THE FACTOR STRUCTURE OF PARENTS' MATH-RELATED TALK AND ITS RELATION TO CHILDREN'S EARLY ACADEMIC SKILLS

by

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A Dissertation

Submitted to the Faculty of Purdue University

In Partial Fulfillment of the Requirements for the degree of

Doctor of Philosophy

Department of Human Development and Family Studies

West Lafayette, Indiana

August 2022

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Dedicated to my favorite early educator, my mother, Heket, aka Doris McAlpin

ACKNOWLEDGMENTS

I want to give a special thank you to my committee members for their guidance in conducting this project. Thank you to Dr. David Purpura for welcoming me into your lab and fostering my curiosity about mathematical language. My love of research mentorship is partially due to you being an excellent example of a mentor. I hope to be as quick as you with getting students' feedback one day! I appreciate your support over the last six years (from weekly meetings to Thanksgiving dinners). Thanks to Dr. Rob Duncan for your mentorship and statistical expertise throughout my graduate training. Having you as an instructor and collaborator has strengthened my research skills. Many thanks to Drs. Sarah Eason and Arielle Borovsky, I am forever grateful for your support, advice, and expertise in analyzing transcriptions.

I want to give a HUGE thank you to the parents and children who participated in my study. Thank you for making this project exciting and for meeting with me on Zoom. Thank you to everyone who shared the study recruitment information, including Head Start and preschool directors. Thank you to the undergraduate students who were terrific at transcribing and coding; I could not have done it without their support. Thank you to the Administration for Children and Families for supporting this study through the Head Start Dissertation Grants program.

I greatly appreciate the support of my Purdue and HDFS friends, who have been there to celebrate every milestone with me. Thanks for encouraging me to take well-deserved breaks. I will forever cherish my closest friends from before I started graduate school who added the Midwest to their summer travel plans to spend time with me. Thanks to my closest loved ones for believing in me and being inspired by my success, especially my mother and my partner, Ify, who were determined to help me reach my recruitment goal. Thank you to my brother, Sheps King-McAlpin, for being my go-to proofreader since I began applying for graduate school. I could not have asked for a better support system than all of you!

This project is supported by the Administration for Children and Families (ACF) of the United States (U.S.) Department of Health and Human Services (HHS) as part of a financial assistance award (Grant #: 90YR0123) totaling $25,000 with 100 percent funded by ACF/HHS. The contents are those of the author(s) and do not necessarily represent the official views of, nor an endorsement, by ACF/HHS, or the U.S. Government. For more information, please visit the ACF website, Administrative and National Policy Requirements.

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ABSTRACT

Early math skills, including numeracy and mathematical language (e.g., “less” and “a few”), are essential for later academic achievement. Children’s mathematical language knowledge is one of the strongest predictors of numeracy skills before kindergarten, suggesting that early exposure to math language is necessary. However, little work is focused on understanding how children are exposed to mathematical language within their early learning environments (e.g., while interacting with parents). The objective of this study was to investigate different constructs of parents’ talk (i.e., general talk, number talk, mathematical language) during math-related activity engagement with young children and examine how parents’ talk relates to children’s general vocabulary, numeracy skills, and mathematical language knowledge. Findings indicate that parents’ talk was best represented by a general talk, number talk, and mathematical language factor. Parents’ talk factors were not significantly related to their respective child outcomes (i.e., general vocabulary, numeracy skills, and mathematical language knowledge). However, parents used more general language when their children had higher numeracy skills but used more mathematical language when they had lower numeracy skills. This study provides initial evidence that parents’ number talk and mathematical language use are distinct constructs of parents’ talk that may expose children to different aspects of mathematical understanding.

INTRODUCTION

Early academic achievement gaps emerge prior to the start of kindergarten, and children who start school with significantly fewer foundational language and mathematics skills than their peers are at risk of underperformance throughout their academic careers (Burchinal et al., 2011; Galindo & Sonnenschein, 2015; Reardon & Portilla, 2016). Early academic deficits have been linked to later academic problems, such as an increased likelihood of mathematics and reading deficits (Berch & Mazzocco, 2007). One strategy for addressing early achievement gaps is investigating children's early learning environments and identifying areas that might benefit from intervention. Studies using direct observation methods have found that parents' talk about numbers during daily routines and math-activity engagement (e.g., mealtime, reading number storybooks) is predictive of children's mathematics ability (Gunderson & Levine, 2011; Ramani et al., 2015; Susperreguy & Davis-Kean, 2016). Early mathematics development appears to be highly language-based, such that number words and quantity descriptions are learned within a language context (Krajewski & Schneider, 2009; Toll & Van Luit, 2014b). Children's content-specific mathematical language knowledge is one of the strongest predictors of numeracy skills before kindergarten (Purpura & Reid, 2016), suggesting that early exposure to mathematical language is necessary. Mathematical language refers to words or concepts that describe approximate quantities (e.g., "Who has fewer cookies?"), however, it does not include direct references to number words. Although studies have investigated children's exposure to number words during interactions with parents, it is necessary to understand parents' use of mathematical language with young children, which may be an appropriate target area for home-based interventions to boost children's mathematics knowledge before the start of kindergarten.

Research findings suggest that children benefit both within and beyond the domain of math when parents use a variety of vocabulary and engage children in more frequent daily conversations (Hindman et al., 2014; Levine et al., 2010; Romeo et al., 2018; Suskind et al., 2016). Parents' general talk has been consistently predictive of children's general vocabulary skills (Hindman et al., 2014; Romeo et al., 2018; Zimmerman et al., 2009), and recent work suggests that parents' mathematics talk is predictive of children's numeracy skills (Casey et al., 2018; Ramani et al., 2015; Susperreguy & Davis-Kean, 2016). Research on parents' mathematics talk has primarily focused on talk involving number names (e.g., number talk; Casey et al., 2018; Gunderson &

Levine, 2011; Levine et al., 2010); however, some work involves a broader focus on mathematical talk including but not limited to number talk (Mutaf Yildiz et al., 2018; Turan & De Smedt, 2022; Zippert et al., 2020). It is important to examine if there is a significant distinction between parents' general talk, number talk, and mathematical language use, considering that children's mathematical language knowledge has been found to be causally related to early numeracy development (Purpura et al., 2017). Specifically, children's mathematical language knowledge is a stronger predictor of later numeracy skills than general vocabulary and early number performance (Purpura & Logan, 2015), and a consistent classifier of low numeracy performance (Purpura et al., 2017). Thus, if children's mathematical language knowledge is a better predictor of numeracy skills than their initial number knowledge, it may be expected that parents' mathematical language use would also be necessary for both children's mathematical language and numeracy development. It is also relevant to understand if parents' number talk and use of mathematical language are separate constructs that are differentially related to children's early academic skills—as it may be advantageous to target these constructs individually or simultaneously to promote mathematics development within the home environment.

This study aimed to determine if parents' number talk and mathematical language talk are distinct constructs and, if so, examine how the distinct constructs of parents' talk may relate to specific areas of children's early academic performance. Specifically, parents' general talk, number talk, and use of mathematical language were coded during observations of parent-child engagement in various math-related activities to test whether these components of parent talk are distinct factors. Further, this study examined if parents' general talk, number talk, and mathematical language use are distinct factors that are concurrently and independently related to children's general vocabulary, numeracy skills, and mathematical language knowledge. Including a broad range of activities and focusing on coding both parent number talk and mathematical language use may provide more opportunities to investigate the variability of parents' talk during math-related discussions. Identifying the factor structure of parent talk is beneficial for understanding if and how parents' use of number talk and mathematical language talk are uniquely related to children's skills. Additionally, understanding if parents' number talk and mathematical language talk are distinct constructs may provide insight for better targeting of parent-child interventions, considering that both constructs should be targeted if they uniquely relate to children's numeracy and mathematical language knowledge. Parents' mathematical language and number talk may play distinct roles in children's math support, and it is necessary to understand the different areas of parents' talk that might be targeted to bolster early learning.

Importance of Early Academic Skills

Before children are enrolled in formal schooling, they develop early academic skills within their everyday informal learning environments (Blevins-Knabe, 2012; Dickinson & Tabors, 2001; Melhuish, 2010). Early academic skills refer to skills from a broad range of developmental domains, including language skills and mathematics knowledge, which are the focus of this study. Early language and mathematics skills are essential for developing communication and problem-solving abilities predictive of academic success and wellbeing (Cohen, 2010; Duncan et al., 2007; Feeney et al., 2012). Longitudinal studies show that children with higher early academic skills during the preschool age have better achievement in reading and mathematics throughout their formal schooling (Nguyen et al., 2016; Storch & Whitehurst, 2002). Though early academic skills are foundational for later success, all children may not be exposed to environments where acquiring these skills is best fostered to begin formal schooling (Burchinal et al., 2011; Hoff, 2013). Therefore, it is critical to investigate how factors of the early environment are related to children's early academic skills to identify potential ways to bolster the development of these skills before kindergarten. Specifically, it is necessary to understand how parents' talk is structured within the context of math-activity engagement due to findings linking early language exposure to children's skills (Casey et al., 2018; Hindman et al., 2014; Ramani et al., 2015). Furthermore, it is important to distinguish if there are distinct constructs of parents' talk (e.g., general, number, and mathematical language) that are uniquely related to children's performance in specific domains (e.g., general vocabulary, mathematical language, numeracy).

Early Numeracy Skills

Mathematics is an early academic domain which includes a broad array of competencies such as numeracy, patterning, geometry, spatial, measurement, and data (Milburn et al., 2019; NRC, 2009). Early numeracy includes the ability to count and identify quantities and is one of the strongest early predictors of subsequent mathematics achievement, even while controlling for other domains of early mathematics such as geometry, patterning, and measurement skills (Nguyen et al., 2016). The current study is focused on early numeracy skills due to the strong relation between numeracy and later academic achievement (Duncan et al., 2007; Rittle-Johnson et al., 2017; Watts et al., 2014). Early numeracy skills, such as counting, typically develop during early childhood when children are as young as two years old (Gelman & Gallistel, 1978; Wynn, 1990, 1992). Similar to the acquisition of language, early numeracy begins to develop within a child's informal learning environments (e.g., at home) and sets the foundation for mathematics development throughout formal schooling (Nguyen et al., 2016; Rittle-Johnson et al., 2017; Watts et al., 2014). These findings suggest that numeracy skills may be the most foundational for developing mathematics ability within a formal school setting.

Early numeracy is comprised of three highly interrelated domains: numbering, numerical relations, and arithmetic operations (Purpura & Lonigan, 2013). Children's numbering skills include understanding the counting sequence and having the ability to identify quantities. The second domain, numerical relations, comprises a child's understanding of how numbers and quantities are related. Finally, the arithmetic operations domain involves a basic understanding of addition and subtraction (Purpura & Lonigan, 2013).

The three domains of early numeracy appear to develop within three phases (Krajewski & Schneider, 2009). The first phase is characterized by children's ability to distinguish between different quantities, recite number words, and verbally count in sequence, which can be thought of as foundational numerical skills (Krajewski & Schneider, 2009; Purpura et al., 2013). Children begin to exhibit these skills at around two years of age (Wynn, 1992). However, during phase 1, many children have not begun to use number words to describe quantities (Krajewski & Schneider, 2009). For example, a child may be able to verbally count to the number three but not yet understand that the number three refers to a collection of three items (e.g., showing that they understand cardinality). The transition to the second phase involves the ability to use number words to describe, compare, and contrast quantities.

The second phase is characterized by children's ability to verbally count a fixed set of objects in sequence, link imprecise number words (e.g., a bit, much) to quantities, and link precise number words (e.g., two, four) to quantities (Krajewski & Schneider, 2009). This phase suggests that a focus on understanding language specific to mathematics is fundamental during this phase (Barner et al., 2009; Krajewski & Schneider, 2009; Purpura et al., 2013). This ability to describe quantities with imprecise and precise number words tends to develop at around three years of age (Wynn, 1990). Children's ability to match quantities with quantifiers (e.g., all, a, another, the others, none, some, both, most) is positively related to their ability to match quantities with number words (Barner et al., 2009). During the second phase, a child may understand that the number five is bigger than the number three but not yet understand that the number five is composed of two and three. The transition to the third phase involves gaining the ability to perform basic addition and subtraction, which is facilitated by understand how to compose and decompose numbers.

During the third phase, children understand the relation between quantities and number words, the composition and decomposition of quantities (e.g., the quantity 3 is composed of 1 and 2), and the differences between numbers (e.g., the difference between 3 and 4 is 1; Krajewski & Schneider, 2009; Purpura et al., 2013). By age five, many children are within the third phase and have gained the ability to solve simple addition and subtraction problems (Litkowski et al., 2020). Children who master basic numeracy skills from these three phases by the start of kindergarten are better set up for later academic achievement (Duncan et al., 2007). Further, national U.S. standards for early mathematics have recognized that the three domains of early numeracy are essential precursors to students' understanding of more advanced math skills such as formal addition, subtraction, and place value (National Association for the Education of Young Children [NAEYC] & National Council of Teachers of Mathematics [NCTM], 2003; National Mathematics Advisory Panel [NMAP], 2008; National Research Council [NRC], 2009).

Relation between language and numeracy

Due to the involvement of language in the development of early numeracy (Krajewski & Schneider, 2009; LeFevre et al., 2010a; Purpura & Ganley, 2014; VuKovic & Lesaux, 2013), there has been an effort to understand the relation between language and mathematics (Powell et al., 2017; Purpura & Reid, 2016; Toll & Van Luit, 2014b). Specifically, as indicated by the three phases of numeracy development (Krajewski & Schneider, 2009), children must learn number words to master the counting sequence, use imprecise number words/phrases (e.g., a few) to describe quantities, and finally use both precise and imprecise number words to solve mathematics problems (such as word problems). Although these are specific examples of how language is used in early numeracy development, a body of research suggests that general language knowledge is an important predictor of numeracy skills.

Early language skills include understanding and producing vocabulary necessary for oral and written communication. Language development begins during infancy and significantly increases between 2 and 3 years of age (Shonkoff et al., 2000). Early vocabulary skills are necessary to develop more advanced language and literacy skills often acquired through formal schooling (Dickinson et al., 2010; Hooper et al., 2010a; Muter et al., 2004; NICHD Early Child Care Research Network, 2005; Storch & Whitehurst, 2002). Additionally, vocabulary skills are foundational for success in other academic areas, such as understanding mathematics concepts (Purpura & Ganley, 2014; Slusser et al., 2019). Notably, studies suggest a strong relation between early vocabulary and mathematics achievement (Hooper et al., 2010b; Purpura & Ganley, 2014; Romano et al., 2010; Toll & Van Luit, 2014b).

Children with strong early numeracy skills tend to have strong general language skills (LeFevre et al., 2010a; Sowinski et al., 2015). Prior work has shown that young children's expressive vocabulary is strongly related to most early numeracy skills (Purpura & Ganley, 2014) and that children with higher vocabulary knowledge had higher scores on numeracy tasks one year later (Purpura et al., 2011). Although there is a close relation between general vocabulary and numeracy skills, more recent evidence indicates that content-specific mathematical language is a stronger predictor of numeracy skills than is general vocabulary (Hornburg et al., 2018; Purpura & Reid, 2016; Toll & Van Luit, 2014a). These findings align with the second phase of early numeracy development, where children gain an understanding of both imprecise and precise quantity descriptions (Barner et al., 2009; Krajewski & Schneider, 2009; Purpura et al., 2013). As measures of general vocabulary often only account for children's knowledge of words representing general objects and actions (Dunn & Dunn, 2007), it is necessary to understand how language is used within the context of mathematical reasoning**.**

Mathematical language knowledge

Mathematical language is a content-specific language that consists of terms and concepts used to describe quantitative and spatial relations (Purpura & Logan, 2015). Quantitative mathematical language includes words such as "more," "less," "many," and "fewer" (Barner et al., 2009; Purpura et al., 2019; Purpura et al., 2017). Importantly, quantitative mathematical language refers to quantifiers or imprecise words (e.g., some stickers; Barner et al., 2009) that describe quantities and do not include references to specific numbers (e.g., five stickers). Children with quantitative mathematical language knowledge understand the meaning of words used to describe quantities and can compare groups of objects or numbers with this knowledge. For example, a preschooler may understand that “fewer” refers to a particular group of objects being a lesser quantity than another group of objects. Although quantitative mathematical language exclusively refers to descriptions of quantity, spatial mathematical language knowledge also develops preschool and is often used to describe objects’ shape, size, and location (Purpura et al., 2019).

Spatial mathematical language consists of words, such as “before” and “above”, that are associated with locations, directions, and ordinal relations (Cannon et al., 2007). Children must understand spatial mathematical language to make and describe comparisons between spatial terms and objects’ sizes or locations (Pruden et al., 2011). Although spatial language is an aspect of mathematical language, the current study focuses on quantitative language, which has been found to be directly linked to the development of number knowledge (Barner et al., 2009, Purpura et al., 2021). Further, there have been similarities found between the way quantitative language and number words are used within sentences, suggesting that children may acquire knowledge of these descriptions of quantity in similar ways (Barner et al., 2009).

Distinction between mathematical language and numeracy

Recent work suggests that mathematical language is distinct from numeracy but strongly related to the development of numeracy skills (Lin et al., 2021). Specifically, Lin et al. (2021) conducted a meta-analysis and found that mathematical language knowledge is significantly related to numeracy skills during preschool and beyond while controlling for comprehension skills such as general vocabulary. Further, these findings suggest that mathematical language knowledge does not simply act as a proxy for general language skills (Lin et al., 2021). Additionally, the moderate correlations found between mathematical language and numeracy skills suggest that mathematical language is distinct from numeracy but is an essential skill for developing numeracy throughout schooling (Lin et al., 2021).

Mathematical language knowledge is one of the strongest predictors of children’s early numeracy skills, above and beyond general language knowledge (Purpura & Logan, 2015; Toll & Van Luit, 2014b). Toll and Van Luit (2014b) demonstrated that mathematical language mediates the relation between general language (e.g., passive vocabulary and writing orientation) and early numeracy skills, suggesting that content-specific mathematical language may be the mechanism by which children use their language knowledge to understand and solve numeracy tasks. Further, Purpura et al. (2017) conducted an intervention that provided evidence that increasing preschoolers' exposure to mathematical language by using storybooks resulted in increased mathematical language knowledge and numeracy skills. Additionally, Hornburg et al. (2018) found that mathematical language knowledge was more strongly related to both basic (e.g., verbal counting) and advanced (e.g., connecting numerals to quantities) early numeracy skills than was general language, measures as expressive vocabulary, which had previously been found to be predictive of numeracy skills (Purpura & Ganley, 2014). Together these studies suggest that early numeracy development is highly language-based, and sociocultural learning theories (Vygotsky, 1978) suggest that social context plays a role in facilitating the development of interrelated early academic skills (e.g., language and numeracy).

Theoretical Frameworks for Children's Acquisition of Early Academic Skills

Vygotsky's (1978) theoretical perspective about learning development posits that children learn from the environments they develop in by interacting with other knowledgeable humans (e.g., parents, older siblings, peers, teachers). In particular, as children develop, they learn to use the psychological and physical tools relevant to the culture and environment in which they are immersed. Vygotsky considers language one of the most important psychological tools because comprehending and producing language are processes that transform thinking and problem-solving. Further, social-pragmatic theory explains that language acquisition results from children's participation in social interaction routines with adults (Tomasello, 2000). Commonly, studies suggest that specific early academic skills are learned when children engage in language-based interactions with parents or caregivers (Ramani et al., 2015; Strouse et al., 2013; Suskind et al., 2016; Susperreguy & Davis-Kean, 2016). These language-based social interactions are a broader context in which parents provide general and content-specific language input that may support children's learning. Therefore, social learning theoretical perspectives may help explain how mathematical language knowledge and early numeracy development may be fostered through everyday informal activities (e.g., storybook reading) and parent input during social interactions (Gibson et al., 2020; Purpura et al., 2021).

Prior to the start of kindergarten, children spend a great amount of time at home, where they participate in and learn from parent input during social interactions (Capizzano & Adams, 2000).

Evidence suggests that children acquire language skills from engaging in conversations with parents during daily routines such as playing games, shared book reading, and talking during car trips (Tomasello, 2000; Whitehurst et al., 1988; Zimmerman et al., 2009). During daily routines, children gain an understanding of the language used by parents and acquire the ability to respond to verbal requests, express themselves, and ask questions (Tomasello, 2000). As children develop, parents may engage children in more educational activities where children may be exposed to language specific to particular academic domains (King & Purpura, 2021; LeFevre et al., 2009; Ramani et al., 2015). For example, parents may expose their children to mathematical language during discussions about using numbers during gameplay. A child may know the names of numbers but may need their parent's assistance with understanding how numbers are related to each other (e.g., "3 is less than 7" or "3 comes before 7") for a task such as assembling a puzzle with numbered pieces. The importance of social context (Tomasello, 2000; Vygotsky, 1978) has led to studies focused on how the home or classroom environment is related to children's early academic performance.

Social Contexts Where Early Academic Skills Develop

Social contexts, such as the home environment or the classroom, play a significant role in developing early academic skills (Anders et al., 2012; Burchinal et al., 2002; Connor et al., 2005; Lehrl et al., 2020; Ribner et al., 2020). Children exposed to high-quality interactive early learning activities early in life develop strong language and numeracy prior to the start of formal schooling (Manolitsis et al., 2013). Studies show that the home learning environment and the preschool environment are common contexts where young children engage in informal learning activities with their caregivers (Anders et al., 2012; Melhuish et al., 2008). Specifically, the frequency of engagement in learning activities (e.g., storybook reading, board game play) with parents or teachers is predictive of children's language and numeracy skills (Anders et al., 2012; Melhuish et al., 2008). Researchers have begun to conduct interventions within these social contexts to determine the causal mechanisms that contribute to the development of early academic skills to facilitate successful academic trajectories (Clements & Sarama, 2008; Hargrave & Sénéchal, 2000; Starkey et al., 2004; Suskind et al., 2016).

Acquisition of vocabulary

Most 3- to 5-year-old children in the U.S. attend some form of childcare or preschool outside of the home (McFarland et al., 2018), where they may be engaging in learning activities that help them build necessary vocabulary skills for communication and literacy development. Due to the decades of research suggesting that language is one of the most fundamental skills for academic success (Dickinson & Tabors, 2001; Durham et al., 2007; Golinkoff et al., 2019; Hoff, 2013), national standards have been established to support the development of early vocabulary within the preschool environment (NAEYC & NAECS/SDE, 2002). The preschool context may play a significant role in children's language acquisition as interventions show that children gain vocabulary skills when teachers expose them to new vocabulary through storybook reading (Hargrave & Sénéchal, 2000; Zevenbergen & Whitehurst, 2003). Additionally, recent work shows that children with stronger vocabulary skills are exposed to preschool environments rich in language interactions characterized by conversational turn-taking between teachers and preschoolers (Duncan et al., 2020). However, due to the preschool classrooms being populated with many students and only a small number of teachers, teachers have limited opportunities to engage in extended conversational turns with individual children (Justice et al., 2008). Further, although many 3- to 5-year-old children attend preschool, they tend to spend much of their time in the home environment where they learn from interactions with parents (Capizzano & Adams, 2000), which makes the home environment a crucial context for the development of vocabulary.

A large body of work suggests that children acquire early vocabulary from the social interactions they experience in their home environment (Cristofaro & Tamis-LeMonda, 2012; Dickinson & Tabors, 2001; Forget-Dubois et al., 2009; Sénéchal & LeFevre, 2002; Son & Morrison, 2010). Children develop language skills when their parents engage them in everyday activities such as conversational turn-taking and shared storybook reading (Whitehurst et al., 1988; Zimmerman et al., 2009). Engagement in literacy activities is also related to children's language knowledge (Niklas & Schneider, 2015). Further, home-based interventions reveal that parents can be trained to implement high-quality language interactions that support the development of children's language and other cognitive skills (Starkey & Klein, 2000; Strouse et al., 2013; Whitehurst et al., 1988). Within these language-based interactions, there may be opportunities to expose children to specific academic content areas, such as mathematics knowledge. Further, recent work provides evidence that young children's acquisition of mathematics skills is related to frequent engagement in social interactions around math-related activities (Blevins-Knabe, 2012; LeFevre et al., 2009; Niklas & Schneider, 2014).

Acquisition of numeracy

A focus on numeracy skill development tends to emerge when children are a few years away from the start of kindergarten (Ginsburg et al., 2008). Preschool intervention studies have indicated that children's numeracy skills significantly improve when their teachers implement a mathematics-focused curriculum in their classrooms (Clements & Sarama, 2007; Clements & Sarama, 2008; Clements & Sarama, 2011; Clements et al., 2011; Sarama & Clements, 2004). Considering the importance of mathematics throughout formal schooling and beyond, national education and research panels have emphasized the importance of comprehensive mathematics curricula to help equip preschool-aged children with the mathematics knowledge necessary throughout life (NAEYC & NCTM, 2002; NRC, 2009). Suggested teaching strategies for fostering numeracy development include teachers guiding children's counting in everyday situations, telling real-life stories involving numbers, and asking word problems using "how many" (Clements, 2001; NAEYC & NCTM, 2002; Piasta et al., 2014; Sarama & Clements, 2004).

