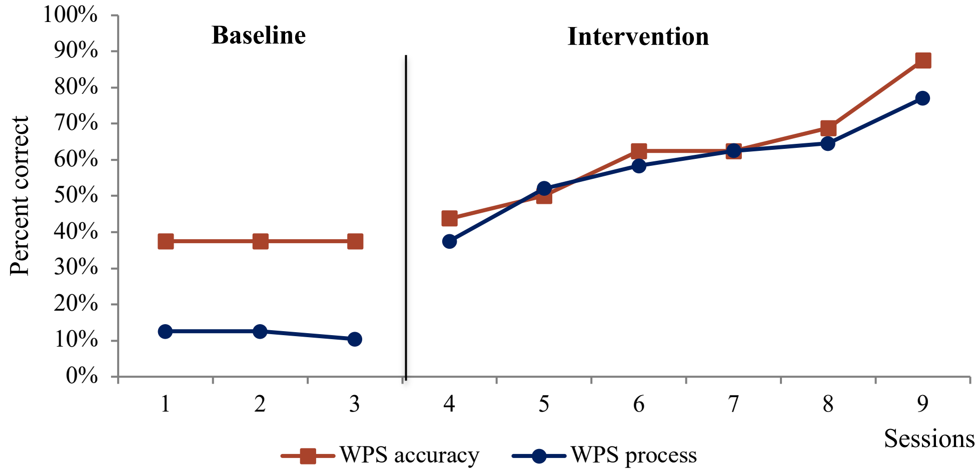

Figure 10**.** Cam's Performance in WPS Accuracy and Processes in the Baseline and the Intervention Condition

In Session One, based on the instruction video, Cam read the problem (Step One) and jumped to Step Six (solving), and then went back to Step Four (representation). Using this sequence to solve for problems showed that Cam did not follow the problem-solving process that the teacher taught during intervention. Therefore, the teacher focused on teaching the sequence of problem-solving process from Session Two. Because the intervention strategy the researcher taught was different from the traditional strategy that he learned from his schoolteachers, Cam was not willing to change his problem-solving strategies in the beginning session and asked "*why do I need to use a*" (see Example 1). When Cam was asked to explain why he put the unknown quantity *a* in the right side of PPW equation, he explained "*Because we don't know it.*" Based on the discourse with the teacher in Session One, he seemed to understand the purpose of using letter *a*, and understood that the unknown quantity can represent either total or part in different situations. For example, he correctly answered whether the unknown quantity represented the total or the part during the intervention (see Example 1).

Example 1 (in Session One):

(*Eva had many cups. Then she gave away 14 cups to her sister. Now she has 38 cups. How many cups did she have in the beginning?*)

Cam: (wrote *a* on the box on the box designated for the total in the PPW equation)

T: You did correctly. Could you tell me why you put *a* in this box?

Cam: Because we don't know it.

T: We don't know the total number of cups or the part?

Cam: The total.

T: That is correct. We don't know the total number, so we use *a* to represent it before you solve for the final answer.

Cam: Why do I need to use *a*?

T: Because using the variable *a* to represent the unknown quantity can help you understand and solve this problem easier, and it also helps you to solve for other types of word problems.

The second step of the scaffolded COMPS intervention was visualization. It was aimed to help him comprehend each part of the word problem by using highlighting, circling, or letters to label the sentences in word problems. Cam started to highlight the question (e.g., "*How many cups did she have in the beginning?*") in Session One. From Session Three, he wrote *part*, *part,* and *whole* next to each sentence, and wrote letter *a* next to the question, which indicated that he can use some PPW strategies combined with visualization to help him comprehend the elements and mathematical relationships expressed in the problems.

Starting from Session Three, Cam started to indicate the elements of the problems (e.g., part, total, or *a*) and explain his reasons of using unknown quantity *a* after reading the problems (see Examples 2 and 3).

Example 2 (in Session Three):
Cam: *Alex had many dolls. Then she gave away 12 of her dolls to her sister. Now Alex has 26 dolls. How many dolls did Alex have in the beginning?* Okay, I know how to do it. I know the method.

T: Don't forget to fill in the PPW equation.

Cam: This is *a*, because we don't know it. (He wrote *a* below the sentence *How many dolls did Alex have in the beginning*?)

T: That's correct, you got it.

Cam: This is 26 (he wrote 26 in the box designated for "part"), and this is *a* (he wrote a in the box designated for the total in the equation), because we don't know it. We don't know the answer.

Cam: (Rewrote the equation for a = 12+26 = 38). I got 38.

Example 3 (in Session Three):
Cam: *Ana had many toys. Then she gave away 16 of her toys to her sister. Now Ana has 36 dolls. How many dolls did Ana have in the beginning*? (Sang the problem instead of reading it.)

T: Remember use the letter *a* to represent the unknown quantity.

Cam: (Highlighted the sentence *How many dolls did Ana have in the beginning*). This is the whole.

Although Cam was able to recognize each component and the unknown quantity *a* independently without the teacher's prompts, he kept using "*because we don't know it*" to explain his reason for choosing letter *a* as the total or the part. According to Table 4, explanation as Example 2 was scored as 0.5 points, as it provided partial and incomplete reasoning regarding using *a* to represent the sentence (e.g., *How many dolls did Alex have in the beginning*). In Session Four (see Example 4), Cam completely explained why he thought *How many cups did she have in the beginning* is the total.

Example 4 (in Session Four):
Eva had many cups. Then, she gave away 14 cups to her sister. Now, she has 38 cups. How many cups did she have in the beginning?

T: Which sentence tells about the total cups?

Cam: How many cups did she have in the beginning?

T: Correct. The number of cups Eva had in the beginning is the total. Could you explain why?
Cam: Because she gives away 14 cups to her sister, and she has 38 now.

Compared with whole or larger group unknown problem type, join-beginning-unknown and separate-change-unknown problems were more difficult for Cam. For example, in Session Three, Cam wrote a = 19 –27 on the paper and used the calculator to solve for the solution to the problem (*Mike made 19 hats. After giving some hats to Linda, he had 8 hats left. How many hats did Mike give to Linda*?). The type of this problem is *change-separate* with change amount unknown. Cam was able to represent the problem on the PPW model and got the correct solution using his calculator, but he had difficulty with the step of solution planning (i.e., Step Five: Rewrite an equation for solving the unknown quantity *a*). Therefore, from Session Three, the teacher focused on scaffolding solution planning, and emphasized elaborating the differences between beginning-unknown and change-unknown problems. Cam used finger counting and standard algorithm (i.e., wrote column-wise addition or subtraction on the paper) across all intervention sessions.

Error Analysis of Cam's WPS Accuracy

Table 11 presents Cam's WPS accuracy in all eight problem situations during baseline and intervention sessions. The results showed that during the baseline condition, Cam only solved combine-whole-unknown, join-ending-unknown, and separate-beginning-unknown problems. During the intervention condition, Cam maintained his performance on the previous three problems and improved on combine-part-unknown, join-change-unknown, and separate-ending-unknown problems. However, he was still struggling with join-beginning-unknown and separate-change-unknown problems after the intervention (see Table 11).

Table 11. Cam's WPS Accuracy in Different Problem Situations During Baseline and Intervention Conditions

Task		Sample Problem Situations	Baseline	Intervention
Combine	Part unknown	Rina and Bill have 80 stickers altogether. Rina says that he has 33 stickers. How many stickers does Bill have?	0%	83.33%
	Whole unknown	Mike made 19 hats in his class. His brother Taylor made 69 hats. How many hats do they make altogether?	100%	100%
Change-join (Increase)	Part unknown (**Change unknown**)	Alex had 80 rubber bands. Then his sister Ivy gave him some more rubber bands. Now he has 99 rubber bands. How many rubber bands did Ivy give Alex?	0%	66.67%
	Part unknown (**Beginning/ start unknown**)	Emily had several crayons. Then, her friend Paul gave her 60 more crayons. Now Emily has 76 crayons. How many crayons did Emily have in the beginning?	0%	33.3%
	Whole unknown (**Ending/result unknown**)	James has 36 candy bars. His brother gives him 26 more candy bars. How many candy bars in total does James have now?	100%	100%
Change-separate (Decrease)	Part unknown (**Ending/result unknown**)	Jessica had 55 fish in her fish tank. Then, one day she lost 16 of them. How many fish does Jessica have now?	0%	66.67%
	Part unknown (**Change unknown**)	Leslie had 40 tickets. She sold some tickets on the first day of her trip. On the second day, she had only 19 tickets left. How many tickets did Leslie sell on the first day?	0%	33.33%
	Whole unknown (**Beginning/start unknown**)	Tony had many stamps. Then he gave away 45 of his stamps to his brother Denzel. Now, Tony has 18 stamps. How many stamps did Tony have in the beginning?	100%	100%

Laura: (1) WPS accuracy: Laura made an immediate improvement in WPS performance in the first session of the intervention condition from 12.50% correct to 39.58% correct, demonstrating a change in level between conditions (Gast & Ledford, 2014). Her WPS accuracy performance increased from an average of 31.25% correct in the baseline phase to an average of 60.42% correct in the intervention phase. During the intervention phase, Laura's WPS accuracy performance increased from 39.58% correct to 75% correct, with the median of 62.50% correct. The trend direction of her data was accelerating. The Tau-U effect size of Laura's WPS accuracy was 1.0 (90% CI = [0.328, 1.0]).

Table 12. Laura's Percentage Correct on the WPS Accuracy and Percentage of WPS Process Elements During the Intervention Condition

	Session 1	Session 2	Session 3	Session 4	Session 5
WPS accuracy	50%	56.25%	68.75%	87.50%	93.75%
WPS process	39.58%	62.50%	62.5%	62.50%	75%

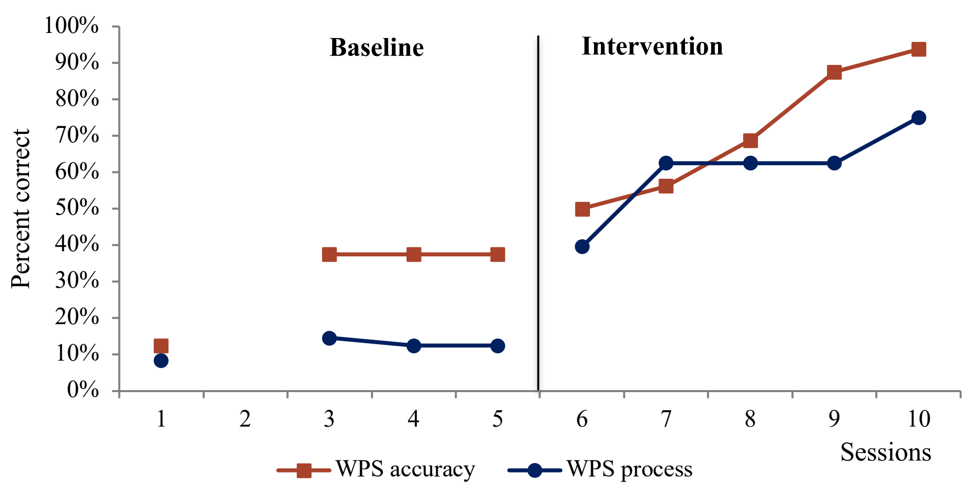

Figure 11. Laura's Performance in WPS Accuracy and Processes in the Baseline and the Intervention Condition

(2) WPS process: The performance of Laura's WPS process made a quick improvement in the first session of the intervention phase from 10.42% correct to 50% correct, and ranged from 50% correct to 93.75% correct, which demonstrating a change in level between conditions (Gast & Ledford, 2014). It increased from an average of 11.98% correct in the baseline phase to an average of 71.25% correct in the intervention phase, with the median of 68.75% correct. The trend direction of her data was accelerating. Based on the video footage of the intervention, the major difficulty of Laura's WPS process was calculation. Specifically, she always made mistakes on calculation when digits in the minuend were smaller than the digits in the same place in the subtrahend. As the result, in addition to teaching PPW strategies, the teacher also spent a long time teaching calculating-related knowledge, such as the meanings of mathematics terms (i.e., minuend, subtrahend, and place value) during the first three intervention sessions.

Laura independently read the problems without prompts after Session Three. She also highlighted each sentence after reading one sentence, following what the teacher taught during the scaffolded COMPS intervention. To recall, the teacher taught reading the problem as the first step, and visualization (e.g., highlighting, labeling) as the second step of solving the problems. Occasionally, Laura artfully combined Steps One and Two together. As shown in Figure 11, although her performance on WPS process did not have many changes, her WPS accuracy improved quickly from 68.75% correct to 87.50% correct. Such progress indicated that Laura was able to accurately solve for the word problems with gradually reduced intervention and scaffolding help. Moreover, she somehow regulated the steps of solving word problems, which indicated that the intervention instruction improved her procedural fluency, and made it more flexible, efficient, and appropriate for herself on solving problems.