Complementary work suggests that young children also learn numeracy skills when their parents engage them in mathematics-focused activities at home (Baroody & Wilkins, 1999; Blevins-Knabe & Musun-Miller, 1996; Ginsburg, 1977; Manolitsis et al., 2013; Melhuish et al., 2008; Niklas & Schneider, 2014; Starkey & Klein, 2000). Within the home environment, parents provide children with educational resources and engage children in learning activities that may facilitate the development of early numeracy skills such as counting objects, comparing sizes, and understanding terms like "more" and "less" (Blevins-Knabe, 2012). Studies suggest that parents who engage their children in more numeracy-related activities have children with higher numeracy skills (Gibson et al., 2020; Hart et al., 2016; LeFevre et al., 2009, Purpura et al., 2021). Other studies indicate that parents' talk about numbers during mathematics activity engagement is important for children's skill development (Gunderson & Levine, 2011; Susperreguy & Davis-Kean, 2016). As suggested by social learning theories (Tomasello, 2000; Vygotsky, 1978), parents may be more likely to introduce their children to mathematical language while engaging in mathematics-focused activities and dialogue with them. However, in contrast to the work examining the broader role of the home environment, less work has been conducted on how specific components of the language environment contribute to the development of early academic skills, though the focus on the parent number talk has increased in recent years (Elliott et al., 2017; Ramani et al., 2015; Thippana et al., 2020).

Parents' mathematical language use may be a distinct yet important factor related to children's acquisition of mathematical language knowledge and numeracy skills. However, it is unclear how distinct parents' mathematical language use (imprecise descriptions of quantity) is from their number talk (precise descriptions of quantity), a more commonly investigated aspect of parents' talk that is related to children's numeracy skills (Casey et al., 2018; Gunderson & Levine, 2011; Levine et al., 2010). Conceptually, research findings suggest that there are distinctions between children's mathematical language knowledge and their numeracy skills (Lin et al., 2021; Purpura & Logan, 2015) and that mathematical language knowledge is predictive of numeracy skills (Purpura & Logan, 2015; Toll & Van Luit, 2014b). Given that this distinction is proposed during investigations of children's independent skills, it may be that it is also reflected in the language input children receive from parents. Parents' mathematical language use may be a distinct factor predictive of mathematical language knowledge and numeracy skills (as mathematical language also underlies numeracy skills; Purpura et al., 2017). This expectation is based on prior evidence suggesting that parents' general talk is predictive of children's general vocabulary (Suskind et al., 2016; Zimmerman et al., 2009), parents' spatial talk is predictive of children's spatial skills (Pruden et al., 2011), and parents' number talk is related to children's number skills (Casey et al., 2018, Susperreguy & Davis-Kean, 2016).

Parent Input During Verbal Interactions

Parents' general talk

Children with strong language skills tend to have parents that use more words during daily parent-child interactions (Golinkoff et al., 2019; Hoff, 2003; Hart & Risley, 2003; Huttenlocher et al., 1991). Generally, studies find that children have stronger vocabulary when they hear their parents use a large variety of vocabulary and spend more time engaging in conversations with their parents (Hindman et al., 2014; Romeo et al., 2018; Zimmerman et al., 2009). Additionally, intervention results suggest that parents who are educated on the importance of diversity in parents' talk demonstrated an increase in the quantity and quality of their child-directed speech (Suskind et al., 2016). As a result, children in the intervention group showed gains in their language ability; however, dyads in the control group did not have the same increase in child-directed speech and language ability (Suskind et al., 2016). Further, studies have begun to investigate the role of parents' content-specific talk (e.g., number words) in the development of other early academic skills (Susperreguy & Davis-Kean, 2016), such as numeracy.

Parents' mathematics talk

The frequency of parent mathematics talk during daily conversations with young children is positively related to children's numeracy skills (Casey et al., 2018; Gunderson & Levine, 2011; Levine et al., 2010; Ramani et al., 2015; Susperreguy & Davis-Kean, 2016; Turan & De Smedt, 2022). Specifically, studies suggest that young children have stronger numeracy skills when their parent uses numbers more during daily conversations with them (Gunderson & Levine, 2011; Levine et al., 2010; Susperreguy & Davis-Kean, 2016).

Number talk

Number talk refers to parents' use of numeral names within a numeracy context (e.g., "you can have three cookies") during conversations with their child. For example, longitudinal studies find that preschoolers have better cardinality knowledge when their parents more frequently use number words (i.e., "one" through "ten") with them between the ages of 14 and 30 months (Gunderson & Levine, 2011; Levine et al., 2010). Specifically, while controlling for parents' general talk, parents' use of number words while referring to large sets (i.e., sets of "four" to "ten" objects) was uniquely predictive of their child's cardinality knowledge at three years of age (Gunderson & Levine, 2011). Though parents' general talk is a strong predictor of children's general vocabulary (Hindman et al., 2014; Zimmerman et al., 2009), these two studies (Gunderson & Levine, 2011; Levine et al., 2010) suggest that parents' number talk is uniquely predictive of young children's number knowledge while controlling for parents' general talk. Together, these two research areas suggest that parents' number talk is potentially a specific domain of parent talk uniquely related to children's numeracy skills.

Studies suggest that the type of activity that parents and children engage in may elicit different levels of numeracy dialogue. Generally, observational work builds on survey data which suggests that parents' reports of frequently engaging their child in numeracy-related activities are related to children's numeracy skills (LeFevre et al., 2009; LeFevre et al., 2010b; Manolitsis et al., 2013). Similarly, explicit numeracy-related activities (e.g., number card/board games and number storybooks; Daubert et al., 2018; Ramani et al., 2015) or implicit numeracy-related activities (e.g., play with blocks or toy cash register; Casey et al., 2018; Zippert et al., 2020) are selected to observe number talk during parent-child play sessions. Separate studies suggest that number talk occurs in various numeracy-related activities that range from least (e.g., mealtime; Susperreguy & Davis-Kean, 2016) to most (e.g., number board game; Ramani et al., 2015) explicit. However, relative to work on number talk, there are few studies examining parents' math language (Chan, Praus-Singh, & Mazzocco, 2020; Eason & Ramani, 2020), and very few, if any, have examined if these two types of talk differentially relate to children's skills.

Studies focused on observing parent-child interactions during the preschool age range have demonstrated relations between the frequency of parents' number talk and broad measures of children's numeracy skills (Casey et al., 2018; Elliot et al., 2017; Ramani et al., 2015; Susperreguy & Davis-Kean, 2016). Ramani et al. (2015) observed parent-child dyads engage in three numeracy-related activities (i.e., a number storybook, a numbered puzzle, and a number board game) and coded instances of parent number talk. In this study, parents advanced number talk, defined as talk describing cardinality, ordinal relations, and arithmetic, was predictive of preschoolers' advanced numeracy skills (counting principles, enumeration/cardinality, number line estimation, number magnitude comparison). Susperreguy and Davis-Kean (2016) observed conversations during family meal time and found that parent number talk was predictive of children's numeracy skills one year later. Further, Casey et al. (2018) found that parents' number talk during a play session (i.e., stencils, cash register, and blocks) with their 3-year-old child was predictive of better numeracy scores when their child was 41/2 and in the first grade.

Although many studies suggest that children's numeracy ability is directly related to parents' talk about numbers in a variety of activity contexts, some studies have not found associations between parent number talk and preschoolers' numeracy skills (Thippana et al., 2020; Zippert et al., 2020) or have found negative relations (Mutaf Yildiz et al., 2018). Thippanna et al. (2020) observed parent-child dyads during free play sessions in the lab and at home; however, parent use of number words ("zero" or higher) in either context was not related to children's numeracy skills. Similarly, Zippert et al. (2020) observed parent-child dyads during play with blocks, beads, and numbered cards and did not find an association between parents' number-related dialogue and children's numeracy skills. These inconsistent findings may be due to the choice of observed activity and focus on number words without considering the variability in other types of mathematics-related talk (e.g., mathematical language). For example, Thippanna et al. (2020) focused on parents' use of number words during ten minutes of unstructured free play at home and in the lab, which may not have provided enough variability to relate to children's broad numeracy skills (e.g., counting, number comparison, and calculation skills). Further, Zippert et al. (2020) only asked dyads to play one number-related game of cards which may have restricted their opportunity to discuss numbers in various ways if they were focused on following instructions for playing the card game. Surprisingly, Mutaf Yildiz et al. (2018) found a negative relation between parents' number talk and children's addition and subtraction, possibly due to observations activity (LEGOs) not allowing for opportunities to talk much about operations that would be related to children's calculation skills. In contrast, previous studies may have captured more instances of number talk when examining a broad selection of math-related activities (e.g., number books, number puzzles, and board games; Ramani et al., 2015) or observing conversations during daily routines (e.g., mealtime; Susperreguy & Davis-Kean, 2016).

Need to consider parents' mathematical language use

In addition to coding parents' use of number words, studies with significant findings appear to have more comprehensive coding scheme categories that may include the use of mathematical language (e.g., talk about ordinal relations "What comes before nine?"; Ramani et al., 2015; Susperreguy & Davis-Kean, 2016). Furthermore, studies include instances where parents elicit a child's number talk (e.g., "How many pennies are there?"; Casey et al., 2018; Eason et al., 2021). These studies showing that parents' number talk is related to children's numeracy performance include coding schemes that appear to go beyond parents' frequency of using number words. It is important to understand if parents' mathematical language use is driving these relations, similar to the way children's mathematical language knowledge appears to be a better predictor of later numeracy skills than initial number knowledge (Purpura & Logan, 2015). If parents' mathematical language use is distinct from number word use, this might indicate that the use of these words serves different purposes that complement each other. For example, mathematical language may often be used to explain how quantities relate, while number words are used to count or identify quantity. However, combining these two types of talk prevents us from understanding their unique relations with children's skills or making specific recommendations to support children's learning at home.

Additionally, it is unclear how similar or distinct parents' mathematical language use is from their general language or number talk. Though previous work suggests that parents' number talk is uniquely related to children's number skills while controlling for parents' general talk (Gunderson & Levine, 2011; Levine et al., 2010), there is a lack of research that formally tests if parents' general talk, number talk, and mathematical language use are separate factors. Evidence suggests that parents' general talk is uniquely related to children's general vocabulary while parents' number talk is uniquely related to children's number skills (Levine et al., 2010). Therefore, based on the strong relations between children's mathematical language knowledge and numeracy skills (Hornburg et al., 2018; Purpura & Reid, 2016; Toll & Van Luit, 2014a), it may be that parents' mathematical language use is essential for children's mathematical language knowledge and numeracy performance.

Gaps in Existing Literature

Prior work on parents' number talk has focused on measuring the proportion or frequency of parents' use of number words and numeracy-specific dialogue (e.g., asking a child to perform addition; see Table 1; Casey et al., 2018; Ramani et al., 2015; Susperreguy & Davis-Kean, 2016; Thippanna et al., 2020). However, it is unknown if parents' use of mathematical language is a construct that should be combined with number talk or if these are separate factors (Turan & De Smedt, 2022). Studies focused on the development of child skills suggest that children's mathematical language knowledge is a distinct skill that is related to numeracy skills (Purpura & Reid, 2016; Lin et al., 2021; Toll & Van Luit, 2014b), and there is evidence of a causal relation between children's exposure to mathematical language and their later numeracy skills (Purpura et al., 2017). Recent work suggests that the frequency of parent-child numeracy engagement is related to children's mathematical language knowledge (King & Purpura, 2021; Purpura et al., 2020). Thus, there is a need to consider how parents' use of mathematical language during parent-child interactions fits into the larger language environment where parents may be using a variety of general talk, number talk, and mathematical language. Particularly, it is necessary to clarify if parents' number talk and mathematical language are distinct factors that are important for specific mathematics skills. Prior work suggests that parents' general talk and number talk are separate constructs uniquely related to children's general vocabulary and numeracy (Levine et al., 2010). However, factor analysis has not been used to formally examine the distinction between parents' general talk and number talk. Examining the factor structure of parents' talk and investigating the relations between parent talk factors and child-specific skills may help explain the inconsistent findings between parent number talk and children's numeracy skills. For example, the limited consideration of parents' mathematical language use in many coding schemes may explain some null findings, considering that exposure to mathematical language is essential for early numeracy development (Purpura et al., 2017). Additionally, if both parents' number talk and mathematical language use are important, understanding the potential factor structure may provide insight for intervening in specific domains of the language environment to support the development of children's early academic skills.

Current Study

The present study is designed to investigate the factor structure of parents' talk during parent-child interactions and examine relations between parents' talk and individual early academic skills: preschooler's general language, numeracy skills, and mathematical language knowledge (see Figures 1 and 2 for conceptual models). Specifically, parent-child engagement in various numeracy-related activities was observed to determine if parents' general talk, number talk, and mathematical language use are distinct factors related to specific academic skills. Including a broad range of activities and focusing on coding both parents' number talk and mathematical language use may provide more opportunities to investigate the variability in numeracy-related discussions. Identifying the factor structure of parents' talk is critical for understanding if and how the frequency of number talk and mathematical language use uniquely relates to children's skills. Additionally, understanding if parents' number talk and mathematical language are distinct constructs may provide insight for better targeting of parent-child interventions. Specifically, the results of this study may suggest that parents' number talk and mathematical language use be individually targeted if they uniquely relate to children's numeracy and mathematical language knowledge. However, if parents' number talk and mathematical language use do not separate into distinct factors, it may be that both types of talk represent a greater breadth of mathematics-related talk that is important for children's early mathematics skills.

There are two primary aims of this study. The first aim is to identify the factor structure of parents' talk during math-related activities. The hypothesis (H1) for this aim was that parents' talk would be best represented by a general language, number talk, and mathematical language factor. This hypothesis was based on research suggesting that parents' use of number talk is uniquely related to children's numeracy skills (Levine et al., 2010), above and beyond parents' general talk, as well as studies suggesting that children's mathematical language knowledge is uniquely related to later numeracy skills while controlling for initial number knowledge (Lin et al., 2021).

The second aim is to examine the relation between parent talk factors and children's early academic skills. Specifically, based on research findings that parents' general language is consistently related to children's general language skills but not early numeracy skills (Hindman et al., 2014; Levine et al., 2010; Romeo et al., 2018; Suskind et al., 2016), it was hypothesized (H2a) that parents' general talk would be uniquely related to children's general vocabulary. Additionally, based on research suggesting that parent number talk is predictive of children's numeracy skills (Casey et al., 2018; Ramani et al., 2015; Susperreguy & Davis-Kean, 2016), it was hypothesized (H2b) that parents' number talk would be uniquely related to children's numeracy skills. Finally, it was hypothesized (H2c) that parents' mathematical language use would be related to children's mathematical language knowledge and numeracy skills. This hypothesis was based on prior work showing that children gain mathematical language and numeracy skills but not general language knowledge when exposed to increased mathematical language use (Purpura et al., 2017).

Table 1

Summary of findings from studies investigating the relation between observed parent number talk and children's numeracy skills

Study Citation	Sample Size	Child Age	Observed Activities	Parent Talk Measures	Child Measures	Relation between Parent Talk and Child Performance	
						Significant findings	**Nonsignificant findings**
1. Casey et al., 2018	140	3 years old	10 minutes of play (cash register and dress up cloths, Duplo blocks)	Frequency of maternal use and elicitation of numerical terms: identify numerals, one-to-one counting, and label sets with number word	Math achievement was directly assessed at $4^{1}/_{2}$ and first grade using the Applied Problems subtest of the Revised Woodcock–Johnson Psycho-Educational Achievement Tests	Maternal use and elicitation of **labeling sets** positively predicted math achievement at $4^{1}/_{2}$ [$r(100) = .30$, $p = .00$] and first grade [$r(97) = .33$, $p = .00$]	Maternal use and elicitation of **identifying numerals, one-to-one counting, and broad numerical support** did *not* predict math achievement (correlations ranged from -.04 to .07)
2. Gunderson & Levine, 2011	44	14 to 30 months	5 sessions (child ages 14, 18, 22, 26, and 30 months) for a total of 7.5 hours of parent-child interaction during daily activities	Instances of parents use of number words "one" through "ten" grouped into the following categories: small=1–3 or large=4–10 (with or without objects present) Parent non-number talk included as a control	Cardinal value knowledge was directly assessed at 46 months using the Point-to-X task	Parents' number talk **(4-10)** with objects present positively predicted children's cardinal value knowledge, while controlling for parents' non-number talk ($\beta = .38$, $p < .01$)	Parents' number talk **(1-3),** number talk without objects present, and non-number talk did *not* predict children's cardinal value knowledge (beta coefficients ranged from .07 to .18)

Table 1 *continued*

Study Citation	**Sample Size**	**Child Age**	**Observed Activities**	**Parent Talk Measures**	**Child Measures**	**Relation between Parent Talk and Child Performance**	
						Significant findings	**Nonsignificant findings**
3. Levine et al., 2010	44	14 to 30 months	5 sessions (child ages 14, 18, 22, 26, and 30 months) for a total of 7.5 hours of parent-child interaction during daily activities	Frequency of parent use of number words “one” through “ten” Parent non-number talk included as a control	Cardinal value knowledge was directly assessed at 46 months using the Point-to-X task	Parents’ cumulative number talk was positively related to children’s cardinal value knowledge (r = .47, p < .01), while controlling for parents’ non-number talk (β = .34, p < .05)	Parents’ non-number talk was *not* significantly related to children’s cardinal value knowledge, while controlling for parents’ cumulative number talk (β = .03, p > .05)
4. Ramani et al., 2015	33	3 to 5 years old	15 minutes of play (number book, number puzzle, number board game)	Instances of parents’ use of *foundational* (counting and number identification) and *advanced* (cardinality, ordinal relations, arithmetic) math talk	*Foundational* numerical knowledge (verbal counting and numeral identification) and *advanced* numerical knowledge (number line estimation, counting principles, numerical magnitude comparison, enumeration and cardinality) were directly assessed	Parents’ **advanced** number talk was significantly related to children’s **advanced** numerical knowledge (r = .38, p < .05), while controlling for parents’ foundational number talk (β = .33, p < .05)	Parents’ **foundational and advanced** number talk were *not* significantly related to children’s **foundational** numerical knowledge (correlations ranged from -.17 to .12)

Table 1 *continued*

Study Citation	Sample Size	Child Age	Observed Activities	Parent Talk Measures	Child Measures	Relation between Parent Talk and Child Performance	
						Significant findings	**Nonsignificant findings**
5. Susperreguy & Davis-Kean, 2016	40	3 to 5 years old	3 days of LENA recordings (2 weekdays and 1 day on the weekend) capturing daily conversations during breakfast and dinner	Instances of mothers' use of cardinal values, counting, naming digits, units of measure, conventional nominatives, number comparisons, ordinal numbers, adding/subtracting, and division/fractions/percentages	Numeracy skills were directly assessed *one year later* using the Test of Early Mathematics Ability– Third Edition	Mothers cumulative number talk positively predicted children's numeracy skills (β = .31, $p < .05$)	
6. Zippert et al., 2020	45	3 to 5 years old	20 minutes of play (number cards, beads and laces, Lego Duplo blocks) Materials were paired with a list of suggested activities to elicit number, patterning, and spatial talk, respectively	Frequency of number talk (advanced operations, magnitude comparison, numeral identification, cardinal values, counting objects, ordinal relations, rote counting, and relative magnitude) Frequency of patterning and spatial talk also captured	Broad math knowledge (including numeracy, geometric and pattern skills) was directly assessed in the fall and spring using the Research-Based Early Mathematics Assessment Short Form Two additional measures assessed pattern and spatial skills		Parents' number, pattern, and spatial talk were *not* significantly related to children's corresponding skills or broad math knowledge at either time Point (correlations ranged from -.24 to .19)

Table 1 *continued*

Study Citation	Sample Size	Child Age	Observed Activities	Parent Talk Measures	Child Measures	Relation between Parent Talk and Child Performance	
						Significant findings	**Nonsignificant findings**
7. Thippana et al., 2020	97	3 years old	10 minutes of free play with standard toys during two separate lab visits and 10 minutes of free play at home during three separate timepoints between the lab visits	Frequency of parents' use of all numbers zero or greater	Numeracy skills were directly assessed *at the last lab visit* using the Test of Early Mathematics Ability– Third Edition		Parents' number talk was *not* significantly related to children's numeracy skills [$r(79) = .20$, $p = .070$]
8. Elliot et al., 2017	44	5 to 6 years old	10 minutes of free play with standard toys during a lab visit	Frequency of parents' use of all numbers one or greater grouped into the following categories: small (1–5), medium (6–10), and large (>10) numbers	Numeracy skills were directly using the Test of Early Mathematics Ability– Third Edition	Parents' number talk with numbers **larger than 10** was positively and significantly related to children's numeracy skills [$r(42) = .39$, $p < .01$], while controlling for parents' small and medium number talk ($\beta = .48$, $p < .01$)	Parents' number talk with numbers **1-10** and overall number talk were *not* significantly related to children's numeracy skills (correlations ranged from -.04 to .16)

Table 1 *continued*

Study Citation	Sample Size	Child Age	Observed Activities	Parent Talk Measures	Child Measures	Relation between Parent Talk and Child Performance	
						Significant findings	**Nonsignificant findings**
9. Mutaf Yildiz et al., 2018	44	4 to 6 years old	10 minutes of play (Building 'LEGO 60072 City Demolition Starter Set', Reading a commercially available storybook)	Frequency of number talk (counting up, counting down, counting wrong, determining the number of a set, operations, sorting things, identifying written numerals, distinguishing quantities, ordering quantities, and other numerical words)	Calculation skills (addition and subtraction) were directly assessed using two calculation subtests of the TediMath	Parents' number talk **during LEGO play** was *negatively* and significantly related to children's calculation skills ($r = -.35$, $p < .05$)	Parents' number talk **during storybook reading** was *not* significantly related to children's calculation skills ($r = -.05$, $p > .05$)
10. Son & Hur, 2020	46	4-year-olds	15 to 20 minutes of a cooking activity	Instances of number talk, operation talk, and measurement talk	Math achievement was directly assessed using the Applied Problems subtest of the Woodcock Johnson Test of Achievement III	Parents' number talk was marginally related to children's math skills in the fall ($r = .27$, $p < .10$) but significantly and positively associated with children's fall early math skills ($\beta = 0.49$, $p < .05$, $f^2 = .32$, while controlling for operation and measurement talk Parents' measurement talk was marginally related to children's math skills in the fall ($r = .33$, $p < .10$)	Parents' operation and measurement talk were *not* significantly related to children's fall early math skills ($rs = .01, .10$, $ps > .10$) Parents' number and operation talk were *not* significantly related to math skills in the spring ($rs = .12, .03$, $ps > .10$)

Table 1 *continued*

Study Citation	Sample Size	Child Age	Observed Activities	Parent Talk Measures	Child Measures	Relation between Parent Talk and Child Performance	
						Significant findings	**Nonsignificant findings**
11. Eason & Ramani, 2020	72	4 to 5 years old	15 minutes of play (a set of toy food) in one of three conditions: unguided play, guided play, and formal learning	Instances of fraction talk (formal, informal, and quantitative) and instances of number talk (1-20)	Study did not investigate the relation between parent math talk and child math skills. Parents' in the formal learning condition engaged in the most math talk followed by the guided play condition.		
12. Chernyak, 2020	115	Between infancy through the end or early childhood	Naturalistic settings (mealtime ad free play)	Frequency of parents' use of number words and quantifiers	Study did not investigate the relation between parent math talk and child math skills. Parents were more likely to talk about numbers and quantifiers within resource distribution (sharing) contexts than outside of them.		
13. Vandermaas-Peeler et al., 2016	23	Mean age 5.58 years	*Flip It, Fold It, Figure It Out! Playing with Math* exhibit in the North Carolina Museum of Life and Science	Instances of math talk (size, quantity comparison, number words, counting, spatial orientation, shape identification) during parent-child conversations	Study did not investigate the relation between parent math talk and child math skills. Parents who received explicit guidance instructions provided more math talk about size than parents in control group.		

METHOD

Recruitment

Multiple methods, including advertising on social media, contacting preschools, and sharing through networks, were used to recruit families to participate in this study. Social media posts were used as one recruitment method by publicly posting study details on social networking websites (e.g., Twitter) and sharing study details in various parenting Facebook groups. Additionally, recruitment emails about the study were sent to the local Head Start division director and local and out-of-state preschools and childcare centers. Preschool directors were asked to share information about the study with parents of 3-5-year-old children attending these programs. Recruitment efforts were also made by sharing study details with the local children's museum, posting the study in a research recruitment newsletter, and posting the study on childrenhelpingscience.com (a recruitment website). The recruitment email and flyer can be found in Appendix A.

Procedure

When parents agreed to participate in the study, they first completed a Qualtrics survey where they consented to the study, provided demographic information, and answered questions about their child's experiences and skills. Next, participants were mailed a USPS package containing activities to be used during a Zoom meeting (a video conferencing platform) with the experimenter. Participants were asked to wait until the Zoom meeting to open the package. When packages were delivered, parents were sent a Calendly (online scheduling software) link and asked to sign up for a time to participate in an approximately 30-minute Zoom meeting. Parents were given up to three email or text reminders to schedule a Zoom meeting if they did not sign up after the first request or missed their scheduled meeting. During the Zoom meeting, parent-child dyads participated in an observed interaction, and children completed brief direct assessments of early academic skills. Participating families received an e-gift card ($10) from Amazon for completing the Zoom meeting.

Participants

One hundred and thirty-three parents completed the Qualtrics survey, were mailed a USPS package, and were invited to select a Zoom meeting time. One hundred and twenty-eight families scheduled a Zoom meeting. Of the 128 families, 120 U.S. parent-child dyads from 31 different states attended the Zoom meeting to complete participation in the study. The sample size was recruited based on the recommended sample size for confirmatory factor analysis with a high level of communality and good fit criterion (Mundfrom et al., 2005). The 120 children (52.5% female) who participated in the study were on average 4.25 years old (SD = .83), 49.2% were White, 23.3% were multiracial/multiethnic, 15.8% were Black, 9.2% were Asian, and 2.5% were Latine. Parent education (95% mothers) varied, with 15.8% of parents having a Doctoral/Postgraduate degree, 36.7% reporting a Master's degree, 34.2% reporting a Bachelor's degree, 4.2% reporting an Associate's degree, and 9.2% reporting some college.

Parent-Child Interaction

Parent-child dyads were asked to engage in three different activities during the first 15-20 minutes of the Zoom meeting. The materials to facilitate the three activities included a wordless storybook, a sharing game, and a numbered puzzle presented in Appendix B. The wordless storybook was created by removing text from a lab-created mathematical language storybook, The Little Elephants Big Adventures: Just Enough Eggs (Isaacs & Dye, 2017). The illustrations from a relatable storyline (e.g., a story about baking a birthday cake) were used so that parent-child dyads could create their own story based on the illustrations. The illustrations included images that provided the opportunity to discuss quantities and comparisons if dyads naturally decided to use mathematical language and number talk. The sharing game included paper cutouts of the characters from the book (two elephants and a teddy bear) and peanuts (a snack in the book). Parent-child dyads were asked to help the characters share the peanuts, which allowed dyads to discuss how to share fifteen peanuts between the characters. Finally, parent-child dyads were asked to complete a numbered puzzle. Precisely, dyads matched numeral pieces with quantity pieces and put the pairs in number order to complete the puzzle. All activities were video recorded through Zoom. These materials were chosen because they are similar to materials and activity contexts that have been used in prior research (e.g., storybooks, number puzzles, sharing; Chernyak, 2020;

Ramani et al., 2015) and represent a broad range of activities that may be played at home that offer varying opportunities for communication about quantities or numbers.

Brief prompts were used to introduce each activity. For the first activity, the experimenter explained that the first activity was a wordless storybook and prompted participants to “create a story by talking about what is happening on each page and asking questions along the way”. For the second activity, the experimenter stated that “the elephants and bear love peanuts, but they need your help sharing them” and prompted participants to “help them share the peanuts”. For the last activity, the experimenter explained that this activity includes puzzle pieces and a sheet of paper with instructions and prompted participants to “lay the puzzle pieces on top of the paper to create a picture”. On average, the wordless storybook took seven minutes for parent-child dyads to complete, the sharing game took three minutes, and the puzzle was completed in six minutes.

Coding Scheme

During the recorded interaction sessions, parent and child speech were first automatically transcribed by the Zoom software. The Zoom transcriptions were then used as a guide/draft for research assistants to manually transcribe each parent-child play session verbatim and correct any errors in the Zoom transcription. A second transcriber reviewed twenty-five percent of the participant transcripts to check for errors. On average, parent-child activity transcripts had a 1% error (ranging from 0% to 4%) which involved misspelled words or missing child or parent speech. These minor errors were corrected. Overall, an average of 99% agreement suggested that the transcription process was adequate (Graham et al., 2012; Hartmann, 1977). Transcriptions of parent and child speech during the wordless storybook, sharing game, and number puzzle activities were formatted and saved at the utterance level using Child Language Analysis (CLAN) software. The Codes for the Human Analysis of Transcripts (CHAT) conventions of the Child Language Data Exchange System (CHILDES, MacWhinney, 2000) were used to code transcripts for mathematics-related talk.

Transcripts were coded for cumulative instances (tokens) of parents’ number talk, mathematical language talk, and general (non-math) talk that occurred during each of the three activities (wordless storybook, sharing game, and a numbered puzzle; Table 2).

Measures

Parents' talk, parents' reports of children's skills, and direct assessments of children's skills were measured to conduct this study.

Parent number talk

Specifically, every use of the number word "zero" or greater was coded as number talk tokens (total amount of number words used) which is consistent with number talk coding practices in previous work (Elliott et al., 2017; Levine et al., 2010; Thippana et al., 2020). However, the number word "one" was only included as number talk when it was used numerically (e.g., "There is only one piece left") and not when used non-numerically (e.g., used as a deictic expression; "I like this one the best"). The word "one" was manually coded by two research assistants to ensure that "one" was counted only when used numerically. Two research assistants double-coded 20% of the uses of "one" and had interrater reliability of 96%. Parent number talk consisted of all instances of numerical "one" and all instances of "two" through "hundred" spoken by parents (see Table 3 for a list of number words).

Parent mathematical language

Following a similar coding procedure, each use of quantitative mathematical language was coded as a mathematical language token. CLAN software was used to search for 48 mathematical language terms that parents may have used, and the search indicated that parents used 38 mathematical language terms (see Table 4 for the complete list of terms). All instances of parents' use of each target term were manually coded to ensure that words were being used in a quantitative context to count as mathematical language. If a target term was used by parents fewer than 150 times, every instance of its use was coded by two separate research assistants. Interrater reliability ranged from 81% to 100% for target terms that were used fewer than 150 times. If a target term was used more than 150 times, 20% of all instances of parents' use were double-coded, followed by one research assistant coding the remaining 80%. Interrater reliability ranged from 92% to 100% for terms parents used more than 150 times. When there was disagreement between the first two coders, a third research assistant decided on how these terms should be coded.

Parent general talk

General talk tokens were created by subtracting cumulative number talk and mathematical language tokens from overall word tokens during each activity (Levine et al., 2010). This coding scheme resulted in nine indicator variables consisting of parents' cumulative general talk, mathematical language, and number talk, which occurred during each of the three play activities (book, sharing, and puzzle). The parent talk variables from each play activity were used to create the general, math language, and number talk factors (Table 2).

Parent-reported measures

Parents completed a Qualtrics survey that reported background demographic variables, including child age, gender, and parent education (scored on a 9-point scale ranging from less than 8th grade to doctoral degree). Parents also reported their child's general vocabulary, mathematical language, and numeracy skills. Parent-reported executive functions were also collected as control variables to account for child characteristics that may be related to variability in target early academic skills. The experimenter also virtually assessed children on brief measures of their general vocabulary, mathematical language, and numeracy skills.

Parent-reported general vocabulary

The Developmental Vocabulary Assessment for Parents (DVAP; Libertus et al., 2015) was used to assess children's expressive vocabulary. The DVAP uses the first 204 words from the PPVT-4, and parents are asked to mark the words they have heard their child say. The DVAP was created for a broad age range (2-to 7-year-olds); therefore, the 36 most advanced words were dropped from the original measure to be used with a more targeted sample of 3- to 5-year-olds. Specifically, in this study, the DVAP included the first 168 words, which include words commonly learned between 2 and 15 years of age. The DVAP typically takes 5-10 minutes for parents to complete and is highly correlated with the MacArthur-Bates Communicative Development Inventories (CDI; $r = .79$; Fenson et al., 2007; Libertus et al., 2015) and the Peabody Picture Vocabulary Test, fourth edition (PPVT-4; $r = .69$; Dunn & Dunn, 2007, Libertus et al., 2015), suggesting that this measure has good convergent validity.

Parent-reported mathematical language knowledge

Parents were presented with nine quantitative mathematical language terms and asked to select all the words they were confident their child understood. These mathematical language terms included *more, most, fewest, fewer, least, less, a lot, a little bit,* and *same* ($\alpha = .80$). The nine parent-reported quantitative mathematical language terms corresponded with direct assessments of children's mathematical language knowledge.

Parent-reported early numeracy

Reports of children's numeracy abilities (verbal counting, numeral identification, and simple addition) were assessed using three questions (Lin et al., 2021) presented in Appendix C. For verbal counting, parents were asked, "How high can your child count?" For numeral identification, parents were presented with numerals 1 through 15 and asked to select all the numerals that their child could identify ($\alpha = .94$). Finally, for simple addition, parents were asked to respond "yes" or "no" to the prompt, "Can your child calculate simple addition, like 1 + 1 =___, 1 + 2 =___)?".

Parent-reported executive functioning

The Childhood Executive Functioning Inventory (CHEXI; Thorell & Nyberg, 2008) was used to assess children's executive functioning. The CHEXI consists of 24 items that are divided into two subscales: working memory (13 items, e.g., "has difficulty with tasks or activities that involve several steps") and inhibition (11 items, e.g., "has clear difficulties doing things he/she finds boring"). Parents indicated how true each item is for their child on a 5-point Likert scale (from 1 = definitely not true to 5 = definitely true). CHEXI items were reverse-scored for analyses so that higher scores indicated higher EF skills. Previous research indicates this measure has good internal consistency ($\alpha > .85$; Catale et al., 2015).

Brief direct assessment of early academic skills

Children were assessed on a brief measure of mathematical language, numeracy skills, and general vocabulary. During virtual data collection, the experimenter verbally prompted children and shared a screen that showed the response option images that the child could choose from (Appendix D). The response option images were lettered so parents could let the experimenter know which option their child chose. Parents were asked to let the child answer independently and refrain from letting the child know if their answer was correct or incorrect.

Mathematical language

Selected items from the original Preschool Assessment of the Language of Mathematics (PALM; Purpura & Reid, 2016) term pool were used to assess children's quantitative mathematical language. Nine items assessing quantitative language (i.e., more, most, fewest, fewer, least, less, a lot, a little bit, same; $\alpha = .65$) were selected to match the parent-reported measure. Children were awarded one point for each correct response. All items were designed to be completed without exact quantitative skills and in a non-numeracy context. For example, the quantitative questions are asked in different ways: (a) comparing dots with such a gross difference that children would be able to respond correctly regardless of numeracy ability as long as they knew the meaning of the language terms (e.g., 10 vs. 2), and (b) using a picture of mostly full and mostly empty glasses when asking “Which glass has the least amount of water?”.

Early numeracy

Selected subscales from Purpura & Lonigan (2013; verbal counting, numeral identification, and simple addition) were used to evaluate children’s numeracy skills. Specifically, items that match the parent-reported measure were used.

Verbal counting

For verbal counting, the experimenter prompted the child, “I want you to count as high as you can, starting with the number one. Go ahead.” Children were stopped when they made an error or when they reached 100. The greatest number they counted without making an error was recorded as their score.

Simple addition

For verbal counting, the experimenter prompted the child, “I want you to count as high as you can, starting with the number one. Go ahead.” Children were stopped when they made an error or when they reached 100. The greatest number they counted without making an error was recorded as their score.

Numeral identification

For numeral identification, the experimenter showed children cards with numerals printed on them and ask, “What is the name of this number?” Numbers assessed included: 1, 2, 3, 7, 8, 10, 12, 14, and 15 (α = .90). If children said the names of individual numbers for double-digit numerals (e.g., “one and five” instead of “fifteen”), the assessor asked, “Do you know what they are called together?” Children received one point for each correct response, and scores were summed for a maximum of nine possible points.

General vocabulary

Children’s general vocabulary skills were assessed using the Expressive Vocabulary subtest from the Clinical Evaluation of Language Fundamentals–Preschool–Second Edition (CELF:P2; Wiig et al., 2004). There are high correlations between the Expressive Vocabulary subtest and the CELF:P2 core language composite score (e.g., r = 0.86; Wiig et al., 2004) which suggests that this subtest may be used as a proxy for general language ability. In the Expressive Vocabulary subtest, children were presented with pictures of people, objects, and actions and asked to name the picture (referential naming). The CELF:P2 has good reliability (αs = 0.77–0.84 for 3- to 5-year-old children; Wiig et al., 2004). Children received one point for each correct response. Basal and ceiling rules were administered per the instruction manual.

Analytic strategy

A series of three confirmatory factor analyses were conducted in Mplus to address the first aim of this study and identify the best-fitting factor structure of parents’ talk during parent-child math-related engagement (Muthén & Muthén, 2012; See Figure 1 for conceptual models). First, a single factor CFA was fitted, including all parent talk items. Second, a 2-factor CFA was fitted, including a factor representing general talk and a factor representing math-related talk (both number talk and mathematical language). Third, a 3-factor CFA was fitted. The 3-factor model included factors representing general talk, number talk, and mathematical language. Global model fit was assessed using a series of indices. A chi-square difference test was used to determine which model was the best fit. The Tucker-Lewis index (TLI), Bentler comparative fit index (CFI), and the root mean square error of approximation (RMSEA) were also used to examine model fit (Hu & Bentler, 1999). Additionally, Akaike Information Criterion (AIC) and sample-size adjusted Bayesian Information Criterion (BIC) were used to evaluate the relative model fit of all models. Lower AIC and BIC values (typically differences of 10 or more) indicate a better fitting model (Kass & Raftery, 1995; Hu & Bentler, 1999; Burnham et al., 2011).

The original plan was to run a structural equation model (SEM) to investigate the relations between the individual parent talk factors and children's general vocabulary, numeracy skills, and mathematical language (the second aim of this study; see Figure 2 for conceptual models). However, the SEM model would not run correctly (positive factor loadings from the CFA model became negative in the SEM model), likely due to insufficient power for the complexity of the full model. Thus, to address this issue, the analytic procedure was modified, and factor score extraction was conducted so that factors could be used as independent variables in regression analyses as an alternative to the SEM approach (Logan et al., 2021). The three parent talk factors were extracted using the ten Berge method in R. The ten Berge method extracts factors while preserving the correlations among factors and maintains factor score determinacy (Krijnen et al., 1996; Logan et al., 2021; Ten Berge et al., 1999).

A combination of parent ratings and direct assessments were used to create the variables for children's general vocabulary, numeracy skills, and mathematical language (z-scores for parent reports and direct assessment were used to create composite scores for each domain). Control variables in the structural equation model included the child's age, gender, executive functioning, and parent's education level.

Maximum likelihood estimation with robust standard errors (MLR) and full information maximum likelihood (FIML) was used to address missing data in the variables used in the CFA and regression analyses, respectively. Specifically, there was 0.8% to 1.6% missing data in the sharing game and puzzle parent talk item. This missing data was due one parent not talking during the puzzle activity and two parents accidently not being sent the peanuts for the sharing game. Additionally, the child sex covariate had 0.8% missing data. Both MLR and FIML use all available information about the data to make missing values consistent with observed data rather than using listwise deletion to address missing values (Acock, 2012).

Table 2

Coding scheme and definitions for parents' talk

Parents' Talk Type/Factors	**Parent-Child Activity**	**Parents' Talk Codes/Indicators**	**Definition**
General Talk	Wordless storybook Sharing game Numbered puzzle	Wordless: General talk tokens Sharing: General talk tokens Puzzle: General talk tokens	General talk tokens were created by subtracting cumulative number talk and mathematical language tokens from overall word tokens during each activity.
Number Talk	Wordless storybook Sharing game Numbered puzzle	Wordless: Number talk tokens Sharing: Number talk tokens Puzzle: Number talk tokens	Every instance of parents' use of number words "zero" or greater, excluding the use of "one" in non-numerical contexts.
Math Language	Wordless storybook Sharing game Numbered puzzle	Wordless: Math language tokens Sharing: Math language tokens Puzzle: Math language tokens	Every instance of parents' use of quantitative math language (e.g., take away, a lot, more, fewer, less, a few, a couple, some, etc.)

Table 3

List of number words that were used at least once by a parent

Parents’ Math Language	Total Tokens	% Book Tokens	% Share Tokens	% Puzzle Tokens
zero	7	14%	0%	86%
one	1181	22%	53%	26%
two	821	39%	37%	24%
three	1096	41%	23%	36%
four	698	31%	28%	42%
five	630	6%	42%	52%
six	462	13%	20%	67%
seven	315	7%	19%	75%
eight	275	6%	9%	85%
nine	322	23%	6%	71%
ten	256	3%	9%	88%
eleven	13	23%	69%	8%
twelve	17	18%	71%	12%
thirteen	15	7%	80%	13%
fourteen	24	0%	83%	17%
fifteen	54	0%	87%	13%
eighteen	1	0%	100%	0%
thirty	1	0%	100%	0%
fifty	1	0%	100%	0%
hundred	3	33%	67%	0%

Table 4

List of quantitative math language terms that were used at least once by a parent

Parents' Math Language	Number of Tokens	% Book Tokens	% Share Tokens	% Puzzle Tokens	Definition	Example
Add/ added/ adding	10	50%	30%	20%	Combine two or more numbers/quantities	"*Add* more peanuts."
All	524	26%	52%	22%	The whole quantity of a group	"Let's count *all* the rest of them."
Any	48	29%	60%	10%	One or some of a group	"Does the bear get *any* peanuts?"
Big/ bigger/ biggest	16	6%	38%	56%	A quantity of considerable size	"Is three *bigger* than nine?"
Different	11	9%	27%	64%	A quantity that is not the same as another quantity	"They all got a *different* number of apples."
Divide/ divided/ dividing/ divvy	14	0%	100%	0%	Separate a quantity into parts	"Fifteen *divided* by three is five."
Each	165	4%	93%	2%	Every one of two or more considered individually	"Give *each* of them one more."
Enough	118	78%	10%	12%	As many as required	"You think that's *enough* peanuts for everybody?"
Equal/ equals/ equally	32	6%	94%	0%	The same in quantity	"They're sharing them *equally*."
Even/ evenly	21	0%	95%	5%	Equal in quantity	"Can we make everybody have an *even* amount?"
Extra	26	65%	35%	0%	Added to an existing amount of number	"We have two *extra* after giving each one four."
Few/ fewer/ fewest	30	13%	27%	60%	A small number/quantity	"Who has the *fewest* apples?"
Less/ Least	36	17%	61%	22%	A smaller amount of	"What is one *less* than seven?"

Table 4 *continued*

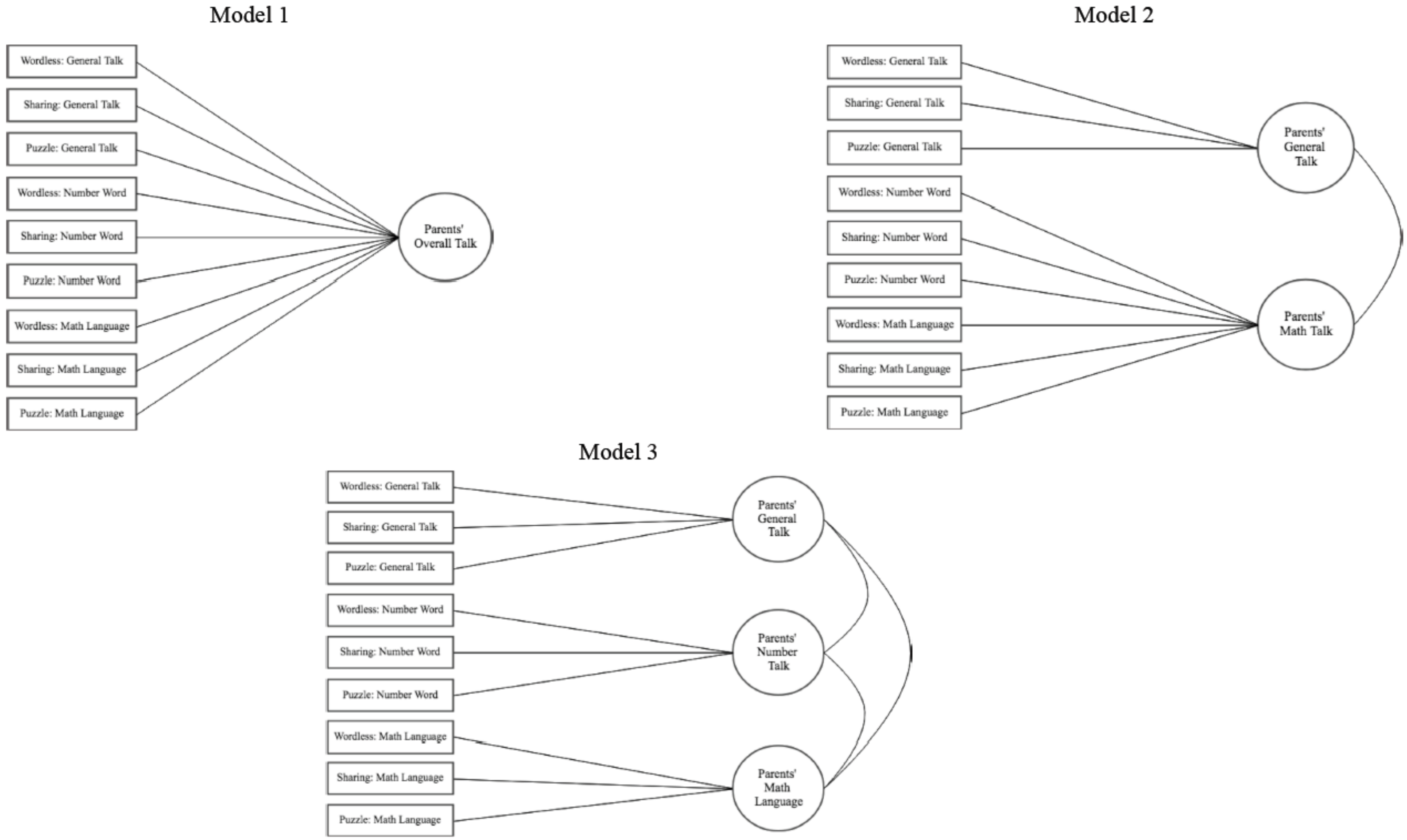
Parents' Math Language	Number of Tokens	% Book Tokens	% Share Tokens	% Puzzle Tokens	Definition	Example
Little	5	60%	20%	20%	A quantity of small amount	"Does she have too many or too *little* eggs?"
Lots	26	8%	62%	31%	A large amount	"We have *lots* of pieces here."
Many	52	60%	23%	17%	A large number of	"We have so *many* peanuts to share."
Minus	4	0%	100%	0%	Less by the subtraction of	"Five *minus* one is four."
More/ most	482	22%	50%	27%	A greater amount	"I think my friends have *more* peanuts than me."
Much	52	71%	17%	12%	A great amount of	"Do you think she has too *much* milk?"
Multiple	2	0%	0%	100%	More than one	"There's *multiple* elephants."
None	2	50%	50%	0%	Not any	"They have two peanuts and I have *none*."
Plenty	1	100%	0%	0%	A large amount	"She's got *plenty* of eggs."
Plus	26	15%	73%	12%	With the addition of	"What's five *plus* five?"
Same	185	4%	61%	35%	Quantities that are identical	"Do those have the *same* number of dots?"
Small/ smaller/ smallest	11	9%	82%	9%	A quantity of an insignificant size	"That's *smaller* than ten."
Some	348	44%	39%	17%	An unspecified amount	"The bear wants *some* peanuts."
Subtract/ subtracted/ subtracting	1	100%	0%	0%	To take away one quantity from another	"They need to *subtract* six eggs."
Times	4	75%	25%	0%	Multiplied by to increase in number/quantity	"Three *times* three is nine total eggs."

Table 4 *continued*

Parents' Math Language	Number of Tokens	% Book Tokens	% Share Tokens	% Puzzle Tokens	Definition	Example
Total	13	38%	62%	0%	The whole number or amount	"We have fifteen *total* peanuts."
Twice	4	25%	50%	25%	Two times	"You counted that one *twice*."
Whole	7	14%	0%	86%	All of	"We have the *whole* top row done."
A bunch	12	33%	33%	33%	A number of things	"We have *a bunch* of puzzle pieces."
A couple	13	31%	15%	54%	Two of something	"We only have *a couple* pieces left."
A little bit/ a tiny bit	8	75%	25%	0%	A small amount	"Is that *a little bit* if peanuts?"
A lot	104	23%	34%	43%	A large amount	"This one has *a lot* of dots."
A whole bunch/ a whole lot	3	33%	67%	0%	A large amount of something	"She has *a whole bunch* of eggs."
How many/ how much	1,408	43%	34%	23%	What number or amount	"*How many* eggs are over here?"
Take away/ took away	39	56%	33%	10%	Process of subtracting	"*Take away* six eggs."
Additional terms that were searched but not present in parent transcripts						
Combine					Add two or more amounts together	
Greater					A number or quantity larger than another	
Large/ larger/ largest					A quantity of a considerable size	

Table 4 *continued*

Additional terms that were searched but not present in parent transcripts	**Number of Tokens**	**% Book Tokens**	**% Share Tokens**	**% Puzzle Tokens**	**Definition**	**Example**
Majority					The greater number	
Multiply					Find the product of two or more numbers	
Pair					A set of two things	
Several					More than two but not very many	
Similar					Quantities that resemble but are not identical	
A number of					More than two but fewer than many	
All gone					The whole quantity is finished or used up	



Note. Model 1 depicts a single factor model, Model 2 is a 2-factor model where items make up a general talk and math talk factor, and Model 3 illustrates a 3-factor model where items represent a general talk, number talk, and math language factor.