Step Five of the intervention is solution planning. This step helped her to solve for the final solution based on the COMPS model used for story mapping. Laura was able to rewrite an equation using the form of "*a* = number – number," but she could not get the correct solution without using her calculator. Figure 12 shows two examples of Laura's algorithm solving processes. Specifically, when subtracting, Laura randomly subtracted the smaller number from the larger one digit by digit, ignoring the place value with ones or tens in the minuend and the subtrahend, which means that when she calculated 15 - 8, she used 8 - 5 to get the ones place value by using the minuend subtracted from the subtrahend. Similarly, in the example of 45 - 27 = 22, she used 7 - 5 for the ones place value, and used 4 - 2 to get the tens place value.

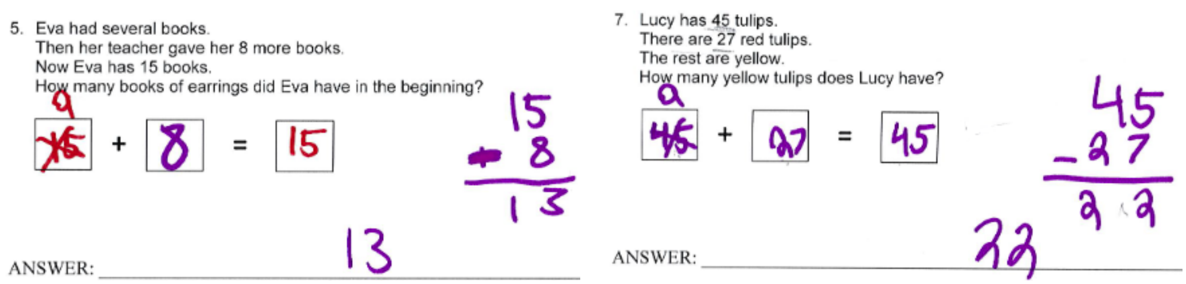

Figure 12**.** Examples of Laura's Algorithm Process

Like Cam, Laura's counting strategy was double-counting by ones. Instead of using finger counting, Laura wrote her counting processes on the worksheet using both standard algorithm and marking down the number of ones as tallies to mark how many times she had added (see Figure 13). From Session Five, she used standard algorithm only (i.e., used column-wise addition or subtraction on the paper) to calculate the solution.

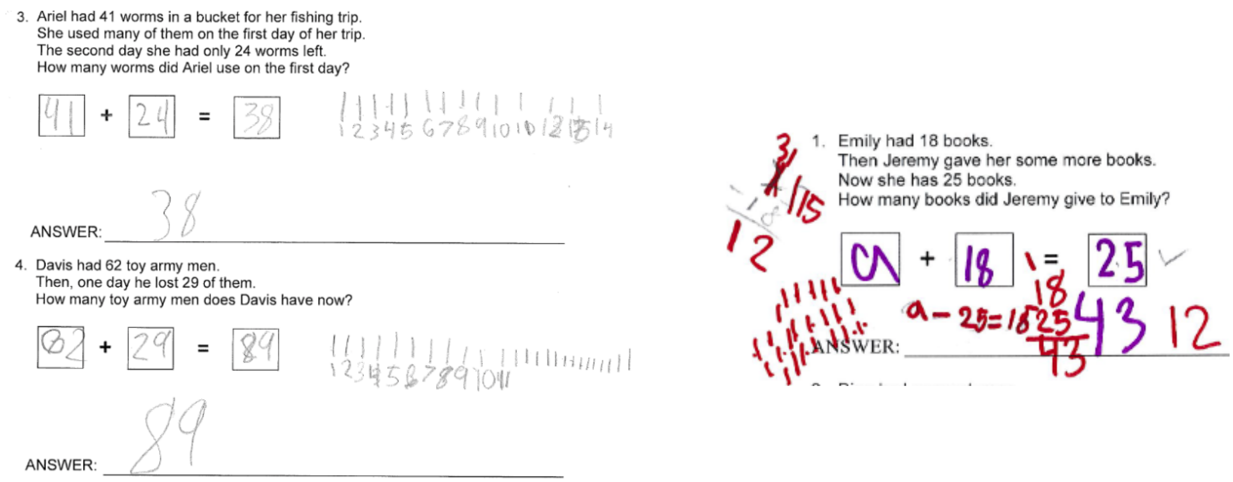

Figure 13. Cases of Laura's Counting Strategy

Moreover, Laura also had difficulty rewriting the equation to solve for unknown quantity *a* (i.e., Step Five, solution planning) during the first two sessions, so she occasionally wrote two versions of the equations. As shown in Figure 14, she could use the representation to label the problem (using letter T to represent the total number, letter P as the part, and letter *a* as unknown quantity). Regarding the rewriting step, she first wrote 11 minus 28, and found that the minuend is smaller than the subtrahend, and she then switched the order of the minuend and subtrahend (see Figure 13).

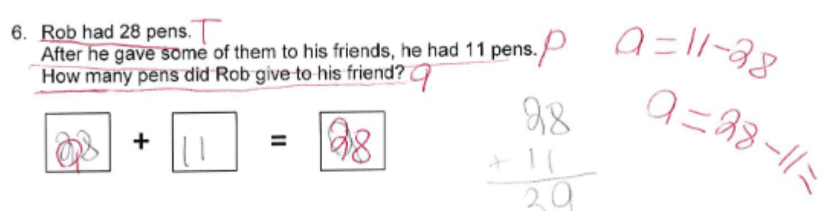

Figure 14. An Example of Laura's WPS Process in Solution Planning

Another important finding is Laura's progress with self-generated actions or answers without prompts. For example, during Session Five, Laura automatically read a new problem, wrote letter *a* next to the sentence (e.g., *How many stairs does Amy go up*), and pointed out which sentence tells about the total or the part. She can also explain why she thought that sentence tells about the total (see Example 5).

Example 5 (in Session Five):
L: (read without prompts). *Together, Amy and Pat went up a total of 92 stairs. Pat says that she goes up 58 stairs. How many stairs does Amy go up*?
L: This is *a*. (wrote letter *a* next the sentence "How many stairs does Amy go up")
L: This is total. (Pointed to 92 using pencil and wrote 92 on the right side of the PPW equation)
T: Good job. Why is 92 total?
L: Because they went up together.

Error Analysis of Laura's WPS Accuracy

Table 13 shows Laura's WPS accuracy in all problem situations during baseline and intervention sessions. Like Cam, Laura did well with combine-whole-unknown, join-ending-unknown, and separate-beginning-unknown problems during the baseline condition.

During the intervention condition, although Laura's WPS performance improved on most types of problems, she still had difficulty with combine-part-unknown, join-change-unknown, join-part-unknown, and separate-ending-unknown problems (see Table 13).

Table 13. Laura's WPS Accuracy in Different Problem Situations During Baseline and Intervention Conditions

Task	Word problem situations	Baseline	Intervention
Combine	Part unknown	0%	50%
	Whole unknown	100%	100%
Change-join (Increase)	Part unknown (**Change unknown**)	0%	50%
	Part unknown (**Beginning/start unknown**)	0%	40%
	Whole unknown (**Ending/result unknown**)	100%	100%
Change-separate (Decrease)	Part unknown (**Ending/result unknown**)	0%	40%
	Part unknown (**Change unknown**)	0%	60%
	Whole unknown (**Beginning/start unknown**)	75%	90%

Sara: (1) WPS accuracy: Sara made an improvement in WPS performance at the first session of the intervention phase from 12.5% correct to 25% correct, demonstrating a change in level between conditions (Gast & Ledford, 2014, see Table 14). The average of Sara's WPS performance increased from 25% correct in the baseline phase to 66.67% correct in the intervention phase. Over the following sessions, her performance increased gradually and finally reached 87.5% correct in the sixth session. The trend direction of her data was accelerating. Sara's median score was 75% correct and the Tau-U effect size of WPS accuracy was 0.8 (90% CI = [0.199, 1.0]).

Table 14**.** Sara's Percentage Correct on the WPS Accuracy and Percentage of WPS Process During the Intervention Condition

	Session 1	Session 2	Session 3	Session 4	Session 5	Session 6
WPS accuracy	25%	56.25%	75%	75%	81.25%	87.50%
WPS process	47.92%	58.33%	62.50%	52.08%	66.67%	60.42%

(2) WPS process: Sara's WPS process made a quick improvement once provided the scaffolded COMPS treatment in the intervention, her performance increased above her baseline score, which demonstrating a change in level between conditions (Gast & Ledford, 2014). The trend direction of her data was accelerating. Her median score was 59.38% correct. The major difficulty for Sara is calculation. She did well with representation, which indicated that Sara could understand the semantics of the word problems but needed more interventions and scaffoldings on calculation and explanation.

Moreover, Sara was able to map numbers onto the PPW equation during the second intervention session, but she could not rewrite the equation to get the final solution. Similar to Laura, Sara also had difficulties with mathematics calculation, especially using column-wise addition and subtraction algorithms to solve for the unknown quantity in the first two sessions. For example, as shown in Figure 15, Sara wrote 45 + 27 = 45 first, and changed 45 to the variable *a* in the box designated for one of the two parts. Then she used both finger counting and a column-wise subtraction algorithm to get 45 – 27 = 22. We can observe that Sara used 7 minus 5 to get the ones place value and 4 minus 2 to get the tens place value. Basically, she subtracted the smaller number from the larger one digit by digit, completely ignoring the place value or regrouping when subtracting (see Figure 15). Sara made the same mistakes that Laura made regarding misconceptions of place value and regrouping.

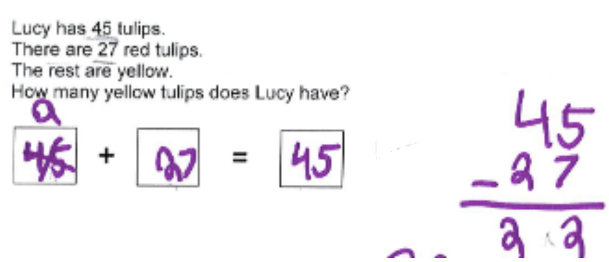

Figure 15. An Example of Sara's Counting Strategies in Session One

Another finding of interest was that among all three participants, only Sara wanted to share her interesting stories with the teacher before and during the intervention, although all students received the same expectations and instructions. Examples 6 and 7 were the interactions between Sara and the teacher in Sessions One and Three.

Example 6 (in Session One):

Sara (S): I wonder why do we get cavities?

T: Do you know why? What is a cavity?

S: Like if you eat candy and then like, you eat a lot, your teeth will be, eh, and you need to go to the dentist. That’s what a cavity is.

T: Oh, I see.

S: Have you been to the dentist before?

T: Yes, I have.

S: When you were a kid?

T: Yes.

Example 7 (in Session Three):

S: Do you know Toys “R” Us is about to close?

T: I don’t know. Where is the Toys “R” Us?

S: Toys “R” Us is close to the mall, it’s NOT in the mall, it’s CLOSE to the mall. It’s like Walmart, and like Target, big stores. Do you know Amazon?

T: Yes.

S: Is Amazon in the United States?

T: It’s a website, Amazon is a website.

S: Is it a store?

T: I think so. They also have stores. Have you been there?

S: My mom has.

S: Do you have Amazon on your phone?

T: Yes, I do.

Regarding visualization skills, Sara initiated visualization skills to label each component in Session Five (e.g., use *t* to represent total, use *p* to represent part, and *a* to represent the unknown quantity). She also wrote "Part" and "Part" in the left two boxes of PPW equation, and "total" in the right side of the PPW equation (see Figure 16).

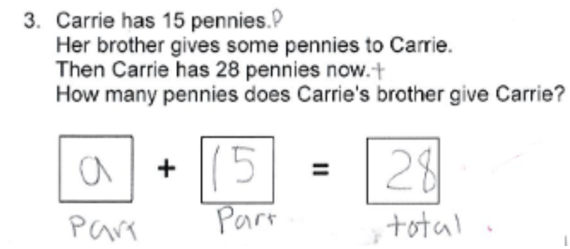

Figure 16. One Case of Sara's Strategy on Using Visualization and Representation

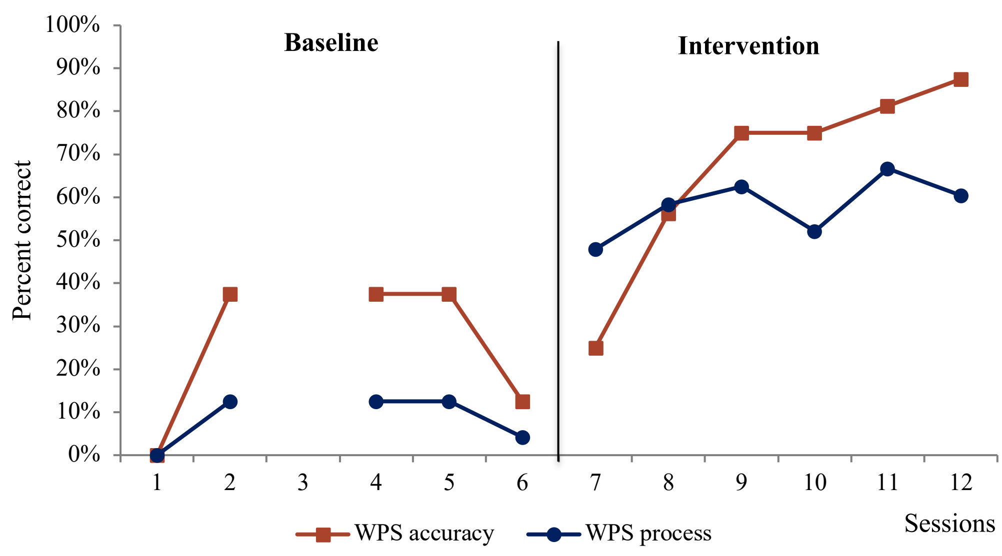

Figure 17. Sara's Performance in WPS Accuracy and Processes in the Baseline and the Intervention Condition

Error Analysis of Sara's WPS Accuracy

Table 15 shows Sara's WPS accuracy in all problem situations during baseline and intervention sessions. Comparing her performance from baseline to intervention, she improved her WPS abilities in all types of problems, especially in combine-part-unknown, join-change-unknown, and separate-ending-unknown problems. However, Sara needed improvement in join-beginning-unknown and separate-change-unknown problems (see Table 15).