Figure 1. *Conceptual models for the factor structure of parents' talk*

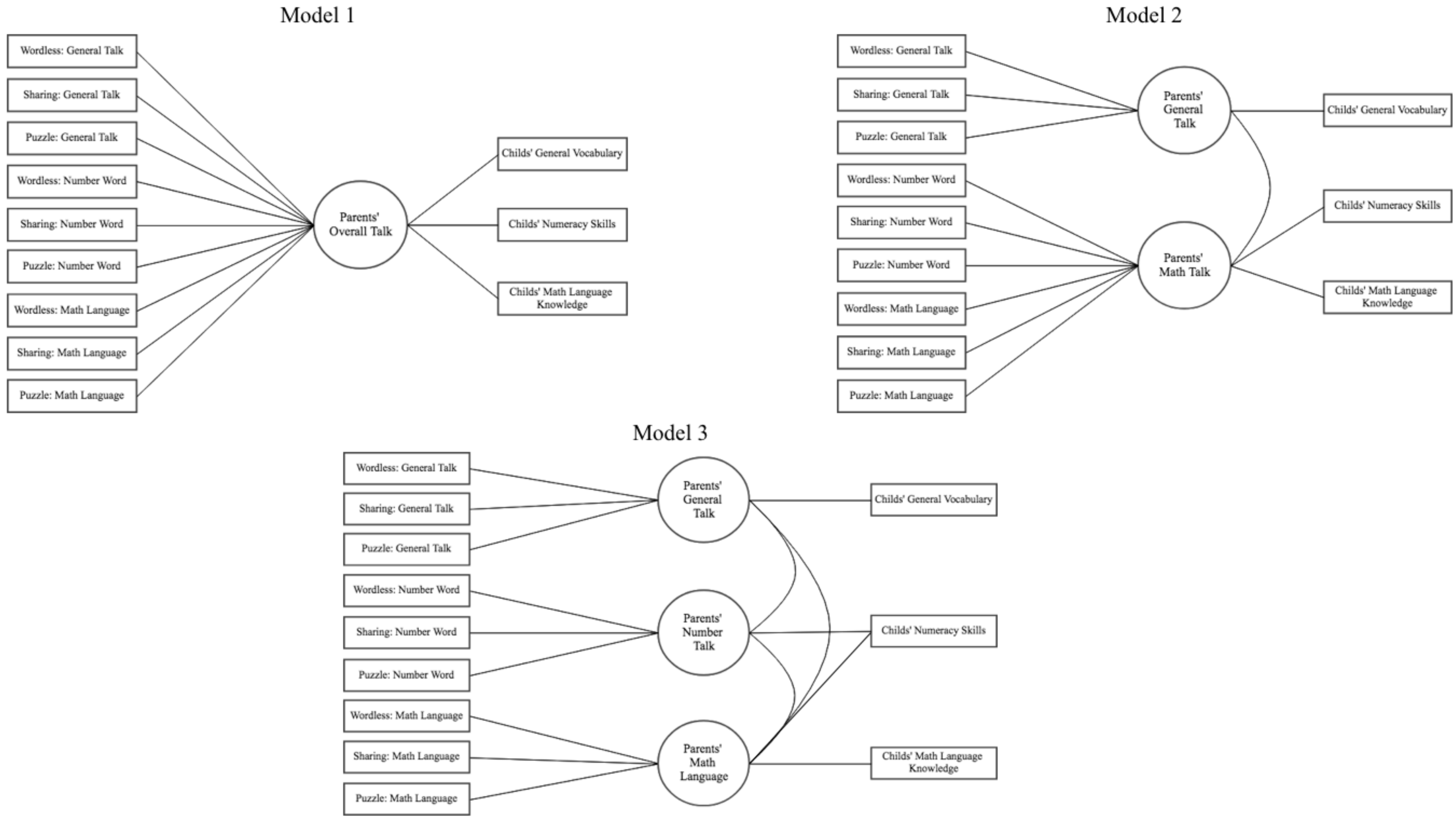


Figure 2. *Conceptual models for the relation between parents' talk and children's performance*

RESULTS

The descriptive statistics for demographic and control variables, children's early academic skills, and parents' talk items are included in Table 5. The skewness and kurtosis for most variables were within an acceptable range (between -2 and +2; George & Mallery, 2010). Five parent talk variables had skewness and kurtosis that were outside the acceptable range (Table 5). Square root transformation was used to normalize the skewed distribution of parents' general and number talk during book reading, general talk during the sharing game, and number and quantitative talk during the puzzle. Correlations between individual parent talk items are presented in Table 6. All parent talk items are significantly and positively correlated, with higher correlations present within activity type. Parent reports and direct assessments of children's general vocabulary, numeracy skills, and mathematical language knowledge were moderate to highly correlated (Table 7).

Individual parent talk items tended to be negatively correlated with children's early academic skills (Table 6). Specifically, parents' general talk during the sharing game was significantly and negatively related to children's expressive vocabulary, numeracy skills, and mathematical language knowledge. Parents' number talk during the sharing game was significantly and negatively related to numeracy skills and mathematical language knowledge but not children's expressive vocabulary. Additionally, parents' general, number, and mathematical language talk items during the puzzle were significantly and negatively related to children's early academic skills. These correlations suggested that parents used higher levels of talk during the sharing game and puzzle when their children had lower early academic skills.

Examining the Factor Structure of Parents' Talk

The first aim of this study was to examine the factor structure of parents' talk. The factor structure of parents' talk was initially tested by comparing a 1-factor model (parents talk as one construct) and a 2-factor model (general talk and math talk [number + math language] as separate constructs) to the hypothesized 3-factor model (general talk, number talk, and math language as three separate constructs). Parent talk factors were correlated in the 2- and 3-factor models. Additionally, for all models, the parent talk indicator errors within each activity were correlated due to talk items being more related when they occurred within the same context (wordless book, sharing game, or puzzle). Fit statistics for the three competing models are presented in Table 9. The 3-factor model provided an excellent fit to the data based on fit statistics (Figure 3; RMSEA ≤ 0.08, CFI and TLI ≥ 0.90, SRMR ≤ 0.10; Hu and Bentler, 1999). Additionally, consistent with the first hypothesis, the 3-factor model was a better fit to the data than the 2-factor model ($\Delta \chi2 = 17.23$, df = 2, $p < .001$; Δ AIC > 10, Δ BIC > 10) and the 1-factor model ($\Delta \chi2 = 35.07$, df = 3, $p < .001$; Δ AIC > 10, Δ BIC > 10).

The three factors on the 3-factor model were significantly and positively correlated (Table 8). Due to the high correlation (r = 0.92) between the general talk and math language factors, a robustness check was conducted to compare an alternative 2-factor model (with general talk and math language combined) to the hypothesized 3-factor model (Table 10). Although the alternative 2-factor model was a good fitting model, the 3-factor model was a significantly better fitting model ($\Delta \chi2 = 7.36$, df = 2, $p < .05$; Δ AIC < 10, Δ BIC < 10), suggesting that parents' general talk, number talk, and math language use are highly related but distinct categories of talk that children were exposed to during conversations with their parents. Although Δ AIC and Δ BIC values less than 10 would indicate that there is no difference between the two competing models, these models are nested, and therefore the significant chi-square difference test indicates that the 3-factor model is a better fit to the data than the simpler model (Bandalos & Finney, 2018). All parent talk indicators significantly and positively loaded (> .4; Brown, 2015) onto their respective factors (Table 11).

Relations Between Parent Talk Factors and Children's Early Academic Skills

The second aim of this study was to examine the relation between parent talk factors and children's early academic skills. The results of the regression analyses with the extracted parent talk factors can be found in Tables 12-14. First, children's general vocabulary, numeracy skills, and mathematical language knowledge were regressed on covariates and one parent talk variable in separate regression models (Models 1 to 3). Then, a fourth model was run that included all three parent talk variables to assess unique relations between parents' talk and children's early academic skills. In models 1 to 3, none of the parent talk variables were significantly related to children's general vocabulary, numeracy skills, or mathematical language knowledge. In model 4, when including all three parent talk variables in the same model, as well as all covariates (Figure 4), results indicated that parents' general talk was not significantly related to children's general vocabulary ($\beta = 0.03$, $p = .881$; Table 12), which was contrary to hypotheses. Similarly, contrary to hypotheses, parents' number talk was not significantly related to children's numeracy skills ($\beta = 0.16$, $p = .203$; Table 13), and parents' mathematical language use was not significantly related to children's mathematical language knowledge ($\beta = -0.54$, $p = .053$; Table 14). However, parents' general talk was positively related to children's numeracy skills ($\beta = 0.67$, $p < .001$; Table 13), which was contrary to hypotheses. Contrary to hypotheses, parents' mathematical language use was significantly and negatively related to numeracy skills ($\beta = -0.87$, $p < .001$; Table 13). These results suggest that parents tended to use more general talk when children had higher numeracy skills. In contrast, parents tended to use more mathematical language when children had lower numeracy skills while engaging in math-related activities.

Table 5

Descriptive statistics for demographic variables, early academic skills, and parent talk items

	N	*M*	*SD*	Range	Skewness	Kurtosis
Age	120	4.25	0.83	3.04-5.93	0.36	-0.84
Child Sex	119 (52.5% female)		–	–	–	–
Parent Education	120	7.46	1.10	5-9	-0.68	0.18
Parent Reported EF						
Working Memory	120	3.73	0.58	2.08-5.00	-0.39	0.13
Inhibitory Control	120	3.22	0.57	1.73-4.73	0.11	0.19
Parent Reported Child Outcomes						
General Vocabulary	120	77.38	28.51	7-139	0.00	-0.39
Verbal Counting	119	33.87	33.62	0-100	1.35	0.12
Numeral Identification	120	10.11	5.02	0-15	-0.72	-0.68
Simple Addition	120	0.58	0.50	0-1	-0.34	-1.92
Math Language	120	6.19	2.06	0-9	-0.34	-0.33
Direct Assessments of Child Outcomes						
General Vocabulary	118	18.37	7.25	2-38	0.31	0.10
Verbal Counting	111	32.14	30.85	2-100	1.38	0.58
Numeral Identification	117	5.88	2.90	0-9	-0.61	-0.79
Simple Addition	114	2.18	1.82	0-5	0.34	-1.29
Math Language	118	5.87	2.01	2-9	-0.08	-0.94
Child Early Academic Skills Standardized Composites						
General Vocabulary	120	0.01	0.86	-1.95-2.21	0.06	0.02
Math Language	120	-0.00	0.85	-2.21-1.46	-0.22	-0.50
Numeracy	120	-0.01	0.82	-1.48-1.43	0.28	-0.84
Parent Talk Items Raw						
General_Book	120	605.69	246.38	96-1628	0.96	2.42
Number_Book	120	12.62	12.86	0-68	1.64	3.10
Math language_Book	120	10.91	8.25	0-38	1.09	1.18
General_Share	118	236.63	144.54	9-885	1.50	3.39
Number_Share	118	16.59	15.24	0-66	1.40	1.51
Math language_Share	118	14.36	10.65	0-52	1.32	1.45
General_Puzzle	119	408.63	232.29	51-1215	0.82	0.84
Number_Puzzle	119	23.28	24.21	0-154	2.18	7.09
Math language_Puzzle	119	7.23	7.09	0-53	2.84	14.28

Table 5 c*ontinued*

	N	*M*	*SD*	Range	Skewness	Kurtosis
Parent Talk Items Transformed/Standardized						
General_Book*	120	0.00	1.00	-2.86-3.25	0.08	0.89
Number_Book*	120	0.00	1.00	-1.68-2.86	0.33	-0.24
Math language_Book	120	0.00	1.00	-1.32-3.28	1.09	1.18
General_Share*	118	0.00	1.00	-2.64-3.37	0.53	0.62
Number_Share	118	0.00	1.00	-1.09-3.24	1.40	1.51
Math language_Share	118	0.00	1.00	-1.35-3.52	1.32	1.45
General_Puzzle	119	0.00	1.00	-1.54-3.47	0.82	0.84
Number_Puzzle*	119	0.00	1.00	-1.78-3.47	0.53	0.44
Math language_Puzzle*	119	0.00	1.00	-1.96-3.99	0.50	1.28

Note. EF = executive function. Parent report and direct assessment of children's general vocabulary skills were used to create the general vocabulary composite. Parent report and direct assessment of children's math language knowledge were used to create the math language composite. Parent report and direct assessment of children's verbal counting, numeral identification, and simple addition skills were used to create the numeracy composite. *Parent talk items were square root transformed to normalize skewed distribution.

Table 6

Correlations between individual parent talk items and child outcomes

	1	2	3	4	5	6	7	8	9	10	11
1. General_Book	–										
2. Number_Book	0.55**	–									
3. Math language_Book	0.63**	0.64**	–								
4. General_Share	0.58**	0.27**	0.35**	–							
5. Number_Share	0.33**	0.42**	0.37**	0.62**	–						
6. Math language_Share	0.38**	0.23*	0.27**	0.74**	0.66**	–					
7. General_Puzzle	0.43**	0.20*	0.22*	0.57**	0.32**	0.39**	–				
8. Number_Puzzle	0.33**	0.33**	0.30**	0.38**	0.45**	0.31**	0.73**	–			
9. Math language_Puzzle	0.42**	0.22*	0.26**	0.38**	0.24**	0.32**	0.76**	0.66**	–		
10. Child general vocab.	-0.18	-0.03	-0.07	-0.21*	-0.15	-0.03	-0.30**	-0.27**	-0.21*	–	
11. Child numeracy	-0.12	0.01	0.04	-0.27**	-0.25**	-0.17	-0.45**	-0.43**	-0.33**	0.59**	–
12. Child math language	-0.13	-0.03	0.03	-0.22*	-0.25**	-0.12	-0.38**	-0.35**	-0.30**	0.69**	0.67**

$^{*}p < 0.05$, $^{**}p < 0.01$.

Table 7

Correlations between individual parent report and direct assessment of child outcomes

	1	2	3	4	5	6	7	8	9
1. PR_General Vocabulary	–								
2. PR_Verbal Counting	0.43**	–							
3. PR_Numeral Identification	0.32**	0.56**	–						
4. PR_Simple Addition	0.33**	0.46**	0.49**	–					
5. PR_Math Language	0.53**	0.43**	0.40**	0.31**	–				
6. DA_General Vocabulary	**0.48****	0.36**	0.34**	0.44**	0.41**	–			
7. DA_Verbal Counting	0.50**	**0.92****	0.54**	0.45**	0.43**	0.39**	–		
8. DA_Numeral Identification	0.44**	0.62**	**0.83****	0.55**	0.37**	0.42**	0.62**	–	
9. DA_Simple Addition	0.48**	0.63**	0.49**	**0.59****	0.42**	0.55**	0.67**	0.60**	–
10. DA_Math Language	0.40**	0.52**	0.43**	0.51**	**0.40****	0.62**	0.56**	0.53**	.62**

Note. PR = parent report. DA = direct assessment. Bolded values are the correlations between the matched parent report and direct assessment of each skill. $^{**}p < 0.01$.

Table 8

Correlations between covariates, parent talk factors, and early academic skills

	1	2	3	4	5	6	7	8	9	10
1. Child age	–									
2. Child sex	-0.22*	–								
3. Parent education	-0.09	0.01	–							
4. Working memory	0.14	-0.14	0.03	–						
5. Inhibitory control	0.05	0.01	0.18*	0.70***	–					
6. Parent general talk	-0.18	0.02	0.16	0.05	0.08	–				
7. Parent number talk	-0.32***	0.11	0.13	0.10	0.10	0.67***	–			
8. Parent math language	-0.32***	0.09	0.16	0.06	0.11	0.92***	0.83***	–		
9. Child general vocab.	0.64***	-0.25**	-0.05	0.21*	-0.02	-0.08	-0.12	-0.17	–	
10. Child numeracy	0.62***	-0.22*	-0.01	0.17	-0.06	-0.11	-0.26**	-0.28**	0.59***	–
11. Child math language	0.57***	-0.17	-0.02	0.16	0.05	-0.13	-0.25**	-0.26**	0.69***	0.67***

Note. Child sex is coded as 1 for female and 0 for male. *p< 0.05, **p< 0.01, ***p< 0.001.

Table 9

Model fit statistics and nested model comparisons

	χ^2 (*df*)	χ^2_{diff} (*df*)	AIC	Adj. BIC	RMSEA	CFI	TLI	SRMR
3 Factors	12.97 (15)	–	2461.23	2446.64	0.00	1.00	1.00	0.03
2 Factors	35.67 (17)	17.23^{***} (2)	2480.08	2466.24	0.10	0.96	0.92	0.05
1 Factor	56.04 (18)	35.07^{***} (3)	2500.30	2486.83	0.13	0.93	0.85	0.10

$^{***}p<0.001$.

Table 10

Robustness check: comparisons between three factor model and alternative two factor model

	χ^2 (*df*)	χ^2_{diff} (*df*)	AIC	Adj. BIC	RMSEA	CFI	TLI	SRMR
3 Factors	12.97 (15)		2461.23	2446.64	0.00	1.00	1.00	0.03
Alt. 2 Factors	20.72 (17)	7.36^* (2)	2466.06	2452.22	0.04	0.99	0.99	0.05

Note. Alternative 2-Factor model combines general and quantitative mathematical language in one factor. $^*p < 0.05$.

Table 11

Standardized factor loadings from the best fitting three-factor model of parents' talk

	General Talk	Number Talk	Math Language
General_Book	0.67	–	–
General_Share	0.90	–	–
General_Puzzle	0.63	–	–
Number_Book	–	0.52	–
Number_Share	–	0.77	–
Number_Puzzle	–	0.59	–
Math language_Book	–	–	0.48
Math language_Share	–	–	0.64
Math language_Puzzle	–	–	0.46

Table 12

Parent talk factors predicting child vocabulary

	Child General Vocabulary							
	Model 1		Model 2		Model 3		Model 4	
	β	*SE*	β	*SE*	β	*SE*	β	*SE*
Child age	0.60***	0.05	0.62***	0.06	0.61***	0.06	0.62***	0.06
Child sex	-0.07	0.07	-0.07	0.07	-0.07	0.07	-0.07	0.07
Parent education	0.05	0.07	0.04	0.07	0.05	0.07	0.04	0.07
Working memory	0.31**	0.09	0.30**	0.09	0.30**	0.09	0.29**	0.10
Inhibitory control	-0.28**	0.10	-0.28**	0.09	-0.28**	0.10	-0.27**	0.10
Parent general talk	0.02	0.07	–	–	–	–	0.03	0.20
Parent number talk	–	–	0.07	0.07	–	–	0.13	0.13
Parent math language	–	–	–	–	0.04	0.07	-0.10	0.26

**p< 0.01, ***p< 0.001.

Table 13

Parent talk factors predicting child numeracy skills

	Child Numeracy Skills							
	Model 1		Model 2		Model 3		Model 4	
	β	*SE*	β	*SE*	β	*SE*	β	*SE*
Child age	0.59***	0.06	0.56***	0.06	0.56***	0.06	0.48***	0.07
Child sex	-0.05	0.07	-0.05	0.07	-0.05	0.07	-0.03	0.07
Parent education	0.01	0.07	0.02	0.07	0.02	0.07	0.003	0.07
Working memory	0.28**	0.10	0.30**	0.10	0.29**	0.10	0.29**	0.09
Inhibitory control	-0.29**	0.10	-0.29**	0.10	-0.29**	0.10	-0.28**	0.09
Parent general talk	0.01	0.07	–	–	–	–	0.67***	0.18
Parent number talk	–	–	-0.08	0.07	–	–	0.16	0.13
Parent math language	–	–	–	–	-0.09	0.07	-0.87***	0.24

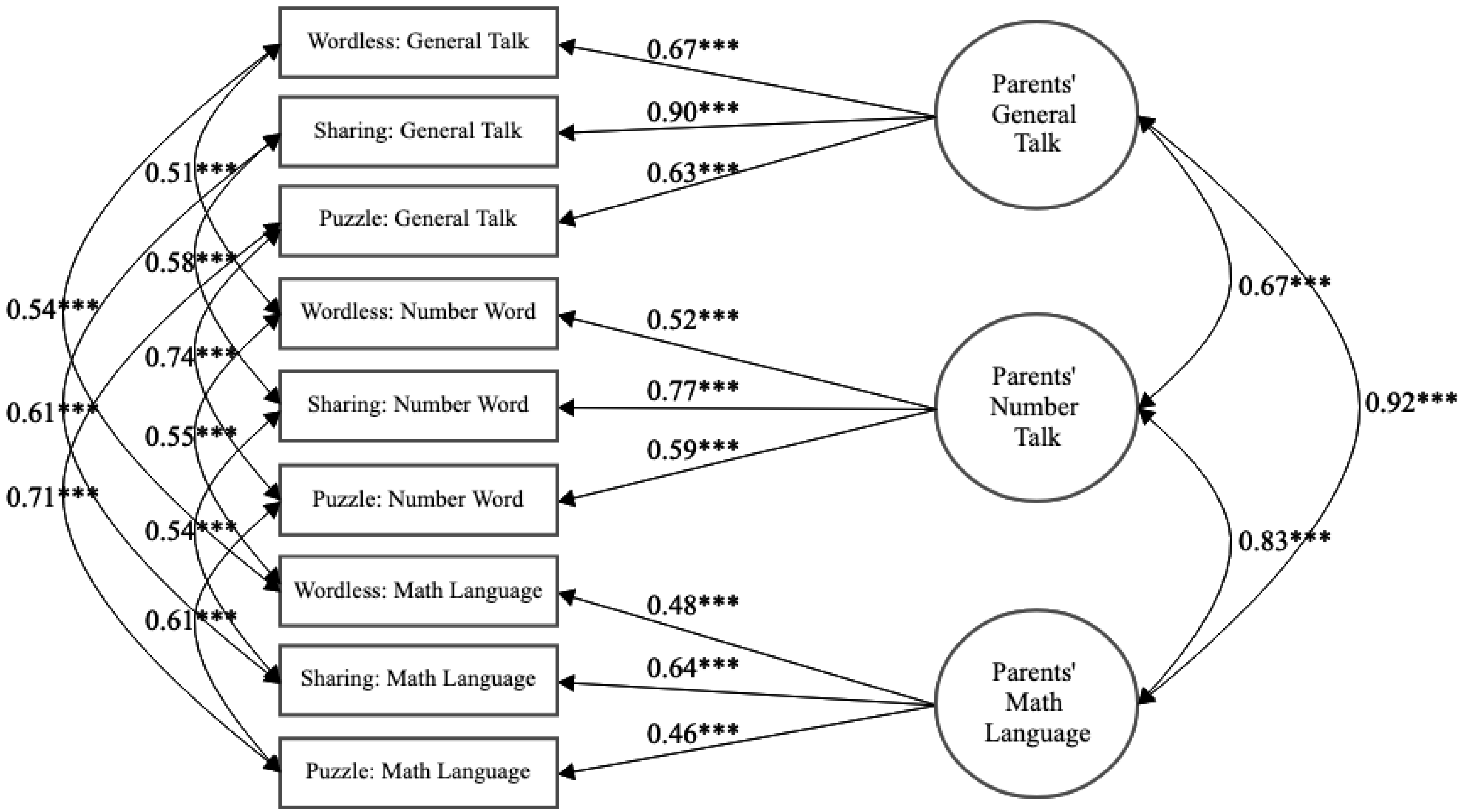
**$p < 0.01$, ***$p < 0.001$.

Table 14

Parent talk factors predicting child mathematical language knowledge

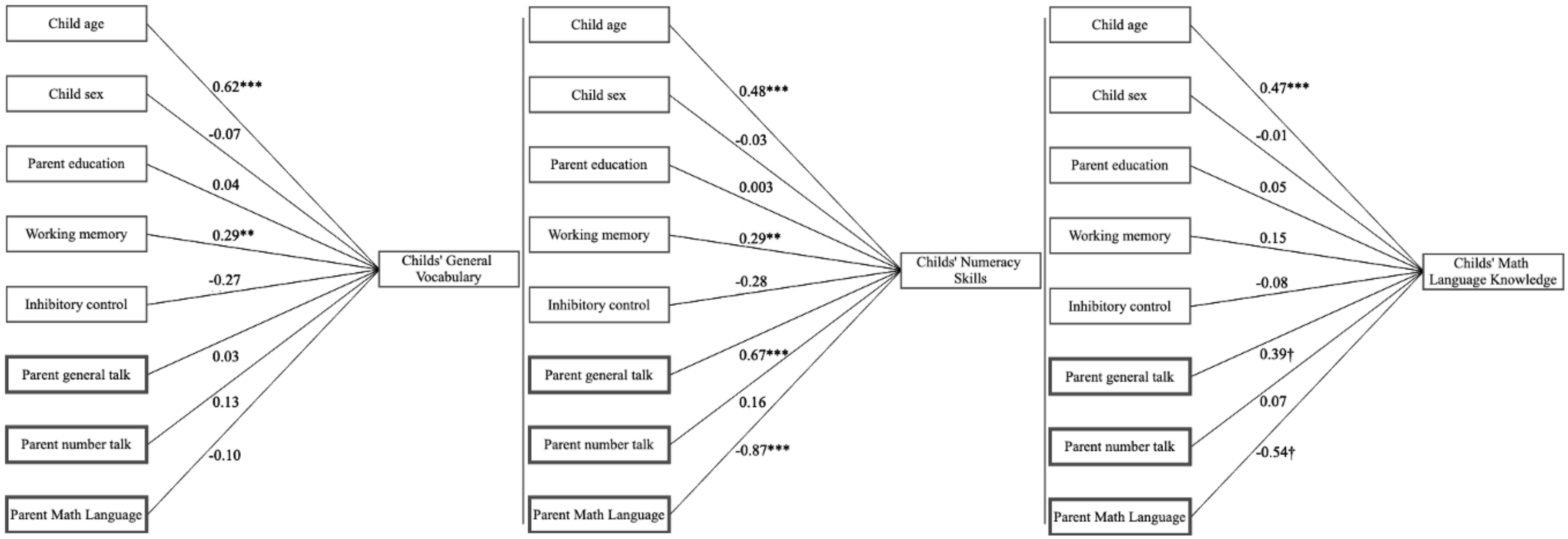
	Child Mathematical Language Knowledge							
	Model 1		Model 2		Model 3		Model 4	
	β	*SE*	β	*SE*	β	*SE*	β	*SE*
Child age	0.54***	0.06	0.52***	0.07	0.51***	0.07	0.47***	0.07
Child sex	-0.03	0.08	-0.02	0.08	-0.03	0.08	-0.01	0.08
Parent education	0.05	0.08	0.06	0.08	0.06	0.08	0.05	0.08
Working memory	0.14	0.11	0.16	0.11	0.15	0.11	0.15	0.11
Inhibitory control	-0.09	0.11	-0.09	0.11	-0.08	0.11	-0.08	0.11
Parent general talk	-0.04	0.08	–	–	–	–	0.39†	0.21
Parent number talk	–	–	-0.10	0.08	–	–	0.07	0.15
Parent math language	–	–	–	–	-0.11	0.08	-0.54†	0.28

†*p*< 0.10, ****p*< 0.001.



Note. Standardized coefficients are presented in the figure. ***p< 0.001.

Figure 3. *Best-fitting model, the 3-factor model (general talk, number talk, and a math language talk factor)*



†*p*< 0.10, ***p*< 0.01, ****p*< 0.001.