Table 15. Sara's WPS Accuracy for Different Problem Situations During Baseline and Intervention Conditions

Task	Word problem situations	Baseline	Intervention
Combine	Part unknown	0%	83.33%
	Whole unknown	60%	66.67%
Change-join (Increase)	Part unknown (**Change unknown**)	0%	66.67%
	Part unknown (**Beginning/start unknown**)	0%	50%
	Whole unknown (**Ending/result unknown**)	50%	75%
Change-separate (Decrease)	Part unknown (**Ending/result unknown**)	0%	75%
	Part unknown (**Change unknown**)	0%	50%
	Whole unknown (**Beginning/start unknown**)	50%	83.33%

4.3 Post-Test

Post-tests were taken immediately after the intervention was completed. In terms of WPS accuracy, the median scores of both Cam and Laura in the post-test (87.5% correct for Cam and 87.5% correct for Laura) were above their medians during the intervention condition (62.5%

correct for Cam and 62.5% correct for Laura). A small decrease was seen in Sara's median score of her WPS accuracy in the post-test (71.88% correct) compared with her performance in the intervention condition (75% correct).

Regarding WPS process performance, all three participants' median scores in the post-test (38.55% correct for Cam, 40.63% correct for Laura, and 28.13% correct for Sara) were lower than their medians in the intervention condition (60.42% correct for Cam, 68.75% correct for Laura, and 75% correct for Sara), which indicated that the students reduced the need for scaffolds and regulated their problem-solving strategies based on their own needs.

Table 16**.** Percentage Correct for the Three Participants' WPS Accuracy and Process Performance in the Post-Test

	Cam		Laura		Sara	
	WPS accuracy	WPS process	WPS accuracy	WPS process	WPS accuracy	WPS process
Post-test 1	87.50%	35.42 %	87.50%	45.83%	56.25%	20.83%
Post-test 2	87.50%	41.67%	87.50%	35.42%	87.50%	35.42%

In terms of the WPS accuracy, all three participants kept their improvement, and their median scores were all above 65% correct (78.13% correct for Cam, 81.25% correct for Laura, and 65.63% correct for Sara). However, Cam's WPS process performance decreased from a median of 60.42% correct in the intervention to 38.55% correct in the post-test. Laura's WPS process performance decreased to a median of 36.46% correct, and Sara's WPS process median score decreased to 27% correct.

4.4 Generalization Tests

The generalization tests were administered before and after the intervention. On the AWPS test, which involved eight types of PPW problems as well as six types of Additive Compare problems. Sara's WPS accuracy performance was the best. Her score increased from 28.57% correct to 92.86% correct after the intervention condition. Cam's performance increased from 32.14% correct to 75% correct following the intervention. Laura also increased her WPS accuracy performance in the post-test from 21.43% correct to 67.86% correct (see Table 17).

Table 17. Percentage Correct for the Three Participants' Performance in the AWPS Generalization Test

	Cam		Laura		Sara	
	Baseline	Post-test	Baseline	Post-test	Baseline	Post-test
AWPS	32.14%	75%	21.43%	67.86%	28.57%	92.86%

Figure 18 presents the participants' performance in WPS accuracy and process on the criterion tests during the baseline, intervention, and post-test assessment phases.

The combined weighted Tau-U score for all three participants' WPS accuracy effect of the scaffolded COMPS intervention was 0.93 (95% CI = [0.4720, 1.0]), indicating the treatment was highly effective for all three participants' WPS accuracy performance (Parker et al., 2011).

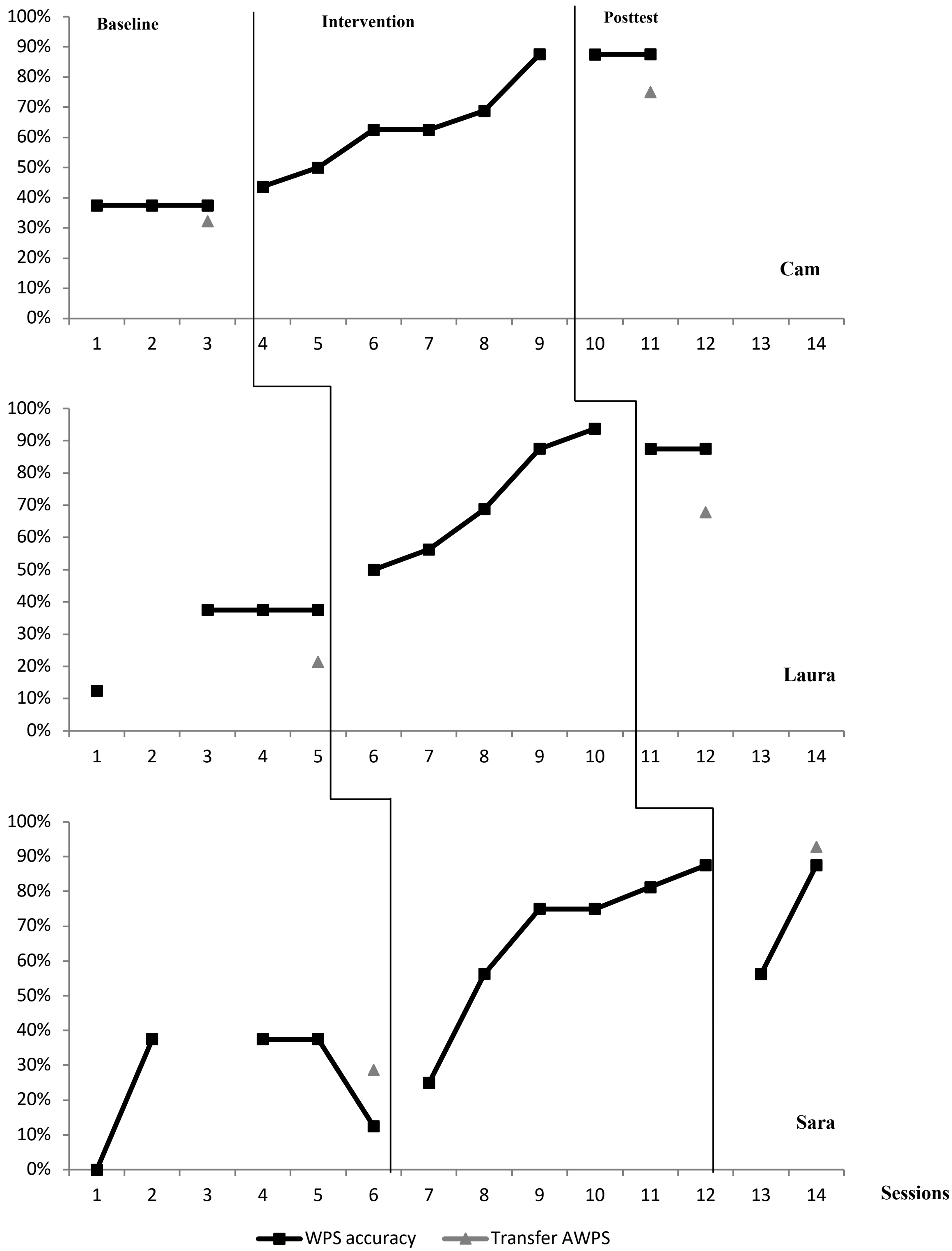

Figure 18. Percentage Correct on the Criterion Test and Generalization Test During Baseline, Intervention, and Post-Test Across the Three Participants

Note. WPS = word problem solving; AWPS = additive word problem solving test

4.5 Relations Between Students' WPS Accuracy and WPS Process

The correlation between the students' performance on WPS accuracy and WPS process were analyzed using Pearson's correlation and linear regression in R. Figure 19 presents the correlations between WPS accuracy and carryout of WPS process for all participants during the baseline, intervention, and post-test conditions. Results showed that there was a significant positive association (Pearson correlation = 0.698, $p < 0.001$) between WPS accuracy and WPS process performances for all three participants, which means that students with higher WPS process perform better in their WPS accuracy.

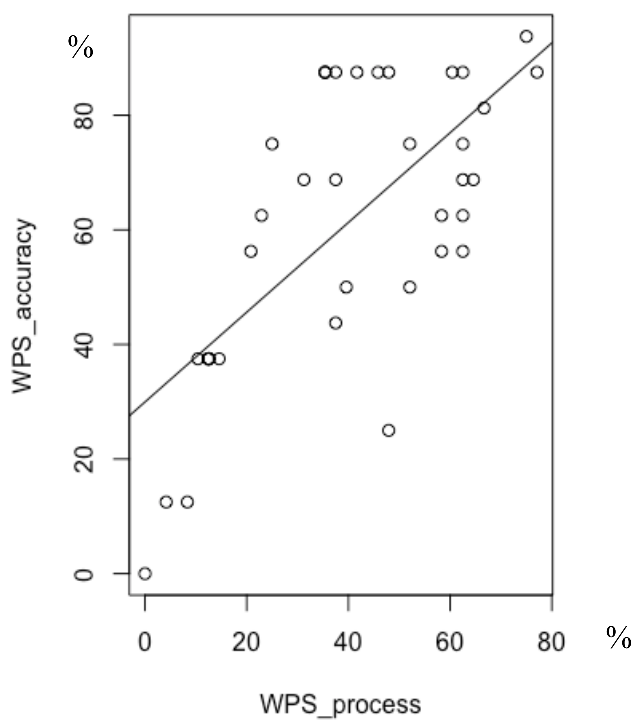

Figure 19**.** Correlation Between WPS Accuracy and WPS Process Performances

4.6 Fidelity of Implementation: Teacher's Usage of Scaffolds

As proposed in the methods section, the teacher's usage of scaffolds was further coded and analyzed to verify the fidelity of implementation for the intervention. First, the frequency of four scaffolds used by the teacher for all participants across sessions was coded and calculated. The teacher did not provide any scaffolds during baseline or post-test phases, and thus this section only analyzed the teacher's scaffolding usage during the intervention condition.

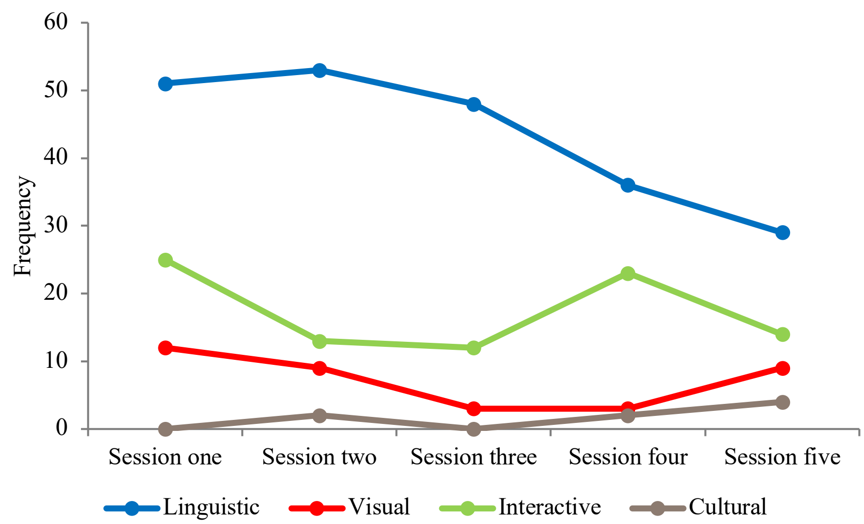

Figure 20**.** Frequency of Four Scaffolds Used by Teacher Across Sessions

The frequency of scaffold usage by the teacher across five sessions was analyzed. As shown in Figure 20, during the first session, the most common scaffold used was linguistic, but its use was gradually reduced from Session Two. The other three scaffolds had a decreasing trend across the sessions. Moreover, within each session, the linguistic scaffolding was the most frequently used by the teacher (see Figure 21). Therefore, there is a need to further distinguish the types of linguistic scaffolds the teacher used during the intervention.

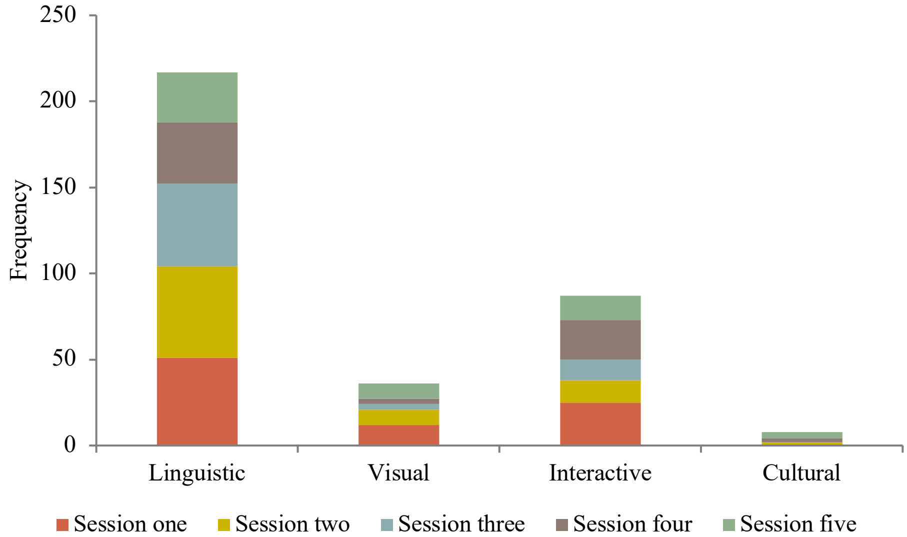

Figure 21. Frequency of Scaffolds Used by Teacher for All Sessions

Based on the 75-page transcript, I further created the subcategories of linguistic scaffolding used in the discourse between the teacher and students during the intervention. Subcategories of linguistic scaffolds were identified and coded as paraphrasing/summarizing/clarifying, teaching vocabulary, COMPS, and precorrection/prompts (see Table 18 for examples).

Table 18. Subcategories of Linguistic Scaffolds Used by the Teacher

Type of Linguistic Scaffolds	Definition	Example
a. Paraphrasing/ summarizing/ clarifying	1) Paraphrasing: Restate the word problems using different words to more concisely and clearly.	1) T: Let's read the problem again. *Mason had 89 flags on Monday. By Friday, he had sold 76 of the flags. How many flags does Mason have now?* Which is also asking how many flags does Mason have left, right?
	2) Summarizing: Identify key ideas in word problems or in what the student said.	2) (*A cart has 14 books. There are 6 on the bottom shelf. The rest are on the top shelf. How many books are on the top shelf*?) T: This cart has a bottom shelf, where had 6 books, and a top shelf, where had some books. And they are asking how many books on the top shelf. Student (S): It's eight.
	3) Clarifying: Ask the student to explain and identify some unclear aspect of what she/he said in order to check their understanding.	3) T: Why you wrote *a* in a box designated for the total in the equation? S: Because we don't know it. T: We don't know the part or we don't know the whole? S: We don't know the whole.
b. Teaching vocabulary	Define vocabulary that showed in word problems; Teach new mathematic terms used for solving word problems.	T: 32 is correct. This is the whole amount because Ben and Dina have 32 cups altogether. *Altogether* means *total*, when we combine, we put things *altogethe*r, which is the same meaning as *total*.
c. COMPS	Use COMPS-related phrases such as "*which sentence tells about the whole*," "*which sentences tell about the two parts*," and "*what are we asked to solve for*" (Xin, 2012) to teach problem solving.	(*Ben and Dina have 32 cups. Ben has 15 cups. How many cups does Dina have*?) T: Which sentence tells about total number of cups? S: (Pointed to "*Ben and Dina have 32 cups*") T: Correct. Which sentences tell about the two parts? S: This (pointed out the second sentence). T: Correct. Which is another part? Cam: How many?
d. Precorrection/ prompts	Precorrect/prompt students' mistakes on solving problems.	T: Can you use the calculator to verify your answer? Remember we are always using the minuend minus the subtrahend.

As shown in Table 19 and Figure 22, among four subcategories of linguistic scaffolds, the teacher provided the highest frequency of COMPS instruction for all participants during the intervention, while the second-highest type was precorrection/prompts. I have combined the three categories of paraphrasing, summarizing, and clarifying as one group because they all have the similar purpose of checking students' comprehension. The teacher used COMPS-related phrases (such as "which sentence tells about the whole or the part") to help students understand the elements and relationships expressed in the word problems and scaffolded them to make connections between mathematical problems and the PPW diagram equation. Precorrection or prompts were intertwined with COMPS instructions. The purpose of precorrection or prompts was reminding students to follow the problem-solving process steps. Teaching vocabulary was focused on teaching mathematical terms that may be difficult for students to comprehend or solve for word problems. The results showed that teaching mathematical vocabulary was used least by the teacher during all sessions.

Table 19**.** Frequency of Subcategories of Linguistic Scaffoldings Used by the Teacher

	COMPS	Precorrection/ prompts	Paraphrasing/ summarizing/clarifying	Teaching vocabulary
Session One	28	12	6	3
Session Two	25	17	5	3
Session Three	24	16	3	2
Session Four	20	6	5	1
Session Five	19	5	1	0
Total	116	56	20	9

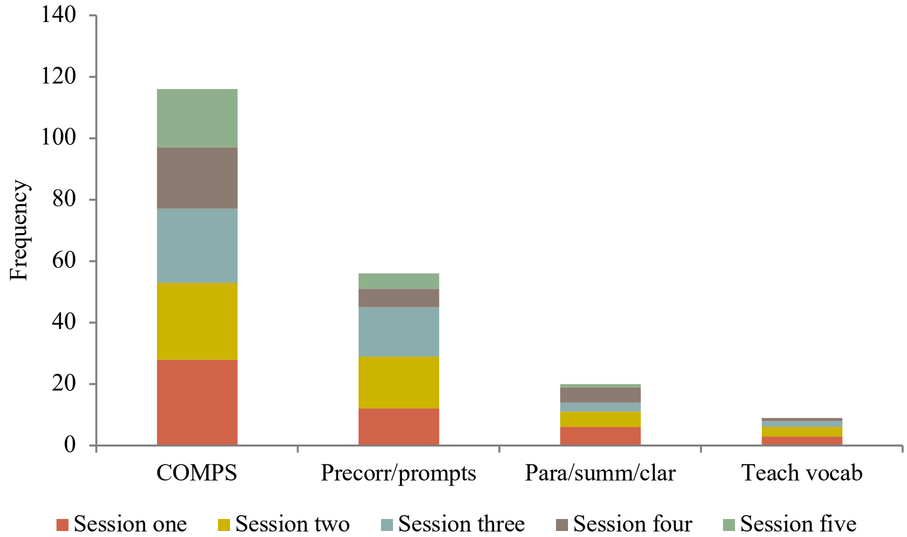

Figure 22. Frequency of Subcategories of Linguistic Scaffoldings Used By the Teacher

4.7 Social Validity

The average scores on the survey questionnaire of all three participants ranged from 2.33 to 5. The summary of the results on each of the items on the social validity form for the three participants is in Table 20. Most of the students reported that they thought participating in this program was helpful for their word problem solving skills. For example, for the statement "How helpful were the mapping of information and Part-Part-Whole diagrams in understanding and solving addition and subtraction word problems," Cam chose "very helpful," and both Laura and Sara chose "helpful." In the early intervention stage, some individuals seemed frustrated when they first learned the PPW strategy, but they all agreed that they will recommend the PPW diagram strategy to their friends, and they believed that the strategy was useful in general.

Table 20**.** Students' Perspectives on the Scaffolded COMPS Intervention

Social validity questions	**Cam**	**Laura**	**Sara**	**Average rating**
1. I like the PPW diagram strategy that I learned	5	4	2	3.67
2. The mapping of information and Part-Part-Whole diagrams in understanding and solving addition and subtraction word problems were helpful.	5	4	4	4.33
3. I will recommend the strategy I learned to my friend.	5	4	3	4.00
4. I am using the PPW diagram strategy in other classes.	5	1	1	2.33
5. In general, I think the PPW diagram strategy is useful.	5	4	3	4.00

CHAPTER 5. DISCUSSION

The present study aims to promote the mathematics word problem solving ability of English learners with LDM. The intervention program was designed to provide explicit sequenced WPS process instruction by using instructional scaffoldings (i.e., linguistics, interactive, visual, and culturally responsive scaffolds) along with the COMPS approach to support students' WPS abilities.

Based on the visual analysis, the functional relation between the scaffolded COMPS intervention and the performance of ELs with LDM on solving addition and subtraction mathematics word problems is evident. The data showed that all participants started with random sequence in WPS process and low-level performance in WPS accuracy on the PPW criterion tests. Once the scaffolded intervention was introduced, all three participants showed immediate improvement and had an increasing trend on both WPS accuracy and WPS process performances. More specifically, Cam had difficulties with representation and explanation during the first two intervention conditions, but by Session Four he was able to use visualization to map the PPW equation and provide complete explanations regarding why that sentence tells about the part or the total. By the end of the intervention, all participants could use column-wise addition or subtraction to get the solution. Not surprisingly, both Laura and Sara's common errors were misconceptions of regrouping and place value principles. After using linguistic and visual scaffolding during the intervention, Sara could self-correct her answers, and Laura could read and visualize the word problems without prompts from Session Four of the intervention.

In sum, both Cam and Laura had strong Tau-U effect sizes on WPS accuracy (Tau-U = 1.0), which means that there were no overlaps of performance between the baseline and the intervention conditions. The intervention also showed a strong effect (Tau-U = .8) on Sara's WPS accuracy performance (Tau-U = 1.0). WPS process, however, was aimed to support the students' procedural fluency on solving word problems. It seemed that once the students could use the intervention strategy as a natural part of solving problems, they started to skip one or two steps (e.g., visualization and explanation) that they no longer relied on when solving word problems, and such reductions did not affect the performance of WPS accuracy on the post-test and generalization tests.

As for the transfer tests, all participants showed significant improvement in the AWPS test from the pre-tests to post-tests. It indicated that the scaffolded COMPS intervention contributed to participants' conceptual understanding of solving both part-part-whole and additive comparing problems. The generalization test not only involved eight PPW problems that were included in the criterion tests, but it also involved six types of compare problems. The results showed that although the teacher did not introduce how to solve the comparing problems, students can transfer the representation skills to solve for compare-change-unknown problems (e.g., *Tom has 15 books. Jim has 33 books more than Tom. How many books does Jim have*?), as well as compare-difference-unknown problems (e.g., *Jessie worked 55 hours in a month. Sam worked 15 hours. How many more hours did Jessie work than Sam*?).

5.1 Effectiveness of Multiple Scaffolds

According to Van de Pol, Volman, and Beishuizen (2010), measuring scaffolding is difficult because of its dynamic nature in interactions. Therefore, the coding methods (i.e., using the four types of scaffolds as well as four subcategories of linguistic scaffolds to code teacher-student discourse) used in this present study may contribute to the field of teacher education and mathematics education. Moreover, the findings from this study demonstrate that using multiple scaffolds contributes to the fields of special education, mathematics education, and English education. According to Montague, Bos, and Doucette (1991), students with learning disabilities did not use paraphrasing or visualization when they attempted to solve word problems. Previous research also found that students with learning disabilities were disadvantaged in representing word problems, which requires transferring both linguistic and numerical information to math equations (Montague & Applegate, 1993; Xin, Jitendra, & Deatline-Buchman, 2005; Zawaiza & Gerber, 1993). In this study, multiple scaffolds, particularly linguistic ones, were integrated with the COMPS approach as the intervention for teaching WPS to ELs with LDM. As is the purpose of the COMPS approach, it provided ELs with LDM with conceptual understanding of the word problems for comprehending the mathematical relations described in a range of real-word-situated word problems, and how the mathematical relations connected to mathematics model equations. As shown in the Results section, the teacher provided linguistic scaffolds, visual scaffolding, interactive scaffolding, and culturally responsive scaffolding for the participants during all intervention conditions. The findings of this present study show that the teacher gradually decreased the frequency of scaffolding supports during the intervention across sessions, which indicated that the students needed fewer scaffolds from the teacher and could regulate the WPS process on their own. The following explains the roles of each scaffold used in the present study.

5.1.1 The Role of Linguistic Scaffolding

Based on the findings of this study, the teacher mainly used four categories of linguistic scaffoldings during the intervention: paraphrasing/summarizing/clarifying, teaching vocabulary, COMPS, and precorrection/prompts.