Figure 4. *Path diagrams of each early academic skill regressed on covariates and parent talk variables*

DISCUSSION

This study aimed to identify the factor structure of parents' talk during math-related engagement with their preschool-aged children and to examine the relation between parents' talk and children's general vocabulary, numeracy skills, and mathematical language knowledge. It was expected that parents' talk would be best represented by a 3-factor model consisting of a general talk, number talk, and mathematical language factor. It was also hypothesized that parents' general talk would be uniquely related to children's general vocabulary skills, parents' number talk would be uniquely related to children's numeracy skills, and parents' use of mathematical language would be predictive of both children's numeracy skills and mathematical language knowledge. This study is the first to formally examine if parents' number talk and mathematical language use are distinct constructs of math input and distinct from parents' general (non-math) talk. Therefore, this study makes an essential contribution to the literature, which has primarily measured parents' math talk as number talk or a combination of number and mathematical language (Casey et al., 2018; Eason et al., 2021; Ramani et al., 2015; Susperreguy & Davis-Kean, 2016; Thippanna et al., 2020). This distinction is important for understanding content-specific categories of children's language exposure and how this exposure uniquely relates to children's skills to potentially inform intervention development and educational practices (Turan & De Smedt, 2022).

In general, the hypotheses of this study were partially supported, and the findings are consistent with and expand previous parent math talk research (Levine et al., 2010; Mutaf Yildiz et al., 2018; Thippana et al., 2020; Zippert et al., 2020). A 3-factor model was found to be the best-fitting model of parents' talk, suggesting that general talk, number talk, and mathematical language use are related but distinct content-specific constructs of talk that children are exposed to during math-related interactions with their parents. Importantly, parents' number talk and mathematical language use which is often combined and investigated as math talk (Ramani et al., 2015; Susperreguy & Davis-Kean, 2016; Zippert et al., 2020), were found to be two separate constructs. It is necessary to further investigate these types of talk as separate constructs to understand if and how they might support children's skills in different ways. Contrary to hypotheses but consistent with some previous findings (Mutaf Yildiz et al., 2018; Thippana et al., 2020; Zippert et al., 2020), the three parent talk factors were not related to children's early academic skills in expected ways and instead had negative or non-significant relations.

The Factor Structure of Parents' Talk

Findings that parents' talk was best represented by a general talk, number talk, and mathematical language factor extends previous work suggesting that parents' number and non-number talk are two separate domains of talk (Levine et al., 2010). This distinction between parent talk factors may be due to the different functions that the talk types may serve during math-related engagement. For example, number words are used to count, numerically describe a quantity, apply a cardinal value to a collection of items, and verbally represent symbolic numerals ("one" is what English speakers call the symbol "1"; Krajewski & Schneider, 2009; Wynn, 1992). However, quantitative mathematical language may serve a different purpose than number words. Quantitative terms are used to compare numeric values and specified or unspecified quantities ("fewer", "same", "most"), to describe unspecified quantities ("lots" refers to a large amount), to request identification of quantities ("how many"), and to discuss the composition and decomposition of number or quantity ("add", "divide", "take away"; see Table 4). Thus, the number talk factor may best represent parents' counting and numeric labelling, while the mathematical language factor may represent parents' imprecise description of quantity and comparisons/relations between quantities (Purpura et al., 2019).

These findings are consistent with studies suggesting that children's mathematical language knowledge is distinct from their early numeracy understanding (Purpura et al., 2017; Turan & De Smedt, 2022), likely because children may understand mathematical language terms without understanding numbers and vice versa even though these skills are related (Krajewski & Schneider, 2009; Wynn, 1992). For example, a child may know how to count to three but may not understand that the word "few" could be used to refer to three items. In contrast, a child may be able to identify "a lot" of dots but may have trouble attaching a numeric cardinal value to a large group of dots.

Although parents' number talk and mathematical language use were found to be separate constructs, they were highly correlated ($r = 0.83$; Table 8), suggesting that parents who use more number talk also tend to use more mathematical language. It may be that parents who use more number talk go beyond basic counting and discuss more complex relations between numbers, which is associated with more mathematical language used to describe these relations. Similarly, previous studies focused on preschoolers' mathematical understanding suggest that children with a better understanding of mathematical language have higher numeracy skills (Lin et al., 2021; Purpura & Logan, 2015; Purpura et al., 2017; Toll & Van Luit, 2014a). There is also a causal link between children’s mathematical language knowledge and later numeracy skills (Purpura et al., 2017; Purpura et al., 2021), suggesting that early exposure to mathematical language is essential for children to understand more complex relations between numbers. Findings from experimental studies (Purpura et al., 2017; Purpura et al., 2021) might indicate that parents’ talk may have a similar function, such that more exposure to parents’ mathematical language use and number talk overtime might be predictive of children’s mathematical language and numeracy skills.

The current study contributes to the parent talk literature by providing evidence suggesting that parents expose their preschoolers to mathematical language during math-related conversations, and this talk type is distinct from number talk. It is important to consider that the 3-factor model of parents’ talk emerged from a controlled set of activities that families were asked to engage in and may not represent the types of activities they engage in daily. Future work should investigate the factor structure of parents’ talk during typical daily routines to understand if the 3-factor model transfers to a real-world context. However, using a controlled set of math-related activities was necessary to ensure that sufficient mathematical language and number talk was used because talk about math does not occur consistently and frequently throughout the day (Susperreguy & Davis-Kean, 2016). Although, there are some structured times (e.g., mealtime) where math talk may occur more frequently (Susperreguy & Davis-Kean, 2016). Having a better understanding of parents’ daily mathematical language use with their children could help inform practical recommendations for increasing parents’ mathematical language use to support children’s learning.

Parent Talk Factors and Children’s Early Academic Skills

After identifying parents’ talk as a 3-factor model, the relations between parent talk factors and children’s general vocabulary, numeracy skills, and mathematical language knowledge were examined. Results from regression analyses with extracted parent talk factors indicated that, parents’ general talk and mathematical language use were significantly related to children’s numeracy skills. Specifically, contrary to hypotheses, when children had high numeracy skills, their parents tended to use more general talk; however, their parents tended to use more mathematical language when children had lower numeracy skills. Further, contrary to hypotheses, no other relations were found between parents’ talk and children’s outcomes. Specifically, general talk was not related to general language, parents’ mathematical language was not related to mathematical language knowledge, and number talk was not related to numeracy skills. This pattern of results may be due to the concurrent nature of the data and the method used to measure parents' talk, which is discussed in the following sections.

Concurrent relations

Results from initial correlations suggested that parents' general talk was not related to children's early academic skills, while parents' number and mathematical language use had small significant negative associations with children's early math skills (Table 8). Specifically, this indicates that, without controlling for other variables, children with lower numeracy and mathematical language knowledge had parents who used more number talk and mathematical language during the parent-child interaction. This finding is consistent with past work suggesting that parents' number talk can sometimes be negatively related to children's numeracy skills, such as addition and subtraction (Mutaf Yildiz et al., 2018). The current study uses concurrent data; therefore, the directionally of the negative relations cannot be determined. For instance, it is unclear if parents use more math talk because of their child's skill level or if children have a particular skill level because of their parents' use of math talk. Future longitudinal work is needed to better understand the directionality of these relations over time.

It is important to consider that the frequency of parents' talk may be in response to the child's skill level in the context of the selected activities. In particular, correlations between individual parent talk indicators and children's skills indicated that parents used higher levels of talk when their children had lower early academic skills, specifically during the sharing game and puzzle. For example, during the sharing game, dyads were asked to help the elephants and the bear share peanuts. During this activity, children with higher numeracy skills may have been more familiar with the concept of fair sharing and may not have needed as much verbal support as children with lower numeracy skills to divide the peanuts among characters. The following examples demonstrate differences in verbal support between a parent who has a child with high numeracy skills and a parent who has a child with low numeracy skills during the sharing game:

Parent of child with high numeracy performance.

> ***Parent****: Fifteen peanuts. Do you know if there's three of them… should we share equally?*
> *Child:* [nods yes]
> ***Parent****: How many would they each get?*

Child: Let's just start… one.
Parent*: One for yellow dress.* [parent narrates as child gives one peanut to each character]
Child: One.
Parent*: One for bear.*
Child: One.
Parent*: One for green elephant.*
Parent*: Okay.*
Parent*: Keep going.*
Child: Two. Two. Two. [child continues giving one peanut to each character and says how many peanuts each character now has]
Child: Three. Three. Three.
Child: Four. Four. Four.
Child: Five. Five. Five.
Parent*: So how many do they each get equal?*
Child: Five.

Parent of child with low numeracy performance.

Parent*: They need a lot. Oh, they're still hungry. Okay. He has two peanuts. Let's see.*
Parent*: She's got some there. How many does she have?*
Child: Hum. Three. [child points to peanuts and says the incorrect cardinal value]
Parent*: One two three four.* [parent points to each peanut to count out the correct amount]
Parent*: Let's give more to the teddy bear. One more.*
Parent*: Okay, now how many does the teddy bear have?*
Parent*: One two…* [parent begins pointing to each peanut and counting with child]
Child: One two three four. [child points to each peanut as they count]
Parent*: Okay, same as her. And now, he needs more peanuts. Ready? Okay, how many does he have? Count his peanuts.*
Child: One two three four.
Parent*: Okay, one more for the girl.*
Parent*: Now, how many does she have?*
Child: Three. [child says incorrect cardinal value]
Parent*: Count.*
Child: One two three four five. [child points to each peanut and counts out the correct amount]
Parent*: Okay. We got one more for the... Let's give it to the teddy bear. How many does the teddy bear has now?*
Child: One two three four five.
Parent*: Okay. And now, one more for this guy.*
Parent*: How many does he have?*
Child: One two three four five.
Parent*: We did it! They all have the same?*
Child: [nods yes]

***Parent**: Five, five, and five.* [parent points to each group of five peanuts]

Parents of children with higher numeracy skills may not have seen the target activities as an opportunity to engage in high levels of math talk with their child because their child may have been independently using the appropriate math talk to facilitate their completion of the task. This potential explanation is consistent with past work suggesting that children's early number talk is predictive of their numeracy performance (Levine et al., 2010). Further, a child with higher numeracy scores may have made fewer mistakes when completing the numbered puzzle, resulting in parents using less math talk because they did not see an opportunity to correct or teach their child something about the numbers or quantities on the puzzle pieces. However, children with lower numeracy skills may have been more likely to make mistakes while counting out the dots on puzzle pieces and matching them with the numeral puzzle pieces, which may have prompted parents to use more math talk to support their child's completion of the puzzle. The following examples demonstrate differences in verbal support between a parent who has a child with high numeracy skills and a parent who has a child with low numeracy skills during the puzzle:

Parent of child with high numeracy performance.

***Parent**: It says, "match the numbers and dots to complete the puzzle".* [parent reads puzzle instructions]
Child: Yes. Oh my. [child starts looking at the puzzle pieces]
Child: Three and three! [child begins independently putting the puzzle together while the parent watches]
Child: Good. Four and four.
Child: Found two fours.

Parent of child with low numeracy performance.

***Parent**: Can you match the dots?*
Child: Two! [child says incorrect cardinal value for the number of dots on the puzzle piece]
***Parent**: How many dots is that?*
Child: Two! [child repeats incorrect cardinal value]
***Parent**: No that's not two.* [parent corrects child]
Child: Three.
***Parent**: Three. Good job.*

Results of regression analyses further suggest that, with covariates in the model, contrary to hypotheses, parents' mathematical language use is negatively related to children's numeracy skills while general talk is positively related to children's numeracy skills. These findings suggest that when children have higher numeracy skills, parents may use the target activities (wordless book, sharing game, and numbered puzzle) to discuss broader topics related to the activities rather than use the activities as an opportunity to discuss quantity. The following example shows how a parent who has a child with high numeracy skills initiated broader discussion as their child completed the puzzle:

> ***Parent****: It looks like the girl elephant.* [parent begins identifying the puzzle image as the child completes the puzzle]
> ***Parent****: What's her name?*
> *Child: I can't remember.*
> ***Parent****: You can't remember. Can you give her a name?*
> *Child: Umm… Grammy?*
> ***Parent****: Her name's Grammy? Okay.*
> ***Parent****: And what's the boy elephant's name?*
> *Child: Umm- Coco.*
> ***Parent****: Coco? Coco and Grammy? Oh, the picture is coming together! What's it look like they're doing?*
> *Child: I don't know.*
> ***Parent****: You don't know, yet?*
> ***Parent****: Good. Just three more.* [parent says how many pieces are left to complete the puzzle]
> ***Parent****: Can you take a guess at what they're doing before you finish the picture?*
> *Child:* [Does not respond but finishes putting the puzzle together]
> ***Parent****: Last piece. Wow! What are they doing?*
> ***Parent****: What's Grammy and Coco elephant doing?*
> *Child: Umm, they're uh… ant!*
> ***Parent****: Do you see an ant?*
> ***Parent****: I see the ant right there. What's it doing?*
> *Child: Taking a peanut.*
> ***Parent****: Taking a peanut!*

Parents were not explicitly asked to have math discussions or teach math during these activities. Therefore, it may be that the activities elicited more dialogue about topics outside of quantity (e.g., birthdays, picnics; Skwarchuk et al., 2014) when children had the numeracy skills needed to complete the activities with less parent support. Although this interpretation of the results assumes that parents are aware of their child's level of numeracy skills (Mutaf Yildiz et al., 2018), correlations between parent-report and direct assessment (rs = 0.59 – 0.92) suggested that parents in this sample were aware of their child's numeracy skills. Future work should investigate if parents adjust their verbal math support based on knowledge of their child's numeracy skills. Future studies should also examine if other types of parent support, such as scaffolding behaviors (Baranovich et al., 2019; Hammond et al., 2012; Huang et al., 2022) and math-related gesturing (e.g., pointing and finger counting; Gordon & Ramani, 2021), are related to parents' frequency of mathematical language use and if these supports change based on their child's level of numeracy skill.

Measurement of parents' talk

Regression analyses resulted in null findings for the hypothesized relations between parents' general talk and child general language, number talk and numeracy skills, and the relation between parents' mathematical language and children's mathematical language knowledge. It should not be interpreted that parents' talk is not important for children's respective skills, but instead, it may be that measuring parents' talk with cumulative word count may not fully capture the aspects of parents' talk that are linked with children's acquisition and knowledge. For example, parents' general talk was measured by cumulative word count (excluding number and math language), but parent report and direct assessment of children's general vocabulary measured the different types (diversity) of words that children knew. It may be that parents' cumulative general talk may be a better predictor of children's word tokens than a measure of the different types of words they know. Further, past work on parents' general talk has found that parents' cumulative word count is not a consistent predictor of children's language skills (Zimmerman et al., 2009). Instead, the number of conversational turns that a parent and child took (Zimmerman et al., 2009) and parents' efforts to make a story relatable to their child's life (Hindman et al., 2014) were better predictors of children's vocabulary skills than other types of general talk, like exposure to adult word count or parents' efforts to define vocabulary, respectively. Thus, the quality of the parent-child interaction or the quality of parents' math talk may be more predictive of children's mathematical language knowledge and numeracy skills than parents' word tokens. Similarly, studies on parent's number talk suggest that parents' use of specific numbers or types of number talk were better predictors of preschooler's numeracy skills than parents' cumulative use of all numbers (Elliot et al., 2017; Ramani et al., 2015, Thippana et al., 2020). Specifically, only parents' advanced number talk (Ramani et al., 2015) and use of numbers higher than ten (Elliot et al., 2017) were predictive of children's numeracy skills when compared to foundational number talk and use of numbers ten and lower, respectively. Further, consistent with null findings in the present study, one study that measured number talk as parents' cumulative use of numbers zero and above did not find a significant relation between parent number talk and preschoolers' numeracy skills (Thippana et al., 2020).

Together this work suggests that the quantity of words alone may not be the best way to measure and understand how parents talk supports children's skills. Future work should investigate the diversity of words spoken and the verbal context (open-ended questions, close-ended questions, statements) of word use within different parent talk factors to understand better how parents' general talk, number talk, and mathematical language use are related to children's early academic skills. For example, children's frequent exposure to a diverse set of mathematical language terms may be more supportive of math achievement than frequent exposure to the same narrow set of mathematical language terms. Although previous work has included some quantitative mathematical terms in their measures of parents' math talk (Casey et al., 2018; Eason & Ramani, 2020; Susperreguy & Davis-Kean, 2016), this is the first study to measure parents' mathematical language as a separate construct from number talk. Notably, the current study identified 38 different mathematical language terms used at least once by parents (see Table 4). Further investigation is needed to understand if parents' use of certain mathematical language words (terms used to compare quantities vs describe unspecified quantities) is more strongly related to children's mathematical language knowledge and numeracy skills. Given that active participation from the child is essential in parent-child conversations (Zimmerman et al., 2009; Eason et al., 2021), it may be beneficial to investigate links between parents' mathematical language diversity, parents' efforts to elicit child participation with questions, and measure children's use of mathematical language. Future work in this area is necessary to better understand how mathematical language use during math-related activities relates to children's numeracy skills.

Limitations and Future Directions

Although the results of this study provide important insights into the structure of parents' talk and the relations between parents' talk and children's early academic skills, limitations that highlight areas for future research should be noted. The first limitation is that the data is concurrent, and the study design is correlational. The study was conducted at a single time point; therefore, directional implications about the relation between parents' talk and children's outcomes cannot be concluded. Though directionality cannot be confirmed, the results of this study suggest that when children had lower numeracy skills, their parents used more mathematical language. One study using concurrent data had similar findings (Mutaf Yildiz et al., 2018), suggesting that more number talk by parents was negatively related to children's numeracy skills. However, work using longitudinal data tends to find a positive relation between parents' math talk and children's numeracy skills (Casey et al., 2018; Levine et al., 2010; Susperreguy & Davis-Kean, 2016). At the moment, parents are likely responding to children's skill level to support the completion of a math-related task, but higher levels of early support may be linked to higher skills over time.

Recent concurrent findings by Eason et al. 2021 also suggest that when parents used more math prompts (e.g., asking questions about a number or eliciting a child's math talk), they had children who used more math talk. These recent findings indicate that parents' math talk may be a significant predictor of children's engagement and participation in math-related conversations, which might be related to children's later mathematical language knowledge and numeracy skills. It is necessary to understand if parents' math talk and prompts are in response to children's math talk or skill level or if children's engagement and skills result from parents' math talk and if these relations change over time. For example, it is possible that early on (e.g., before the start of preschool), children's numeracy skills are in response to their exposure to parents' math talk, but as children develop autonomy and interest in math, their math talk and skill level may predict parents' math talk and support. Future research should measure parents' talk factors, children's math talk, and children's early academic skills at multiple time points throughout the preschool years to investigate differences between concurrent and longitudinal relations and possible bidirectional relations.

The second limitation was that the sample size was relatively small for the intended statistical analyses. Although the current study has a large sample size (N = 120) relative to much of the prior research on parents' math talk (Elliot et al., 2017; Levine et al., 2010; Mutaf Yildiz et al., 2018; Susperreguy & Davis-Kean, 2016; Zippert et al., 2020), this sample size is considered small for advanced statistical analyses such as SEM (Logan et al., 2021). This sample size was sufficient for running a CFA model; however, it was not sufficient for conducting the proposed SEM analyses. However, alternative comparable regression models were run using extracted factor scores. Specifically, factor score extraction was conducted using the recommended ten Berge approach so that the identified parents' talk factors could be used as independent variables in regression analyses (Logan et al., 2021). Future research should use a larger sample to conduct SEM and simultaneously estimate all necessary parameters in one model.

The third limitation is that the sample was relatively homogeneous. The majority of parents (86.7%) who participated in this study had a bachelor's degree or higher; therefore, results may not generalize to a sample of parents with less education. Although this study did not find an association between parent education and parents' talk, prior work investigating a more economically diverse sample indicates that parent SES (education and income) is positively associated with parent number talk and children's numeracy skills (Levine et al., 2010; Thippana et al., 2020). These findings suggest that parents with lower education may use number words in ways that are not captured during brief observations or that these parents tend to align their talk with what they know their children understand. It is crucial to determine if parents' talk is best represented as a 3-factor model in a more economically diverse sample. In particular, understanding the factor structure of parents' talk in families with low SES is essential for informing intervention and educational practices, specifically for children who may benefit the most from more support prior to formal schooling (Blevins-Knabe & Musun-Miller, 1996; Turan & De Smedt, 2022). Additionally, the findings of this study are specific to English-speaking families in the U.S.; therefore, future work is needed to understand the factor structure of parents' talk in culturally and linguistically diverse families.

The fourth limitation was that the study was conducted virtually during the COVID-19 pandemic. Although virtual data collection was the most feasible option for conducting an observational study during the pandemic, this method of data collection may have limited this study's sample to families who had access to communication devices with video chat features, who had reliable access to WIFI or mobile data, and who were comfortable video chatting with an experimenter. It is necessary to replicate parent talk factor findings in a sample where data is collected in person in a lab setting and a natural setting such as the home with inconspicuous recording devices (e.g., LENA) to capture parent-child verbal interactions during daily routines. For example, a LENA device is a digital language processor that audio records and provides reports of word counts and conversational turns between an adult and child. The LENA device is small enough to fit in a pocket and may not be as distracting or noticeable as a traditional video camera which is beneficial for recording multiple days' worth of naturalistic parent-child speech to capture how math talk occurs in daily life (Susperreguy & Davis-Kean, 2016). The present study may have felt artificial to participants who knew they were being watched over Zoom. This setting may have led to parents talking more, especially if their child was talking less, to avoid silence during the observation. Examining the factor structure of parents' talk in different settings is necessary to establish the consistency or stability of these content-specific categories of language exposure.

The final limitation is that brief direct assessments of children's numeracy and mathematical language had to be adapted for and conducted over video chat, limiting our understanding of children's broader numeracy and mathematical language knowledge in relation to parents' talk factors. The brief assessments were composites of both parent-reported skills and direct assessment. Parents' reports of children's skills may have been subject to reporter bias and findings may reflect the relation between parents talk and their perception of their child's skills. However, moderate to high correlations between parent-report and direct assessment suggest that parents' perceptions were consistent with direct assessments of children's skills. Future research should use full direct assessments of children's numeracy and mathematical language knowledge, conducted in-person as intended, to investigate the relations between parents' talk and children's skills.

CONCLUSION

The findings from this study fill an important gap in the parent math talk literature. Parents' mathematical language use was found to be a related but distinct construct of parents' talk that is separate from general talk and number talk. This finding extends prior research, which has focused on investigating parents' number talk or a broader measure of math talk, by identifying parents' mathematical language use as an aspect of language exposure potentially necessary for children's early mathematics understanding and development. Identifying the distinction between parents' mathematical language use and number talk is important because it may indicate that parents are using these types of talk in different ways that may expose children to different aspects of mathematical understanding. Notably, this study suggests that parents use more mathematical language when engaging in math-related activities with children who have lower numeracy skills. The results of this study provide a rationale for future studies to investigate the factor structure of parents' talk longitudinally to better understand the relation between parents' talk and children's skill development. Overall, findings indicate that parents' mathematical language use is a distinct area of children's language exposure, and more research is needed to understand the nuances of parents' talk that may be targeted to support children's early mathematics learning.

APPENDIX A. RECRUITMENT EMAIL AND FLYER

Study Title: Parent Child Playful Learning Study

Parents of three- to five-year-old children,

The Purdue Early Achievement Research Labs (PEARL) invite you to participate in a project investigating parent and children's conversations during play! During this study, you will first be asked to complete a roughly 30-minute online survey asking you questions about your child's early academic skills. After you have completed the survey, you will be mailed materials for three play activities and asked to schedule a 30-minute video call over Zoom with a member of our lab. During the video call, you will complete the activities with your child and then your child will complete a few short assessments. These assessments will focus on children's early academic skills. After the Zoom session, you will receive a $10 gift card to Amazon for participating!

Use the following link to sign up:
https://purdue.ca1.qualtrics.com/jfe/form/SV_3yfUIPoYQnDbBpI?Q_CHL=qr

Further questions can be directed to the study researchers:
Doctoral Candidate: Yemimah King, M.S. (kingy@purdue.edu)
Principal Investigator: Dr. David Purpura (purpura@purdue.edu)

PARENT CHILD PLAYFUL LEARNING STUDY

IRB-2021-161

We are interested in how conversations during play help young children learn!

- Who can participate? **3-** to **5-**year-old children and their parents
- Participation includes:
  1. You will complete a **30-minute online survey** about your child's early academic skills
  2. You will be **mailed materials for 3 play activities**
  3. You and your child will attend a **30-minute Zoom (video chat) session** to complete the 3 play activities and your child will complete a few short educational assessments
- For participation, after the Zoom session you will receive a $10 gift card

Use the following link to signup!
https://purdue.ca1.qualtrics.com/jfe/form/SV_3yfUIPoYQnDbBpl

PEARL
Purdue Early Achievement Research Labs

Email Yemimah King (kingy@purdue.edu) for more information

APPENDIX B. ACTIVITIES FOR PARENT-CHILD INTERACTION

Wordless Storybook

The experimenter explained to participants that this is a wordless storybook. **Prompt**: "Create a story by talking about what is happening on each page and asking questions along the way." See full wordless book below.

The Little Elephants' Big Adventures

Just Enough Eggs

Written by Angela M. Isaacs
with the Purdue Early Achievement Research Labs
Illustrated by Matt Dye

Development of this book was funded by the Heising-Simons Foundation

Attention!