Paraphrasing/Summarizing/Clarifying. Since students with learning disabilities lack necessary paraphrasing abilities (Montague et al., 1991), the teacher emphasized paraphrasing or summarizing the complicated sentences of problems to the participants when teaching the semantics of word problems. On the other hand, if the students had unclear self-explanations, the teacher usually asked the student to paraphrase or clarify what they said. Using paraphrasing, summarizing, and clarifying, the teacher will have a clear picture about what students thought and what difficulties they have and, therefore, provide them with appropriate instructions based on their needs.

Teaching Vocabulary. Teaching vocabulary was not commonly used by the teacher compared to the other three categories. Once the teacher noticed that the students had difficulty comprehending the problems, the teacher taught them the definitions and usage of mathematics terms/vocabulary (such as minuend, subtrahend, altogether, several, place value, algorithm, regroup) before and during the intervention. Example 8 is an excerpt of the conversation in which the teacher found that Laura switched the minuend and subtrahend when rewriting the equation from the PPW model in Step Five. Thus, the teacher taught Laura the usage of the minuend and the subtrahend during Session One.

Example 8 (in Session One):
(*Kevin has 17 spelling words to learn this week. Then he learned 9 words. How many words does he still need to learn?*)

T: To solve for the unknown quantity a, since a is the part, you need to use the total minus the other part. For this problem, we have $a = 17 - 9$.

T: The first number 17 in this subtraction problem is called the minuend. From the minuend we separate the subtrahend. The subtrahend is the number after the minus sign for this problem. What is the subtrahend?

L: 9.

T: Correct. Remember that we always use the minuend minus the subtrahend, which is also the total minus the part. So, for this problem, you use 17 minus 9 instead of 9 minus 17. Remember that?

L: Yes.

Another excerpt of conversation is between the teacher and Sara during Session Three (see Example 9). In the word problem (*Adam had several insects in his collection. Then Rex gave him 50 more insects. Now Adam has 67 insects. How many insects did Adam have in the beginning?*), there was the vocabulary word "several," which was irrelevant information for solving the problem, but Sara did not know the meaning of the word. If students cannot comprehend the semantics of a word problem, they may have difficulty solving it, especially those students who are English learners (Lei et al., 2020b; Lei, 2016). Thus, providing vocabulary instruction before and during teaching problem-solving strategies is critical for English learners.

Example 9 (in Session Three):
(*Adam had several insects in his collection. Then Rex gave him 50 more insects. Now Adam has 67 insects. How many insects did Adam have in the beginning*?)

T: Do you remember if we don't know the amount, we use the letter?

S: *a*

T: Yes, very good. Adam had several insects in his collection. Several means a number that is not large but is greater than two. Do we know "several" means how many insects? Do we know it?

S: No.

T: That is correct, we don't know it, so we use *a* to represent the number of insects Adam had in the beginning.

COMPS. As the present study's conceptual framework, COMPS was used most frequently during the intervention. Findings of this study showed that during the baseline, all students simply grabbed the two numbers and used addition operations for all types and situations of problems, including *change-join*, *change-separate*, and *combine,* regardless of the semantics and constructions of the word problems. Cam and Laura also used keyword strategies to solve for word problems during the baseline condition. Teaching the COMPS instruction, the teacher repeated use of the five W's, simple phrases such as "which sentence tells about the total/part" to teach students reading comprehension of word problems (Xin, 2012). Such simple sentence instruction as well as the unique Word Problem (WP) Story Grammar builds a linguistic scaffold environment for teaching English learners to comprehend different types of situations of word problems and to represent word problems through mathematical model equations (i.e., Part-Part-Whole diagram equation) (Xin, 2012). In contrast to story grammar in reading comprehension literature, WP Story Grammar used a "mathematical word problem structure that is common across a range of WP situations for a particular problem type" (Xin et al., 2020, p. 119). To illustrate, using a generalizable mathematical model "Part + Part = Whole" (PPW) and prompted by WP Story Grammar questions (e.g., "*Which sentence tells about the total* and *which statement tells about the part*", Xin et al., 2020, p. 112), students are able to identify the three elements (Part, Part, and Whole) of a variety of story problems and thus, they would construct the PPW mathematical model equations for problem solving (Xin et al., 2020).

Precorrections or prompts. In addition to providing COMPS instruction, precorrections or prompts were used frequently during the intervention. Precorrections were different from other types because the purpose of providing prompts is to help students regulate the problem-solving process rather than explicitly teaching students how to solve the problems in detail. For example, before students solved the problem, the teacher prompted them to read the problem first. From Session Three, both Cam and Sara could read the problem and rewrite the equation independently and spontaneously without the teacher's prompts. Based on the findings of the present study, such prompts were gradually diminished through the intervention sessions.

5.1.2 The Role of Visual Scaffolding

According to Walqui and vanLier (2010), visual scaffolds are the most commonly used scaffold with ELs because this type of scaffold is easy for teachers to access. Previous research has demonstrated the effectiveness of visualization as a powerful representation process for solving mathematics word problems for students with LD (Van Garderen & Montague, 2003). As a component of the intervention in this study, visual scaffolds were implemented through highlighting, labeling the sentences, or writing the corresponding letters "W" or "P" next to the sentence to represent the elements of the problem story ("whole", "part", or "a"). In this study, using visual scaffolds helped ELs with LDM to convert the linguistic information into mathematical model equations more easily (Xin et al., 2020). Furthermore, previous research has noted that students with LDM have limited working memory and short attention spans (Zentall, 2014), so using visual scaffolding such as labeling or highlighting serves the role of "permanent prompt" (Snell, 1993) in linking the complex mathematical content knowledge with equations.

5.1.3 The Role of Interactive Scaffolding

Interactive scaffolding, as used in the present study, refers to the strategic back and forth between the teacher and ELs to cultivate and facilitate the instructions of the target content. The study only included conversations with at least three turns between the teacher and students, which was to make sure that the participant provided enough information to show their understanding. In the following example, the teacher and the student had four turns of discourse. During this conversation, the teacher asked the students the elements of the word problem and asked them to explain their answers. Through interactive scaffolding, the teacher taught students how to use WP story grammar prompting questions to regulate the problem representation and solving process.

Example 10 (in Session One):

(*Ben and Dina have 32 cups altogether. Ben has 15 cups. How many cups does Dina have?*)

Cam: *Ben and Dina have 32 cups altogether. Ben has 15 cups. How many cups does Dina have?*

T: Could you tell me which sentence tells about the whole?

Cam: This is whole. (Pointed out the last sentence and put number 47 into PPW equation)

T: Read this problem, tell me which number represents the *whole*?

Cam: 32.

T: Correct, so you can write 32 in the box designated for the total in the equation as a *whole*. So this number is not 47, it should be 32. Could you tell me why it is 32?

Cam: Because we don't know it right now.

T: We know it right now. Let's read the problem again. Can you read this sentence with me? *Ben and Dina have 32 cups.* So, Ben and Dina have how many cups?

Cam: 32.

T: 32 is correct. This is the whole amount because Ben and Dina both have 32 cups altogether.

5.1.4 The Role of Culturally Responsive Scaffolding

Using culturally relevant teaching examples in the WPS intervention to help students more easily recognize the problem situation and generate correct solutions was recommended by previous research (Kim et al., 2015). The present study also included Hispanic names such as Rosita, Mariana, Daniel, Lucas, and Natalia as the content in the word problems. Cam was the only student who recognized the Hispanic name used in the problem (see Example 11). Such real-word examples in the word problems provide more motivations for ELs to learn mathematics WPS.

Example 11 (in Session Three):

Rosita had 65 books. Then, Rosita gave some books to Jay. Now, Rosita has 27 books left. How many books did Rosita give to Jay?

Cam: Rosita, Rosita, Rosita.

T: Do you know the meaning?

Cam: Rosita, it's a name. Rosita, Rosita, Rosita.

T: Yes, it's a name. Rosita means rose, it's a Hispanic girl's name.

On the other hand, CRS as the component of the intervention was also involved in the teacher-student daily discourse. Sara was the student who shared her daily life and experiences with the teacher during the intervention. Sara talked about her experience seeing a dentist because of her cavities, her feelings about a local toy store that was about to close, and some daily stories about her big brother. Such communications built a close rapport between the student and the teacher during the intervention, and also created a good learning environment.

5.2 Implications of the Study

5.2.1 The Development of WPS Accuracy and WPS Process Performances

Findings of the study showed that students' WPS process performance positively correlated with their WPS accuracy performance. In this study, the WPS self-regulation checklist was created based on the theoretical framework COMPS (Conceptual Model-based Problem Solving) (Xin, 2012), which required students to read the problems, comprehend the underlying problem structure, and understand the relationship between the key elements in the problems and the mathematical model equations. Such a self-regulation checklist was able to guide students "through the process as they execute the solution" (Montague, 2006). The final step of this process was to find the solution, which also tested students' WPS accuracy performance. Therefore, in order to accurately solve for the word problems, students need to carry out these self-regulation steps. To some extent, the self-regulation checklist is a series of steps that may lead students to accurate problem solving. Previous research has indicated that students with LD have difficulty selecting, organizing, and executing appropriate strategies when solving word problems and, thus, they need help acquiring self-regulation strategies that "underline effective and efficient problem solving" (Montague, 2006, p. 91).

It was surprising that both Cam and Sara’s performance on WPS accuracy and process had an interaction in the second session of the intervention (see Figure 18). One potential explanation is that the progress of the WPS accuracy was slower than the WPS process in the beginning of the intervention whereas, after the second intervention session, the performance of WPS accuracy increased quickly but students did not need more supports on process. According to Raduan (2010), students usually compare the current mathematical problems with prior problems that they learned before and select which knowledge they will use or not use during the process of solving problems. From the findings of the present study, we also found that some of the WPS steps were not commonly used during the post-tests by the participants (i.e., Step Two: visualization, and Step Three: explanation). Although students skipped two or three steps, their performance on WPS accuracy was still at high levels. Therefore, the finding indicated that after several sessions of practice, students were able to modify the WPS regulation checklist and only keep the steps necessary for them. Those four necessary WPS steps were reading, mapping, rewriting, and solving.

In addition to choosing her own WPS steps, Laura also integrated her own visualization during the post-tests. As shown in Figure 23, instead of writing “part,” “whole,” and “a” next to the sentences to help understanding, she used two “a”s to represent the unknown quantity (i.e., the number of candy bars Lucas gave Sam), which was also part of the problem (*Sam had 8 candy bars. Then Lucas gave him some more candy bars. Now he has 15 candy bars. How many candy bars did Lucas give Sam*?). Moreover, based on the video footage, we also observed that after reading the problem, Laura wrote “*a*” next to the sentence “*Then Lucas gave him some more candy bars*” and said, “This is *a*.” Then she wrote the second “*a*” next to the question and said, “This is also *a*.” Such findings support the suggestions of NCTM (2014) that students need experience and opportunities to create their own problem-solving strategies and processes.

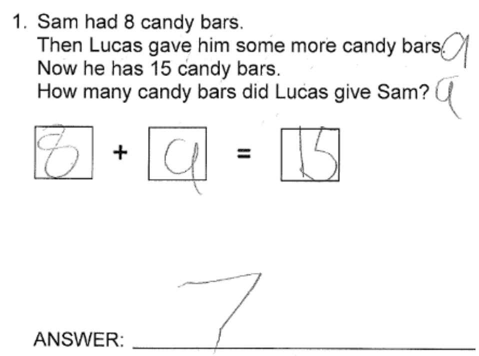

Figure 23. Laura’s WPS Process Development

5.2.2 Error Analysis of Students’ WPS Performance

Error analysis has been used by educators to analyze students’ mathematical errors in order to correct their misconceptions, thereby improving mathematical instructions (Mastropieri & Scruggs, 2002). Researchers also proposed that evaluating students’ work to identify an appropriate instruction focusing on correcting their errors is one of the core principles in remedial education for all students, especially for those with LD (Fuchs, Fuchs, & Hamlett, 1994; Riccomini, 2005). This study targeted to two main aspects to analyze the participants’ WPS errors during the intervention. The first aspect was analyzing the problem types on which students made the most mistakes. Through comparing their baseline and intervention performances, we found that the join-beginning-unknown problem (*Bruce had several cookies. Then Tom gave him 30 more cookies. Now Bruce has 68 cookies. How many cookies did Bruce have in the beginning*?) was the most difficult problem type for all three participants. As previous research has documented, students were taught “keyword strategy” at their school (Cathcart, Pothier, Vance, & Bezuk, 2006; Xin et al., 2020). The keyword in the above problem is “*gave him 30 more,*” which implies an operation of “addition,” so all three students used 30 + 68 to solve for this problem. This finding suggested that teachers need to provide more scaffolding on teaching students conceptual understanding of word problems rather than using keyword strategies to solve problems regardless of the mathematical relations in word problems.

Table 21. Error Analysis of Students on Solving Word Problems

Common Errors	Sample Task	Sample Correct Expression	Student Answer
Keyword strategy	*Bruce had several cookies. Then Tom gave him 30 more cookies. Now Bruce has 68 cookies. How many cookies did Bruce have in the beginning?*	a + 30 = 68 a = 68-30 = 38	30 + 68 = a a = 30 + 68 = 98
Misconceptions of regrouping	*Rosita had 27 books. Then her sister Laura gave her some more books. Now she has 43 books. How many books did Laura give Rosita?*	a + 27 = 43 a = 43 - 27 = 16	a + 27 = 43 a = 43 - 27 = 24

The second aspect of error analysis in this study was related to counting strategy. Several researchers have reported that subtraction is particularly problematic for students, especially for problems requiring borrowing (e.g., Drucker, McBride, & Wilbur, 1987; Riccomini, 2005). In this study, misconceptions regarding regrouping or place value, as one type of bridging errors, were the mistakes made most by the participants (see Table 21 for examples).

Misconceptions of regrouping in the context of this study also used the term “smaller-from-larger (SFL)” (National Research Council, 2002) to define if students made mistakes when subtracting the smaller digit in a column from the larger digit, regardless of which appears in the top line. Through analyzing the intervention video transcripts and students’ paperwork, it was found that Laura and Sara misunderstood regrouping and place value principles. In the sample task shown in Table 21 (*Rosita had 27 books. Then her sister Laura gave her some more books. Now* *she has 43 books. How many books did Laura give Rosita?*), the solution should be 43 - 27 = 16, but Sara and Laura used 7 - 3 to get 4 instead of 13 - 7 = 6. Such calculation skills were taught in Steps Five and Six during the intervention to those students who made mistakes.

Laura used the keyword strategies (“more” is addition, and “less” is subtraction) as her main counting strategy during her baseline. Cam and Laura could not solve for problems with *combine*, *change-join*, and *change-separate* situations with part or smaller group unknown during the baseline since these conditions did not use the addition operation. Therefore, teaching conceptual understanding is critical, especially to solve part or smaller group unknown problems.

5.2.3 The Decision-Making Tree for ELs Teachers

Students usually go through two phases when solving problems: “interpretation of the mathematical language” and “the calculation process” (Raduan, 2010, p. 3838). Moreover, as suggested by the CCSS, EL teachers should provide instructions that target balancing and connecting both conceptual and procedural knowledge (Moschkovich, 2013). In order to help teachers determine the remedial steps that may enable ELs with LDM to learn WPS more effectively, a decision-making tree is provided in this study (see Figure 24). It focuses on teaching conceptual and procedural development to ELs with LDM, targeting understanding of both mathematical language and calculation skills. The teacher first poses one situation of a word problem, and if a student gets a correct answer, the teacher will provide different type/situated problems (i.e., combine, change-join, and change-separate) to that student; however, if the student gets a wrong answer, the teacher should provide appropriate instructions on conceptual understanding (which include Steps One to Four). If this student gets the correct answer on conceptual understanding, the teacher should move to teach more specific and explicit instructions on counting strategies, which are Steps Five and Six. Figure 24 provides the flowchart of the decision-making tree for ELs teachers.

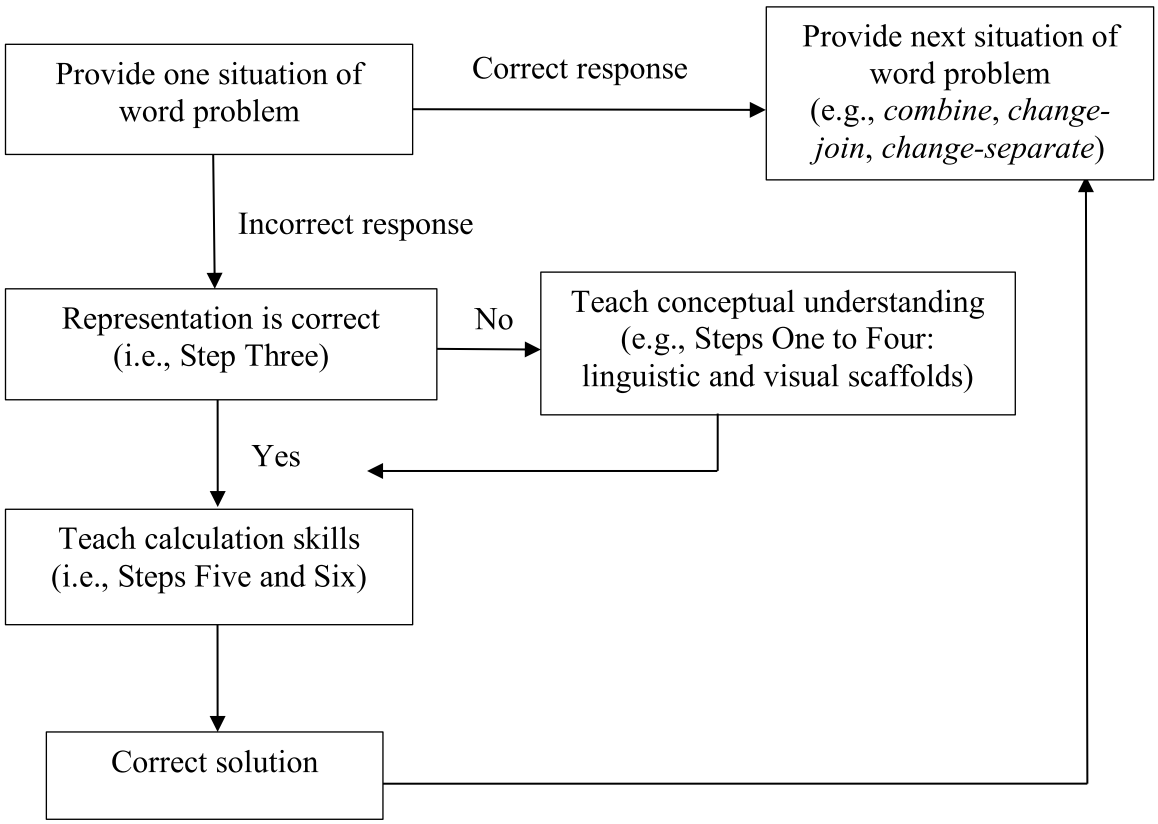

Figure 24. The Decision-Making Tree for EL Teachers Teaching WPS

5.3 Limitations and Implications for Future Research

The present study contains several limitations to inform future research. First, this study only involved a sample size of three participants, which restrained the generalization of the intervention effects as well as the accuracy of correlation effects between WPS accuracy and WPS process performances. Future studies might explore the intervention effects using a group design to examine the relations between the scaffolded COMPS intervention and ELs' performance changes in both WPS accuracy and process. Second, since all participants were English language learners, their English proficiency may affect their performance on comprehending and solving word problems. However, the present study did not measure the English proficiency or vocabulary skills before the intervention and, the intervention did not systematically teach language or vocabulary to the participants based on their individual needs. Future research should consider ELs' English proficiency as potential confounding variables to the intervention study. Finally, the findings of the present study show that the least common scaffold used by the teacher was culturally responsive scaffolding (CRS), which is because only one student really wanted to share her stories with the teacher. Future research may involve CRS in multiple aspects of instruction, such as involving group activities for ELs to communicate their culture and hobbies, and/or posing word problems based on their own stories.

Footnote

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APPENDIX A. TEACHING SCRIPT FOR SCAFFOLDED COMPS INTERVENTION

6. Mom baked 62 cookies for the class party.
   Dad baked 26 cookies for the party.
   How many cookies did the parents bake altogether?

$\square + \square = \square$

Teacher (T): We are going to talk about some interesting stories together. You can tell me any interesting things you want to let me know about your daily life. There are several colors of pens and blank paper in front of you on your desk. You can draw on the paper if you want.

Student (S): Share interesting things......

T: Great! It is very interesting. We are going to use your story for today's math problems. We will read the problem first—remember in PPW problems, there are three parts, one part and another part will make up the whole—the total (point to corresponding diagram boxes). Here is a cheat sheet that you can use to solve the PPW problems—we will follow the steps on the cheat sheet. (show the cheat sheet). Can you read the problem for me?

S: "*Mom baked 62 cookies for the class party. Dad baked 26 cookies for the party. How many cookies did the parents bake altogether?*"

T: Let's think about what is *altogether*?

S: Total...

T: That's right, when we combine, we put things altogether, which is the same meaning as total. This problem is asking the total number of cookies the parents baked. Do we know the number of cookies the parents baked?

S: No, we don't.

T: Correct, we do not know how many cookies the parents baked. Let's highlight this sentence and use letter *a* to represent the unknown quantity. Then we write letter "*a*" next to this sentence.

T: What we are asked to solve for?

S: "Do we know the number of cookies the parents baked?"

T: That's right, this is the unknown quantity that we need to solve for. So, from reading the problem, do you know which quantity is the total?

S: "*a*"

T: Great, let's write *a* in the box for "whole" or total on the right side of the equals sign by itself.

T: Now you will help me highlight the two sentences that talk about the two parts: the cookies Mom baked and the cookies Dad baked. Write the letter "P" next to these sentences.

S: (Students highlight both of these sentences: "*Mom baked 62 cookies for the class party. Dad baked 26 cookies for the party.*") (T will help if students are confused).

T: Now you will find the number of cookies Mom baked and the number of cookies Dad baked. So, which sentences tell about the two parts?

S: "*Mom baked 62 cookies for the class party. Dad baked 26 cookies for the party.*"

T: Good job! Now you will help me to write them in the two boxes marked as "Part;" these two boxes are side by side connected by the plus sign on the same side of the equals sign. Remember, on the right side of the equals sign is the unknown quantity "*a*".

T: Wonderful. We are done with the representation! The completed diagram tells us the number of cookies Mom baked and the number of cookies Dad baked make up the cookies the parents baked altogether. Now let us solve for the unknown quantity in the equation.

T: Let's read the diagram—"part plus part equals the whole or total." To solve for the unknown part, write "a =…" (e.g. a = 62 + 26).

Student will try mental math first to solve for the unknown quantity *a*.

T: Let's see if you can solve for the unknown quantity *a* in your head. (If not successful, ask student to use the calculator to solve for the unknown. Teacher also will monitor that student is correctly using a calculator.)

Student uses a calculator and finds the answer "88".

T: Do you think 88 is a reasonable answer to this problem?

S: Yes, it is a reasonable answer.

T: Ok, let's write down the answer on the answer line. "*Parents baked 88 cookies altogether*".

APPENDIX B. FIDELITY CHECKLIST

Elements	Implemented [(1) present; (2) absent]
1. Students read the problem with the teacher, ensure students' understanding that this is a PPW problem.	
2. Example question: Bobby had 87 cards in a box. (W) He gave some cards to his brother, Jeff. (P) Then, Bobby had 62 cards left in his box. (P) How many cards did he give to his brother Jeff? (*a*) Ask the student to identify and highlight "which sentence tells about the 'whole'" (in red), provide instruction as needed. Ask the student to write a letter "W" next to the sentence. Ask the student to identify and highlight "the two sentences that talk about the two parts—the cards he gave Jeff and the cards he had left" (in yellow), provide instruction as needed. Ask the student to write a letter "P" next to each of the sentences.	
3. Ask students "What are we asked to solve for?", provide instruction as needed. Tell that is the unknown quantity that we need to solve for. Write a letter "*a*" next to that sentence.	
4. Ask students to represent all numbers into the PPW diagram equation, provide instruction as needed.	
5. Ask students to rewrite the equation to solve for the unknown quantity "*a*", provide instruction as needed.	
6. Ask students to solve for the unknown quantity "*a*" in the equation, provide instruction as needed.	

APPENDIX C. SOCIAL VALIDITY QUESTIONNAIRE (FOR PARTICIPANTS)

Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
1	2	3	4	5

1) I like the PPW diagram strategy that I learned.

____

2) The mapping of information and Part-Part-Whole diagrams in understanding and solving addition and subtraction word problems were helpful.

____

3) I will recommend the strategy I learned to my friend.

____

4) I am using the PPW diagram strategy in other classes.

____

5) In general, I think the PPW diagram strategy is useful.

____

REFERENCES

Abedi, J., Hofstetter, C. H., & Lord, C. (2004). Assessment accommodations for English language learners: Implications for policy-based empirical research. *Review of Educational Research*, *74*(1), 1–28. doi:10.3102/00346543074001001

Asher, J. J. (1969). The total physical response approach to second language learning. *The Modern Language Journal*, *53*(1), 3–17. doi:10.1111/j.1540-4781.1969.tb04552.x

Association for Supervision and Development & Centers for Disease Control and Prevention. (2014). *Whole school, whole community, whole child: A collaborative approach to learning and health*. Retrieved from https://www.ascd.org/ASCD/pdf/siteASCD/publications/wholechild/wscc-a-collaborative-approach.pdf

August, D., Spencer, S., Fenner, D. S., & Kozik, P. (2012). *The evaluation of educators in effective schools and classrooms for all learners*. Washington, DC: American Federation of Teachers.

Beed, P. L., Hawkins, E. M., & Roller, C. M. (1991). Moving learners toward independence: The power of scaffolded instruction. *The Reading Teacher*, *44*(9), 648–655.