This is a wordless picture book, which means you get to create the story. Make storytelling a conversation with your child! Here are some tips:

1. Ask questions.
2. Keep the conversation going.
   Be flexible! Build on what your child says and relate the conversation to his or her interests.
3. Have fun!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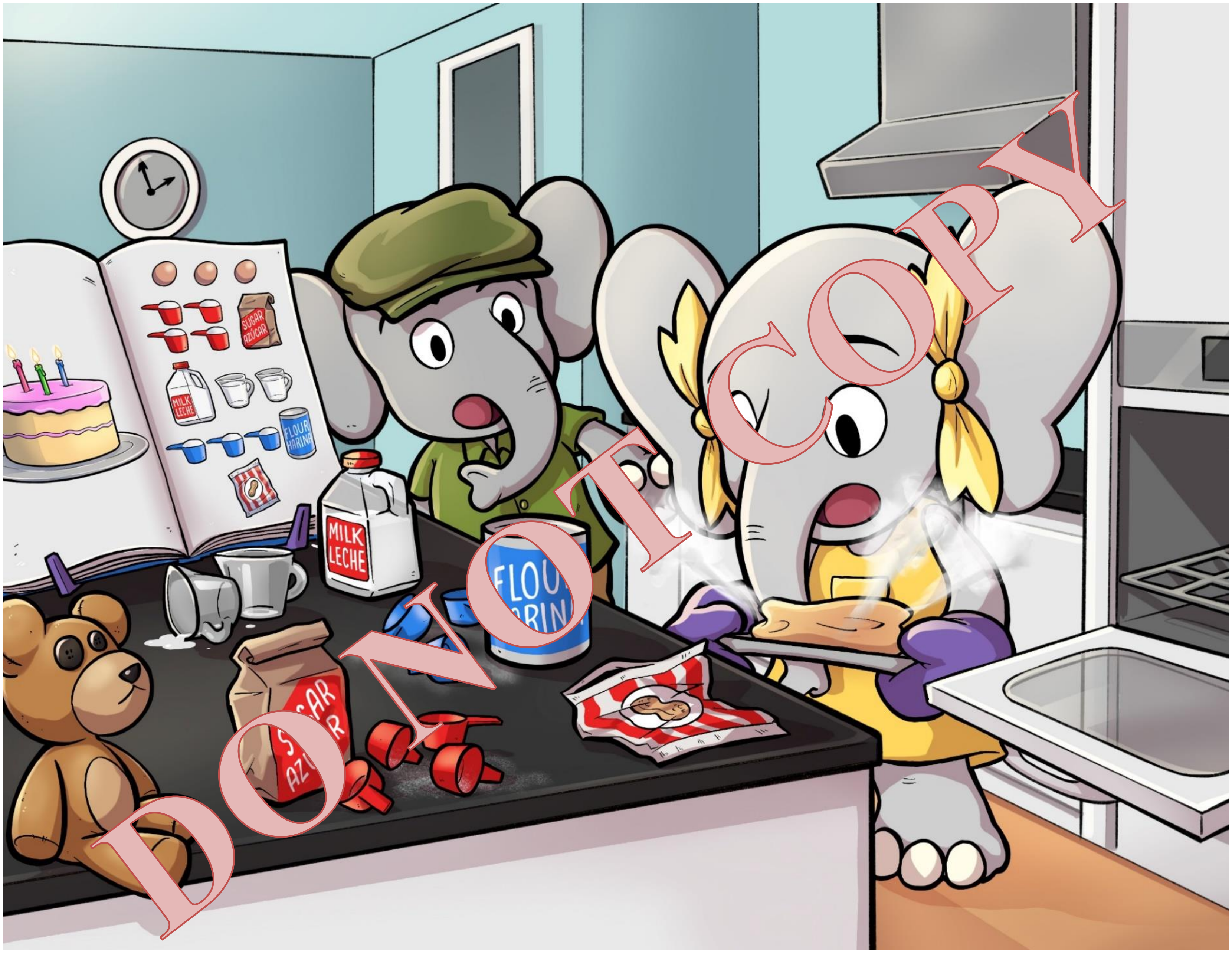
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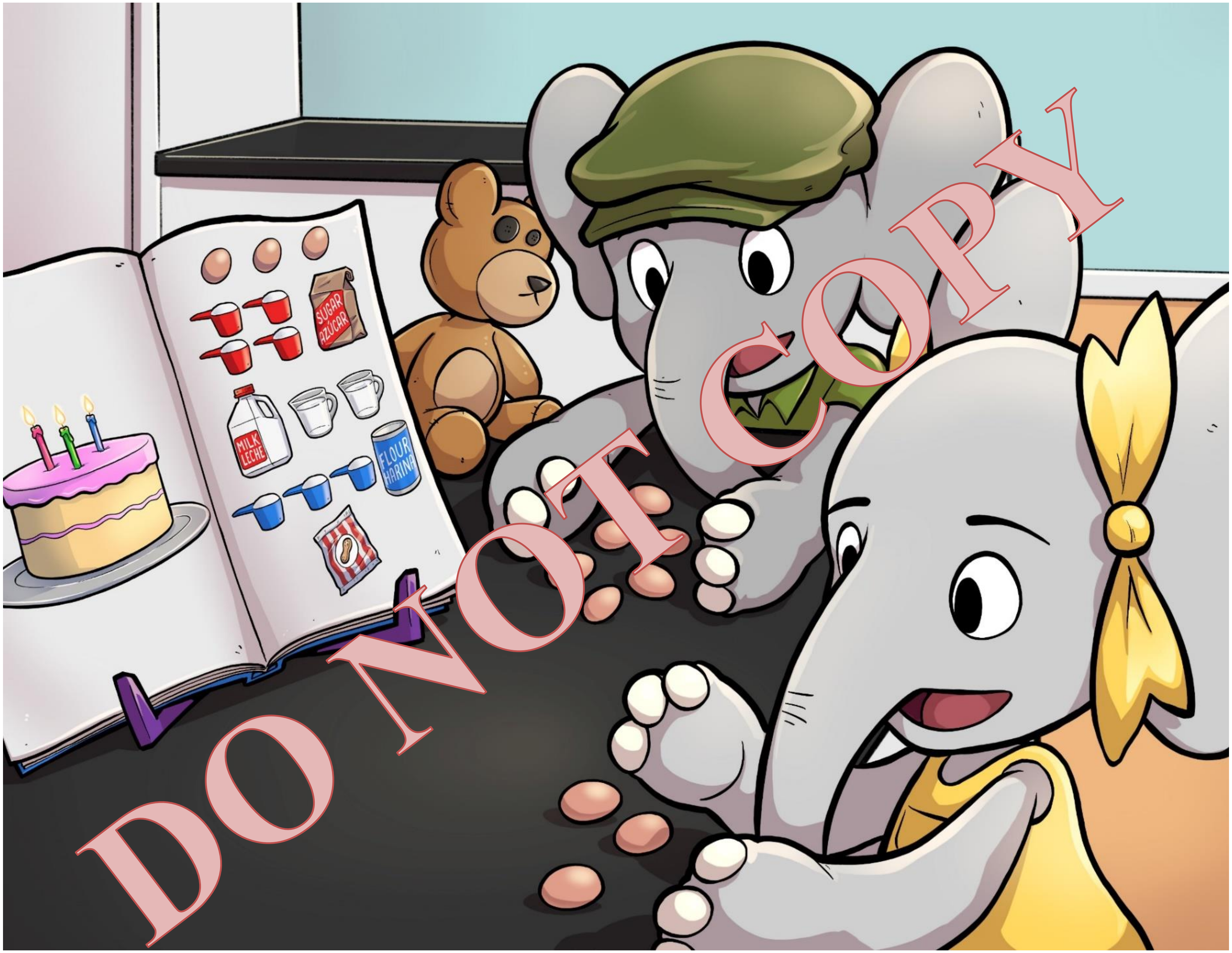
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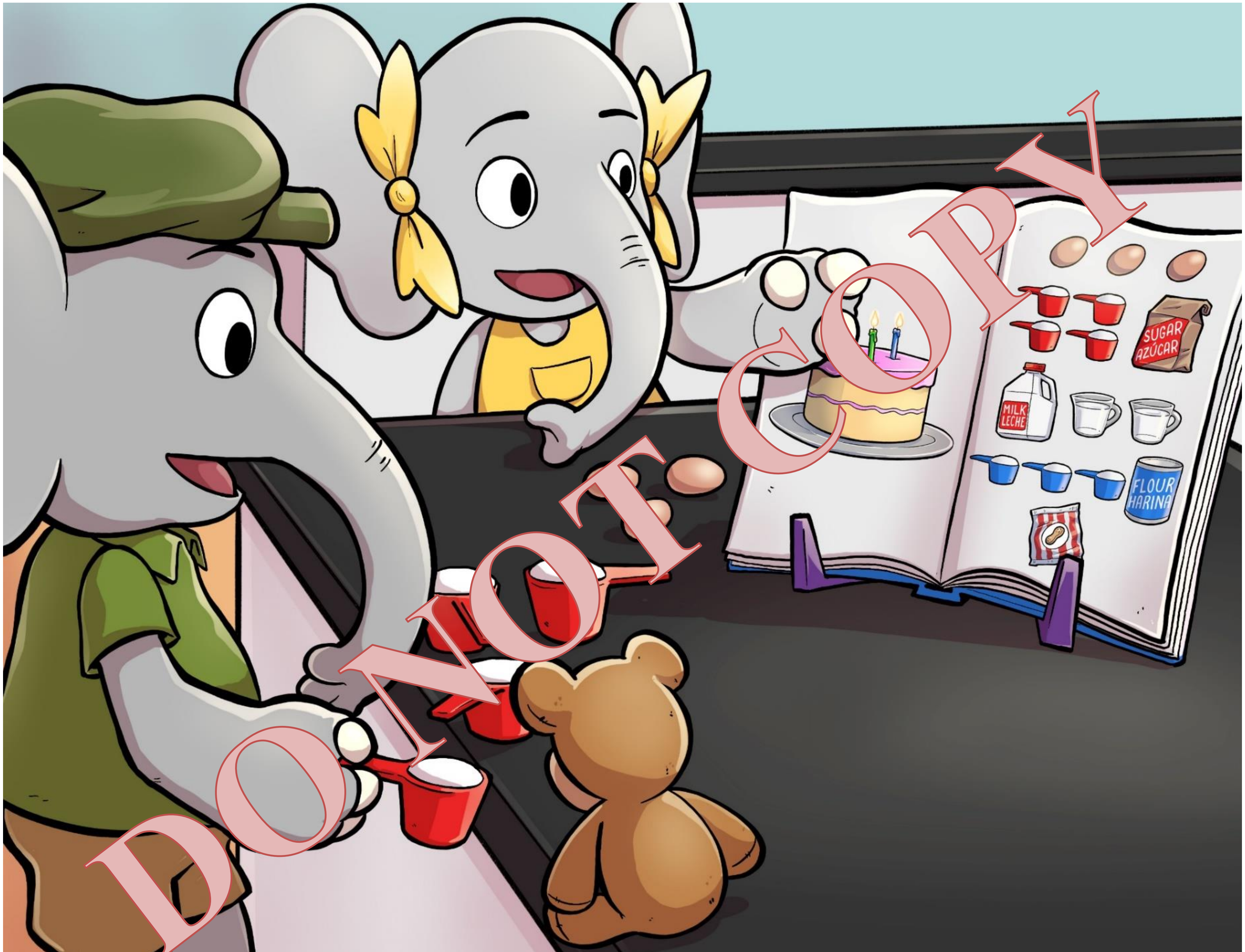
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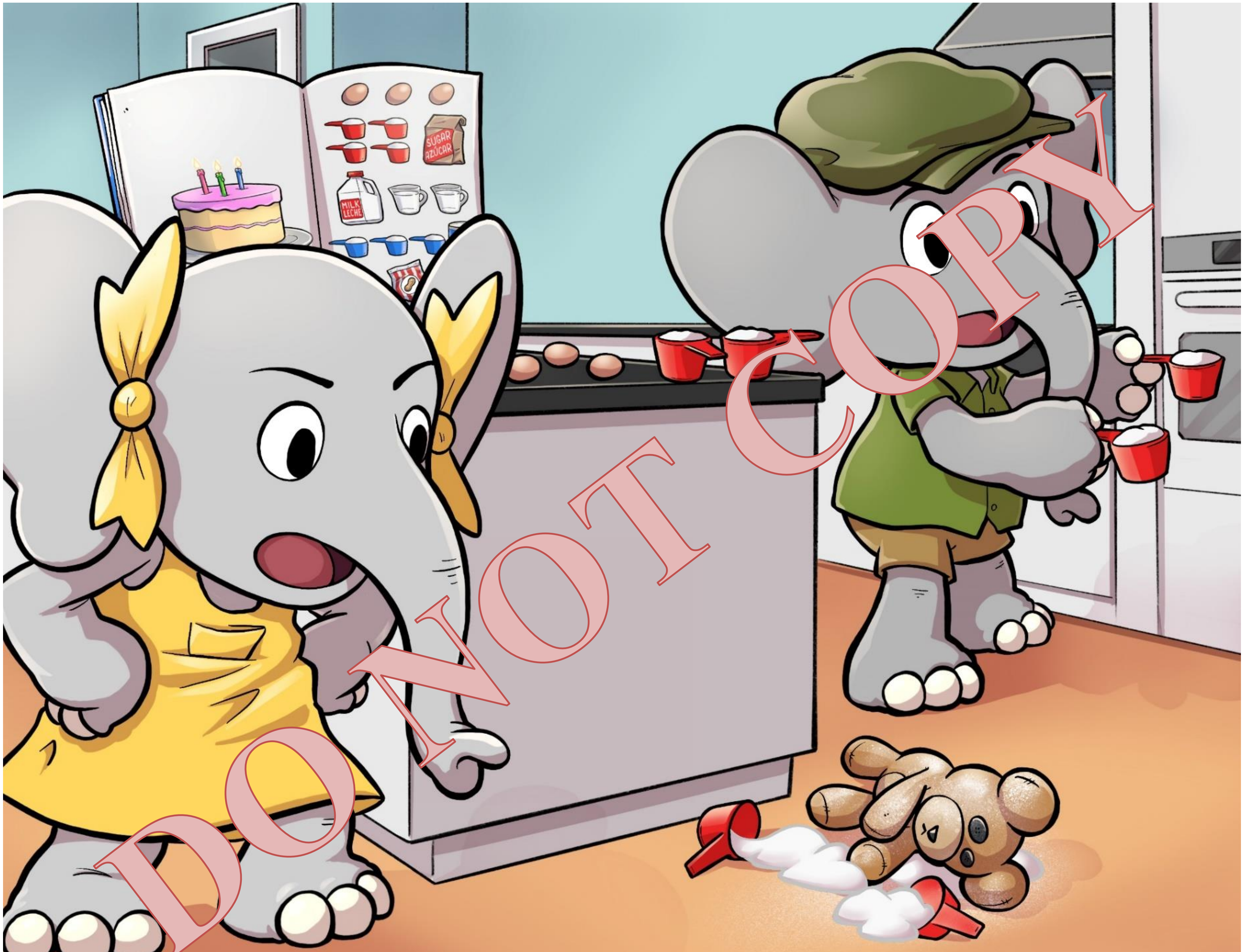
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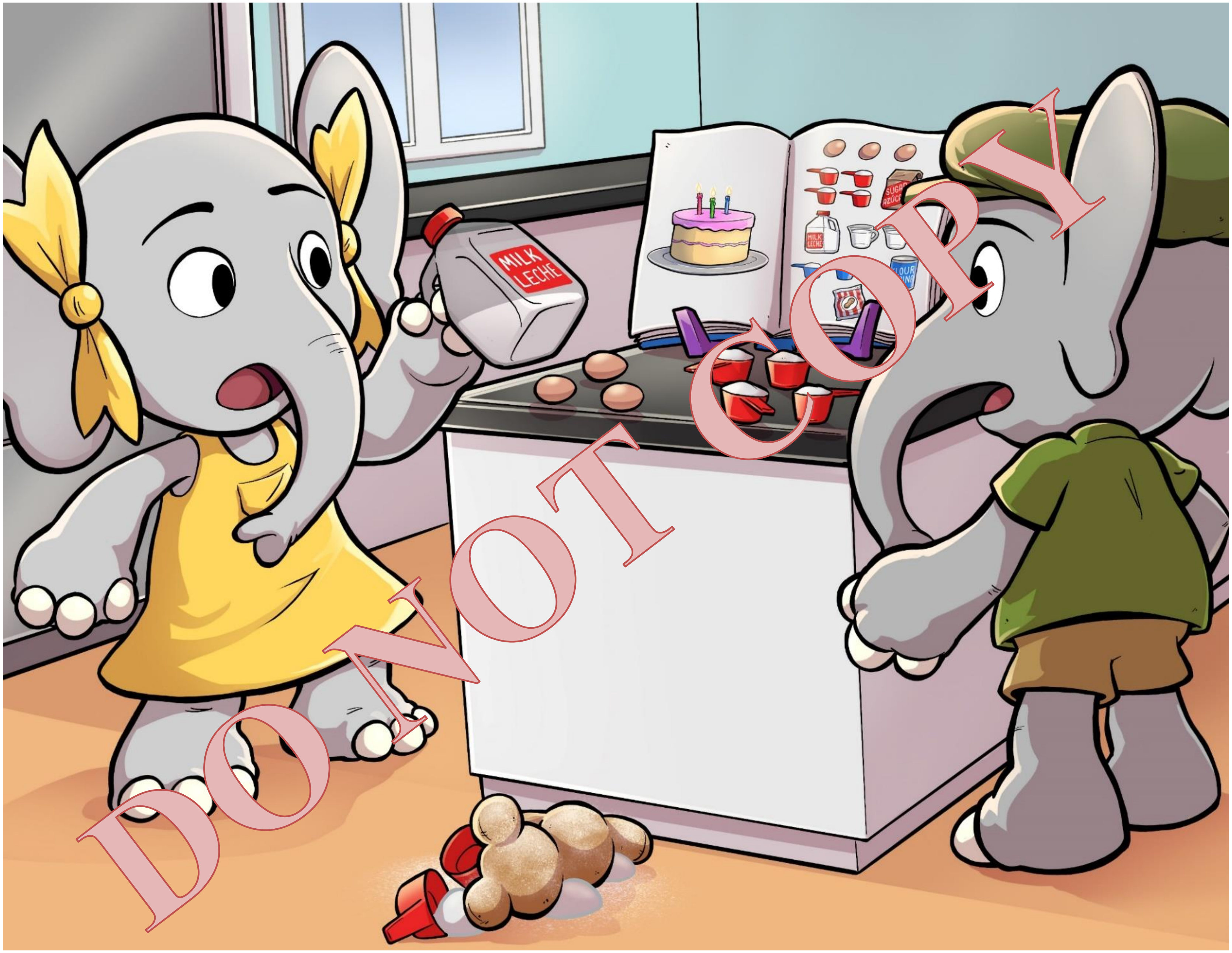
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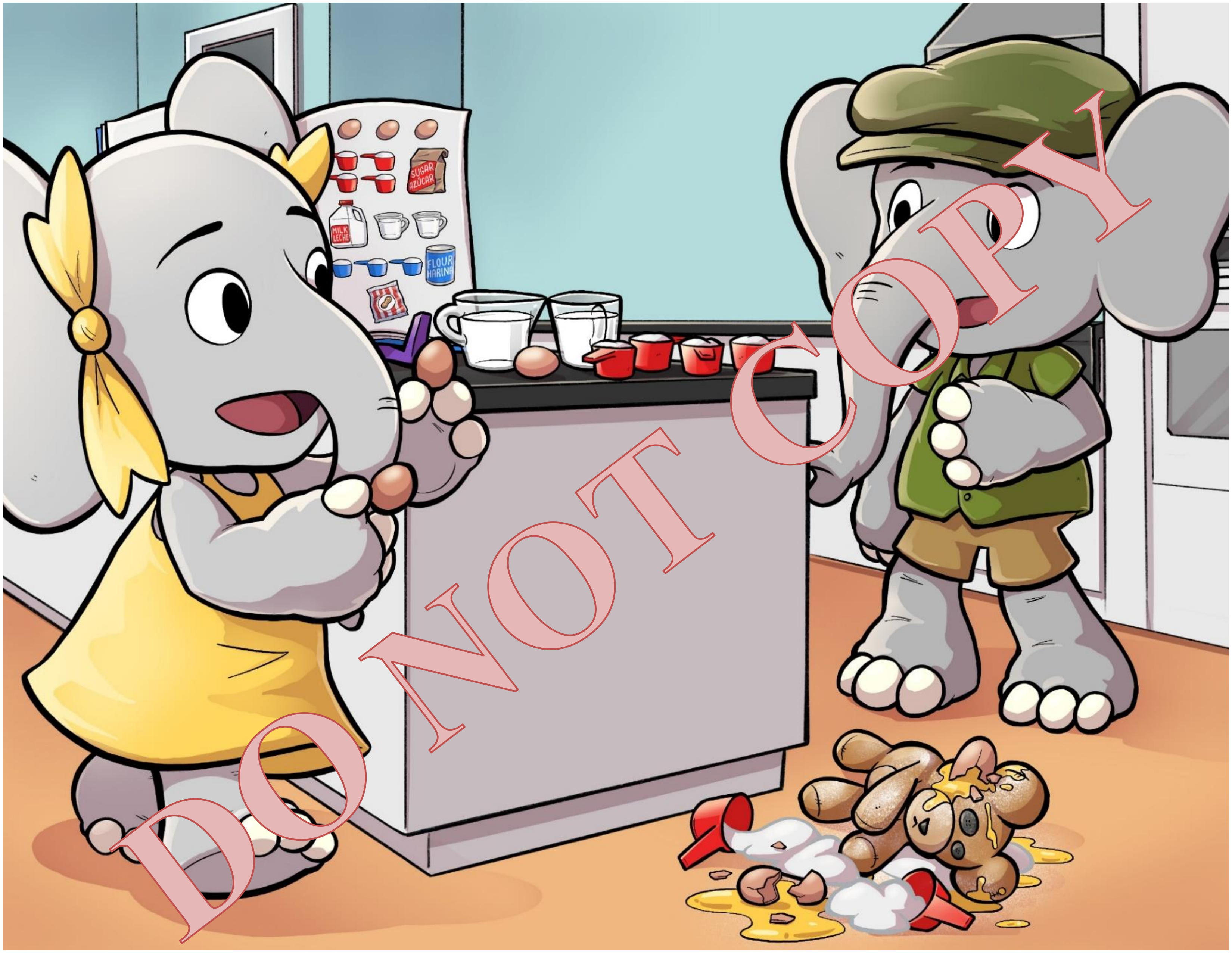
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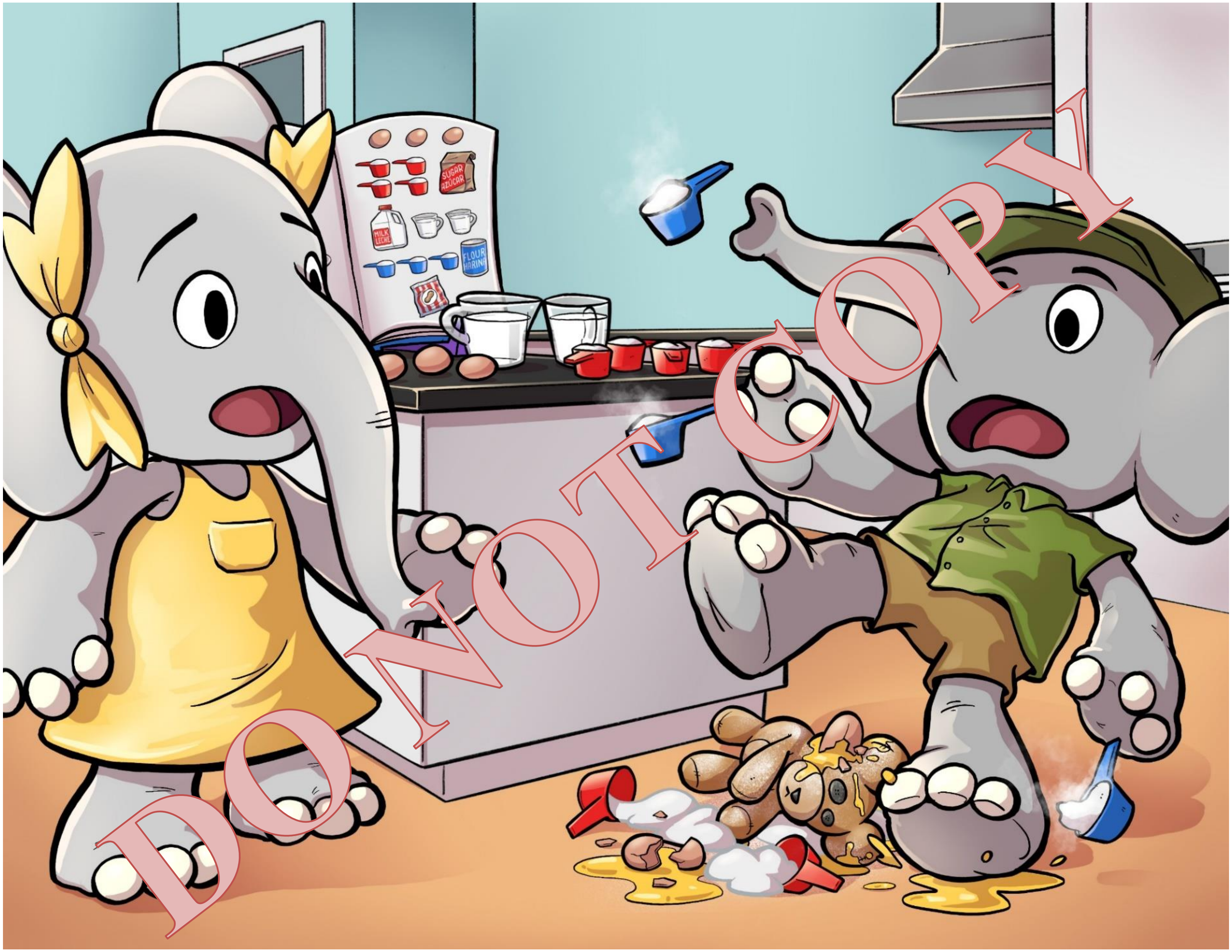
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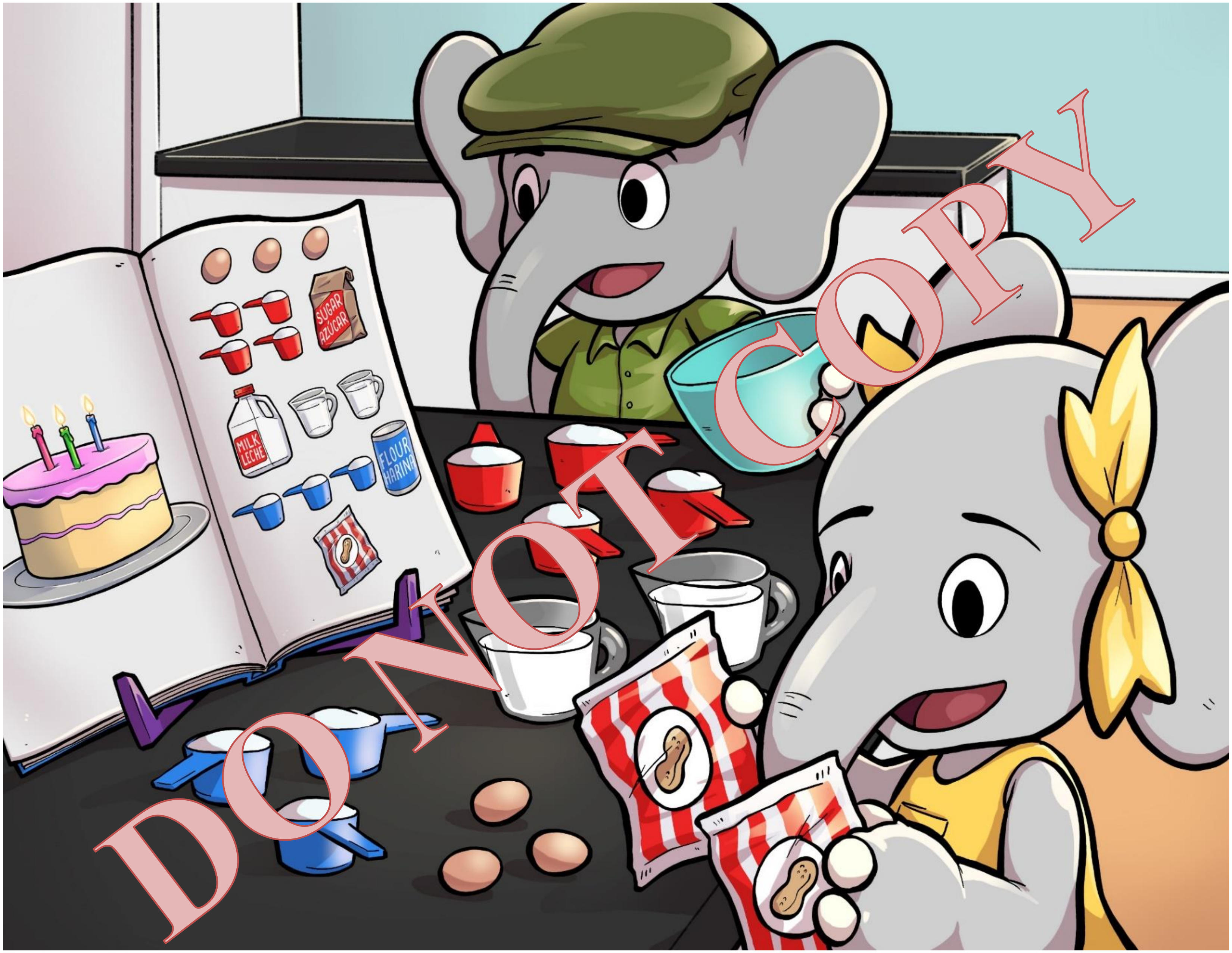
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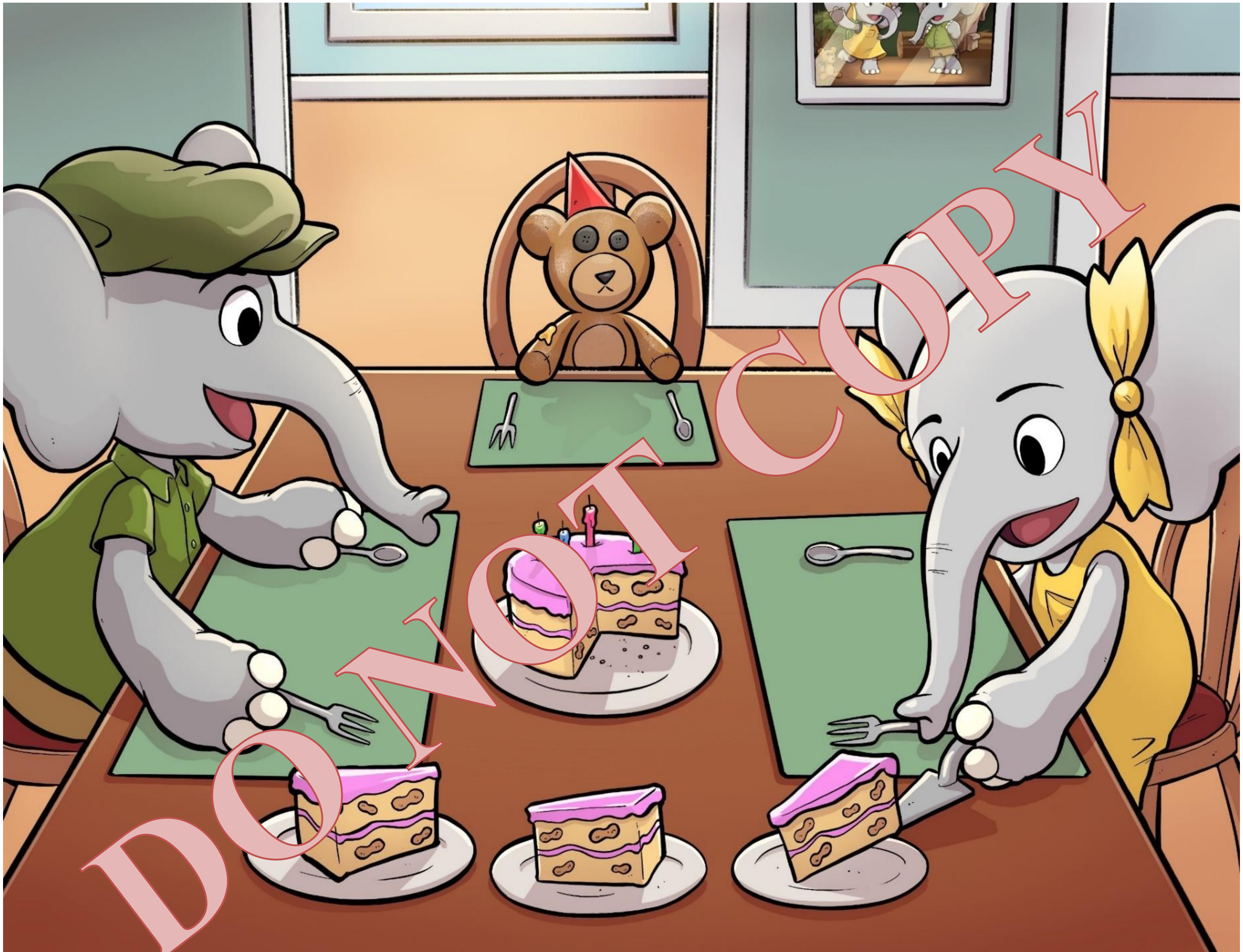
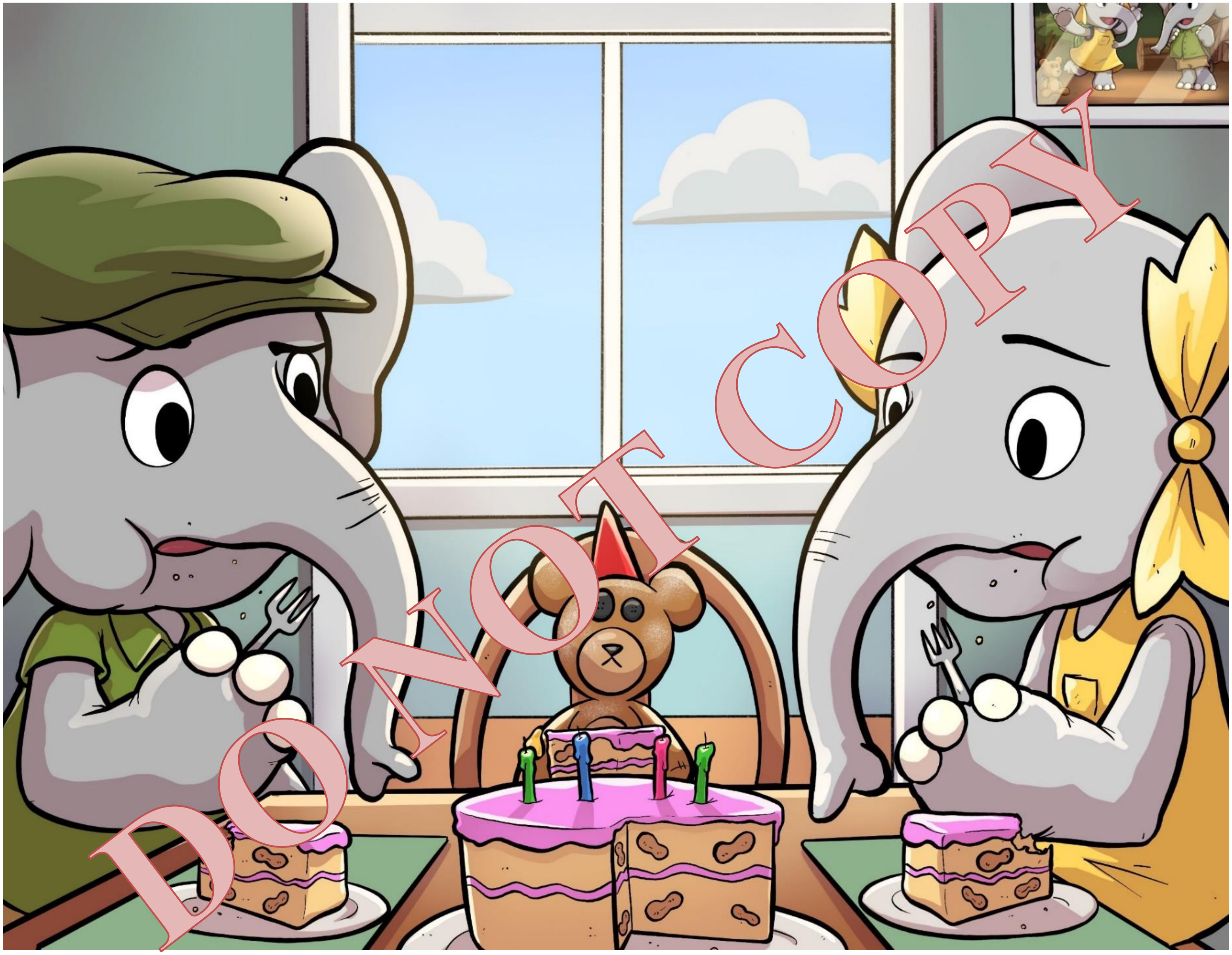
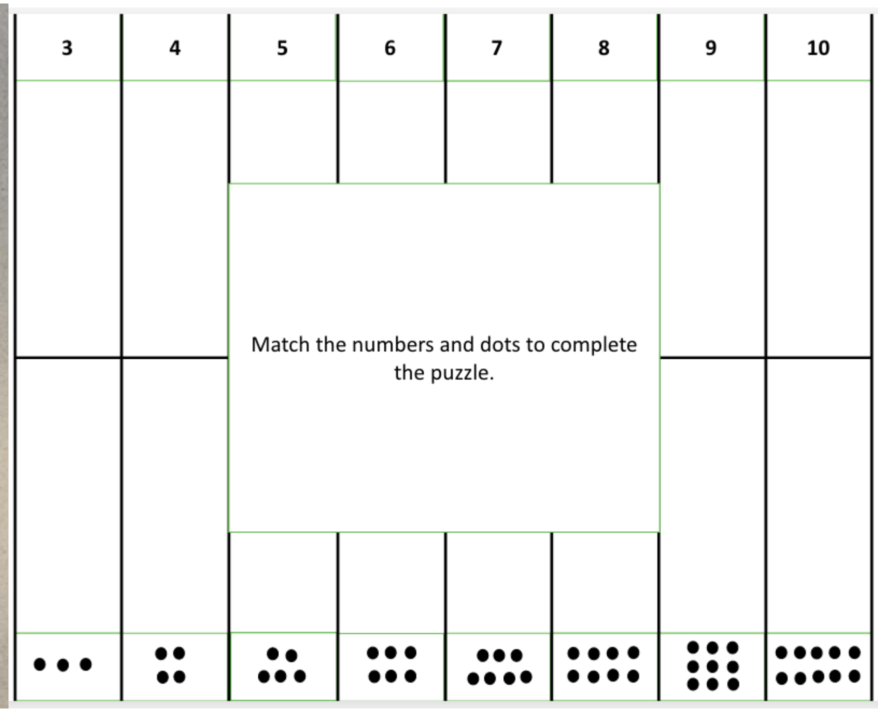