Belland, B. R. (2017). *Instructional scaffolding in STEM education: Strategies and efficacy evidence*. Springer Nature.

Boulineau, T., Fore, C, Hagan-Burke, S., & Burke, M. D. (2004). Use of story-mapping to increase the story-grammar text comprehension of elementary students with learning disabilities. *Learning Disability Quarterly*, 27, 105–121. doi:10.2307/1593645

Bradley, B. A., & Reinking, D. (2011). A formative experiment to enhance teacher-child language interactions in a preschool classroom. *Journal of Early Childhood Literacy*, *11*(3), 362–401. doi:10.1177/1468798411410802

Brand, T., S., Favazza, A. E., & Dalton, E. M. (2012). Universal design for learning: A blueprint for success for all learners. *Kappa Delta Pi Record*, *48*(3), 134–139. doi:10.1080/00228958.2012.707506

Brown, J. E., & Ault, P. C. (2015). Disentangling language differences from disability: A case study of district-preservice collaboration. *Journal of Multilingual Education Research*, *6*, 111–136.

Brown, J., & VanLehn, K. (1980). Repair theory: A generative theory of bugs in procedural skills. *Cognitive Science*, *4*(4), 379–426. https://doi.org/10.1207/s15516709cog0404_3

Bruner, J. S. (1960). *The process of education*. Harvard University Press.

Bryant, D. P., Bryant, B. R., Roberts, G., Vaughn, S., Pfannenstiel, K. H., Porterfield, J., & Gersten, R. (2011). Early numeracy intervention program for first-grade students with mathematics difficulties. *Exceptional Children*, *78*, 7–23. doi:10.1177/001440291107800101

Bryant, D. P., Bryant, B. R., & Smith, D. D. (2019). *Teaching students with special needs in inclusive classrooms*. Thousand Oaks, CA: Sage Publications

Canobi, K. H. & Bethune, N. E. (2008). Number words in young children's conceptual and procedural knowledge of addition, subtraction and inversion. *Cognition*, 108, 675–686. doi: 10.1016/j.cognition.2008.05.011

Carrasquillo, A., & Rodrigues, V. (2002). *Language minority students in the mainstream classroom*. Clevedon, UK: Multilingual Matters. doi:10.21832/9781853595660

Cathcart, W. G., Pothier, Y. M., Vance, J. H., & Bezuk, N. S. (2006). *Learning mathematics in elementary and middle schools: A learner-centered approach* (4th ed). Upper Saddle River, NJ: Person Merrill Prentice Hall.

Cazden, C. B. (2001). *Classroom discourse: The language of teaching and learning*. Pearson Education Canada.

Chiu, C. W., & Pearson, P. D. (1999). *Synthesizing the effects of test accommodations for special education and limited English proficient students*. Paper presented at the National Conference on Large Scale Assessment, Snowbird, UT.

Collier, C. (2011). *Seven steps to separating difference from disability*. Thousand Oaks, CA: Corwin. doi:10.4135/9781452219424

de Araujo, Z., Roberts, S. A., Willey, C., & Zahner, W. (2018). English learners in K–12 mathematics education: A review of the literature. *Review of Educational Research, 88*(6), 879–919. doi:10.3102/0034654318798093

Dixon, R., Carnine, D., & Kameenui, E. J. (1993). Using scaffolding to teach writing. *Educational Leadership*, *51*(3): 100–101.

Dowker, A. (2008). Individual differences in numerical abilities in preschoolers. *Developmental Science*, 11, 650–654. doi: 10.1111/j.1467-7687.2008.00713.x

Drucker, H., McBride, S., & Wilbur, C. (1987). Using a computer-based error analysis approach to improve basic subtraction skills in the third grade. *The Journal of Educational Research*, *80*(6), 363–365.

Durkin, K., Rittle-Johnson, B., & Star, J. R. (2011). *Procedural flexibility matters for student achievement: How procedural flexibility relates to other outcomes*. Paper presented at the 14th Biennial Conference of the European Association for Research on Learning and Instruction, August, Exeter, England, UK.

Every Student Succeeds Act (ESSA) Act of 2015, Pub. L. No. 114-195, § 115, Stat. 1425 (2015).

Fisher, D., & Frey, N. (2010). *Guided instruction: How to develop confident and successful learners*. ASCD.

Fuchs, L. S., Fuchs, D., Compton, D. L., Hamlett, C. L., & Wang, A. Y. (2015). Is word-problem solving a form of text comprehension? *Scientific Studies of Reading*, *19*(3), 204–223. https://doi.org/10.1080/10888438.2015.1005745

Fuchs, L. S., Fuchs, D., & Hamlett, C. L. (1994). Strengthening the connection between assessment and instructional planning with expert systems. *Exceptional Children*, *61*(2), 138–146.

Fuchs, L. S., Gilbert, J. K., Fuchs, D., Seethaler, P. M., & Brittany, B. L. (2018). Text comprehension and oral language as predictors of word-problem solving: Insights into world-problem solving as a form of text comprehension. *Scientific Studies of Reading*, *22*(2), 152–166. https://doi.org/10.1080/10888438.2017.1398259

Fuchs, L. S., Powell, S. R., Cirino, P. T., Schumacher, R. F., Marrin, S., Hamlett, C. L., ... & Changas, P. C. (2014). Does calculation or word-problem instruction provide a stronger route to prealgebraic knowledge? *Journal of Educational Psychology*, *106*(4), 990.

Gast, D. L., & Ledford, J. R. (2014). *Single case research methodology: Applications in special education and behavioral sciences*. Routledge.

Gee, J. (2015). *Social linguistics and literacies: Ideology in discourses* (5th ed.). London, UK: Routledge. doi:10.4324/9781315722511

Gerena, L., & Keiler, L. (2012). Effective intervention with urban secondary English language learners: How peer instructors support learning. *Bilingual Research Journal*, *35*(1), 76–97. doi:10.1080/15235882.2012.667372

Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., & Flojo, J. (2009). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research*, *79*(3), 1202–1242.

Gibbons, P. (2002). *Scaffolding language, scaffolding learning: Teaching second language learners in the mainstream classroom*. Portsmouth, NH: Heinemann.

Gibbons, P. (2003). Mediating language learning: Teacher interactions with ESL students in a content-based classroom. *TESOL Quarterly*, *37*(2), 247–273. doi:10.2307/3588504

Gibbons, P. (2008). “It was taught good and I learned a lot”: Intellectual practices and ESL learners in the middle years. *Australian Journal of Language and Literacy*, *31*(2), 155–173.

Gibbons, P. (2014). *Scaffolding language, scaffolding learning: Teaching English language learners in the mainstream classroom* (2nd ed.). Portsmouth, NH: Heineman.

Goffman, E. (1967). *Interaction ritual: Essays on face to face behavior*. Garden City, NY: Doubleday.

Gottlieb, M. (2012). *Essential actions: A handbook for implementing WIDA's framework for English language development standards*. Madison, WI: WiDA Consortium.

Gottlieb, M. (2013). *Essential actions*: *A handbook for implementing WIDA's framework for English language development standards*. Madison, WI: WiDA Consortium.

Gottlieb, M. (2016). *Assessing English language learners: Bridges to educational equity: Connecting academic language proficiency to student achievement*. Thousand Oaks, CA: Corwin.

Gunning, T. G. (2010). *Creating literacy instruction for all students*. Boston, MA: Pearson.

Indiana Department of Education. (2006). *Indiana math standards*. Retrieved from https://web.archive.org/web/20101007024333/http://dc.doe.in.gov/Standards/AcademicStandards/PrintLibrary/docs-math/2006-math-grade05.pdf

Indiana Department of Education. (2014). *Indiana math standards*. Retrieved from https://www.doe.in.gov/sites/default/files/standards/2014-07-21-math-g5-architecturewith-front-matterbr.pdf

Jaciw, A. P., Hegseth, W. M., Lin, L., Toby, M., Newman, D., Ma, B., & Zacamy, J. (2016). Assessing impacts of math in focus, a "Singapore math" program. *Journal of Research on Educational Effectiveness*, *9*(4), 473–502. doi:10.1080/19345747.2016.1164777

Jitendra, A. K., & Star, J. R. (2012). An exploratory study contrasting high-and low-ability students' word problem solving: The role of schema-based instruction. *Learning and Individual Differences*, *22*(1), 151–158.

Jordan, N. C. (2007). Do words count? Connections between mathematics and reading difficulties. In D. B. Berch & M. M. M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 107-120).

Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child Development*, *74*(3), 834–850. https://doi.org/10.1111/1467-8624.00571

Kangas, S. E. N. (2014). When special education trumps ESL: An investigation of service delivery for ELLs with disabilities. *Critical Inquiry in Language Studies*, *11*(4), 273–306. doi:10.3102/0002831214544716

Kangas, S. E. N. (2017). "That's where the rubber meets the road": The intersection of special education and bilingual education. *Teachers College Record*, *119*(7), 1–36.

Kangas, S. E. N. (2019). English learners with disabilities: Linguistic development and educational equity in jeopardy. In X. Gao (Ed.), *Second handbook of English language teaching. Springer international handbooks of education.* New York, NY: Springer. doi:10.1007/978-3-319-58542-0_48-1

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in school children*. University of Chicago Press.

Lei, Q. (2016). A research of English article errors in writings by Chinese ESL learners. *Teaching Together, Learning Together: English Language Teaching Ideas*, *1*, 65-75.

Lei, Q., Xin, Y. P., Morita-Mullaney, T., & Tzur, R. (2018). Analyzing a discourse of scaffolds for mathematics instruction for an ELL with learning disabilities. In T. E. Hodges, G. J. Roy, & A. M. Tyminski (Eds.), *Proceedings of the 40th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 460–467). Greenville, SC: University of South Carolina & Clemson University. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&site=eds-live&db=eue&AN=133248617

Lei, Q., & Xin, Y. P. (2019). *Mathematics word problem solving scaffolding interventions for English learners at risk for learning disabilities: A research synthesis*. Poster presented at 2019 CEC conference.

Lei, Q., Mason, R. A., Xin, Y. P., Davis, J. L., David, M., & Lory, C. (2020a). A meta-analysis of single-case research on mathematics word problem-solving interventions for English learners with learning disabilities and mathematics difficulties. *Learning Disabilities Research & Practice*, *35*(4), 201–217.

Lei, Q., Xin, Y. P., Morita-Mullaney, T., & Tzur, R. (2020b). Instructional scaffolds in mathematics instruction for English learners with learning disabilities: An exploratory case study. *Learning Disabilities: A Contemporary Journal*.

Lesh, R., Cramer, K., Doerr, H. M., Post, T., & Zawojewski, J. S. (2003). Model development sequences. In R. Lesh & H. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning and teaching* (pp. 35–58). Mahwah, NJ: Erlbaum.

Madison Metropolitan School District. (2015). MMSD culturally and linguistically responsive strategies: Project overview.

Mariani, L. (1997). Teacher support and teacher challenge in promoting learner autonomy. *Perspectives. A Journal of TESOL Italy*, 23(2).

Mayer, R. E. (1998). Cognitive, metacognitive, and motivational aspects of problem solving. *Instructional science*, *26*(1), 49–63.

McCloskey, M. L. (2005). Visual scaffolding to support ELL reading. *Keeping the Promise*, 1–7.

McGhee, B. M. D. (2011). *Profiles of elementary-age English language learners with reading-related learning disabilities (LD) identified as speech and language impaired prior to, at, or after identification as LD* (Doctoral dissertation). Retrieved from ProQuest Dissertations & Theses Global (1432171790).

Montague, M. (2006). Self-regulation strategies for better math performance in middle school. *Teaching mathematics to middle school students with learning difficulties*, 89-107.

Montague, M., & Applegate, B. (1993). Middle school students' mathematical problem solving: An analysis of think-aloud protocols. *Learning Disability Quarterly*, *16*, 19–30.

Montague, M., Bos, C., & Doucette, M. (1991). Affective, cognitive, and metacognitive attributes of eighth-grade mathematical problem solvers. *Learning Disabilities Research & Practice*, *6*, 145–151.

Montague, M., Enders, C., & Dietz, S. (2011). Effects of cognitive strategy instruction on math problem solving of middle school students with learning disabilities. *Learning Disability Quarterly*, *34*(4), 262–272.

Montague, M., & Jitendra, A. K. (Eds.). (2006). *Teaching mathematics to middle school students with learning difficulties* (Vol. 2). New York, NY: Guilford Press.

Moran, A. S., Swanson, H. L., Gerber, M. M., & Fung, W. (2014). The effects of paraphrasing interventions on problem-solving accuracy for children at risk for math disabilities. *Learning Disabilities Research & Practice*, *29*(3), 97–105.