Sharing Game

Parent-child dyads were prompted to help the characters share the peanuts. **Prompt**: “The elephants and bear love peanuts but they need your help sharing them. For this activity you will help them share the peanuts.”

Numbered Puzzle



The experimenter explained that this activity includes puzzle pieces and a sheet of paper with instructions. **Prompt**: “Lay the puzzle pieces on top of the paper to create a picture.”

APPENDIX C. PARENT REPORTED MEASURES

Numeracy

Now you will be asked some questions about skills that children learn in preschool or kindergarten. Some of these things may be beyond your child's skill level for his/her age.

Verbal Counting

How high can your child count, consistently and accurately? (if can't count, enter 0)

____

Numeral Identification

Which of these written numbers would your child be able to name? (select all that apply)

▢ 1 (1)
▢ 2 (2)
▢ 3 (3)
▢ 4 (4)
▢ 5 (5)
▢ 6 (6)
▢ 7 (7)
▢ 8 (8)
▢ 9 (9)
▢ 10 (10)
▢ 11 (11)
▢ 12 (12)

▢ 13 (13)
▢ 14 (14)
▢ 15 (15)

Simple Addition

Can your child calculate simple sums, like 1 + 1 and 1 + 2?
o Yes (1)
o No (2)

Mathematical Language

Please select all of the words/terms that you're sure your child knows the meaning of.
▢ More (1)
▢ Less (2)
▢ Fewest (6)
▢ Most (7)
▢ Least (10)
▢ Fewer (11)
▢ A little bit (12)
▢ A lot (13)
▢ Same (14)

General Vocabulary

The next set of questions are designed to assess your child's current vocabulary. Please mark each word you have heard your child SAY. If your child uses a different pronunciation of a word (e.g., "duckie" instead of "duck") or a different part of speech (e.g., "walked" instead of "walking"), mark it anyway. Please do not ask your child if they know a word while you complete this section. The list includes words that children tend to learn at some point between 2 and 15 years of age.

Please click the corresponding box to select each word you have heard your child say.

- ☐ boy (1)
- ☐ chair (2)
- ☐ puppy (3)
- ☐ bike (4)
- ☐ laughing (5)
- ☐ sleeping (6)
- ☐ hugging (7)
- ☐ walking (8)
- ☐ ball (9)
- ☐ dog (10)
- ☐ spoon (11)
- ☐ foot (12)
- ☐ duck (13)
- ☐ banana (15)
- ☐ shoe (16)
- ☐ cup (17)
- ☐ eating (18)
- ☐ bus (19)
- ☐ flower (20)
- ☐ mouth (21)
- ☐ pigeon (22)
- ☐ ankle (23)
- ☐ flaming (24)
- ☐ wrench (25)
- ☐ aquarium (26)
- ☐ refueling (28)
- ☐ safe (29)
- ☐ boulder (30)
- ☐ reptile (31)
- ☐ canoe (32)
- ☐ athlete (33)
- ☐ towing (34)
- ☐ pedestrian (35)
- ☐ interior (36)
- ☐ garment (37)
- ☐ departing (38)
- ☐ feline (39)
- ☐ hedge (40)
- ☐ citrus (41)
- ☐ florist (42)
- ☐ hovering (43)
- ☐ aquatic (44)
- ☐ reprimanding (45)
- ☐ carpenter (46)
- ☐ pencil (47)
- ☐ cookie (48)
- ☐ drum (49)
- ☐ turtle (50)
- ☐ red (51)
- ☐ jumping (52)
- ☐ carrot (53)
- ☐ reading (54)
- ☐ toe (55)
- ☐ belt (56)
- ☐ fly (57)
- ☐ painting (58)
- ☐ dancing (59)
- ☐ whistle (60)
- ☐ kicking (61)
- ☐ lamp (62)
- ☐ square (63)
- ☐ fence (64)
- ☐ empty (65)

- ☐ happy (66)
- ☐ inhaling (67)
- ☐ links (68)
- ☐ polluting (69)
- ☐ archeologist (70)
- ☐ coast (71)
- ☐ injecting (72)
- ☐ fern (73)
- ☐ mammal (74)
- ☐ demolishing (75)
- ☐ isolation (76)
- ☐ clamp (77)
- ☐ dilapidated (78)
- ☐ hyena (79)
- ☐ plumber (80)
- ☐ river (81)
- ☐ timer (82)
- ☐ catching (83)
- ☐ trunk (84)
- ☐ vase (85)
- ☐ harp (86)
- ☐ bloom (87)
- ☐ horrified (88)
- ☐ swamp (89)
- ☐ heart (90)
- ☐ fire (91)
- ☐ castle (92)
- ☐ squirrel (93)
- ☐ throwing (94)
- ☐ farm (95)
- ☐ penguin (96)
- ☐ gift (97)
- ☐ feather (98)
- ☐ cobweb (99)
- ☐ elbow (100)
- ☐ juggling (101)
- ☐ fountain (102)
- ☐ net (103)
- ☐ shoulder (104)
- ☐ dressing (105)
- ☐ roof (106)
- ☐ peeking (107)
- ☐ ruler (108)
- ☐ tunnel (109)
- ☐ branch (110)
- ☐ violin (111)
- ☐ group (112)
- ☐ globe (113)
- ☐ vehicle (114)
- ☐ chef (115)
- ☐ squash (116)
- ☐ ax (117)
- ☐ flamingo (118)
- ☐ chimney (119)
- ☐ sorting (120)
- ☐ waist (121)
- ☐ vegetable (122)
- ☐ interviewing (123)
- ☐ pastry (124)
- ☐ assisting (125)
- ☐ fragile (126)
- ☐ solo (127)
- ☐ snarling (128)
- ☐ puzzled (129)
- ☐ beverage (130)
- ☐ inflated (131)
- ☐ tusk (132)
- ☐ trumpet (133)
- ☐ rodent (134)
- ☐ envelope (135)
- ☐ diamond (136)
- ☐ calendar (137)
- ☐ buckle (138)
- ☐ sawing (139)
- ☐ panda (140)
- ☐ vest (141)
- ☐ arrow (142)
- ☐ picking (143)
- ☐ target (144)
- ☐ dripping (145)
- ☐ knight (146)

- [ ] delivering (147)
- [ ] cactus (148)
- [ ] dentist (149)
- [ ] floating (150)
- [ ] claw (151)
- [ ] uniform (152)
- [ ] gigantic (153)
- [ ] furry (154)
- [ ] luggage (155)
- [ ] directing (156)
- [ ] vine (157)
- [ ] digital (158)
- [ ] dissecting (159)
- [ ] predatory (160)
- [ ] hydrant (161)
- [ ] surprised (162)
- [ ] palm (163)
- [ ] clarinet (164)
- [ ] valley (165)
- [ ] kiwi (166)
- [ ] primate (167)
- [ ] glider (168)
- [ ] weary (169)
- [ ] hatchet (170)
- [ ] transparent (171)
- [ ] sedan (172)
- [ ] constrained (173)
- [ ] valve (174)
- [ ] parallelogram (175)
- [ ] pillar (176)
- [ ] consuming (177)
- [ ] currency (178)

Executive Function

Below, you will find a number of statements. Please read each statement carefully and indicate how well that statement is true for your child. Indicate your response by selecting one of the numbers (from 1 to 5) after each statement.

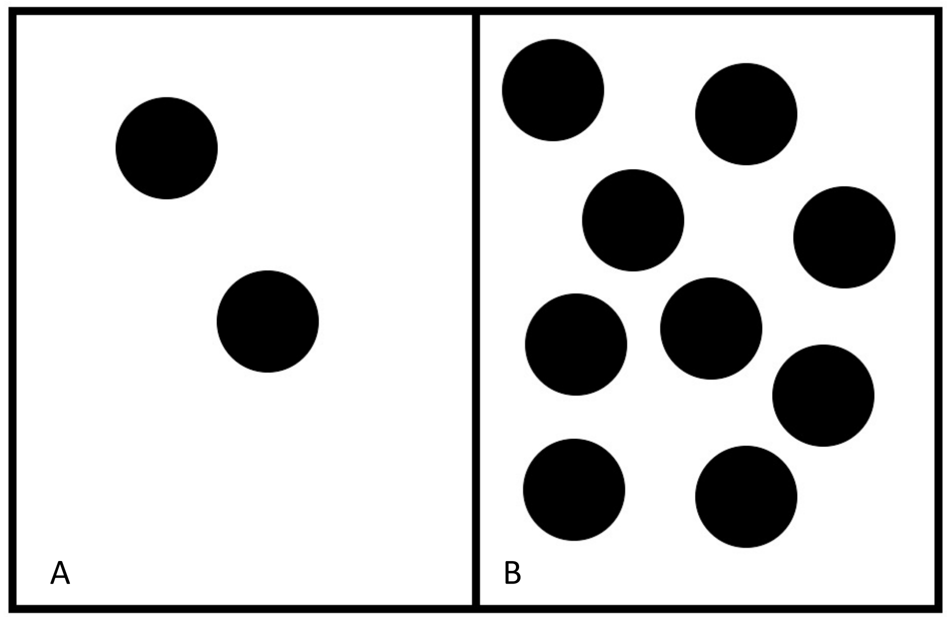
	Definitely not true (1) (1)	Not true (2) (2)	Partially true (3) (3)	True (4) (4)	Definitely true (5) (5)
Has difficulty remembering lengthy instructions (1)	○	○	○	○	○
Seldom seems to be able to motivate him-/herself to do something that he/she doesn't want to do (2)	○	○	○	○	○
Has difficulty remembering what he/she is doing, in the middle of an activity (3)	○	○	○	○	○
Has difficulty following through on less appealing tasks unless he/she is promised some type of reward for doing so (4)	○	○	○	○	○
Has a tendency to do things without first thinking about what could happen (5)	○	○	○	○	○
When asked to do several things, he/she only remembers the first or last (10)	○	○	○	○	○
Has difficulty coming up with a different way of solving a problem when he/she gets stuck (11)	○	○	○	○	○
When something needs to be done, he/she is often distracted by something more appealing (12)	○	○	○	○	○

Easily forgets what he/she is asked to fetch (13)	○	○	○	○	○
Gets overly excited when something special is going to happen (e.g., going on a field trip, going to a party) (14)	○	○	○	○	○
Has clear difficulties doing things he/she finds boring (25)	○	○	○	○	○
Has difficulty planning for an activity (e.g., remembering to bring everything necessary for a field trip or things needed for school) (15)	○	○	○	○	○
Has difficulty holding back his/her activity despite being told to do so (16)	○	○	○	○	○
Has difficulty carrying out activities that require several steps (e.g., for younger children, getting completely dressed without reminders; for older children, doing all homework independently) (17)	○	○	○	○	○
In order to be able to concentrate, he/she must find the task appealing (18)	○	○	○	○	○
Has difficulty refraining from smiling or laughing in situations where it is inappropriate (19)	○	○	○	○	○
Has difficulty telling a story about something that has happened so that others may easily understand (20)	○	○	○	○	○

Has difficulty stopping an activity immediately upon being told to do so. For example, he/she needs to jump a couple of extra times or play on the computer for a little longer after being asked to stop. (21)					
Has difficulty understanding verbal instructions unless he/she is also shown hoe to do something (6)					
Has difficulty with tasks or activities that involve several steps (7)					
Has difficulty thinking ahead or learning from experience (8)					
Acts in a wilder way compared to other children in a group (e.g., at a birthday party or during a group activity) (9)					
Has difficulty doing things that require mental effort, such as counting backwards (23)					
Has difficulty keeping things in mind while he/she is doing something else (24)					

Instructions

- In this game you will answer questions about pictures and numbers!
- You will point to the screen to answer some of the questions
- Parents please let us know which picture your child pointed to by telling us the letter (A, B, C, D)
  - Please let your child answer on their own!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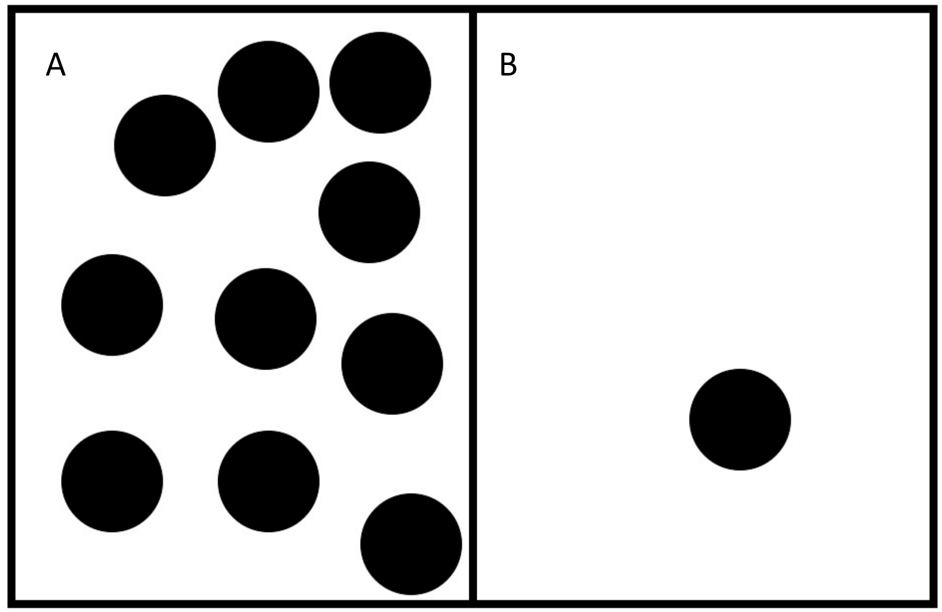
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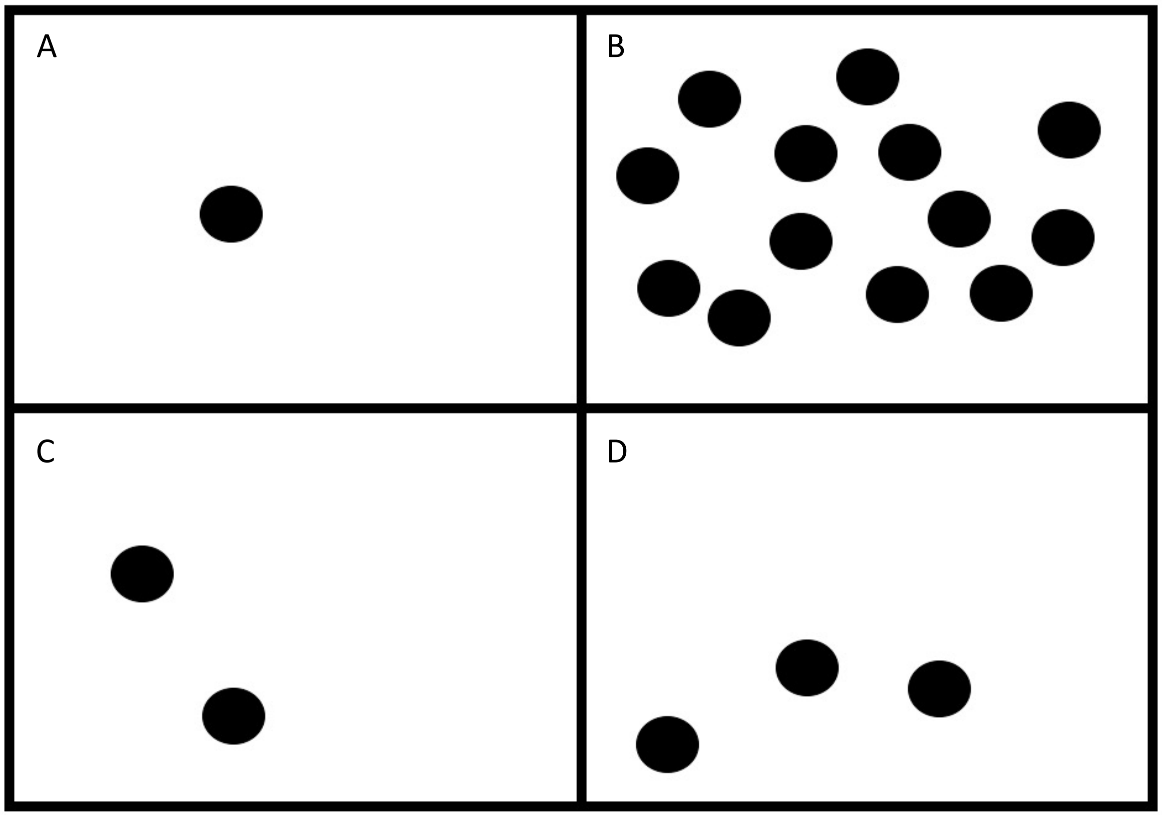
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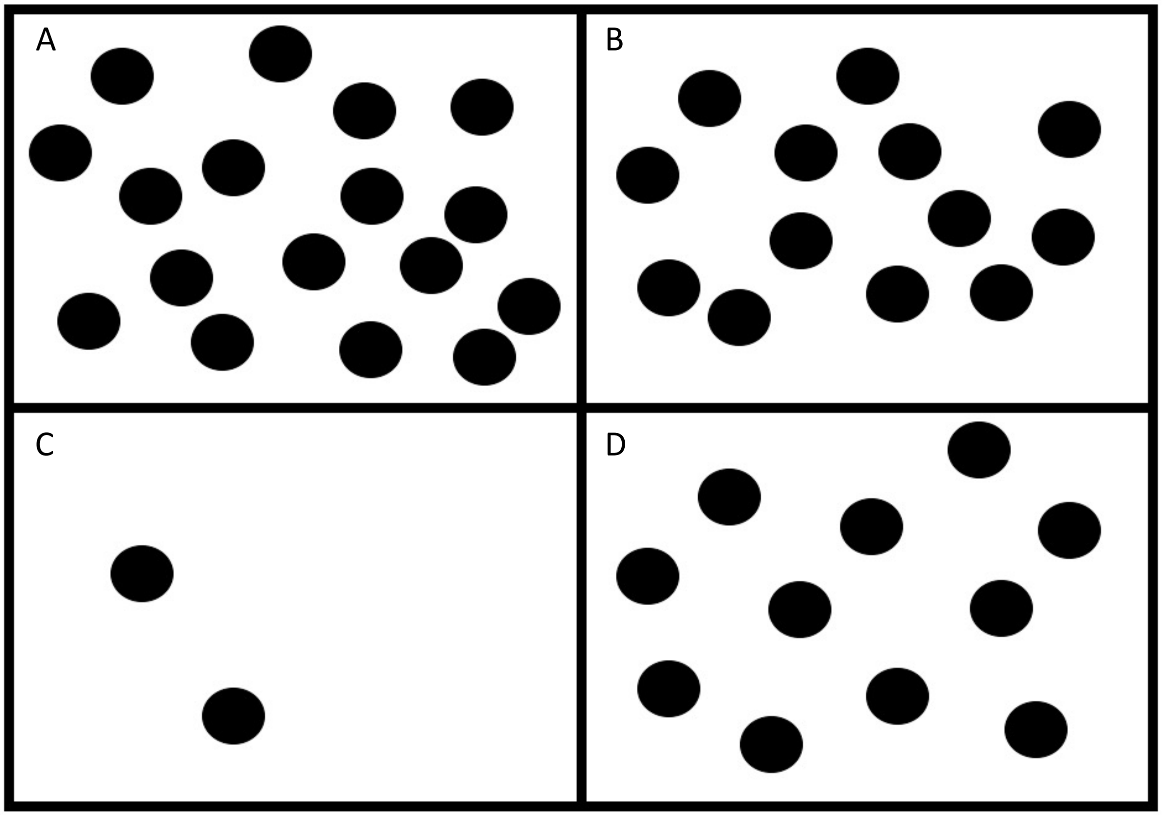
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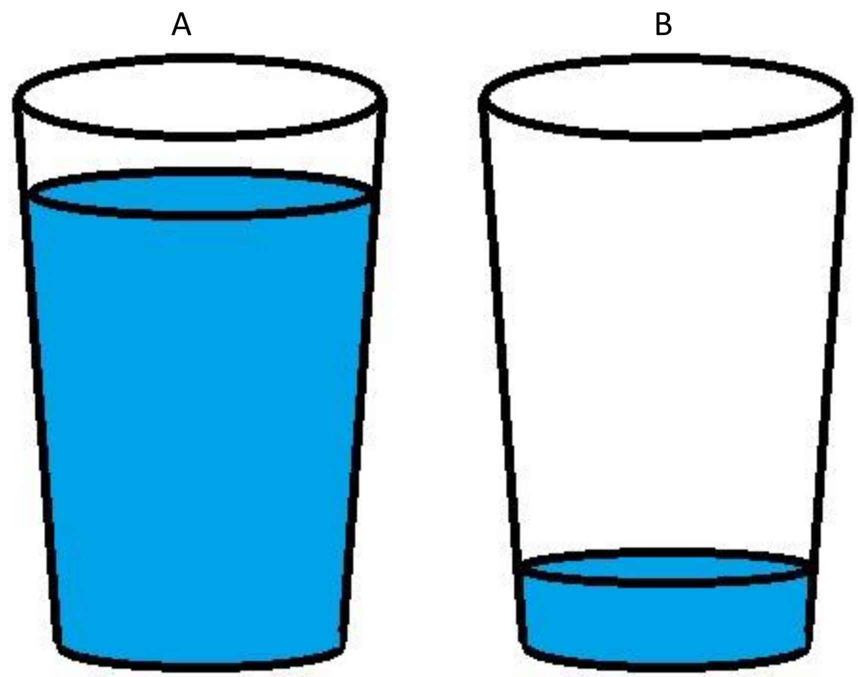
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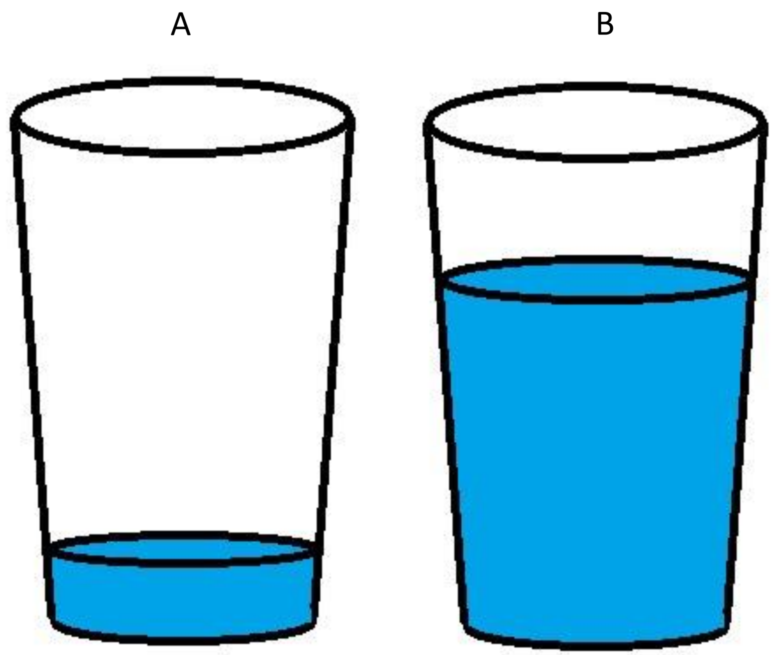
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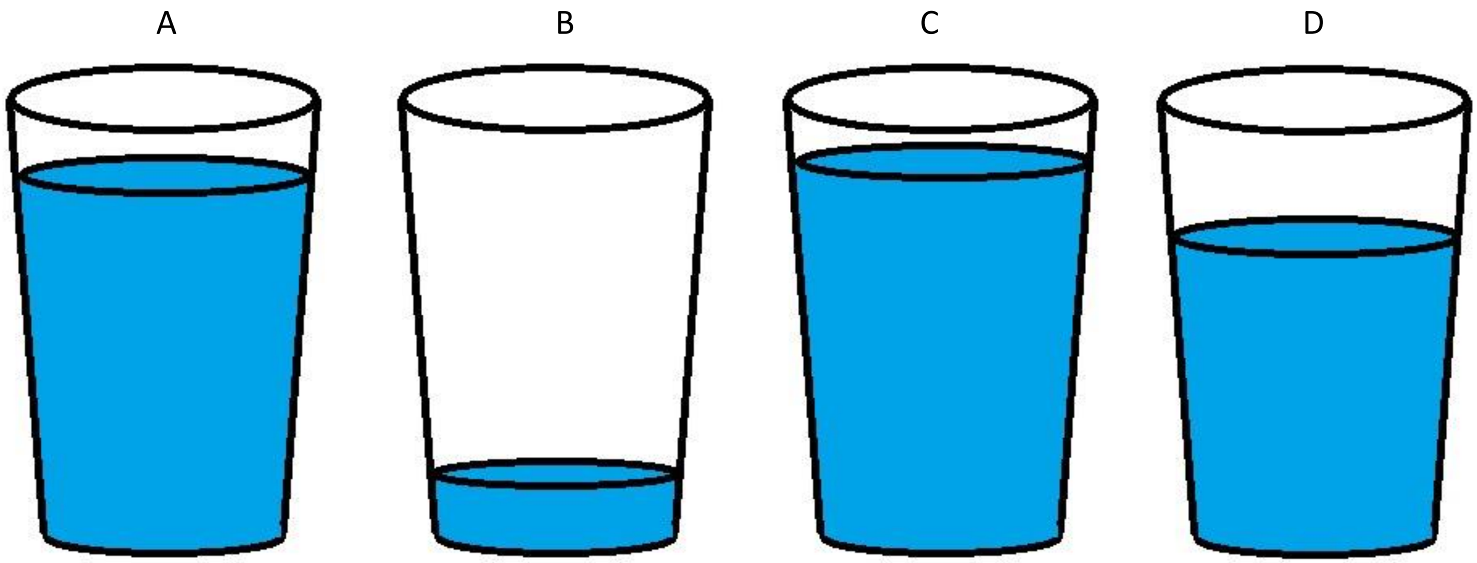
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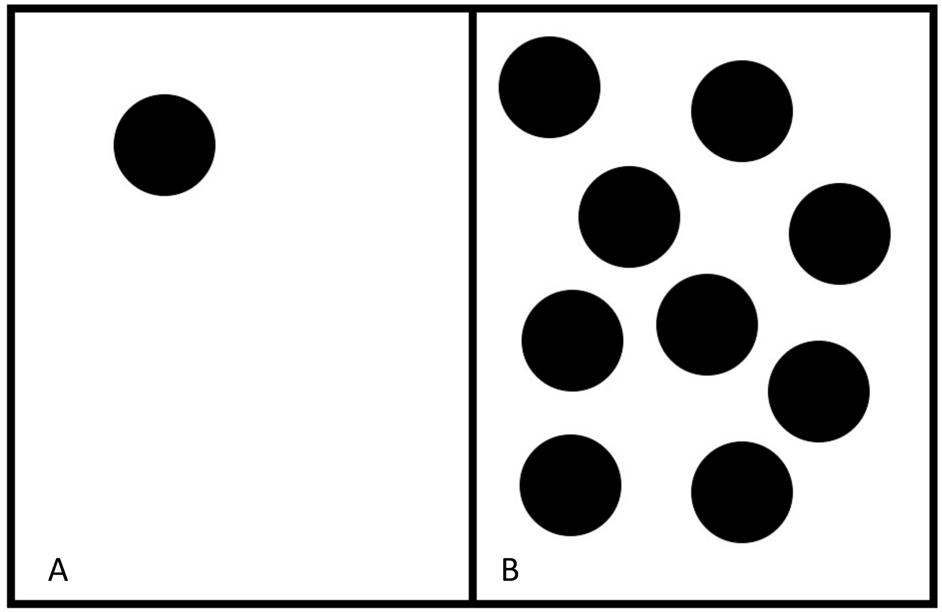
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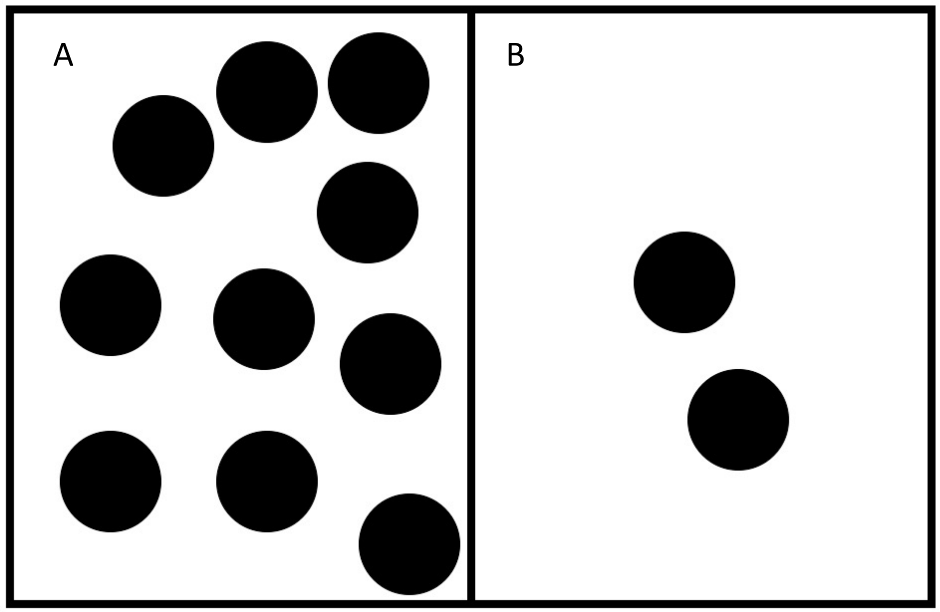
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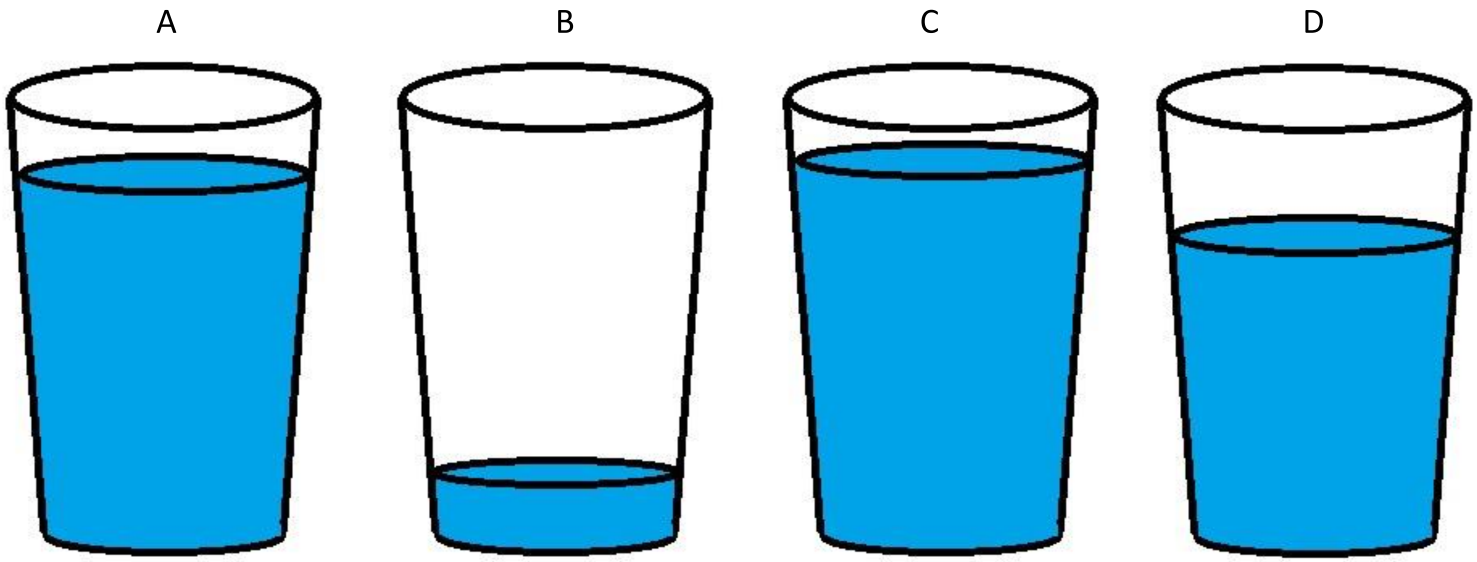
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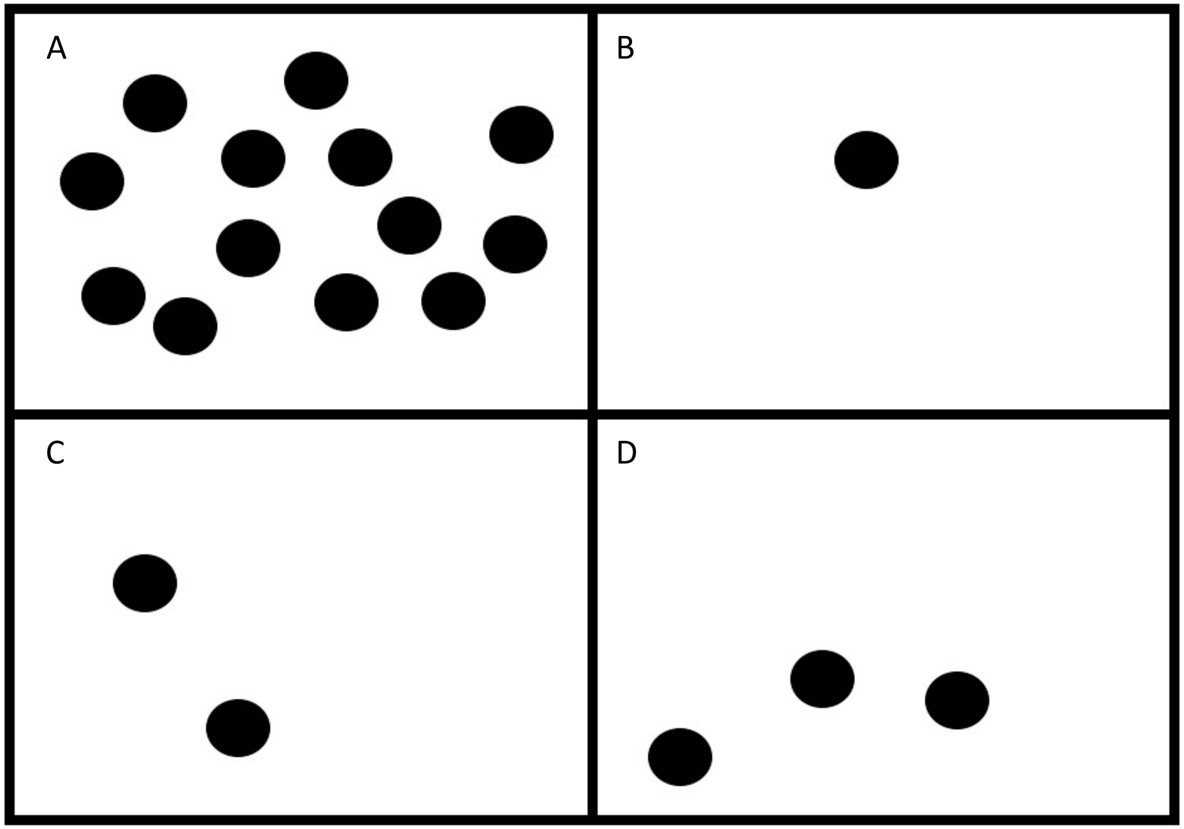
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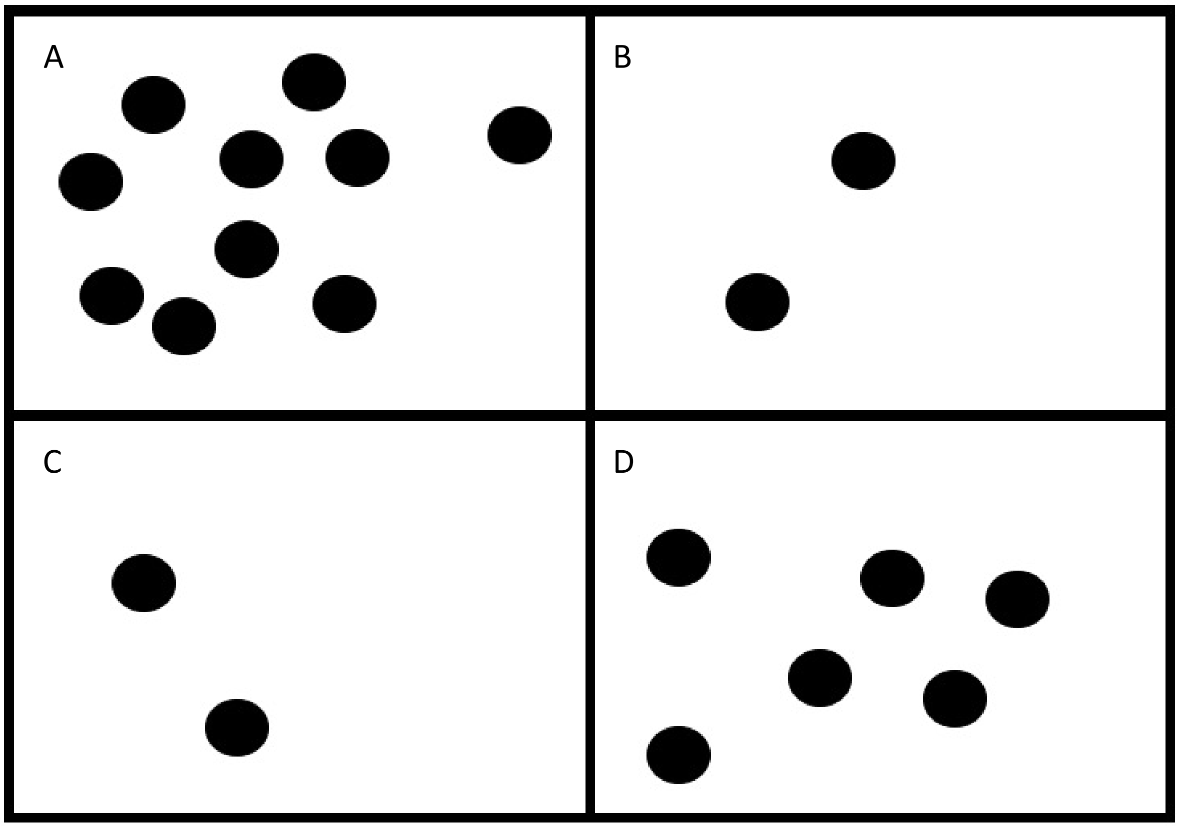
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