More, C. M., Spies, T. G., Morgan, J. J., & Baker, J. N. (2016). Incorporating English language learner instruction within special education teacher preparation. *Intervention in School and Clinic*, *51*(4), 229–237. doi:10.1177/1053451215589183

Morita-Mullaney, T., & Singh, M. (2019). Obscuring English learners from state accountability: The case of Indiana's language blind policies. *Educational Policy*, 1–25. doi:10.1177/0895904818823751

Morita-Mullaney, T., & Stallings, L. (2018). Serving Indiana's emergent bilingual immigrant (EBI) youth: A collective case study of EBI teacher educators. *The Teacher Educator*, *53*(3), 293–312. doi:10.1080/08878730.2018.1462422

Moschkovich, J. (2013). Principles and guidelines for equitable mathematics teaching practices and materials for English language learners. *Journal of Urban Mathematics Education*, *6*(1), 45–57.

Moschkovich, J. (2015). Scaffolding student participation in mathematical practices. *ZDM: International Journal on Mathematics Education*, *47*, 1067–1078. doi:10.1007/s11858-015-0730-3

Narciss, S., & Huth, K. (2006). Fostering achievement and motivation with bug-related tutoring feedback in a computer-based training for written subtraction. *Learning and Instruction*, *16*(4), 310–322. https://doi.org/10.1016/j.learninstruc.2006.07.003

National Center for Education Statistics. (2017). *NAEP data explorer*. Retrieved from https://nces.ed.gov/pubs2017/2017144.pdf.

National Council of Teachers of Mathematics (NCTM). (2000). Principles and standards for school mathematics electronic version. Retrieved from http://standards.nctm.org

National Council of Teachers of Mathematics (NCTM). (2014). Procedural fluency in mathematics: A position of the National Council of Teachers of Mathematics. Retrieved from https://www.nctm.org/uploadedFiles/Standards_and_Positions/Position_Statements/Procedural%20Fluency.pdf

Non-Regulatory Guidance. (2016). *English learners and Title III of the elementary and secondary education Act (ESEA), as amended by the Every Student Succeeds Act (ESSA)*. Retrieved from https://www2.ed.gov/policy/elsec/leg/essa/essatitleiiiguidenglishlearners92016.pdf

Northwest Evaluation Association, (n.d.). RIT Reference Charts. Retrieved October 20, 2021, from https://www.nwea.org/content/uploads/2014/10/RIT-Reference-Brochure-Standard-Digital-MAY15-2.pdf

OECD. (2012). Programme for international student assessment (PISA): Results from PISA 2012. Retrieved from http://www.oecd.org/pisa/keyfindings/PISA-2012-results-US.pdf

Olsen, L. (2010). *Reparable harm: Fulfilling the unkept promise of educational opportunity for California's long term English language learners*. Long Beach, CA: Californians Together.

Orosco, M. J. (2013). The development of a math strategy in Spanish for Latino English language learners at risk for math disabilities. *International Journal for Research in Learning Disabilities*, *1*(2), 86–109. https://doi.org/10.1177/0731948713504206

Orosco, M. J., Swanson, H. L., O'Connor, R., & Lussier, C. (2013). The effects of dynamic strategic math on English language learners' word problem solving. *The Journal of Special Education*, *47*(2), 96–107. https://doi.org/10.1177/0022466911416248

Orosco, M. (2014a). Word problem strategy for Latino English language learners at risk for math disabilities. *Learning Disability Quarterly*, *37*(1), 45–53. https://doi.org/10.1177/0731948713504206

Orosco, M. (2014b). A math intervention for third grade Latino English language learners at risk for math disabilities. *Exceptionality*, *22*(4), 205–225. https://doi.org/10.1177/0731948713504206

Pfannenstiel, K. H., Bryant, D. P., Bryant, B. R., & Porterfield, J. A. (2015). Cognitive strategy instruction for teaching word problems to primary-level struggling students. *Intervention in School and Clinic*, *50*(5), 291–296.

Piaget, J. (2006). The stages of the intellectual development of the child. In B. A. Marlowe & A. S. Canestrari (Eds.), *Educational psychology in context: Readings for future teachers* (pp. 98–106). Thousand Oaks, CA: Sage Publications, Inc.

Powell, S. R., & Fuchs, L. S. (2018). Effective word-problem instruction: Using schemas to facilitate mathematical reasoning. *Teaching Exceptional Children*, *51*(1), 31–42.

Raduan, I. H. (2010). Error analysis and the corresponding cognitive activities committed by year five primary students in solving mathematical word problems. *Procedia-Social and Behavioral Sciences*, *2*(2), 3836–3838.

Riccomini, P. J. (2005). Identification and remediation of systematic error patterns in subtraction. *Learning Disability Quarterly*, *28*(3), 233–242.

Rittle-Johnson, B. (2017). Developing mathematics knowledge. *Child Development Perspectives*, *11*(3), 184–190. https://doi.org/10.1111/cdep.12229

Rittle-Johnson, B., & Schneider, M. (2015). Developing conceptual and procedural knowledge of mathematics. *Oxford Handbook of Numerical Cognition*, 1118-1134.

Sáenz, L. M., Fuchs, L. S., & Fuchs, D. (2005). Peer-assisted learning strategies for English language learners with learning disabilities. *Exceptional Children, 71*(3), 231–247. doi:10.1177/001440290507100302

Short, D., & Fitzsimmons, S. (2007). *Double the work: Challenges and solutions to acquiring language and academic literacy for adolescent English language learners – A report to Carnegie Corporation of New York*. Washington, DC: Alliance for Excellent Education.

Shyyan, V., Thurlow, M. L., & Liu, K. K. (2006). Instructional strategies for improving achievement in reading, mathematics, and science for English language learners with disabilities. *Assessment for Effective Intervention, 33*(3), 145–155. doi:10.1177/1534508407313239

Snell, M. (1993). *Instruction of students with severe disabilities* (4th ed.). New York: Merrill.

Sorrell, A. L. (1990). Three reading comprehension strategies: TELLS, story mapping, and QARs. *Academic Therapy*, *25*, 359–368.

Steffe, L. P., & Thompson, P. W. (2000). Interaction or intersubjectivity? A reply to Lerman. *Journal for Research in Mathematics Education*, *31*(2), 191–209. doi:10.2307/749751

Stone, C. A. (1998). The metaphor of scaffolding: Its utility for the field of learning disabilities. *Journal of Learning Disabilities*, *31*(4), 344–364. doi:10.1177/002221949803100404

Sullivan, A. L. (2011). Disproportionality in special education identification and placement of English language learners. *Exceptional Children*, *77*(3), 317–334. doi:10.1177/001440291107700304

Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics* (Vol. 2101). Washington, DC: National Academy Press.

Swanson, H. L., & Jerman, O. (2006). Math disabilities: A selective meta-analysis of the literature. *Review of Educational Research*, *76*(2), 249–274. doi:10.3102/00346543076002249

Swanson, H. L., Kong, J., & Petcu, S. D. (2020). Math problem-solving and cognition among emerging bilingual children at risk and not at risk for math difficulties. *Child Neuropsychology*, *26*(4), 489–517.

Szabo, Z. K., Körtesi, P., Guncaga, J., Szabo, D., & Neag, R. (2020). Examples of problem-solving strategies in mathematics education supporting the sustainability of 21st-century skills. *Sustainability*, *12*(23), 10113.

TestPrep-Online. (n.d.). Retrieved from https://www.testprep-online.com/olsat-scores

Torres-Velasquez, D., & Rodriguez, D. (2005). *Improving mathematics problem solving skills for English language learners with learning disabilities*. University of South Florida, Project LASER.

Tracey, D., & Morrow, L. M. (2012). *Lenses on reading: An introduction to theories and models*. New York, NY: Guilford Press.

Van de Pol, J., Volman, M., & Beishuizen, J. (2010). Scaffolding in teacher–student interaction: A decade of research. *Educational Psychology Review*, *22*(3), 271–296.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2012). *Elementary and middle school mathematics: Teaching developmentally* (8th ed.). Boston, MA: Pearson.

Van Garderen, D., & Montague, M. (2003). Visual-spatial representation, mathematical problem solving, and students of varying abilities. *Learning Disabilities Research & Practice*, *18*(4), 246–254.

Vannest, K. J., Parker, R. I., Gonen, O., & Adiguzel, T. (2016). Single case research: Web-based calculators for SCR analysis. (Version 2.0) [Web-based application]. College Station, TX: Texas A&M University. Retrieved from singlecaseresearch.org

Vermeulen, J. A., Béguin, A., Scheltens, F., & Eggen, T. J. (2020). Diagnostic assessment in third-grade subtraction: The relation between bridging errors, number of errors and mathematical ability. *Assessment in Education: Principles, Policy & Practice*, *27*(6), 687–706.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press.

Vygotsky, L. (1962). *Studies in communication*. *Thought and language* (E. Hanfmann & G. Vakar, Eds.). MIT Press. doi:10.1037/11193-000

Walqui, A. (2010). Principles of quality teaching for English learners. In A. Walqui & L. vanLier (Eds.), *Scaffolding the academic success of adolescent English language learners* (pp. 80–85). San Francisco, CA: WestEd.

Walqui, A., & vanLier, L. (2010). *Scaffolding the academic success of adolescent English language learners*: *A pedagogy of promise*. San Francisco, CA: WestEd.

Wang, A. Y., Fuchs, L. S., & Fuchs, D. (2016). Cognitive and linguistic predictors of mathematical word problems with and without irrelevant information. *Learning and Individual Differences*, *52*, 79–87.

Watkins, E., & Liu, K. K. (2013). Who are English language learners with disabilities? *Impact*: *Feature Issue on Educating K-12 English Language Learners with Disabilities*, *26*(1), 2–3.

Wertsch, J. V. (1979). From social interaction to higher psychological processes. *Human Development, 22*, 1–22.

Wood, D., Bruner, J. S., & Ross, G. (1976). The role of tutoring in problem solving. *Journal of Child Psychology and Psychiatry*, *17*(2), 89–100. doi:10.1111/j.1469-7610.1976.tb00381.x

Wyndhamn, J., & Säljö, R. (1997). Word problems and mathematical reasoning—a study of children's mastery of reference and meaning in textual realities. *Learning and Instruction*, *7*(4), 361–382.

Xin, Y. P. (2012). *Conceptual model-based problem solving*: *Teach students with learning difficulties to solve math problems*. Rotterdam, Netherlands: Sense. doi:10.1007/978- 94-6209-104-7_1

Xin, Y. P. (2015). Research related to modeling and problem solving: Conceptual model-based problem solving: Emphasizing pre algebraic conceptualization of mathematical relations. In E. A. Silver & P. A. Kenney (Eds.), *More lessons learned from research: Useful and useable research related to core mathematical practices* (pp. 235–246). National Council of Teachers of Mathematics (NCTM).

Xin, Y. P., Jitendra, A. K., & Deatline-Buchman, A. (2005). Effects of mathematical word problem-solving instruction on middle school students with learning problems. *The Journal of Special Education*, *39*(3), 181–192.

Xin, Y. P., Kim, S. J., Lei, Q., Wei, S., Liu, B., Wang, W., Kastberg, S., Chen, Y., Yang, X., Ma, X., & Richardson, S. E. (2020). The impact of a conceptual model-based intervention program on math problem-solving performance of at-risk English learners. *Reading and Writing Quarterly: Overcoming Learning Difficulties, 36*(2), 104–123.

Xin, Y. P., Liu, J., Jones, S. R., Tzur, R., & Si, L. (2016). A preliminary discourse analysis of constructivist-oriented mathematics instruction for a student with learning disabilities. *The Journal of Educational Research*, *109*(4), 436–447. doi:10.1080/0022 0671.2014.979910

Xin, Y. P., Liu, J., & Zheng, X. (2011). A cross-cultural lesson comparison on teaching the connection between multiplication and division. *School Science and Mathematics*, *111*(7), 354–367. doi:10.1111/j.1949-8594.2011.00098.x

Xin, Y. P., Tzur, R., Hord, C., Liu, J., Park, J. Y., & Si, L. (2017). An intelligent tutor-assisted mathematics intervention program for students with learning difficulties. *Learning Disability Quarterly*, *40*(1), 4–16. doi:10.1177/0731948716648740

Xin, Y. P., Tzur, R., & Si, L. (2008). *Nurturing multiplicative reasoning in students with learning disabilities in a computerized conceptual-modeling environment* (National Science Foundation Funded Project). West Lafayette, IN: Purdue University.

Xin, Y. P., Wiles, B., & Lin, Y. (2008). Teaching conceptual model-based word-problem story grammar to enhance mathematics problem solving. *The Journal of Special Education, 42,* 163–178.

Xin, Y. P., Zhang, D., Park, J. Y., Tom, K., Whipple, A., & Si, L. (2011). A comparison of two mathematics problem-solving strategies: Facilitate algebra-readiness. *The Journal of Educational Research*, *104,* 381–395.

Zawaiza, T. R. W., & Gerber, M. M. (1993). Effects of explicit instruction on math word-problem solving by community college students with learning disabilities. *Learning Disability Quarterly*, *16*, 64–79.

Zentall, S. S. (2014). *Students with mild exceptionalities: Characteristics and applications*. Los Angeles, CA: Sage. doi:10.4135/9781506374604